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# Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education

**Editors:** Jeremy Hodgen, Eirini Geraniou, Giorgio Bolondi, Federica Ferretti

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Jeremy Hodgen and Eirini Geraniou

IOE, UCL's Faculty of Education and Society, University College London, UK

Giorgio Bolondi and Federica Ferretti

Faculty of Education, Free University of Bozen-Bolzano, Italy

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## Preface: CERME 12 in virtual Bolzano

Carl Winsløw

ERME President

University of Copenhagen, Denmark; [winslow@ind.ku.dk](mailto:winslow@ind.ku.dk)

The Conferences of ERME (European society for Research in Mathematics Education) have been held roughly biannually since 1998. The core of the conference is the *thematic working groups* (TWG) in which new, related research studies are discussed, based on papers which participants read in advance. While this format has remained almost unchanged since 1998, the scale and scope of CERME have developed considerably over the years: from 120 participants and 7 TWG at CERME 1, to 915 participants and 27 TWG at CERME 12. Moreover, CERME has gone from being a mainly regional congress (with only a few participants from other continents) to being a truly global event in mathematics education research, known for fostering high quality scientific communication, cooperation and collaboration. At CERME12, no less than 48 nations were represented (Table 1).

At the same time, it is evident that CERME12 was a very special – and historically difficult – congress to organize. It was first scheduled for February 3-7, 2021. In May 2020, the ERME board announced its decision to postpone the congress by one year, due to the then roaring outbreak of the COVID-19 pandemic. Indeed, large parts of the world – and most of Europe – continued to experience lockdowns and restrictions that would have made the scheduled congress impossible. Instead, an online *Pre-CERME12 event* was organized in February 2021, hosted by the Institute of Education at University College London, and made possible by the tireless efforts of the team led by Jeremy Hodgen and Eirini Geraniou (also chair resp. co-chair of the IPC of CERME12). The Pre-CERME12 event allowed the 27 TWGs to meet and prepare for conference, now postponed to 2022.

The biannual General Assembly of ERME was also held during this event. We warmly thank Susanne Prediger for her service as President of ERME from 2017 to 2021! Her leadership also contributed crucially to the organisation of CERME12, and thus to the results presented in these proceedings.

During the summer and fall of 2021, we all continued to plan for CERME12 as an onsite event in Bolzano, Italy. The YESS summer school was held near Bolzano in August, with great success. More than 700 papers and posters were submitted for CERME12 in September. But in November 2021, new and unknown variants of the virus appeared. Their alarming spread forced us to reconsider the situation. Finally, the LOC, the IPC and the ERME board jointly decided that CERME12 would be held as an online congress, as announced in a mail sent to all members of ERME on December 1<sup>st</sup>:

It is with great sadness that we must communicate a decision which is forced upon us by the current developments of the COVID epidemic in Europe, and which has been taken by the ERME board in full agreement with us: CERME12 will be organized by the Bolzano team as an online conference, on the same dates as originally foreseen. For a long time we hoped for the much desired possibility of having the first CERME in three years as a normal, face to face event. Organizing a virtual CERME – which we will strive to hold as much “CERME spirit” as possible – will be a very demanding task, both in terms of finding good technical solutions, and in terms of organizing the programme and preparing the many TWG teams in a good way.

Germany	209	Turkey	15	Iceland	3
Norway	85	Ireland	13	Lithuania	3
Italy	81	Czech Rep.	11	New Zealand	3
Spain	62	Slovakia	11	China	2
Sweden	62	Chile	9	Colombia	2
USA	41	Brazil	8	Hong Kong	2
UK	38	Croatia	8	Poland	2
Israel	31	Finland	6	Algeria	1
Netherlands	26	South Africa	6	Egypt	1
Denmark	24	Switzerland	6	Faroe Islands	1
France	21	Australia	5	Malta	1
Austria	20	Malawi	5	Romania	1
Canada	19	Mexico	5	Russia	1
Greece	19	Belgium	4	Thailand	1
Portugal	17	Japan	4	Tunisia	1
Hungary	15	Cyprus	3	Ukraine	1

**Table 1: The success of CERME12 in numbers – 915 participants from 48 countries**

Indeed, it took a unique *tour de force* for all organisers to prepare – in just two months – an online version of CERME, based as it is on group work and interaction, rather than on one-way presentations (which are relatively easy to transmit online). These effort was crowned by the best success the new conditions could possibly allow: an online congress with more participants than ever, with virtually no technical problems, and not least with a high level of participant satisfaction.

In the history of ERME, CERME12 will be remembered as a scientific highlight during the long and hard pandemic. First of all, that is due to the plenary speakers and panelists, and to the contributors of papers and posters. Your efforts shine through the quality of the scientific texts offered by these proceedings. ERME, as a society of scholars, was not stopped – hardly delayed – by the pandemic, thanks to your ingenuity and unfailing determination to do and share first class research.

The realization of CERME12 was made possible also by the many people who organised the congress, under the difficult conditions alluded to above, namely:

- The Local Organizing committee, led by Giorgio Bolondi and Federica Ferretti, and all of the Bolzano team, including also the technicians who made the online congress run smoothly;
- The International Programme committee, led by Jeremy Hodgen and Eirini Geraniou;
- The leader teams of all 27 Thematic Working Groups.

For all your tireless and unselfish work during the three years between CERME 11 and CERME 12, the community owes you immense and extraordinary gratitude.

And the story goes on: ERME invites all interested researchers to CERME 13 (Budapest , Hungary, July 2023), and after that, to CERME 14 to be held in *real* Bolzano in February 2025.

# Introduction to the Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)

Jeremy Hodgen<sup>1</sup>, Eirini Geraniou<sup>1</sup>, Giorgio Bolondi<sup>2</sup> and Federica Ferretti<sup>3</sup>

<sup>1</sup>IOE, UCL’s Faculty of Education and Society, University College London, UK;

[jeremy.hodgen@ucl.ac.uk](mailto:jeremy.hodgen@ucl.ac.uk), [e.geraniou@ucl.ac.uk](mailto:e.geraniou@ucl.ac.uk)

<sup>2</sup>Faculty of Education, Free University of Bozen-Bolzano, Italy; [Giorgio.Bolondi@unibz.it](mailto:Giorgio.Bolondi@unibz.it),

<sup>3</sup>Department of Mathematics, University of Ferrara, Italy, [federica.ferretti@unibz.it](mailto:federica.ferretti@unibz.it)

## About CERME12

The Eleventh Congress of European Research in Mathematics Education (CERME 12) took place virtually, hosted by the Free University of Bozen-Bolzano, Italy, from 2<sup>nd</sup> to 6<sup>th</sup> of February 2022, after a year’s delay due to the pandemic. Jeremy Hodgen (UK) and Eirini Geraniou (UK) were chair and co-chair of the International Programme Committee (IPC), which comprised Giorgio Bolondi (Italy), Jason Cooper (Israel), Ana Donevska-Todorova (Germany / North-Macedonia), Çiğdem Haser (Finland / Turkey), Uffe Thomas Jankvist (Denmark), Leander Kempen (Germany), Esther Levenson (Israel), Nuria Planas (Spain) and Michiel Veldhuis (The Netherlands). Giorgio Bolondi and Federica Ferretti were chair and co-chair, respectively, of the Local Organizing Committee (LOC).

CERME12 hosted 27 Thematic Working Groups, listed in the table below. The TWGs 11 and 27 were new TWGs, created following a call launched just after CERME11, and a selection process involving the CERME12 IPC and the ERME board. They have both been very successful. Nine of the TWGs received so many submissions that they had to be split in two – more precisely the TWGs 01, 03, 05, 09, 14, 16, 18, 19 and 20. In the end, CERME12 had 27 TWG leaders and 110 TWG co-leaders.

Thematic Working Group	Leader	Co-Leaders
TWG1: Argumentation and Proof	Andreas Moutsios-Rentzos (Greece)	Orly Buchbinder (USA); Jenny Christine Cramer (Germany); Nicolas Leon (YR) until Aug 2021; and from Sep 2021: Viviane Durand-Guerrier (France); David A. Reid (Norway); Mei Yang



		(British Indian Ocean Territory/UK) YR
TWG2: Arithmetic and Number Systems	Elisabeth Rathgeb-Schnierer (Germany)	Judy Sayers (UK); Beatrice Vargas Dorneles (Brazil) until Sep 2021; Pernille Bødtker Sunde (Denmark) from Sep 2021; Renata Carvalho (Portugal) YR
TWG3: Algebraic Thinking	Dave Hewitt (UK)	Maria Chimoni (Cyprus); Cecilia Kilhamn (Sweden); Luis Radford (Canada) from Sep 2021; Jorunn Reinhardtsen (Norway) YR
TWG4: Geometry Teaching and Learning	Michela Maschietto (Italy)	Alik Palatnik (Israel); Lina Brunheira (Portugal); Chrysi Papadaki (Germany) YR
TWG5: Probability and Statistics Education	Caterina Primi (Italy)	Sibel Kazak (Turkey); Aisling Leavy (Ireland); Orlando Rafael Gonzalez (Thailand); Daniel Frischemeier (Germany) YR
TWG6: Applications and Modelling	Berta Barquero (Spain)	Susana Carreira (Portugal); Jonas Bergman Ärlebäck (Sweden); Katrin Vorhölter (Germany); Gilbert Greefrath (Germany) from Sep 2021; Britta Eyrih Jessen (Denmark) YR
TWG7: Adult Mathematics Education	Kees Hoogland (The Netherlands)	Javier Díez-Palomar (Spain); Fiona Faulkner (Ireland); Beth Kelly (UK) YR
TWG8: Affect and the Teaching and Learning of Mathematics	Stanislaw Schukajilow (Germany)	Inés M <sup>a</sup> Gómez-Chacón (Spain); Çiğdem Haser (Finland); Peter Liljedahl (Canada); Chiara Andrà (Italy); Hanna Viitala (Sweden) YR
TWG9: Mathematics and Language	Jenni Ingram (UK)	Kirstin Erath (Germany); Aurélie Chesnais (France); Ingólfur Gíslason (Iceland) YR

TWG10: Diversity and Mathematics Education: Social, Cultural and Political Challenges	Laura Black (UK)	Anette Bagger (Sweden); Anna Chronaki (Greece); Nina Bohlmann (Germany); Sabrina Bobsin Salazar (Brazil) YR
TWG11: Algorithmics	Christof Weber (Switzerland)	Janka Medova (Slovakia); Ulrich Kortenkamp (Germany); Simon Modeste (France); Piers Saunders (UK) YR until Oct 2021; Maryna Rafalska (France) from Oct 2021
TWG12: History in Mathematics Education	Renaud Chorlay (France)	Antonio M. Oller-Marcén (Spain); Jenneke Krüger (The Netherlands); Tanja Hamann (Germany) YR
TWG13: Early Years Mathematics	Bożena Maj-Tatsis (Poland)	Marianna Tzekaki (Greece); Esther Levenson (Israel); Martin Carlsen (Norway); Andrea Maffia (Italy) YR
TWG14: University Mathematics Education	Alejandro González-Martín (Canada)	Ghislaine Gueudet (France); Olov Viirman (Sweden); Athina Thoma (UK) YR; and from Sep 2021: Irene Biza (United Kingdom); Chris Rasmussen (United States); Ignasi Florensa (Spain) YR
TWG15: Teaching Mathematics with Technology and Other Resources	Alison Clark-Wilson (UK)	Ornella Robutti (Italy); Melih Turgut (Norway); Daniel Thurm (Germany) from Sep 2021; Gülay Bozkurt (Turkey) YR
TWG16: Learning Mathematics with Technology and Other Resources	Paul Drijvers (The Netherlands)	Florian Schacht (Germany); Nathalie Sinclair (Canada); Osama Swidan (Israel); Eleonora Faggiano (Italy) from Sep 2021; Seçil Yemen Karpuzcu (Turkey) YR
TWG17: Theoretical Perspectives and Approaches in Mathematics Education Research	Angelika Bikner-Ahsbahs (Germany)	Heather Johnson (USA); Anna Shvarts (The Netherlands); Abdel Seidouvy (Togo/Sweden) YR

TWG18: Mathematics Teacher Education and Professional Development	Janne Fauskanger (Norway)	Libuse Samkova (Czech Republic); Andreas Ebbelind (Sweden); Marita Eva Friesen (Germany) YR; and from Sep 2021: Tracy Helliwell (UK); Macarena Larrain (YR)
TWG19: Mathematics Teaching and Teacher Practice(s)	Reidar Mosvold (Norway)	Mark Hoover (USA); Siún Nic Mhuiri (Ireland); Edyta Nowinska (Poland/Germany); Helena Grundén (Sweden) YR
TWG20: Mathematics Teacher Knowledge, Beliefs and Identity	Fatma Aslan-Tutak (Turkey)	Miguel Montes (Spain); Francesca Martignone (Italy); Miguel Ribeiro (Brazil); Veronika Hubeňáková (Slovakia) YR
TWG21: Assessment in Mathematics Education	Paola Iannone (UK)	Francesca Morselli (Italy); Michal Ayalon (Israel); Michiel Veldhuis (The Netherlands); Gözde Kaplan Can (Turkey) YR
TWG22: Curricular Resources and Task Design in Mathematics Education	Shai Olsher (Israel)	Sebastian Rezat (Germany); Annalisa Cusi (Italy); Nataly Essonnier (France) YR
TWG23: Implementation of Research Findings in Mathematics Education	Mario Sánchez Aguilar (Mexico)	Boris Koichu (Israel); Morten Misfeldt (Denmark); Rikke Maagaard Gregersen (Denmark) YR until Aug 2021; Linda Marie Ahl (Sweden) YR from Sep 2021
TWG24: Representations in Mathematics Teaching and Learning	Anna Baccaglini-Frank (Italy)	Carla Finesilver (UK); Michal Tabach (Israel); Kate O'Brien (USA/UK) YR
TWG25: Inclusive Mathematics Education – Challenges for Students with Special Needs	Petra Scherer (Germany)	Hana Moraova (Czech Republic); Michael Gaidoschik (Italy); Helena Roos (Sweden) YR
TWG26: Mathematics in the Context of STEM Education	Behiye Ubuz (Turkey)	Michele Stephan (USA); Clelia Cascella (Italy/UK); Nelleke Den Braber (The Netherlands) YR