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Who can benefit from multi-license oil concessionaires valuation?

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ABSTRACT

In this paper, we study the interactions between a public body and several potential concession holders. We propose a dynamic stochastic optimization problem providing the optimal exercise price of the "expropriation" option, which safeguards the social interest from over-exploitation of the resource. Other crucial quantities are also determined, such as the optimal value of the option and the condition for mutual convenience to enter the deal. We apply the model to a Southern Italy oilfield finding that the deal was in the common interest and that the optimal expropriation value is quite high.

1. Introduction

Hydrocarbon extraction is still a major source of energy supply for most countries in the world, despite pledges to progressively reduce these operations, to meet the needs of the energy transition.

The history of modern-day oil extraction dates back to the mid-19th century. Since then, such activities have had a strong impact on the economies of both producing and importing countries. A crucial role would apparently be played by the oil price, although there is no unanimous consensus in the literature on its impact on macroeconomic quantities, such as GDP, see e.g. Kilian (2008) and Kallis and Sager (2017). All the natural resources belong to the government, resulting in a potential asset for the economic and social development of the country and prompting the need to regulate the implementation of projects for the exploration, development, and sale of hydrocarbons.

Licensing the exploitation of fossil natural resources is currently a hot topic for both academics and policy-makers. The latter are faced with the need to mediate between social interests, environment protection, and revenue management of private operations. From a practical perspective, the government is required to define the conditions and who should be allocated the licenses for the development and exploitation of natural resources. The allotment is accomplished by stipulating proper agreements with third parties, see e.g. D'Alpaos and Moretto (2013), D'Alpaos et al. (2006), Fan and Zhu (2010), Jin et al. (2021), Monjas-Barroso and Balibrea-Iniesta (2013), Randall et al. (1989), Pindyck (1984), Saito et al. (2001), Scandizzo and Ventura (2010) and Wang and Pallis (2014). Based on the pillars of public international law, there exist three contractual systems regulating the agreements between the parties and granting exclusive rights for a specific area and for a given period of time. Such types are a *concession* or *license agreement,* a *joint venture,* and a *production sharing agreement* (PSA).

In the present paper, we focus on the first kind of agreement, namely oil licenses. Licenses and concessions are two terms used interchangeably to refer to partnerships between the public sector and private companies. According to the definition of the EU Commission,[2] concession contracts are used by public authorities to deliver services or construct infrastructures. Concessions involve a contractual arrangement between a public authority and at least one economic operator, i.e. the concession holders. The latter provide services or carry out works and are remunerated by being permitted to exploit state properties. The bidder pays the bid price and burdens himself with both current operating and capital investment costs, as well as production taxes. The latter are established *a priori,* and are paid regardless of the actual activation of the extraction process. When production starts,

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[2] EU definition of concession contracts.

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the bidder further pays an exercise tax, according to the production volumes.

Concessions are a particularly attractive way of carrying out projects in the public interest when central or local authorities need to mobilize private capital and know-how to supplement scarce public resources. Studying concession contracts and the allotment of licenses is a crucial issue given the broad diffusion of such agreements, especially in the public utility sector, and also given the high amount of money at stake. For this reason, in 2014 the EU Commission regulated the award of concessions by means of Directive2014/23 forcing the member states to adopt this directive into their national legislation by April 2016.

For the exploitation of natural resources, such as crude oil, the economic literature claims that the agents are required to choose whether to start, continue, or abandon the development project, depending on the circumstances prevailing at various stages, such as the exploitation of new drilling and production technologies, see e.g. Mauritzen (2017). In our work, we model the starting, or entry stage. The right, but not the obligation, to decide which direction to take means that the agents may decide whether to invest now, take preliminary stages while reserving the right to invest in the future, or even do nothing. Since each of these choices is contingent upon the state of the world and can be postponed all management decisions can be thought of in terms of options, as in the financial markets. The uncertainty surrounding future payoffs invariably affects the expected payoff from the investment, see e.g. Ampomah et al. (2017), Cerqueti and Ventura (2013) and Dai et al. (2020).

In practice, providing a model that perfectly describes all the variables involved in the whole process is demanding. The related literature extensively uses options on the market for goods and services, the so-called *real options.* The use of real options to model natural resource development problems dates back to the mid-1980s, see e.g. Brennan and Schwartz (1985), Dixit and Pindyck (1994), Randall et al. (1989) and Schwartz and Trigeorgis (2001). The real options theory has been extensively applied to derive models for oil development problems, see e.g. Chorn and Croft (2000), Laughton (1998), Paddock et al. (1988), Smith and McCardle (1996, 1999) and Tang et al. (2017).

The salient features of this kind of derivative can be summarized under the following three headings: *(i)* the irreversibility component of the investment considered, either physical or economic irreversibility or both; *(ii)* the uncertainty surrounding the expected returns from the investment; *(iii)* the term structure of the contract (concession in our case), i.e. the expiration date. This makes real options the most suitable tool to fairly evaluate oil licenses.

Within such a framework, three types of actors play a crucial role: a public authority, commonly referred to as a *government,* which must choose whether or not to assign the possible development project of the natural resource, n private companies i.e. the would-be operators or potential license holders and a state-owned company the so-called National Oil Company (NOC), as in the case of the Kuwait Oil Company in Kuwait, Saudi Aramco in Saudi Arabia, or ENI in Italy up to 1992.[3] The issue of valuing the *buyout clause,* i.e., the possibility for the public partner to redeem a concession from the private partner before the end of the concession period should be solved before signing the concession agreement, as this clause has to be included in the concession agreement, see e.g. Caselli et al. (2009).

The licensing system entails that all the risks associated with the project must be assumed by the bidder. Consequently, the potential concessionaire may decide to offer a price for the license lower than the actual one, resulting in a potential reduction in the government's revenue. Therefore, it is essential to be able to uniquely establish a methodology to determine the optimal price of the concession. The problem of optimally engaging in concession contracts has been examined from the government's point of view by Scandizzo and Ventura (2010) and Cerqueti and Ventura (2020). In particular, the latter highlight that the public agent has to choose between *(a)* preserving the resource as it is, i.e. not assigning any license; *(b)* assigning exclusive rights to the NOC; *(c)* assigning licenses to the $n + 1$ companies, the private n plus the NOC.

In the present paper, we are focused on deriving lower bounds of optimal cash flow to enter the project, within a continuous-time framework, when the government decides to award oil concessions to n private concession holders. Our work would like to enrich the extant literature in three directions. From a financial standpoint, we reconcile the real-option framework and standard dynamic programming techniques to obtain closed-form formulae featuring the leading variables of the problem, namely the optimal value of the option for the government to revoke the license and the optimal amount of royalties the concessionaires are required to pay to compensate for the land exploitation. From a decision-making perspective, we offer a blueprint for shaping appropriate public asset management policies, also within the realm of contract renegotiation. From a numerical point of view, we test our proposal on real data, covering a specific case study. To the best of our knowledge, a full approach bringing together all of the above features has not been analyzed yet. The results we obtain are relevant to the literature as, in spite of its apparent complexity, the model is computable and can be applied to real-world cases so as to help the stakeholders find an agreement based on the objective features of the deal. In addition, the model is sufficiently general to be applied to any concession contract, not necessarily pertaining to oilfields. A second leverage point of the model consists in the fact that running the business additional information is released over time and it can be conveniently used to precisely estimate the sensitivity parameters. Renegotiation of the contracts is quite a common event (see Guasch et al., 2008) and the estimates are a valid input to tailor the new covenants for each stakeholder according to the sensitivity parameters. For instance, it is possible to determine new royalties close to the willingness to pay and to be compensated. This result stems from the study of the behavior of some quantities, e.g. the royalties, with respect to the crucial parameters, such as volatility and time to maturity. Finally, we apply the model to *Tempa Rossa*, an oilfield located in the South of Italy, showing how to implement the model and how to exploit information unfolded over time. Even though *Tempa Rossa* does not perfectly fit the stylized situation depicted in the model, it offers a solid basis to root the theoretical findings into a real-world case.

Establishing the best approach for deciding whether to assign the task of cultivating hydrocarbons is a key issue. To this end, Cillari et al. (2021) provide an overview and discuss a comparison among the different procedures implemented by some developing countries, stressing the relevance of benchmarking against the so-called *best practices.* As a matter of fact, a profitable licensing mechanism should settle good management and ensure benefits accrue to the country. First, we should distinguish between experienced and inexperienced countries. For example, Egypt belongs to the first category, where the government devotes special attention to updating policies, and licenses are awarded by public auction through NOCs. Therefore, we witness high participation of companies, and consequently good performance on economic returns. Within the Mediterranean area, Tunisia can also be qualified as an expert country, whose activities are supported by non-governmental organizations (NOGs) and NOC. The latter handles license allotment to private companies through a direct allocation mechanism of *beauty contest*-type. This resulted in medium private participation, which led to an inability to maximize profits, also due to the geo-morphology of the oil basins. A different picture applies to countries that have recently discovered to possess similar natural resources. Although necessarily inexperienced, these countries displayed different approaches. For example, Sri Lankan licenses are awarded by the government, with minimal support from the NOC. There are disappointing results from the company participation standpoint, resulting in potential extra profit

[3] The Italian Government currently holds about 30% of the shares.

losses. On the other hand, Jamaica has no NOC. Licensing is directly conducted by the government on a *First-In-First-Out* basis to a single company. Consequently, the preliminary exploration phase is ineffective, causing the loss of possible exploration benefits. A hybrid strategy is implemented by Mozambique. The latter is a relatively inexperienced country, which requires the involvement of external advisory bodies and does not have a proper NOC.

Based on these insights, it is impossible to establish a clear-cut policy decision: the latter depends on the availability or scarcity of resources, the government's experience, the technological/economic capabilities of the companies, the geo-morphological knowledge of the territory, and the NOC's expertise. Cillari et al. (2021) argue that it would be sound to adopt the public auctions mechanism to assign one or more blocks to different lessors, to increase competition, attract a large number of potential investors, and magnify the economic benefits of resource exploitation. Therefore, a model capable of accommodating all the above aspects is required to succeed in maximizing the returns from resource exploitation and minimizing the opportunity costs associated with sub-optimal strategies. Our proposal works in this same vein, with strong attention to the financial aspects of the assessment of possible exit strategies. We provide a first attempt within a stylized context; nevertheless, we include all relevant factors that may affect or influence the policy assessment.

The paper is organized as follows. Section 2 presents the model, while Sections 3 and 4 solve for the optimal values of the expropriation option and the feasibility condition, respectively. Subsequently, Section 5 applies the theoretical model to a real-world case study and Section 6 draws some concluding remarks.

2. The model

Throughout the paper, we will consider a filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\right)$, where $\{\mathcal{F}_t\}_{t\geq 0}$ is a complete and right continuous filtration. Let $\{X(t)\}_{t\geq 0}$ be the payoff, e.g. the cash flow generated by the project, whose dynamics are given by

$$dX(t) = \alpha X(t)dt + \sigma X(t)dW(t),\ t > 0\ , \tag{2.1}$$

where, with a slight abuse of notation for the sake of readability, we set $X(0) := X > 0$ as the current value of the cash flow, used as a basis to estimate the expected present value of the process under all alternative hypotheses, see e.g. Dixit and Pindyck (1994). The process $W = \{W(t)\}_{t\geq 0}$ appearing in (2.1) is a Wiener process, while $\alpha > 0$ and $\sigma > 0$ are the drift and the instantaneous volatility rate of the cash flow, respectively.

We notice that Eq. (2.1) entails a log-normal distribution of the cash flow, whose expected value is easily obtained as

$$\mathbb{E}[X(t)] = Xe^{\alpha t}.$$

Thus, the expected present discounted value is obtained by discounting at rate ρ, yielding

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}X(t)dt\right] = \int_0^\infty e^{-\rho t}\mathbb{E}\left[X(t)\right]dt = \int_0^\infty e^{-\rho t}Xe^{-(\rho-\alpha)s}ds = \frac{X}{\rho-\alpha}\ ,$$

with $\rho > \alpha$. We set $\delta := \rho - \alpha$, where δ represents the so-called implicit *oil convenience yield,* namely the interest rate, denominated in barrels of oil, for borrowing a single barrel of oil, measuring the value of storing crude oil over the borrowing period.

We aim to investigate the issue of optimally managing a natural resource (in this case, the hydrocarbon extraction) owned by a government, in terms of assessing costs and investment opportunities. We will assume, throughout the work, that any investment has irreversible consequences. As far as costs are concerned, first, a feasibility study must be accomplished. On the chance that such a study provides the desired aims, the development phase might be carried out. Broadly speaking, the government may choose either not to pursue the exploitation of the natural resource postponing the launch of the operation, or to bear the costs of the project development, or, finally, to grant the license to one or more private companies.

Here, we will focus on the economic implications of the project development, and we consider the case in which the Government chooses to assign licenses to n private companies and one NOC. We denote by $T < +\infty$ the expiry date of oil concessions. The net cash flow of the project consists of a systematic term, generally normalized to unity, and a stochastic term, described through the process $\{X(t)\}_{t\in[0,T]}$, whose dynamics are described in Eq. (2.1). The payoff for the ith concessionaire, $i = 1,\dots,n$, is

$$C_i(n+1) = \mathbb{E}\left[\int_0^T \gamma_i X(s)e^{-\rho s}ds\right] - I_i - P_i^m - G_i\left(X^*\right)\ , \tag{2.2}$$

with $C_i(n+1) \geq 0$, for $i = 1,\dots,n$, where $\gamma_i \in (0,1)$ is the share of the expected cash flow accruing to the ith concessionaire up to maturity T, I_i is the sunk cost[4] of the investments borne by the ith concessionaire, P_i^m is the price paid by the ith company to the government, typically referred to as *royalties,* and $G_i\left(X^*\right)$ is the option in the hand of the government to undertake some actions against the private company to safeguard local interests. Specifically, such an option is a function of

$$X^* := \underset{X>0}{argsup}\left\{G_i(X)\right\},$$

representing the (optimal) entry threshold, i.e. the *exercise price.* This option plays a crucial role in many concession contracts and arises from the conflicting interests between the public body, which must trade public and private interests off, and the private party, which aims at maximizing its expected net discounted value of the project. For instance, it prevents the private party from over-exploiting the site or polluting it or the air. In other words, this option captures the *buyout clause*. It is worth stressing that we deal with a *real* option, that gives the owner (i.e., the government) the right to exercise according to some conditions about the concessionaires, that is according to the concessionaires' behavior. We will define later in detail what these conditions are, but for the time being it is important to stress that the option acts as a threat to the concessionaires should they violate the signed agreement, making *excess profits.* In turn, this implies $G_i(X)$ to be an increasing function of X, i.e. $G_i'(X) > 0$, for $i = 1,\dots,n.$

To make a possible deal, the expected value of the project accruing to the private party must be not less than the value expected by the government, otherwise, the latter would not enter into the deal. Thus, we can formalize this condition as:

$$\sum_{i=i}^{n} C_i(n+1) \geq B(n+1)\ , \tag{2.3}$$

where $B(n+1) \geq 0$ in the RHS of (2.3) is the expected value of the project for the government. The latter is given by

$$B(n+1) = \mathbb{E}\left[\int_0^T \gamma X(s)e^{-\rho s}ds\right] + \mathbb{E}\left[\int_T^{+\infty} X(s)e^{-\rho s}ds\right] + F\left(X^*\right) + P - I\ , \tag{2.4}$$

where $\gamma > 0$ with $\gamma + \sum_{i=1}^n \gamma_i = 1$ is the share of the expected cash flow accruing to the NOC. Therefore, the first expected value term is the cash flow accruing to the NOC, i.e. the government. Such a payoff is in force only over the concession period $[0,T]$. After T, the n licenses expire and the entire project is taken over by the government. Such a second phase of the deal is captured by the second term in (2.4). Moreover, P represents the revenues from the prices paid by the n private firms,

[4] We recall that the sunk cost for oil projects is defined as the initial capital outlays associated with starting an investment project, such as seismic exploration, early experimental costs, and investigation and assessment costs. See e.g. Li et al. (2021) for further details.

I is the sunk cost of investments borne by the NOC, and F captures the option to undertake some action against the private companies to safeguard local interests, with a reference to the entry threshold. Notice that F and G represent the same option in the hands of the government. Loosely speaking, we refer to the option to expropriate or somehow interrupt the project, and we have

$$F(X^*) = \sum_{i=1}^{n} G_i(X^*)\,, \tag{2.5}$$

where $X = \underset{X>0}{argsup}\{F(X)\}$.

One could look at these options as implicitly bought (resp., sold) by the government (resp., the concessionaires) when signing the agreement. To make the model as general as possible, we use different notation, namely, we either indicate the option by F when the government assigns its value to the option and by G when the concession holders evaluate the option, as in (2.2). Notice also that, when a natural resource is exploited, its public flow of amenities, i.e., its socio–economic value, is lost forever, entailing an indirect cost for society. It is of primarily importance to recall that externalities, such as environmental impacts and other social considerations, may be relevant in the context of extraction. In terms of the model, such quantities can be considered as components of I, therefore, including direct and indirect development costs.

Once the framework has been defined, we focus on the interaction between a government and n would-be private agents. Hence, we prove the following

Proposition 2.1. *Let $B(n+1)$ be the expected value of the development project for the government. Assume there are $n \geq 1$ private concessionaires and denote by I_i (resp., P_i^m) the sunk cost of the investment borne (resp., the price paid) by the ith company to the government, for $i = 1, \ldots, n$.*

Then, the expected value of the project for the government must not exceed the opportunity cost, namely,

$$B(n+1) < \sum_{i=1}^{n} \int_0^T \gamma_i \mathbb{E}\left[X(s)\right] e^{-\rho s} ds - \sum_{i=1}^{n} I_i - \sum_{i=1}^{n} P_i^m\,. \tag{2.6}$$

Proof. To study the possible interactions between the parties, and more specifically between a government and n would-be private agents, we have ruled out situations in which no potential concessionaires are willing to enter the deal. This is easily captured by the condition $C_i(n+1) \geq 0$, for $i = 1, \ldots, n$. Exploiting such a non-negativity constraint, we can rewrite (2.2) as:

$$G_i\left(X^*\right) \leq \mathbb{E}\left[\int_0^T \gamma_i X(s) e^{-\rho s} ds\right] - I_i - P_i^m\,, \tag{2.7}$$

for $i = 1, \ldots, n$. At the same time, this condition makes clear that the RHS in (2.7) represents the upper bound to the real option value, namely, as long as the value of the threat is not too high, i.e. (2.7) applies, the ith company has the incentive to be in the deal. We observe that we can substitute (2.2) in (2.3), giving rise to

$$\sum_{i=1}^{n} \left(\mathbb{E}\left(\int_0^T \gamma_i X(s) e^{-\rho s} ds\right) - I_i - P_i^m - G_i\left(X^*\right)\right) \geq B(n+1)\,, \tag{2.8}$$

or, equivalently,

$$0 < \sum_{i=1}^{n} G_i\left(X^*\right) \leq \sum_{i=1}^{n} \mathbb{E}\left[\int_0^T \gamma_i X(s) e^{-\rho s} ds\right] - \sum_{i=1}^{n} I_i - \sum_{i=1}^{n} P_i^m - B(n+1)\,. \tag{2.9}$$

Fubini's theorem and Eq. (2.9) imply that

$$B(n+1) < \sum_{i=1}^{n} \int_0^T \gamma_i \mathbb{E}\left[X(s)\right] e^{-\rho s} ds - \sum_{i=1}^{n} I_i - \sum_{i=1}^{n} P_i^m\,. \tag{2.10}$$

The proof is now completed. □

Inequality (2.6) is a meaningful expression, as it represents the upper bound to the governments' value of the project. Interestingly, it can be regarded as an *opportunity cost* given by the (net) forgone benefits from assigning n licenses to private companies, instead of assigning the entire project to the NOC, which is, substantially, as developing the resource on its own. Indeed, the first term in (2.6), $\sum_{i=1}^{n} \int_0^T \gamma_i \mathbb{E}\left[X(s)\right] e^{-\rho s} ds$, represents the forgone benefit. In contrast, the remaining two terms, namely $\sum_{i=1}^{n} I_i$ and $\sum_{i=1}^{n} P_i^m$, are the costs to be incurred by the license holders that partially compensate the government for the foregone benefit. At first sight, it may be puzzling that the feasibility condition of the deal depends upon an upper bound to the government's value of the project. At least it seems an opportunistic condition on the part of the private party. However, this is a consequence of the buyout covenant. On the one hand, the private party has the incentive not to be expropriated, i.e. to keep G as small as possible. On the other hand, it is in the public interest to avoid excess natural resource exploitation and keep G as small as possible, for $i = 1, \ldots, n$. Therefore, the feasibility condition reflects this common interest in terms of the upper bound.

3. The optimal value of real options

The value $G_i(X^*)$, for $i = 1, \ldots, n$, can be optimally determined from (2.7) by studying the condition of indifference between the value of the option and its (net) expected value from exercising. To this aim, by exploiting the properties of X as a geometric Brownian motion, along the lines of Dixit and Pindyck (1994), we can write the condition of indifference for the ith private agent, with $i = 1, \ldots, n$, as:

$$G_i(X) = \int_0^T \gamma_i \mathbb{E}\left[X(s)\right] e^{-\rho s} ds - I_i - P_i^m = \gamma_i \int_0^T X e^{-\delta s} ds - I_i - P_i^m\,,$$

with $\delta = \rho - \alpha$. Thus, the exercise price, for $i = 1, \ldots, n$, is

$$\begin{aligned} G_i(X) &= -\frac{\gamma_i X}{\delta} \int_0^T -\delta e^{-\delta s} ds - I_i - P_i^m = -\frac{\gamma_i X}{\delta}\left(e^{-\delta T} - 1\right) - I_i - P_i^m \\ &= \frac{\gamma_i X}{\delta}\left(1 - e^{-\delta T}\right) - I_i - P_i^m\,. \end{aligned} \tag{3.1}$$

Eq. (3.1) provides the value matching, since it matches the value of the unknown function G_i to the expected value of the investment from granting the ith license, for $i = 1, \ldots, n$.

On the other hand, the no-arbitrage condition guarantees that

$$\mathbb{E}[dG_i(X)] = \rho G_i(X)\, dt\,, \tag{3.2}$$

where, for $i = 1, \ldots, n$,

$$\begin{aligned} dG_i(X) &= G_i'(X) dX(t) + \frac{1}{2} G_i''(X) d\langle X(t), X(t)\rangle \\ &= G_i'(X)\left(\alpha X(t) dt + \sigma X(t) dW(t)\right) + \frac{\sigma^2}{2} G_i''(X) X^2(t) dt \\ &= \left(\alpha G_i'(X) X(t) + \frac{\sigma^2}{2} G_i''(X) X^2(t)\right) dt + \sigma X(t) dW(t)\,. \end{aligned} \tag{3.3}$$

Eq. (3.2) and the Itô formula (3.3) imply that

$$\alpha G_i'(X)\mathbb{E}[X] + \frac{\sigma^2}{2} G_i''(X)\mathbb{E}[X^2] = \rho G_i(X), \;\; \text{for } i = 1, \ldots, n\,. \tag{3.4}$$

We notice that $\mathbb{E}[X] = X$ and $\mathbb{E}\left[X^2\right] = X^2$, hence Eq. (3.4) is a homogeneous second-order Ordinary Differential Equation (ODE), which can be solved by exploiting standard techniques. The general solution to (3.4) is of the form

$$G_i(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2}, \;\; \text{for } i = 1, \ldots, n\,, \tag{3.5}$$

where β_1 and β_2 are the solutions to the associated characteristic function

$$\alpha\beta + \frac{\sigma^2}{2}\beta(\beta - 1) - \rho = 0.$$

In particular, we have

$$\beta_1 = \frac{-\alpha + \frac{\sigma^2}{2} + \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2\rho\sigma^2}}{\sigma^2} > 1\,, \tag{3.6}$$

$$\beta_2 = \frac{-\alpha + \frac{\sigma^2}{2} - \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2\rho\sigma^2}}{\sigma^2} < 0\,. \tag{3.7}$$

To obtain the constants A_1 and A_2, we exploit the boundary condition: when the underlying is zero, the option must be worthless for each company, i.e. $G_i(0) = 0$ for $i = 1,\dots,n$. As a consequence, $A_2 = 0$ and we have

$$G_i(X) = A_1 X^{\beta_1}, \;\text{ for } i = 1,\dots,n\,. \tag{3.8}$$

The first order conditions from (3.1) and (3.5) allow us to obtain the smooth pasting for $i = 1,\dots,n$, namely

$$\begin{cases} G_i'(X) &= \beta_1 A_1 X^{\beta_1 - 1} \\ G_i'(X) &= \frac{\gamma_i}{\delta}(1 - e^{-\delta T}) \end{cases}. \tag{3.9}$$

By equating the RHS of (3.9), we get

$$A_1 X^{\beta_1} = \frac{\gamma_i X}{\delta\beta_1}(1 - e^{-\delta T}), \;\text{ for } i = 1,\dots,n\,. \tag{3.10}$$

Therefore, we replace (3.10) in (3.1), so that

$$\frac{\gamma_i X}{\delta\beta_1}(1 - e^{-\delta T}) = \frac{\gamma_i X}{\delta}(1 - e^{-\delta T}) - I_i - P_i^m, \;\text{ for } i = 1,\dots,n\,, \tag{3.11}$$

Finally, the optimal value for X associated to the ith license holder, say X_i^*, $i = 1,\dots,n$, is obtained from (3.11), and we have

$$X_i^* = \frac{\delta}{\gamma_i}\frac{\beta_1}{\beta_1 - 1}(1 - e^{-\delta T})^{-1}\left(I_i + P_i^m\right)\,. \tag{3.12}$$

Moreover, Eq. (3.12) can be used to determine the explicit expression for the constant A_1, so that

$$A_1 = \left(\frac{\gamma_i}{\delta\beta_1}\left(1 - e^{-\delta T}\right)\right)^{\beta_1}\left(\frac{I_i + P_i^m}{\beta_1 - 1}\right)^{1-\beta_1}\,. \tag{3.13}$$

From (3.12) we argue that X_i^* is the optimal value of the cash flow that verifies (3.1). On the one hand, in analogy with financial options, the trigger value is greater than the strike price, given by $(I_i + P_i^m)$, for $i = 1,\dots,n$, and the discrepancy accounts for the inherent risk implied by the project and its length. In particular, it is interesting to notice that, for the ith license, $i = 1,\dots,n$, X_i^* increases as the uncertainty related to the investment, represented by σ, increases. Indeed, the term $\beta_1/(\beta_1 - 1)$ is greater than 1 and accounts for risk. On the other hand, we observe that $\partial X_i^*/\partial\sigma > 0$, since β_1 is a function of σ and $\partial\beta_1/\partial\sigma < 0$, according to (3.6). Thus, in this case, the greater the uncertainty, the greater G_i and the lower the value of the investment for the license holder from $C_i(n+1)$ in (2.2). This result might seem counter-intuitive at first sight, but we recall that G_i is the counterpart of F, i.e., the buyout clause: the higher the incentive for the government to expropriate the private party, the lower $C_i(n+1)$ will be. The sensitivity analysis of (3.10) fully captures such a feature.

It is also worth noticing that the greater the share of benefits accruing from the deal, indicated by γ_i, for $i = 1,\dots,n$, the lower the expropriation threshold, i.e. $\partial X_i^*/\partial\gamma_i < 0$. Such a sensitivity reflects the fact that the greater the share of public resources held by a private party, the higher the risk for society from environmental damage and, accordingly, the stronger the threat from the government with a more likely exercise of the option. Finally, in complete analogy with the sensitivity to γ_i, we find $\frac{\partial X_i^*}{\partial T} < 0$.

Thanks to (3.8), we are able to provide the optimal value of the expropriation option that the ith concessionaire bears. Such a value is

$$G_i(X_i^*) = \frac{I_i + P_i^m}{\beta_1 - 1}\,, \tag{3.14}$$

for $i = 1,\dots,n$, which increases as uncertainty surrounding future benefits increases, i.e.

$$\frac{\partial G_i(X_i^*)}{\partial\sigma} = \frac{\partial G_i(X_i^*)}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma} > 0,$$

being $\frac{\partial G_i(X_i^*)}{\partial\beta_1} < 0$ and $\frac{\partial\beta_1}{\partial\sigma} < 0$.

4. The optimal value of the feasibility condition

We can now rewrite the feasibility condition $\sum_{i=1}^n C_i(n+1) \geq B(n+1)$ at its optimal values, i.e. by replacing the optimal values of G and F. To this aim, we can easily determine the latter by recalling that $F(X^*) = \sum_{i=1}^n G_i(X^*)$ from (2.5), and we obtain

$$F(X^*) = \sum_{i=1}^n G_i\left(X_i^*\right) = \sum_{i=1}^n \frac{I_i + P_i^m}{\beta_1 - 1} = \frac{1}{\beta_1 - 1}\left(P + \sum_{i=1}^n I_i\right)\,. \tag{4.1}$$

Remark 4.1. As for the sensitivities of $F(X)$ with respect to the parameters, they trivially follow from (3.14).

The optimal value of G_i in (3.14) can be substituted into the definition of $C_i(n+1)$ in (2.2) giving rise to the optimal value of $C_i(n+1)$, for $i = 1,\dots,n$. In doing this, we exploit the fact that $C_i(n+1) \geq 0$, so that

$$\int_0^T \gamma_i \mathbb{E}[X(s)]e^{-\rho s}ds - I_i - P_i^m - G_i(X_i^*) \geq 0\,, \tag{4.2}$$

or, equivalently,

$$\frac{\gamma_i X}{\delta}\left(1 - e^{-\delta T}\right) - \left(I_i + P_i^m\right) - \frac{I_i + P_i^m}{\beta_1 - 1} \geq 0\,. \tag{4.3}$$

Hence, we have, for $i = 1,\dots,n$,

$$P_i^m \leq \frac{\gamma_i X}{\delta}\frac{(\beta_1 - 1)}{\beta_1}\left(1 - e^{-\delta T}\right) - I_i =: \tilde{P}_i^m\,. \tag{4.4}$$

Thanks to Eq. (4.4), we recover that $\tilde{P}_i^m$ is an upper bound and it reveals the ith maximum willingness to pay to enter a concession contract of length T, in other words, it is a reservation price. Such a price is higher, the higher the future expected discounted value from the project, adjusted for risk, i.e. $\frac{\gamma_i}{\delta}X\frac{(\beta_1 - 1)}{\beta_1}$, the longer the concession length, T, and the lower the initial investment, I_i. In addition, uncertainty is detrimental to the willingness to pay, indeed,

$$\frac{\partial\tilde{P}_i^m}{\partial\sigma} = \frac{\partial P_i^m}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma} < 0,$$

being $\frac{\partial P_i^m}{\partial\beta_1} > 0$ and $\frac{\partial\beta_1}{\partial\sigma} < 0$.

We can now rewrite the feasibility condition $\sum_{i=i}^n C_i(n+1) \geq B(n+1)$ in (2.3) by replacing the optimal value of G_i, $i = 1,\dots,n$, and F from (3.14) and (4.1), respectively, and we have

$$\begin{aligned} &\sum_{i=1}^n \frac{\gamma_i X}{\delta}\left(1 - e^{-\delta T}\right) - \sum_{i=1}^n I_i - \sum_{i=1}^n P_i^m - \sum_{i=1}^n G_i(X_i^*) \\ &\quad \geq \frac{\gamma X}{\delta}\left(1 - e^{-\delta T}\right) + \frac{Xe^{-\delta T}}{\delta} - I + P - \sum_{i=1}^n G_i(X_i^*)\,. \end{aligned} \tag{4.5}$$

Recalling that $P = \sum_{i=1}^n P_i^m$ and with some algebra, we get

$$\begin{aligned} P &\leq \frac{X(\beta_1 - 1)}{2\delta\beta_1}\left(1 - e^{-\delta T}\right)\sum_{i=1}^n\left(\gamma_i - \frac{\gamma}{n}\right) \\ &\quad - \frac{Xe^{-\delta T}(\beta_1 - 1)}{2\delta\beta_1} + \frac{\beta_1 - 1}{2\beta_1}\left(I - \sum_{i=1}^n I_i\right) =: \tilde{P}\,. \end{aligned} \tag{4.6}$$

Whenever condition (4.6) holds, the public and the private party are willing to enter the agreement. Condition (4.6) provides an upper bound and, while it is intuitive on the part of the concession holders, it is less intuitive when considering the government. However, to make things as simple as possible we can say that it is the other side of the

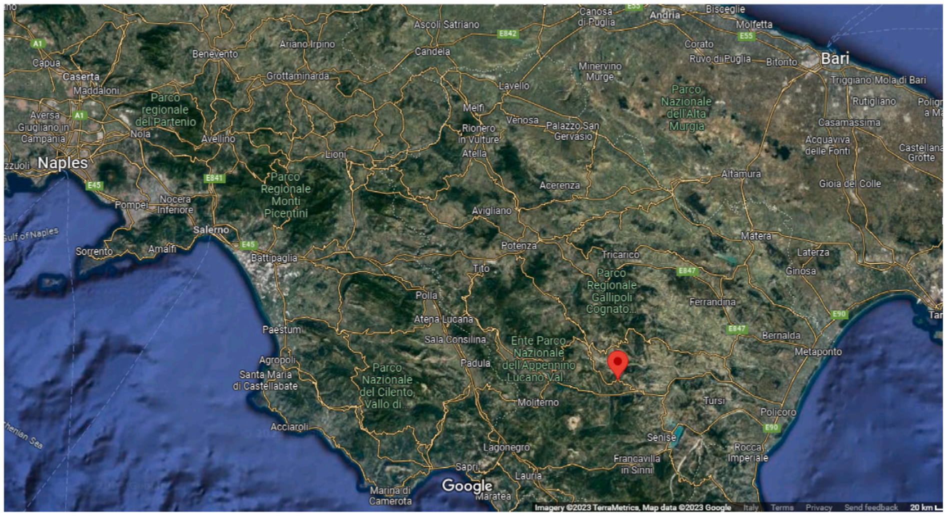


Fig. 5.1. *Tempa Rossa*: Location.
Source: Google Maps.

coin of condition (2.9), re-written in terms of prices. It follows that the government's interest not to exceed a certain value of the royalties comes from its interest to keep the value of the threat, G_i, $i = 1, \dots, n$, as low as possible for the same reason put forth above. Some comparative statics shows that $\frac{\partial \tilde{P}}{\partial T}$ and $\frac{\partial \tilde{P}}{\partial \gamma_i}$ are both positive as the willingness to pay increases as time and share of prospected profits increase. As a mirror image, the same applies to the government whose willingness to be compensated increases as the time length and share of foregone profits increase. Differently, $\tilde{P}$ is negatively related to uncertainty,

$$\frac{\partial \tilde{P}}{\partial \sigma} = \frac{\partial P}{\partial \beta_1} \frac{\partial \beta_1}{\partial \sigma} < 0,$$

being $\frac{\partial P}{\partial \beta_1} > 0$ and $\frac{\partial \beta_1}{\partial \sigma} < 0$.

5. A case study: *Tempa Rossa*

In this section, we apply the model to a real-world case, offering some interesting features that can be conveniently exploited to calibrate the model on the basis of actual parameters. We focus on *Tempa Rossa* license, namely one of the oil concessions released by the Italian government for one of the oilfields located in Basilicata, a region in Southern Italy.

Tempa Rossa is an oilfield located in the Alta Valle del Sauro, covers 27 hectares, and is located at an altitude of 1050 m, see Fig. 5.1 for further details. The production site comprises six wells drilled at a depth of 7148 meters and equipped with dual-pump electrical systems. The site extends mainly on the territory of the municipality of Corleto Perticara (province of Potenza), 4 km from where the treatment center was built. Five of the six wells are also located on the territory of the municipality of Corleto Perticara, while the sixth well is in the municipality of Gorgoglione (province of Matera). When fully operational, the plant is expected to produce approximately 50,000 barrels of oil per day, 230,000 m^3 of commercial gas, 240 tonnes of liquefied petroleum gas (LPG), and 50 tonnes of sulfur per day.

The master agreement for granting licenses (*Accordo Quadro*) dates back to 2006 (subsequently amended on 6th February 2020) and was concluded between the Basilicata Regional Authority and three concessionaires, namely Total E&P Italia S.p.A, Mitsui E&P Italia B S.r.l., and Shell Italia E&P S.p.A. The role of NOC is carried out by Società Energetica Lucana S.p.A. (SEL), a company wholly owned by the Basilicata Regional Authority, which also deals with periodically calling public tenders for the choice of the economic operator for electricity and gas supplies.

Based on real data, we intend to determine *(i)* the optimal (maximum) price of the concession contracts such that the parties are mutually willing to enter the contract (the so-called *royalties*), and *(ii)* the optimal exercise value of the expropriation option, X_i^*, for each license owner involved in the agreement.

5.1. Empirical analysis

All the key parameters that have been used to carry out the estimation/simulation exercise are reported in Table A.1 in the Appendix, along with a detailed description of the sources. Even if the model is quite close to fully capturing the case of *Tempa Rossa*, some actual features must be tailored. In particular, the project is twofold, in that both oil and gas are produced, the NOC is involved only concerning gas, while a direct transfer from the private party to the public bodies is due for oil. In particular, the master agreement sets that, since the beginning of production, the license holders must pay €0.80 for each barrel. This periodical transfer can be regarded as a right of the government on a fraction of the cash flow generated by the project, without incurring the investment cost. Hence, in the *Tempa Rossa* oilfield, the public authority enjoys the benefit from the private investment but does not bear the investment cost. In terms of the model, we are able to determine γ, but we cannot recover I. This makes some variables of the models not perfectly computable in closed-form solutions, e.g. X_i^*, and calls for simulated results.

The value of oil barrels produced lends itself as the natural candidate to proxy $X(t)$ and, for this reason, we have taken the Europe spot Brent price, expressed in dollars and converted to Euro by using the exchange rate averaged over the concession period. In addition, the Regione Basilicata website reports the daily production which amounts on average to 35,000 barrels. Therefore, $X(t)$ is given by the average production times the Brent price per barrel.

According to the master agreement, the duration of the concession nominally amounts to 24 years, from September 19, 1999, to September 19, 2023, but oil production started on the 1st of December 2020. Consequently, to match the model we must consider December 1, 2020–September 19, 2023, as the period for the three concessions, and the estimation sample ends on December 31st. Consistently, the parameters α, σ, ρ have been estimated over such a sample span, and the corresponding values are reported in Table A.1. The share of benefits accruing to each firm is given by the master document in terms of 50%, 25%, and 25% for Total, Shell, and Mitsui, respectively. However, given the flow of payments accruing to the public body, set equal to €0.8 per barrel, we must consider the aforementioned shares with reference to the net value of X. Consistently, the share for the NOC is computed as $\gamma = 0.80/P_{bbl}$ where P_{bbl} is the oil price per barrel in December 2020 (€52.26). It follows that $\gamma = 1.53\%$, while the values of γ_i, for $i = \{Total, Shell, Mitsui\}$ are

$$\gamma_{Total} = (100 - 1.53) \times 0.5 = 49.2\%,$$
$$\gamma_{Shell} = \gamma_{Mitsui} = (100 - 1.53) \times 0.25 = 24.6\%.$$

The total investment cost of the production site amounts to €15bln and it has been used to impute the investment cost of each license holder proportionally to their share of benefits,[5] i.e. the γ's. It is worth mentioning that the total investment covers the entire duration of the concession (24 years) and, in order to be used in our analysis, it must be adjusted for the period of extraction activity (25 months, from December 1st, 2020, to December 31st, 2022, the estimation sample). Furthermore, the indicated investment must be adjusted for several additional costs. In particular, we refer to *(i)* contributions for compensation of alternative land use loss and reinstatement of environmental and territorial balance (items A and C of the granting agreement), *(ii)* contributions for environmental monitoring (item B of the granting agreement), and *(iii)* contributions for sustainable development (item D of the granting agreement). Concerning *(i)*, we consider an amount equal to €3mln/year pro-rata, as a contribution to the planning and design of an environmental monitoring network, as well as a contribution of €1mln/year pro-rata for the first four years of production. Concerning *(ii)*, we include a contribution to the operating costs of the reservoir, equal to €1.5mln/year pro-rata, for the maintenance of the environmental monitoring network. Concerning *(iii)*, we incorporate a contribution of €250,000/year pro-rata, to organize events for promoting the environment and the territory.

The royalties paid by the firms are publicly available on the Italian Ministry of Economic Development's website and constitute the prices for the licenses. We recall that the royalties are due to the government only for the period in which the oil extraction actually takes place, namely 25 months. Thus, they are obtained as the sum of the premia paid by each concessionaire over the years 2020, 2021, and 2022, only concerning the *Tempa Rossa* oilfield. It is worth stressing that, in the case of Shell, we must not take into account the royalties paid by the company over the same time period to municipalities not pertaining to the *Tempa Rossa* concession.

Given these premises, our first interest rests upon the comparison between the royalties actually paid and the maximum willingness to pay, $\tilde{P}_i^m$, as in Eq. (4.4), for each concession holder. In particular, we obtain that the three companies paid €42.4mln (Total), €51.9mln (Shell) and €22.9mln (Mitsui), respectively, against the computed maximum willingness to pay $\tilde{P}_{Total}^m$ = €127.6mln, $\tilde{P}_{Shell}^m$ = €63.8mln, and $\tilde{P}_{Mitsui}^m$ €63.8mln. Such a comparison implies that the maximum willingness to pay was approximately three times greater than the actual price.

Another crucial quantity that can be computed is the value of the underlying process that triggers expropriation, i.e. X_i^*, $i = \{Total, Shell, Mitsui\}$, as in (3.12). According to our estimates, we have X_{total}^* = €31.4mln, X_{shell}^* = €48.3mln, and X_{mitsui}^* = €32.3mln.

These estimates are now useful to calibrate and discuss a sensitivity analysis of the maximum upper bounds for the royalties w.r.t. the time-to-maturity and the volatility, and of the optimal entry/exit threshold. The simulations are shown in Figs. 5.2 and 5.3, respectively. Fig. 5.2(a) plots $\tilde{P}_i^m$ from Eq. (4.4) as a function of the concession length, T. The continuous line refers to Total, while the hyphened and the dotted line refer to Shell and Mitsui, respectively. These last two curves are overlapped because the two companies hold the same share of benefits, $\gamma_{shell} = \gamma_{mitsui} = 24.6\%$. The map also reports the actual royalties paid and the ones estimated from the model, for given $T = 25$. Such actual values are marked by a blue diamond, a black dot, and a red square for Total, Shell, and Mitsui, respectively, assessed at $T = 25$. For the sake of completeness, in the chart we also display the maximum values obtained via our proposal, by using the asterisks on the respective curves. As expected from the sensitivity analysis in Section 4, the curves are positively sloped, but the simulation shows that the gap between Total and the other two concessionaires widens over time. While it is barely noticeable at $T = 0$, it becomes wider and wider as the time to maturity increases. This widening gap is a sort of compounded effect stemming from the fact that both $\frac{\partial \tilde{P}_i^m}{\partial T}$ and $\frac{\partial \tilde{P}_i^m}{\partial \gamma_i}$ are positive. We recall that $\gamma_{Total} > \gamma_{Mitsui} = \gamma_{Shell}$. In turn, this gap leads also to another interesting phenomenon. For $T = 25$ the distance between the maximum optimal affordable price, $\tilde{P}_i^m$, and the one actually paid is larger for Total than for Mitsui and Shell. In terms of the figure, the distance between the asterisk on the curve and the rhombus below the curve is greater for Total than for the other two companies, represented by the distance between the asterisk and the dot for Shell and the square for Mitsui. Indeed, the feasibility condition is an aggregate condition, and it implicitly implies that *bigger* concessionaires, where *big* is defined in terms of greater γ_i, benefit proportionally more than smaller concessionaires and can afford a proportionally higher royalty. From this standpoint, the tiny share of benefits accruing to the government could have been tailored according to the share of the license holders, rather than setting it independently of the γ's.

Fig. 5.2(b) is a mirror image of 5.2(a) in the sense that, other things being equal, the *biggest* company is particularly risk-averse because for low levels of σ the gap between its willingness to pay and the one by the other two license holders is large, but it decreases as σ increases. Put differently, we have

$$\left|\frac{\partial \tilde{P}_i^m}{\partial \sigma}\right| > \left|\frac{\partial \tilde{P}_j^m}{\partial \sigma}\right|$$

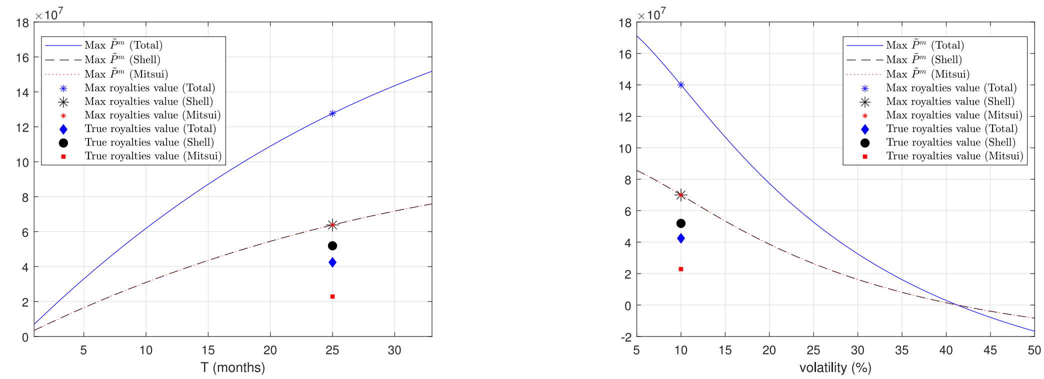
for $\gamma_i > \gamma_j$. For values of σ roughly greater than 40% the Total curve lies below the other two, indicating that there exists a value of σ, let us say σ^*, such that $\tilde{P}_i^m = \tilde{P}_j^m$ even if $\gamma_i > \gamma_j$.

As far as the ith optimal trigger point X_i^* is concerned, Fig. 5.3 plots its sensitivity w.r.t. the concession length, see Fig. 5.3(a), and uncertainty, see Fig. 5.3(b), spotting at $T = 25$ the values estimated from the model. As predicted by comparative statics, we have a downward-sloping relationship w.r.t. T and increasing w.r.t. σ. The three estimated thresholds are quite close to each other, revealing a limited sensitivity of X_i^* to γ_i and P_i^m, the two parameters that make the thresholds differ from company to company. In addition, it is possible to observe a remarkable convexity in Fig. 5.3(a), meaning that a given increase in T will produce a higher drop in X_i^* for low values of T, than for higher values. In the simulation, the value of the threshold almost stabilizes at around $T = 15$, making the curve almost flat from that point onward. As far as the positive relationship between X_i^* and σ is concerned, the gap between the Total curve and the other two concessionaires barely widens as σ increases. In terms of the model, we can say that the second derivative of X_i^* w.r.t. σ is roughly constant along the curves, showing a substantial steadiness of the relationship between the trigger value and the change in uncertainty. We can conclude that the level of uncertainty is much more relevant in determining X_i^*, rather than its changes, whereas the opposite holds true for P_i^m.

6. Conclusions

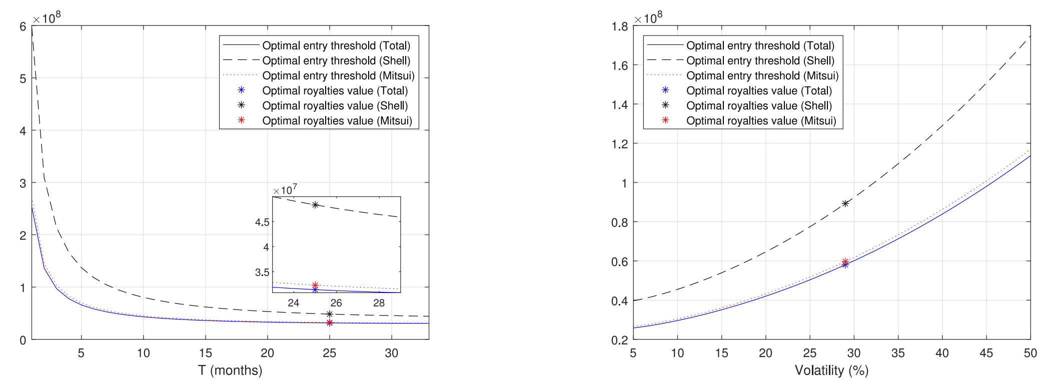
In this paper, we model concession contracts as a stochastic optimal control problem with standard dynamic programming techniques. The model demonstrates its value in determining the most efficient ways to use public resources by establishing optimal decision strategies. Notably, we propose a model to study the interaction between the government and n potential license holders, providing a closed-form formula for the entry/exit threshold. The model is flexible and can be easily adapted to accommodate actual instances and applied to

[5] Source: *Il Sole 24ore* 19-12-2020.



(a) Maximum P^m vs time to maturity (b) Maximum P^m vs volatility

Fig. 5.2. Sensitivity analysis: Royalties upper bound vs. time to maturity and volatility.



(a) Optimal entry threshold vs. time to maturity (b) Optimal entry threshold vs volatility

Fig. 5.3. Sensitivity analysis: Optimal threshold vs. time to maturity and volatility.

the exploitation of any public resource under a concession regime. Given the importance of hydrocarbon extraction, we apply the model to *Tempa Rossa*, an Italian oilfield. Even though *Tempa Rossa* does not perfectly fit the stylized situation depicted in the model, it offers a solid basis to root the theoretical findings into a real-world case. Our findings show that the model is a good fit with the real data and suggests improvements in local and national policies. In particular, as a novel result in the literature, we find that the royalties can be tailored to each potential license holder by considering the rate at which the maximum affordable price increases as the time to maturity increases. The higher the rate, the higher the benefit from the contract to the company and the higher the affordable price. *Mutatis mutandis,* analogous considerations can be made for the optimal trigger value. Some policy implications arise from our study, not only from the model itself but also from its implementation and simulation. In particular, while the model can help write the terms of the agreement and find a mutual benefit before the signature, it can be of even greater help when the contract is renegotiated or restructured (see Guasch et al., 2008) on the renegotiation of concession contracts). Indeed, after the business starts, actual data will progressively become available and can be used to tailor the renegotiation of the contract by carrying out a study similar to the one presented for *Tempa Rossa*. Therefore, the sensitivity of the maximum entry threshold and the optimal trigger can be precisely estimated and used to re-determine the royalties and/or other parameters of interest. In our study, we focus only on concessions, without investigating other types of agreements, such as Production Sharing Agreements (PSA) or joint ventures. Moreover, we consider diffusive, continuous-time dynamics for the project, without including more general assumptions for underlying dynamics. This research represents a first attempt where we did not address the potential effects on the model of exogenous market shocks, such as the impact of geopolitical changes due to the conflicts in the Middle East and Ukraine, the unexpected bursting of inflation resulting in higher interest rates or the occurrence of pandemics. Finally, thanks to the presence of n competitors in the market, a similar framework might be broadened to other fields of application, such as the issue of beach concessions. The topics above are left for further research.

CRediT authorship contribution statement

I. Oliva: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **M. Ventura:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

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Table A.1
Data and estimates of model parameters. The Table reports the estimated parameters of the model along with a detailed description of the sources.

Parameter	Description	Value	Source
$X(t)$	Underlying process	Average daily production (35k) times Europe spot Brent Price	Brent price Avg daily production
α	Drift rate of the process	2%	Our estimates of the drift rate of Europe spot Brent price
σ	Volatility rate of the process	11.8%	Our estimates of the standard deviation of Europe spot Brent price
ρ	Risk-free rate	3.77%	Our estimates of Italian Treasury Bond (BTP) from Datastream
	Average Eur/USD exchange rate	1.0454€	Our estimates of Italian Treasury Bond (BTP) from Datastream
T	Concession length: December 1st, 2020 September 19th, 2023	34.07 months	Regione Basilicata website
P_{bbl}	Europe spot Brent price ($/bbl), converted in €(December 2022)	52.26€	clal website
γ	Share of benefit for public body	1.53%	0.8/52.26 (see the text for further details)
γ_i	Share of benefit for the concessionaires	49.2 (Total) 24.6% (Shell) 24.6% (Mitsui)	$(100 - 1.53) * 0.5$ $(100 - 1.53) * 0.25$ $(100 - 1.53) * 0.25$ (see the text for further details)
$I + \sum_{i=1}^{n} I_i$	Investment cost of the production site	15bln€	Il Sole 24 ore
I_i	Investment cost for the concessionaires	71,800,453€ (Total) 35,900,226€ (Shell) 35,900,226€ (Mitsui)	See the text for further details
P_i^m	Price paid by the concessionaires (Royalties)	42,445,953€ (Total) 51,921,383€ (Shell) 22,856,394€ (Mitsui)	Ministry of Economic Development website

Appendix A

See Table A.1.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eneco.2024.107640.

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