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# Models of accidental dark matter with a fundamental scalar

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Abstract: We consider models of accidental dark matter, namely models in which the dark matter is a composite state that is stable thanks to an accidental symmetry of the theory. The fundamental constituents are vectorlike fermions, taken to be fragments of representations of the grand unifying gauge group SU(5), as well as a scalar singlet. All the new fields are charged under a new confining gauge group, which we take to be SU($N$), leading to models with complex dark matter. We analyse the models in the context of SU(5) grand unification with a non-standard approach recently proposed in the literature. The advantage of including the scalar mainly resides in the fact that it allows several undesired accidental symmetries to be broken, leading to a larger set of viable models with respect to previous literature, in which only fermions (or only scalars) were considered. Moreover these models present distinct novelties, namely dark states with non-zero baryon and lepton number and the existence of composite *hybrid* states of fermions and scalars. We identify phenomena that are specific to the inclusion of the scalar and discuss possibilities to test this setup.

Keywords: Models for Dark Matter, Particle Nature of Dark Matter, Confinement, Grand Unification

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## Contents



## 1 Introduction

The exact nature of dark matter (DM) is yet unknown, and a great effort has been made on the theoretical side to imagine and explore a variety of possible scenarios, ranging from extended objects such as primordial black holes [1] to modifications of gravity [2]. However, the most extensively studied option to solve this puzzle is that in which the DM is comprised by new particles, whose relic abundance is set by various mechanisms. Much work has been devoted to special cases in which the DM puzzle may be solved in combination with other fundamental issues of the standard model of particle physics (SM). Two examples are axions, which are perfectly suitable DM candidates [3], or supersymmetry, which naturally provides for a DM candidate [4]. In other cases one can introduce new particles or entire new sectors specifically to address the DM puzzle, perhaps gaining insights or solutions for other issues on the side.

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One key feature that any theory of DM must present is an explanation of why the DM is stable on cosmological time scales, as required by observations. In our universe, there are other forms of matter which are stable on such long time scales, namely ordinary protons and electrons. Indeed one of the greatest successes of the SM is the understanding of the observed global symmetries in terms of *accidental* symmetries of the Lagrangian at renormalizable level. These symmetries are the baryon number and the three lepton flavors, and they guarantee the stability of the lightest charged states.

These symmetries are called accidental since they are not symmetries of the theory if one considers higher dimensional operators in the Lagrangian, which are suppressed by increasing powers of the energy scale of ultraviolet (UV) physics. In other words, they are only symmetries of the theory at low enough energies when the contributions from these operators can be ignored, analogously to the spherical symmetry of the electric field produced by a charge distribution whose size is much smaller than the distance from which it is observed, so that higher multipoles are irrelevant.

In this work we consider extensions of the SM that can solve the DM puzzle with new particles that are stable thanks to an accidental symmetry of the theory. In order to do so, we introduce a non-Abelian dark sector that undergoes confinement. Its accidental symmetries determine which of the low energy states (dark hadrons) are stable and can play the role of DM. This way of tackling the DM puzzle has been given serious consideration in the literature (see, among many others, [5–14]). The advantage is that such theories are UV complete, and that the desired properties of the DM descend directly and solely from the quantum numbers of the fundamental constituents and the ensuing accidental symmetries. Moreover, the richness of their low energy spectrum makes them especially promising from a phenomenological standpoint, especially at colliders, since generally these dark partners carry SM charges and may be light enough to be produced, and in cosmology, where they may realize various non-standard scenarios.

In classifying the models, we follow [10], in which the SM was extended with fermionic *dark quarks* (Dq) in the fundamental representation of a new *dark color* (DC) gauge group (they considered both $SU(N_{DC})$ and $SO(N_{DC})$ groups; we consider only the former). These new fields transformed in vector-like representations of the SM gauge group, in such a way that, unlike in technicolor models or even in quantum chromodynamics (QCD), the confinement does not break the electroweak gauge symmetry. This scenario is known in the literature as vector-like confinement [15, 16]. In [10] only a relatively small number of models were found to be viable. This was due essentially to difficulties in breaking the large number of undesired accidental symmetries that the models generally possess. Indeed any stable symmetry leads to the stability of a dark state, which is in general electrically charged and/or colored, rendering models in which they arise unacceptable. We are interested in grand unification as a criterion for the selection of the models. With this further restriction, essentially one model was found to be acceptable in [10]. The novelty of our work in this sense is twofold. On the one hand, along the fermionic Dqs, we consider a scalar field transforming in the fundamental representation of $SU(N_{DC})$ but as a singlet under the SM gauge group.[1] On the other, following [17], we relax the criterion for grand unification, which results in a more generous selection of models.

[1] More complicated representations for this scalar may of course be considered.

The presence of light elementary scalars notoriously introduces what is known as hierarchy problem, as is also very famously the case for the SM. We postpone entirely the discussion on the hierarchy problem that arises here, as we consider a dark fundamental scalar, assuming that a mechanism exists that stabilizes its mass — and possibly that of the Higgs boson as well. We focus instead on several advantages provided by this setup. First of all we show how, by allowing new kinds of interactions between the SM and the dark sector, a larger set of viable models is found. Furthermore, Dqs acquire baryon or lepton numbers depending on their SM representations, and new types of bound state arise, made either of only scalars[2] or both of scalars and fermions. In light of these peculiarities we discuss the phenomenological consequences of the presence of the scalar in accidental composite dark matter models.

The rest of this work is organized as follows. In section 2 we discuss the generalities of accidental composite DM, reviewing the content of [10] and highlighting the novelties of our setup, with some examples. We discuss various possible mass orderings and the scenarios they produce. In section 3 we furnish a full classification of the models. In section 4 we analyse the models in the context of SU(5) grand unification, employing the relaxed criterion proposed in [17]. In section 5 we discuss aspects of phenomenology, with focus on the impact of the dark scalar. In section 6 we summarize and discuss our results.

## 2 General aspects of the models

We extend the SM with a dark sector containing several Dqs in the fundamental representation of a new SU($N_{\rm DC}$) gauge symmetry. In all the models we consider a scalar Dq $\phi$ which is a total singlet under the SM gauge group[3]

$$\phi \sim (\mathbf{1},\mathbf{1})_0\,. \tag{2.1}$$

We also consider a number of fermionic Dqs. Following [10], since we wish to study these extensions in the context of a SU(5) grand unification scheme, we consider fermionic Dqs belonging to fragments of the lowest SU(5) representations, as shown in table 1. We assume that they are much lighter than their GUT partners, which we assume to not come into play in the cosmological history because they were never populated, being heavier than the reheating temperature. We name the models by just their light fermionic Dq content, always implicitly considering the dark scalar to be light. Since the models are taken to be vector-like with respect to the SM, if we say that a model contains the left-handed Dq field $\Psi$, we implicitly mean that another field $\Psi^c$ is light, which is a left-handed field in the anti-fundamental of SU($N_{\rm DC}$), belonging to the conjugate of the SM representation of $\Psi$.[4] Vectorlike mass terms in the Lagrangian will then be $M_\Psi \Psi\Psi^c$. In this way, the condensation of the dark sector does not break the electroweak symmetry [15]. It is also possible to consider Dqs in the conjugate of the SU(5) representations in table 1. We denote their fragments with a tilde: if $\Psi$ is a left-handed Dq in the fundamental of SU($N_{\rm DC}$) transforming as $\mathcal{R}_{\rm SM}$, then $\widetilde{\Psi}$ is a

[2]Such a possibility was already explored in [18]. In that work, no fundamental fermions were considered, and the interesting *complementarity* between the higgsed and confined phases of non-Abelian theories exploited [19–21]. We leave the treatment of this duality in the context of the models we here propose to future works.

[3]We use the notation $(\mathcal{R}_c, \mathcal{R}_L)_Y$ for a field transforming under the SM gauge group SU(3)$_c$⊗SU(2)$_L$⊗U(1)$_Y$ .

[4]For example, the model $Q \oplus D$ will have $Q$, $Q^c$, $D$, $D^c$, and $\phi$ as light Dq, and their GUT partners $U$, $U^c$, $E$, $E^c$, $L$ and $L^c$ as heavy Dqs.



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| SU(5) | Name | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ | $\Delta b_3$ | $\Delta b_2$ | $\Delta b_Y$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{1}$ | $N$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | 0 | 0 |
| $\bar{\mathbf{5}}$ | $D$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 0 | $\frac{2}{9}$ |
| | $L$ | $\mathbf{1}$ | $\mathbf{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $\mathbf{10}$ | $U$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | 0 | $\frac{8}{9}$ |
| | $E$ | $\mathbf{1}$ | $\mathbf{1}$ | 1 | 0 | 0 | $\frac{2}{3}$ |
| | $Q$ | $\mathbf{3}$ | $\mathbf{2}$ | $\frac{1}{6}$ | $\frac{2}{3}$ | 1 | $\frac{1}{9}$ |
| $\mathbf{15}$ | $Q$ | $\mathbf{3}$ | $\mathbf{2}$ | $\frac{1}{6}$ | $\frac{2}{3}$ | 1 | $\frac{1}{9}$ |
| | $T$ | $\mathbf{1}$ | $\mathbf{3}$ | 1 | 0 | $\frac{4}{3}$ | 2 |
| | $S$ | $\mathbf{6}$ | $\mathbf{1}$ | $-\frac{2}{3}$ | $\frac{5}{3}$ | 0 | $\frac{8}{9}$ |
| $\mathbf{24}$ | $V$ | $\mathbf{1}$ | $\mathbf{3}$ | 0 | 0 | $\frac{4}{3}$ | 0 |
| | $G$ | $\mathbf{8}$ | $\mathbf{1}$ | 0 | 2 | 0 | 0 |
| | $X$ | $\bar{\mathbf{3}}$ | $\mathbf{2}$ | $\frac{5}{6}$ | $\frac{2}{3}$ | 1 | $\frac{25}{9}$ |
| | $N$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | 0 | 0 |
| $\mathbf{1}$ | $\phi$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | 0 | 0 |

**Table 1.** Left-handed Dqs are taken to be fragments of SU(5) representations (first column), decomposed under the SM gauge groups (middle columns). The rightmost columns show the contributions of each of the Dqs to the three SM $\beta$-functions. Each of the reported values is to be multiplied by $2N_{\mathrm{DC}}$, the Dqs being fundamentals of $SU(N)_{\mathrm{DC}}$ and vectorlike with respect to the SM. Last row: SM singlet scalar Dq.

| | Poincaré | $SU(N)_{\mathrm{DC}}$ | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|---|
| $\Psi$ | $(\mathbf{1/2},\mathbf{0})$ | $\square$ | $\mathcal{R}_c$ | $\mathcal{R}_L$ | $Y$ |
| $\Psi^c$ | $(\mathbf{1/2},\mathbf{0})$ | $\overline{\square}$ | $\overline{\mathcal{R}}_c$ | $\overline{\mathcal{R}}_L$ | $-Y$ |
| $\widetilde{\Psi}$ | $(\mathbf{1/2},\mathbf{0})$ | $\square$ | $\overline{\mathcal{R}}_c$ | $\overline{\mathcal{R}}_L$ | $-Y$ |
| $\widetilde{\Psi}^c$ | $(\mathbf{1/2},\mathbf{0})$ | $\overline{\square}$ | $\mathcal{R}_c$ | $\mathcal{R}_L$ | $Y$ |

**Table 2.** Summary of the notation used in this work for the Dqs representations.

fundamental of $SU(N_{\mathrm{DC}})$ transforming as $\overline{\mathcal{R}}_{\mathrm{SM}}$. As above, a model containing $\widetilde{\Psi}$ contains $\widetilde{\Psi}^c$ as well. The notation is summarized in table 2.

The organization of the fermionic Dqs as in table 1 is to be understood as following. The models contain an (approximate, thanks to Dq masses) chiral dark flavor (DF) symmetry: $SU(N_{\mathrm{DF}})_\Psi \otimes SU(N_{\mathrm{DF}})_{\Psi^c}$. Here $N_{\mathrm{DF}}$ is half the overall number of new left-handed fermionic degrees of freedom. Turning on the weak SM interactions breaks the symmetry explicitly, so that the various flavors organize in *species* belonging to definite SM representations. At a scale $\Lambda_{\mathrm{DC}}$, the dark sector confines. The Dq condensate spontaneously breaks the above

| Dq | $\phi$ Yukawa | Higgs Yukawa |
|---|---|---|
| $N$ | none | $L^c H^\dagger N$, $N^c H^\dagger \tilde{L}$ |
| $D$ | $D^c \phi d^c$ | $\widetilde{Q} H D^c$ |
| $L$ | $L^c \phi l$ | $L^c H \widetilde{E}$, $N^c H L$, $V^c H L$ |
| $U$ | $U^c \phi u^c$ | $\widetilde{Q} H^\dagger U^c$ |
| $E$ | $E^c \phi e^c$ | $L H^\dagger E^c$ |
| $Q$ | $Q^c \phi q$ | $Q^c H \widetilde{D}$, $Q^c H^\dagger \widetilde{U}$ |
| $V$ | none | $V^c H L$ |
| any $\Psi$ | $\phi \Psi \widetilde{\Psi}$ if $N_{\rm DC} = 3$ | — |

**Table 3.** Allowed Yukawa terms involving the scalar Dq $\phi$ and the Higgs. Lowercase letters are the left-handed SM fields in a standard notation. Note that for each Higgs Yukawa term there is an analogous term with $\Psi \leftrightarrow \Psi^c/\widetilde{\Psi}$ and $H \leftrightarrow H^\dagger$ (this is not true for $\phi$ Yukawa terms). Yukawa terms with tilded Dq $\tilde{\Psi}$ can be obtained starting from those in the table with the substitutions $\Psi^c \to \tilde{\Psi}$, $\phi \to \phi^\dagger$.

chiral symmetry to the vectorial subgroup SU($N_{\rm DF}$), which we call DF group. As in QCD, the dark confinement produces $N_{\rm DF}^2 - 1$ pseudo-Nambu-Goldstone bosons (pNGB) in the adjoint representation of SU($N_{\rm DF}$) which we call interchangeably scalar dark mesons or dark pions (D$\pi$). The D$\pi$s may be much lighter than the other dark bound states, whose mass is of the order of $\Lambda_{\rm DC}$, since they are pNGB. The dark equivalent of the $\eta'$ is, as in QCD, expected to be heavier than the other mesons because of the axial anomaly. In particular in the model containing only $N$ and the scalar, because the full DF group is anomalous, there are no light pNGB as the $N^c N$ state plays the role of the $\eta'$. Bound states of two Dqs containing $\phi$ are not pNGB arising from any symmetry breaking pattern, and thus are not expected to be much lighter than the other dark states.[5]

The most important terms for both model building and phenomenology are those connecting the dark sector with the SM, namely

$$\mathcal{L}_{\text{DS-SM}} = -\lambda_{\phi H} \phi^\dagger \phi \, H^\dagger H + \mathcal{L}_{\text{Dark Yukawa}} \,. \tag{2.2}$$

As for the SM, the Yukawa terms are the largest source of breaking of the global symmetries. Thus, before describing the content of $\mathcal{L}_{\text{Dark Yukawa}}$, let us discuss what are the accidental symmetries enjoyed by the theory in their absence.

**Species Number** An independent U(1) symmetry for each species of Dq as in table 1 (including $\phi$). This symmetry leads to the stability of the lightest dark mesons made of different species of Dqs ${\Psi^c}_i \Psi_j$ or ${\Psi^c}_i \phi$ ($i$ and $j$ are species indices), which are the equivalent of charged QCD pions.

[5] In this sense these states are not etymologically *mesons*, and perhaps should be called *baryons* (this time-honored nomenclature is even worse if one considers that the $\tau$ *lepton* is heavier than some of the baryons). However in the literature the term meson is used to identify bound states of two quarks regardlessly of wether they are pNGB — e.g. the $\rho$ particles are usually called vector-mesons, even if they are as heavy as baryons. We abide to this convention.

**G-Parity** Equivalently to QCD, if Dqs belong to non-trivial weak isospin representations with vanishing hypercharges, the theory is invariant under the discrete symmetry acting on Dqs as $\Psi \to \exp\left\{i\pi\frac{\sigma^2}{2}\right\}\Psi^c$, and trivially on the SM fields. Consequently, the lightest G-odd dark state is kept stable.

**Dark Baryon Number** A subgroup of the species number symmetry under which all the Dqs rotate with the same phase. This symmetry is responsible for the stability of the lightest dark baryon (DB), i.e. the lightest composite state of $N_{\rm DC}$ Dqs in a totally antisymmetric DC combination. This is equivalent to the SM baryon number symmetry, which is responsible for the stability of the proton.

Species and G-parity may be broken either at renormalizable level by Yukawa interactions (which generally preserve the DB symmetry) or at the level of dimension five or higher. The Yukawa terms in $\mathcal{L}_{\text{Dark Yukawa}}$ are essentially of two kinds, as shown in table 3. The first involves the SM Higgs field and two dark fermions: there are only a few possibilities, which lead in [10] to a relatively small number of viable models. This changes drastically in the presence of the scalar Dq $\phi$: for any fermionic Dq with a SM counterpart, one can write a term involving them and the scalar Dqs, as in table 3. These terms allow to break the species number in almost all the relevant cases as we shall see. Interestingly, thanks to them, the Dqs $Q$, $D$, and $U$ inherit the SM baryon number. In the presence of Dqs $L$ and $E$, since only one family of Dqs is introduced coupling to all three SM families, the three SM lepton numbers are broken to a single lepton number: $\mathrm{U}(1)^3_\ell \to \mathrm{U}(1)_\ell$; $L$ and $E$ acquire a charge under this symmetry. Similar considerations hold for the tilded versions of the mentioned Dqs. The quantum numbers are summarized in table 4. Other light Dqs may then inherit the SM numbers through other interactions, e.g. if both $N$ and $L$ are light, $N$ acquires SM lepton number 1 via the term $HLN^c$. In models in which $N$ is light but $L$ is not, as we shall discuss later on, the lepton flavor symmetry is preserved up to dimension five operators, at the level of which $N$ acquires lepton number 1.

In the absence of the dark scalar, the DB symmetry is more robust than the other symmetries, since it may be broken only at dimension six or higher. An estimate of the lifetime of DBs is then

$$\tau_{\rm DB} \sim \frac{8\pi\Lambda^4_{\not{D}\not{B}}}{M^5_{\rm DM}} \sim 10^{26}\text{s}\left(\frac{\Lambda_{\not{D}\not{B}}}{M_{\rm Pl}}\right)^4\left(\frac{100\ \text{TeV}}{M_{\rm DM}}\right)^5 \tag{2.3}$$

If we take the scale suppressing these operators to be around the Planck scale, this lifetime evades the bounds from indirect searches $\tau > 10^{26\div 28}$s [22–25] if the mass of the DM is in the ballpark of 100 TeV or below. As we shall see in section 2.1, this value is also selected to reproduce the correct thermal relic abundance in several scenarios.

Including a scalar Dq may introduce dimension five operators that break the DB number, depending on the specific model, such as

$$LH^\dagger E\phi \quad \text{or} \quad QH^\dagger D\phi \quad \text{for } N_{\rm DC} = 3\,. \tag{2.4}$$

If the only stable symmetry is the DB number, the lightest DB can act as the DM. On the contrary, all charged dark mesons, which arise in general and whose relic abundance is

| | $D$ | $U$ | $Q$ | $L$ | $E$ | $N$ | $\phi$ |
|---|---|---|---|---|---|---|---|
| $\mathrm{U}(1)_{\mathrm{DB}}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mathrm{U}(1)_b$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $\mathrm{U}(1)_\ell$ | 0 | 0 | 0 | 1 | $-1$ | $1^*$ | 0 |

**Table 4.** Quantum numbers of the Dqs under the stable DB number and the SM accidental symmetries. Tilded Dqs have opposite numbers since their SM representation is conjugated. The Dq $N$ only inherits lepton number at the level of dimension five unless $L$ is also light.

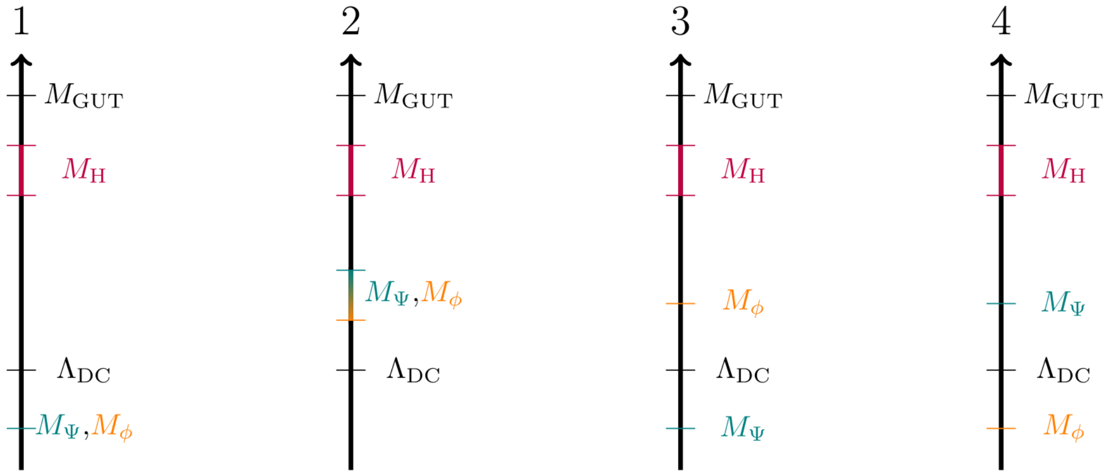



**Figure 1.** Cartoons with the four scenarios for the mass orderings in our models. Here $M_{\mathrm{GUT}}$ is the scale of grand unification of the SM couplings, $\Lambda_{\mathrm{DC}}$ is the scale of confinement of the dark sector, $M_\phi$ is the mass of the scalar Dq, while $M_\Psi$ and $M_H$ are the mass scales of the light and heavy fermion Dqs, respectively. The latter is presented as a range, since, as discussed in section 4, our relaxed criterion for the unification does not fix it univocally as a function of the other mass scales.

heavily constrained by observations, decay immediately. In some models species numbers are only broken at dimension five, in which case the mesons are meta-stable. If they are charged, they must decay soon enough in the cosmological history.

### 2.1 Hierarchy of mass scales

In this section, we discuss the various mass scales of our models, and how their ordering is relevant for identifying general aspects of accidental composite dark matter models. There are four basic scenarios, depicted in figure 1.

There are two scales which are determined dynamically, namely the scale of confinement of the dark sector $\Lambda_{\mathrm{DC}}$, and the scale of grand unification $M_{\mathrm{GUT}}$. The value of the former is selected by cosmology as we shall describe shortly, depending on its relative value with respect to the masses of the Dqs, which is also crucial in understanding the spectrum of the bound states. The latter scale is determined by the unification of the SM couplings, which in turn depends on the masses $M_H$ of the heavy fermions, assumed to lie between $M_{\mathrm{GUT}}$ and $\Lambda_{\mathrm{DC}}$. The scale $M_H$ is not a dynamical scale, and it is represented as a range in figure 1 as a consequence of our approach to grand unification, which is described in section 4.2. There are two more scales that are free parameters of the model, whose position with respect to the others determines the qualitative behaviour of the models: the mass of the scalar $M_\phi$ and the mass of the light fermions $M_\Psi$. Let us discuss the scenarios in figure 1 from left to right.

If both $M_\phi$ and $M_\Psi$ lie beneath the confinement scale, the bound states are Coulomb-like. The DB have masses of the order of $N_{\rm DC}\,\Lambda_{\rm DC}$, while the masses of dark mesons can be estimated as described in the next section. As we shall argue later, the dark matter candidate (DMC) of our models is the lightest of the DBs. If the DM relic abundance is determined by a geometric cross section

$$\langle \sigma v_{\rm rel.} \rangle \sim \frac{\pi}{{\Lambda_{\rm DC}}^2}\,, \tag{2.5}$$

one finds that the observed relic abundance of DM [26] is reproduced with a confinement scale in the ballpark of 100 TeV.[6] Recall that this is the same range selected from comparing the naively estimated lifetime of DBs with the bounds from indirect searches, see eq. (2.3). There is little to say about the hierarchy between the bound states made of only fermions (or only scalars) and the *hybrid* ones made of both fermions and scalars, and thus about the nature of the DMC, without resorting to lattice calculations or other numerical means.

If both $M_\phi$ and $M_\Psi$ lie above the confinement scale [11], on the other hand, the bound states are Coulomb-like, and their hierarchy (including whether the hybrid states are lighter than the purely fermionic ones) depends on the precise orderings of the Dq masses: dark hadrons made of heavier Dqs will be heavier. We shall refer to this configuration as weakly coupled scenario. The lightest dark states would be glueballs with mass $7\Lambda_{\rm DC}$ [29]. They cannot comprise the DM, lest overclosing the universe, thus must decay before BBN. Higher up in mass there would be dark mesons of mass $\sim 2M_{\Psi,\Phi}$, and finally DB with mass $\sim N_{\rm DC}M_{\Psi,\Phi}$. The DMC would be the DB with the lightest constituents and lowest spin. As described in [11], the dynamics of the DM freeze out and the lifetime of the dark glueballs lead to values of $\Lambda_{\rm DC}$ lower than in the previous case.

The third scenario, in which the mass of the scalar lies above or in the ballpark of $\Lambda_{\rm DC}$, while the light fermions are much lighter, is the one that we shall consider in the rest of this work unless otherwise specified. As far as fermionic Dqs are concerned this is another case of strongly coupled scenario, like the first. In this case, however, there is a clear hierarchy between the heavier hybrid bound states and the lighter bound states with fermionic Dqs as constituents. The latter annihilate with geometrical cross-section as in the first scenario, so cosmology once again selects $\Lambda_{\rm DC} \sim 100\,$TeV.

In the last scenario, in which the light fermions lie above $\Lambda_{\rm DC}$ with the scalar being much lighter, the nature of the DM is similar to the case of [18], with differences due to the existence of the fermionic Dqs. We shall not consider this scenario any further.

### 2.2 Dark hadrons

At energies below the scale of confinement $\Lambda_{\rm DC}$, the degrees of freedom of the dark sectors will be dark hadrons. There are essentially three types, with different mass scales and properties: dark mesons, DBs, and dark glueballs.

[6] Smaller values of $\Lambda_{\rm DC}$ are allowed if one assumes a matter-antimatter asymmetry in the dark sector [27, 28]. In this case, the cosmological implications of the formation of dark nuclei must be taken into account [12, 13].

| Scenario | $\Lambda_{\rm DC}$ | Type of DMC |
|---|---|---|
| 1 | $\sim 100\,\text{TeV}$ | Need non-perturbative calculations |
| 2 | $< 100\,\text{TeV}$ (see [11]) | Depends on the Dq masses |
| 3* | $\sim 100\,\text{TeV}$ | fermions as constituents |
| 4 | $\sim 100\,\text{TeV}$ | scalars as constituents |

**Table 5.** Summary of the value of the confinement scale $\Lambda_{\rm DC}$ and the nature of the DMC in the four scenarios of figure 1. We mostly consider the third scenario in this work, marked with an asterisk.

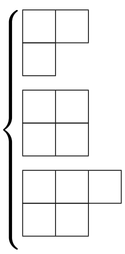
### Dark glueballs

The lightest glueballs have mass $7\Lambda_{\rm DC}$ [29], which means that in scenarios in which the Dqs are heavier than the confinement scale, they might be the lightest dark states. In order not to overclose the universe, they cannot be stable on cosmological timescales. In particular one requires that they all decay before Big Bang Nucleosynthesis (BBN), that is, before 1 s after the big bang [30, 31]. We shall not consider dark glueballs any further, and we refer to [11, 32] for details on their impact on cosmology and collider phenomenology.

### Dark baryons

DBs are bound states of $N_{\rm DC}$ Dqs in an antisymmetric DC combination. As discussed above, in strongly coupled scenarios their mass are given by the dimensional transmutation scale $M_{\rm DB} \sim N_{\rm DC}\,\Lambda_{\rm DC}$, while in weakly coupled scenarios they are determined by their constituents $M_{\rm DB} \sim N_{\rm DC}\,M_{\Psi,\phi}$. Let us restrict, for now, to DBs with only fermionic Dqs as constituents. By addition of the constituents' spins, the DBs are fermions for odd values of $N_{\rm DC}$, and bosons for even values. Their SM quantum numbers are determined by their DF representation and its decomposition under the SM gauge group.

In this work we seek the DM candidate (DMC) among lightest DBs, which are accidentally stable. We make the assumption that the lightest multiplet of DBs is the one with the lowest spin and vanishing orbital angular momentum of the constituents. Since the DC wave function is anti-symmetric, by Fermi's statistics this means that its representation must be symmetric in spin and DF, meaning that their spin and DF representations have the same Young tableau. For the lowest values of $N_{\rm DC}$, the smallest Young tableaux for both spin and DF are



for $N_{\rm DC} = 3$

for $N_{\rm DC} = 4$ ,   (2.6)

for $N_{\rm DC} = 5$

meaning that they are either spin $\frac{1}{2}$ or spin 0, if $N_{\rm DC}$ is odd or even, respectively.

DB masses receive contributions from both the masses of their constituents and the SM gauge interactions. As discussed in section 2.1, the former is particularly relevant in the weakly coupled scenario in which $\Lambda_{\rm DC} \ll M_\Psi$, since the lightest DBs will be those with



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lightest constituents as these contributions are even larger than the spin-spin splittings. Upon the spontaneous breakdown of the electroweak symmetry, the mass of charged DBs within a multiplet is lifted by electromagnetism by $\sim 166\,\mathrm{MeV}$ [33], which means that the lightest DB within the (lightest) multiplet is the one with the smallest SM charge and may be a viable DMC.

There are two notable exceptions to the above criterion, namely cases in which the DMC:

- **is $N^{N_{\mathrm{DC}}}$.** In this case flavor cannot be anti-symmetrized, and therefore the spin must be totally symmetric and equal to $\frac{N_{\mathrm{DC}}}{2} \geq \frac{3}{2}$, which is not the lowest possible (see section 3.2).
- **contains dark scalars.** In this case the spin of the DMC depends on the number of scalars. We briefly discuss this possibility in section 2.4.

### Dark mesons

Dark mesons are bound states of a Dq and an anti-Dq. If the constituents are both fermions, they are bosons, most interestingly vectors (dark $\rho$s) or scalars (D$\pi$s), transforming in the adjoint representation of the DF group SU($N_{\mathrm{DF}}$). If one of the constituents is a scalar, they are fermionic in nature and have the quantum numbers of the SU(5) fragments of table 1, meaning that they mix with SM fermions, in such a way that the asymptotic states are an admixture of elementary SM fermions and composite dark states. Finally there is a scalar SM singlet $\mathcal{S} \sim \phi^\dagger\phi/\frac{\Lambda_{\mathrm{DC}}}{4\pi}$, which mixes with the Higgs through the portal of eq. (2.2). We neglect this effect as it is small except for very large values of $\lambda_{\phi H}$ which are anyways excluded by direct searches (see section 5.1). D$\pi$s are the pNGB associated with the spontaneous breakdown of dark chiral symmetries as described in the previous section and, as such, are expected to be much lighter than the confinement scale.[7] The same cannot be said for mesons with $\phi$ as a constituent (be they fermions or bosons), whose mass is therefore expected to be of the order of $\Lambda_{\mathrm{DC}}$ in strongly coupled scenarios.

The D$\pi$ masses receive a contribution from the SM interactions (if they are charged) arising from loops in the low energy effective field theory (EFT), and one from the constituent masses, which can be estimated via standard chiral perturbation theory techniques:

$$\Delta_{\mathrm{SM}} m_{\pi_D}^2 \sim \left(\frac{g_{\mathrm{SM}}}{4\pi}\Lambda_{\mathrm{DC}}\right)^2 \quad \text{and} \quad \Delta_{\mathrm{mass}} m_{\pi_D}^2 \sim M_\Psi \Lambda_{\mathrm{DC}}\,. \tag{2.7}$$

In strongly coupled scenarios these contributions are much smaller than $\Lambda_{\mathrm{DC}}$: the gauge contribution alone gives, for charged D$\pi$, $m_{\pi_D} \sim 0.1\Lambda_{\mathrm{DC}} \sim 10\,\mathrm{TeV}$. As opposed to this, fermionic mesons as well as $\mathcal{S}$ and mesons containing $N$ have masses of the order of $\Lambda_{\mathrm{DC}}$ as any other dark state. In weakly coupled scenarios the masses of dark mesons are determined mostly by the constituent masses (schematically: $m_{\pi_D} \sim 2M_{\Psi,\phi}$) and receive smaller contributions from the SM gauge interactions.

As discussed above, most models feature charged mesons, which can be at most meta-stable, that is to say, they must decay through dimension four or five operators. In the latter case, requiring that they decay before BBN poses a lower limit on their mass:

$$\tau_{\mathrm{dim.}\ 5} \sim \frac{8\pi\Lambda_{\mathrm{UV}}^2}{m_{\pi_D}^3} \lesssim 1\,\mathrm{s} \implies m_{\pi_D} \gtrsim 1\,\mathrm{TeV}\left(\frac{\Lambda_{\mathrm{UV}}}{10^{16}\,\mathrm{GeV}}\right)^{\frac{2}{3}}\,. \tag{2.8}$$

[7]Recall that the model with only $N$ is an exception since the only meson receives mass contribution from the chiral anomaly.

### 2.3 Model selection

Models are considered viable if they comply with the following requests:

- The DC interactions exhibits asymptotic freedom, i.e. its $\beta$-function is negative. This poses a lower bound on the number of DFs depending on $N_{\rm DC}$, which is easily verified.
- The contribution of light Dqs to the running of the SM gauge couplings does not produce Landau poles below the Planck scale. This poses an upper bound on $N_{\rm DC}$ which is typically the strongest limitation to the model selection.
- The model has a viable DMC, namely a dark hadron that is stable thanks to an accidental symmetry, which:
  - is electrically neutral;
  - has vanishing hypercharge [33];
  - is uncolored;[8]

As a consequence of the first two requirements, it must have integer isospin. Let us discuss these requirements in detail.

#### Asymptotic freedom

The first term in the $\beta$-function of the SU($N_{\rm DC}$) coupling with $N_{\rm DF}$ flavors of fermionic Dqs and one dark scalar $\phi$ is

$$b^0_{\rm DC} = -\frac{11}{3}N_{\rm DC} + \frac{2}{3}N_{\rm DF} + \frac{1}{6}\,. \tag{2.9}$$

Requiring that the dark sector exhibits asymptotic freedom is equivalent to requiring $b^0_{\rm DC} < 0$. This only excludes $N_{\rm DC} = 3$ for more than sixteen light DFs, and we found no viable model with more than fifteen, cf. table 9.

#### Running of the Standard Model gauge couplings

The contribution of the Dqs to the SM $\beta$-functions are summarized in the rightmost columns of table 1. Since each Dq is a left-handed fundamental of SU($N_{\rm DC}$) and vector-like with respect to the SM, the numbers in that columns are to be multiplied by $2N_{\rm DC}$. Assuming that in strongly coupled scenarios the Dqs start contributing to the running when the scale reaches $\Lambda_{\rm DC} \sim 100\,\text{TeV}$ [35], one finds the following conditions [10]:

$$N_{\rm DC}\sum_\Psi \Delta b^\Psi_Y \lesssim \frac{11}{2} \quad N_{\rm DC}\sum_\Psi \Delta b^\Psi_2 \lesssim 5 \quad N_{\rm DC}\sum_\Psi \Delta b^\Psi_3 \lesssim 5\,, \tag{2.10}$$

where the sum extends over all the light species of Dqs. The Dqs $T$, $S$, $G$, and $X$ are automatically excluded since they contribute too much to the running.

In weakly coupled scenarios the contributions of Dqs to the running may start at higher scales, resulting in weaker restrictions on the content of the model and one must therefore check the perturbativity of the SM gauge couplings case by case.

[8] Note that this request may be challenged if one takes into account the QCD confinement of the colored DBs. Such a scenario was considered in [34]. We do not consider such a possibility in this work.



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### Dark matter candidate

A dark state is stable thanks to an accidental symmetry if it is the lightest particle charged under that symmetry. If, as we are requiring, all global symmetry are broken (at most at the level of dimension five) except for the DB symmetry, the DMC must be sought among the lightest DBs.

In strongly coupled scenarios $m_\Psi \ll \Lambda_{\rm DC}$, these are simply those of eq. (2.6). In order for one of those DBs to be a viable candidate, a color singlet with zero hypercharge must be found in the decomposition of the DF representation under the SM gauge group. The model must be discarded if this is not the case, as it will have stable particles in the spectrum that do not evade observational bounds. In appendix A we furnish a few examples of both models that exhibit such a state and models that do not.

If on the other hand, all the Dqs are heavier than $\Lambda_{\rm DC}$, we assume that the lightest DB is that of lowest spin among those containing the lightest Dqs as constituents. Indeed in this case the splittings due to spin-spin interactions are expected to be smaller than in the previous case and the mass orderings of the fundamental Dqs dictates the hierarchy between the bound states.

## 2.4 Hybrid candidates

To the best of our knowledge, there is no literature regarding the dynamics of the formation of hybrid bound states with both fermion and scalar constituents in confining theories, which may for instance be attained through lattice simulations. This makes it challenging to address the question of the mass hierarchy between the bound states, which is needed for instance to assess the nature of the DMC in a specific model in the strongly coupled scenario. In QCD, hybrid hadrons are those with both quarks and gluons as valence constituents. There is plenty of literature on hybrid mesons, but hybrid baryons have attracted less attention for phenomenological reasons [36]. Lattice simulations seem to indicate that baryons with a gluonic component lie slightly heavier than the lightest ordinary baryons [37].

We would like to have an argument to understand the hierarchy between hybrid and regular states in our DM models. In the third and fourth scenarios of section 2.1 the hierarchy of the DBs is dictated by the hierarchy between the scalar and fermionic Dqs. In the other two scenarios if the masses of the two types of constituents are comparable, one may argue once more that the DMC will be the DB with the smallest spin. If, then, a hybrid DB has smaller spin than the lightest DB with only fermionic or only scalar constituents, it may be the DMC. A rough prescription for understanding whether this is the case is the following.

Let a hybrid DB contain $N_F$ fermionic Dqs and $N_S$ scalars, with $N_{\rm DC} = N_F + N_S$, and let us temporarily disregard whether the dark fermions are in a flavor representation that contains a viable DMC. We treat the fermions and the scalars separately, establishing their total angular momenta $J_F$ and $J_S$. The spin of the lightest hybrid state with $N_F$ fermions and $N_S$ scalars will then be the smallest representation arising from the combination of $J_F$ and $J_S$. One then in principle compares this value with the spin of the lightest DB with only fermionic or only scalar constituents. Establishing what is the spin of the latter is challenging and beyond the scope of this work, which unfortunately means that we are not able to give a definitive answer to the question.

The fermions will arrange in the spin configurations arising from the composition of $N_F$ spin $\frac{1}{2}$. Indeed the total antisymmetry of the wavefunction is enforced by taking a flavor representation whose symmetry matches that of the spin representation. If $N_F$ is odd, $J_F$ always starts from $\frac{1}{2}$, while if $N_F$ is even, $J_F = 0, 1$ are always possible. An exception is the case in which the Dq $N$ is the lightest fermionic Dq, with $M_N \sim M_\phi$, in which case the spin configurations is necessarily $\frac{N_F}{2}$ (see eq. (3.3); these models are only viable in the weakly coupled scenario). The scalars, on the other hand, have to arrange in a totally antisymmetric configuration in order to respect the correct statistics, and since they enjoy no DF symmetry, orbital angular momenta of the constituents have to be introduced.

Let us consider the two easiest cases of $N_S = 1, 2$, and first look at the case in which $N$ is not among the lightest fermionic Dqs. If there is only one scalar, it will clearly contribute with $J_S = 0$. If there are two scalars, in order to respect the bosonic symmetry they must have an odd angular momentum, $J_S = 1, 3, 5, \ldots$, since an eigenstate of the total angular momentum has parity $(-1)^J$ under the exchange of the two particles. In light of what we observed above on $J_F$, we see that in these two easiest cases the smallest value arising from the combination of $J_S$ and $J_F$ is

$$J_{N_F+N_S} = \begin{cases} \dfrac{1}{2} & \text{for odd } N_F \\ 0 & \text{for even } N_F \end{cases} \tag{2.11}$$

Recall that DBs with fermionic constituents have $J = \frac{1}{2}$ for odd $N_{\rm DC}$ and $J = 0$ for even $N_{\rm DC}$, which means that if $N_S = 1$ and $N_F$ is even, the hybrid DBs have lower spin than the purely fermionic DBs, and may be the DMC. If $N_F = 2$ the only possible viable hybrid DMC would be $\Psi\widetilde{\Psi}\phi$. However in models containing $\Psi \oplus \widetilde{\Psi}$ for $N_{\rm DC} = 3$ the DB number is broken explicitly by the last interaction of table 3, so this case must be discarded. One may then have hybrid DMC for $N_S = 1$, $N_{\rm DC} = 5, 7$.

If, instead, $N$ is the lightest fermionic Dq, one has,

$$J_{N_F+N_S} = \begin{cases} \dfrac{N_F}{2} & \text{for } N_S = 0, 1 \\ \left|1 - \dfrac{N_F}{2}\right| & \text{for } N_S = 2 \quad \text{up to } N_F = 4 \end{cases} \tag{2.12}$$

and one sees that $N_S = 2$ is always preferred with respect to $N_S = 1, 0$ until $N_{\rm DC} = 6$. For instance for $N_{\rm DC} = 3$ the Dirac DB $N\phi^2$ may be the DMC.

In the weakly coupled scenario, in which the fundamental constituents are non-relativistic, one may be able to deduce the total angular momentum $J_S$ of the subset of scalar Dqs inside the DB even for $N_S \geq 3$ [18]. We leave a more exhaustive treatment of hybrid DBs to a future work.

## 3 Model classification

In this section we show a classification of models that are viable according to the criteria of section 2.3. This classification is only viable in the strongly coupled scenario. Recall, indeed, that in the weakly coupled scenario, if the mass of the light Dqs is large enough, several more models may pass the selection, since the bounds arising from the perturbativity of the SM are weakened. We also give a separate discussion on certain models with light $N$ that are only viable in the weakly-coupled scenario.

### 3.1 Minimal models

In principle, one can construct every possible model compatible with our assumptions of section 2.3 by considering all possible combinations of Dqs taken from table 1 and excluding those that do not meet the requirements, arriving at the list in table 9 in appendix C. Many models will present the same DMC, as there are but a few possibilities to combine the fragments of table 1 to form a DB that is a color singlet with vanishing hypercharge. In particular, if fragments from more than three different SU(5) representations are considered, one either encounters subplanckian Landau poles or falls back to the case of a smaller model, extended with some light Dqs that do not constitute the DMC. Indeed, one can interpret the list in terms of *minimal models*, namely those models with the smallest light Dq content needed to produce a given DMC, and their extensions. The minimal models with the respective DMC (from which one can read off the allowed number of DCs) are envisioned in table 6 with the exclusion of those containing $N$, which are discussed separately.

Minimal models can be extended in two ways (see appendix B for examples):

1. Adding any light Dq (including $N$) and keeping $N_{\text{DC}}$ fixed to find a model with the same DMC

2. Adding the Dq $N$ and increasing $N_{\text{DC}}$ by one to form a new minimal model whose DMC contains the $N$ Dq in addition to the previous content[9]

Moreover if it is possible to rise $N_{\text{DC}}$ without spoiling any of the requirements, one can in principle realize a minimal model in which the DMC is a hybrid state containing scalars in addition to the previous content, as shown in the table. In section 2.4 we give a discussion on hybrid states, arguing that assessing if hybrid DBs may be the DMC is a difficult task. For completeness, however, we do include in both table 6 and table 9 models in which the DMC is a hybrid DB.

The requirement for the SM couplings to stay perturbative below the Plank scale implies that there are no minimal models with more than three different species of light Dqs (excluding $N$: indeed, the minimal model $N$ and its extensions are in some sense special and deserve a separate treatment). Note that no minimal model with $V$ in the light Dq spectrum is quoted. The reason for this is that in order to avoid subplanckian Landau poles in the SM, the only possibility for such models would be $N_{\text{DC}} = 3$. However, in that case, the combination of the dimension five operators

$$V^c \phi H l \quad \text{and} \quad V \sigma^\mu V^{c\dagger} (D_\mu \phi) \tag{3.1}$$

[9]There are only three viable models with $N$ that cannot be obtained directly by extension from minimal models listed in table 6: for $N_{\text{DC}} = 4$ $L \oplus E \oplus N$ and $L \oplus \widetilde{L} \oplus N$ with candidate of the type respectively $LLEN$ and $L\widetilde{L}NN$, and for $N_{\text{DC}} = 5$ the model $L \oplus E \oplus N$ with candidate $LLENN$.

| SU(5) | Minimal Model | DMC | $\mathrm{SU}(2)_L$ Multiplet |
|---|---|---|---|
| $\bar{\mathbf{5}} \oplus \mathbf{5}$ | $D \oplus \tilde{D}$ | $D\tilde{D}\phi^2$, $D\tilde{D}\phi^3$, $D\tilde{D}\phi^4$, $D\tilde{D}\phi^5$ | $\mathbf{1}$ |
| | | $D\tilde{D}D\tilde{D}$, $D\tilde{D}D\tilde{D}\phi$, $D\tilde{D}D\tilde{D}\phi^2$, $D\tilde{D}D\tilde{D}\phi^3$, | $2 \times \mathbf{1}$ |
| | | $D\tilde{D}D\tilde{D}D\tilde{D}$, $D\tilde{D}D\tilde{D}D\tilde{D}\phi$ | $2 \times \mathbf{1}$ |
| | $L \oplus \tilde{L}$ | $L\tilde{L}\phi^2$, $L\tilde{L}\phi^3$, $L\tilde{L}\phi^4$, $L\tilde{L}\phi^5$ | $\mathbf{1} \oplus \mathbf{3}$ |
| | | $L\tilde{L}L\tilde{L}$, $L\tilde{L}L\tilde{L}\phi$, $L\tilde{L}L\tilde{L}\phi^2$, $L\tilde{L}L\tilde{L}\phi^3$, | $2 \times \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$ |
| | | $L\tilde{L}L\tilde{L}L\tilde{L}$, $L\tilde{L}L\tilde{L}L\tilde{L}\phi$ | $2 \times \mathbf{1} \oplus 2 \times \mathbf{3} \oplus \mathbf{5} \oplus \mathbf{7}$ |
| | $D \oplus \tilde{D} \oplus L \oplus \tilde{L}$ | $D\tilde{D}\phi^2$, | $\mathbf{1}$ |
| | | $L\tilde{L}\phi^2$, | $\mathbf{1} \oplus \mathbf{3}$ |
| | | $D\tilde{D}D\tilde{D}$, | $2 \times \mathbf{1}$ |
| | | $L\tilde{L}L\tilde{L}$, | $2 \times \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$ |
| | | $D\tilde{D}L\tilde{L}$ | $2 \times \mathbf{1} \oplus 2 \times \mathbf{3}$ |
| $\mathbf{10} \oplus \overline{\mathbf{10}}$ | $E \oplus \tilde{E}$ | $E\tilde{E}E\tilde{E}$, $E\tilde{E}\phi^2$ | $\mathbf{1}$ |
| $\bar{\mathbf{5}} \oplus \mathbf{10}$ | $D \oplus U$ | $DDU$, $DDU\phi$ | $\mathbf{1}$ |
| | $L \oplus E$ | $LLE\phi$, $LLE\phi^2$ | $\mathbf{1} \oplus \mathbf{3}$ |
| $\mathbf{5} \oplus \mathbf{10}$ | $Q \oplus \tilde{D}$ | $QQ\tilde{D}$, $QQ\tilde{D}\phi$ | $\mathbf{1} \oplus \mathbf{3}$ |
| | $\tilde{D} \oplus E \oplus U$ | $\tilde{D}EU$ | $2 \times \mathbf{1}$ |

**Table 6.** Minimal models, i.e. models with the smallest Dq content that produce a given DMC (third column). The allowed value of $N_{\mathrm{DC}}$ can be read off the number of constituents of the various DMCs. In the last column we show the $\mathrm{SU}(2)_L$ representation of the DMC (recall that it must be a color singlet with vanishing hypercharge).

breaks the DB symmetry, rendering any model containing $V$ unacceptable. The same operators can be written with $N$ in place of $V$:

$$N^c \phi H l \quad \text{and} \quad N \sigma^\mu N^{c\dagger} (D_\mu \phi) , \tag{3.2}$$

which means that models with light $N$ are only acceptable for $N_{\mathrm{DC}} \geq 4$.

### 3.2 $N$ models

In this section we describe the $N$ *models*, in which the DMC has only $N$ as fermionic constituents. It is easy to see that the minimal model with this DMC contains only $N$ as a light Dq, and indeed any $N$ model can be seen as an extension of it. Since $N$ is a total SM singlet, it does not contribute to the running of the couplings, and this model could exist for any value of $N_{\mathrm{DC}} \geq 4$ (see above), safe from subplanckian Landau poles. Hence, the allowed values for $N_{\mathrm{DC}}$ in models that are its extensions are dictated by the other light Dqs.

First let us discuss the accidental symmetries of this kind of models. If the Dq $L$ is light, the species symmetry $\mathrm{U}(1)_N \otimes \mathrm{U}(1)_L \otimes \mathrm{U}(1)_\phi$ is broken to the DB symmetry $\mathrm{U}(1)_{\mathrm{DB}}$ at renormalizable level by the Yukawa coupling with the SM Higgs field: $LHN^c$. The same

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can be said if $\widetilde{L}$ is light. Otherwise, the breaking takes place at dimension five through the operator $N^c\phi\tilde{H}l$, regardless of the number of DCs. This means that, in this second case, the model will present metastable dark mesons. In general these states will be charged under QCD or even electromagnetic interaction, thus their abundance must be very small and they must have all decayed before BBN. If they come to dominate the energy budget of the universe their decay will inject significant entropy in the SM potentially leading to a phase of dilution of the DM relic abundance (see [32] and references therein). Incidentally, these are the only models that we can construct that present this feature, since in every other model the species symmetries are broken at the renormalizable level, and we can assume that mesons decay immediately into SM particles. Note that the operator that breaks the species of $N$ at dimension five also breaks the SM lepton flavor symmetry to a single lepton number, under which $N$ inherits charge 1 as shown in table 4.

Let us now discuss the properties of the DMC, which can be $N^{N_{\rm DC}}$ or any hybrid baryon composed of $N$ and $\phi$, such as $\phi N^{N_{\rm DC}-1}$. In selecting viable models, we have so far assumed that the lightest DB multiplets were those of lowest spin, whose DF representation had the Young tableaux depicted in eq. (2.6). We concluded then that for odd (even), $N_{\rm DC}$ the DM has spin $\frac{1}{2}$ (spin 0). When only the Dq $N$ constitutes the DM, however, one cannot antisymmetrize flavor, so $N^{N_{\rm DC}}$ belongs to a higher spin multiplet, namely:

$$\underbrace{\square\square\cdots\square}_{N_{\rm DC}} \quad \text{with spin } \frac{N_{\rm DC}}{2}\,. \tag{3.3}$$

Hybrid candidates may have lower spin, but in general larger than $\frac{1}{2}$ or 1. As a consequence, unless $N$ is the only light Dq, in which case $N^{N_{\rm DC}}$ is the only (purely fermionic) DB and as such it is stable, one has to enforce that the lightest DB does not have any fermionic Dq other than $N$ as consitutents. To do so one may require that the mass of all the Dqs be much larger than the confinement scale $\Lambda_{\rm DC}$ — which would correspond to the weakly coupled scenario, the second of section 2.1 — and that $N$ is much lighter than all the others. If the mass of the scalar $\phi$ is comparable to the mass of $N$, hybrid DBs might be lighter than $N^{N_{\rm DC}}$, as discussed in section 2.4.

Thus, a model with $N$ and other light Dqs has to be considered in two separate regimes. In the case we have just described, the DMC would be either $N^{N_{\rm DC}}$ or a hybrid baryon, which will be much lighter than any possible DB made of the other Dqs. In any other scenario the DMC is to be sought among the DB made of the other Dqs, including, possibly, hybrid DB containing $\phi$.

## 4 SU(5) grand unification

In this section we analyse the models found in the previous sections in the context of an SU(5) grand unification scheme [38]. This was done in [10] as well, finding that essentially only one of their viable models, namely $Q \oplus \widetilde{D}$, produced a successful unification. In line with [39], the authors observe that in order for a model to succeed in the unification, either the Dq $Q$ or the Dq $V$ (or both) must be light.

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| Model | DMC | $\alpha_{\rm GUT}$ | $M_H$ (GeV) | $M_{\rm GUT}$ (GeV) |
|---|---|---|---|---|
| $Q \oplus \widetilde{D}$ | $QQ\widetilde{D}$ | $6 \times 10^{-2}$ | $2 \times 10^{11}$ | $2 \times 10^{17}$ |
| $Q \oplus \widetilde{D}$ | $QQ\widetilde{D}\phi$ | $2.29 \times 10^{-1}$ | $4 \times 10^{9}$ | $2 \times 10^{17}$ |
| $\widetilde{Q} \oplus D \oplus U \oplus L$ | $\widetilde{Q}\widetilde{Q}D$ or $DDU$ | $8.43 \times 10^{-2}$ | $2 \times 10^{17}$ | $2 \times 10^{17}$ |
| $Q \oplus \widetilde{D} \oplus E$ | $QQ\widetilde{D}$ | $4.5 \times 10^{-2}$ | $2 \times 10^{17}$ | $2 \times 10^{17}$ |
| $Q \oplus \widetilde{D} \oplus E$ | $QQ\widetilde{D}\phi$ | $1.13 \times 10^{-1}$ | $2 \times 10^{14}$ | $2 \times 10^{17}$ |

**Table 7.** Models in which the unification of the SM couplings is exact. The DMC is shown, as well as the value of the GUT coupling constant, the intermediate scale of the heavy dark fermions, and the unification scale.

### 4.1 Standard approach to unification

The one-loop level running of the gauge couplings is given by

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i^{\rm SM}}{2\pi}\log\frac{\mu}{M_Z} + \delta_i(\mu)\,, \tag{4.1}$$

where $\alpha_1 = \frac{3}{5}\alpha_Y$, the values of the coupling constants at the $Z$ boson mass $M_Z$ [40], and $b_i^{\rm SM}$ are the three SM beta-function coefficients

$$b_1^{\rm SM} = \frac{41}{10}, \quad b_2^{\rm SM} = -\frac{19}{6}, \quad b_3^{\rm SM} = -7\,. \tag{4.2}$$

The new physics contributions are all encoded in the $\delta_i$, the SM corresponding to $\delta_i = 0$. Only two independent combinations of the $\delta_i$ are relevant for unification, for instance $\delta_{12} = \delta_1 - \delta_2$ and $\delta_{32} = \delta_3 - \delta_2$.

We perform this analysis in the strongly coupled scenario, and posit that the light fermionic Dqs start to contribute to the running at $\Lambda_{\rm DC} \sim 100\,$TeV. The heavy GUT partners are assumed to start contributing from their (common, for simplicity) mass scale $M_H$, which we take to lie between $\Lambda_{\rm DC}$ and the grand unification scale $M_{\rm GUT}$. Then:

$$\delta_i(\mu) = -2N_{\rm DC}\frac{\Delta b_i}{2\pi}\log\frac{M_H}{\Lambda_{\rm DC}} - 2N_{\rm DC}\frac{\Delta b_{\rm full}}{2\pi}\log\frac{\mu}{M_H}\,, \tag{4.3}$$

where $\Delta\, b_i$ is the sum of the contributions from all light Dqs to the $i$-th $\beta$-function as in table 1, and $\Delta b_{\rm full}$ is the contribution from the full SU(5) multiplet above $M_H$.

Typically, one requires that the new physics make the unification of couplings exact at a certain energy scale. As a further criterion, one requires that GUT coupling is in the perturbative range $[0, 4\pi]$. This fixes $M_{\rm GUT}$ and $M_H$. In this way, following the same steps of [10], we find that the models shown in table 7, obtained extending minimal models in table 6, provide a successful unification.

In the weakly coupled scenario one can perform a similar analysis, and take the light Dq masses as the scale where they start contibuting to the running. In principle many more models could be found that provide a successful unification, thanks to the interplay between the light Dq masses and $M_H$.



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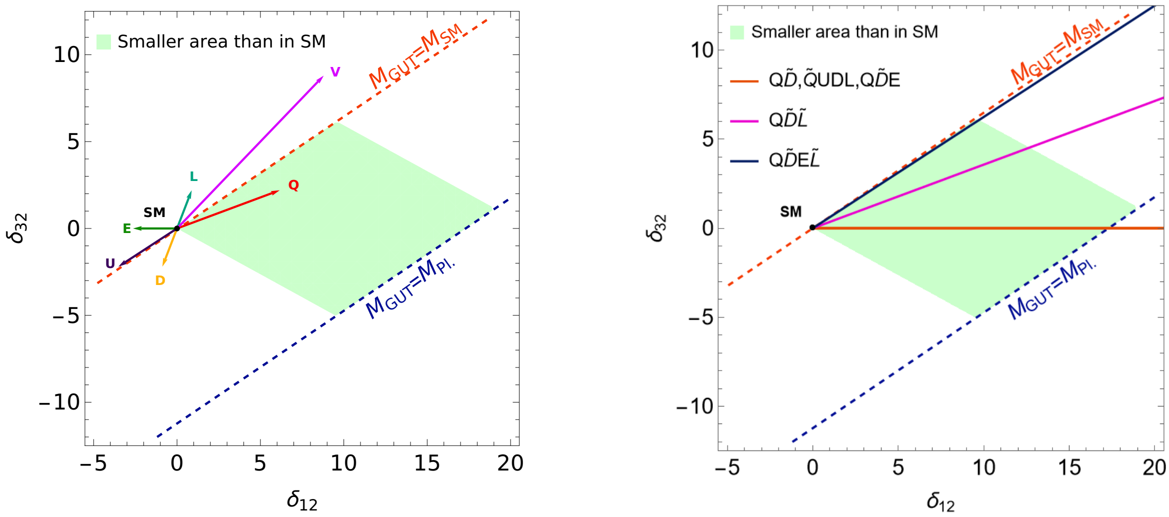


**Figure 2.** Modification to the grand unification of the SM couplings as a function of the contributions to the $\beta$-functions from new physics (see main text). *Left*: the contribution from single Dq are shown assuming $N_{\rm DC} = 3$, and $M_H = 10^3 \Lambda_{\rm DC}$. We see that only the Dqs $Q$ and $V$ point in the right direction for improving unification. *Right*: strongly coupled models with improved unification are shown as continuous half lines. The larger $M_H$, the farther one moves away from the origin along the line.

### 4.2 Relaxed criterion for unification

We also follow a different approach [17]. We relax the condition that the grand unification is exact, and only require that it is in some sense *better* than in the SM, albeit possibly still imperfect. In particular we require that the area of the triangle formed by the intersections of the lines drawn by the running in the $(\log \mu,\, \alpha^{-1})$ plane is smaller than in the SM. One can then take $M_{\rm GUT}$ and $\alpha^{-1}_{\rm GUT}$ to be the coordinates of the barycenter of the triangle. As we shall see momentarily, this approach allows to select a larger number of models as successful with respect to those in table 7. As a check of its validity we verify that the results of table 7 are recovered if one requires that the area formed by the triangle vanishes, which would correspond to exact unification with lines meeting at one point.

In figure 2 we summarize the results of this analysis. On the left, the contribution arising from single Dqs are shown as arrows in the $(\delta_{12},\, \delta_{32})$ plane, whose slopes are $\frac{\Delta\, b_{32}}{\Delta\, b_{12}}$, independently of $N_{\rm DC}$. The direction where the arrows point depends on the sign of $\Delta\, b_{12}$, and their length are obtained taking $M_H = 10^3 \Lambda_{\rm DC}$ and $N_{\rm DC} = 3$ as an example. The orange (blue) dashed line excludes regions in the plane in which the grand unification scale is smaller than that of the SM (larger than the Planck scale), and the shaded green region is that in which the unification is improved according to our criterion. The arrows stem from the origin, where the SM ($\delta_i = 0$) lies. According to eq. (4.3), any model lies in the origin if $M_H = \Lambda_{\rm DC}$: indeed in such case the lines in the $(\log \mu,\, \alpha^{-1})$ plane (and thus the triangle they form) are just moved rigidly along the $\alpha^{-1}$ axis, and the area equals that of the standard case. By inspecting the arrows we see that relaxing the criterion for unification does not change the conclusion that, in order for a model to provide an acceptable unification, either the Dq $Q$ or the Dq $V$ must be light [39] (as discussed in section 3.1, however, we exclude models with $V$ altogether).

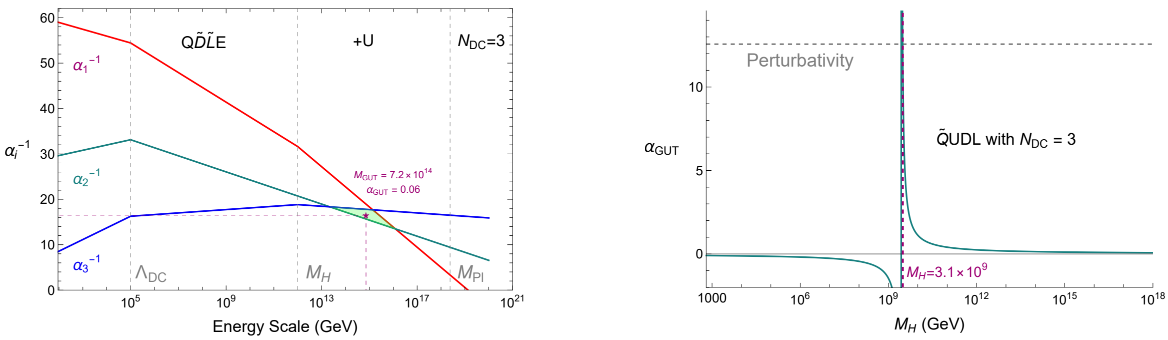


**Figure 3.** *Left*: running of the SM gauge couplings in the model $Q \oplus \widetilde{D} \oplus E \oplus \widetilde{L}$ with $N_{\rm DC} = 3$ and $M_H = 10^{12}$ GeV. The purple star inside the highlighted triangle is its barycenter, whose coordinates correspond to the GUT scale and the inverse of the GUT coupling. *Right*: value of the GUT coupling as a function of $M_H$ in the model $\widetilde{Q} \oplus U \oplus D \oplus L$ with $N_{\rm DC} = 3$. The coupling is negative at first, and then non-perturbative until the value $M_H \sim 3.1 \times 10^9$ is reached, so the model cannot be considered successful for smaller values of the heavy Dq mass scale.

On the right of figure 2 we show models as continuous half lines stemming from the origin, whose slopes and directions depend on the model content as described just above. The models shown are representatives of all the models among the viable ones of table 9 whose lines cross the green region, which means that they may improve the unification according to our criterion for some $M_H$. Replacing one or more of the Dqs with their tilded versions one obtains models with the same lines, that may or may not be viable depending on dimension five operators or the existence of a viable DMC. In total, we count twentyfour models that pass this selection. The value of the GUT coupling must be checked in all cases: typically, requiring that $\alpha_{\rm GUT}$ is perturbative poses a lower limit on the value of $M_H$, as can be seen the right panel of figure 3. As $M_H$ becomes larger, points along the lines are scanned farther and farther away from the origin, and eventually they leave the green area. This poses an upper limit on the heavy fermion scale $M_H$, depending both on the Dq content and on $N_{\rm DC}$.

To give a concrete example, let us consider the model $Q \oplus E \oplus \widetilde{D} \oplus \widetilde{L}$ with $N_{\rm DC} = 3$. This model would be viable even without the fundamental scalar, as Yukawa couplings with the Higgs field suffice in breaking the species symmetries. However, it does not produce a successful exact unification, and indeed neither is it reported in [10] nor in our table 7.[10] Therefore it is a genuine case in which relaxing the criterion for unification makes the model successful. Taking $M_H = 10^{12}$ GeV, we find

$$M_{\rm GUT} = 7.4 \times 10^{14}\,{\rm GeV} \quad \text{and} \quad \alpha_{\rm GUT} = 5.9 \times 10^{-2} \sim \frac{1}{17}\,. \tag{4.4}$$

The running of the SM couplings in this model are shown in the left panel of figure 3.

[10]Actually, in [10] there is no mention of this model because it has $N_{\rm DF} = 12$ and no model is considered with a number of DFs larger than ten in that work. One can easily be convinced that it would indeed be a *golden* model in their language as all accidental symmetries except for DB number are explicitly broken at renormalizable level.

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## 5 Phenomenology

Composite DM setups present very rich phenomenologies. As far as direct detection is concerned, the DM may have sizeable electromagnetic dipole moments because of its charged consitutents [10]. Furthermore, they naturally feature light particles (dark pions and dark glueballs) interacting in number changing interactions, which produce non-standard cosmologies [14, 41, 42], and may be testable in indirect detection experiments [32]. Being some of these light states potentially charged under the SM, they may be observable at colliders [43, 44]. For general features of the phenomenology of models of accidental DM we refer the reader to previous literature (for the weakly coupled scenario, see [11]). Here, we shall focus instead on the impact that the dark scalar $\phi$ has on phenomenology, both directly, by looking at dark states that contain $\phi$ as a constituent, or indirectly through the interactions it mediates, which break the accidental symmetries of the dark sector in a special way, transferring the SM accidental symmetries to the Dqs as envisioned in table 4.

### 5.1 Direct detection of hybrid dark matter candidates

As pointed out in [10], the typical DM-nucleon cross sections for weak interactions [33] are too weak to be detected at current DD experiments (see also [45]), and the best hope for DD would be interactions with photons through the electromagnetic dipoles of DBs. Such interactions only arise at the level of dimension six for scalar DM and of dimension five for Dirac DM, but spin one DM may have such interactions at the renormalizable level. Another possibility is to directly observe Higgs-mediated interactions between the nucleons and the DM, which naturally interact with the Higgs boson as its constituents have the tree-level Yukawa interactions of table 3.

An interesting possibility for the detectability of our setup is to exploit the scalar portal $\lambda_{\phi\, H}\, |\phi|^2\, |H|^2 \supset \lambda_{\phi\, H}\, v\, |\phi|^2\, h$. Even in models in which no Yukawa couplings with the Higgs is allowed, if the DM is a hybrid state containing both fermionic and scalar Dqs (see section 2.4 for a discussion) it interacts with nuclei through this portal with the exchange of one Higgs boson. The DM also inherits a quartic coupling with two Higgs boson legs, which however mediates interactions with nuclei that are suppressed both by a loop factor and by the small coupling oh the Higgs with the nuclei. For Dirac DM, unless its gyromagnetic factor is especially small or $\lambda_{\phi H}$ is especially large, this interaction will be subdominant with respect to the dipole interaction. For scalar DM, however, the dipole interactions are suppressed, and this interaction may be more important.

Considering the weakly coupled scenario in this case, with $M_\phi \simeq M_\Psi \gg \Lambda_{\rm DC}$, the (dimensionless) coupling of a hybrid Dirac DM to the Higgs boson can be computed by matching the matrix elements of the energy-momentum tensor on the hybrid DM states:

$$\lambda_{h\text{-DM}} = v \frac{\partial M_{\rm DM}}{\partial \left(H^\dagger H\right)} = \frac{\lambda_{\phi H} v}{2 M_{\rm DM}} \langle DM |\, \phi^\dagger \phi \,| DM \rangle = \lambda_{\phi H} N_\phi N_{\rm DC} \frac{v}{M_{\rm DM}}\,. \tag{5.1}$$

For a scalar hybrid DM, the dimension one coupling is obtained by multiplying the above equation by $2M_{\rm DM}$. The spin-independent (SI) cross-section for DD is then, for DM with any spin,

$$\sigma_{\rm SI} = \frac{\lambda_{h\text{-DM}}^2 m_{\mathcal{N}}^4 f_{\mathcal{N}}^2}{2\pi m_h^4 v^2} = \frac{\lambda_{\phi H}^2 N_\phi^2 {N_{\rm DC}}^2 m_{\mathcal{N}}^4 f_{\mathcal{N}}^2}{2\pi m_h^4 M_{\rm DM}^2}\,, \tag{5.2}$$

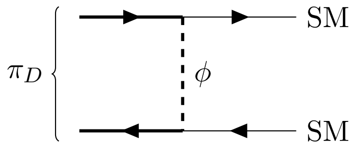


**Figure 4.** Interaction between a dark pion and two SM fermions. Thick continous (dashed) lines represent fermionic (scalar) Dqs.

where $f_{\mathcal{N}} \sim 0.3$ is a nuclear form factor, $m_{\mathcal{N}}$, $m_h$, and $M_{\rm DM}$ are the masses of the nucleon, Higgs boson, and DM, respectively. The LUX-ZEPLIN bound [46] on the SI DD cross-section translates into $\lambda_{\phi H} < 30 \frac{4}{N_{\rm DC}} \frac{1}{N_\phi} \left(\frac{M_{\rm DM}}{100\,{\rm TeV}}\right)^{\frac{3}{2}}$, showing the elusiveness of this scenario.

### 5.2 Dark pions at colliders

In the strongly coupled scenario the DBs, dark vector mesons, and dark glueballs are much too heavy to be produced at a collider; however, D$\pi$s may be much lighter than the DM, and thus are the most promising to be probed. According to eq. (2.7), the gauge contribution gives roughly $m_{\pi_D} \sim 0.1\,\Lambda_{\rm DC} \sim 10\,{\rm TeV}$ for charged D$\pi$s. If we assume that the Dq masses are some factor of a hundred smaller than the confinement scale $M_\Psi \sim \frac{\Lambda_{\rm DC}}{100} \sim 1\,{\rm TeV}$ (analogous to the first generation of QCD quarks), the contribution from the constituent masses is of the same order. We take this to be the benchmark value for the D$\pi$ masses.

Dark mesons without species number have anomalous couplings to vector bosons, allowing to produce them through vector boson fusion. Any dark meson may also be pair produced through their EW interactions or through the mixing of the dark $\rho$ with the EW bosons [43]. The novelty of our setup is that thanks to the new Yukawa interactions mediated by the scalar Dq $\phi$ the dark pions can decay directly to SM fermions through the interaction of figure 4.

Because of the chiral structure of the SM, Dqs couple to SM fermions with specific chiralities, and depending on the constituents of the dark meson, the decay rates may or may not be chiral suppressed. For instance in a model containing $L \oplus E$, the dark mesons transforming as $(\mathbf{1}, \mathbf{2})_{-\frac{3}{2}}$ couple to $\ell^i_L + e^{i-}_R$, so the decay rate features the helicity suppression by a small SM lepton mass, in the same fashion as the QCD pion decay. Indeed in both cases the decay proceeds through an axial vector current. The same is true for dark pions without species number in any model. It is not the case for instance in models containing $L \oplus \widetilde{E}$, in which the dark mesons transforming as $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$ couple directly to $\bar{\ell}^i_R + e^{i,-}_R$ and decay without chiral suppression, through the dark psuedoscalar density $J^5_D$. We can estimate the rate as

$$\Gamma\left(\pi_D \to {\rm SM\,SM}\right) = \left(G_D^{L\widetilde{E}}\right)^2 \frac{m_\pi {\Lambda_{\rm DC}}^4}{128\pi^2}\,, \tag{5.3}$$

where we defined the strength of the four fermion interaction to be

$$G_D^{L\widetilde{E}} = \frac{y_L y^*_{\widetilde{E}}}{{\Lambda_{\rm DC}}^2}\,. \tag{5.4}$$

As this decay width is not chiral-suppressed it may be rather large, and portions of the parameter space of some models in which the D$\pi$s are especially light are likely to be excluded already by existing searches. On the other hand, future colliders may produce a large number of these states resonantly. For instance, under the reasonable assumptions on the mass scales described above, a muon collider with a realistic beam energy spread [47] would produce these states with cross sections at the level of a few to tens of fb in the Breit-Wigner approximation.

In most models, as all the spurious species symmetries are broken at renormalizable level, the decay of the mesons is prompt on collider scales. If, on the contrary, displaced vertices are observed, one can deduce that the D$\pi$s arise from a model with residual species symmetries broken only at higher dimension level. The only models among those we found that have this characteristic are those that feature $N$ but not $L$ in the light Dq spectrum, for which the breaking of the species number happens through the leftmost operator in eq. (3.2).

### 5.3 Dark meson leptoquarks

Models where both Dqs with baryon number and Dqs with lepton number are present in the light Dq spectrum produce dark mesons with both baryon and lepton number. They are leptoquarks (LQ) candidates, as they will couple to both SM leptons and quarks through the dark scalar.

In the strongly coupled scenario ($\Lambda_{\rm DC} \sim 100\,\text{TeV}$), the D$\pi$s have masses around $\sim 10\,\text{TeV}$ (see above), out of current reach of dedicated LHC searches [48]. Let us take, as examples, models that include $Q \oplus \widetilde{L}$ in the light Dq spectrum. In these cases two LQs appear, with quantum numbers $(\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\overline{\mathbf{3}}, \mathbf{3})_{\frac{1}{3}}$, usually referred to in the literature as $S_1$ and $S_3$. In this scenario the quadratic splitting between $S_1$ and $S_3$ dark meson masses is determined only by gauge interaction since the constituent Dq are the same, and is of order $m^2_{S_3} - m^2_{S_1} \sim (5\,\text{TeV})^2$. The $S_3$ is an example of non-genuine LQ (for a reference on leptoquarks see [49]), that is, a LQ that forms baryon-number breaking terms with SM fermions:

$$\mathcal{L} \supset y_{ij} q_i (\epsilon\, \vec{\tau} \cdot \vec{S}_3) l_j + z_{ij} q_i (\epsilon\, \vec{\tau} \cdot \vec{S}_3)^\dagger q_j + h.c. \tag{5.5}$$

It is important to notice that the baryon number breaking term is suppressed as $\sim \frac{1}{M^2_{\rm GUT}}$, because in the UV theory where Dqs and the scalar are the fundamental degrees of freedom, the baryon symmetry is preserved, so the only contribution comes from the GUT scale. On the other hand, the couplings $y_{ij}$ are only suppressed by the mass of the dark scalar, or the confinement scale.

The phenomenology of the dark meson LQs in composite DM theories should be subject of further investigations. For instance, the mixing between the charge $Q = \frac{1}{3}$ states in the two LQs mentioned above, which proceeds through their couplings with the Higgs boson, depending in turn onnly on the Yukawa couplings of the model, may be exploited to ameliorate the muon $g-2$ problem [50].

### 5.4 Lepton flavor and CP violation

As described in section 2, the presence of the scalar Dq allows to write the Yukawa couplings of table 3, which may violate flavor and $CP$. Here we recast the analysis of [51] and discuss the consequences on the lepton sector. The relevant Dqs to consider are $L$ and $E$ (or $\widetilde{E}$), that couple to the SM particles via the Yukawa terms $L^c \phi l$ and $E^c \phi e^c$ ($\widetilde{E} \phi^\dagger e^c$). We write the Yukawa coupling joining a SM fermion $\psi^{\rm SM}$ and a fermionic Dq $\Psi$ through the dark scalar as $y_\Psi^{\psi^{\rm SM}}$. If we are describing, for example, interactions involving left-handed muons

| Effective operator | Wilson coefficient |
|---|---|
| $Q_{e\gamma}^{ij} = (l_i \sigma^{\mu\nu} e_j^c) H^\dagger F_{\mu\nu}$ | $\frac{C^{e\gamma}}{\Lambda^2} = \frac{c_{ij}^{e\gamma}}{16\pi^2 m^{*2}} e y_L^{i*} y_{\widetilde{E}}^j y_{HL\widetilde{E}}^*$ or $\frac{C^{e\gamma}}{\Lambda^2} = \frac{c_{ij}^{e\gamma}}{256\pi^4 m^{*2}} e Y_{\rm SM}^{k\,k} y_L^{i\,*} y_L^k y_E^{k\,*} y_E^j$ |
| $Q_{ll}^{ijmn} = (l_i \sigma_\mu l_j^\dagger)(l_m \sigma^\mu l_n^\dagger)$ | $\frac{C^{ll}}{\Lambda^2} = \frac{c_{ijmn}^{ll}}{16\pi^2 m^{*2}} y_L^{i*} y_L^j y_L^{m*} y_L^n$ |
| $Q_{le}^{ijmn} = (l_i \sigma_\mu l_j^\dagger)(e_m^c \sigma^\mu e_n^{c\dagger})$ | $\frac{C^{le}}{\Lambda^2} = \frac{c_{ijmn}^{le}}{16\pi^2 m^{*2}} y_L^{i*} y_L^j y_E^{m*} y_E^n$ |
| $Q_{ee}^{ijmn} = (e_i^c \sigma_\mu e_j^{c\dagger})(e_m^c \sigma^\mu e_n^{c\dagger})$ | $\frac{C^{ee}}{\Lambda^2} = \frac{c_{ijmn}^{ee}}{16\pi^2 m^{*2}} y_E^{i*} y_E^j y_E^{m*} y_E^n$ |

**Table 8.** Relevant dimension-six operators for the flavour violating processes. Lower-case latin indices are lepton family indices.

or muonic neutrinos, the relevant coupling is $y_L^{\ell_2}$. In the case of $L \oplus \widetilde{E}$, there is also the coupling $y_{L\widetilde{E}} L^c H \widetilde{E}$.

We estimate the contributions of our dark sectors to the Wilson coefficients of operators in the SM EFT through spurionic arguments and naïve dimensional analysis. We consider the strongly coupled scenario and assume for simplicity that all resonances of the dark sector are controlled by just one massive parameter, which we take conservatively to be the smallest mass scale of the theories, namely the D$\pi$ mass scale $m^* \sim m_{\pi_D} \sim 10\,\text{TeV}$. We thus neglect the mass differences between the various dark hadrons. We assume that all the dark Yukawa couplings are of the same order. Furthermore, we make the same assumptions as in [51], namely that the UV theory contains the full basis of dimension six operators at the scale $\mu = m^*$, and that renormalization group effects and the interference between the operators can be ignored.

We use the experimental bounds [51] on processes that violate flavor (such as lepton decays $\mu \to eee$) as well as on observables such as the electic dipole moment (EDM) to derive upper bounds on the Yukawa couplings. The latter allow to derive constraints on both the real and the imaginary parts of the products of Yukawa couplings, as they violate CP. In our models these observables depend on the product of several of the Yukawa couplings, and in general it is not possible to put bounds on any one coupling. We thus assume that they are all of the same order in order to estimate the bounds.

The relevant dimension six operators (written as in the Warsaw basis [52]) are enlisted in table 8. We take a flavor-diagonal basis for our fields

Processes arising from four-fermions operators, such as $\mu \to eee$, do not allow to put any stringent bound as all coefficients satisfy the constraints with $y_D$ as large as $\sim \mathcal{O}(1)$. The same is true for almost all the processes arising from two-fermion operators, which include the decay of muons and tau leptons into lighter leptons and photons, as well as the EDMs. The only exception are the coefficients of the operator $Q_{e\gamma}^{11}$, which is responsible for the EDM of the electron

$$\frac{C_{11}^{e\,\gamma}}{\Lambda_{\rm UV}^2} (l_i \sigma^{\mu\nu} e_j^c) H^\dagger F_{\mu\nu} \tag{5.6}$$

This bound on a $CP$ violating process is the most stringent: at $m^* = 10\,\text{TeV}$ it gives $\text{Im}C_{11}^{e\,\gamma} < 3.8 \times 10^{-10}$ [51].

In the absence of a symmetry suppressing the violation of CP,[11] one finds, in the case of $L \oplus E$, in which the operator is generated at two loops, that the bound translates into $\mathrm{Im}\left[(y_L)^2(y_{\widetilde{E}})^2\right] \lesssim 4.5\times 10^{-3}$, which is passed with $y_{L,E} \lesssim 0.26$. In the case of $L\oplus\widetilde{E}$, instead, the operator is generated at one loop, and the bound is more stringent: $y_L y_{\widetilde{E}} y_{L\,\widetilde{E}} \lesssim 2\times 10^{-7}$, which requires $y_L, y_{\widetilde{E}}, y_{L\,\widetilde{E}} \lesssim 6\times 10^{-3}$. Even if CP violation is suppressed, the first operator in table 8 poses the most stringent bound, from [53]

$$\mathrm{BR}\,(\mu \to e\gamma) < 3.1\times 10^{-13} \quad \text{at } 90\%\ \mathrm{CL}\,, \tag{5.7}$$

translating into $y_L, y_E \lesssim 0.7$ and $y_L, y_{\widetilde{E}}, y_{L\,\widetilde{E}} < 0.02$ for $L\oplus E$ and $L\oplus\widetilde{E}$, respectively.

The only question then becomes if these upper bounds on the Yukawa couplings are in conflict with the requirement that the D$\pi$s decay before BBN. In models featuring $L\oplus\widetilde{E}$, as discussed in section 5.2, the decays mediated by the scalar Dq are not chirally suppressed, and are thus quite fast. With the usual assumptions of $\Lambda_{\mathrm{DC}} \sim 100\,\mathrm{TeV}$, $m_\pi \sim 10\,\mathrm{TeV}$, $M_\phi \gtrsim \Lambda_{\mathrm{DC}}$, the D$\pi$s decay before BBN with yukawa couplings as small as $\sim 10^{-6}$. In models with $L\oplus E$, on the other hand, the decay is chiral suppressed and lifetimes may be larger. Indeed larger yukawa couplings of order $10^{-5}$ are required. In neither case does this simplified analysis lead to an evident exclusion. We leave a more detailed analysis on selected models to a future work.

## 6 Conclusions and outlook

In this work we studied extension of the SM with *dark color* sectors with special unitary gauge groups. The novelty with respect to previous literature on the subject is that we included both fermionic and scalar Dqs. The advantage of such dark sectors in general is that they constitute fundamental, UV complete theories in which all the desired properties of the DM — most importantly its stability on cosmological time-scales and its lack of color and hypercharge interactions — are a consequence of merely the quantum numbers of the fundamental degrees of freedom and the ensuing accidental symmetries of the theory. Indeed they naturally provide a plethora of states, the analog of the QCD hadrons, the lightest of which are stable as a consequence of the accidental symmetries of the theory. In this work we considered the lightest DBs as the DMC. Since the SM gauge interactions lift the masses of the charged DB it comes automatically that the DM is uncharged if the quantum numbers of the constituents allow for it. Yet one has a number of charged partners, some of which may be light; these states may be exploited to test the models in various experiments.

It is the ease with which one can break undesired accidental symmetries, as needed to avoid spurious stable states whose existence would conflict with observations, that drove us to include the scalars. Indeed we found many more models to be viable with respect to the previous literature. We postponed entirely the discussion of the hierarchy problem that one

[11] Since $C^{e\gamma}_{ij}$ depends on all six $y^i_L$,$y^i_E$, in order to have enough CP suppression one needs a symmetry breaking pattern allowing for the redefinition of six phases. The pattern in general will be

$$\mathrm{U}(1)^3_l \otimes \mathrm{U}(1)^{n_{\mathrm{sp}}+1}_D \to \mathrm{U}(1)_l \otimes \mathrm{U}(1)_{\mathrm{DB}}\,,$$

where $n_{\mathrm{sp}}$ is the number of fermionic Dq species, and the plus 1 is the species number of the scalar $\phi$. The total number of unphysical phases is then $n_{\mathrm{sp}}+2$, from which we find that models with four or more fermionic species are free from CP violation in the lepton sector at the two loop level.



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comes across when including fundamental scalars in the theory. The standard configuration we assumed is one in which the fermionic Dqs are lighter than the confinement scale, analogously to the lightest quarks in QCD, realizing a strongly coupled scenario. Cosmology in this case fixes the mass of the DM to be around a few hundreds of TeV. We also discussed, however, the possibility of shuffling the order of the masses to realize weakly-coupled scenarios.

We analyzed the models in the context of an SU(5) grand unification scheme. We followed a recent proposal by [17] and considered a *relaxed* criterion for unification: we consider as successfully unifying models in which the unification of the SM couplings is *better* than in the SM, yet possibly still imperfect. Practically speaking, rather than having the three couplings to match precisely at a certain energy scale, we accept models in which the area of the triangle drawn by the running in the energy scale-inverse coupling plane is smaller than in the SM. Overall, we find twenty-four models to be successfully unifying, to be compared with essentially a single model in previous literature. This is both thanks to a larger set of models as allowed by the existence of the scalar and thanks to this relaxed criterion.

In the section dedicated to phenomenology, we discussed aspects of the testability of the models. Given that many features are common with those discussed in the existing literature on accidental composite dark matter, we focused exclusively on how phenomenology is impacted by the presence of the scalar, with special attention to direct detection experiments and particle colliders. The most interesting features are provided by Yukawa couplings between the Dqs and the SM fermions mediated by the dark scalar. As far as collider phenomenology is concerned, they allow for the production and subsequent decay of dark pions directly to SM fermions. Another crucial consequence of these Yukawa interactions is the transfer of the SM accidental symmetries to the dark sector, with the result that some of the dark states possess both SM lepton and baryon numbers, making them LQs candidates: the extensive literature on LQs suggests several phenomenological application to be studied on specific models.

A feature of any of the models here discussed is the existence of *hybrid* dark states containing both fermions and scalars. It is beyond the scope of this work to determine the dynamics of the formation of these states and to assess the hierarchy between them and the "regular" dark states, which is challenging especially in strongly coupled scenarios. We have however argued that it is possible under some conditions that even the DMC may be of hybrid nature. This would lead to Higgs-portal interactions with nucleons to be constrained at DD experiments. Interestingly if these interactions are leading with respect to dipole/charge-radius interactions, hybrid DM is quite elusive of stringent DD bounds.

Hybrid sates with two Dqs (hybrid dark mesons) are fermionic in nature and mix with the SM fermions. The new Yukawa couplings that govern this mixing violate the SM symmetries in general. We discussed the case of models with Dq that carry lepton number, in which LFV and CP violating processes are mediated by these interactions, to translate the stringent bounds on the lepton sector into bounds on the parameters of the models. There are a large number of these parameters, which always enter in the observables in products of three or more, hence it is impossible at the time to constrain single parameters. However, under certain reasonable assumptions, it is possible to establish whether some models are already excluded by observations. A simplified analysis shows that this is not the case. It would be compelling to delve deeper and explore these phenomenological possibilities in future works.

## Acknowledgments

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## A Examples of model calculations

In this appendix we show the details of one model that passes the selection, one model whose viability is spoiled by dimension five operators peculiar to our setup, and one model that does not pass the selection. In all cases we consider only two light Dq species and make use of the Mathematica package LieART [54, 55] to compute the decomposition of the DF representation into SM representation. The procedure can be iterated to decompose the DF representation of models with more than two light Dq species.

### a) $\boldsymbol{Q \oplus \widetilde{D}}$

This model has two species and $N_{\rm DF} = 9$. Species numbers are broken thanks to the dark Yukawa couplings $y_Q Q^c \phi q$, $y_{\widetilde{D}} \widetilde{D} \phi^\dagger d^c$, and $y_{Q\widetilde{D}} Q^c H \widetilde{D}$. The Landau poles constraint gives $N_{\rm DC} \leq 4$. Let us consider $N_{\rm DC} = 3$. The DB multiplet to decompose is the representation **240** of $\mathrm{SU}(9)_{\rm DF}$, under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$:

$$\begin{aligned}\mathbf{240} = (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{2})_{\pm 1/2} \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{8},\mathbf{1})_{2\times 0,-1} \oplus (\mathbf{8},\mathbf{2})_{2\times -1/2,1/2} \\ 2(\mathbf{8},\mathbf{3})_0 \oplus (\mathbf{8},\mathbf{4})_{1/2} \oplus (\mathbf{10},\mathbf{1})_0 \oplus (\mathbf{10},\mathbf{2})_{\pm 1/2} \oplus (\mathbf{10},\mathbf{3})_0\end{aligned} \tag{A.1}$$

The fermionic DMC is the singlet or the neutral state of the triplet in $QQ\widetilde{D}$:

$$QQ\widetilde{D} \sim (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \tag{A.2}$$

Another valid candidate to include in the classification is the hybrid $QQ\widetilde{D}\phi$, obtained by attaching a scalar to the fermionic one and raising $N_{\rm DC}$ to 4.

### b) $\boldsymbol{E \oplus L}$

This model has $N_{\rm DF} = 3$ and it avoids the Landau poles for $N_{\rm DC} = 3, 4, 5$. All species numbers are broken by Yukawa terms $E^c \phi e^c + h.c$ and $L^c \phi l + h.c$, and the D$\pi$s are unstable. For $N_{\rm DC} = 3$, the DB multiplet to decompose under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ is:

$$\mathbf{8} = \mathbf{1}_0 \oplus \mathbf{2}_{\pm 3/2} \oplus \mathbf{3}_0 \tag{A.3}$$

The possible DMCs are

$$ELL \sim \mathbf{1}_0 \oplus \mathbf{3}_0 \tag{A.4}$$

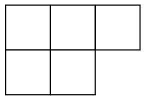
However, the dimension five operator

$$LH^\dagger E \phi \tag{A.5}$$

breaks the DB number explicitly for $N_{\rm DC} = 3$. Regarding hybrid DM candidates, given the bounds from the Landau poles, one finds the singlets $ELL\phi$ ($N_{\rm DC} = 4$) and $ELL\phi^2$ ($N_{\rm DC} = 5$): in these cases there is no dimension five operator that breaks the DB number.

**c) $D \oplus L$**

In this model we can build no Yukawa coupling with the Higgs, but the terms $L^c \phi l + h.c$ and $D^c \phi d^c + h.c$ break respectively $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_D$ species numbers, as well as the $\phi$ species number. The Landau poles constraint allows to build models up to $N_{\mathrm{DC}} = 9$, and a state with $Y = 0$ exists with $N_{\mathrm{DC}} = 5$, schematically $DDDLL$, which would be stable thanks to $\mathrm{U}(1)_{\mathrm{DB}}$. Nonetheless, this state does not belong to the lightest DB multiplet. In fact, for $N_{\mathrm{DC}} = 5$ the Young tableau for DF and spin is

, (A.6)

which is the representation $\mathbf{175'}$ of $\mathrm{SU}(5)_{\mathrm{DF}}$, whose decomposition under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ is

$$\begin{aligned}\mathbf{175'} = &(\mathbf{1},\mathbf{2})_{-5/2} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-5/3} \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{3})_{-5/3} \oplus \\ &(\mathbf{6},\mathbf{2})_{5/6} \oplus (\overline{\mathbf{6}},\mathbf{2})_{-5/6} \oplus (\mathbf{8},\mathbf{1})_{0} \oplus (\overline{\mathbf{6}},\mathbf{4})_{-5/6} \oplus (\mathbf{8},\mathbf{3})_{0} \oplus \\ &(\overline{\mathbf{10}},\mathbf{3})_{0} \oplus (\mathbf{15},\mathbf{1})_{5/3} \oplus (\overline{\mathbf{15}},\mathbf{2})_{5/6}\,.\end{aligned} \tag{A.7}$$

Since there is no state in the multiplet that is uncoloured and with $Y = 0$, there is no viable DMC.

## B Extending models

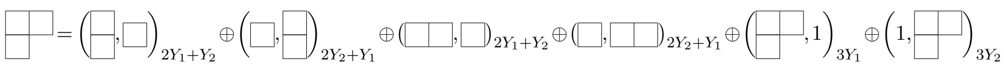
We can extend models starting from the observation of the following property of the decompositions. Let's take a model of the form:

$$\Psi = \Psi_1 \oplus \Psi_2 \tag{B.1}$$

following the method exposed in [10] the first DF decomposition is:

$$\mathrm{SU}(N_{\mathrm{DF}})_\Psi \longrightarrow \mathrm{SU}(N_1)_{\Psi_1} \otimes \mathrm{SU}(N_2)_{\Psi_2} \otimes \mathrm{U}(1)_X \tag{B.2}$$

where $\mathrm{SU}(N_{\mathrm{DF}})_\Psi$ is the DF group of the whole model, $\mathrm{SU}(N_q)_{\Psi_1}$ is the DF group of the species $\Psi_1$, $\mathrm{SU}(N_2)_{\Psi_2}$ is the DF group of the species $\Psi_2$, and $\mathrm{U}(1)_X$ is related to the hypercharge. We can represent this decomposition by means of Young tableaux:

 (B.3)

in the last two terms the same tableau appears that corresponds to the $\mathrm{SU}(N_{\mathrm{DC}})_\Psi$ representation, but this time for the $\Psi_1$ and $\Psi_2$ representations. This kind of terms always appear in the decompositions. This procedure can be iterated if we have more than 2 light Dq species: for example with three Dq representation

$$\Psi = \Psi_1 \oplus \Psi_2 \oplus \Psi_3 \tag{B.4}$$

we can group Dqs as $\Psi = \Psi_M \oplus \Psi_3$, where $\Psi_M = \Psi_1 \oplus \Psi_2$, and make the decomposition exposed above, then repeat the procedure for $\Psi_M$. Let's focus on models with an arbitrary number of Dq species of the form:

$$\Psi = \Psi_M \oplus \Psi_S \tag{B.5}$$

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where $\Psi_M$ is a minimal model and $\Psi_S$ is a spectator Dq, that is, a Dq that is not a constituent of the DMC of the model. We say that $\Psi$ is an extension of the minimal model $\Psi_M$. The last two terms in eq. (B.3) imply that we only need to study minimal models: if we add a spectator light Dq $\Psi_S$ (or a set of spectators Dq), we use the first step in order to decompose the total DF group into the product of the DF groups of the minimal model and the spectator Dqs:

$$\mathrm{SU}(N)_{\mathrm{DF}} \longrightarrow \mathrm{SU}(d_M)_M \otimes \mathrm{SU}(d_S)_S \otimes \mathrm{U}(1)_Y \tag{B.6}$$

where $d_M$ ($d_S$) is the dimension of the SM representation of $\Psi_M$ ($\Psi_S$). In the decomposition of representations of $\mathrm{SU}(d_M)_M \otimes \mathrm{SU}(d_S)_S$ we always get the right representation for $\mathrm{SU}(d_M)_M$ in order to get the DMC associated to the minimal model that respect the Fermi statistic as we saw in the example with the Young tableau.

It is also possible that the extended model is the extension of two different minimal model at the same time, in this case both candidates associated to the two different minimal models can be realized. Let us consider two minimal models, e.g. $Q \oplus \widetilde{D}$ and $\widetilde{D} \oplus \widetilde{U}$. What can we say about $Q \oplus \widetilde{D} \oplus \widetilde{U}$? Let us consider the two models separatly:

- $Q \oplus \widetilde{D}$:

  The DF decomposition for $N_{\mathrm{DC}} = 3$ made in the last section gives the result in eq. (A.1), with DMC in $QQ\widetilde{D} \sim (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0$. We will refer to the full SM decomposition of the dark flavor representation (the right hand side of eq. (A.1)) so obtained as $M_{Q\oplus\tilde{D}}$.

- $\widetilde{D} \oplus \widetilde{U}$:

  There is a candidate for $N_{\mathrm{DC}} = 3$. The first decomposition is $\mathrm{SU}(6)_{\widetilde{D}\oplus\widetilde{U}} \longrightarrow \mathrm{SU}(3)_{\widetilde{D}} \otimes \mathrm{SU}(3)_{\widetilde{U}} \otimes \mathrm{U}(1)_Y$. The second decomposition is trivial since the DF of $\widetilde{D}$ comes all from the color representation, and the same holds for $\widetilde{U}$. So the decomposition is:

$$\mathbf{70} = (\mathbf{1},\mathbf{1})_{0,1} \oplus (\mathbf{8},\mathbf{1})_{2\times 1,2\times 0,-1,2} \oplus (\mathbf{10},\mathbf{1})_{0,1} = M_{\widetilde{D}\oplus\widetilde{U}} \tag{B.7}$$

  the DMC is inside $\widetilde{D}\widetilde{D}\widetilde{U} = (\mathbf{1},\mathbf{1})_0$ multiplet.

Now in order to study the model $Q \oplus \widetilde{D} \oplus \widetilde{U}$ for $N_{\mathrm{DC}} = 3$ we make the first decomposition in two different ways: first we decompose $\mathrm{SU}(12)_{Q\oplus\widetilde{D}\oplus\widetilde{U}} \longrightarrow \mathrm{SU}(9)_{Q\oplus\widetilde{D}} \otimes \mathrm{SU}(3)_{\widetilde{U}} \otimes \mathrm{U}(1)_Y$, so (omitting the hypercharge)

$$\mathbf{572} = (\mathbf{1},\mathbf{8}) \oplus (\mathbf{9},\bar{\mathbf{3}}) \oplus (\mathbf{9},\mathbf{6}) \oplus (\mathbf{36},\mathbf{3}) \oplus (\mathbf{45},\mathbf{3}) \oplus (\mathbf{240},\mathbf{1}) \tag{B.8}$$

The $(\mathbf{240},\mathbf{1})$ is the one we decomposed in the minimal model $Q \oplus \widetilde{D}$. Second we make the decomposition is $\mathrm{SU}(12)_{Q\oplus\widetilde{D}\oplus\widetilde{U}} \longrightarrow \mathrm{SU}(6)_Q \otimes \mathrm{SU}(6)_{\widetilde{D}\oplus\widetilde{U}} \otimes \mathrm{U}(1)_Y$. This must lead to the same result because it cannot depend on the way we factorize the decomposition. So the decomposition of the flavor representation in SM is:

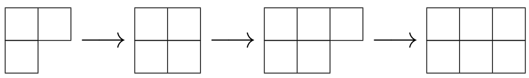
$$\mathbf{572} = (\mathbf{6},\mathbf{15}) \oplus (\mathbf{15},\mathbf{6}) \oplus (\mathbf{21},\mathbf{6}) \oplus (\mathbf{6},\mathbf{21}) \oplus (\mathbf{70},\mathbf{1}) \oplus (\mathbf{1},\mathbf{70}) \tag{B.9}$$

the $(\mathbf{1},\mathbf{70})$ is the representation we decomposed before in the $\widetilde{D} \oplus \widetilde{U}$ minimal model. This means that both candidates need to appear in the model $Q \oplus \widetilde{D} \oplus \widetilde{U}$. To check let us consider the decomposition of the DF representation for $Q \oplus \widetilde{D} \oplus \widetilde{U}$ with $N_{\rm DC} = 3$ [10]:

$$\mathbf{572} = M_{Q\oplus\widetilde{D}} \oplus (\mathbf{1},\mathbf{1})_{0,2\times 1} \oplus (\mathbf{1},\mathbf{2})_{2\times\frac{1}{2},\frac{3}{2}} \oplus (\mathbf{1},\mathbf{3})_{1} \oplus (\mathbf{8},\mathbf{1})_{2\times 0,4\times 1,2} \oplus \\ \oplus (\mathbf{10},\mathbf{1})_{0,2\times 1} \oplus (\mathbf{8},\mathbf{2})_{4\times\frac{1}{2},2\times\frac{3}{2}} \oplus (\mathbf{10},\mathbf{2})_{2\times\frac{1}{2},\frac{3}{2}} \oplus 2\times(\mathbf{8},\mathbf{3})_{1} \oplus (\mathbf{10},\mathbf{3})_{1}\,. \tag{B.10}$$

Both $M_{Q\oplus\widetilde{D}}$ and $M_{\widetilde{D}\oplus\widetilde{U}}$ are contained in the multiplet. It would seem that one $(\mathbf{8},\mathbf{1})_{-1}$ representation, contained in $M_{\widetilde{D}\oplus\widetilde{U}}$ is missing, but it is actually already included in $M_{Q\oplus\widetilde{D}}$. Indeed it is a $\widetilde{D}\widetilde{D}\widetilde{D}$ DB, which is common to both models.

Another way to extend minimal models is to add the Dq $N$ and increase $N_{\rm DC}$ by one. With this procedure we can build a minimal model starting from another minimal model. For example starting from a model $Q \oplus \widetilde{D}$ for $N_{\rm DC} = 3$ we can build a model $Q \oplus \widetilde{D} \oplus N$ and $N_{\rm DC} = 4$. The former model has $QQ\widetilde{D}$ as DMC, while the latter has $QQ\widetilde{D}N$. Of course one needs to make sure that this new model is not discarded because of dimension five operators that break the DB number or because it produces subplanckian Landau poles. Note that this procedure cannot be iterated arbitrarily: for example iterating three times this procedure starting from the DF representation of the minimal model $Q \oplus \widetilde{D}$ we have in terms of Young tableaux:



(B.11)

the last tableau with $N_{\rm DC} = 6$ is associated to a DF representation that, once we decompose it into SM representation, do not include any DMC of the form $QQ\widetilde{D}NNN$: this is because in the last tableau two anti-symmetrized boxes have to correspond necessarily to two $N$ Dqs, which cannot, however, be antisymmetrized.

## C Full list of viable models

In table 9 and table 10, we show all the models that we found to be viable, with the exclusion of those models in which $N$ is light. Remember that exchange each fermion with its tilded version

$$\Psi \leftrightarrow \widetilde{\Psi}$$

results in another viable model with the same features. The DMC is taken to be the least charged DB with smallest spin.

Generally, one can extend these models with singlet fermions $N$ if one makes sure that the DB number remains unbroken. Essentially this boils down to excluding $N_{\rm DC} = 3$, which is spoiled by the dimension five operators of eq. (3.2). Unless the Dq $L$ is light, the $N$ species number is only broken at dimension five leading to metastable mesons.

| SU(5) | Model | $N_{\rm DC}$ | Dark Matter Candidate |
|---|---|---|---|
| $\bar{\mathbf{5}} \oplus \mathbf{5}$ | $D \oplus \tilde{D}$ | $4 \leq N_{\rm DC} \leq 7$ | $D\tilde{D}\phi^{N_{\rm DC}-2}$, $D\tilde{D}D\tilde{D}\phi^{N_{\rm DC}-4}$, $D\tilde{D}D\tilde{D}D\tilde{D}\phi^{N_{\rm DC}-6}$ |
| | $D \oplus \tilde{D} \oplus L$ | $4 \leq N_{\rm DC} \leq 7$ | $D\tilde{D}\phi^{N_{\rm DC}-2}$, $D\tilde{D}D\tilde{D}\phi^{N_{\rm DC}-4}$, $D\tilde{D}D\tilde{D}D\tilde{D}\phi^{N_{\rm DC}-6}$ |
| | $L \oplus \tilde{L}$ | $4 \leq N_{\rm DC} \leq 7$ | $L\tilde{L}\phi^{N_{\rm DC}-2}$, $L\tilde{L}L\tilde{L}\phi^{N_{\rm DC}-4}$, $L\tilde{L}L\tilde{L}L\tilde{L}\phi^{N_{\rm DC}-6}$ |
| | $L \oplus \tilde{L} \oplus D$ | $4 \leq N_{\rm DC} \leq 6$ | $L\tilde{L}\phi^{N_{\rm DC}-2}$, $L\tilde{L}L\tilde{L}\phi^{N_{\rm DC}-4}$, $L\tilde{L}L\tilde{L}L\tilde{L}$ |
| | $D \oplus \tilde{D} \oplus L \oplus \tilde{L}$ | 4 | $D\tilde{D}\phi^2$, $L\tilde{L}\phi^2$, $D\tilde{D}D\tilde{D}$, $L\tilde{L}L\tilde{L}$, $D\tilde{D}L\tilde{L}$ |
| $\bar{\mathbf{5}} \oplus \mathbf{10}$ | $D \oplus U$ | $\leq 4$ | $DDU$, $DDU\phi$ |
| | $D \oplus U \oplus L$ | 3 | $DDU$ |
| | $D \oplus U \oplus E$ | 3 | $DDU$ |
| | $L \oplus E$ | $\leq 5$ | $LLE\phi$, $LLE\phi^2$ |
| | $L \oplus E \oplus D$ | 4 | $LLE\phi$ |
| $\mathbf{5} \oplus \mathbf{10}$ | $Q \oplus \tilde{D}$ | $\leq 4$ | $QQ\tilde{D}$, $QQ\tilde{D}\phi$ |
| | $Q \oplus \tilde{D} \oplus \tilde{L}$ | 3 | $QQ\tilde{D}$ |
| | $Q \oplus \tilde{D} \oplus E$ | $\leq 4$ | $QQ\tilde{D}$, $QQ\tilde{D}\phi$ |
| | $Q \oplus \tilde{D} \oplus \tilde{L} \oplus E$ | 3 | $QQ\tilde{D}$ |
| | $\tilde{D} \oplus E \oplus U$ | 3 | $\tilde{D}EU$ |

**Table 9.** Complete list of the viable models according to the criterion of section 2.3 with the exclusion of the models in which $N$ is light. It is worth stressing that this list is valid in the strongly coupled scenario. In the first column we show the SU(5) representation to which the light Dqs (second column) belong. In the third column we show the allowed values for $N_{\rm DC}$ and in the last column the potential DMCs for all the possibilities. Continues in table 10.

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| SU(5) | Model | $N_{\mathrm{DC}}$ | DMC |
|---|---|---|---|
| $\mathbf{10} \oplus \overline{\mathbf{10}}$ | $E \oplus \widetilde{E}$ | 4 | $E\widetilde{E}E\widetilde{E}$, $E\widetilde{E}\phi^2$ |
| $\mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{10}$ | $D \oplus \widetilde{D} \oplus U$ | 4 | $D\widetilde{D}D\widetilde{D}$,$D\widetilde{D}\phi^2$, $DDU\phi$ |
| | $D \oplus \widetilde{D} \oplus E$ | 4 | $D\widetilde{D}D\widetilde{D}$,$D\widetilde{D}\phi^2$ |
| | $L \oplus E \oplus \widetilde{D}$ | 4 | $LLE\phi$, $LLE\phi^2$ |
| | $L \oplus \widetilde{L} \oplus E$ | 4 | $LLE\phi$, $L\widetilde{L}L\widetilde{L}$, $L\widetilde{L}\phi^2$ |
| | $Q \oplus \widetilde{D} \oplus L$ | 3 | $QQ\widetilde{D}$ |
| $\mathbf{5} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$ | $Q \oplus \widetilde{D} \oplus \widetilde{U}$ | 3 | $QQ\widetilde{D}$, $\widetilde{D}\widetilde{D}\widetilde{U}$ |
| | $\widetilde{D} \oplus \widetilde{U} \oplus \widetilde{L} \oplus Q$ | 3 | $QQ\widetilde{D}$, $\widetilde{D}\widetilde{D}\widetilde{U}$ |
| | $Q \oplus \widetilde{D} \oplus \widetilde{E}$ | 3 | $QQ\widetilde{D}$ |
| | $\widetilde{D} \oplus \widetilde{U} \oplus E$ | 3 | $\widetilde{D}\widetilde{D}\widetilde{U}$ |
| $\bar{\mathbf{5}} \oplus \overline{\mathbf{10}} \oplus \mathbf{5} \oplus \mathbf{10}$ | $Q \oplus \widetilde{D} \oplus L \oplus \widetilde{U}$ | 3 | $QQ\widetilde{D}$, $\widetilde{D}\widetilde{D}\widetilde{U}$ |
| | $Q \oplus \widetilde{D} \oplus L \oplus \widetilde{E}$ | 3 | $QQ\widetilde{D}$ |

**Table 10.** Continuation of table 9.



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