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Performance measures of industrial symbiosis with inventory

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ABSTRACT

Industrial symbiosis is a production practice that can mitigate the environmental impacts of production processes while generating economic benefits. However, the produced and used waste flows are usually unsynchronized and unbalanced, and depend on the main output demands of the production processes involved in the industrial symbiotic exchanges. Waste inventory can increase the environmental and economic effectiveness of this industrial practice.

In this paper, we analyze the role of inventory under purely stochastic waste flows, which are assumed to be the worst cases of synchronization. Technical and economic models for a simple industrial symbiotic relationship and network are proposed, viewing inventory as a M/M/1/K queueing system. Then, environmental and economic performance measures, expressed in the closed form, are obtained, which permit us to answer research questions regarding the environmental and economic benefits and the optimal size of inventory for both industrial symbiotic relationships and networks, as well as to determine the best inventory location option for the network.

Results: from numerical case examples based on the end-of-life tire case show that the environmental performance measures always improve when the inventory increases. On the contrary, the optimal size of the inventory is affected by the environment- and inventory-related costs. As the waste production and use rates become more balanced, the economic benefit and inventory effectiveness increase. In the network, the optimal inventory location is affected by how the waste flows are designed and balanced for each specific relationship in the network. The limitations of this work and further research areas are indicated.

1. Introduction

1.1. Industrial symbiosis: context and research questions

Nowadays, it is essential for companies to mitigate the environmental impacts of their production processes, such as raw material consumption, waste disposal, and greenhouse gas emissions. Industrial symbiosis (IS) is a valuable approach to address such challenges (Chertow, 2000). Specifically, two companies establish an IS relationship when (at least) one waste product from a company can be used as a traditional production input by the other company (Lombardi and Laybourn, 2012). Through the adoption of IS, waste producers can reduce the volume of waste destined for landfills, while waste users can curtail their reliance on raw materials for their production processes. Consequently, companies contribute to environmental benefits for the community, while reducing their production costs (Martin, 2020). IS can be viewed as a practical implementation of the circular-economy principles within industrial networks, fostering collaboration and innovation to achieve sustainability goals. Furthermore, IS recognized as one of the most effective strategies to facilitate the transition toward a circular economy (Dominguez et al., 2021; Turken et al., 2020). For these reasons, the adoption of IS practices is strongly advocated nowadays (European Commission, 2020).

One of the most relevant issues hampering the emergence of IS relationships is the quantity mismatch between the demand and potential supply of waste (Fraccascia, 2019; Herczeg et al., 2018). Such a mismatch is caused by the problems of balancing and synchronizing waste production and use. Specifically, balancing refers to how close the average value of the waste-production rate (which determines the potential waste supply from the waste producer) is to the waste-use rate

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(which determines the potential demand of the waste user). In an IS relationship, these values may be unbalanced – i.e., the average value of waste demand may be (much) lower or higher than the average value of the waste supply – because production processes that generate and use waste are designed and planned to support the main output production of the company. Waste is not produced upon the demand of waste users, but results as a secondary output of the production processes. Specifically, the amount of waste produced by a generic company depends on the main output of the company (Yazan et al., 2016). Nevertheless, even fully balanced IS relationships can suffer from quantity mismatch, owing to synchronization issues. Specifically, synchronization deals with the time and quantity variability of the waste produced and used. Because companies are involved in dynamic business environments, both supply and demand of waste may fluctuate over time, as the result of changes in the market demand for companies' outputs, as well as because of disruptive events (e.g., natural disasters and operational failures) (Chopra and Khanna, 2014; Fraccascia, 2019; Li and Shi, 2015). Hence, the problem of a quantity mismatch can even emerge over time in IS relationships that have been successfully balanced. On the contrary, unbalanced flows can even impact the problem of variability.

A quantity mismatch between the demand and supply of waste can become a serious problem (Fraccascia et al., 2017). When two companies try to operate an IS relationship under a quantity-mismatch condition, an incentive misalignment problem may arise (Albino et al., 2016). If the amount of waste produced is higher than the demand for the waste, the excess waste is destined to be disposed of in a landfill, which might reduce the willingness of the waste producer to cooperate (Yazan and Fraccascia, 2020). Alternatively, if the demand for waste is higher than the amount of waste produced, the waste users will be forced to buy additional amounts of input from traditional suppliers and to use a different mix of waste and primary input in their production processes, which may undermine their willingness to cooperate with the waste producers.

While the problem of balancing can be solved during the design phase of an IS relationship, synchronization issues remain. The introduction of a waste inventory can contribute to better synchronization between the supply and demand for waste, owing to its ability to smooth the variability. Inventories are traditionally used by companies as a tool to synchronize the demand and supply levels of resources and to cope with disruptive events, such as failures, unavailability of production plants, and delays in transportation (e.g., Dolgui and Ivanov, 2020). Companies usually determine their input stock based on the uncertainties that may arise in both the supply of materials and demands for subsequent production processes (Alvarez et al., 2021; Cannella et al., 2014; Lee and Park, 2016; Urlu and Erkip, 2020). However, holding inventory requires additional space and incurs additional costs for companies, including storage, obsolescence, damage risk, deterioration, insurance, management costs, and traditional cost of capital invested (Christopher, 2016). The higher the amount of input stocked, the more protected the companies will be against uncertainties in the supply chain; however, the inventory costs will also be higher, *ceteris paribus*.

In the IS context, waste stock can be used when the supply is not synchronized with the demanded amount. This practice can reduce the amount of traditional input that a company purchases from traditional suppliers or reduce the cost of waste disposal in the landfill, hence providing firms with additional economic advantages, which might be higher than the inventory costs. However, in the literature there have been no case studies addressing the practice of stocking waste for IS purposes. Although two recently published articles (Fussone et al., 2025a, 2025b, 2025a) permit, within their models, the stocking of wastes by companies engaged in IS relationships, they do not delve into the behavior of waste inventory. Furthermore, they do not examine the impact that waste inventory has on the performance of IS.[1] Nevertheless, several studies acknowledge that this issue calls for *ad-hoc* investigation and that there are ample opportunities for further research on waste-inventory management and production planning combining different wastes with traditional input (Densley Tingley et al., 2017; Herczeg et al., 2018). Specifically, there is a need to understand how inventory can address the synchronization problem and impact the performance of IS. In particular, it is important to model IS with inventory to consider the variability of time and quantity of the produced and used waste, its impact on the environmental and economic performance, and the balance of average flows of produced and used waste. Three research questions arise:

(RQ1) Can inventory effectively improve the performance of IS?
(RQ2) What should be the optimal size of the inventory?
(RQ3) Where is the best location for the inventory?

1.2. Proposed approach

Answering the above questions requires the development of models that can capture the variabilities in the waste flows and inventory levels, and provide simple performance measures to compare designed and operational solutions. Different modelling approaches are available to model the material flows in IS (Demartini et al., 2022; Turken and Geda, 2020). In particular, discrete-event simulation (Fussone et al., 2024), agent-based modeling (Fraccascia et al., 2020), input–output modeling (Fang et al., 2017), material flow analysis (Sendra et al., 2007; Sun et al., 2017), and system dynamics (Cui et al., 2018) can be considered to represent the material flows and inventory in IS relationships. However, these modelling approaches exhibit certain limitations with regards to the aim of this paper. It is essential to consider the stochastic variability of waste flows to represent and model the inventory behavior, as well as produce closed-form expressions for the performance measures – that the above-mentioned approaches are unable to provide – to compare the different design and operational parameters of the IS with inventory.

Dealing with variability and uncertainty when matching stochastic production and demand flows of materials is one of the main problems in inventory management (Williams and Tokar, 2008). In fact, the primary purpose of inventory is to serve as a buffer between the supply and demand processes. When such processes are stochastic, as in the case of waste inventory in IS, a queueing-systems approach seems to be a more suitable modelling. The systematic review of the literature on the queueing-inventory approach developed by Salini et al. (2023) reveals different types of queueing models wherein the orders in a queue wait to be served under different inventory disciplines and hypotheses. In production systems with stochastic processes, buffer sizing and allocation problems have also been largely explored (Amjath et al., 2023; Dallery and Gershwin, 1992; Demir et al., 2014). The current literature on queueing-inventory and buffer sizing and allocation does not contain any specific application to IS.

To answer the three research questions, we propose a finite-queueing model (Cruz and van Woensel, 2014) that considers the variability of the produced and used waste and the balance of the average flow rates of the produced and used waste when inventory is adopted in IS. This model permits us to evaluate the impact of inventory on the technical performance of IS, as well as its effectiveness in reducing the flow of waste to landfill and the flow of primary input purchase from a traditional supplier, i.e., the environmental performance of IS. Then, from the economic perspective, we propose a model for the considered queueing model, taking into account only the cost affected by the IS with

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[1] Specifically, these studies do not compare scenarios with and without waste inventories. Consequently, the marginal contribution of waste inventory cannot be determined.

inventory. Based on these technical performances and costs, the costs for a unit of time are the economic performance measures defined to compare the different configurations of inventory in IS relationship and network.

The remainder of this paper is organized as follows. Section 2 presents the model of IS with inventory and introduces the performance measures used to assess the effectiveness of inventory in IS. Section 3 presents two numerical case examples associated with the end-of-life tire (ELT) industry, which help demonstrate how inventory can be effective in IS practices. The paper ends with a discussion (Section 4) and conclusions (Section 5).

2. Model of industrial symbiosis

The simplest IS relationship considers two processes, *i* and *j*, where the production process *i* produces a waste that can be used as a substitute of a primary input of the production process *j*. If the amount of waste produced in each period is greater than the quantity of waste demanded in the same period, the excess waste is sent to a landfill. Conversely, if the amount of waste produced is lower than the quantity of waste demanded in that period, the missing primary input is purchased for the process *j* from a traditional supplier. Both processes can agree to an IS relationship, depending on their exchange costs.

However, IS may involve complex scenarios wherein a network of supply and demand processes (IS network) exists for the same type of waste. In the following subsections, two models are proposed to represent different IS cases. Model A considers the basic IS relationship between two production processes, *i* and *j*. Model B considers an IS network described as a set of basic IS relationships.

2.1. Model A: basic IS relationship

Let us assume two production processes, *i* and *j*, such that, in each time period a waste produced by process *i* can substitute a primary input used by process *j*. For the sake of simplicity, let us assume that in any period.

- a symbiosis with a perfect substitution of primary input by waste can be established; i.e., the waste produced by process *i* can be directly used by process *j* to replace the primary input, without a need for further transformation (Fraccascia et al., 2017);
- one unit of waste produced by process *i* can replace one unit of primary input required by process *j* (Fraccascia, 2019);
- waste transportation is neglected, as usually IS occurs between processes located close by (Jensen et al., 2011);
- the landfill is always available for the excess of waste from process *i*;
- the traditional supply for the primary input for process *j* is always instantaneously available.

Fig. 1 displays the basic IS model (Model A).

Regarding the flows of produced and used waste, we assume the maximum level of variability in each period because they are affected by different sources of uncertainty (Zhu and Ruth, 2014). Then, the flow of waste produced by process *i* can be represented as a Poisson stochastic process with rate λ_i. Accordingly, the amount of waste arriving in a period is counted in terms of an integer number of units. Similarly, the flow of waste required by process *j* is represented as a Poisson process with rate μ_j. The units of waste arriving to process *j* are stored in a buffer if process *j* is already using a unit of waste, which then wait to be utilized. If the buffer is full, the excess waste is disposed in a landfill. If process *j* requires a unit of waste and the buffer is empty, the required primary input is always and instantaneously available from a traditional supplier.

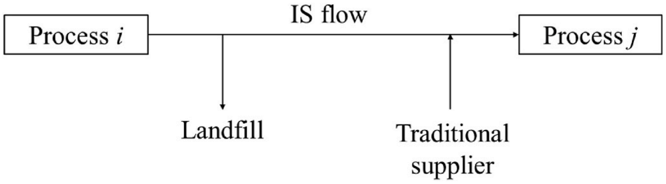


Fig. 1. Basic IS model (Model A).

The resulting aggregated process is a birth–death stochastic process that models the production and use of waste as well as the buffer of waste waiting to be used by process *j*. It can be represented by a Markov discrete-state continuous-time stochastic process commonly employed to model specific queuing systems. In this context, the system's state is delineated by the integer quantity of waste units awaiting to be used by process *j*, inclusive of that undergoing transformation. Fig. 2 shows the inventory model (a) and corresponding queue model (b).

The selected queue model is M/M/1/K, where the size of the queue system (queue plus server, i.e., inventory) is finite and equal to *K*. We define $\rho = \lambda_i/\mu_j$ as the traffic intensity that measures the balance between the average rates of waste produced and used. When $\rho = 1$, the system is balanced; otherwise, it is unbalanced. This model has steady-state solutions for all values of ρ. The steady-state probability, $\pi(s)$, that the system is in state *s* (i.e., it contains *s* units of waste, including that in transformation via process *j*) is:

$$\pi(s) = \rho^s\left[\frac{1-\rho}{1-\rho^{K+1}}\right] \mathit{for}\ \rho \neq 1, \pi(s) = \frac{1}{K+1}\ \mathit{for}\ \rho = 1 \tag{1}$$

Let us consider the following technical performance measures of the queue system.

- $\pi(0)$: probability that no waste units are in the system and that the process *j* (server) needs to receive primary input from the traditional supplier (idle condition);
- $\pi(K)$: probability that an arriving unit of waste finds *K* units in the system, because of which it is disposed in the landfill;
- *L*: average number of units of waste in the queue system;
- L_q: average number of units of waste in the queue (i.e., in the buffer).

These performance measures can be computed as follows:

$$\pi(0) = \frac{1-\rho}{1-\rho^{K+1}}\ \mathit{for}\ \rho \neq 1, \pi(0) = \frac{1}{K+1}\mathit{for}\ \rho = 1, \tag{2}$$

$$\pi(K) = \rho^K\left[\frac{1-\rho}{1-\rho^{K+1}}\right] \mathit{for}\ \rho \neq 1, \pi(K) = \frac{1}{K+1}\mathit{for}\ \rho = 1, \tag{3}$$

$$L = \frac{\rho}{1-\rho} - \frac{(K+1)\cdot\rho^{K+1}}{1-\rho^{K+1}}\ \mathit{for}\ \rho \neq 1, L = \frac{K}{2}\mathit{for}\ \rho = 1, \tag{4}$$

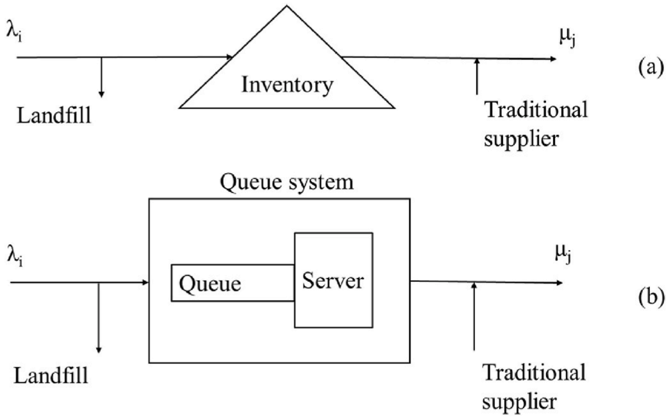


Fig. 2. Inventory model (a) and the corresponding queue model (b).

$$Lq = \frac{\rho}{1-\rho} - \frac{K\rho^{K+1}+\rho}{1-\rho^{K+1}}\, for\ \rho \neq 1, Lq = \frac{K}{2} - \frac{K}{K+1} for\ \rho = 1. \tag{5}$$

Some environmental performance measures can be directly derived from the technical ones. In particular, two noteworthy measures are related to the fraction of waste disposed of to the landfill, F_{W}, and of the primary input purchased from a traditional supplier, F_{PI}.

$$F_W = \frac{\lambda_i \cdot \pi(K)}{\lambda_i} = \pi(K) \tag{6}$$

$$F_{PI} = \frac{\mu_j \cdot \pi(0)}{\mu_j} = \pi(0) \tag{7}$$

Furthermore, some cost performance measures can be defined to support the answer to the research questions. We only define the following IS-related production costs, i.e., the costs that can be affected by the IS: cost of a unit of waste sent to the landfill ($C\omega$), cost of a unit of primary input bought from the traditional supplier ($C\alpha$), cost of a unit of waste in the buffer for a unit of time (CL), and cost of a unit size of the buffer for a unit of time (CK).

If the IS relationship does occur, the following costs, with an inventory of size K, must be considered.

- average cost for waste disposal to the landfill in a unit of time = $\lambda_i \cdot \pi(K) \cdot C\omega$;
- average cost of primary input purchased from a traditional supplier in a unit of time = $\mu_j \cdot \pi(0) \cdot C\alpha$;
- average cost of waste in the buffer in a unit of time = $L_q \cdot CL$;
- cost of the buffer size in a unit of time = $(K-1) \cdot CK$.

Then, the economic performance measures of the model can be evaluated in terms of the total IS-related costs. This performance considers a unit of time and measures the cost for managing the inventory, cost for discharging the waste to the landfill when the inventory is full, and cost of purchasing the primary input from a traditional supplier when the inventory is empty.

Accordingly, if the size of the inventory is equal to K, the cost for a unit of time (CUT) when IS does occur is equal to:

$$CUT = \lambda_i \cdot \pi(K) \cdot C\omega + L_q \cdot CL + (K-1) \cdot CK + \mu_j \cdot \pi(0) \cdot C\alpha. \tag{8}$$

Of course, if IS does not occur, the cost for a unit of time is equal to:

$$CUT = \lambda_i \cdot C\omega + \mu_j \cdot C\alpha, \tag{9}$$

which is always greater than the cost for a unit of time when IS occurs for $K = 1$ ($L_q = 0$). Let us consider that in the model, for $K = 1$, when a unit of waste is produced by process i, it is immediately sent to process j if process j is not working. Otherwise, it is disposed of in the landfill as no buffer is allowed.

Now, it is possible to show if and how the inventory is effective in improving the environmental and economic performance of IS (RQ1). Regarding the environmental performance, F_{W} and F_{PI} are decreasing functions of K. Then, inventory is always effective in reducing the environmental impact. For economic-performance measures, answering the above question means evaluating the value of K for which the cost for a unit of time is lower than the corresponding cost for $K = 1$, denoted as $CUT(K{=}1)$ (i.e., no buffer available). Then, for any $K > 1$, inventory is effective if the following cost difference is positive:

$$\begin{aligned}
&\lambda_i \rho \frac{1-\rho}{1-\rho^2} C_\omega + \mu_j \frac{1-\rho}{1-\rho^2} C_\alpha - \lambda_i \rho^K \frac{1-\rho}{1-\rho^{K+1}} C_\omega - \left[\frac{\rho}{1-\rho} - \frac{K\rho^{K+1}+\rho}{1-\rho^{K+1}}\right] C_L \\
&\quad - (K-1)C_K - \mu_j \frac{1-\rho}{1-\rho^{K+1}} C_\alpha \\
&\quad > 0\ for\ \rho \\
&\quad \neq 1, \lambda_i \frac{1}{2} C_\omega + \mu_j \frac{1}{2} C_\alpha - \lambda_i \frac{1}{K+1} C_\omega - \left(\frac{K}{2} - \frac{K}{K+1}\right) C_L - (K-1)C_K \\
&\qquad - \mu_j \frac{1}{K+1} C_\alpha \\
&\quad > 0\ for\ \rho = 1.
\end{aligned} \tag{10}$$

Now, we can evaluate the optimal size of the inventory (RQ2). From the environmental perspective, the optimal size of the inventory is trivially obtained for $K \to \infty$. From the economic point of view, if the inventory is effective, the size of the inventory can be optimized. Let us consider the cost per unit time when IS occurs. Then, we can find the optimal value of K, denoted as K_{opt}, which minimizes the following functions:

$$\begin{aligned}
&\min_K \left\{ \lambda_i \rho^K \frac{1-\rho}{1-\rho^{K+1}} C_\omega + \left[\frac{\rho}{1-\rho} - \frac{K\rho^{K+1}+\rho}{1-\rho^{K+1}}\right] C_L + (K-1)C_K + \mu_j \frac{1-\rho}{1-\rho^{K+1}} C_\alpha \right\} for\ \rho \neq 1, \\
&\qquad \min_K \left\{ \lambda_i \frac{1}{K+1} C_\omega + \left(\frac{K}{2} - \frac{K}{K+1}\right) C_L + (K-1)C_K + \mu_j \frac{1}{K+1} C_\alpha \right\} for\ \rho = 1.
\end{aligned} \tag{11}$$

Finally, regarding the location of the inventory (RQ3) in Model A, if the inventory is effective, it does not matter where the inventory is located. With respect to the environmental and economic performance, there is no difference whether it is located at the exit of the waste producer (process i) or at the entrance of the waste user (process j).

2.2. Model B: IS network of supply and demand processes for the same type of waste

In Model B, we assume that the IS network regards the relationships between the supply processes (i_1, i_2, etc.), that produce the same type of waste, and demand processes (j_1, j_2, etc.) that use this waste as a primary input. For the sake of simplicity, in Fig. 3 we depict some IS network models with different inventory location options (Model B1, B2, B3, and B4) in the case of three supply processes (i_1, i_2, i_3) and two demand processes (j_1, j_2). More specifically, we consider the following cases: one inventory for all the IS network (B1), inventory at the entrance of user processes only (B2), inventory at the exit of production processes only (B3), and inventory at the exit of processes i_1 and i_3 and entrance of processes j_1 and j_2 (B4).

Then, the IS network is modeled as a specific set of basic IS relationships, depending on the inventory location considered. The inventory location may result from a real existing case or from the designed option set of an inventory system in an IS network. In the former case, the inventory location and flows are known. In the latter case, different inventory location options are possible and the flow rates for each couple of producer and user are known. For each option, adopting the assumptions and results of Model A, we can define the corresponding basic IS relationship for each inventory in the option. Then, each inventory can be modeled as a M/M/1/K queue model, where the rate of waste production is the sum of the rates of all flows

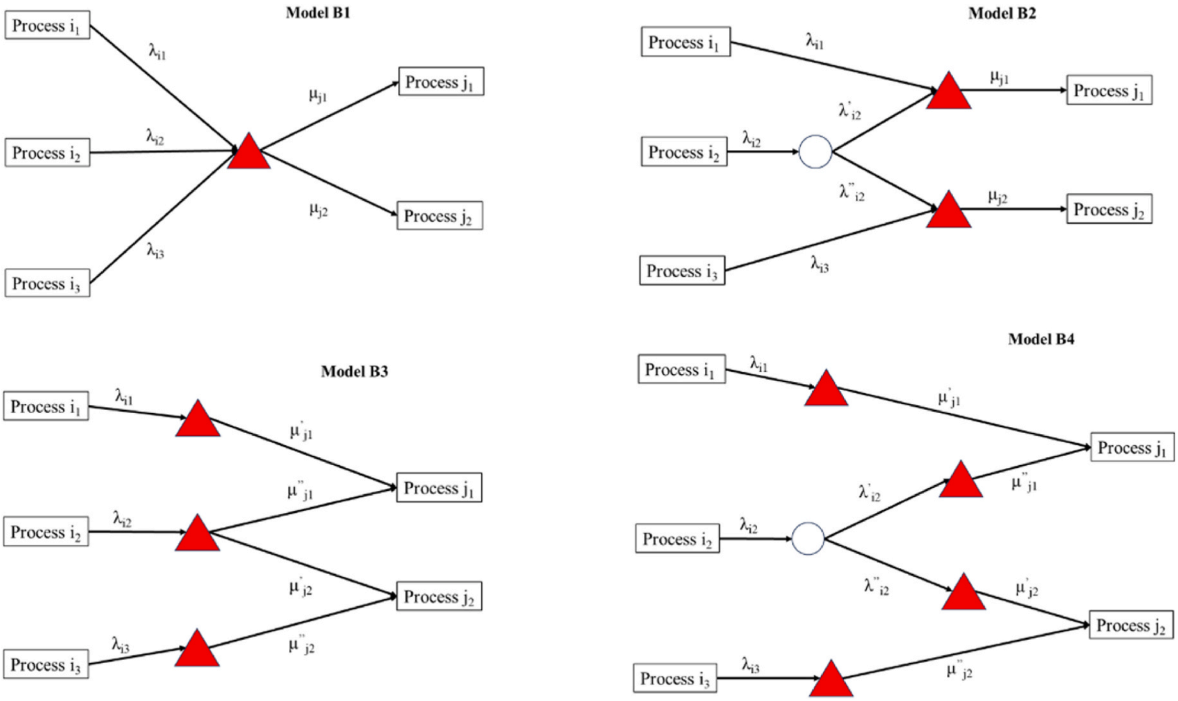


Fig. 3. Models of a IS network for different inventory location options (Models B1, B2, B3, and B4).

arriving at the inventory. This stands for the superposition property of merging Poisson processes (Chan, 2014). Similarly, the rate of waste required is the sum of the rates of all wastes required from the inventory, which corresponds to the property of splitting Poisson processes (Chan, 2014). In fact, both merging and splitting Poisson processes result in Poisson processes. Then, for each inventory location option, the average flow of waste that can be supplied from process i_1 to an inventory, and so on, and the average flow of waste that can be demanded from process j_1 from an inventory, and so on, are known.

Considering Fig. 3, for Model B1, one queue model (Q1) exists, in the form of M/M/1/K$_1$, with $\lambda_i = \lambda_{i1} + \lambda_{i2} + \lambda_{i3}$ and $\mu_j = \mu_{j1} + \mu_{j2}$.

For Model B2, two queue models (Q1 and Q2) exist, in the forms of M/M/1/K$_1$ and M/M/1/K$_2$, with $\lambda_i = \lambda_{i1} + \lambda'_{i2}$, $\mu_j = \mu_{j1}$ and $\lambda_i = \lambda''_{i2} + \lambda_{i3}$, $\mu_j = \mu_{j2}$, respectively.

For Model B3, three queue models (Q1, Q2, and Q3) exist, in the forms of M/M/1/K$_1$, M/M/1/K$_2$, and M/M/1/K$_3$, with $\lambda_i = \lambda_{i1}$, $\mu_j = \mu'_{j1}$; $\lambda_i = \lambda_{i2}$, $\mu_j = \mu''_{j1} + \mu'_{j2}$; and $\lambda_i = \lambda_{i3}$, $\mu_j = \mu''_{j2}$, respectively.

For Model B4, four queue models (Q1, Q2, Q3, and Q4) exist, in the forms of M/M/1/K$_1$, M/M/1/K$_2$, M/M/1/K$_3$, and M/M/1/K$_4$, with $\lambda_i = \lambda_{i1}$, $\mu_j = \mu'_{j1}$; $\lambda_i = \lambda'_{i2}$, $\mu_j = \mu''_{j1}$; $\lambda_i = \lambda''_{i2}$, $\mu_j = \mu'_{j2}$; and $\lambda_i = \lambda_{i3}$, $\mu_j = \mu''_{j2}$, respectively.

The environmental performance measures F_{W} and F_{PI} can be computed for each inventory location option, considering the weighted average of the corresponding performance of all IS relationships involved in that option. For instance, for Model B2, they result in the weighted averages for the two corresponding queues, Q1 and Q2:

$$F_W = \frac{(\lambda_{i1} + \lambda'_{i2}) \cdot \pi_{Q1}(K1) + (\lambda''_{i2} + \lambda_{i3}) \cdot \pi_{Q2}(K2)}{\lambda_{i1} + \lambda'_{i2} + \lambda''_{i2} + \lambda_{i3}} \tag{12}$$

$$F_{PI} = \frac{\mu_{j1} \cdot \pi_{Q1}(0) + \mu_{j2} \cdot \pi_{Q2}(0)}{\mu_{j1} + \mu_{j2}} \tag{13}$$

For all the models of the IS network (B1, B2, B3, and B4), all the costs *Cω, CL, CK*, and *Cα* for each queue model are assumed to be the same, for the sake of simplicity. Then, the *TCUT* of an IS network model, defined as the sum of the *CUT* of each queue in the corresponding inventory location option, can be computed.

If IS does not occur, *TCUT* is equal to:

$$TCUT = (\lambda_{i1} + \lambda_{i2} + \lambda_{i3}) \cdot C\omega + \left(\mu_{j1} + \mu_{j2}\right) \cdot C\alpha. \tag{14}$$

Demonstrating the effectiveness of IS in Model B1, B2, B3, and B4 is as straightforward as that in Model A.

Now, it is easy to answer the three RQs. In particular, it is possible to examine whether and how the inventory system enhances the performance of IS (RQ1). For each model, from an environmental perspective, conclusions similar to for Model A can be obtained. From the economic perspective, for each inventory location option and for each corresponding queue system, the inventory is effective if a value of *K* does exist for which the *CUT* is less than the value of the *CUT(K=1)*. For an IS network model, it means that *TCUT* should be less than the *TCUT* for all queue systems with *K=1*, denoted as *TCUT(K=1)*.

Once the effectiveness of the inventory is confirmed, the size of each inventory option can be individually optimized from the economic point of view (RQ2). The value of *K* that minimizes its corresponding *CUT*, denoted as K_{opt}, can be obtained using the same approach adopted in Model A. Then, by encompassing all inventories, the comprehensive optimization of *TCUT* for that option can be obtained. From the environmental point of view, the conclusions are the same as that for Model A.

Finally, the best inventory location option (RQ3) corresponds to the one from among Model B1, B2, B3, or B4 having the minimum F_W, F_{PI}, or *TCUT* depending on the specific optimization goal.

In the examples of Fig. 3, it is easy to demonstrate that, from both environmental and economic perspectives, the best inventory location option corresponds to that of the model having only one inventory (Model B1) with *Cω, CL, CK*, and *Cα* being the same for all inventory options.

3. Numerical case examples

To show how inventory can be effective in IS practice, the case of end-of-life tires (ELTs) is considered. This type of waste must be collected and recovered in specific plants. In these case examples, ELTs may be channeled toward one of two different destinations: sent to the

Table 1
Economic scenarios (ESs) considered.

	ES1 (Base case)	ES2	ES3	ES4	ES5
$C\omega$ [€/t]	100	100	50	100	50
$C\alpha$ [€/t]	100	100	50	50	100
CK [€/t·day]	1	10	1	1	1
CL [€/t·day]	1	10	1	1	1

landfill or recovered as energy in *ad-hoc* plants, mainly cement factories (World Business Council for Sustainable Development, 2021). ELTs possess a heating value similar to that of coal, but with lower climate-changing emissions (Maga et al., 2023).

The case example examined in this section is based on an actual instance of IS between two companies: an ELT collector and cement plant, both situated in Southern Italy. The cement factory uses ELT as an alternative energy source. For the sake of simplicity, but without compromising generality, we assume a perfect substitution (no transformation processes required) of coal (pet-coke) with ELT. We examine two different numerical cases: (1) one ELT collecting and recovering plant and one cement factory (case I), which models the actual case of IS mentioned above; and (2) three ELT collecting and recovering plants and two cement factories (case II), which has been used to conduct a strategic analysis aimed at addressing the potential outcomes of multisource and multiuser conditions.[2]

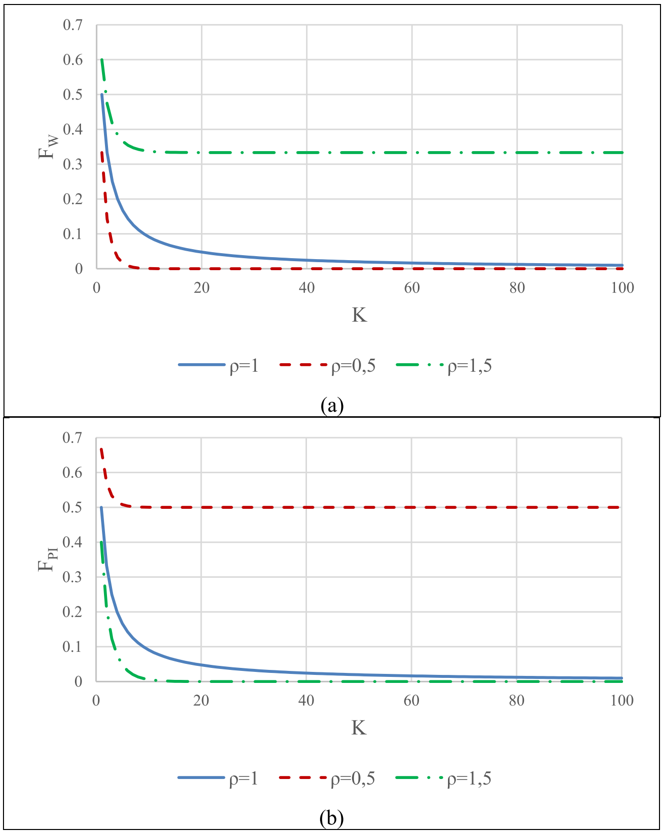
3.1. Case I

In this case, we have one ELT collecting and recovering plant (process *i*) and one cement factory (process *j*). This case can be approached using Model A.

As the base case, with reference to the technical parameters, we consider the system fully balanced, i.e., $\rho = 1$, with $\lambda_i = \mu_j = 10$ t/day. With reference to the economic parameters, we consider $C\omega = C\alpha = 100$ €/t and $CK = CL = 1$ €/t·day. These economic values have been set after an interview with the business players involved and are thus considered realistic in this context. We examine some further scenarios to investigate how the system behaves under different technical and economic conditions. Specifically, to investigate the impact of different system balances, i.e., the change in the ratio between waste production and demand rates, we consider two further values of ρ: (1) $\rho = 0,5$ with $\lambda_i = 5$ t/day and $\mu_j = 10$ t/day, and (2) $\rho = 1,5$ with $\lambda_i = 15$ t/day and $\mu_j = 10$ t/day. To investigate the impact of the economic values, which may be different in different contexts, we consider four scenarios in addition to the base case, as displayed in Table 1.

Compared to the base case (ES1), in ES2 and ES3, an increase in the inventory-related costs and reduction in waste-disposal and primary input purchase costs, respectively, are considered. In ES4 and ES5, a reduction in the primary input purchase cost and waste-disposal cost, respectively, are assumed, compared to the base case.

First, let us focus on the environmental performance. F_W and F_{PI} as a function of K are shown in Fig. 4a and b, respectively, for three different values of ρ. It can be noted that the relationship between both the performance measures and K is affected by the system balance. Specifically, for $\rho = 1$, both performance measures tend to zero as K increases. Differently, for $\rho = 0,5$, the fraction of waste disposed of in landfills (F_W) tends to zero as K increases, while it is impossible to eliminate the fraction of inputs purchased from the traditional supplier (F_{PI}).



(a)

(b)

Fig. 4. (a) F_W as a function of K, for three different values of ρ; (b) F_{PI} as a function of K, for three different values of ρ.

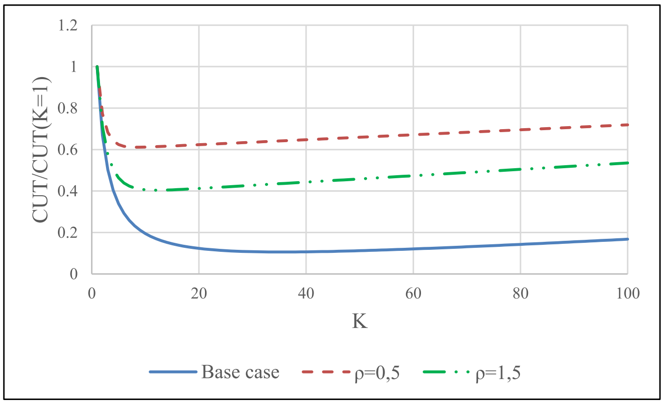


Fig. 5. *CUT/CUT(K=1)* vs. K in the base case compared with $\rho = 0,5$ and $\rho = 1,5$.

Conversely, for $\rho = 1,5$, the fraction of inputs purchased from the traditional supplier tends to zero as K increases, whereas it is impossible to eliminate the fraction of waste disposed of in landfills.

Let us focus now on economic performance for the base case.

Fig. 5 displays the values of the ratio between *CUT* and *CUT(K=1)* – i.e., when the buffer is not implemented – as a function of K for three values of ρ and for ES1. As K changes, all these relationships exhibit a minimum value, which corresponds to the optimal value of K, K_{opt}. Such a minimum value, together with the value of K_{opt}, can depend on the system balancing. Indeed, in the base case (characterized by $\rho = 1$), implementing an inventory with the optimal value of K ($K_{opt} = 36$) might be able to reduce the costs of IS, compared to the one with $K = 1$ (no buffer) by up to approximately 90 %, whereas such a percentage

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[2] In this regard, both companies have expressed interest in moving away from the traditional one-to-one cooperation model, to explore the opportunity of a multisource and a multiuser strategy. On the one hand, the cement factory can benefit from a multisource strategy with additional ELT collectors; on the other hand, the ELT collector can benefit from a multiuser strategy to boost demand for ELT and mitigate quantity variability (Fraccascia et al., 2020).

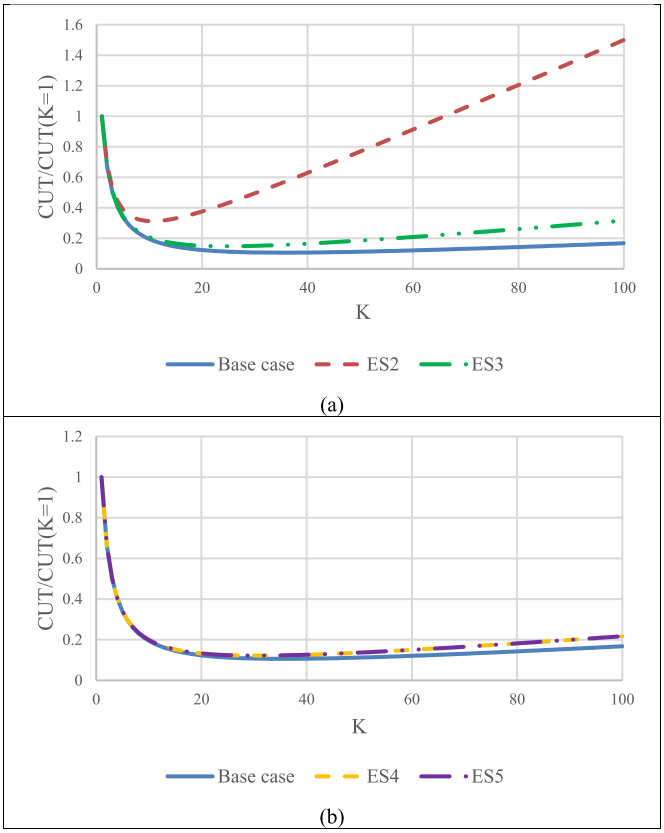

Fig. 6. *CUT/CUT(K=1)* vs. *K* in the base case, compared with (a) ES2 and ES3, (b) ES4 and ES5.

becomes approximately 40 % (with $K_{\text{opt}} = 8$) when $\rho = 0{,}5$ and approximately 60 % (with $K_{\text{opt}} = 12$) when $\rho = 1{,}5$.

Deepening the base case ($\rho = 1$ and ES1), in Fig. 6, the values of the ratio between *CUT* and *CUT(K=1)* – i.e., when the buffer is not implemented – as a function of *K* are compared for the different economic scenarios considered. In particular, Fig. 6a compares, *ceteris paribus*, the base case with ES2 and ES3, aimed at highlighting the impact of increased values of *CL* and *CK* (ES2) and decreased values of *Cω* and *Cα* (ES3), respectively. Fig. 6b compares, *ceteris paribus*, the base case with ES4 and ES5, aimed at highlighting the impact of a single reduction in *Cα* (ES4) and *Cω* (ES5), respectively. As *K* changes, all these relationships exhibit a minimum value, which corresponds to the optimal value of *K*, K_{opt}. Such a minimum value, together with the value of K_{opt}, can depend on the economic scenario considered. In this regard, implementing an inventory with the optimal value of *K* might allow reducing the costs of IS, compared with the one with $K = 1$ (no buffer) between 70 % and 90 %, depending on the scenario considered. Furthermore, *ceteris paribus*, the higher the inventory cost *CK* and *CL*, compared to *Cω* and *Cα* – in the base case, CL and CK account for 1 % of Cω and Cα, in ES2 they account for 10 %, and in ES3 they account for 2 % – the lower the inventory will be effective in reducing the costs of IS and the lower the optimal value of *K* will be (K_{opt} is equal to 36 for the base case, 11 for ES2, 25 for ES3, and 31 for ES4 and ES5).

We further analyze how the system balancing can impact the optimal economic performance of inventory, as well as the optimal value of *K*.

Fig. 7 displays the optimal value of *CUT/CUT(K=1)* (Fig. 7a and b) and K_{opt} (Fig. 7c and d) as a function of ρ. Specifically, Fig. 7a and c compare the ES1 case with the economic scenarios ES2 and ES3, while Fig. 7b and d compare the ES1 case with the economic scenarios ES4 and ES5. In general, it can be appreciated that the maximum cost reduction resulting from the use of an inventory occurs when the system is balanced ($\rho = 1$ or close to one), for all economic scenarios considered, and this corresponds to the maximum value of K_{opt}. This happens because, when the system is balanced, both F_{W} and F_{PI} can be minimized simultaneously (Fig. 4). Specifically, both the amount of waste to be disposed of in the landfill and amount of input purchased from the conventional suppliers can be minimized.

Specifically, Fig. 7a underlines that, in ES1, ES2, and ES3 (when *Cω*=*Cα*), the higher the inventory costs *CL* and *CK* are, compared to *Cω* and *Cα* (in ES1, *CL* and *CK* account for 1 % of *Cω* and *Cα*; in ES2, they account for 10 %; and in ES3, they account for 2 %), the higher the optimal value of *CUT/CUT(K=1)* will be, for all values of ρ, i.e., the lower the effectiveness of the inventory will be in reducing the costs of IS. Fig. 7b displays that, in ES4 and ES5 (when *Cω*≠*Cα*), the effectiveness of the inventory, from the economic perspective, is higher in ES4 (i.e., when *Cω*>*Cα*) when $\rho < 1$; alternatively, when $\rho > 1$, the effectiveness of the inventory, from the economic perspective, will be higher in ES5 (i.e., when *Cω* < *Cα*). Fig. 7c and d displays that the higher the inventory costs *CL* and *CK* are, compared to *Cω* and *Cα*, the lower the value of K_{opt} will be, for all values of ρ and with all economic scenarios considered.

3.2. *Case II*

In this case we have three ELT collecting and recovering plants (process i_1, i_2, and i_3) and two cement factories (process j_1 and j_2) with the following technical parameters: $\lambda_{\text{i}1} = 10$ t/day, $\lambda_{\text{i}2} = 10$ t/day, $\lambda_{\text{i}3} = 5$ t/day, $\mu_{\text{j}1} = 10$ t/day, and $\mu_{\text{j}2} = 10$ t/day. Then, we can adopt Model B.

Four different inventory location options, corresponding to Models B1, B2, B3, and B4, are examined with the related flow rates as shown in Fig. 8. As for the economic parameters, we consider the same five different economic scenarios adopted in Case I (Table 1).

The values of F_{W} and F_{PI} for all models B1–B4 are presented in Fig. 9. As none of the options have all queues fully balanced (being $\lambda_{i1} + \lambda_{i2} + \lambda_{i3} > \mu_{j1} + \mu_{j2}$), it is impossible to eliminate both the fraction of waste disposed of in landfills and fraction of inputs purchased from the traditional supplier. Specifically, for models B1, B2, and B4, the fraction of inputs purchased from the traditional supplier tends to zero as *K* increases. Thus, the environmental performance might depend on the inventory location option; i.e., how queues are balanced in the option.

In Fig. 10, for each economic scenario and for each model B1–B4, the value of the ratio between *TCUT*, when each queue of the option is characterized by its K_{opt}, and *TCUT*, when each queue has no buffer (i.e., when each queue has *K* equal to one), is reported. The corresponding values of K_{opt} for each queue of models B1–B4 are reported in Table 2.

It can be noted that inventory with K_{opt} might reduce the costs of IS by 50 %–80 %, depending on the specific inventory location option and the specific economic scenario. For all economic scenarios, model B1 is the one with the lowest value of K_{opt} and highest economic performance, followed by models B2, B4, and B3. The higher performance of model B1 results from the higher environmental performance of this model, compared to that of the other models, which drives a higher reduction in costs to dispose of the remaining amount of waste in the landfill and to purchase the amount of input, not replaced by wastes, from traditional suppliers. Furthermore, the lower value of K_{opt} would allow us to reduce the inventory costs, compared to that of the other models. It is noteworthy that model B3 exhibits the worst economic performance for all economic scenarios. This result reflects the environmental performance measures of this model, which are worse than the ones of all the other models. Both environmental and economic performance measures of model B3 are strongly affected by the unbalanced queues, which characterize this inventory option. It can also be noted that the economic performance of model B2 is always better than that of model B4, although the environmental performances of these models are similar. The difference in economic performance can be explained by the value of K_{opt}, which is higher in B4 than in B2; hence, inventory costs are higher for B4 than for B2.

Finally, from Table 2 it can be noted that the value of K_{opt} can be significantly affected by the economic scenario. Specifically, the lower

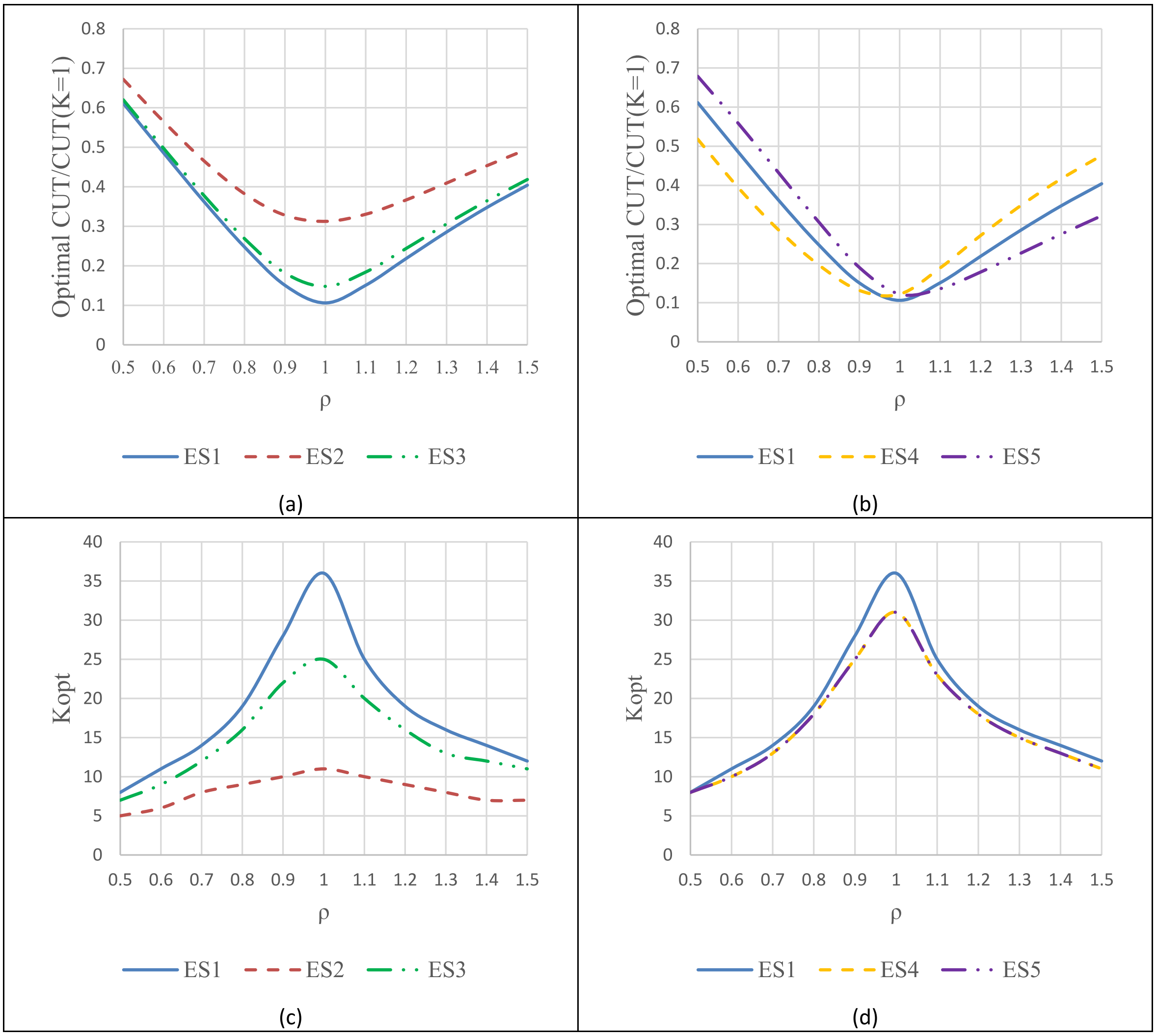

Fig. 7. (a) Optimal *CUT/CUT(K=1)* vs. ρ for ES1, ES2, and ES3; (b) optimal *CUT/CUT(K=1)* vs. ρ for ES1, ES4, and ES5; (c) K_{opt} vs. ρ for ES1, ES2, and ES3; (d) K_{opt} vs. ρ for ES1, ES4, and ES5.

the inventory costs will be, compared to $C\omega$ and $C\alpha$, the higher the K_{opt} will be, *ceteris paribus*. This outcome is similar to what has been described for Case I.

4. Discussion

The proposed approach based on the finite-queueing model can support the evaluation of the technical and environmental performance of IS relationships and networks with inventory, when the level of the inventory and waste flows produced and used are stochastic variables, with the flows being eventually unbalanced. Moreover, assuming the maximum level of uncertainty in waste production and used processes permits us to adopt the M/M/1/K model, and then to express the IS performance using simple closed-form expressions. They can support the design and operations management of the relationship and network, overcoming the limitations of other approaches, such as discrete-event simulation, agent-based modeling, and system dynamics.

Associated with the finite-queueing model, the proposed economic model considers the more important costs related to the IS relationship, i.e., the cost of waste disposal in the landfill and the cost of primary input, as well as the major costs related to the inventory design, i.e., the cost of waste in the buffer and cost of buffer size. Unlike other economic models available in the literature, the economic model developed in this paper permits us easy comparison of the different configurations of inventory in IS relationships and networks. This is a peculiar feature of such a model, when compared with the other inventory-queueing models available in the literature.

Then, we can specifically address the three research questions.

Regarding RQ1 (Can inventory effectively improve the IS performance?), two separate discussions must be conducted for environmental and economic performances. From the environmental-performance perspective, considering F_W and F_{PI}, the inventory effectiveness increases when the size K of the inventory increases, as shown in equations (6) and (7). This result is consistent with that of Fussone et al. (2025b), who highlighted that policies with unconstrained capacity for waste storage resulted in higher environmental IS performance than policies with constrained capacity for waste storage. From the economic-performance perspective, considering the *CUT*, equation (10) can be used to evaluate the effectiveness of the inventory for a simple IS relationship and then for any IS relationship in an IS network. Specifically, it depends on the size K of the inventory, rates λ_i and μ_j of the produced and used waste flows, and costs considered in the model – specifically, the cost of a unit of waste sent to the landfill ($C\omega$), the cost of a unit of primary input bought from the traditional supplier ($C\alpha$), the cost of a unit of waste in the buffer for a unit of time (CL), and the cost of a unit size of the buffer for a unit of time (CK). Based on the specific values of the $C\omega$, $C\alpha$, CL, and CK, in Case I and Case II, the inventory will always be effective if the value of K for a single IS relationship is not too

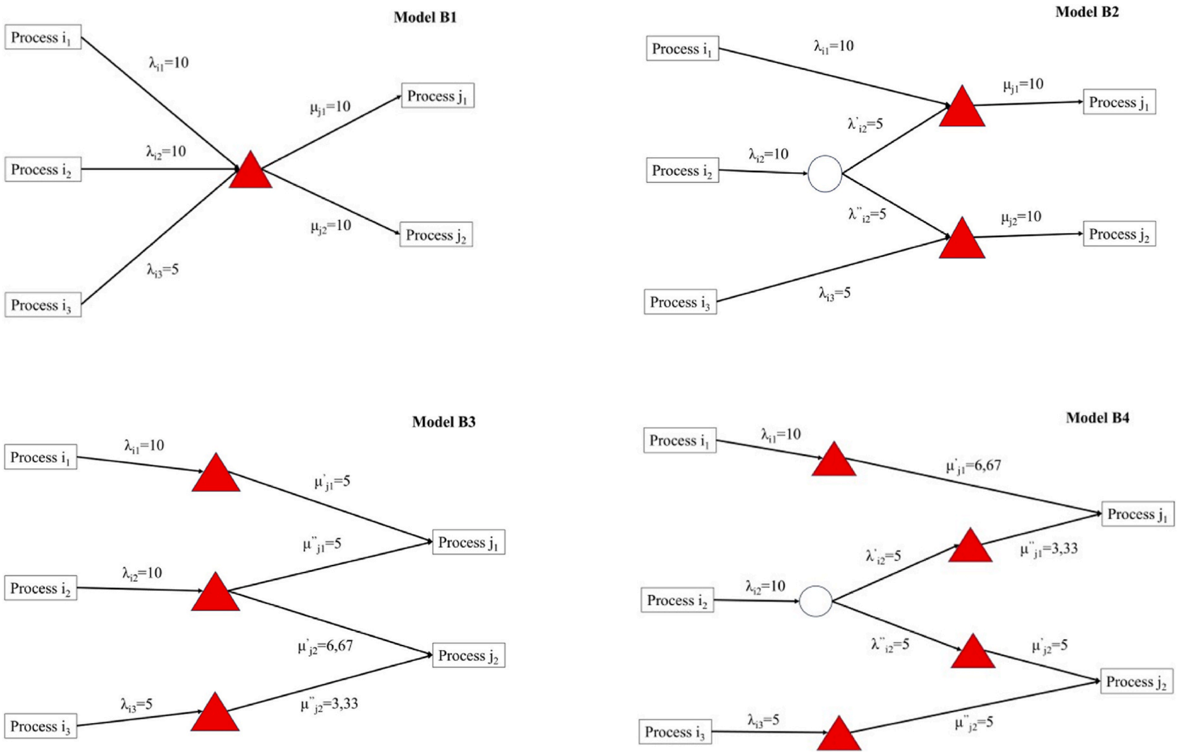

Fig. 8. Inventory location options considered in Case II.

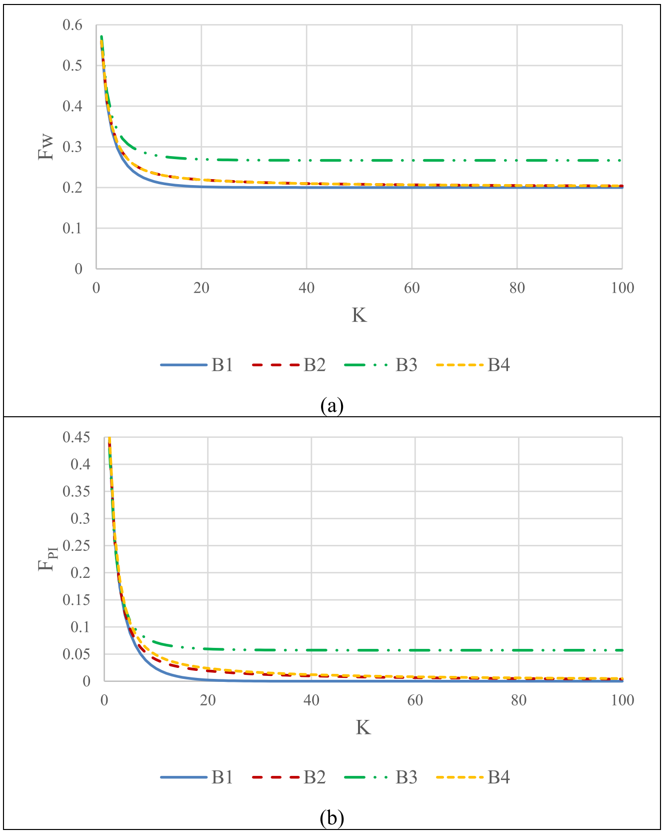

Fig. 9. (a) F_W vs. K for all models B1–B4; (b) F_{PI} vs. K for all models B1–B4.

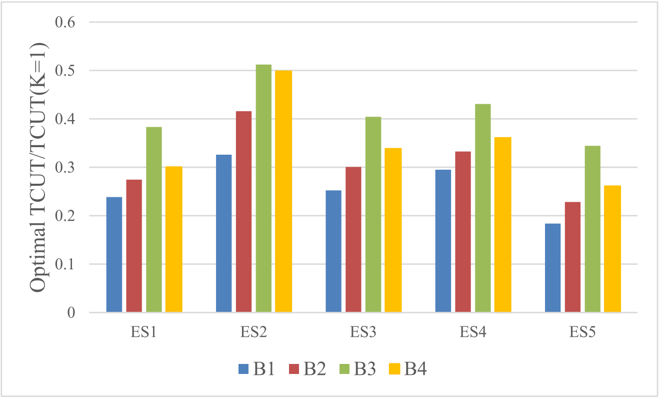

Fig. 10. Optimal *TCUT/TCUT(K=1)* for different inventory location options (models B1–B4) and for different economic scenarios.

high. For other values of *Cω, Cα, CL*, and *CK*, the economic effectiveness of the inventory has to be evaluated case by case. This outcome is reminiscent of the findings by Fraccascia et al. (2020) concerning the effectiveness of the redundancy strategy in IS. Redundancy pertains to the number of partners involved in exchanging a given waste. Partners engaged in IS relationships can utilize redundancy to achieve a more balanced flow of IS and effectively manage disruptions. Similar to the effects observed with the inventory size *K*, increasing redundancy enhances the environmental performance of IS. However, its economic effectiveness is contingent upon the variability of the waste market, as well as the costs associated with managing IS relationships. Furthermore, apart from IS, inventory costs can impact the economic efficiency of closed-loop supply chains (Ben-Daya et al., 2019; Hariga et al., 2017; Mawandiya et al., 2020). Ultimately, while it can be stated that inventory is always effective in boosting the environmental performance of IS, there is no definite answer to the question concerning economic performance.

Table 2
Values of K_{opt} for each queue of models in B1–B4.

Model			ES1	ES2	ES3	ES4	ES5
B1	Queue 1	$\lambda_{i1} + \lambda_{i2} + \lambda_{i3} = 25$ $\mu_{j1} + \mu_{j2} = 20$	20	11	17	19	19
	Total		**20**	**11**	**17**	**19**	**19**
B2	Queue 1	$\lambda_{i1} + \lambda'_{i2} = 15$ $\mu_{j1} = 10$	12	7	11	11	11
	Queue 2	$\lambda''_{i2} + \lambda_{i3} = 10$ $\mu_{j2} = 10$	36	11	25	31	31
	Total		**48**	**18**	**36**	**42**	**42**
B3	Queue 1	$\lambda_{i1} = 10$ $\mu'_{j1} = 5$	7	4	7	7	7
	Queue 2	$\lambda_{i2} = 10$ $\mu''_{j1} + \mu'_{j2} = 11{,}67$	24	11	20	23	23
	Queue 3	$\lambda_{i3} = 10$ $\mu''_{j2} = 3{,}33$	10	5	8	9	9
	Total		**41**	**20**	**35**	**39**	**39**
B4	Queue 1	$\lambda_{i1} = 10$ $\mu'_{j1} = 6{,}67$	11	6	10	11	11
	Queue 2	$\lambda'_{i2} = 5$ $\mu''_{j1} = 3{,}33$	10	5	8	9	9
	Queue 3	$\lambda''_{i2} = 5$ $\mu'_{j2} = 5$	25	7	17	21	21
	Queue 4	$\lambda_{i3} = 5$ $\mu''_{j2} = 5$	25	7	17	21	21
	Total		**71**	**25**	**52**	**62**	**62**

Regarding RQ2 (What should be the optimal size of the inventory?), from the environmental-performance perspective, the performance measures improve as the size K increases. The optimal size is affected only by the economic performance. Using equation (11), the economic optimal size of the inventory is evaluated in a simple IS relationship and then in any IS relationship in an IS network. Specifically, it depends on the rates λ_i and μ_j of the produced and used waste flows, and on the costs $C\omega$, $C\alpha$, CL, and CK. Hence, such an optimal size is highly case-specific. This result is consistent with the findings of Fraccascia et al. (2020) regarding the optimal redundancy strategy in IS, which is case-specific, depending on the variability of the waste market, as well as the costs associated with managing IS relationships. With reference to the numerical case examples, based on specific technical and economic data from Case I, the optimal size of the inventory is determined. The existence of this value is not affected by the different ESs considered to take into account the specific costs of different contexts. The optimal size of the inventory can have a significant impact on the reduction of the *CUT*, and this impact is greater for balanced relationships ($\rho = 1$). In Case II, similar findings are obtained for each IS relationship of the network.

The problem of RQ3 (Where is the best location for the inventory?) arises in an IS network where different inventory location options are considered. In this case, the best inventory location corresponds to the option having the minimum value of *TCUT*. In Case II, it has been pointed out that the best inventory location corresponds to the option of a centralized inventory. *Ceteris paribus*, the uncertainty of the waste flows can be reduced by centralizing the inventory into a single entity. This finding aligns with the research conducted by Dominguez et al. (2021), who explored a similar issue in the context of remanufacturing centers within closed-loop supply chains. However, if more inventories are considered, specific technical and economic data may affect the best option, as it also depends on how the waste flows are designed and balanced in the network (see, for instance, model B3 vs. B4). This result is consistent with the results of Öksüz et al. (2025), whose research was conducted in the context of collection points for end-of-life products. They emphasized that it was not feasible to select a centralized or decentralized structure *a priori*, as the effectiveness could vary on a case-by-case basis.

In conclusion, it is evident that the responses to the three research questions are highly context-dependent, indicating that a general solution is not feasible.

Regarding the validation of the results, the queueing model proposed is a well-known queue model having closed-form solutions. For the economic model, the selected costs of the inventory have been widely adopted in the queueing-model literature (see, for instance, Kumar et al., 2024); similarly, the environmental costs considered are the ones usually adopted in the IS literature (see, for instance, Guo et al., 2016; Jacobsen, 2006). Concerning the data adopted in the numerical-case examples, the case base has been discussed with business actors operating in the Italian context. As such costs can be affected by the context, different economic scenarios have been designed to validate the results.

5. Conclusions

IS is a production practice that offers considerable advantages, particularly in reducing the consumption of natural resources and minimizing the waste sent to landfills. However, given that the flows of produced and used waste are often unsynchronized and unbalanced, implementing inventory stocks can enhance the performance of this practice. In this paper, our analysis focused on the effectiveness of inventory stocks, assuming waste flow rates characterized by the maximum level of uncertainty. Such an assumption was based on the intrinsic variability of the IS relationships.

Three research questions were addressed: Can inventory effectively improve the IS performance? (RQ1) What should be the optimal size of the inventory? (RQ2) Where is the best location for the inventory? (RQ3). To answer these research questions, we modeled inventory stocks as a queue in a finite-queueing system (M/M/1/K). Initially, the simple IS relationship was modeled to demonstrate when and how inventory could increase the environmental and economic performance measures. This basic relationship model was then used to model a generic IS network where each inventory located between waste producers and users was modeled as a finite-queueing system. This approach permitted us to answer the research questions using IS performance measures represented by simple closed-form expressions, overcoming the limitations of other approaches, such as discrete-event simulation, agent-based modeling, and system dynamics. In this way, the design and operations management of the IS relationship could be better supported in actual cases.

The numerical case examples, based on the IS relationship (Case I) and network (Case II) of a specific ELT case, demonstrated that inventory was always effective in enhancing the environmental performance, while its economic effectiveness depended on the specific context, i.e., on the inventory-related costs and environment-related costs.

Regarding the optimal size of the inventory, from an economic perspective, it was shown that, when the average values of the produced and used waste flows in an IS relationship were balanced, inventory could be more effective in reducing the *CUT* as the IS flow increased. Similar results were obtained for each relationship in a network. As the optimal size was affected by the environment- and inventory-related costs, the adoption of different economic scenarios was useful to evaluate how the results could be extended to other contexts.

In IS networks, as different producers and users can exchange wastes, different inventory-location options have been considered. The centralized inventory has been determined to be the best option. However, if options with more inventories are considered, it has been pointed out that the specific technical and economic data can affect the best option, as it also depends on how the waste flows are designed and balanced in each specific relationship in the network.

This study has both theoretical and managerial implications.

From the theoretical perspective, to the best of our knowledge, this is the first application of queueing theory to model IS relationships and networks with inventory. Based on this technical model, an economic model with closed-form solutions for IS relationships and networks has been developed introducing IS-related environmental and inventory costs. Additionally, this study contributes to the inventory design in IS considering both environmental and economic performance measures.

From the managerial perspective, insights have been provided into the environmental and economic effectiveness of the inventory as well as its optimal size. Managers should be aware that an effective design of waste inventories can mitigate the issues arising from unsynchronized waste flows. Utilizing a waste inventory in IS relationships can enhance the environmental performance of the symbiosis by reducing the amount of waste sent to landfills (for waste producers) and quantity of primary input purchased from conventional suppliers (for waste users). These environmental benefits can translate into economic advantages, such as lower production costs, but only if the inventory is adequately designed with regard to size and location. Hence, managers can use the model developed in this study to conduct thorough cost – benefit analyses to understand the trade-offs between benefits and inventory-related costs. Moreover, as the rates of produced and used waste in IS relationships may not be balanced, this model suggests to IS players how to obtain more profitable relationships. Our model can also assist managers in designing inventory strategies at the level of is networks (e.g., how many inventories to establish and where). Managers should thus focus on balancing waste production and usage rates to enhance the overall effectiveness of IS.

Some limitations of this study are acknowledged here. The finite-queueing model adopted assumes the maximum level of uncertainty in waste production and use processes, and the basic policy for queue management. This has permitted us to easily show how IS relationships can be modeled using the queueing theory and to explore its performance measures. Nevertheless, in actual cases, the level of uncertainty can be lower, and a more complex queue-management policy can be implemented. Consequently, more complex queueing models may be needed. This can result in more difficult analytical treatments or the requirements for simulation tools. Similarly, the economic model proposed considers the basic environmental and inventory costs, disregarding other costs that may be relevant in actual cases (for example, transportation and treatment costs). Such costs can be included in a more complex economic model. Finally, all costs are associated with the IS relationship and are not shared among IS players. Then, the roles of the players in the IS design is not considered in this paper.

Further research can address the problem of performance optimization in an IS network with inventory where a hierarchical (the network) or decentralized (single player) perspective is adopted. Furthermore, more complex IS relationships, including waste treatment and transportation, and networks with different types of waste, and roundput flows can be modeled using the queueing theory. This can enhance the design and operations management of actual IS cases.

CRediT authorship contribution statement

Vito Albino: Writing – review & editing, Writing – original draft, Methodology, Funding acquisition, Data curation, Conceptualization. **Luca Fraccascia:** Writing – original draft, Methodology, Funding acquisition, Conceptualization. **Devrim Murat Yazan:** Writing – review & editing, Writing – original draft.

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Data availability

No data was used for the research described in the article.

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