



SAPIENZA
UNIVERSITÀ DI ROMA

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DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING

PH.D. IN AERONAUTICAL AND SPACE ENGINEERING

Turbulent flows on curved walls – insights from DNS

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ABSTRACT

Turbulent flows on curved walls arise in many aerospace applications and combine curvature-driven centrifugal instabilities, non-uniform pressure gradients and – at high Mach number – shock-boundary layer interactions, producing complex and unsteady flow fields. In this context we present FLEW, a high-fidelity Direct Numerical Simulation (DNS) solver for compressible flows on body-fitted curvilinear meshes, based on high-order finite-difference discretisations in generalized curvilinear coordinates. After verification on the inviscid Taylor–Green vortex and validation against canonical benchmarks, we employ FLEW (and an additional incompressible solver) to study via DNS three canonical problems:

1. turbulent flow in curved channels under mild and strong curvature conditions across a wide range of Reynolds numbers;
2. transonic buffet over a supercritical aerofoil up to chord-based Reynolds number 6×10^5 with increasing angle of attack to span stable and fully-buffet flow conditions;
3. supersonic turbulent flow on a compression ramp at Mach number 2.9 with imposed crossflow to simulate Shock Wave-Boundary Layer Interactions (SBLI) with separation under cylindrical symmetry.

The curved channel simulations show that concave wall curvature systematically reorganises the flow generating streamwise-aligned large-scale structures, which influences significantly flow transition and wall shear stress. Strong convex curvature suppresses the near-wall regeneration cycle while promoting spanwise-coherent motions.

DNS of transonic buffet reproduce the experimentally observed inversion of shock motion with increasing angle of attack and the onset of low-frequency shock oscillations past the critical condition. Modal analysis reveals a dominant coherent mode at Strouhal number 0.1 (based on the chord length and freestream velocity), which couples the shock motion with the unsteady shedding of large vortical structures from the separated shear layer. These structures travel downstream towards the trailing edge, where acoustic scattering generates upstream-traveling waves that perturb the shock wave, consistent with a feedback loop model.

Regarding the swept SBLI over compression ramp, increasing sweep enlarges the separation region, increases the coherence of pressure fluctuations and shifts the characteristic low-frequency dynamics upward. These effects are explained by a coupling between breathing mode of the separation bubble and the spanwise convection of large-scale coherent structures. We derive a scaling law for the low-frequency unsteadiness that is corroborated by DNS and experimental results.

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ACRONYMS

AMD	Advanced Micro Devices
CFL	Courant-Friedrichs-Lewy
CUDA	Compute Unified Device Architecture
DES	Detached Eddy Simulation
DNS	Direct Numerical Simulation
FD	Finite-Difference
GPU	Graphics Processing Units
HIP	Heterogeneous-computing Interface for Portability
IBM	Immersed Boundary Method
JPDF	Joint Probability Density Function
LES	Large Eddy Simulation
MPI	Message Passing Interface
OpenMP	Open Multi-Processing
PDF	Probability Density Function
POD	Proper Orthogonal Decomposition
PSD	Power Spectral Density
RANS	Reynolds-averaged Navier-Stokes
URANS	Unsteady Reynolds-averaged Navier-Stokes
RK	Runge-Kutta
RMS	Root Mean Square
SBLI	Shock Wave-Boundary Layer Interactions
SPOD	Spectral Proper Orthogonal Decomposition
TKE	Turbulent Kinetic Energy
WENO	Weighted Essentially Non-Oscillatory

INTRODUCTION

Turbulent flows developing over curved walls arise across a broad range of aerospace components: from the external surfaces of wings, fuselages, control surfaces and high-lift devices, to the internal passages of compressor and turbine blade channels and the cooling passages of propulsion nozzles. In these settings, surface curvature generates non-uniform pressure distributions that locally accelerate or decelerate the flow and thereby shape boundary-layer dynamics and the global flow topology. The resulting physics is rich and multi-faceted. Adverse pressure gradients and flow deceleration promote boundary-layer thickening and separation, while favourable pressure gradients and flow acceleration can facilitate transition to turbulence. At sufficiently high Mach numbers, shock waves produce abrupt compression and steep pressure rises that trigger [SBLI](#), causing flow separation, enhanced wall-pressure fluctuations, heat-transfer spikes and unsteady reattachment dynamics. Concave curvature further introduces centrifugal instabilities and secondary motions that drive streamwise vortical structures and cross-stream redistribution of momentum and heat. These phenomena coexist and interact, producing highly unsteady flow fields that affect the performance and integrity of aerospace components. Among other effects, such unsteady flows can drive large fluctuations in the aerodynamic coefficients, produce severe heat-transfer loads on thermal-protection and cooling systems, and influence stability margins for compressors and control surfaces. Inaccurate prediction of the key features of this coupled physics – e.g., the onset and extent of separated regions, the strength of [SBLI](#)-driven pressure loads, or the intensity of curvature-induced secondary flows – can undermine aerodynamic design, reduce component life, or precipitate structural failure.

A key tool in this context is [DNS](#), whose predictive capabilities can aid both fundamental research and technological development. However, numerical algorithms for compressible flow simulations are less standardized than those for incompressible flows (Bernardini et al., [2021](#)). The hyperbolic character of the compressible Navier-Stokes equations permits propagating disturbances and discontinuities that interact with the underlying turbulence. Accurate simulation is therefore challenging because methods suitable for turbulence must minimise numerical dissipation, while shock-capturing schemes must add dissipation to stabilise discontinuities (Johnsen et al., [2010](#)). Geometric complexity compounds the difficulty: Cartesian-based local refinement and unstructured-mesh approaches trade cost for accuracy in different ways (Piquet et al., [2020](#)), and Immersed Boundary Method ([IBM](#)) struggle to capture accurately the boundary layer dynamics required in [DNS](#) (Johnson et al., [2014](#)). Body-fitted meshes in a generalized curvilinear-coordinate framework offer an alternative that facilitates high-order finite-difference discretisations and accurate boundary layer resolution.

Within this framework, we developed the high-fidelity solver FLEW, tailored for [DNS](#) of turbulent compressible flows over curvilinear geometries. Following its development, Graphics Processing Units ([GPU](#)) porting and validation, we employ the solver to study three canonical problem classes: turbulent flow in curved channels, transonic

buffet over supercritical aerofoils, and supersonic compression ramps. The thesis is organised as follows.

In Chapter 2 we outline the numerical algorithms underpinning FLEW, with emphasis on the formulation and implementation of high-order schemes in generalized curvilinear coordinates. In Chapter 3 we present a series of validation cases, starting with the inviscid Taylor–Green vortex to assess energy preservation via time reversibility, and progressing to viscous simulations in a curved channel, around a transonic aerofoil, and over a supersonic compression corner. In Chapter 4 we examine the turbulent flow in curved channels, which serves as a model configuration to isolate the effects of concave and convex curvature on flow stability, transition and near-wall turbulence. In Chapter 5 we analyse transonic turbulent buffet over a supercritical aerofoil, with the aim of clarifying the mechanisms that trigger, sustain, and modulate the low-frequency shock oscillations. In Chapter 6 we investigate SBLI over supersonic compression ramps with imposed crossflow, tracing the characteristic temporal and spatial scales of the interaction and deriving scaling laws for the associated unsteadiness. Finally, in Chapter 7 we draw the conclusions.

COMPUTATIONAL APPROACH 2

In this chapter we outline the numerical algorithms underlying FLEW, a high-fidelity solver developed for [DNS](#) of turbulent compressible flows over curved geometries. Emphasis is placed on the formulation of energy-preserving schemes within a generalized curvilinear coordinate framework, extending conventional Cartesian approaches.

2.1 BALANCE EQUATIONS

The code has been developed in a generalized curvilinear coordinate system to accommodate arbitrary geometries. This approach involves a transformation between the Cartesian framework (*physical space*) and the curvilinear framework (*computational space*). As shown in Fig. 2.1, a body-fitted mesh is generated in the physical domain to accurately conform to the curved surface of the solid body, e.g., a NACA 0012 airfoil. The Cartesian coordinates (x_i) are expressed as functions of the curvilinear coordinates (ξ_j) via the transformation $x_i(\xi_j)$, with $i, j = 1, 2, 3$. This mapping is constructed such that domain boundaries coincide with coordinate surfaces $\xi_j = \text{const}$, producing a regular hexahedral mesh in the computational space, where standard discrete differential operators can be applied. The solution is then mapped back to the physical space for time integration. The governing equations are transformed from physical to computational space by expressing the derivatives in their curvilinear form. The compressible Navier-Stokes equations for a perfect gas in generalized curvilinear coordinates read

$$\frac{1}{J} \frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial}{\partial \xi_j} (\hat{\mathbf{F}}_j - \hat{\mathbf{F}}_j^v) = 0, \quad j = 1, 2, 3, \quad (2.1)$$

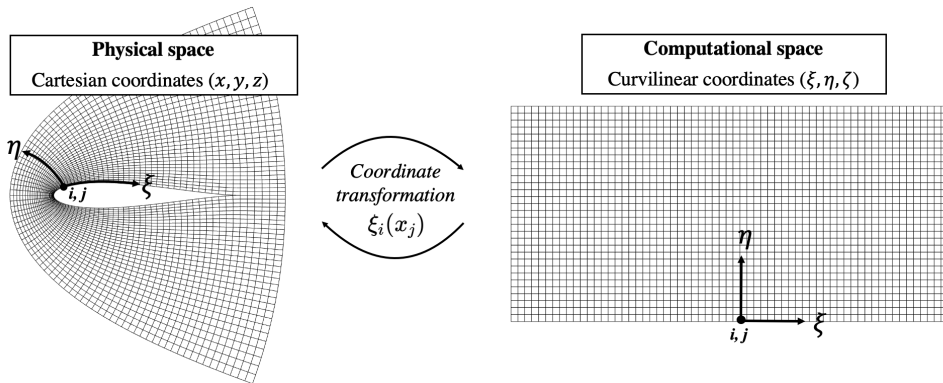


Figure 2.1: Coordinate transformation from the physical space (x_j) to the computational space (ξ_j). The mapping $x_i(\xi_j)$ allows the curvilinear physical geometry to be represented on a uniform grid.

where the repeated index convention is used to indicate summation. The vectors of conservative variables, convective and viscous fluxes are

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u_i \\ \rho E \end{bmatrix}, \quad \hat{\mathbf{F}}_j = \begin{bmatrix} \rho \hat{u}_j \\ \rho u_i \hat{u}_j + p \hat{J}_{ji} \\ \rho \hat{u}_j H \end{bmatrix}, \quad \hat{\mathbf{F}}_j^{\mathbf{v}} = \begin{bmatrix} 0 \\ \sigma_{ik} \hat{J}_{jk} \\ (\sigma_{ik} u_k - q_i) \hat{J}_{ji} \end{bmatrix}, \quad (2.2)$$

where $E = c_v T + u_i u_i / 2$ is the total energy, $H = E + p / \rho$ the total enthalpy, u_i the velocity component in the i^{th} Cartesian direction, $\hat{u}_j = u_i \hat{J}_{ji}$ the velocity component (normalized by J) in the j^{th} curvilinear direction, with $\hat{J}_{ji} = J_{ji} / J$, being $J_{ji} = \partial \xi_j / \partial x_i$ the Jacobian matrix of the transformation $\xi_j(x_i)$, and J its determinant. The constitutive equations defining the viscous stress tensor and heat flux vector are

$$\sigma_{ik} = \mu \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3} \frac{\partial u_\ell}{\partial x_\ell} \delta_{ik} \right), \quad q_i = -\kappa \frac{\partial T}{\partial x_i}, \quad (2.3)$$

where μ is the dynamic viscosity, $\kappa = c_p \mu / \text{Pr}$ the thermal conductivity, and the Prandtl number Pr is constant. The system is completed by the equation of state $p = \rho R T$, and the viscosity-temperature relationship is defined by the Sutherland's law. The Navier-Stokes equations are solved in non-dimensional form. From the minimal set of four independent reference variables, ρ_0, T_0, l_0, R , all the other reference quantities are derived as $p_0 = \rho_0 R T_0$, $u_0 = \sqrt{R T_0}$, $\mu_0 = \rho_0 u_0 l_0$, $t_0 = l_0 / u_0$, where l_0 and t_0 are the reference length and time.

2.2 SPATIAL DISCRETIZATION

The code employs high-order central Finite-Difference (FD) schemes. Central approximations are particularly suitable for DNS, as they do not introduce artificial dissipation; however, their inherent sensitivity to numerical instabilities requires special attention. This non-linear instability arises from the accumulation of aliasing errors due to the discrete evaluation of the convective terms (Coppola et al., 2019). To mitigate this issue, Pirozzoli (2010) introduced an energy-preserving formulation of the convective terms, achieved by fully expanding the derivatives of triple products through the product rule. This approach enables accurate representation of the entire turbulence spectrum without extra stabilization mechanisms, such as artificial diffusivity (Kawai et al., 2008), explicit filtering of flow variables (Visbal et al., 2002), or reliance on Weighted Essentially Non-Oscillatory (WENO)-based discretizations (Sun et al., 2011). For years, these stabilization strategies have set the benchmark for DNS of compressible flows, particularly on body-fitted grids, as most of the existing energy-preserving schemes have been traditionally formulated in Cartesian coordinates (Kuya et al., 2021).

By recasting the governing equations from Cartesian into curvilinear form (2.1), skewed cells are stretched into cubic uniform cells, allowing the application of standard

FD schemes designed for uniform stencils. Going into detail, the convective derivative in the j^{th} direction – temporarily omitting pressure terms – can be expressed as

$$\frac{\partial \rho \hat{u}_j \varphi}{\partial \xi_j}, \quad j = 1, 2, 3, \quad (2.4)$$

where φ is the transported scalar quantity, namely 1 for the continuity equation, u_i for the momentum equation, H for the energy equation. Following Pirozzoli (2011a), the convective derivatives are expanded as

$$\frac{\partial \rho \hat{u}_j \varphi}{\partial \xi_j} = \frac{1}{4} \frac{\partial \rho \hat{u}_j \varphi}{\partial \xi_j} + \frac{1}{4} \left(\rho \frac{\partial \hat{u}_j \varphi}{\partial \xi_j} + \hat{u}_j \frac{\partial \rho \varphi}{\partial \xi_j} + \varphi \frac{\partial \rho \hat{u}_j}{\partial \xi_j} \right) + \frac{1}{4} \left(\rho \hat{u}_j \frac{\partial \varphi}{\partial \xi_j} + \rho \varphi \frac{\partial \hat{u}_j}{\partial \xi_j} + \hat{u}_j \varphi \frac{\partial \rho}{\partial \xi_j} \right). \quad (2.5)$$

At this point, the numerical discretization relies on identifying a numerical flux at the intermediate nodes such that

$$\left(\frac{\partial \rho \hat{u}_j \varphi}{\partial \xi_j} \right)_N \approx (\hat{f}_{j;1/2} - \hat{f}_{j;-1/2})_N, \quad (2.6)$$

where the subscript $j;1/2$ denotes a shift by $1/2$ about N in the positive j^{th} direction. A locally conservative and computationally efficient approximation of the convective derivatives is obtained from the following expression for the numerical flux:

$$(\hat{f}_{j;1/2})_N = 2 \sum_{\ell=1}^L a_\ell \sum_{m=0}^{\ell-1} \left[\begin{array}{c} (\rho, \widetilde{\hat{u}_j}, 1)_{j;-m,\ell} \\ (\rho, \widetilde{\hat{u}_j}, u_i)_{j;-m,\ell} + (p, \widetilde{\hat{J}_{ji}}, 1)_{j;-m,\ell} \\ (\rho, \widetilde{\hat{u}_j}, H)_{j;-m,\ell} \end{array} \right]_N, \quad (2.7)$$

where

$$(f, \widetilde{g}, h)_{j;n,\ell} = \frac{1}{8} (f_{j;n} + f_{j;n+\ell}) (g_{j;n} + g_{j;n+\ell}) (h_{j;n} + h_{j;n+\ell}) \quad (2.8)$$

is the two-point, three-variable discrete averaging operator, a_ℓ are the standard coefficients for central approximations of the first derivative, and $2L$ is the order of accuracy, which can range from second to eighth.

The numerical flux (2.7) is evaluated using the velocity components in the curvilinear directions ($\hat{u}_j$). These components are derived from the Cartesian velocities through the metric terms, defined as the elements of the Jacobian matrix ($J_{ji} = \partial \xi_j / \partial x_i$) of the coordinate transformation $\xi_j(x_i)$. Being generally unknown, J_{ji} is obtained by inverting the Jacobian matrix of the inverse transformation $x_i(\xi_j)$. That is,

$$\begin{bmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{bmatrix} = \begin{bmatrix} x_\xi & x_\eta & x_\zeta \\ y_\xi & y_\eta & y_\zeta \\ z_\xi & z_\eta & z_\zeta \end{bmatrix}^{-1}. \quad (2.9)$$

The coordinate transformation is assumed to be locally invertible and time-invariant. The inverse Jacobian matrix $\partial x_i / \partial \xi_j$ is obtained by numerically differentiating the

Cartesian coordinates of the mesh along each curvilinear direction. Freestream preservation is a critical aspect in the computation of metric terms on highly skewed meshes. To satisfy this constraint, identical discrete operators to those used for the convective derivatives are employed. Consequently, the metric terms are evaluated at each node $\mathcal{N}$ as

$$\left(\frac{\partial x_i}{\partial \xi_j}\right)_{\mathcal{N}} = \left(\hat{f}_{j;1/2} - \hat{f}_{j;-1/2}\right)_{\mathcal{N}}, \quad i = 1, 2, 3, \quad (2.10)$$

where $(x_i)_{\mathcal{N}}$ are the Cartesian coordinates of the grid node $\mathcal{N}$, and $\hat{f}_{j;1/2}$ is the numerical flux at the intermediate location. Uniform spacing is assumed in the computational space, such that grid nodes coincide with integer values of ξ_j . The numerical flux is computed as

$$\left(\hat{f}_{j;1/2}\right)_{\mathcal{N}} = 2 \sum_{\ell=1}^L a_{\ell} \sum_{m=0}^{\ell-1} \left[\left(x_i, \widetilde{1}, 1\right)_{j;-m,\ell} \right]_{\mathcal{N}}, \quad i = 1, 2, 3, \quad (2.11)$$

where the order of accuracy matches that of convective derivatives. By using these scheme, the following metric identities are numerically satisfied

$$\frac{\partial \hat{J}_{ij}}{\partial \xi_i} = 0, \quad i = 1, 2, 3, \quad (2.12)$$

which is necessary and sufficient condition to ensure freestream preservation (Visbal et al., 2002).

The locally energy-preserving formulation (2.7) is compatible with shock-capturing techniques, thereby enabling the construction of hybrid schemes that currently represent an optimal strategy for the numerical simulation of shocked flows (Pirozzoli, 2011b). In the present work, shock capturing is achieved through Lax-Friedrichs flux vector splitting combined with a WENO reconstruction. Implementation of such schemes requires expressing the Euler equations in their characteristic form within the curvilinear coordinate system ξ_j . This is accomplished by introducing a metric normalization such that the *contravariant velocity*, i.e. the physical velocity projected along the curvilinear directions, appear in the convective flux vectors. Specifically,

$$\tilde{\mathbf{F}}_j = \begin{bmatrix} \rho \tilde{u}_j \\ \rho u_i \tilde{u}_j + p \tilde{J}_{ji} \\ \rho \tilde{u}_j H \end{bmatrix}, \quad i = 1, 2, 3, \quad (2.13)$$

where $\tilde{u}_j = u_i \tilde{J}_{ji}$ is the j^{th} component of the contravariant velocity, and the metrics terms are normalized by their magnitude m_j , namely $\tilde{J}_{1i} = J_{1i}/m_1$, $\tilde{J}_{2i} = J_{2i}/m_2$, $\tilde{J}_{3i} = J_{3i}/m_3$, where

$$m_1 = \left| \frac{\partial \xi}{\partial x_i} \right|, \quad m_2 = \left| \frac{\partial \eta}{\partial x_i} \right|, \quad m_3 = \left| \frac{\partial \zeta}{\partial x_i} \right|. \quad (2.14)$$

The Jacobian matrix of the normalized flux vector can be diagonalized as $\partial \tilde{\mathbf{F}}_j / \partial \mathbf{Q} = \mathbf{R}_j \mathbf{\Lambda}_j \mathbf{L}_j$, where $\mathbf{R}_j$ and $\mathbf{L}_j$ are the right and left eigenvectors matrices, and $\mathbf{\Lambda}_j$ is the diagonal matrix of the eigenvalues,

$$\mathbf{\Lambda}_{j;1/2} = \text{diag} \left[\tilde{u}_{j;1/2} - c, \tilde{u}_{j;1/2}, \tilde{u}_{j;1/2}, \tilde{u}_{j;1/2}, \tilde{u}_{j;1/2} + c \right] \quad (2.15)$$

where c is the speed of sound. For each j^{th} direction, eigenvectors and eigenvalues are evaluated at the interface (denoted by the subscript $j; 1/2$) using the Roe's average for flow variables and the arithmetic mean for metric terms. Positive and negative flux components are projected along the characteristic directions using a local Lax-Friedrichs flux splitting

$$\left(\tilde{\mathbf{g}}_{j;1/2}^{\pm} \right)_{\mathcal{N}} = \frac{1}{2} \left[\mathbf{L}_{j;1/2} \left(\tilde{\mathbf{F}}_j \pm |\lambda_{\max}| \mathbf{Q} \right) \right]_{\mathcal{N}}, \quad (2.16)$$

where $|\lambda_{\max}|$ is the maximum eigenvalue over the entire stencil. First, the characteristic fluxes are consistently normalized as

$$\hat{\mathbf{g}}_1^{\pm} = \frac{m_1}{J} \tilde{\mathbf{g}}_1^{\pm}, \quad \hat{\mathbf{g}}_2^{\pm} = \frac{m_2}{J} \tilde{\mathbf{g}}_2^{\pm}, \quad \hat{\mathbf{g}}_3^{\pm} = \frac{m_3}{J} \tilde{\mathbf{g}}_3^{\pm}. \quad (2.17)$$

Then, the $\hat{\mathbf{g}}_j^{\pm}$ are reconstructed by non-linearly combining the interpolation polynomials on [WENO](#) sub-stencils. Finally, the [WENO](#) reconstructions $\hat{\mathbf{g}}_j^{\text{WENO}\pm}$ are projected back into physical space, obtaining the numerical flux at the interface

$$\left(\hat{\mathbf{f}}_{j;1/2} \right)_{\mathcal{N}} = \left[\mathbf{R}_{j;1/2} \left(\hat{\mathbf{g}}_{j;1/2}^{\text{WENO}+} + \hat{\mathbf{g}}_{j;1/2}^{\text{WENO}-} \right) \right]_{\mathcal{N}}. \quad (2.18)$$

The accuracy of [WENO](#) schemes reaches up to seventh order. The transition between energy-preserving and shock-capturing discretization is controlled by an improved version of the shock sensor introduced by Ducros et al. (1999)

$$\theta = \max \left(\frac{-\partial u_i / \partial x_i}{\sqrt{(\partial u_i / \partial x_i)^2 + \omega_i \omega_i + u_0^2}}, 0 \right), \quad i = 1, 2, 3, \quad (2.19)$$

where ω_i is the vorticity and u_0 is the reference velocity. The sensor is designed to be $\theta \approx 0$ in smooth flow regions and $\theta \approx 1$ in the presence of shock waves. The shock-capturing scheme is locally activated when $\theta > \theta_0$, where θ_0 is a customized threshold.

Spatial derivatives of the viscous terms are discretized using central schemes applied in the computational space. The objective is to derive the analytical relation required to reconstruct derivatives from computational to physical space. To this end, we first consider the viscous terms in the momentum equation expressed in Cartesian coordinates. To avoid odd-even decoupling phenomena, we expand them into Laplacian form

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \mu \left[\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1}{3} \frac{\partial}{\partial x_i} \left(\frac{\partial u_k}{\partial x_k} \right) \delta_{ij} \right] + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right) \frac{\partial \mu}{\partial x_j}. \quad (2.20)$$

As for the energy equation,

$$\frac{\partial (\sigma_{ij} u_i - q_j)}{\partial x_j} = u_i \frac{\partial \sigma_{ij}}{\partial x_j} + \sigma_{ij} \frac{\partial u_i}{\partial x_j} + \frac{c_p}{Pr} \left(\mu \frac{\partial^2 T}{\partial x_j \partial x_j} + \frac{\partial \mu}{\partial x_j} \frac{\partial T}{\partial x_j} \right). \quad (2.21)$$

The relationship between the first derivative of a generic variable φ expressed in physical (x_i) and computational (ξ_j) space is obtained applying the chain-rule

$$\frac{\partial \varphi}{\partial x_i} = \frac{\partial \xi_j}{\partial x_i} \frac{\partial \varphi}{\partial \xi_j}, \quad j = 1, 2, 3, \quad (2.22)$$

while the Laplacian, assuming that the ζ -axis is normal to the (ξ, η) -surface, reads

$$\begin{aligned} \frac{\partial^2 \varphi}{\partial x_i \partial x_i} = & J \left(\frac{\partial \hat{g}_{11}}{\partial \xi} \frac{\partial \varphi}{\partial \xi} + \hat{g}_{11} \frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial \hat{g}_{12}}{\partial \xi} \frac{\partial \varphi}{\partial \eta} + \hat{g}_{12} \frac{\partial^2 \varphi}{\partial \xi \partial \eta} + \frac{\partial \hat{g}_{21}}{\partial \eta} \frac{\partial \varphi}{\partial \xi} + \right. \\ & \left. + \hat{g}_{21} \frac{\partial^2 \varphi}{\partial \eta \partial \xi} + \frac{\partial \hat{g}_{22}}{\partial \eta} \frac{\partial \varphi}{\partial \eta} + \hat{g}_{22} \frac{\partial^2 \varphi}{\partial \eta^2} \right) + \zeta_z^2 \left(\frac{\partial^2 \varphi}{\partial \zeta^2} - \zeta_z \frac{\partial \varphi}{\partial \zeta} \frac{\partial^2 z}{\partial \zeta^2} \right), \end{aligned} \quad (2.23)$$

where

$$\hat{g}_{11} = \frac{\xi_x^2 + \xi_y^2}{J}, \quad \hat{g}_{22} = \frac{\eta_x^2 + \eta_y^2}{J}, \quad \hat{g}_{12} = \hat{g}_{21} = \frac{\xi_x \eta_x + \xi_y \eta_y}{J}. \quad (2.24)$$

Equations (2.22) and (2.23) enable the reconstruction of physical-space derivatives from their discrete approximations in computational space, where central **FD** schemes are employed. At each node N , the first and second derivatives along the j^{th} direction are computed as

$$\left(\frac{\partial \varphi}{\partial \xi_j} \right)_N \approx \sum_{\ell=-L}^L a_\ell (\varphi_{j;\ell})_N, \quad \left(\frac{\partial^2 \varphi}{\partial \xi_j^2} \right)_N \approx \sum_{\ell=-L}^L b_\ell (\varphi_{j;\ell})_N, \quad (2.25)$$

where b_ℓ are the coefficients for the second derivative operator, providing formal accuracy up to the eighth order.

2.3 BOUNDARY CONDITIONS

Acoustic phenomena that are intrinsic in compressible flows generate waves traveling towards the computational boundaries. When steady boundary conditions are simply enforced, these waves are partially reflected towards the interior of the domain, producing spurious oscillations in the flow field. Following the methodology of Pirozzoli et al. (2013), unsteady boundary conditions are prescribed through a one-dimensional characteristic decomposition of the governing equations at the boundaries. Outgoing characteristic waves are discretized via inward finite-difference derivatives, while incoming waves are estimated from reference primitive variables defined at the first ghost point outside the domain. This characteristic-based relaxation scheme achieves exact non-reflection for one-dimensional acoustic waves, without requiring tuned relaxation parameters.

Considering the boundaries at the coordinate surface $\xi = \text{const}$, the convective derivative in the ξ direction reads

$$\frac{\partial \hat{\mathbf{F}}_1}{\partial \xi} = \frac{\partial}{\partial \xi} (\hat{J}_{1i} \mathbf{F}_i), \quad \mathbf{F}_i = \begin{bmatrix} \rho u_i \\ \rho u_i u_j + p \delta_{ij} \\ \rho u_i H \end{bmatrix}, \quad i = 1, 2, 3, \quad (2.26)$$

where the metric terms contribution associated to the ξ direction ($\hat{J}_{1i}$) is split from the Cartesian convective fluxes ($\mathbf{F}_i$). Expanding the product derivative, we get

$$\frac{\partial \hat{\mathbf{F}}_1}{\partial \xi} = \hat{J}_{1i} \frac{\partial \mathbf{F}_i}{\partial \xi} + \mathbf{F}_i \frac{\partial \hat{J}_{1i}}{\partial \xi}. \quad (2.27)$$

The first term of the right-hand-side can be recast as

$$\hat{J}_{1i} \frac{\partial \mathbf{F}_i}{\partial \xi} = \frac{m_1}{J} \frac{\partial \tilde{\mathbf{F}}_1}{\partial \xi} = \frac{m_1}{J} \mathbf{R}_1 \mathbf{\Lambda}_1 \mathbf{L}_1 \frac{\partial \mathbf{Q}}{\partial \xi}, \quad (2.28)$$

where the Jacobian matrix of the flux, $\partial \tilde{\mathbf{F}}_1 / \partial \mathbf{Q}$, has been diagonalized as outlined above. Hence the conservative variables are updated at the boundary according to

$$\frac{1}{J} \frac{\partial \mathbf{Q}}{\partial t} + \frac{m_1}{J} \mathbf{R}_1 \mathcal{L}_1 \frac{\partial \mathbf{Q}}{\partial \xi} + \mathbf{F}_i \frac{\partial \hat{J}_{1i}}{\partial \xi} = 0, \quad (2.29)$$

where $\mathcal{L}_1 = \mathbf{\Lambda}_1 \mathbf{L}_1 \partial \mathbf{Q} / \partial \xi$ are the amplitudes of the characteristic waves. Analogous considerations can be made for boundaries at the surface $\eta = \text{const}$.

2.4 TIME INTEGRATION

The spatial discretization leads to a semi-discrete system of ordinary differential equations

$$\frac{1}{J} \frac{d\mathbf{Q}}{dt} = \mathbf{R}(\mathbf{Q}), \quad (2.30)$$

where $\mathbf{R}$ is the vector of the residuals. The system is advanced in time using a three-stage, third-order Runge-Kutta ([RK](#)) scheme ([Spalart et al., 1991](#))

$$\mathbf{Q}^{(\ell+1)} = \mathbf{Q}^{(\ell)} + \alpha_\ell \Delta t \mathbf{R}^{(\ell-1)} + \beta_\ell \Delta t \mathbf{R}^{(\ell)}, \quad \ell = 0, 1, 2, \quad (2.31)$$

where $\mathbf{Q}^{(0)} = \mathbf{Q}^n$ and $\mathbf{Q}^{(3)} = \mathbf{Q}^{n+1}$ are the solution vectors at the time-step n and $n+1$, respectively. The integration coefficients are $\alpha_\ell = (0, 17/60, -5/12)$ and $\beta_\ell = (8/15, 5/12, 3/4)$. The time-step Δt is updated at each iteration as the minimum value among the convective, viscous and thermal time limitations

$$\Delta t = \min(\Delta t_{\text{conv}}, \Delta t_{\text{visc}}, \Delta t_{\text{thrm}}), \quad (2.32)$$

which are computed, in turn, as the minimum value among all cells. Respectively,

$$\Delta t_{\text{conv}} = \min_{\text{cells}} \left(\frac{1}{|\tilde{u}_j| + m_j c} \right), \quad \Delta t_{\text{visc}} = \min_{\text{cells}} \left(\frac{1}{m_j^2 \nu} \right), \quad \Delta t_{\text{thrm}} = \min_{\text{cells}} \left(\frac{1}{m_j^2 \alpha} \right), \quad (2.33)$$

where $j = 1, 2, 3$ is the curvilinear direction, c is the speed of sound, $\nu = \mu/\rho$ is the kinematic viscosity, $\alpha = \kappa/\rho$ is the molecular conductivity.

2.5 GPU IMPLEMENTATION

Following the development of the curvilinear solver, it was adapted for GPU acceleration using CUDA and Message Passing Interface (MPI) to enable distributed-memory parallel execution. A subsequent effort has been devoted to its integration within the Cartesian code STREAMS, leading to the current version STREAMS-2.1 (Salvatore et al., 2025). In addition to the original CUDA Fortran implementation, two new computational backends have been introduced, namely HIP for AMD GPU and OpenMP-offload for Intel GPU. Furthermore, an in-situ visualization layer based on Catalyst2 technology has been integrated into the solver. STREAMS-2.1 is publicly available on GitHub (<https://github.com/STREAMS-CFD/STREAMS-2>).

SOLVER VALIDATION

In this chapter we present a systematic benchmarking of the solver with a focus on turbulent flows in curved geometries. The energy-preserving properties are first assessed in curvilinear coordinates using the inviscid Taylor-Green vortex as a time-reversibility benchmark. The analysis then progresses to turbulent flow simulations in a curved channel, around a symmetric aerofoil, and over a compression corner, covering a wide spectrum of flow regimes from low-subsonic through transonic to supersonic conditions.

3.1 TAYLOR-GREEN VORTEX

The energy-preserving property of the solver was preliminarily assessed using the inviscid Taylor-Green vortex, adopted from Duponcheel et al. (2008) as a time-reversibility benchmark. The key novelty lies in demonstrating exact conservation of total kinetic energy – in the inviscid, subsonic limit – within curvilinear domains, thereby confirming that the coordinate transformation does not introduce numerical dissipation. Simulations are performed on a triply periodic domain composed of a wavy two-dimensional mesh, extruded in the third dimension (Fig. 3.1). The mesh is generated following the mapping procedure of Kawai et al. (2008)

$$\begin{aligned} x(\xi, \eta) &= \frac{\xi - 1}{N_\xi - 1} L_x + 0.4 \sin \left(2\pi \frac{\eta - 1}{N_\eta - 1} \right), \\ y(\xi, \eta) &= \frac{\eta - 1}{N_\eta - 1} L_y + 0.2 \sin \left(4\pi \frac{\xi - 1}{N_\xi - 1} \right), \\ z(\zeta) &= \frac{\zeta - 1}{N_\zeta - 1} L_z, \end{aligned} \tag{3.1}$$

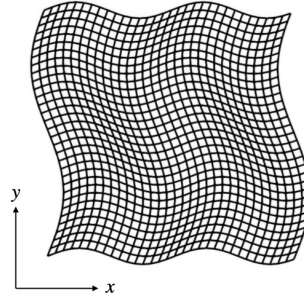


Figure 3.1: Cross-section of the mesh used for the Taylor-Green vortex simulation.

where $L_x = L_y = L_z = 2\pi$ and $N_x = N_y = N_z = 32$. Initial conditions are

$$\begin{aligned} \rho &= \rho_0 \\ u &= u_0 \sin(x) \cos(y) \cos(z) \\ v &= -u_0 \cos(x) \sin(y) \cos(z) \\ w &= 0 \\ p &= p_0 + u_0^2 \rho_0 [\cos(2x) + \cos(2y)] [\cos(2z) + 2] / 16 \end{aligned} \tag{3.2}$$

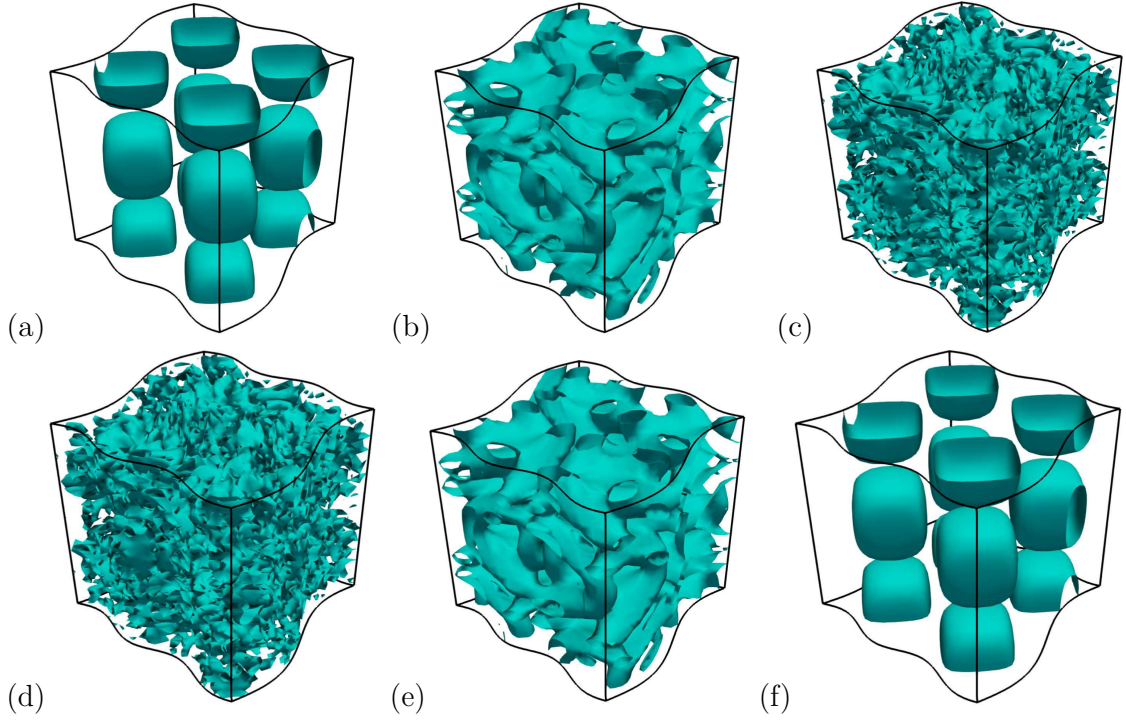


Figure 3.2: Flow visualization of the inviscid Taylor-Green vortex using one isosurface of Q -criterion. The snapshots are captured at subsequent times: $t/t_0 = 0.0$ (a), $t/t_0 = 3.2$ (b), $t/t_0 = 6.4$ (c), $t/t_0 = 9.6$ (d), $t/t_0 = 12.8$ (e), $t/t_0 = 16.0$ (f).

where $u_0 = M_0 c_0$ is the reference velocity, $M_0 = 0.01$ is the Mach number, and c_0 , p_0 , ρ_0 are the reference speed of sound, pressure, and density.

Due to the absence of viscosity, small-scale turbulent structures rapidly develop from the initially ordered flow, accompanied by a corresponding growth of vorticity. The solution is advanced to $t/t_0 = 8$, at which point the velocity field is reversed, and then further integrated up to $t/t_0 = 16$, the reference time and length being $t_0 = l_0/u_0$ and $l_0 = 1$. Exploiting the time-reversibility of the Euler equations, the initial velocity field is expected to be exactly recovered at the end of the simulation. Fig. 3.2 shows a sequence of flow-field snapshots, qualitatively illustrating the faithful reconstruction of the initial coherent structures at the final time. The temporal evolution of total kinetic energy and enstrophy is reported in Fig. 3.3. Kinetic energy is exactly conserved throughout, confirming the absence of numerical dissipation. Total enstrophy grows sharply up to the velocity reversal point at $t/t_0 = 8$, after which it decreases symmetrically, with the initial velocity field fully recovered at the final time $t/t_0 = 16$.

3.2 CURVED CHANNEL FLOW

The first validation case evaluates the fidelity of the numerical method in reproducing turbulent flows subjected to curvature-induced pressure gradients. For this purpose, we considered the classical study of Moser et al. (1987), which investigated pressure-driven flow in a mildly curved channel. The same configuration has recently been employed as

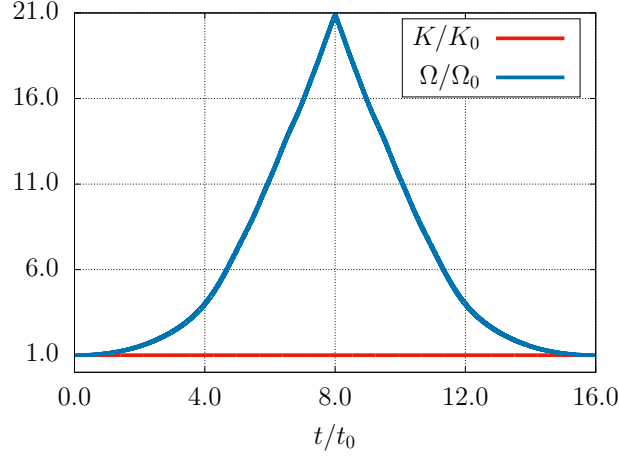


Figure 3.3: Results of the Taylor-Green vortex simulation: time evolution of the total kinetic energy (K/K_0 , red line), and enstrophy (Ω/Ω_0 , blue line).

<i>Study</i>	$Re_{\tau,i}$	$Re_{\tau,g}$	$Re_{\tau,o}$
Present DNS	155	167	179
Moser et al. (1987)	155	168	180
Brethouwer (2022)	154	168	180

Table 3.1: Comparison of local and global values of friction Reynolds number.

a benchmark by Brethouwer (2022). The computational domain is bounded by sectors of concentric cylinders. The curvature parameter is set to $\delta/r_c = 1/79$, where δ is the channel half-width and r_c is the radius of curvature measured at the centreline. The domain has a length of $4\pi\delta/3$ in the spanwise (z) direction and subtends an angle of 0.16 rad in the azimuthal (θ) direction, yielding a length of 12.64δ along the centreline. An imposed mean-pressure gradient in the azimuthal direction drives the flow. No-slip isothermal conditions are imposed at the walls, while periodic boundary conditions are enforced in the azimuthal and spanwise directions, so that the simulated flow is the fully evolved turbulent flow in a curved channel. The bulk Reynolds number is $Re_b = \rho_b u_b \delta / \mu_0 = 2600$ and the bulk Mach number is $M_b = u_b / c_0 = 0.1$, where μ_0 and c_0 are the viscosity and speed of sound at the wall temperature. The bulk density and velocity are defined as

$$\rho_b = \frac{1}{\mathcal{V}} \int_{\mathcal{V}} \rho \, d\mathcal{V}, \quad u_b = \frac{1}{\rho_b \mathcal{V}} \int_{\mathcal{V}} \rho u \, d\mathcal{V}, \quad (3.3)$$

where $\mathcal{V}$ is the domain volume. The computation is carried out with $N_\theta \times N_r \times N_z = 216 \times 72 \times 144$, corresponding to grid spacings of $\Delta z^+ = 5$, $r_c^+ \Delta\theta = 10$, $\Delta r^+ = 0.2$ at the wall. The mesh in the radial direction is staggered such that the wall coincides with an intermediate node, where the convective fluxes are identically zero. This approach guarantees correct telescoping of the numerical fluxes and exact conservation of the total mass (Modesti et al., 2016). Centrifugal instabilities due to wall curvature break the symmetry of the flow in wall-normal direction. Specifically, the convex curvature

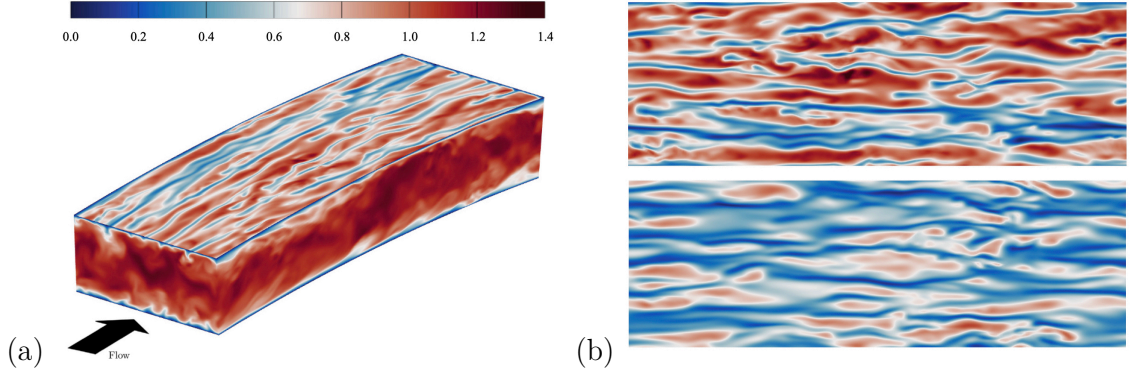


Figure 3.4: (a) Instantaneous field of the velocity magnitude normalized by the bulk velocity (u/u_b) for the turbulent curved channel flow. (b) Wall-parallel slices of u/u_b at $y^+ = 15$, locally computed from the concave (top) and convex wall (bottom).

has a stabilizing effect, leading to turbulence attenuation at the inner wall, whereas the concave curvature leads to turbulence amplification at the outer wall. Hence, the friction velocity depends on the wall curvature. The local friction velocity is defined as

$$u_{\tau,i}^2 = \nu \left. \frac{\partial \bar{u}_\theta}{\partial r} \right|_{r_i}, \quad u_{\tau,o}^2 = \nu \left. \frac{\partial \bar{u}_\theta}{\partial r} \right|_{r_o}, \quad (3.4)$$

where $\bar{u}_\theta$ is the mean streamwise velocity. A global friction velocity $u_{\tau,g}$ can be defined based on the mean-pressure gradient $(dP/d\theta)/r$, which must be equal to $-\rho u_{\tau,g}^2/\delta$. From the streamwise momentum balance we obtain $dP/d\theta = -\rho (r_i^2 u_{\tau,i}^2 + r_o^2 u_{\tau,o}^2) / (2r_c\delta)$, hence

$$u_{\tau,g}^2 = \frac{r_i^2 u_{\tau,i}^2 + r_o^2 u_{\tau,o}^2}{2r_c^2}. \quad (3.5)$$

According to the various definitions of friction velocity, we can define the friction Reynolds number as $Re_\tau = u_\tau \delta / 2\nu$. Table 3.1 presents a comparison with the results of Moser et al. (1987) and Brethouwer (2022). Excellent agreement is observed in terms of friction Reynolds number, with the local friction Reynolds number being higher at the convex (outer) wall, as expected. The instantaneous field of velocity magnitude is visualized in Fig. 3.4, showing its three-dimensional organization (a), as well as two wall-parallel slices (b), highlighting the predominance of high-speed streaks near the concave wall and low-speed streaks near the convex wall. One-point statistics are presented in Fig. 3.5, showing the mean velocity profile in local wall units (a) and turbulence intensities (b). In the log-law region, the mean velocity is higher at the convex wall (black line in Fig. 3.5a) due to normalization with the local friction velocity, which is lower at the convex wall. Velocity fluctuations, particularly in the azimuthal direction (red line in Fig. 3.5b), indicate turbulence amplification near the concave wall induced by centrifugal instability. Overall, the results exhibit excellent agreement with reference data.

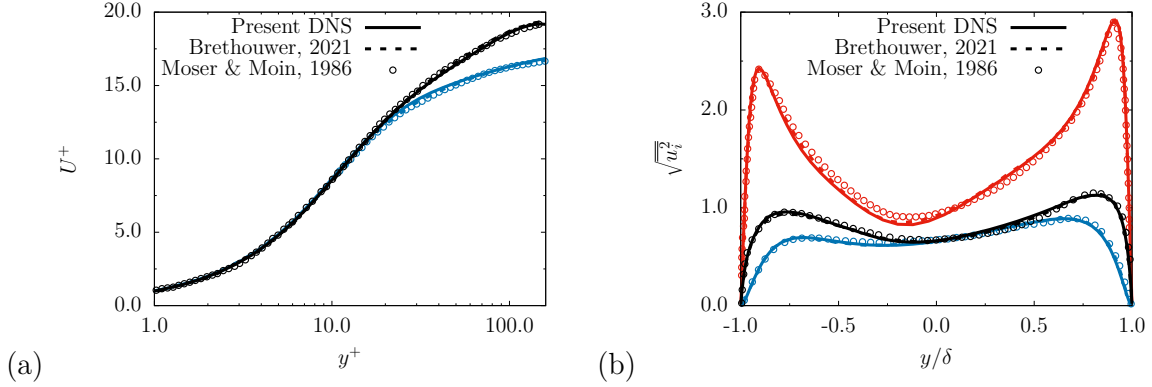


Figure 3.5: One-point statistics for the turbulent curved channel flow. (a) Mean velocity profile in local wall units: black line refers to the convex wall, blue line to the concave one. (b) Root mean square of velocity fluctuations: red line refers to $u_{\theta,rms}$, blue line to $u_{r,rms}$, black line to $u_{z,rms}$.

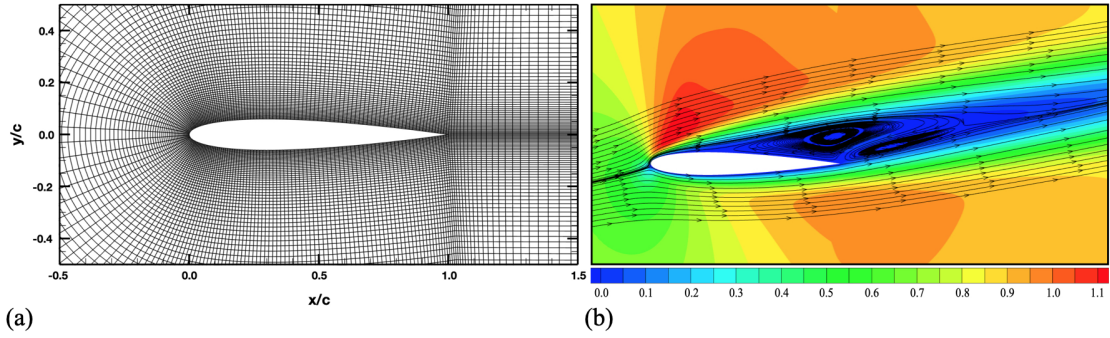


Figure 3.6: (a) Near field view of the 320×128 grid for laminar flow over a NACA 0012 airfoil. (b) Mach contours and streamlines (1280×512 grid).

3.3 TRANSONIC AIRFOIL FLOW

Laminar flows over airfoils are widely employed to assess the accuracy, stability, and convergence of numerical algorithms for the Navier–Stokes equations. Following the study of Swanson et al. (2016), the present investigation considers a two-dimensional laminar flow over a NACA 0012 airfoil at free-stream Mach $M_\infty = 0.8$, Reynolds number $Re = 500$ (based on the chord length c), and angle of attack $\alpha = 10^\circ$. The fore part of the domain has a radius of curvature $20c$ and the outlet is located $20c$ downstream of the trailing edge. Four structured grids with C-type topology are employed to assess grid sensitivity, generated using the open-source software Construct2D Prosser, 2018. The finest mesh comprises $N_x \times N_y = 1280 \times 512$ grid points, with 768 cells along the airfoil clustered near the leading and trailing edges. Successive halving of the grid points in both directions yields the coarsest mesh with $N_x \times N_y = 160 \times 64$. Fig. 3.6(a) depicts the near-field region of the 320×128 grid. No-slip adiabatic conditions are imposed on the airfoil surface, while non-reflective boundary conditions are applied at the far field. The C-mesh closure introduces an artificial boundary in the wake region, which is treated by modifying only the ghost nodes, setting their values according

<i>Study</i>	<i>Grid</i>	C_L	C_D	C_p	C_f
Present DNS	160×64	0.459	0.287	0.166	0.121
Present DNS	320×128	0.442	0.276	0.151	0.125
Present DNS	640×256	0.440	0.276	0.150	0.126
Present DNS	1280×512	0.437	0.275	0.149	0.127
Swanson et al. (2016)	1280×512	0.436	0.275	0.148	0.128

Table 3.2: Comparison of the aerodynamic coefficients for the laminar flow over a NACA 0012.

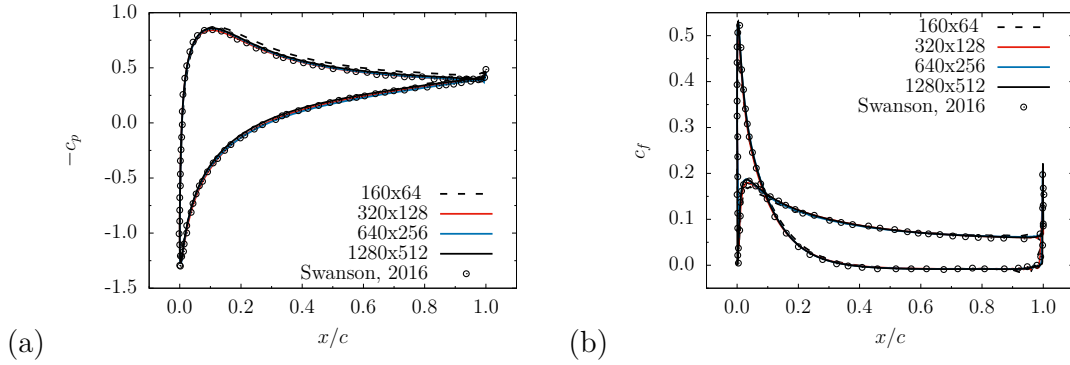


Figure 3.7: Distribution of pressure coefficient (a) and friction coefficient (b).

to the corresponding internal nodes. This procedure ensures that the wake boundary remains fully transparent to the flow, without affecting the solution in any internal node. The inflow conditions induce sufficient acceleration to generate a small supersonic region, without the formation of a shock wave. The supersonic zone is highlighted in the Mach number contours shown in Fig. 3.6(b), where streamline patterns around the airfoil are also depicted. Two extensive recirculation regions are observed on the suction side of the airfoil and in the wake.

The comparison of aerodynamic coefficients (Tab. 3.2) and the distributions of surface pressure and skin friction (Fig. 3.7) show excellent agreement with the results of Swanson et al. (2016). Accurate predictions of C_L and C_D are obtained on relatively coarse grids.

3.4 SUPERSONIC RAMP FLOW

A canonical configuration for the study of shock wave–boundary layer interactions is the compression ramp. Supersonic turbulent flows over such ramps have been extensively investigated experimentally, and a number of DNS are available in the literature for this configuration. One of the most relevant computational studies is that of Wu et al. (2007), who performed a DNS of supersonic turbulent flow over a 24° compression corner. The present results are compared with their study as well as with the experimental measurements of Bookey et al. (2005) for the same configuration. A scale drawing

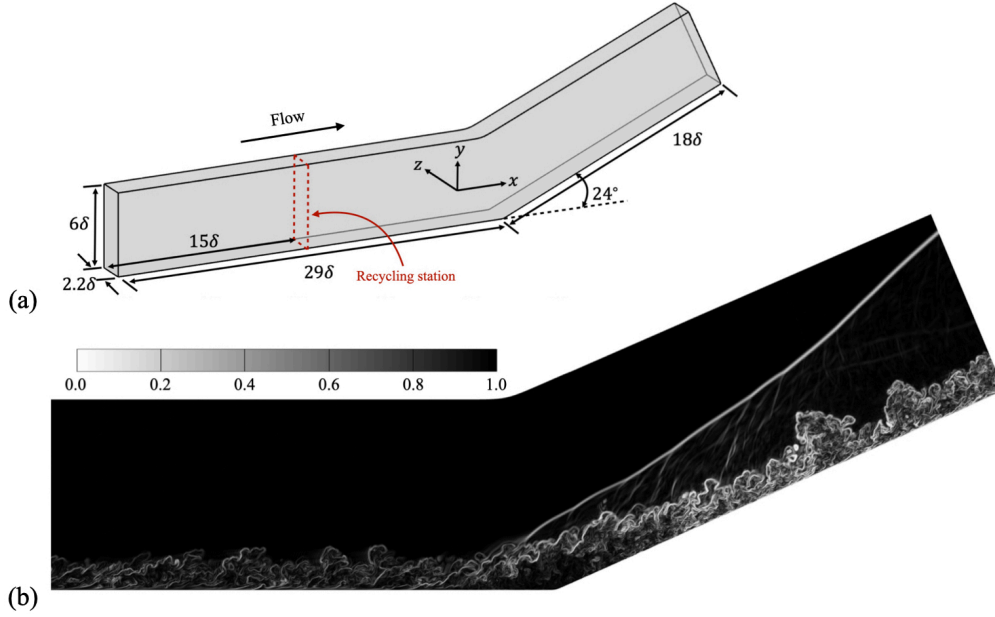


Figure 3.8: (a) Scale drawing of the computational domain for the 24° compression ramp simulation. (b) Instantaneous numerical Schlieren visualization of turbulent flow at $M_\infty = 2.9$.

of the computational domain is shown in Fig. 3.8(a). The dimensions of the domain are 29δ upstream and 18δ downstream of the corner in the streamwise direction, 2.2δ in the spanwise direction and 6δ in the wall-normal direction, where δ is the 99% thickness of the incoming boundary layer. The mesh is generated analytically following the procedure of Martín et al. (2003), who provides a complete description of the coordinate mapping. Grid points are clustered near the wall in the wall-normal direction, while a finite curvature is prescribed at the ramp corner to avoid a geometric singularity. The DNS is performed with $N_x \times N_y \times N_z = 2432 \times 256 \times 160$ points, corresponding to grid spacings of $\Delta x^+ \simeq 5$, $\Delta z^+ \simeq 3$ and $\Delta y^+ = 0.5$ at the wall. The incoming flow conditions are $M_\infty = 2.9$ and $Re_\delta = 31500$. A recycling-rescaling method is applied to feed the inflow turbulence Ceci et al., 2022, with the recycling station located 15δ downstream of the inlet. Non-reflecting boundary conditions are used at the outlet and top boundary, while adiabatic, no-slip conditions are enforced at the bottom wall.

The approaching turbulent boundary layer is deflected at the corner, where the shock system originates. Turbulence intensity amplifies as the flow crosses the shock wave, resulting in intense pressure fluctuations. This interaction is depicted in Fig. 3.8(b), where an instantaneous visualization of the numerical Schlieren is shown. The coherent flow structures visible across the corner mark the presence of an extended separated region. Mean flow separation and reattachment are identified as the streamwise locations where the mean friction coefficient changes sign. In the present DNS, these are predicted at $x_s = -3.2\delta$ and $x_r = 1.3\delta$, with the ramp corner positioned at $x = 0$. comparison with the data of Wu et al. (2007) and Bookey et al. (2005) is provided in Table 3.3. To assess the predictive accuracy of our DNS, the wall-pressure distribution and mean velocity profiles across the interaction region are examined. The

<i>Study</i>	<i>Separation point</i>	<i>Reattachment point</i>
Present DNS	-3.2δ	1.3δ
Wu et al. (2007)	-3.2δ	1.6δ
Bookey et al. (2005)	-3.0δ	1.3δ

Table 3.3: Comparison of separation and reattachment points for supersonic ramp flow. The origin of the reference system is located at the corner.

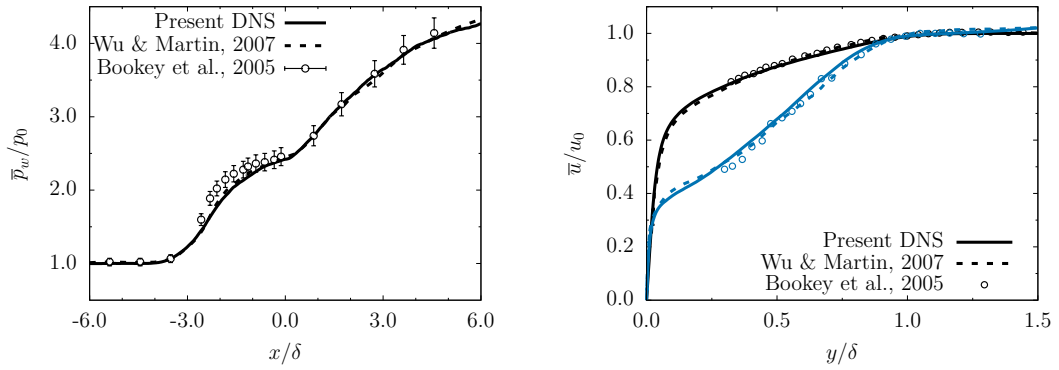


Figure 3.9: One-point statistics for the turbulent ramp flow: mean wall-pressure distribution (a) and mean velocity profiles (b) located in the incoming boundary layer (black line) and 4δ downstream of the corner (blue line).

mean wall-pressure distribution (Fig. 3.9a) falls within the experimental uncertainty of 5% (Bookey et al., 2005). Two representative velocity profiles are shown in Fig. 3.9b, corresponding to the incoming boundary layer and a location 4δ downstream of the corner, both exhibiting excellent agreement.

CURVED CHANNELS

In this chapter we examine the fully developed flow in curved channels to explore the interaction between turbulence and curvature-driven coherent structures. In contrast to the remainder of the thesis, which employs the solver introduced above, the simulations reported herein are performed using a dedicated incompressible solver. This methodological deviation is motivated by the specific focus on low-subsonic flows: removing sound propagation from the governing equations, the incompressible formulation drops the requirement to solve acoustic time scales. Since the ratio between acoustic and convective time scales is proportional to the Mach number, the Courant-Friedrichs-Lewy (CFL) time-step limitation is relaxed by one order of magnitude compared to the compressible solver, allowing faster time-to-solutions and enabling a more extensive DNS investigation.

4.1 INTRODUCTION

Turbulence bounded by curved walls is a key aspect of various engineering applications, such as highly cambered aerofoils, turbo-machinery blades and cooling channels. In these configurations, curvature alters significantly turbulence structures, impacting friction, heat transfer and flow stability. The spatially evolving and geometry-dependent nature of these flows poses major challenges in developing a unified and comprehensive description of curvature effects on turbulence, which can be achieved by considering the time-evolving curved channel flow. Direct Numerical Simulation (DNS) of plane channel flow led to a scientific breakthrough in the theory of wall-bounded turbulence through the pioneering work of Kim et al., 1987, and subsequent efforts at higher Reynolds numbers were made by Lee et al., 2015 and by Pirozzoli et al., 2021a for the circular pipe flow. In a similar vein, we scrutinise the effect of the Reynolds number on flow in curved channels, with the aim of providing a benchmark dataset for Reynolds-averaged Navier-Stokes (RANS) and Large Eddy Simulation (LES) of turbulent flows over curved surfaces.

Compared to flat walls, surface curvature produces additional strain rates an order of magnitude larger than what dimensional arguments would suggest (Bradshaw et al., 1973). The laminar solution of the equations of motion indicates that the effects of surface curvature (κ) on quantities such as the friction coefficient and boundary layer thickness (δ) are of the order $\mathcal{O}(\kappa\delta)$. However, experimental measurements in turbulent flow reveal much stronger effects, underscoring the direct impact of curvature on turbulence. Rayleigh (1917) first investigated the stability of inviscid disturbances in curved channel flow around its laminar solution, deducing flow stability as a balance between the centrifugal force per unit mass ($\rho U^2/r$, where U is the streamwise velocity at distance r from the centre of curvature) and the radial pressure gradient ($\partial P/\partial r$). Specifically, he found that for flow over a concave surface the centrifugal force of the

displaced fluid element is greater than the centripetal pressure gradient, leading to flow instability, whereas the reverse occurs for convex walls. This idea is known as ‘Rayleigh’s criterion for centrifugal instability’, and was later supported by experimental evidence (Wattendorf, 1935; S. et al., 1956; Ellis et al., 1974). Using the narrow-gap approximation, Dean (1928) showed analytically that fully developed flow between two concentric cylinders becomes centrifugally unstable when the Reynolds number exceeds a critical value, which decreases as the channel curvature increases. When the instability sets in, large-scale secondary motions are generated in the form of streamwise-aligned counter-rotating vortices, commonly referred to as Dean vortices. Due to these flow structures, a downwash (upwash) motion is generated between a vortex pair, resulting in a corresponding peak (trough) in the friction coefficient. Footprints of large-scale streamwise-aligned vortices were found also in the turbulent regime: experiments on curved duct flow (Ellis et al., 1974; Brewster et al., 1959; Hunt et al., 1979) highlighted regular spanwise variations in mean velocity and friction coefficient, which were considered evidence for the existence of Dean vortices-like structures. Other experiments (S. et al., 1956; Ramaprian et al., 1978; Kobayashi et al., 1989), however, did not detect any spanwise variation.

The dynamics of streamwise vortices may be unsteady due to events of splitting and merging, depending on channel curvature and Reynolds number. Through experiments in a curved duct, Ligrani et al., 1988 and Ligrani et al., 1994 observed the splitting of vortex pairs, during which new vortices seem to emerge from the concave wall between existing pairs. Splitting and merging phenomena were also detected by Alfredsson et al., 1989 in channels with spanwise rotation and by Matsson et al., 1990 in a channel with both curvature and rotation. The origin of this phenomenon was investigated by Guo et al., 1991 who found that vortex splitting and merging in curved channels are caused by Eckhaus instability, namely a secondary instability of steady periodic flows (such as the Dean vortex flow) with respect to spanwise perturbations (Eckhaus, 1965).

Flow in curved channels is further complicated by secondary instabilities with respect to streamwise perturbations. In fact, in numerical simulations of curved channels with roll cells, Finlay et al., 1988 observed a ‘wavy Dean vortex flow’ in the form of travelling waves superposed on the secondary flow (the Dean vortices). Those authors suggested that travelling waves are originated by a shear-layer instability induced by the Dean vortices. Consistently, the stability analysis carried out by Yu et al., 1991 identified the inflectional profile of the mean velocity as the mechanism driving the secondary instability. The same secondary instability was found also experimentally by Matsson et al., 1990, who described it as a ‘wave train riding on the primary instability’ (Matsson et al., 1992).

The distinctive three-dimensional effects associated with concave curvature are absent in the case of convex curvature, whose main effect is to reduce turbulence intensity (Gillis et al., 1983). Indeed, So et al. (1973) found experimentally that the turbulent stress decreases near the wall and vanishes about midway between the convex surface and the edge of the velocity gradient layer. The reduction in turbulent shear stress due to convex curvature leads to a shift of the zero-crossing point closer to the convex wall. This asymmetry is associated with a displacement between the locations where turbulent and viscous shear stresses vanish, giving rise to a region with negative production of Turbulent Kinetic Energy (TKE). This phenomenon was examined

by Eskinazi et al., 1969, who related it to a local ‘energy reversal’ mechanism, whereby energy is transferred from turbulent fluctuations to the mean flow.

Despite the large amount of laboratory data available in the literature, the effects of curvature on turbulence have neither been fully quantified nor understood. Due to the narrow range of both Reynolds numbers and curvatures spanned by the experiments conducted so far, it is difficult to draw general conclusions. Additionally, it is often unclear whether the flow is fully developed, inflow conditions are free of disturbances and three-dimensional effects due to side walls are negligible. DNS is the obvious candidate to understand the flow physics in greater detail. The first major contribution from DNS was made by Moser et al., 1987, who studies fully developed turbulent flow at bulk Reynolds number $Re_b = u_b \delta / \nu = 5200$ (where u_b is the bulk velocity, δ the channel height and ν the kinematic viscosity) in a curved channel with curvature radius $r_c / \delta = 39.5$ at the channel centreline. They concluded that most turbulence quantities, except for the turbulent shear stress, are equivalent on the convex and concave sides of the channel when scaled in local wall units, inferring the existence of flow similarity. However, Nagata et al., 2004, who carried out several DNS of mildly ($r_c / \delta = 39.5$) to moderately ($r_c / \delta = 2.5$) curved channel flow at $Re_b \approx 4600$, did not observe near-wall similarity, especially for the strongest curvature considered. Contrary to previous studies, they suggested that large-scale roll cells primarily contribute to turbulence production near the convex wall, rather than the concave wall. Ambiguities in assessing the effects of large-scale structures may arise from the use of periodicity conditions in the streamwise direction combined with short computational domains, artificially enhancing their coherence. In addition, narrow domains in the spanwise direction can constrain the wavelength of Dean-like vortices. These issues were addressed by Brethouwer, 2022, who employed DNS to investigate fully developed turbulent flow in channels with mild ($r_c / \delta = 30$) to moderate ($r_c / \delta = 3$) curvature using significantly larger domains. The Reynolds number was increased to $Re_b = 40000$, ensuring clear separation between near-wall turbulence scales and large-scale streamwise vortices. Based on this improved setup, velocity fluctuations in local scaling turned out to collapse into a single curve in the viscous wall region, whereas they were found to depend heavily on curvature in the outer layer. The contribution of large-scale streamwise vortices to turbulence production was found to be substantial for all curvatures considered, especially near the concave wall. Despite accurate data and valuable insights on curvature effects on turbulence, that study was restricted to a single Reynolds number and did not cover the case of strong wall curvature.

Given this background, in this work we aim to extend our understanding of turbulence in curved channels by widening the range of Reynolds numbers and examining cases with extreme curvature, thus filling existing gaps in the current literature. This is well illustrated in figure 4.1, which reports an overview of the controlling flow parameters, namely bulk Reynolds number (Re_b) and relative curvature radius (r_c / δ), from previous experimental and computational studies of curved channel flow, along with our DNS. We investigate the effects of channel curvature by focusing on two extreme cases, namely a mildly ($r_c / \delta = 40.5$) and strongly ($r_c / \delta = 1$) curved channel. According to the definition proposed by Hunt et al., 1979, these two cases correspond to ‘shear-dominated’ and ‘inertia-dominated’ flows, respectively. For both cases, we examine the effect of the Reynolds number – which has never been studied systematically for this flow configuration – by carrying out a DNS campaign covering a wide range of

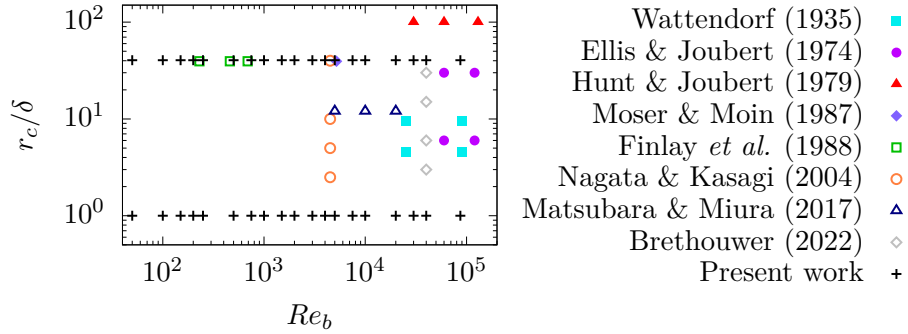


Figure 4.1: Overview of previous experimental and computational studies of curved channel flow in terms of bulk Reynolds number (Re_b), and relative curvature radius (r_c/δ). Crosses indicate the flow cases computed in the present work.

flow regimes. Special attention is paid to friction, specifically to how the changes in flow organisation and turbulence structures induced by curvature affect its behavior. This analysis reveals important insights into the flow transition, which varies according to the type and magnitude of curvature. In addition, we focus on characterising the secondary motions induced by large-scale structures that develop in curved channel flow.

4.2 METHODOLOGY

The turbulent flow in curved channels is simulated in a computational domain bounded by sectors of concentric cylinders, as shown in figure 4.2. The velocity components along the streamwise (θ), radial (r) and spanwise (z) directions are denoted by u , v and w , respectively. The flow is driven by a mean-pressure gradient ($\partial P/\partial\theta$), which is imposed as a volumetric forcing to maintain constant mass flow rate in time. Numerical simulations are carried out assuming periodicity conditions in the streamwise and spanwise directions, so that the flow is fully developed.

The in-house code used for DNS, which solves the incompressible Navier-Stokes equations in cylindrical coordinates, stems from a previous solver developed by Verzicco et al., 1996 and used for DNS of pipe flow by Orlandi et al., 1997. The switch from the pipe setup to the curved channel was attained implementing two main modifications to the code. The first is the addition of an inner cylinder (the inner wall of the channel) concentric to the outer cylinder. The second is the change in direction of the mean-pressure gradient, which is imposed along the azimuthal direction (θ), namely the streamwise direction of the curved channel. In this respect, we note that the pressure term in the streamwise momentum equation is $(\partial P/\partial\theta)/r$, with constant $\partial P/\partial\theta$, hence the volumetric forcing varies along the radial direction. The spatial discretisation is based on second-order finite-difference schemes, which are implemented in the classical marker-and-cell framework (Harlow et al., 1965). The pressure is located at the cell centres, whereas the velocity components at the cell faces, thus removing odd-even decoupling phenomena and guaranteeing discrete conservation of the total kinetic energy in the inviscid limit (Pirozzoli, 2023). The governing equations are advanced in

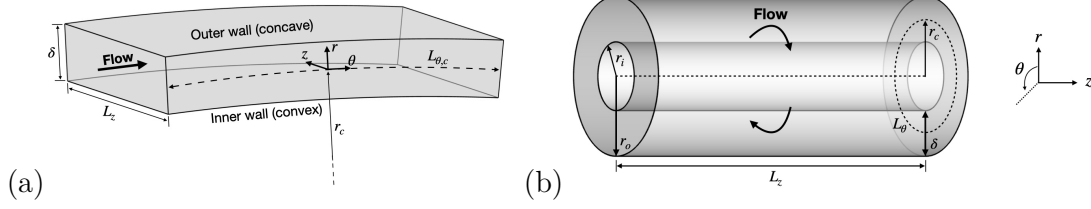


Figure 4.2: Computational setup for flow in mildly (a) and strongly (b) curved channels.

time by means of a hybrid third-order low-storage Runge-Kutta algorithm, whereby the diffusive terms are handled implicitly and convective terms explicitly. Further details regarding the numerical methods implemented in the code can be found, e.g., in Pirozzoli et al., 2021a.

The flow field is controlled by two parameters, namely the bulk Reynolds number, Re_b , and the radius of curvature at the centreline, r_c/δ . To investigate the effect of both parameters, we split the simulation campaign in two main groups: 1) shear-dominated flow with mild curvature ($r_c/\delta = 40.5$), and 2) inertia-dominated flow with strong curvature ($r_c/\delta = 1$). Throughout the paper, we will refer to the first group as R40 and to the second group as R1. Within each group, we vary the Reynolds number from $Re_b = 25$ up to 87000. The domain extends in the radial direction from $r = r_i$ (inner wall), to $r = r_o$ (outer wall), where $r_o - r_i = \delta$ and $r_c = (r_i + r_o)/2$. To compare cases with different values of the curvature radius, we consider the y -coordinate, aligned with the radial direction and origin shifted at the inner wall, i.e. $y = y_i = r - r_i$. In all cases, the computational domain has a streamwise length $L_\theta/\delta = 2\pi$ along the centreline, meaning that for the R1 cases the domain covers a full circumference (see figure 4.2). The spanwise width is set to $L_z/\delta = 4$ for the R40 cases, and it is doubled for R1 cases, in order to minimise the influence of spanwise periodicity on the large-scale structures.

Domain sizes, number of grid points and grid spacings are listed in table 5.1, along with the resulting friction Reynolds number at the inner ($Re_{\tau,i} = u_{\tau,i}\delta/2\nu$) and outer wall ($Re_{\tau,o} = u_{\tau,o}\delta/2\nu$). Distinction between inner and outer wall is necessary as the friction velocity changes depending on the wall at which it is evaluated. Inner and outer friction velocities are defined respectively as

$$u_{\tau,i} = \sqrt{\nu \left| \frac{\partial U}{\partial r} \right|_{r_i}}, \quad u_{\tau,o} = \sqrt{\nu \left| \frac{\partial U}{\partial r} \right|_{r_o}}. \quad (4.1)$$

A global friction velocity can also be defined based on the mean-pressure gradient, as derived by Brethouwer (2022) starting from the mean momentum balance. We believe that it may be instructive to illustrate briefly how it relates to the mean-pressure gradient. The analytical expression of the streamwise pressure gradient as a function of the shear stresses at the two walls can be derived in the simplest way from the balance of moments of forces about the centre of curvature, which, referring to the sketch of figure 6.1, reads

$$\int_{r_i}^{r_o} \left(-P - \frac{\partial P}{\partial \theta} \Delta \theta + P \right) r dr + \left(\tau_{w,i} r_i^2 + \tau_{w,o} r_o^2 \right) \Delta \theta = 0. \quad (4.2)$$

Mild curvature: $r_c/\delta = 40.5$, $L_\theta/\delta \times L_z/\delta = 2\pi \times 4$				
Re_b	$Re_{\tau,i}$	$Re_{\tau,o}$	$N_\theta \times N_r \times N_z$	$r_o^+ \Delta\theta \times \Delta r^+ \times \Delta z^+$
25	6	6	$129 \times 65 \times 129$	$0.6 \times 0.01 \times 0.4$
50	9	9	$129 \times 65 \times 129$	$0.9 \times 0.01 \times 0.5$
100	12	12	$129 \times 65 \times 129$	$1.2 \times 0.01 \times 0.8$
150	15	15	$129 \times 65 \times 129$	$1.5 \times 0.01 \times 0.9$
200	17	17	$129 \times 65 \times 129$	$1.7 \times 0.01 \times 1.0$
250	19	19	$129 \times 65 \times 129$	$1.9 \times 0.01 \times 1.2$
500	28	31	$129 \times 65 \times 129$	$3.1 \times 0.01 \times 2.0$
750	35	40	$129 \times 65 \times 129$	$4.0 \times 0.02 \times 2.5$
1000	41	50	$129 \times 65 \times 129$	$5.0 \times 0.01 \times 3.1$
1500	51	65	$161 \times 97 \times 161$	$5.2 \times 0.01 \times 3.2$
2000	61	78	$193 \times 97 \times 193$	$5.2 \times 0.01 \times 3.3$
2500	69	92	$225 \times 97 \times 225$	$5.3 \times 0.02 \times 3.3$
3000	82	112	$257 \times 129 \times 257$	$5.6 \times 0.01 \times 3.5$
4000	103	143	$321 \times 129 \times 321$	$5.8 \times 0.01 \times 3.6$
5000	139	170	$321 \times 145 \times 353$	$6.8 \times 0.01 \times 3.8$
7500	211	241	$449 \times 145 \times 449$	$6.9 \times 0.02 \times 4.3$
10000	270	305	$513 \times 161 \times 513$	$7.7 \times 0.02 \times 4.8$
20000	506	583	$769 \times 205 \times 1025$	$9.8 \times 0.03 \times 4.6$
30000	720	839	$1153 \times 261 \times 1537$	$9.4 \times 0.03 \times 4.4$
40000	922	1085	$1409 \times 303 \times 1793$	$9.9 \times 0.03 \times 4.8$
87000	1824	2177	$3073 \times 489 \times 3585$	$9.1 \times 0.03 \times 4.8$

Strong curvature: $r_c/\delta = 1.0$, $L_\theta/\delta \times L_z/\delta = 2\pi \times 8$				
Re_b	$Re_{\tau,i}$	$Re_{\tau,o}$	$N_\theta \times N_r \times N_z$	$r_o^+ \Delta\theta \times \Delta r^+ \times \Delta z^+$
25	8	5	$129 \times 65 \times 129$	$0.8 \times 0.01 \times 0.7$
50	11	8	$129 \times 65 \times 129$	$1.1 \times 0.01 \times 1.0$
75	14	10	$129 \times 65 \times 129$	$1.5 \times 0.01 \times 1.3$
100	16	12	$129 \times 65 \times 129$	$1.8 \times 0.01 \times 1.5$
150	20	15	$129 \times 65 \times 129$	$2.2 \times 0.01 \times 1.9$
200	23	18	$129 \times 65 \times 129$	$2.6 \times 0.01 \times 2.2$
250	26	21	$129 \times 65 \times 129$	$3.0 \times 0.01 \times 2.6$
500	39	35	$129 \times 65 \times 129$	$5.1 \times 0.01 \times 4.4$
750	49	47	$161 \times 65 \times 193$	$5.5 \times 0.02 \times 3.9$
1000	58	58	$193 \times 97 \times 257$	$5.7 \times 0.01 \times 3.6$
1500	73	79	$225 \times 97 \times 289$	$6.7 \times 0.01 \times 4.4$
2000	86	98	$225 \times 129 \times 385$	$8.2 \times 0.01 \times 4.0$
2500	98	115	$289 \times 129 \times 449$	$7.6 \times 0.01 \times 4.1$
3000	110	132	$385 \times 129 \times 513$	$6.5 \times 0.01 \times 4.1$
4000	132	165	$449 \times 145 \times 513$	$6.9 \times 0.01 \times 5.1$
5000	152	193	$449 \times 145 \times 769$	$8.1 \times 0.02 \times 4.0$
7500	199	263	$577 \times 145 \times 961$	$8.6 \times 0.02 \times 4.4$
10000	244	327	$641 \times 161 \times 1153$	$9.6 \times 0.04 \times 4.5$
20000	403	563	$1153 \times 205 \times 2049$	$9.2 \times 0.03 \times 4.4$
30000	551	784	$1665 \times 261 \times 2817$	$8.8 \times 0.02 \times 4.4$
40000	697	995	$2049 \times 303 \times 3585$	$9.2 \times 0.02 \times 4.4$
87000	1362	1952	$4097 \times 489 \times 7169$	$9.0 \times 0.02 \times 4.4$

Table 4.1: Flow parameters: bulk Reynolds number, friction Reynolds number at the inner and outer wall, number of grid points and grid spacing in inner units in the streamwise, radial and spanwise directions, respectively. The title line reports curvature radius and domain extension in the streamwise (along the centreline) and spanwise directions.

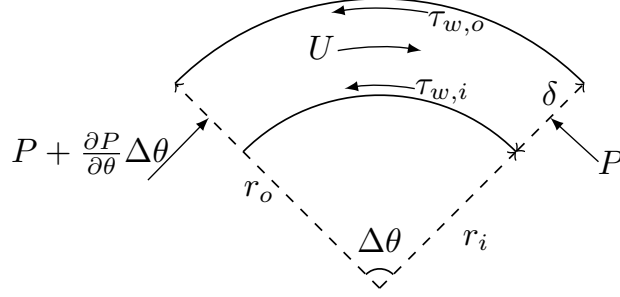


Figure 4.3: Illustrative sketch of the curved channel flow.

Hence, recalling that $r_c = (r_o + r_i)/2$ and $\delta = r_o - r_i$, we obtain

$$-\frac{dP}{d\theta} = \frac{\tau_{w,i}r_i^2 + \tau_{w,o}r_o^2}{r_c\delta}. \quad (4.3)$$

To define a global friction velocity, $u_{\tau,g}$, an equivalent length of the curved channel must be defined. Considering the length of the channel at the centreline, $L_\theta = r_c\Delta\theta$, one obtains

$$-\frac{1}{\rho r_c} \frac{dP}{d\theta} = 2 \frac{u_{\tau,g}^2}{\delta} \Rightarrow u_{\tau,g}^2 = \frac{u_{\tau,i}^2 r_i^2 + u_{\tau,o}^2 r_o^2}{2r_c^2} \quad (4.4)$$

where $u_{\tau,i}^2 = \tau_{w,i}/\rho$ and $u_{\tau,o}^2 = \tau_{w,o}/\rho$.

Throughout the paper, we will use two types of normalisation: local wall scaling, based on $u_{\tau,i}$ and $u_{\tau,o}$ (denoted with the ‘plus’ superscript), and global wall scaling, based on $u_{\tau,g}$ (denoted with the ‘star’ superscript). Capital letters denote flow properties averaged in the homogeneous spatial directions and in time, and lower-case letters denote instantaneous values. Hence, $u = U + u'$, where u' is the fluctuation from the mean. Brackets denote the averaging operator (with subscripts indicating the space coordinates over which averaging is done) and the overline is reserved for temporal averages, namely $\bar{u} = \frac{1}{T} \int_{t_1}^{t_2} u dt$, where the averaging period is $T = t_2 - t_1$. The grid spacings in local wall scaling ($r_o^+ \Delta\theta$ and Δr_o^+) listed in table 5.1 are evaluated at the outer wall, which is the most critical as the wall shear stress is higher and the grid spacing in the streamwise direction is larger. All simulations are run up to $t_1 = 600\delta/u_b$, sufficient to reach a statistically steady state, and further for $T = 600\delta/u_b$ to ensure convergence of statistics. Whether the statistics are well-converged can be verified by comparing the analytical profile of the total shear stress with the DNS results. Starting from the mean momentum equation in streamwise direction for the curved channel flow, which reads

$$0 = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + \nu \left(\frac{1}{r} \frac{\partial U}{\partial r} - \frac{U}{r^2} + \frac{\partial^2 U}{\partial r^2} \right) - \left(\frac{\partial \bar{u}v}{\partial r} + \frac{2}{r} \bar{u}v \right), \quad (4.5)$$

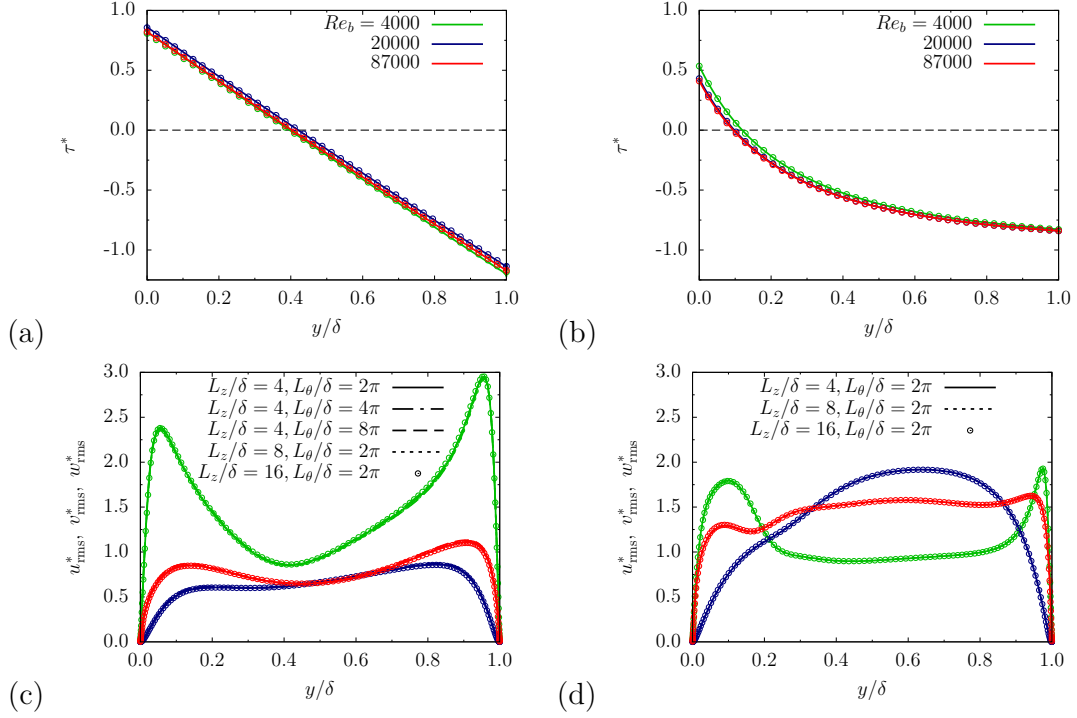


Figure 4.4: Mean profile of the total shear stress (τ^*) at various Reynolds numbers for the R40 flow cases (a) and R1 flow cases (b). Circles denote the analytical profile (4.6). RMS profiles of streamwise (green), wall-normal (blue) and spanwise (red) velocity fluctuations at $Re_b = 5000$ for the R40 flow cases (c) and R1 flow cases (d).

one can derive the following equation for the total shear stress distribution

$$\tau(r) = \frac{r_i^2 \tau_{w,i} (r_o^2 - r^2) + r_o^2 \tau_{w,o} (r_i^2 - r^2)}{2r^2 r_c \delta}. \quad (4.6)$$

In figure 4.4 we show the analytical profile of the total shear stress (circles) and the DNS results at the corresponding Reynolds numbers (solid lines). The DNS results effectively collapse to the analytical profile, which corroborates that satisfactory statistical convergence is achieved. Moreover, six additional simulations have been carried out to verify a-posteriori the adequacy of the domain sizes used in the DNS campaign. Here, we consider the Reynolds number $Re_b = 5000$, which is the least to have fully turbulent flow. As for the R40 flow cases, from the baseline configuration ($L_\theta/\delta = 2\pi$, $L_z/\delta = 4$) we have doubled and quadrupled both L_θ (for fixed L_z) and L_z (for fixed L_θ), by retaining the same grid spacing. As for the R1 flow cases, the streamwise extent of the domain covers a full circumference, hence it was not increased. From $L_z/\delta = 8$ used in the baseline DNS, the spanwise extent was decreased to $L_z/\delta = 4$ and increased to $L_z/\delta = 16$. In figure 4.4 the Root Mean Square (RMS) profiles of streamwise (green), wall-normal (blue) and spanwise (red) velocity fluctuations for the R40 (c) and R1 (d) flow cases. One can see that all the profiles obtained with the different combinations of domain size (marked by the different line types) nearly collapse to a single line, showing that the domain size used in the DNS campaign is adequate.

4.3 FLOW TRANSITION

The friction coefficient is a key parameter for characterising the transition from laminar to turbulent flow in curved channels. According to the various definitions of friction velocity given in (4.1), we define local friction coefficients at the inner and at the outer wall, as well as a global friction coefficient

$$C_{f,i} = 2 \left(\frac{u_{\tau,i}}{u_b} \right)^2, \quad C_{f,o} = 2 \left(\frac{u_{\tau,o}}{u_b} \right)^2, \quad C_{f,g} = 2 \left(\frac{u_{\tau,g}}{u_b} \right)^2, \quad (4.7)$$

shown in figure 4.5 as a function of the bulk Reynolds number. The global friction (in the inset) is related to the mean-pressure gradient (4.4), hence to the power expenditure to drive the flow. For comparison with the plane channel, we also report the friction law for laminar flow, $C_f = 12/\text{Re}_b$ (dashed line in the inset), and the logarithmic friction relation,

$$\sqrt{\frac{2}{C_f}} = \frac{1}{k} \left[\ln \left(\frac{\text{Re}_b}{2} \sqrt{\frac{C_f}{2}} \right) - 1 \right] + A \quad (4.8)$$

(solid line), obtained by Zanoun et al. (2009) from the logarithmic law of the wall, with log-law constants $k = 0.41$, $A = 5.17$.

In the turbulent regime, the friction at the outer wall is consistently higher than at the inner wall due to increased turbulence intensity. For the R40 flow cases (a), the friction coefficient of a plane channel at the same flow rate (black solid line) lies approximately midway between the values at the two walls of the curved channel and is comparable to the global friction. In contrast, for the R1 flow cases (b), the inner-wall friction is significantly lower than that of an equivalent plane channel. However, the global friction is higher and aligns with the plane channel only for $\text{Re}_b \geq 40000$.

To comment more clearly on the friction trend in the laminar and transitional regime, it is useful to consider the mean velocity profiles at representative Reynolds numbers, as reported in figure 4.6. At low Reynolds number, the flow in a curved channel is laminar and the velocity profile can be derived analytically as

$$U(r) = \alpha r - \frac{\beta}{r} - r \ln r, \quad (4.9)$$

where

$$\alpha = \frac{r_o^2 \ln r_o - r_i^2 \ln r_i}{r_o^2 - r_i^2}, \quad \beta = \frac{r_i^2 r_o^2 \ln(r_o/r_i)}{r_o^2 - r_i^2}. \quad (4.10)$$

In the R40 flow cases, the laminar velocity profile is nearly identical to the parabolic profile of plane channel flow, meaning that the shear stress distribution is almost symmetrical. As the curvature increases, the pressure gradient in the radial direction becomes stronger and the location of maximum velocity shifts towards the inner wall. As a result, the shear stress is larger at the inner wall than at the outer wall. In the laminar regime, the friction coefficient at the two walls can be determined analytically from equation (4.9), thus obtaining

$$\frac{dU(r)}{dr} = \alpha + \frac{\beta}{r^2} - \ln r - 1. \quad (4.11)$$

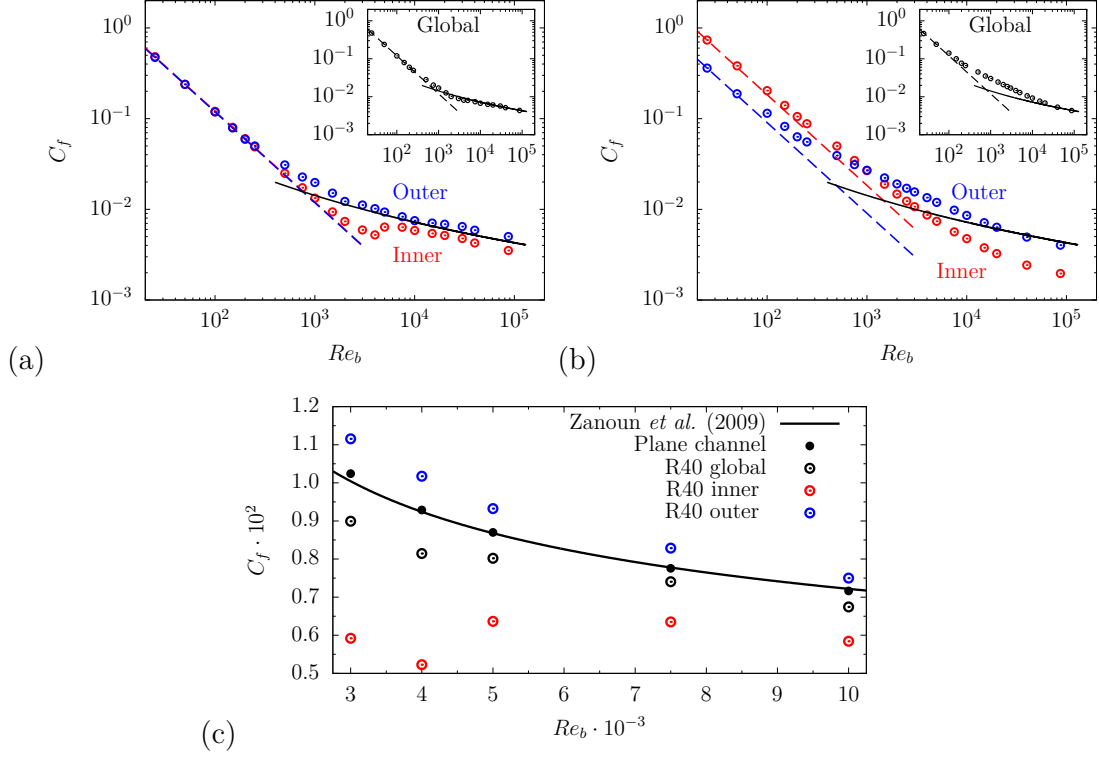


Figure 4.5: Friction coefficient as a function of the bulk Reynolds number for the R40 (a) and R1 flow cases (b). Red and blue circles denote the local friction coefficient at the inner ($C_{f,i}$) and outer ($C_{f,o}$) wall, respectively; black circles in the insets denote the global friction coefficient ($C_{f,g}$); dashed lines denote the friction law for laminar flow (red/blue for the inner/outer wall, black for plane channel); black solid lines denotes the logarithmic friction relation for plane channel (4.8). A magnified view of the friction trend for the R40 flow cases is shown in panel (c).

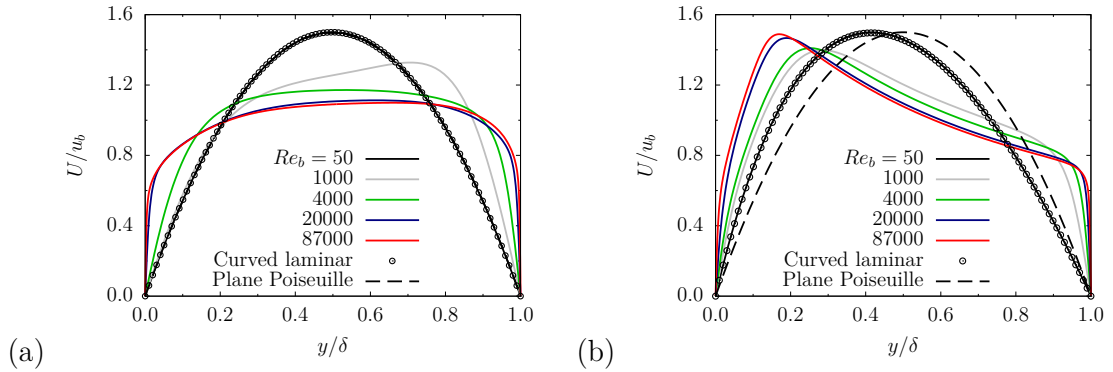


Figure 4.6: Mean streamwise velocity (U/u_b) at various Reynolds numbers for the R40 flow cases (a) and R1 flow cases (b). Black dashed lines refer to the Poiseuille profile, black circles to the velocity profile for laminar curved channel flow (4.9).

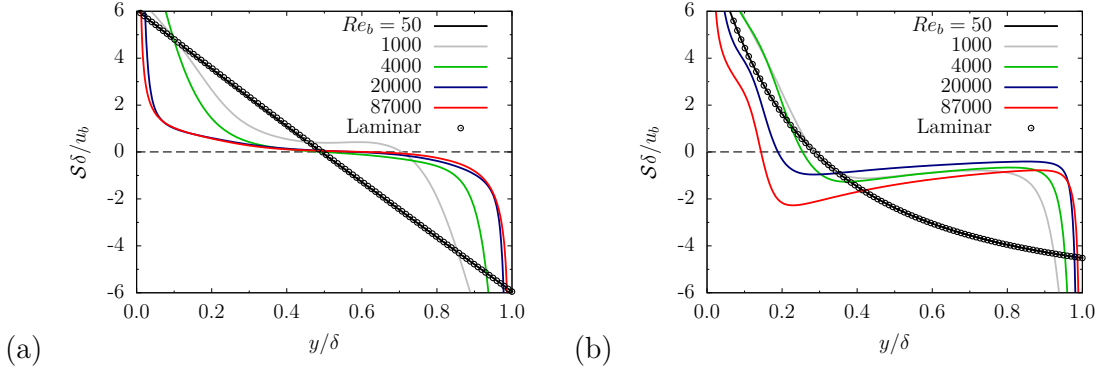


Figure 4.7: Mean shear rate ($S\delta/u_b$) at various Reynolds numbers for the R40 flow cases (a) and R1 flow cases (b). Circles denote the analytical profile for the laminar case.

For the R40 flow cases, we find $C_{f,i} \approx 12.03/Re_b$, hence nearly identical to plane Poiseuille flow, and $C_{f,o} \approx 11.83/Re_b$. In contrast, in the R1 flow cases the friction coefficient at the inner wall, $C_{f,i} \approx 18.34/Re_b$, is about twice as at the outer wall, $C_{f,o} \approx 9.04/Re_b$. Flow transition occurs when the Reynolds number attains a critical value at which centrifugal instability sets in and Dean vortices form, similarly to the Taylor vortices in flow between two rotating cylinders (Taylor, 1923). Here, the critical Reynolds number is defined as the point where the friction coefficient deviates from the laminar trend by at least 2%, an amount higher than the uncertainty of time-averaged measurements. For the R40 flow cases the critical Reynolds number is $Re_b \approx 250$, which is in good agreement with the value of 228.5 found by Finlay et al., 1988, whereas for the R1 flow cases transition starts earlier, at $Re_b \approx 50$. The transitional regime is greatly affected by curvature, with convex curvature (inner wall) tending to stabilise the flow, and concave curvature (outer wall) having the opposite effect. This is particularly evident in the R40 flow cases: at $Re_b = 1000$, the flow near the inner wall is laminar, as visible from the mean velocity profile in figure 4.6(a). In contrast, secondary motions are generated near the outer wall, which results in stronger momentum exchange. The peak velocity then shifts towards the outer wall, resulting in higher friction. At $Re_b = 4000$, the flow near the outer wall is fully turbulent (as will be shown later through flow visualisations), whereas the velocity profile near the inner wall remains close to the laminar state. Hence, mild convex curvature tends to delay transition, as plane channel flow becomes turbulent at $Re_b \approx 2600$ (Yimprasert et al., 2021). Due to this stabilising effect, the transitional flow regime near the inner wall extends well into the range of Reynolds numbers for which fully turbulent flow is expected in plane channels. As a result, the global friction in the range $3000 \leq Re_b \leq 10000$ is less than predicted from equation (4.8), as clearly shown from the magnified view in figure 4.5(c). This deviation reaches its maximum at $Re_b = 4000$, with a drag reduction of 12.2% with respect to the plane channel data (Orlandi et al., 2015). By increasing the Reynolds number to 5000, the flow near the inner wall undergoes abrupt transition to a fully turbulent state, as indicated by the jump in the friction trend indicated by red circles in figure 4.5(a). This type of transition from laminar to turbulent regime is typical of canonical wall flows (see e.g. Patel et al., 1969), however it quite different than at the outer wall, where transition is facilitated by centrifugal instabilities and occurs more smoothly.

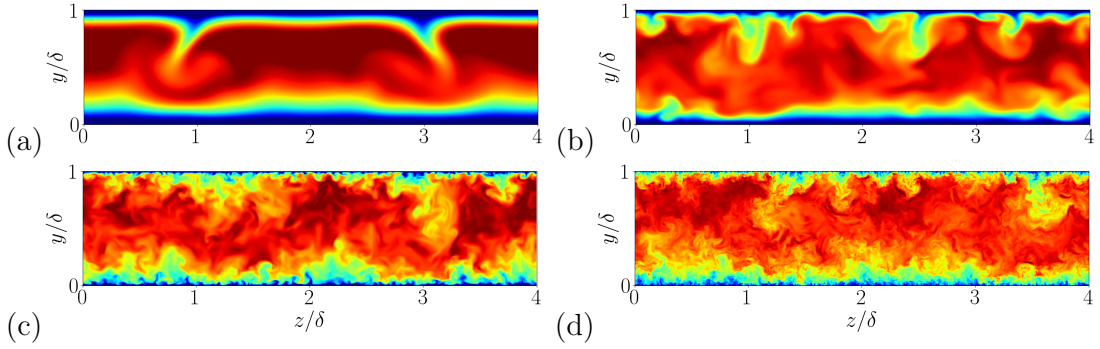


Figure 4.8: Instantaneous streamwise velocity fields in a cross-stream plane for the R40 flow cases at $Re_b = 1000$ (a), 4000 (b), 20000 (c), 87000 (d). The flooded contours range from $0.2 u_b$ (blue) to $1.2 u_b$ (red).

For the R1 flow cases, friction in the laminar regime is higher at the inner wall. By increasing the Reynolds number, the velocity profiles tend to flatten at the outer wall, where friction increases. Turbulence is inhibited near the inner wall, and friction deviates less markedly from the laminar trend. An inversion occurs past $Re_b \approx 1000$, whereby friction at the outer wall becomes higher than at the inner wall. Flow transition is facilitated by strong channel curvature, and it occurs smoothly at both walls. At the outer wall, the different transition from canonical wall flows is due to centrifugal (primary) instabilities. As we will show, in the R1 flow cases secondary motions associated with centrifugal instabilities do not reach the inner wall, and transition is affected by streamwise (secondary) instabilities, which lead to the formation of large-scale cross-stream structures.

In figure 4.7 we show the mean shear rate, $\mathcal{S} = dU/dr - U/r$. Similar to the case of the plane channel, the location where the mean shear vanishes can be used to define a bound for the regions of influence of the two walls. In the R40 flow cases (a) the shear rate profile is nearly symmetrical, hence this location coincides roughly with the channel centreline, with exception of the case $Re_b = 1000$, for which it occurs at $y/\delta \approx 0.71$. In contrast, for the R1 flow cases (b) the shear rate profile is strongly asymmetrical, the zero crossing shifting towards the inner wall and approaching it as the Reynolds number increases. Hence, the region of influence of the inner wall is much narrower than for the outer wall. In case of strong curvature, the shear rate profile exhibits a local maximum near the outer wall and a local minimum near the inner wall, corresponding to inflectional points of the mean velocity profiles. This showcases the presence of shear layers that can support inflectional instabilities.

The previous discussion on flow transition can be visualised through the instantaneous fields of streamwise velocity in cross-stream and wall-parallel planes, which we show in figures 4.8 and 4.9 for the R40 flow cases and in figures 4.10 and 4.11 for R1 flow cases.

As for the R40 flow cases, the cross-stream and longitudinal planes at $Re_b = 1000$, shown in figures 4.8(a) and 4.9(a), highlight the onset of two symmetric large-scale ejections carrying low-speed fluid from the outer wall towards the channel core. These secondary motions are generated by two pairs of counter-rotating roll cells, which resemble closely Dean vortices. The inner-wall flow region is almost unaffected by these vortices, as one can see from figure 4.9(b). Figures 4.8(b) and 4.9(c), depicting the cross-stream and longitudinal planes at $Re_b = 4000$, show that fine-scale ejections and

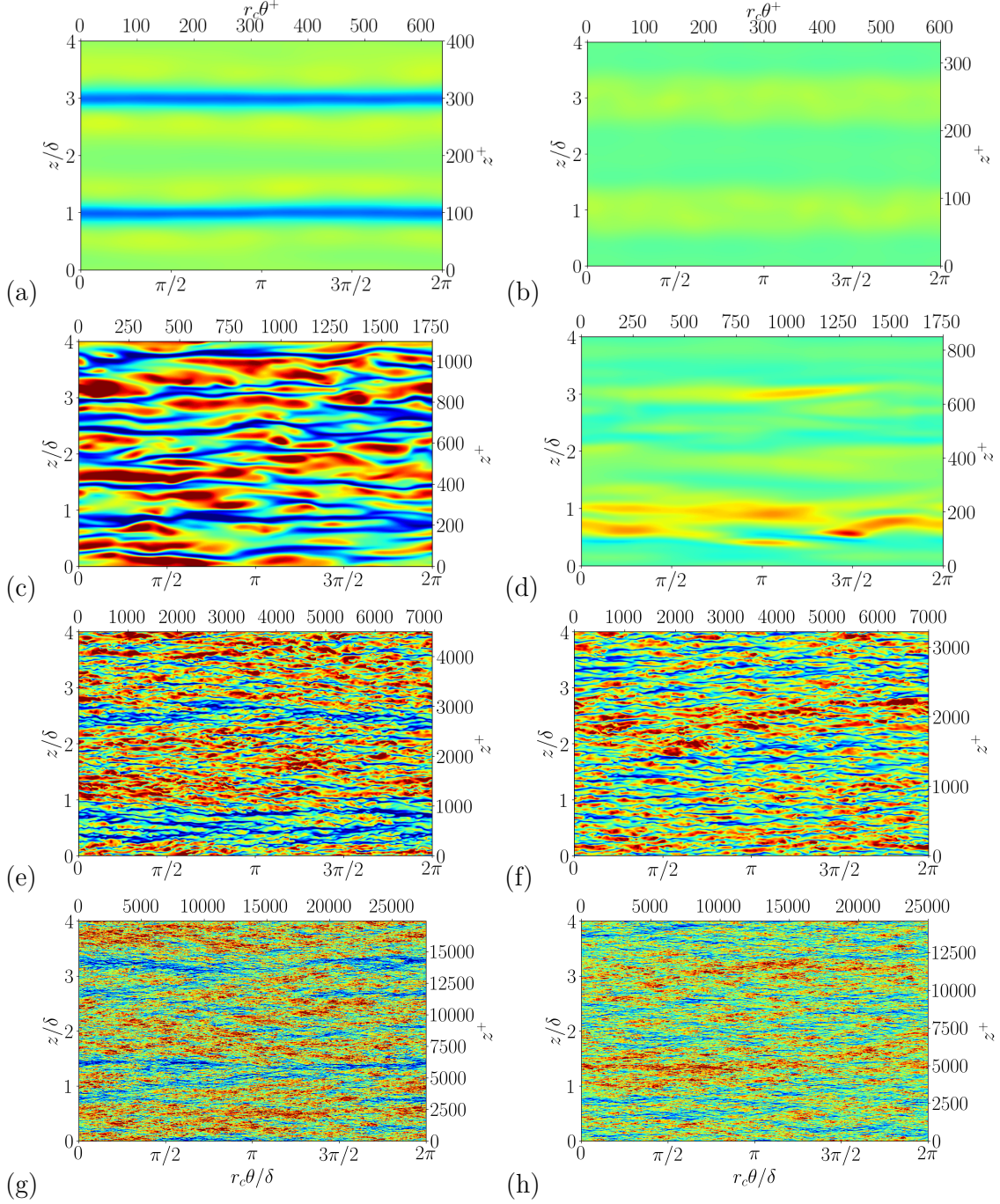


Figure 4.9: Instantaneous fields of streamwise velocity fluctuations for the R40 flow cases in wall-parallel planes near the outer (left panels) and inner wall (right panels) at $y^+ \approx 12$. From top to bottom, the panels correspond to $\text{Re}_b = 1000$ (a, b), 4000 (c, d), 20000 (e, f), 87000 (g, h). The flooded contours range from $-0.3 u_b$ (blue) to $0.3 u_b$ (red). Streamwise and spanwise coordinates are shown in both outer units $(r_c\theta/\delta, z/\delta)$, and local wall units $(r_c\theta^+, z^+)$, where the streamwise domain extent is measured at the centreline. Mean flow goes from left to right.

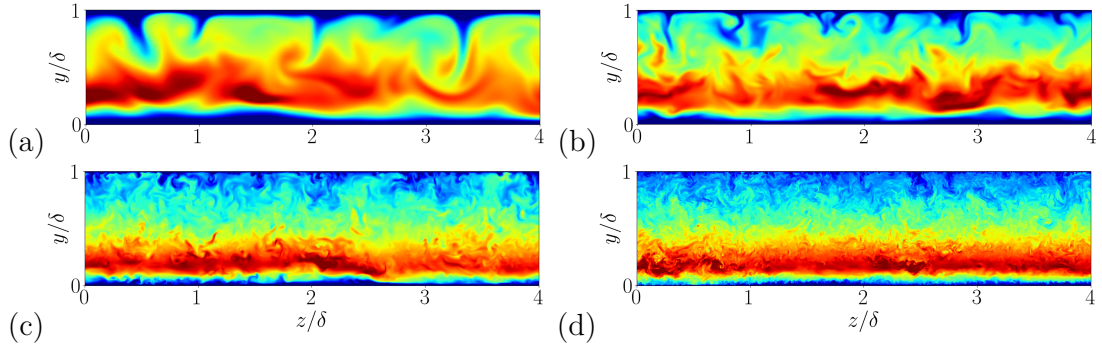


Figure 4.10: Instantaneous streamwise velocity fields in a cross-stream plane for the R1 flow cases at $Re_b = 1000$ (a), 4000 (b), 20000 (c), 87000 (d). The flooded contours range from $0.4 u_b$ (blue) to $1.5 u_b$ (red). Only half of the domain is shown.

organised structures of momentum streaks characterise the outer wall, from which one can infer that the flow is fully turbulent. Traces of the streamwise vortices appear as streamwise-aligned bands of high-speed fluid near the inner wall, as visible in figure 4.9(d). Being centrifugally stable, the inner-wall flow region is still transitional, as no clear signs of turbulent activity appear. The cross-stream planes at $Re_b = 20000$ and 87000 (panels (c) and (d) of figure 4.8) show clear signs of streaky patterns at both walls. Overlaying the small scales of turbulence, streamwise large-scale structures are found near the outer wall, whose footprint appears in the wall-parallel planes of figures 4.9(e) and 4.9(g) as wide regions of alternating high and low speed fluid. These large-scale regions fill the entire streamwise extent of the domain, and their spanwise width is comparable with the channel height, hence two pairs of roll cells are present. The influence of the latter is found to extend to the inner wall at high Reynolds numbers. In fact, low-speed regions at the outer wall, corresponding to large-scale ejections, are found at the same spanwise location as high-speed regions at the inner wall, corresponding to large-scale sweeps (panels (f) and (h) of figure 4.9).

Cross-stream visualisations show clearly how the region with higher momentum shifts towards the inner wall for the R1 flow cases (figure 4.10), as expected from the mean velocity profiles (figure 4.6). As well as for the R40 flow cases, signatures of streamwise roll cells are visible as large-scale ejections from the outer wall, although they are now more numerous and chaotic. As displayed in the wall-parallel plane of figure 4.11(a), streaky structures appear to be quite distinct at the outer wall at $Re_b = 1000$, confirming that increasing curvature promotes the transition to turbulence. Panels (c), (e), (g) of figure 4.11 show that momentum streaks at the outer wall exhibit a wavy pattern with a streamwise wavelength comparable to the domain size. This pattern resembles the *undulating Dean vortex flow* found by Finlay et al., 1988, which is likely associated to inflectional instabilities of the wall-normal velocity profile (Le Cunff et al., 1993), as evidenced by figures 4.6(b) and 4.7(b). The streaks are wider for the R1 flow cases than for R40, whereas streamwise large-scale structures are narrower, making the distinction between fine-scale turbulence and large-scale structures less clear-cut. In fact, streamwise roll cells do not clearly show up in the wall-parallel planes of fluctuating streamwise velocity. This, however, does not convey that they are absent: as we shall see, in case of strong curvature these secondary motions are organised differently and mainly impact the wall-normal velocity. As visible from the

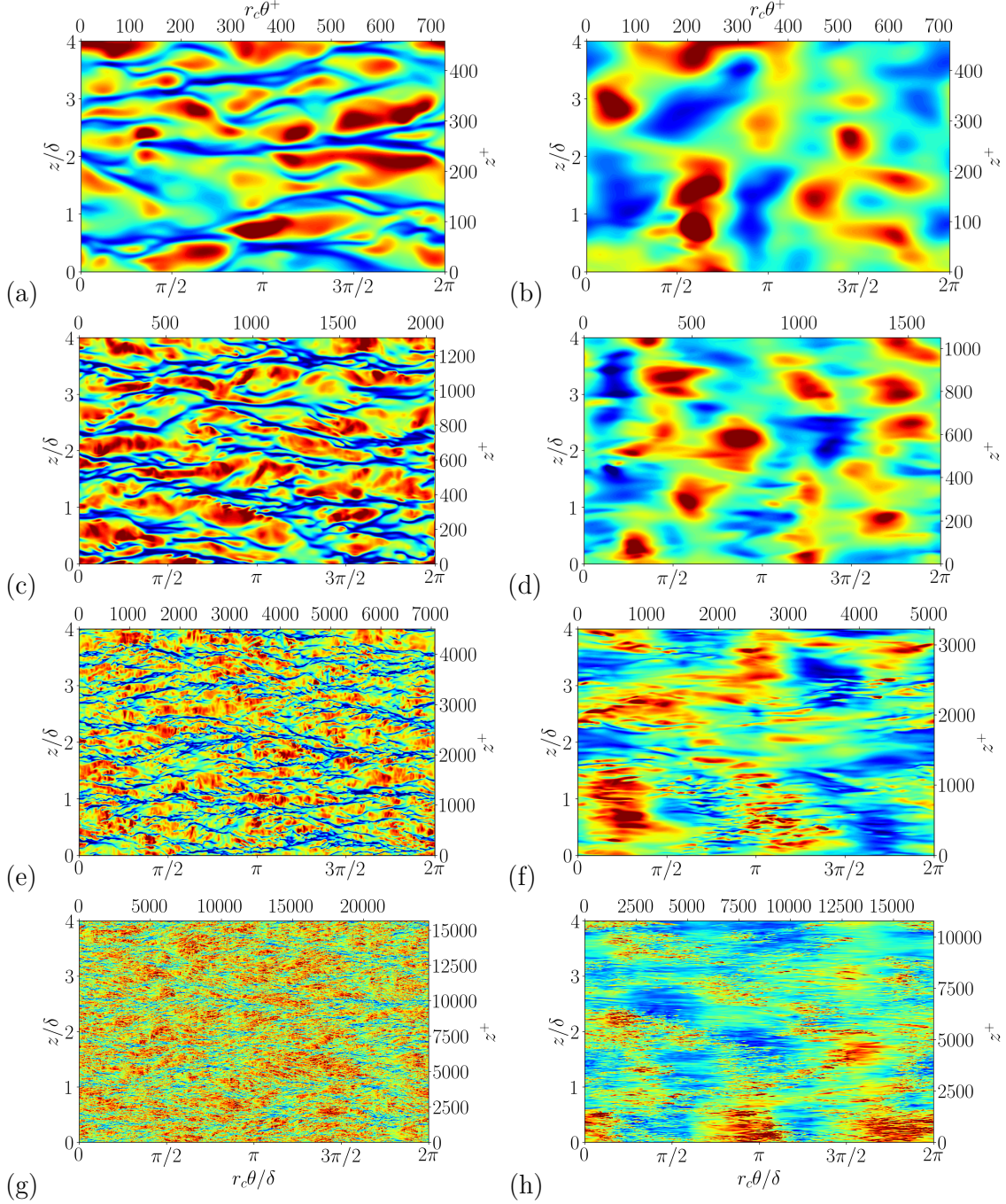


Figure 4.11: Instantaneous fields of streamwise velocity fluctuations for the R1 flow cases in wall-parallel planes near the outer (left panels) and inner wall (right panels) at $y^+ \approx 12$. From top to bottom, the panels correspond to $Re_b = 1000$ (a, b), 4000 (c, d), 20000 (e, f), 87000 (g, h). The flooded contours range from $-0.3 u_b$ (blue) to $0.3 u_b$ (red). Streamwise and spanwise coordinates are shown in both outer units ($r_c \theta / \delta, z / \delta$), and local wall units ($r_c^+ \theta, z^+$), where the streamwise domain extent is measured at the centreline. Only half of the domain is shown.

wall-parallel planes shown in panels (b), (d), (f) of figure 4.11, the inner-wall flow region is devoid of turbulent structures, or nearly so for the highest Reynolds number (h), whereas it is characterised by the presence of large-scale structures elongated in the spanwise direction. The latter are visible in the form of alternating regions of high and low speed fluid aligned along the spanwise with length comparable with δ . The streamwise extent of each pair of spanwise structures is about $\pi\delta$, meaning that the computational domain can fit two pairs, with exception of the case $Re_b = 4000$, at which the streamwise wavelength seems slightly shorter.

4.4 SPECTRAL ANALYSIS

A clearer understanding of the energetic relevance of the various scales of motions can be achieved from inspecting the spectra of the velocity fluctuations. Spectra of both streamwise and wall-normal velocity fluctuations are presented, the former to detect large-scale structures typical of plane channel flows (Lee et al., 2015), whereas the latter are instrumental to reveal the presence of streamwise roll cells (Dai et al., 2016). Figure 4.12 displays the pre-multiplied spectra of streamwise velocity fluctuations in the spanwise ($k_z^* E_{uu}^*$) and streamwise ($k_\theta^* E_{uu}^*$) directions as a function of the distance from the wall for R40 and R1 flow cases. For clarity, we refer to the distance from the inner wall as $y_i = y$, and from the outer wall as $y_o = \delta - y$, both displayed on the left vertical axis. The distance from the inner and outer wall is also reported on the right vertical axis in local wall units, y_i^+ and y_o^+ , respectively.

Looking at panels (a), (b), (c), we observe a primary energy peak in the spanwise spectra of R40 cases near both walls at $y_i^+ \approx y_o^+ \approx 12$, $\lambda_z^+ \approx 100$, which is the typical signature of the regeneration cycle of momentum streaks (Hamilton et al., 1995). An exception is the case at $Re_b = 4000$ where the inner-wall primary peak is nearly absent. The streamwise spectra in panels (d), (e), (f) shows that the streaks-related peak is at $\lambda_\theta^+ \approx 800$, meaning that the mean wavelength of momentum streaks is slightly shorter than the typical value of plane channel flows, namely $\lambda_x^+ \approx 1000$ (Monty et al., 2009). A secondary peak appears in the spanwise spectra farther from walls, at $y_i \approx y_o \approx 0.1\delta$ at $Re_b = 4000$ (a), and $y_i \approx y_o \approx 0.2\delta$ at $Re_b = 20000$ (b) and 87000 (c). As seen from the intersection of the dashed lines, the energy of the peaks is concentrated at $\lambda_z/\delta \approx 2$. If the domain contains n pairs of roll cells, we expect to find a peak in the spanwise spectra at $\lambda_z = L_z/n$, where $L_z = 4\delta$ for the R40 flow cases. Flow visualisations indeed revealed the presence $n = 2$ roll cells pairs, hence we interpret these peaks as the footprint of streamwise large-scale structures. A third peak at $\lambda_z/\delta \approx 1$ appears faintly at $Re_b = 20000$, and sharply emerges at $Re_b = 87000$. This peak corresponds to $n = 4$ pairs of streamwise cells, indicating that roll cells with various sizes could be present, which we will investigate below. The streamwise spectra reveal that the scales of motion with the maximum streamwise wavelength, $\lambda_\theta = L_\theta = 2\pi$, are quite energetic, suggesting that the streamwise vortices span the whole domain along the streamwise direction.

As for the R1 flow cases, the panels (g), (h), (i) reveal that the primary peak in the spanwise spectra due to momentum streaks appears on the outer wall only. Hence the near-wall turbulence regeneration cycle still takes place at the outer wall. As noted in flow visualisations, streaky structures have an enlarged spanwise wavelength

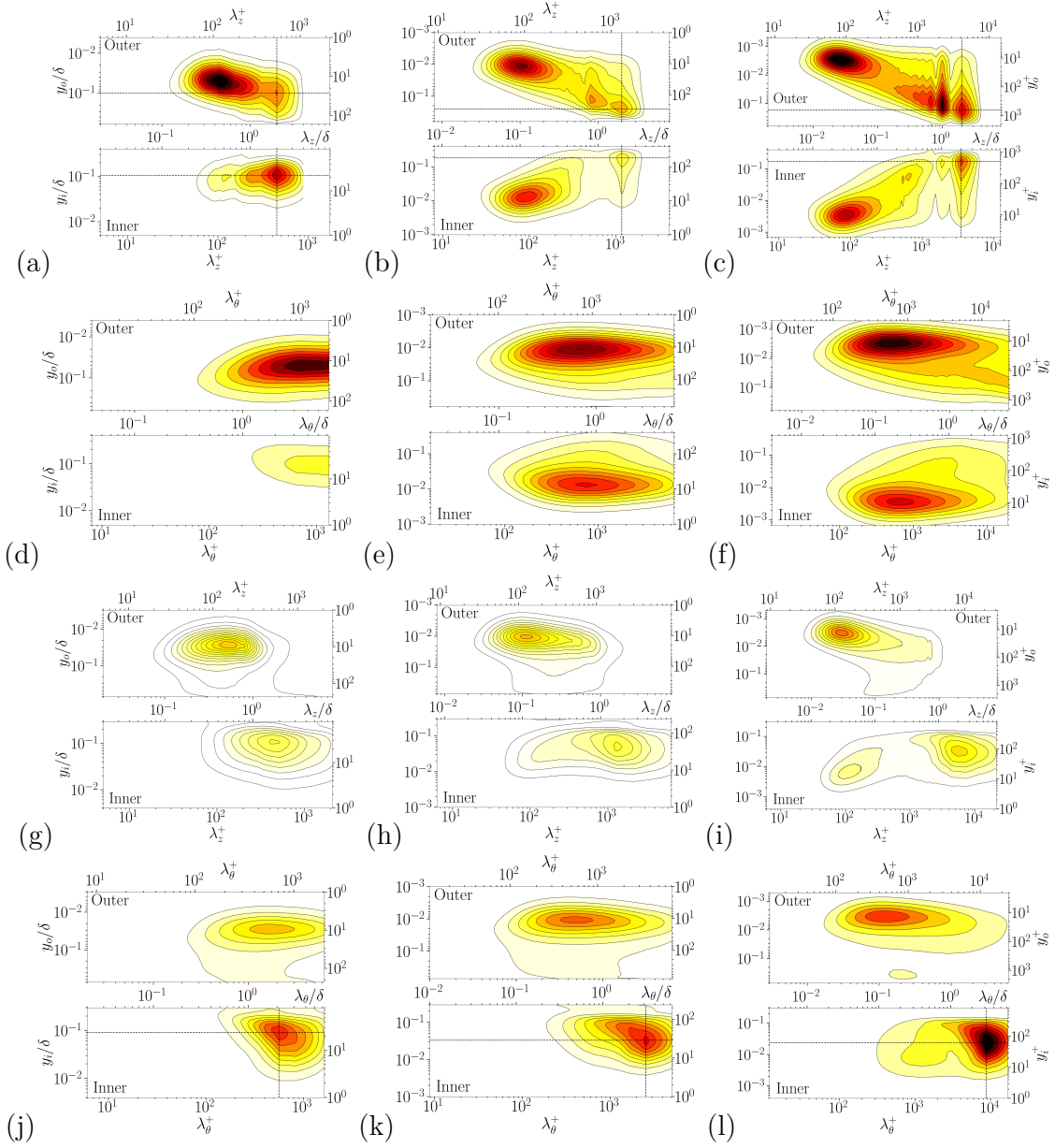


Figure 4.12: Pre-multiplied spectra of streamwise fluctuating velocity as a function of spanwise wavelength ($k_z^* E_{uu}^*$) and of streamwise wavelength ($k_\theta^* E_{uu}^*$) as the wall distance varies. Panels (a, b, c, d) refer to R40 flow cases, while (e, f, g, h) to R1 flow cases. From left to right, the panels correspond to $Re_b = 4000$ (a, d, g, j), 20000 (b, e, h, k), 87000 (c, f, i, l). Wall distance from the inner and outer wall, spanwise and streamwise wavelengths are reported in outer units (y_i/δ , y_o/δ , λ_z/δ , λ_θ/δ), and in local wall units (y_i^+ , y_o^+ , λ_z^+ , λ_θ^+). Dashed lines mark wavelength and wall distance of the energy peak associated with streamwise large-scale structures (a, b, c) and with spanwise large-scale structures (j, k, l).

($\lambda_z^+ \approx 120$) whereas their streamwise wavelength is shorter ($\lambda_\theta^+ \approx 500$), as visible from the streamwise spectra in the panels (j), (k), (l). The absence of a primary peak near the inner wall in the spanwise spectra highlights the stabilising effect of convex curvature. However, both spanwise and streamwise spectra, suggest that this inhibiting effect becomes gradually less intense as the Reynolds number increases. In contrast to the R40 flow cases, never does a secondary peak emerge in the spanwise spectra near the outer wall. This implies that the streamwise roll cells, if present, have a negligible effect on the streamwise velocity fluctuations. A secondary peak is instead present near the inner wall at $\lambda_z/\delta \approx 2$, which moves closer to the wall (in δ units) as Re_b increases. This peak cannot be related to streamwise roll cells for two reasons: 1) they are most intense at the outer wall, yet no secondary peak is detected there, and 2) the spanwise wavelength of streamwise vortices must be shorter, as one can infer from figure 4.10. Rather, we can ascribe this peak to the presence of spanwise large-scale structures. To substantiate this statement, we consider the streamwise energy spectra, which highlight energetic modes near the inner wall at long wavelengths: $\lambda_\theta/\delta \approx 2\pi/3$ at $Re_b = 4000$ (j) and $\lambda_\theta/\delta \approx \pi$ at 20000 (k) and 87000 (l), as marked by the dashed lines. These energy peaks cannot be related either to turbulence structures, which are absent or strongly inhibited near the inner wall, or to streamwise roll cells, as we have just seen via the spanwise spectra. Flow visualisations in wall-parallel planes near the inner wall (figure 4.11, right panels) revealed the presence of spanwise structures, organised in pairs with a streamwise length of about $\pi\delta$. The wavelength of these peaks corresponds to $n = L_\theta/\lambda_\theta = 3$ pairs of eddies at $Re_b = 4000$ and $n = L_\theta/\lambda_\theta = 2$ pairs at 20000 and 87000. Hence, the energy peaks in the streamwise spectra are the signature of cross-flow structures. The spanwise size of these structures is comparable with δ , explaining the peaks in the spanwise spectra at $\lambda_z/\delta \approx 2$.

In figure 4.13 we show the pre-multiplied spectral densities of wall-normal velocity fluctuations in the spanwise direction ($k_z^* E_{vv}^*$). In the R40 flow cases (upper panels) energy peaks related to near-wall turbulent structures form at $y_i^+ \approx y_o^+ \approx 50$, $\lambda_z^+ \approx 100$, except for the inner wall at $Re_b = 4000$, in agreement with the flow visualisations. Two secondary peaks in the channel core, more prominent at $Re_b = 87000$, highlight the presence of energetic modes with $\lambda_z/\delta \approx 1$ and $\lambda_z/\delta \approx 2$ at $y_o/\delta \approx 0.3$. Following the same reasoning about the streamwise velocity spectra, we trace these energetic modes to streamwise roll cells, which generate large-scale radial sweeps and ejections.

The effect of strong curvature on the energy distribution of the wall-normal velocity fluctuations (lower panels) is rather striking. Contrary to the streamwise velocity spectra, the energy content is now much higher in the case of strong curvature, which points to substantial structural changes. No distinct peak is observed in the inner part of the channel, which conveys that the spanwise large-scale structures that we noted above have no impact (on average) on the wall-normal velocity fluctuations. A prominent energy peak is located at $y_o \approx 0.3\delta$, $\lambda_z \approx 0.8\delta$, marked by the intersection of the dashed lines. This energetic mode is not present in plane channel flow (see e.g. Cho et al., 2018) and cannot be related to cross-flow structures, which are more intense near the inner wall. Hence, we interpret them as the signatures of streamwise roll cells. This would mean that the typical configuration of streamwise large-scale structures consists of $n = L_z/\lambda_z = 10$ pairs of roll cells (with $L_z = 8$, $\lambda_z = 0.8\delta$), which we will ascertain in the following.

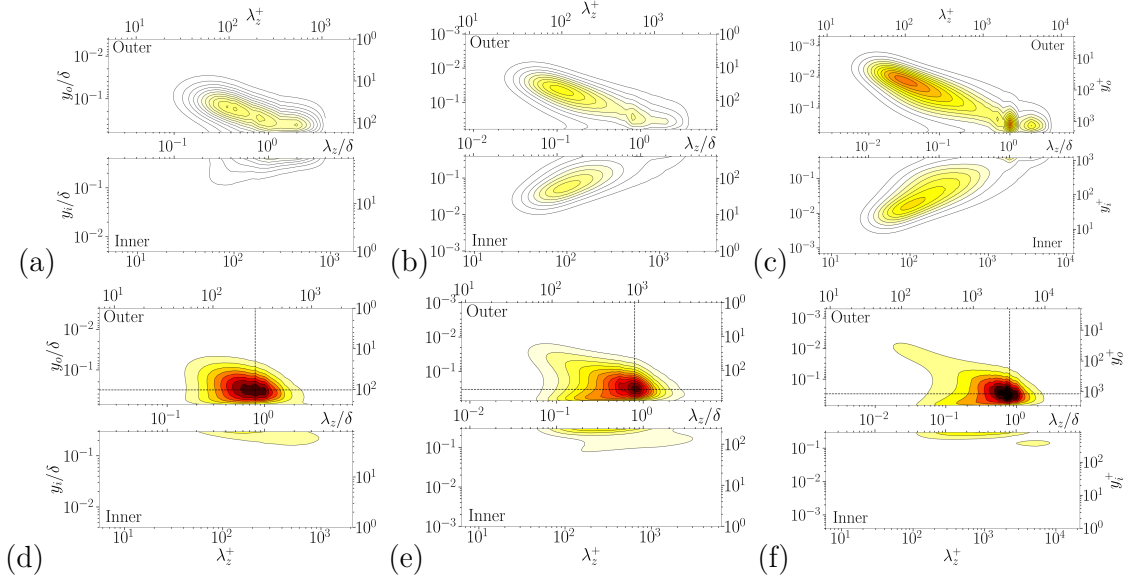


Figure 4.13: Pre-multiplied spectra of fluctuating wall-normal velocity ($k_z^* E_{vv}^*$) as a function of the spanwise wavelength and wall distance for the R40 flow cases (upper panels) and R1 flow cases (lower panels). From left to right, the panels correspond to $Re_b = 4000$ (a, d), 20000 (b, e), 87000 (c, f). Wall distance from the inner and outer wall and spanwise wavelength are reported in outer units (y_i/δ , y_o/δ , λ_z/δ) and local wall units (y_i^+ , y_o^+ , λ_z^+), respectively. Dashed lines mark the wavelength and wall-distance of the energy peak related to streamwise large-scale structures.

4.5 STREAMWISE LARGE-SCALE STRUCTURES

Flow visualisations and energy spectra reveal the presence of streamwise large-scale structures, which are akin to the Dean vortices observed in laminar flow (Dean, 1928). To understand how these structures depend on curvature and Reynolds number and how they affect the flow field, we separate the coherent contribution from the underlying turbulence, where coherence is intended in the sense given by Hussain (1986). For that purpose we use the triple decomposition of Hussain et al. (1970), whereby a generic field variable, $\varphi(\theta, r, z, t)$, is decomposed as

$$\varphi(\theta, r, z, t) = \Phi(r) + \tilde{\varphi}(r, z, t) + \varphi''(\theta, r, z, t), \quad (4.12)$$

where Φ is the space average over the homogeneous directions (θ, z) and time (t),

$$\tilde{\varphi}(r, z, t) = \langle \varphi(\theta, r, z, t) \rangle_\theta - \Phi(r), \quad (4.13)$$

is the coherent contribution from the streamwise vortices, $\langle \varphi(\theta, r, z, t) \rangle_\theta$ is the average over the streamwise direction, and

$$\varphi''(\theta, r, z, t) = \varphi(\theta, r, z, t) - \langle \varphi(\theta, r, z, t) \rangle_\theta \quad (4.14)$$

is the instantaneous turbulent fluctuation. The total fluctuation is then $\varphi' = \tilde{\varphi} + \varphi''$. The instantaneous streamwise-averaged fields ($\tilde{\varphi}$) are then averaged over time to extract the coherent contribution, namely $\bar{\tilde{\varphi}}$.

One should note that, in principle, the averages defined in (4.13) would be zero in infinitely long domains. This effect is illustrated in figure 4.14(a), where the RMS profiles of coherent streamwise velocity ($\tilde{u}_{\text{rms}}^*$) are plotted for the R40 flow case at $\text{Re}_b = 10000$, for increasing values of the domain extension L_θ . As expected, the intensity of coherent fluctuations decreases for longer domains. Moreover, the streamwise vortices are free to move in the spanwise direction, whereby their time averages would tend to zero as the averaging period (T) approaches infinity. This effect is assessed in figure 4.14(b), where we show the kinetic energy associated with the time-averaged coherent motions, defined as $\bar{E}_k = \langle (\tilde{u}^2 + \tilde{v}^2 + \tilde{w}^2)/2 \rangle_{r,z}$ as a function of T (solid lines). The time-averaging is carried out over incremental time periods from $T = \delta/u_b$ to $T = 1000\delta/u_b$. The temporal decay of $\bar{E}_k$ is evident, occurring more rapidly in the R1 flow case due to increased unsteadiness of streamwise vortices. However, the time-averaged kinetic energy of coherent motions, defined as $\tilde{E}_k = \langle (\tilde{u}^2 + \tilde{v}^2 + \tilde{w}^2)/2 \rangle_{r,z}$ is nearly constant for increasing T (dashed lines), implying that the variance of (4.12) is independent of the averaging period. The same conclusion can be drawn from figure 4.14(c), which shows the RMS profiles of coherent wall-normal velocity ($\tilde{v}_{\text{rms}}^*$) for the R1 flow case at $\text{Re}_b = 10000$. All profiles obtained using different values of T are nearly overlapping. The present results obtained with $L_\theta = 2\pi\delta$ for the R40 flow case at $\text{Re}_b = 40000$ are compared to those reported in Brethouwer (2022) for the flow in a curved channel with similar wall curvature ($r_c/\delta = 30$) at the same Reynolds number. As shown in figure 4.14(d), the profiles of the turbulent shear stress are virtually overlapping (solid lines) and, despite the curvature is not identical, the coherent contribution is comparable (dashed lines).

The coherent motions herein considered refer to a finite streamwise length associated with the extent of the computational domain ($L_\theta = 2\pi\delta$), and to a finite time horizon ($600\delta/u_b$). Whereas the intensity of those coherent motions is inevitably affected by these choices, the conclusions regarding the influence of Reynolds number and wall curvature still stand.

In figure 4.15 we show the Stokes streamfunction for streamwise-coherent motions ($\tilde{\psi}$), defined such that $\tilde{w} = \partial\tilde{\psi}/\partial r$, $\tilde{v} = -\partial\tilde{\psi}/\partial z$. The overlaid flooded contours represent the mean coherent streamwise velocity ($\tilde{u}$). In addition, we report the spanwise distribution of the mean shear at the inner and outer wall,

$$\bar{\tau}_{w,i}^+(z) = \frac{\nu}{u_{\tau,i}^2} \frac{\partial \langle u \rangle_\theta}{\partial r} \bigg|_{r_i}, \quad \bar{\tau}_{w,o}^+(z) = \frac{\nu}{u_{\tau,o}^2} \frac{\partial \langle u \rangle_\theta}{\partial r} \bigg|_{r_o}. \quad (4.15)$$

In the R40 flow cases (upper panels) two pairs of counter-rotating vortices appear which nearly span the whole channel thickness and with a spanwise wavelength $\lambda_z/\delta \approx 2$, which is in agreement with the energy spectra in figure 4.12. Streamwise roll cells with the same size were detected by Moser et al., 1987. The spanwise inhomogeneity due to these secondary eddies has a strong impact on the distribution of the wall shear stress. Indeed, large-scale ejections are generated between any pair of counter-rotating vortices (blue contours), where low-speed fluid is diverted away from the wall, and local friction attains a minimum. Correspondingly, large-scale sweeps generate at the

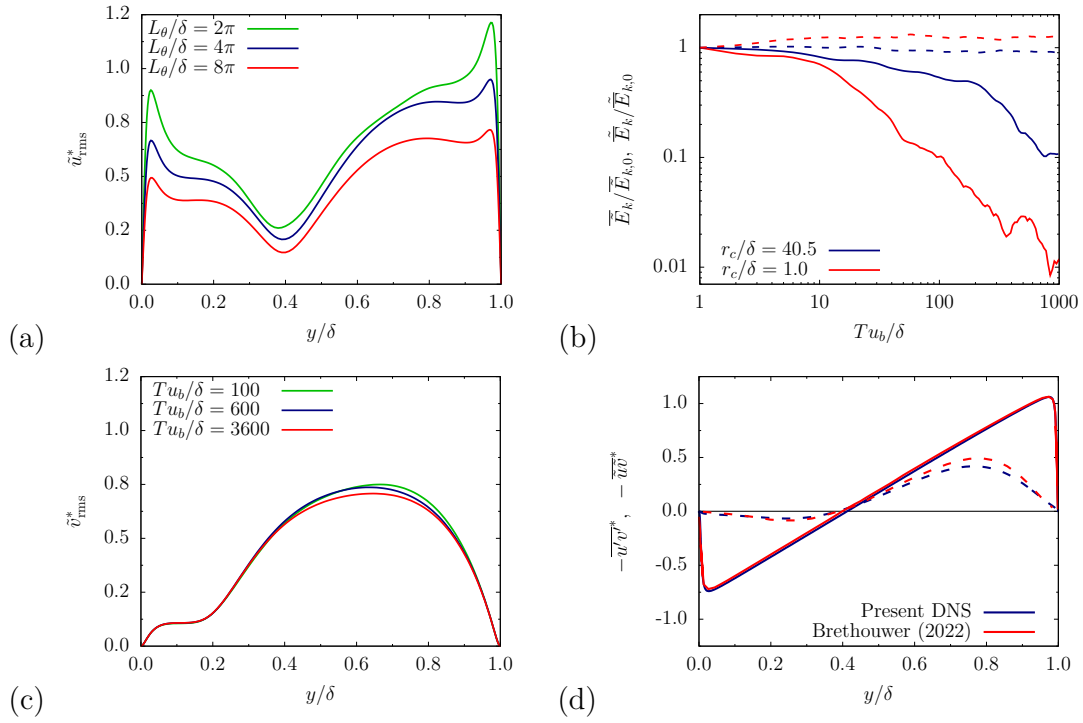


Figure 4.14: (a) **RMS** profiles of coherent streamwise velocity fluctuations for the R40 flow case at $Re_b = 10000$ for various domain lengths. (b) Kinetic energy of time-averaged coherent motions (solid lines) and time-averaged kinetic energy of coherent motions (dashed lines) scaled by the initial value as a function of the averaging time period. (c) **RMS** profiles of coherent wall-normal velocity fluctuations for the R1 flow case at $Re_b = 10000$ for various averaging periods. (d) Total (solid lines) and coherent (dashed lines) turbulent shear stress for the R40 flow case at $Re_b = 40000$, compared to the results obtained by Brethouwer (2022) for $r_c/\delta = 30$ at $Re_b = 40000$.

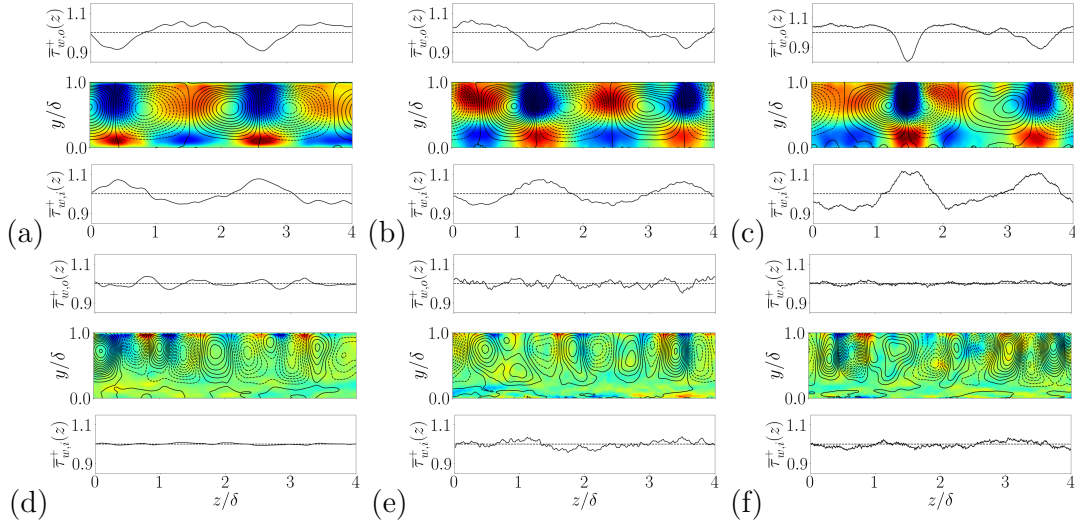


Figure 4.15: Mean Stokes streamfunction ($\bar{\psi}$) for streamwise-coherent disturbances in an (r, z) -plane overlaid to flooded contours of coherent streamwise velocity ($\tilde{u}$), for the R40 flow cases (upper panels) and for the R1 flow cases (lower panels). From left to right, the panels correspond to $Re_b = 4000$ (a, d), 20000 (b, e), 87000 (c, f). Positive values of $\bar{\psi}$ (solid lines) indicate a clockwise-rotating roll cell, and vice-versa for negative values (dashed lines). The flooded contours range from $-1.5 u_{\tau,g}$ (blue) to $1.5 u_{\tau,g}$ (red). Each panel shows the spanwise distribution of the mean shear stress at the two walls, $\bar{\tau}_{w,i}^+(z)$ and $\bar{\tau}_{w,o}^+(z)$, defined in (4.15). Only half of the domain is shown for the R1 flow cases.

opposite wall whereby high-speed fluids is pushed towards the wall (red contours), causing local increase of wall friction. This tendency is clearer as the Reynolds number increases, as one can infer from the spanwise distribution of the local wall shear, which shows excursions of about 10% at $Re_b = 4000$ (a) and $Re_b = 20000$ (b), and reaching up to 20% at $Re_b = 87000$ (c).

As for the R1 flow cases (lower panels), the number of pairs of counter-rotating vortices increases to about ten (only five are visible as half of the domain is shown). Hence, the spanwise wavelength of the roll cells is $\lambda_z/\delta \approx 0.8$, which again corresponds to what found from the spectral analysis in figure 4.13. Spanwise shortening of the streamwise roll cells is also observed in rotating channel flows, in which similar vortices develop on account of Coriolis forces (Matsson et al., 1990). Through DNS of rotating channel flow, Brethouwer, 2017 found that the size of the streamwise roll cells is smaller at higher rotation numbers (the rotation number in rotating channel flow is the counterpart of the curvature ratio in curved channels), in agreement with previous numerical studies (Kristoffersen et al., 1993; Yang et al., 2012). In addition, Brethouwer, 2017 found that the size of the roll cells is independent of the Reynolds number, which is also the case here. Due to strong channel curvature, the roll cells are pushed towards the outer wall and cannot fill the entire channel thickness. Hence, the flow region near the inner wall (say, $y/\delta < 0.2$) is not affected, as seen in the spanwise distribution of the wall shear, which is nearly flat. As for the shear at the outer wall, spanwise excursions of $\tau_{w,o}^+(z)$ have a maximum amplitude of about 4% at $Re_b = 4000$ (d) and $Re_b = 20000$ (e), whereas they are very small at $Re_b = 87000$ (f), at which the effect of

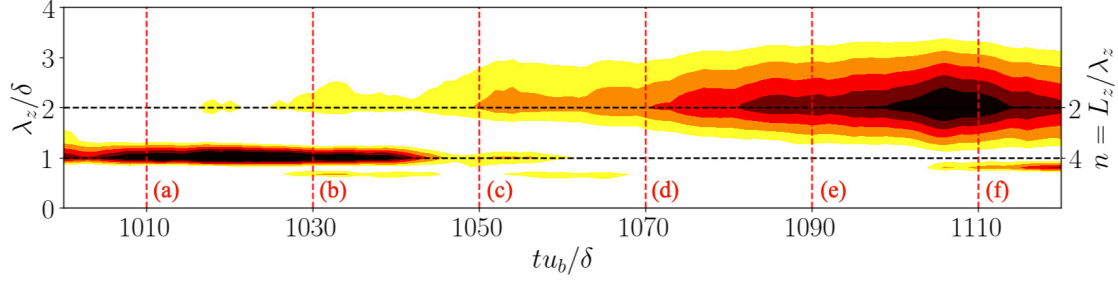


Figure 4.16: Time history of the pre-multiplied spanwise energy spectra of fluctuating streamwise velocity ($k_z^* E_{uu}^*$) for the R40 flow case at $Re_b = 87000$, at $y/\delta = 0.8$. The black dashed lines correspond to $\lambda_z/\delta = 2$ and $\lambda_z/\delta = 1$, at which the expected number of streamwise vortices are indicated on the right vertical axis.

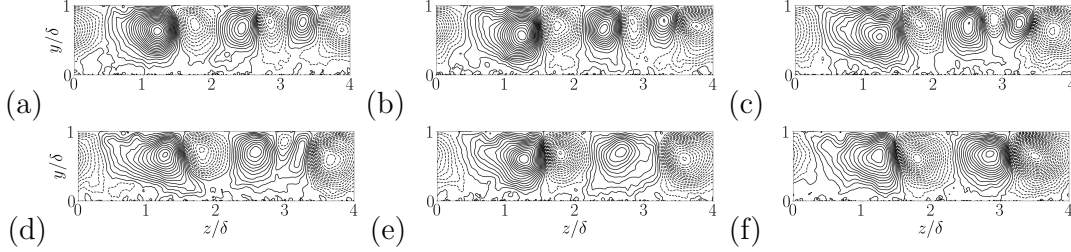


Figure 4.17: Coherent streamfunction ($\tilde{\psi}$) at the time instants marked by the red dashed lines in figure 4.16, for the R40 flow case at $Re_b = 87000$.

the streamwise vortices is outweighed by turbulence. The reason why the outer-wall shear stress is barely affected by streamwise vortices is their unsteadiness. The coherent streamfunction shows indeed that roll cells are less organised in the R1 flow cases than in R40, and on average they do not affect the streamwise velocity. Nonetheless, the impact of those eddies on the instantaneous flow is certainly not negligible in the case of strong curvature. The strength of streamwise large-scale structures can be quantified in terms of the maximum amplitude of the coherent wall-normal velocity (Canton et al., 2016), which results in $\max|\tilde{v}|/u_b \approx 6\%$ for the R40 flow cases, nearly independent of the Reynolds number. Streamwise vortices in the R1 flow cases are much stronger, their strength being $\max|\tilde{v}|/u_b \approx 20\%$ at $Re_b = 4000$ and 12% at 87000 .

The eduction procedure based on triple decomposition allowed to quantify the statistical organization of streamwise large-scale structures. However, the energy spectra highlighted the presence of multiple peaks at large wavelengths, which are particularly evident for the R40 flow case at $Re_b = 87000$ (figure 4.12). Multiple peaks suggest either the coexistence of large-scale structures of different sizes, or the occurrence of splitting and merging events. To clarify this point, we inspect the time history of the spanwise energy spectra of streamwise velocity fluctuations, shown in figure 4.16 for the R40 flow case at $Re_b = 87000$, at the wall distance where the peak of the energy associated with the streamwise vortices occurs. A time window from $tu_b/\delta = 1000$ to 1120 was selected for the analysis and we took six subsequent snapshots of $\tilde{\psi}$, shown in figure 4.17, at intervals of $20\delta/u_b$ marked by the red dashed lines in figure 4.16. From $tu_b/\delta \approx 1000$ to 1040 most energy is clustered around $\lambda_z/\delta = 1$, hence $n = 4$ pairs

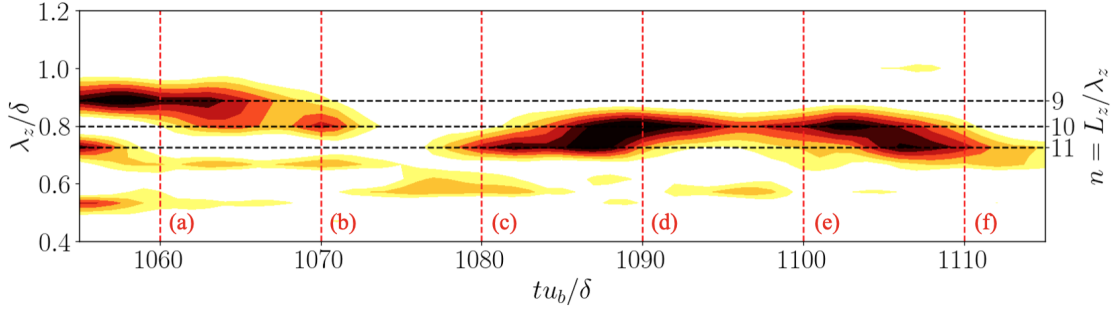


Figure 4.18: Time history of the pre-multiplied spanwise energy spectra of fluctuating wall-normal velocity ($k_z^* E_{vv}^*$) for the R1 flow case at $Re_b = 87000$, at $y/\delta = 0.7$. The black dashed lines correspond to $\lambda_z/\delta = 0.73$, $\lambda_z/\delta = 0.80$ and $\lambda_z/\delta = 0.89$, at which the expected number of streamwise vortices are indicated on the right vertical axis.

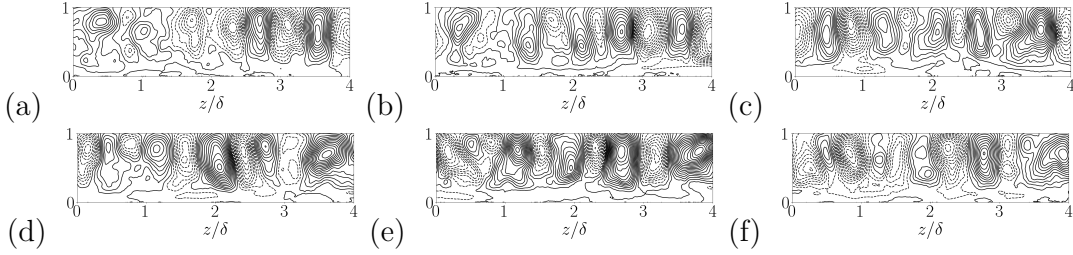


Figure 4.19: Coherent streamfunction ($\tilde{\psi}$) at the time instants marked by the red dashed lines in figure 4.18 for the R1 flow case at $Re_b = 87000$. Only half of the domain is shown.

of vortices would be expected (the number of vortices pairs is indicated on the right vertical axis). However, the snapshots of $\tilde{\psi}$ at $tu_b/\delta \approx 1010$ (a) and 1030 (b) reveal the occurrence of $n = 3$ pairs. In this respect, we note that the roll cells are not uniform in size, as the pair centred at $z/\delta \approx 0.5$ has a wavelength $\lambda_z/\delta \approx 2$, whereas the two remaining pairs have a wavelength $\lambda_z/\delta \approx 1$. A possible explanation is that the two pairs of small-size vortices are stronger than the large-size one, hence more energy is concentrated at $\lambda_z/\delta = 1$. A vortex-merging event is observed between $tu_b/\delta \approx 1050$ and 1090 . Panels (c) and (d) depict that the pair of smallest vortices located at $z/\delta \approx 3$ decrease in size and strength, as the iso-lines of the streamfunction get sparser. This process continues until the smallest pair is embedded within the adjacent clockwise rotating vortex (e), which appears more regular and strong at $tu_b/\delta \approx 1110$ (f). From $tu_b/\delta \approx 1080$ on, the energy peak is clustered around $\lambda_z/\delta = 2$ and, as expected, $n = 2$ pairs of roll cells are found.

The results obtained for the R1 flow case at $Re_b = 87000$ are presented in figure 4.18, which reports the time evolution of the spanwise spectra of the wall-normal velocity at the peak position of energy associated with the streamwise vortices. The time window under scrutiny is half as for the R40 flow case, and the six subsequent snapshots of $\tilde{\psi}$, shown in figure 4.19, are sampled at intervals of $10\delta/u_b$ instead of 20 on account of stronger unsteadiness in cases with strong curvature. The time evolution of the spectra shows that the energy peak is clustered around $\lambda_z/\delta \approx 0.89$ from $tu_b/\delta = 1055$ to 1065 .

Consistently, figure 4.19(a) displays nine vortices in the region under scrutiny ($n = 9$ pairs are present in the whole domain). This configuration seems to be unstable because of the lack of organization of the vortex pair located at $z/\delta \approx 1$, and because two counterclockwise vortices are adjacent straddling $z/\delta \approx 2$. At $tu_b/\delta = 1070$ the energy peak shifts at $\lambda_z/\delta \approx 0.8$, indeed the related figure 4.19(b) shows that a new clockwise rotating vortex emerged at $z/\delta \approx 2.2$ fitting in between the two counterclockwise rotating vortices and increasing the number of vortices to 10 (i.e. $n = 10$). The number of vortex pairs increase further to $n = 11$ at $tu_b/\delta \approx 1080$, as visible in figure 4.19(c), and the energy peak decreases accordingly to $\lambda_z/\delta \approx 0.73$. From $tu_b/\delta \approx 1085$ to 1105 the energy peak settles at $\lambda_z/\delta \approx 0.8$, and panels (d) and (e) highlight that the vortex configuration consists of $n = 10$ pairs. The energy peak shifts again to $\lambda_z/\delta \approx 0.73$ at $tu_b/\delta = 1110$. Figure 4.19(f) shows that the main vortices are still ten, however small secondary vortices tend to split off from the primary ones, bringing more energy to smaller scales. In the case of strong curvature, the transitions from one vortex configuration to the other can be attributed to the unsteady dynamics of the vortices, which can be inferred from their spanwise motions, distorted shapes and different sizes, rather than to splitting and merging phenomena, which could not be clearly identified.

In figure 4.20 we show the Root Mean Square (RMS) of the streamwise (a, b) and wall-normal (c, d) velocity fluctuations, as well as the turbulent shear stress (e, f). Total fluctuations are reported along with the contributions due to the streamwise vortices, which we have determined by taking the root-mean-square of the coherent contribution (4.13) along the spanwise direction and in time. For the R40 flow cases, the streamwise velocity fluctuations (a) are higher at the outer than at the inner wall. As made clear from the increased intensity of the coherent fluctuations, this asymmetry is mainly due to the streamwise vortices, which are stronger near the outer wall. An exception is the case at $Re_b = 4000$ (green line), in which the coherent contribution at the outer wall is comparable to that at the inner wall. In general, the coherent contribution is about half of the total. The peak of the coherent contribution occurs between $y/\delta \approx 0.8$ and $y/\delta \approx 0.9$, corresponding to a bump in the profile of the total fluctuations. As for the R1 flow cases, the profiles of streamwise velocity fluctuations (b) show a near-wall peak on the concave side comparable to that of the corresponding R40 flow cases, and little contribution from coherent fluctuations. The profile of the velocity fluctuations is almost flat in the channel core, and the near-wall peak on the convex side is very different from the case with mild curvature, extending much farther from the wall. Wall-normal velocity fluctuations (c) are only relevant in the channel core, with a peak at $y/\delta \approx 0.7$. For the R1 flow cases (d) fluctuations of wall-normal velocity are up to two times larger than the streamwise velocity. Substantial contribution is found to be provided by the coherent fluctuations, which also attain a peak at $y/\delta \approx 0.7$, as observed in the velocity spectra.

As for the turbulent shear stress, it is nearly symmetrical in fully-turbulent R40 flow cases (e), resembling the case of a plane channel (see e.g. Kim et al., 1987). However, two differences should be noted: 1) the point of zero crossing is shifted towards the inner wall at $y/\delta \approx 0.4$, and 2) the peak value increases near the outer wall and decreases near the inner wall. The effect of convex curvature is particularly evident at $Re_b = 4000$, at which the zero crossing is even closer to the inner wall. The coherent turbulent stress is about half of the total at $y/\delta \approx 0.8$, pointing to significant contribution of streamwise vortices to momentum transport. Similar conclusions were also reached

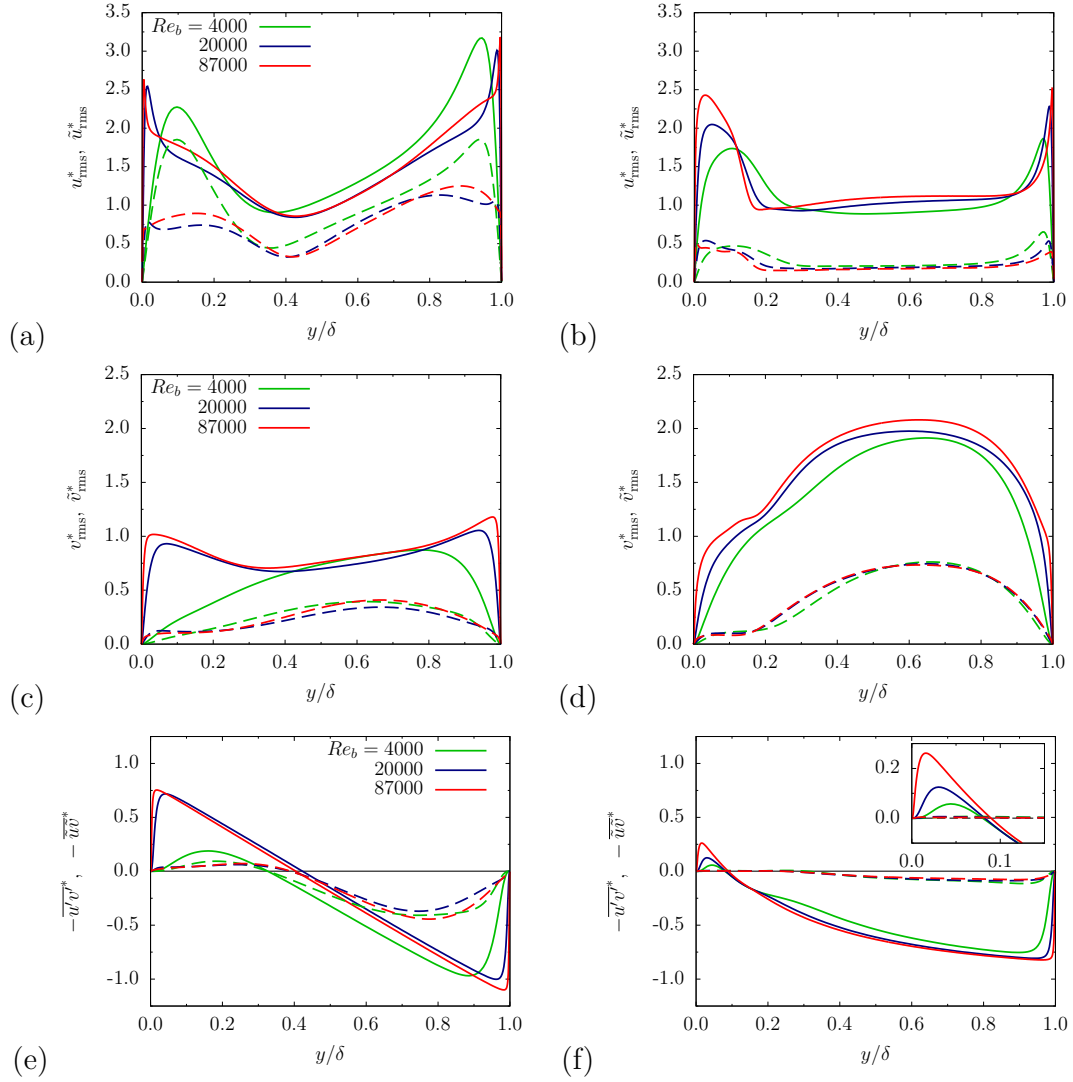


Figure 4.20: Profiles of root-mean-square streamwise (a, b) and wall-normal (c, d) velocity fluctuations and mean turbulent shear stress (e, f) at various Reynolds numbers for the R40 flow cases (left panels) and R1 flow cases (right panels). Solid lines refer to total fluctuations and dashed lines refer to coherent fluctuations due to streamwise vortices.

by Moser et al., 1987 and Brethouwer, 2022. The effects of strong curvature are shown in panel (f). In the channel core, where viscous effects are negligible, the turbulent stress profile becomes quadratic rather than linear, as after the analytical distribution of the total shear stress (4.6). As seen in the inset of the panel (f), the peak value is greatly reduced near the inner wall, which aligns with the findings of So et al., 1973 in experiments of turbulent boundary layers over a convex surface. The reduction in turbulent shear stress at the inner wall is not only due to the stabilizing effect of convex curvature but also results from the influence of spanwise large-scale structures, as will be discussed below. The coherent Reynolds stress is negligible and vanishes entirely near the inner wall, as the streamwise vortices are displaced towards the outer wall (see §4.5). This analysis demonstrates that strong curvature significantly alters the role of streamwise-aligned coherent structures in turbulence. Specifically, their contribution to wall-normal fluctuations becomes substantially larger than to streamwise fluctuations, while their contribution to momentum transport is suppressed.

4.6 SPANWISE LARGE-SCALE STRUCTURES

Visualisations of the flow near the inner wall of strongly curved channels (figure 4.11) revealed the presence of alternating regions with positive/negative velocity fluctuations elongated along the spanwise direction. Those wavy patterns are the footprint of spanwise large-scale structures, which are originated from streamwise instabilities (Finlay et al., 1988) and which are convected at the mean flow speed (Matsson et al., 1992). As we did for the streamwise vortices, we exploit triple decomposition to separate the effects of the cross-stream structures from those of turbulence. A generic field variable, $\varphi(\theta, r, z, t)$, is decomposed as

$$\varphi(\theta, r, z, t) = \Phi(r) + \hat{\varphi}(\theta, r, t) + \varphi'''(\theta, r, z, t), \quad (4.16)$$

where

$$\hat{\varphi}(\theta - tu_c/r, r, t) = \langle \varphi(\theta, r, z, t) \rangle_z - \Phi(r), \quad (4.17)$$

is the spanwise-coherent contribution, $\langle \varphi(\theta, r, z, t) \rangle_z$ is the average along the spanwise direction, and $\varphi'''(\theta, r, z, t)$ is the instantaneous turbulent fluctuation. Since spanwise-aligned structures are advected with the flow, phase alignment is required to reduce the associated coherent contribution. Hence, the time averages are evaluated by shifting consecutive spanwise-averaged field by an angle $\Delta t_s u_c / r_c$, where Δt_s is the sampling interval, set equal to a time-unit, and u_c is the convection velocity of coherent disturbances. The convection velocity was estimated from the analysis of the wavenumber-frequency spectra of the streamwise velocity fluctuations for the R1 flow case at $Re_b = 4000$, resulting in $u_c \approx 0.7u_b$, which is comparable to the value of $u_c \approx 0.80u_b$ found in experiments (Matsson et al., 1992). Moreover, the convection velocity in inner units is $u_c^+ \approx 11$, which is similar to the mean speed of the near-wall energy-containing eddies (Kim et al., 1993; Jimenez et al., 2001). This value of u_c was also considered for the flow cases at higher Reynolds number, as it was found to ensure maximum coherence in time.

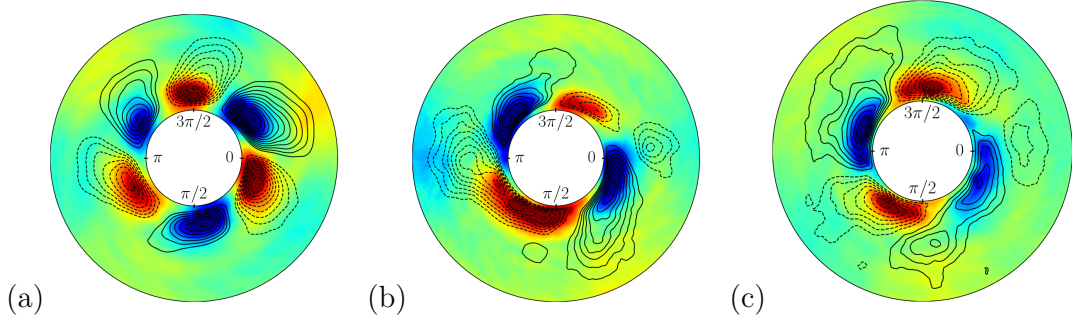


Figure 4.21: Mean Stokes streamfunction ($\hat{\psi}$) for spanwise-coherent disturbances, overlaid to flooded contours of the mean coherent pressure ($\hat{p}$) in a (θ, r) -plane for the R1 flow cases at $Re_b = 4000$ (a), 20000 (b), 87000 (c). The flooded contours range from $-0.3u_{\tau,g}^2$ (blue) to $0.3u_{\tau,g}^2$ (red). Positive values of $\hat{\psi}$ (solid lines), indicate a clockwise-rotating roll cell, associated with negative coherent pressure (blue contours), whereas negative (dashed lines) correspond to counter-clockwise rolls and positive coherent pressure (red contours). The mean flow is clockwise.

In figure 4.21 we show the mean Stokes streamfunction for spanwise-coherent motions ($\hat{\psi}$), defined such that $\hat{v} = (\partial\hat{\psi}/\partial\theta)/r$, $\hat{u} = -\partial\hat{\psi}/\partial r$, overlaid to flooded contours of the mean coherent pressure ($\hat{p}$) in the (θ, r) -plane. Alternating high- and low-pressure regions are observed, marking the presence of spanwise large-scale structures. Those are organised into three pairs of roll cells at $Re_b = 4000$ (a), whereas only two pairs are found at $Re_b = 20000$ (b) and 87000 (c). Although the radial extent of the spanwise structures is comparable with δ , they are most intense near the inner wall and weaker towards the outer wall. The streamfunction shows that the cross-stream structures have an irregular shape and tend to split up at high Reynolds number. This more chaotic organisation yields reduced strength of the spanwise structures, as measured in terms of the magnitude of the coherent streamwise velocity, $\max|\hat{u}|$. Indeed, we found $\max|\hat{u}|/u_b \approx 9\%$ at $Re_b = 4000$ and $\max|\hat{u}|/u_b \approx 7\%$ at 20000 and 87000, hence spanwise large-scale structures are about half as strong as the streamwise ones (see §4.5). The centres of the roll cells are approximately at the same location as the local minima of the mean shear rate (figure 4.7), supporting the idea that the spanwise structures may originate from a shear-layer instability (Finlay et al., 1988; Yu et al., 1991).

Figure 4.22(a) displays the distributions of the RMS pressure fluctuations for the R1 flow cases, and includes the contribution of coherent fluctuations due to the spanwise structures, which are obtained from the spanwise-coherent contribution according to equation (4.17). Near the inner wall, pressure fluctuations attain a peak which is twice as high as the outer wall at $Re_b = 4000$, and almost four times as high at $Re_b = 87000$. The impact of the spanwise-aligned structures on pressure fluctuations is substantial, as the peak value of the coherent contribution near the inner wall is about one third of the total. Another effect of spanwise structures, which is related to the increase of pressure fluctuations, is the suppression of the turbulent shear stress. This can be ascertained in figure 4.22(b), where we show the total turbulent stress (solid lines) and the coherent turbulent stress due to spanwise structures (dashed lines). The coherent turbulent shear stress yields a negative contribution, conveying that the

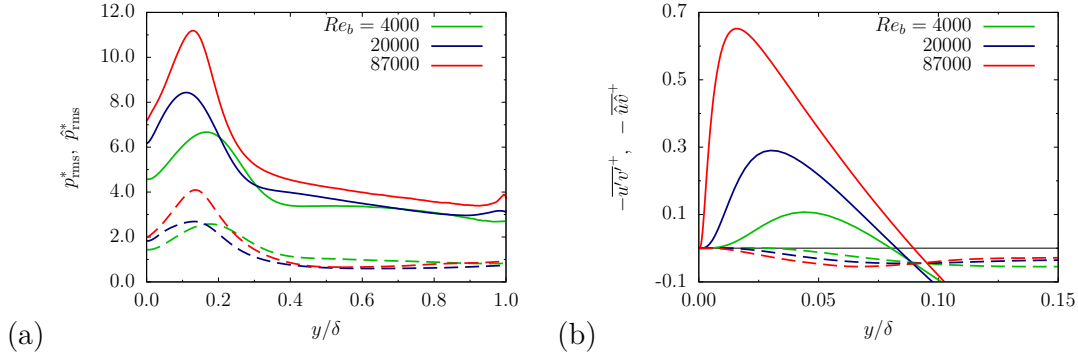


Figure 4.22: Profiles of RMS pressure fluctuations (a) and mean turbulent stress near the inner wall (b) at various Reynolds numbers, for the R1 flow cases. Solid lines refer to total fluctuations and dashed lines refer to coherent fluctuations due to spanwise large-scale structures (the latter are denoted with the hat symbol).

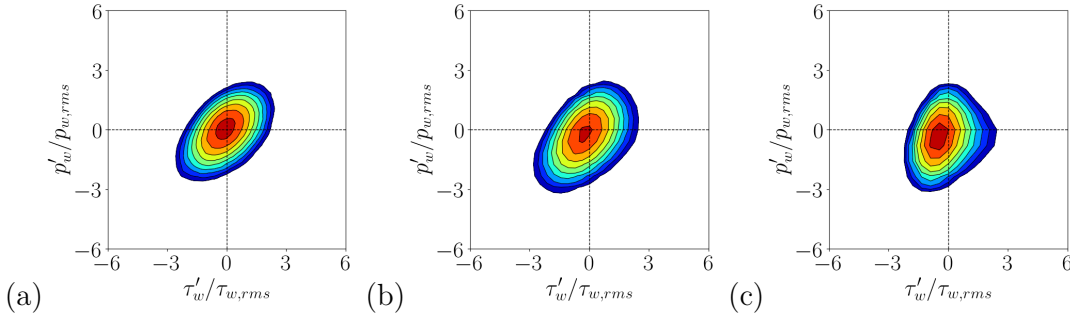


Figure 4.23: Joint Probability Density Function (PDF) of wall shear stress and pressure fluctuations at the inner wall, $P(\tau'_w, p'_w)$, for the R1 flow cases at $Re_b = 4000$ (a), 20000 (b) and 87000 (c).

spanwise structures tend to suppress ejections and sweeps near the inner wall. A similar result was reported by Kuwata, 2022 simulating a turbulent flow over high-aspect-ratio streamwise ribs, in which case the spanwise large-scale structures (originated by a Kelvin-Helmholtz instability) were found to increase locally pressure fluctuations and suppress the turbulent shear stress.

Spanwise-coherent structures were associated with suppression of the turbulent shear stress by many authors (Koumoutsakos, 1999; Fukagata et al., 2005; Mamori et al., 2011). In addition, experiments on turbulent boundary layers revealed that characteristic and identifiable variation of the wall pressure accompanies the advection of large organised structures, which are responsible for variation of the wall shear stress (Thomas et al., 1983). To verify whether those quantities are correlated, we inspect the joint PDF of the wall shear stress and of the fluctuating pressure at the inner wall, which we report in figure 4.23. Strong positive correlation emerges at $Re_b = 4000$ (a) and $Re_b = 20000$ (b), at which flow near the inner wall is dominated by spanwise-coherent structures. This correlation becomes less distinct at $Re_b = 87000$ (c), at which turbulent fluctuations start reach down to the near-wall region.

Further insights into the relationship between wall pressure and wall shear are provided in figure 4.24, where we show the streamwise distribution of the coherent

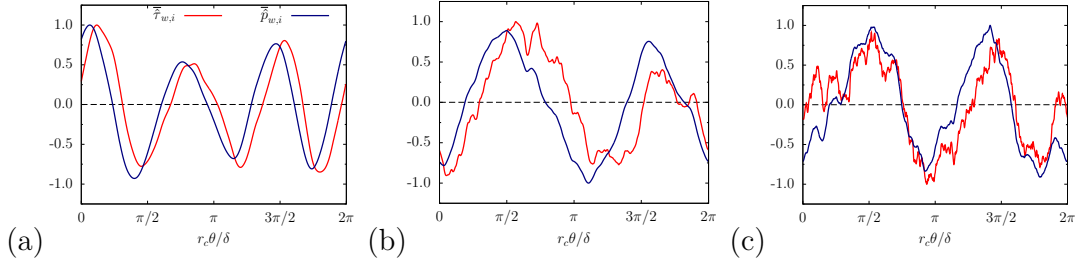


Figure 4.24: Streamwise distribution of mean coherent shear stress ($\bar{\tau}_{w,i}$, red lines), and mean coherent pressure ($\bar{p}_{w,i}$, blue lines) at the inner wall, for the R1 flow cases at $Re_b = 4000$ (a), 20000 (b), 87000 (c). Both quantities are normalised by their maximum value.

pressure and of the coherent shear stress at the inner wall, both normalised by their maximum value. Streamwise inhomogeneity induced by spanwise-coherent structures imposes clear imprint on the wall pressure (blue lines), which features peaks and troughs corresponding to the high- and low-pressure regions observed in figure 4.21. The coherent wall shear stress (red lines) also features peaks and troughs, which we explain as follows. The coherent streamwise velocity of a clockwise-rotating roll cell (positive values of the streamfunction in figure 4.21) opposes the mean flow near the inner wall, reducing locally the streamwise velocity and hence the wall shear. Counter-clockwise rotating roll cells act in the opposite way.

The phase shift between the wall shear stress and the wall pressure elucidates the mechanism through which spanwise-coherent structures reduce turbulent shear stress at the convex wall. The wavy pressure distribution generates local pressure gradients in the streamwise direction, which tend to accelerate and decelerate the fluid and result in alternating regions with high and low wall shear stress. Looking back at figure 4.21, one can see that high-pressure regions correspond to counter-clockwise rotating eddies, and vice-versa for the low-pressure regions. Hence, between any pair of counter-rotating eddies – where the flow is locally subjected to an adverse pressure gradient ($u' < 0$) – high-speed fluid is pushed towards the inner wall ($v' < 0$). Between any neighbouring pair, there is a favourable pressure gradient ($u' > 0$) and simultaneously low-speed fluid is diverted away from the inner wall ($v' > 0$). In both cases, the combination of these motions yields to $-u'v' < 0$, explaining why spanwise structures make a negative contribution to the production of Reynolds shear stress, which we have highlighted in figure 4.22(b).

To provide a quantitative basis for the above analysis, we consider the Joint Probability Density Function (JPDF) of streamwise and wall-normal fluctuations, $P(u', v')$, defined such that

$$-\overline{u'v'} = \int_{-\infty}^{+\infty} u'v' P(u', v') du' dv', \quad (4.18)$$

where the covariance integrand, $u'v'P(u', v')$, is a measure of the contribution of each pair of u' and v' to the turbulent shear stress (Wallace et al., 1977). Each quadrant of the (u', v') plane corresponds to a class of motions, specifically Q2 and Q4 quadrant events correspond to ‘ejections’ ($u' < 0$ and $v' > 0$) and ‘sweeps’ ($u' > 0$ and $v' < 0$), yielding positive contribution to the overall turbulent shear stress, whereas Q1 and

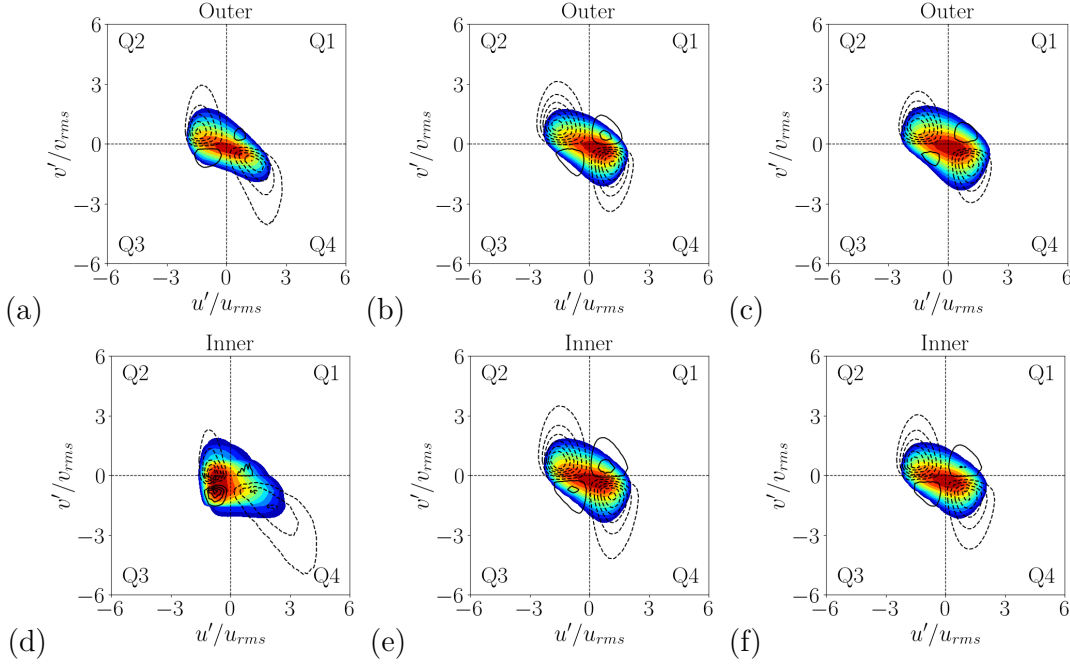


Figure 4.25: Joint PDF of streamwise and wall-normal velocity fluctuations, superimposed to flooded contours of the covariance integrand, near the outer wall (upper panels) and near the inner wall (lower panels) at $y^+ \approx 12$, for the R40 flow cases. From left to right, the panels correspond to $Re_b = 4000$ (a, d), 20000 (b, e), 87000 (c, f).

Q3 quadrants correspond to ‘outward interactions’ ($u' > 0$ and $v' > 0$), and ‘inward interactions’ ($u' < 0$ and $v' < 0$), which yield negative contribution to it.

In figures 4.25 and 4.26 we show flooded contours of $P(u', v')$ superimposed to iso-lines of $u'v'P(u', v')$ for the R40 and R1 flow cases, respectively. The joint PDF is evaluated in wall-parallel planes near the outer and the inner wall at $y^+ \approx 12$. In plane channel flow, this location is the ‘balance point’ where the contributions of ejections and sweeps are equal (Kim et al., 1987). In the R40 flow cases, the JPDF at the outer wall (upper panels) has a roughly elliptical shape with major axis inclined along the Q2 and Q4 quadrants, pointing to high probability of sweeps and ejections. Similar observations apply to the fully turbulent R40 flow cases near the inner wall (panels (e) and (f) of figure 4.25), and also for the R1 flow cases near the outer wall (upper panels of figure 4.26). However, strong curvature seems to increase the probability of sweeps more than ejections, which can be inferred from the shift of the JPDF peak towards the Q4 quadrant. As previously noted, this tendency is associated with strong secondary motions that transport high-velocity fluid from the channel core towards the outer wall, increasing near-wall velocity and thereby suppressing the formation of regions with negative velocity.

The scenario is different at the inner wall. In the R40 flow case at $Re_b = 4000$, as depicted in figure 4.25(d), the peak probability is mainly concentrated in the Q2 and Q3 quadrants, and to a lesser extent in the Q4 quadrant, on account of stronger, less probable values of u' and v' . The roughly circular shape of the JPDF at the inner wall in the R1 flow cases (figure 4.26, lower panels) points to the suppression of the negative

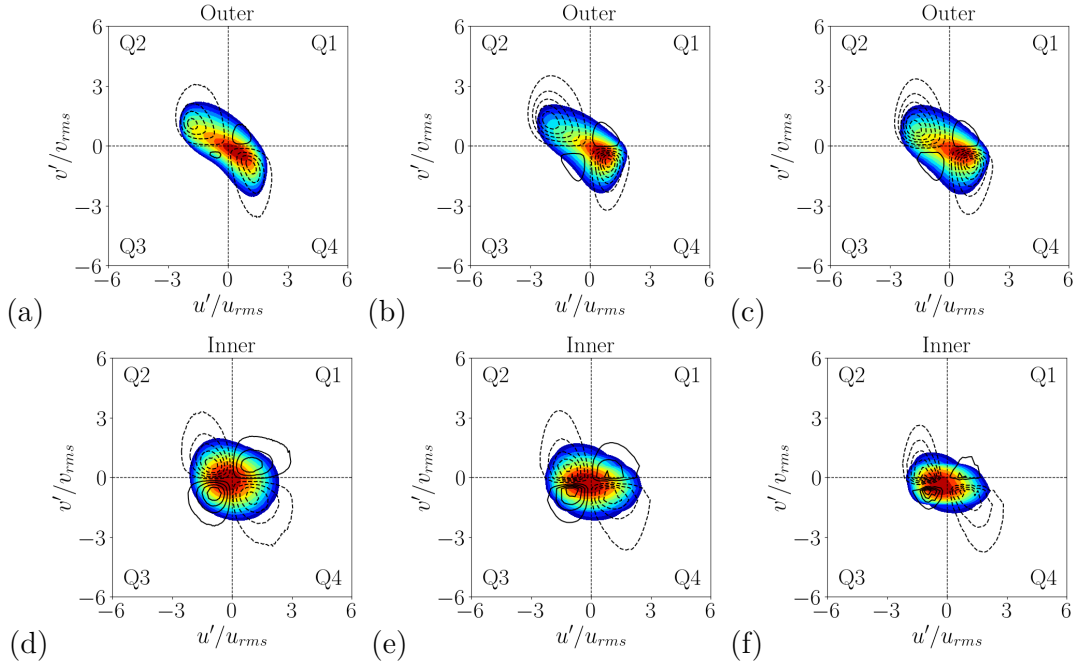


Figure 4.26: Joint PDF of streamwise and wall-normal velocity fluctuations, superimposed to flooded contours of the covariance integrand, near the outer wall (upper panels) and the inner wall (lower panels) at $y^+ \approx 12$, for the R1 flow cases. From left to right, the panels correspond to $Re_b = 4000$ (a, d), 20000 (b, e), 87000 (c, f).

correlation between u' and v' which is typical of near-wall turbulent flow. In addition, the covariance integrand highlights that the contribution to the turbulent stress is not dominated by the Q2 and Q4 motions, but rather strong contributions also come from Q1 and Q3 motions.

The same results are quantified in table 4.2, where we list the integrated contributions to the turbulent shear stress from each quadrant, at $y^+ \approx 12$. For the R40 fully turbulent flow cases the results are comparable with experimental results for plane channel flow (Wallace et al., 1972), namely the contribution of both ejections and sweeps is about 70%, whereas inward and outward interactions each contribute negatively by about 20%. At $Re_b = 4000$, the Q2 contribution near the inner wall reduces to 30%, which is comparable with the Q1 contribution but opposite in sign, whereas the Q4 contribution exceeds 110% of the shear stress. Combining this value with the shape of the JPDF in figure 4.25(d), one can infer that larger, energetic but infrequent motions are the main contributors to the turbulent shear stress. These strong sweeps can be attributed to the streamwise vortices pushing the high-speed mean flow from the channel core towards the inner wall. As for the R1 flow cases, the quadrant contributions at the outer wall are similar to those of the R40 flow cases, except for greater contribution of ejections as compared to sweeps. Hence, the most probable motions (i.e. sweeps, as visible in figure 4.26) do not contribute as much to the turbulent shear stress as the ejections, see table 4.2. At the inner wall, the fractional contributions of the outward (Q1) and inward (Q3) interactions exceeds half the contributions of ejections (Q2) and sweeps (Q4). This result confirms that the spanwise large-scale structures, which were

Re_b	Q1	Q2	Q3	Q4
R40 outer wall				
4000	-8.90	+47.56	-7.90	+69.24
20000	-12.67	+66.64	-10.10	+56.13
87000	-16.60	+66.59	-11.63	+61.64
R40 inner wall				
4000	-30.76	+31.38	-11.82	+111.19
20000	-14.10	+63.82	-9.57	+59.86
87000	-17.26	+63.75	-10.81	+64.33
R1 outer wall				
4000	-5.09	+56.27	-2.43	+51.29
20000	-7.01	+72.47	-7.09	+41.64
87000	-9.58	+68.95	-9.51	+50.14
R1 inner wall				
4000	-75.25	+115.39	-65.18	+125.04
20000	-60.01	+95.88	-49.38	+113.52
87000	-39.40	+59.72	-21.18	+100.86

Table 4.2: Percent contribution of quadrants to turbulent shear stress $(\overline{u'v'}_{Qi}/\overline{u'v'})$, for the R40 and R1 flow cases near the inner and outer wall, at $y^+ \approx 12$.

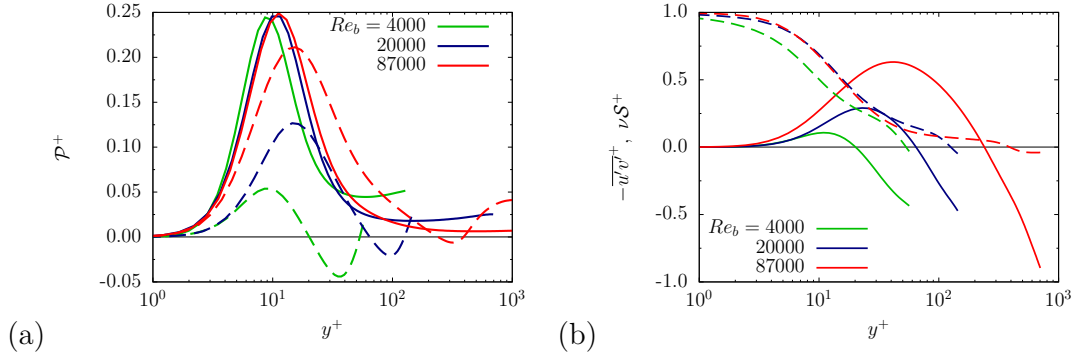


Figure 4.27: (a) TKE production ($\mathcal{P}^+$) near the outer wall (solid lines) and near the inner wall (dashed lines). (b) Turbulent shear stress ($-\overline{u'v'}^+$, solid lines) and viscous shear stress (νS^+ , dashed lines) near the inner wall. All quantities are reported in local wall units for the R1 flow cases.

found to generate Q1 and Q3 motions in §4.6, dampen the turbulent shear stress on the convex side of strongly curved channels.

Q1 and Q3 motions contribute negatively to turbulent shear stress, hence they yield negative contribution to the production of TKE, $\mathcal{P} = -\overline{u'v'}\mathcal{S}$, where $\mathcal{S} = dU/dr - U/r$ is the mean shear rate. Based on the quadrant analysis, we expect that TKE production be reduced near the inner wall in flow cases with strong curvature. In figure 4.27(a) we then show the TKE production for the R1 flow cases. Near the outer wall (solid lines) all distributions tend to collapse, especially at high Reynolds number, revealing that the flow similarity is preserved near the outer wall. The peak production is located at $y^+ \approx 12$ (except at $Re_b = 4000$, for which the peak occurs at $y^+ \approx 9$) and the peak value is $\mathcal{P}^+ \approx 0.25$, similar to plane channel flow (Kim et al., 1987; Laadhari, 2002). In contrast, TKE production near the inner wall (dashed lines) depends heavily on the Reynolds number, implying that classical wall scaling no longer holds near highly convex surfaces. Notably, a region appears where the production is negative, with an extent of 0.12δ at $Re_b = 4000$ and gradually reducing at higher Reynolds numbers. For the production to be positive everywhere, the turbulent shear stress and the viscous shear stress must have the same sign and vanish at the same location. This is not the case for the strongly curved channel, as clearly illustrated in figure 4.27(b), which shows profiles of the turbulent shear stress, $-\overline{u'v'}^+$ (solid lines), and of the viscous shear stress, νS^+ (dashed lines), near the inner wall in the R1 flow cases. The displacement between the zero-crossings of the turbulent shear stress and of the viscous shear is related to asymmetry of the mean velocity profile (Beguier et al., 2005) and leads to a region of opposing shear where $-\overline{u'v'}^+ < 0$ and $\nu S^+ > 0$, hence $\mathcal{P}^+ < 0$.

As pointed out in §4.3, the zero crossing of the mean shear marks the interface between the two distinct flow structures developing at each wall. Straddling this interface there is a diffusive transfer of shear stress from the outer- to the inner-wall region that has a sign opposite to the locally produced shear stress, leading to negative net TKE production (Hanjalić et al., 1972). In the region with negative production local energy reversal takes place, whereby energy is transferred from turbulent fluctuations to the mean flow (Eskinazi et al., 1969). Besides its physical significance, this result provides useful caveat for use of RANS models for simulations of turbulent flows over

convex walls. In fact, standard eddy-viscosity models would clearly fail if turbulent shear stress and mean shear do not go to zero at the same location.

4.7 CONCLUSIONS

We have investigated fully developed flow in a curved channel to get insight into turbulence bounded by curved walls. This setup showcases rich physics due to the interplay of fine-scale turbulence with large-scale coherent structures driven by centrifugal instabilities, which break the symmetry of the flow resulting in distinctly different behaviour near convex and concave walls. We have focused on the effects of curvature by examining two extreme cases, a mildly curved channel with radius of curvature $r_c/\delta = 40.5$ and a strongly curved channel with $r_c/\delta = 1$, with δ the channel height. For each geometry, we have studied the effect of Reynolds number (Re_b) through an extensive series of DNS, covering flow regimes from laminar up to the moderately high value of $Re_b = u_b\delta/\nu = 87000$, where u_b is the bulk velocity and ν the kinematic viscosity. Our analysis has shown that in laminar flow the friction coefficient is higher at the convex wall, whereas the opposite holds in the turbulent regime. Flow transition is anticipated by concave curvature and delayed by convex curvature, thus preventing turbulence to fully develop near the inner wall of strongly curved channels. For mild curvature, the global friction based on the mean-pressure gradient is less than in plane channel flow at the same flow rate within the range of ‘transitional’ Reynolds numbers $3000 \leq Re_b \leq 10000$, with a maximum frictional drag reduction of 12.2% at $Re_b = 4000$. This behaviour is not observed in strongly curved channels, for which friction is higher than the planar case in the transitional regime, and becomes roughly equivalent at $Re_b \geq 40000$. Through spectral analysis and flow field visualisations, we managed to detect the typical hallmarks of the near-wall turbulence regeneration cycle at both walls in the case of mild curvature, whereas wall turbulence is virtually suppressed on the convex side of strongly curved channels. Clear footprints of streamwise- and spanwise-aligned large-scale structures were also found.

Streamwise large-scale structures, originating from centrifugal instabilities and reminiscent of the Dean vortices found in laminar flow, were identified through an eduction scheme based on triple decomposition, which also allowed to quantify their effects on turbulence. Specifically, streamwise coherent structures were found to depend weakly on Reynolds number and highly on the channel geometry. In case of mild curvature, streamwise vortices modify the spanwise distribution of the wall shear and contribute significantly to streamwise velocity fluctuations and turbulent shear stress, consistent with previous studies. For strong curvature, these vortices are displaced towards the outer wall, becoming smaller in size and less stable, as we have shown by combined use of spectral analysis and triple decomposition. As a consequence, the impact of streamwise roll cells on the wall shear and momentum transfer is marginal, whereas the wall-normal velocity is strongly affected.

Spanwise-aligned large-scale structures were also observed on the convex side of strongly curved channels, leaving a footprint characterized by pressure disturbances being advected downstream at about the mean flow speed. These structures were analysed using triple decomposition and phase averaging, revealing their role in amplifying pressure disturbances. This amplification correlates with large fluctuations of

the wall shear stress, which may contribute to friction reduction. The link between enhanced pressure fluctuations and reduced wall friction is ascribed to the generation of alternating favourable and adverse pressure gradients. These gradients interact with upward and downward radial motions, respectively, leading to suppression of the turbulent shear stress near the convex wall.

This mechanism was further investigated and quantified through quadrant analysis of streamwise and wall-normal velocity fluctuations, which highlighted enhancement of inward and outward interactions (i.e. Q3 and Q1 motions), resulting in the emergence of a region with negative turbulence production. The reduction in size of streamwise vortices and the onset of spanwise structures – specific to strongly curved channels and not observed in previous studies – suggest that the flow physics undergo substantial changes when the curvature exceeds a critical threshold. Further study encompassing a wider range of relative curvature is required to determine the exact value at which these changes take place and the coherence of near-wall streaks on the inner wall breaks down.

These results offer valuable information regarding the fundamental physics of flows over curved walls, as well as for turbulence modelling, e.g. subgrid-scale models for [LES](#) and [RANS](#) closures. We also believe that insights gained from analysis of curved channel flow in a temporal setting may be relevant to understand the case of spatially developing boundary layers over convex and concave surfaces. In fact, our analysis has shown that the interaction between the inner and outer wall regions is concentrated in the channel core, whereas the inner (outer) wall region is dominated by convex (concave) curvature effects.

TRANSONIC BUFFET

In this chapter we analyse the transonic turbulent buffet over a supercritical aerofoil, used as a canonical model for the more complex buffeting flow that develops over a finite aircraft wing. Transonic buffet is a global flow instability that arises under specific aerodynamic conditions and gives rise to self-sustained shock oscillations, producing large fluctuations in the aerodynamic coefficients among others detrimental effects. Special attention is devoted to the coupling between shock motion and the shear-layer unsteadiness associated with shock-induced flow separation. In addition, upstream-travelling acoustic disturbances are identified and their role in sustaining shock buffet is examined, with the aim of clarifying the mechanisms underlying the instability.

5.1 INTRODUCTION

Modern commercial aircraft typically cruise in the high-subsonic to transonic regime. Under these conditions, the geometry of supercritical airfoils produces strong acceleration on the suction surface that generates a locally supersonic region terminated by a shock. At a fixed flight Mach number (M), increasing the angle of attack (α) amplifies the local acceleration, steepens the shock and induces shock-induced boundary-layer separation.

Beyond a critical α the flow undergoes transonic buffet, characterised by low-frequency shock oscillations coupled with coherent unsteady motion of the separated shear layer. These oscillations are self-sustained and persist without external forcing (Lee, 2001). The intense wall-pressure fluctuations associated with shock unsteadiness produce large variations in the aerodynamic coefficients on time scales at least two orders of magnitude longer than those typical of wall-bounded turbulence. Such low-frequency buffet poses an operational hazard, as it can excite structural modes of aircraft wings or tails and lead to self-sustained vibrations (commonly referred to as *buffeting*), which degrade performance, increase the risk of fatigue failure, and cause discomfort to crew and passengers. As a result, conservative safety margins are imposed that restrict the usable flight envelope (Sansica et al., 2022).

Shock buffet instability occurs only for specific combinations of Mach number and angle of attack, defining a bounded region in the (M, α) parameter space (Lee, 2001). Stanewsky et al. (1990) originally defined the buffet boundary as the locus in the (M, α) plane beyond which the flow downstream of the shock is fully separated. Several mechanisms have been proposed to explain the onset of shock unsteadiness, including: (i) bursting of the separation bubble at the shock foot (Pearcey, 1958); (ii) merging of the shock-induced separation bubble with trailing-edge separation to produce a fully separated boundary layer (Pearcey et al., 1962); and (iii) attainment of a critical Mach number immediately upstream of the shock (Mabey, 1981). Subsequent experimental investigations have consistently identified flow separation as a necessary condition

for buffet development (Lee, 1990a; Jacquin et al., 2009; D’Aguanno et al., 2021). Depending on the airfoil geometry and operating conditions, however, separation may first appear at the shock foot or near the trailing edge (Lee, 1990b).

Unsteady RANS simulations by Raghunathan et al. (1998) underlined that the shock must be sufficiently strong to generate a separation bubble for buffet to grow. In this picture, the shock periodic motion is sustained by the alternating expansion and collapse of the separation bubble, which in turn alters the effective camber of the airfoil. Nevertheless, several numerical studies did not observe a clear connection between buffet onset and flow separation (Crouch et al., 2009; Iovnovich et al., 2012; Sartor et al., 2015). A possible explanation is that these studies relied on RANS closures with turbulence models, which often struggle to predict separated boundary layers accurately. In fact, significant sensitivity to numerical schemes and turbulence-model choices has been reported, leading to large discrepancies in the prediction of the shock-induced separated region (Goncalves et al., 2004). On the other hand, Unsteady Reynolds-averaged Navier-Stokes (URANS) simulations capture the global dynamics of shock buffet reasonably well, implying that they should also capture its driving mechanisms.

Recently, Accorinti et al. (2022) showed that a necessary (but not sufficient) condition for the buffet onset is an inversion of the shock motion, with the actual onset occurring at a slightly larger angle of attack. As α increases, the flow accelerates more on the suction side and the upstream pressure is reduced, which intensifies the shock – shock strength can be quantified by the downstream-to-upstream pressure ratio – and in a first stage tends to shift the shock downstream. At a critical α , trailing-edge separation forms and concurrently the shock motion is reversed. An explanation is that trailing-edge separation raises the downstream pressure and increases the shock strength, which forces the shock towards the leading edge to raise the upstream relative Mach number and re-establish the Rankine–Hugoniot jump relations. As this upstream shift progresses, the steady solution eventually becomes unstable and the shock begins to oscillate. However, the mechanism that converts the steady shock configuration into a self-sustained unsteady oscillation at buffet onset remains an open question.

A physical explanation for the self-sustained nature of shock buffet was first proposed by Lee (1990a). In Lee’s formulation, the phenomenon is cast as an acoustic feedback loop: unsteady shock motion generates pressure disturbances that propagate downstream and amplify while convecting through the separated shear layer. Upon reaching the trailing edge, these amplified disturbances excite upstream-traveling pressure waves that propagate through the subsonic region above the separated flow and interact with the shock, exchanging energy that closes the loop and sustains the periodic shock motion. In this picture, the spatial growth of disturbances in the separated shear layer, together with the upstream and downstream propagation speeds, determine the buffet frequency.

Subsequent computational and experimental investigations have provided mixed quantitative support for Lee’s model. Good agreement between modelled and measured reduced frequencies was reported for the BGK No.1 aerofoil in several studies, and comparable buffet frequencies were obtained using URANS (Xiao et al., 2006) and zonal Detached Eddy Simulation (DES) (Deck, 2005) simulations. By contrast, Jacquin et al. (2009) found poor frequency agreement, and proposed an extension of the feedback picture in which upstream waves also travel along the lower surface and around the leading edge, thereby impinging on the shock from both sides. Based on Spectral Proper

Orthogonal Decomposition (SPOD) analysis of their LES results, however, Moise et al. (2022) found that the oscillation period predicted using Lee’s model (even considering Jacquin’s modification) is substantially shorter than the buffet period obtained from LES results. On the other hand, Hartmann et al. (2013) argued that the characteristic length traversed by upstream-traveling disturbances is the distance between the trailing edge and the shock tip, where the shock is weaker. Substituting this length into the original model yields good agreement between modelled and measured Strouhal numbers. A more robust estimation of the upstream travel time was obtained by Memmolo et al. (2018): using geometrical acoustics, they traced the path of upstream-traveling acoustic rays and showed that these rays can circumvent the shock tip and impact the shock from the upstream region, where perturbations are more efficiently amplified. In their analysis, the predicted buffet frequency resulted virtually coincident with the computed one.

An alternative perspective on the mechanism driving transonic buffet was proposed by Crouch et al. (2009). Using global linear stability analysis, they associated buffet onset with the emergence of an unstable aerodynamic mode. In this view, shock oscillations appear once the angle of attack exceeds a critical threshold, in analogy with the onset of vortex shedding in a cylinder wake when the Reynolds number crosses a critical value (Jackson, 1987). When the unstable mode is triggered, pressure perturbations originating at the shock foot propagate in the wall-normal direction along the shock and through the boundary layer, rather than within the separated shear layer as in Lee’s model. Upon reaching the upper portion of the shock, these perturbations propagate forward and dissipate into the incoming flow. Other disturbances travel aft, wrap around the trailing edge, and propagate upstream along the pressure side of the airfoil (Crouch et al., 2007). The interpretation of buffet as a global flow instability has since been widely employed to compute buffet boundaries numerically (Sartor et al., 2015; Sansica et al., 2023; Timme, 2020; Plante et al., 2021; Paladini et al., 2019).

More recently, Iwatani et al. (2023) used resolvent analysis to isolate two different mechanisms that can sustain shock oscillations. The first involves modulation of the trailing edge separation: a thicker separated area leads to flow acceleration above the shear layer and reduced post-shock pressure, vice versa for a thinner separated region. These periodic variations alter the shock-jump balance and drive the oscillations. The second mechanism emphasises pressure disturbances that travel around the shock tip and influence the shock from the upstream region, consistent with the ray-tracing arguments of Memmolo et al. (2018).

Although transonic buffet is of primary importance for commercial aviation and has been investigated for many decades, its driving mechanisms remain elusive and disagreement persists within the scientific community. In this work, we produce a high-fidelity DNS database of turbulent transonic flow over a supercritical airfoil. The angle of attack is gradually increased from stable conditions into the buffet regime. The objective is to characterise in detail the reorganisation of the flow that occurs when the angle of attack crosses the critical threshold and sustained buffet develops. In addition to providing physical insights into the mechanisms sustaining buffet, the resulting DNS dataset will serve as a benchmark for lower-order modelling approaches – especially RANS closures – which remain one of the most widely used investigative tools in both industrial and academic research on transonic turbulent buffet.

α	M	Re	$N_\xi \times N_\eta \times N_z$	$\bar{x}_s/c$	$\Delta x_{s,\max}/c$	St_B
4°	0.7	3×10^5	$5536 \times 491 \times 296$	0.460	0.031	0.09
5°	0.7	3×10^5	$5536 \times 491 \times 296$	0.486	0.016	0.09
6°	0.7	3×10^5	$5536 \times 491 \times 296$	0.483	0.042	0.09
7°	0.7	3×10^5	$5536 \times 491 \times 296$	0.436	0.179	0.09
4°	0.7	6×10^5	$10496 \times 652 \times 556$	0.469	0.026	0.13
5°	0.7	6×10^5	$10496 \times 652 \times 556$	0.472	0.029	0.17
6°	0.7	6×10^5	$10496 \times 652 \times 556$	0.449	0.070	0.09
7°	0.7	6×10^5	$10496 \times 652 \times 556$	0.402	0.177	0.11

Table 5.1: Flow parameters: angle of attack, freestream Mach number, freestream Reynolds number, number of grid points in ξ , η , z directions.

5.2 METHODOLOGY

The direct numerical simulations presented here were performed with the STREAMS-2 solver (see Chapter 2), in which its predecessor FLEW has been fully integrated. A major advantage of STREAMS-2 is its support for AMD GPU acceleration, which enabled a large-scale DNS campaign on the LUMI supercomputer. The robustness and stability of the computing environment proved essential when individual simulations required parallel execution across 32 GPU nodes for a total duration of few months. The sectional geometry is the supercritical V2C aerofoil, designed within the European TFAST project and previously investigated experimentally (Placek et al., 2018) and numerically (Zauner et al., 2019).

The flow is controlled by three parameters, namely the angle of attack (α), the freestream Mach number (M), and the chord-based Reynolds number ($Re = cu_0/\nu$, where c is the chord length, u_0 is the freestream velocity, and ν the kinematic viscosity). In all cases M is fixed at 0.7, representative of typical cruise conditions for commercial transport aircraft. The initial simulation campaign was performed at $Re = 3 \times 10^5$. Based on those results, a subsequent campaign was carried out at $Re = 6 \times 10^5$ to investigate cases in which the oncoming boundary layer is fully turbulent prior to interacting with the suction-side shock. The angles of attack considered span from stable ($\alpha = 4^\circ, 5^\circ$) to buffet conditions ($\alpha = 6^\circ, 7^\circ$). The complete set of flow parameters for each simulation is reported in Table 5.1.

The numerical setup mainly follows the procedures adopted in previous studies by our group (Memmo et al., 2018; Quadrio et al., 2022; Berizzi et al., 2025). The computational domain, sketched in the left panel of Fig. 5.1, extends radially to approximately $26c$, where c is the chord length. In the following, we will use both Cartesian (x, y) and curvilinear (ξ, η) coordinates, where ξ and η denote the directions tangent and normal to the wall. The third axis is normal to the surface, hence $\zeta = z$. The spanwise extent is $0.1c$, which is sufficient to decorrelate three-dimensional structures over the airfoil and in the wake (Hosseini et al., 2016). Spanwise periodicity is assumed and characteristic-based non-reflecting boundary conditions are applied at the far field

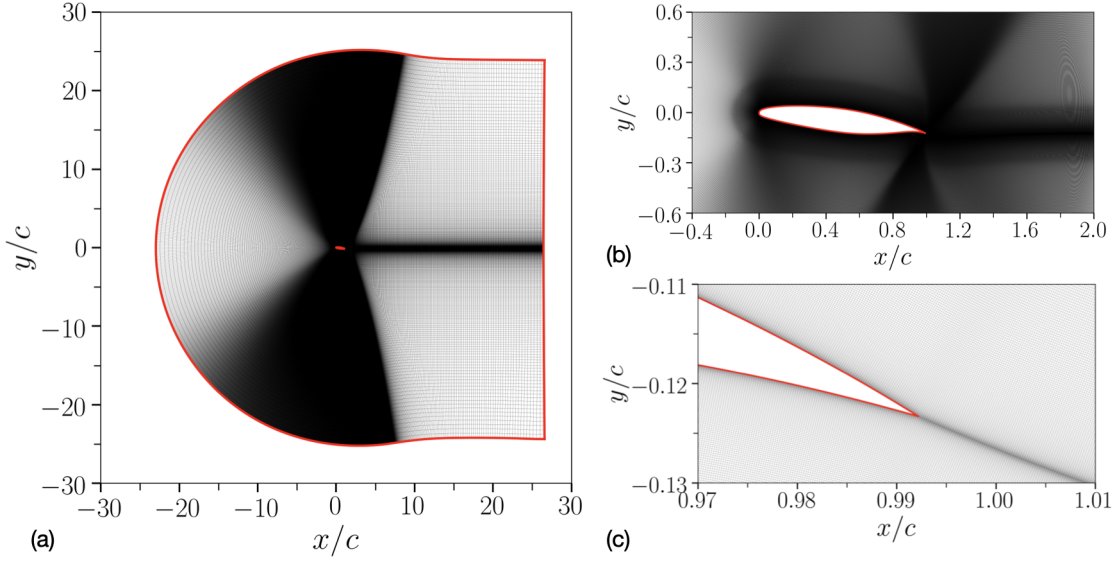


Figure 5.1: Computational grid for the V2C supercritical aerofoil at $M = 0.7$, $\text{Re} = 6 \times 10^5$ and $\alpha = 7^\circ$. (a) Global view of the body-fitted C-type mesh and computational domain in the (x, y) plane. (b) Close-up of the near field, showing the grid clustering around the airfoil. (c) Further zoom on the trailing-edge region, showing the smooth transition into the wake region.

(Pirozzoli et al., 2013). At the downstream rectilinear boundary, a subsonic extrapolation with imposition of a reference pressure is used, together with purely non-reflecting characteristic conditions. At the upstream curvilinear boundary, the characteristic conditions in conjunction with a narrow sponge region relax the flow variables towards a reference far-field state, obtained from a precursor [RANS](#) calculation.

Boundary-layer transition is forced on both airfoil surfaces at $x = 0.1c$ by applying a time-varying wall-normal body force that emulates experimental roughness elements, dampening the numerical artefacts associated with conventional tripping techniques (Schlatter et al., 2012). The time and length scales of the forcing are defined based on the local displacement thickness of the boundary layer, which is $\delta^* \approx 10^{-3}$ and $\delta^* \approx 7 \times 10^{-4}$ for the two Reynolds numbers considered. The spatial extent of the forcing region is defined by a Gaussian blob centred at $x = 0.1c$, with length scales of $4\delta^*$ and δ^* along ξ and η directions. The spanwise wavelength is $1.7\delta^*$, the time scale is $4\delta^*/u_0$, and the tripping amplitude is $150u_0^2$.

A body-fitted curvilinear mesh with C-type topology is used to conform to the airfoil geometry. The C-mesh in the (x, y) plane was generated with an in-house hyperbolic grid generator, based on the open-source Construct2D library (Prosser, 2018). The algorithm minimises cell non-orthogonality, which reaches a maximum value of 4° in the trailing edge region. Moreover, the most refined region of the C-mesh, clustered around the computational boundary that connect the trailing edge with the downstream boundary, is aligned with the wake behind the airfoil, thus optimising grid resolution.

A key challenge in [DNS](#) of inhomogeneous flows is resolving the local Kolmogorov length scale η_K , which varies substantially across the domain. To address this, we adopt a two-step strategy. First, the near-wall grid distribution is designed under the

assumption of local boundary-layer equilibrium. Under this assumption, the streamwise and spanwise spacings are chosen according to standard wall-unit constraints, namely $\Delta\xi^+ < 9$ and $\Delta z^+ < 4.5$. Wall-normal clustering is prescribed using the stretching function of Pirozzoli et al. (2021b), which guarantees $y^+ < 1$ at the wall and provides adequate resolution of both the buffer and logarithmic layers. Second, a precursor **RANS** computation at the same flow conditions is used to estimate the local **TKE** production, from which the local dissipation rate (ε) is inferred assuming a local production–dissipation balance. The Kolmogorov scale is then obtained via

$$\eta_K = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4},$$

which permits explicit sizing of the mesh. The two approaches produce similar target resolutions over the aerofoil surface. However, only the **RANS**-based estimate enables a practical prescription of the wake resolution, which we enforce by requiring $\Delta x < 6\eta_K$ (Pope, 2001) for at least one chord length downstream of the trailing edge, and $\Delta y < 6\eta_K$ for $0.2c$ in η direction. The fidelity of the **RANS** results was confirmed *a posteriori* against **DNS**.

The computational meshes comprise $5536 \times 491 \times 296$ points for flow cases at $\text{Re} = 3 \times 10^5$, and $10496 \times 652 \times 556$ points for cases at $\text{Re} = 6 \times 10^5$. For practical and physical alignment between the high-resolution region of the mesh and the wake, each simulation uses a case-specific grid constructed by rotating the airfoil of α degrees, rather than the incoming velocity vector. The computational mesh for the case at $\text{Re} = 6 \times 10^5$ and $\alpha = 7^\circ$ is shown in Fig. 5.1. This case can serve as an illustrative example for the distribution of grid points. Along ξ direction, 5598 points lie on the airfoil surface, 2085 points are allocated to the near wake ($1 < x/c < 2$), and 364 points cover the far wake ($2 < x/c < 26$), where the grid spacing increases according to a geometric progression. In η direction, 153 points are located within the boundary-layer thickness ($y/c < 0.025$), 233 points populate the intermediate region ($0.025 < y/c < 0.2$) to resolve wake structures, and the remaining 266 grid planes extend toward the far-field boundary. The most refined mesh regions are visible as the dark patches in Fig. 5.1.

5.3 FLOW ORGANIZATION

Snapshots of the instantaneous flow field are shown in Fig. 5.2 and Fig. 5.3 for representative non-buffet/incipient-buffet and fully-buffet flow cases, respectively, at $\text{Re} = 6 \times 10^5$. The numerical Schlieren, computed as $\|\nabla\rho\|$, clearly highlights the suction-side shock system, the small-scale turbulent structures that develop downstream of the tripping location, and the turbulent wake behind the trailing edge. At $\alpha = 4^\circ$ (Fig. 5.2a) the post-shock flow contains a narrow supersonic region, which is terminated by a small secondary shock. This weak shock wave is perturbed by upstream-travelling waves and merges intermittently with the primary shock.

Increasing the angle of attack to $\alpha = 5^\circ$ (Fig. 5.2b) enlarges the supersonic pocket and strengthens the main shock. The mean shock position $\bar{x}_s$ for $\alpha = 5^\circ$ is closer to the trailing edge than for $\alpha = 4^\circ$ (see Table 5.1), consistent across both Reynolds numbers considered. This result aligns with the inversion of shock motion observed

by Accorinti et al. (2022), according to which a first increase in the angle of attack produces a downstream shift of the shock. At $\alpha = 6^\circ$ the shock becomes unsteady and low-frequency oscillations develop. Although the shock excursions remain limited in amplitude, with a maximum displacement of $0.07c$, their effect on the boundary layer and wake is pronounced: when the shock is in its most downstream position (Fig. 5.2c) the wake is relatively thin, whereas after an upstream shift of the shock (Fig. 5.2d) the boundary-layer thickness increases and the wake becomes thicker. Compared with non-buffet flows, the shock extends to a higher distance from the wall and exhibits a curved shape, with concavity facing downstream.

Further increasing the angle of attack to $\alpha = 7^\circ$ produces a fully developed buffet state, in which the shock oscillation amplitude is roughly four times larger than in the $\alpha = 6^\circ$ case. When the shock lies in its most downstream position (Fig. 5.3a) the boundary layer remains relatively thin and the wake is narrow. As the shock moves upstream, namely the shock Mach number $M_s < 0$ (Fig. 5.3b), the relative Mach number ahead of the shock increases and the shock strength attains its maximum. At this stage of the buffet cycle, the adverse pressure gradient is increased, which thickens the boundary layer and enlarges the separated region.

The increased unsteadiness of the separated shear layer is visible as steeper gradients in the trailing-edge region, associated with the formation of large vortical structures that detach from the trailing edge and emit upstream-travelling acoustic disturbances. These upstream-travelling disturbances begin to impinge upon the shock when it is in its most upstream position (Fig. 5.3c). The disturbances circumvent the shock tip and deflect from above the shock toward the upper surface of the aerofoil, interacting with the shock within the supersonic flow region. Fig. 5.3d completes the buffet cycle. At this stage, the shock is weakened as it moves downstream again ($M_s > 0$), reducing the upstream relative Mach number. Throughout the cycle, pressure perturbations propagating upstream along the pressure side are also observed. Upon reaching the leading edge, these perturbations are convected further into the upstream flow rather than wrapping around the airfoil nose.

The shock motion is compared across various angles of attack in Fig. 5.4. The instantaneous shock position x_s is identified as the location of the maximum pressure gradient in the wall-parallel direction $dp/d\xi$, evaluated at a wall-normal coordinate $\eta \approx 5^{-3}c$, i.e. just above the δ_{99} boundary-layer thickness of the incoming flow.

At low Reynolds number (Fig. 5.4a) the mean shock position for $\alpha = 5^\circ$ and $\alpha = 6^\circ$ lies, on average, closer to the trailing edge than for $\alpha = 4^\circ$. Small-amplitude shock excursions are already present at $\alpha = 4^\circ$ and $\alpha = 5^\circ$, their amplitude increasing slightly at $\alpha = 6^\circ$, which can be identified as an incipient-buffet flow state. The shock cycles at $\alpha = 6^\circ$ are highly irregular: both amplitude and period vary noticeably from cycle to cycle, further motivating the terminology of *incipient* buffet condition. At $\alpha = 7^\circ$ the amplitude of shock oscillations grows further, although significant cycle-to-cycle variability remains.

At high Reynolds number (Fig. 5.4b) the shock is sharper and the x_s trace therefore appears smoother. Relative to the low-Re cases, the mean shock position is shifted downstream at $\alpha = 4^\circ$, is essentially unchanged at $\alpha = 5^\circ$, and is shifted upstream at $\alpha = 6^\circ$, with comparable oscillation amplitudes to the low-Re case. For the fully developed buffet case ($\alpha = 7^\circ$) the low-frequency variations become less irregular and the x_s time series approaches a nearly periodic behaviour.

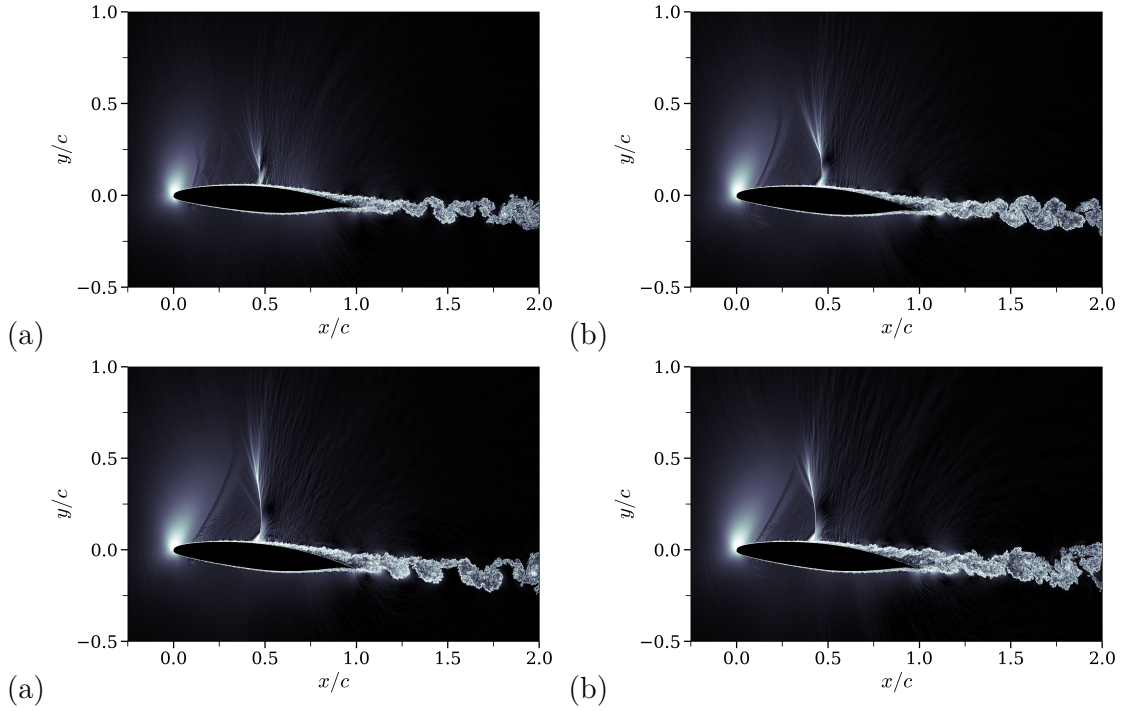


Figure 5.2: Snapshots of numerical Schlieren for non-buffet flow (a,b) and incipient-buffet flow (c,d) at $Re = 6 \times 10^5$. The angle of attack is 4° (a), $\alpha = 5^\circ$ (b), and 6° (c,d). For incipient-buffet flow the snapshots are taken at instants corresponding to the most downstream (c) and most upstream (d) shock location.

A direct comparison of the shock position (Fig. 5.4a,b) and the lift coefficient C_L (Fig. 5.4c,d) reveals a strong positive correlation between the two signals: when the shock shifts upstream toward the leading edge, the supersonic, low-pressure region expands on the suction side and C_L increases; when the shock moves downstream the opposite occurs. Because C_L is an integral quantity, its time trace also contains higher-frequency contributions arising from smaller-scale phenomena on the airfoil surface, which appear as superimposed high-frequency oscillations.

The spectral content of both signals is summarised in Fig. 5.4(e *rightarrow* h) via their frequency spectra. For the low-Reynolds-number cases (Fig. 5.4e) the dominant peak associated with the buffet corresponds to a Strouhal number $St = fc/u_0 \approx 0.09$ for all examined angles of attack. This observation suggests that the buffet instability is already present as a weak signature below the fully developed buffet threshold, even when the low-frequency shock excursions have very small amplitude. In contrast to the shock-position spectra, the Power Spectral Density (PSD) of the lift coefficient (Fig. 5.4g) exhibits a clear secondary energy bump in the frequency band $1 < St < 4$. This feature is associated with trailing-edge vortex shedding or ‘wake mode’ (Moise et al., 2022). At $\alpha = 7^\circ$ the wake mode also appears in the spectra of the shock-position, indicating a coupling between a wake-shedding mechanism and the low-frequency buffet dynamics.

At $Re = 6 \times 10^5$ (Fig. 5.4f) the buffet signature is less clear-cut: the dominant Strouhal number varies non-monotonically with α and the spectral peak is actually more pronounced at $\alpha = 4^\circ$ than at $\alpha = 5^\circ$. The characteristic frequency of the wake

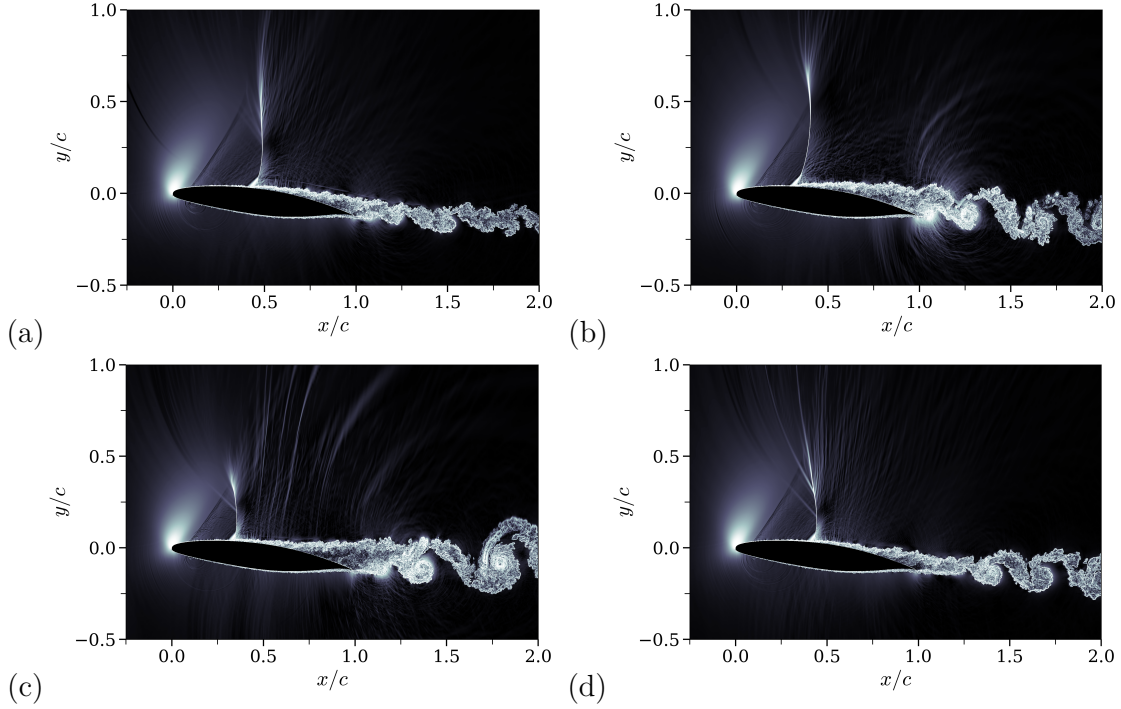


Figure 5.3: Numerical Schlieren for fully-buffet flow at $\text{Re} = 6 \times 10^5$, $M = 0.7$, $\alpha = 7^\circ$. The snapshots are taken at time instants corresponding to: (a) most downstream shock position ($M_s = 0$); (b) shock moving upstream ($M_s < 0$); (c) most upstream shock position ($M_s = 0$); (d) shock moving downstream ($M_s > 0$).

mode decreases with increasing α (Fig. 5.4g), consistent with an enlarged wake length scale. Possible causes for the non-monotonic behaviour of the peak Strouhal number include changes in the effective length and convection speed that set the buffet time scale, and modal competition between buffet and wake-shedding dynamics.

We analyse the boundary-layer properties across the computed flow cases, with more emphasis on the suction side of the aerofoil. Figure 5.5(a, b) shows the pressure coefficient

$$c_p = \frac{2(p_w - p_0)}{\rho_0 u_0^2},$$

on the airfoil surface, where the subscript 0 refers to freestream quantities and the subscript w refers to the wall. The direction of the y -axis is reversed from the usual convention in order to display the upper side (where c_p is negative) at the top. A pronounced decrease of the c_p on the upper side with increasing angle of attack is evident. This progressive increase in suction accounts for the rise in lift observed at high α (Fig. 5.4c, d), despite the mean shock position shifting closer to the trailing edge. For each α , the most upstream and most downstream shock locations are marked by dashed lines. Except for $\alpha = 7^\circ$, the mean slope of c_p across the shock region is similar across all flow cases. At $\alpha = 7^\circ$, the milder mean slope reflects the larger instantaneous shock excursions. In that case, the c_p distribution shows an incomplete pressure recovery downstream of the shock, with $c_p < 0$ at the trailing edge. The systematically larger jump in c_p at $\text{Re} = 6 \times 10^5$ (Fig. 5.5b) relative to $\text{Re} = 3 \times 10^5$ (Fig. 5.5a) indicates a stronger shock at the higher Reynolds number, as observed above.

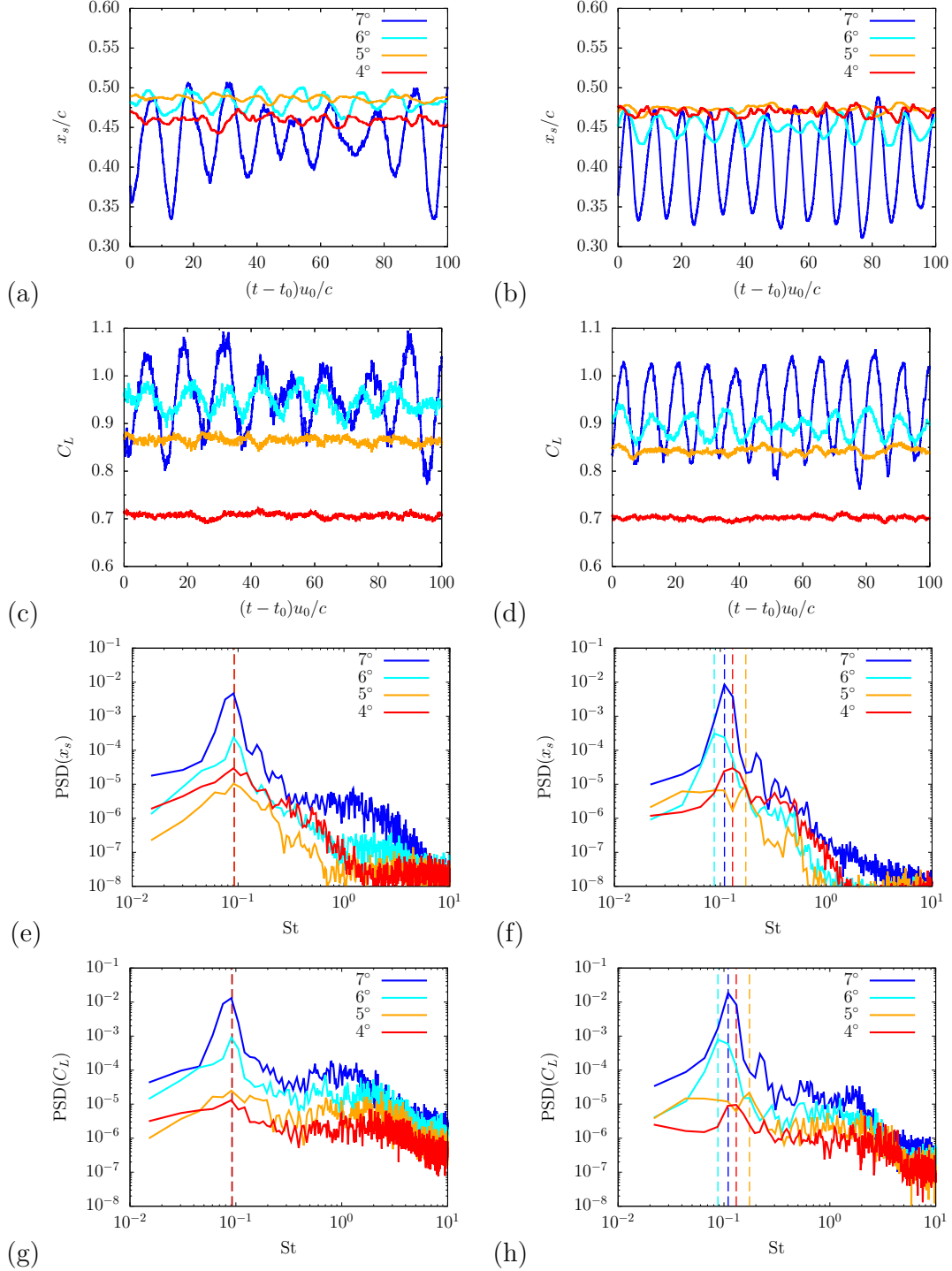


Figure 5.4: Time evolution of the shock position x_s (a,b) and of the lift coefficient C_L (c,d) for flow cases at $Re = 3 \times 10^5$ (a,c) and $Re = 6 \times 10^5$ (b,d). PSD of x_s (e,f) and C_L (g,h) as a function of the Strouhal number St for flow cases at $Re = 3 \times 10^5$ (e,g) and $Re = 6 \times 10^5$ (f,h).

The friction coefficient is defined as

$$c_f = \frac{2\mu_w}{\rho_0 u_0^2} \left. \frac{d\tilde{u}}{d\eta} \right|_w,$$

where $\tilde{u}$ denotes the velocity component along the ξ (wall-parallel) direction. The c_f distribution along the suction side exhibits a clear Reynolds-number dependence. At low Re (Fig. 5.5c) the tripping forcing produces a marked local peak in c_f , downstream of which c_f continues to increase. This behaviour signals that the boundary layer has not reached a canonical fully turbulent state: in a zero-pressure-gradient turbulent boundary layer one expects a gradual downstream decay of c_f . This observation is consistent with the strongly localised favourable pressure gradient on the suction side. Although the local Mach number rises from 0.7 to ~ 1.3 , that acceleration is confined to a narrow region immediately downstream of the leading edge. Accordingly, the boundary layer in the interval $0.1c < x < 0.3c$ effectively experiences near-zero pressure gradients. Conversely, high-Re cases (Fig. 5.5d) are less influenced by the tripping force and display a downstream reduction of c_f , consistent with a genuinely turbulent state. An exception is the fully-buffet case, where the proximity of the shock most upstream location to the trip, combined with stronger flow acceleration, hinders full development of turbulence in the upstream boundary layer.

A key quantity for characterising buffet is the extent of the separated flow region, since its central role in buffet onset has long been recognised. Flow separation is identified as the location where $c_f = 0$, highlighted by solid vertical lines in Fig. 5.5. Owing to the relatively steep negative slope of the V2C aerofoil near the trailing edge compared to other supercritical aerofoils, separation occurs in all flow cases considered here, with the separation point moving upstream as α increases. Somewhat counter-intuitively, the mean separation point at high Re lies farther upstream than in the corresponding low-Re flow cases.

However, the backflow probability is lower at high Re, and never exceeds 50% within the shock region (Fig. 5.5e,f). This rules out the presence of a mean shock-induced separation bubble as a necessary condition for buffet onset. Nevertheless, the present results reinforce that backflow events remain closely tied to buffet (the reason will be clarified later). As shown in Fig. 5.5(g,h), a shock-induced separation bubble in a time-averaged sense appears only at low Re and only for the higher α cases, reaching a maximum height of $\eta_{\psi=0} \approx 10^{-3}c$. The separation height is computed as the wall-normal distance to the dividing streamline ($\psi = 0$), obtained by integrating $\partial\psi/\partial\eta = \bar{\rho}\tilde{u}$ in the η (wall-normal) direction. In addition to exhibiting an upstream shift of the detachment point, the trailing-edge separation at high Re also attains a larger height, extending to more than 5% of the chord.

5.4 SPECTRAL ANALYSIS

The spectral characteristics of wall-pressure fluctuations on the suction side are examined through pre-multiplied PSD distributions. At each spatial location, the spectra are normalised by the local wall-pressure variance, allowing a direct comparison of the dominant temporal scales across the flow field. The resulting wall-pressure spectra,

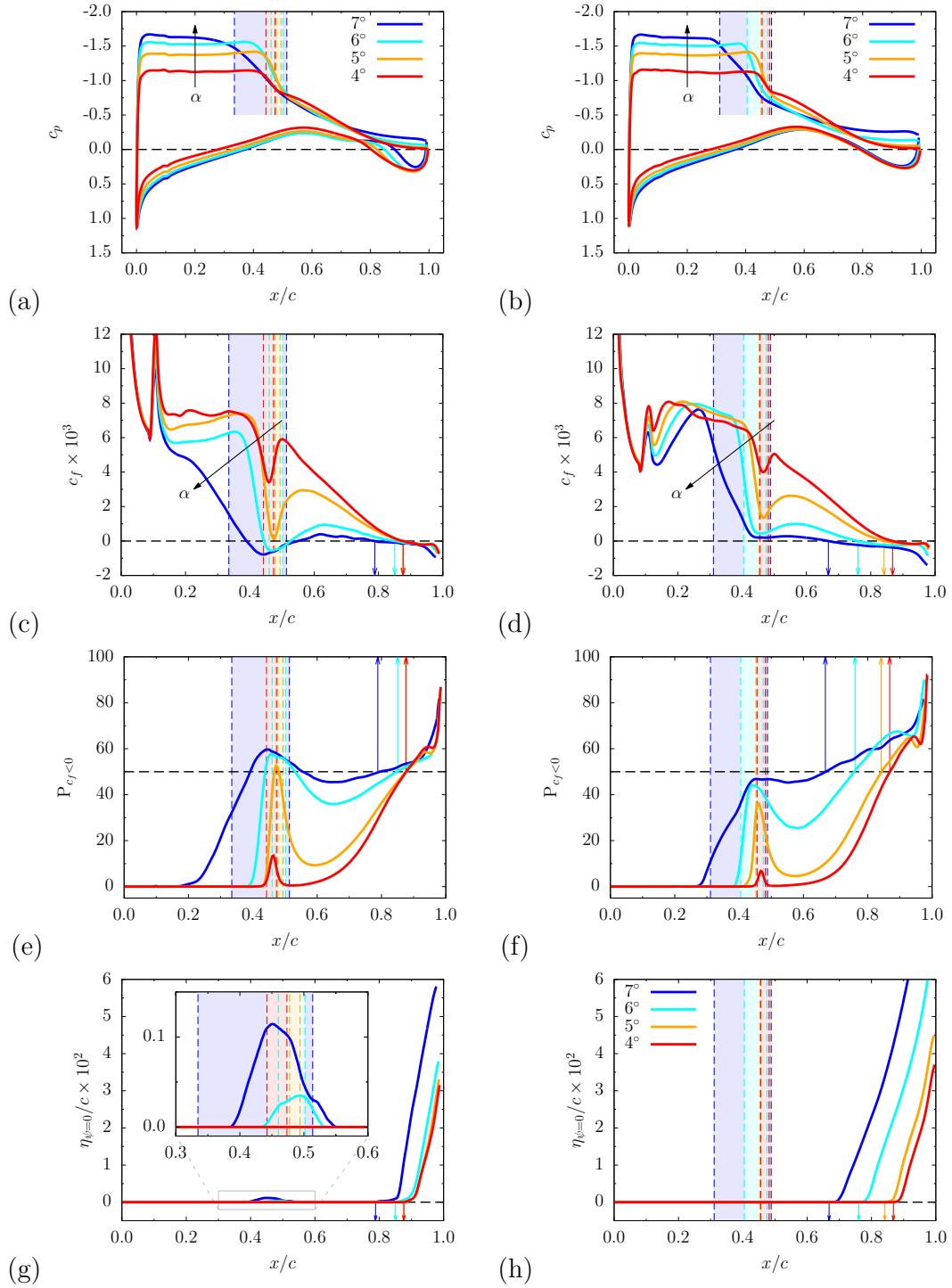


Figure 5.5: Distributions along x -direction of the pressure coefficient c_p (a,b), friction coefficient c_f (c,d), backflow probability $P_{c_f < 0}$ (e,f), and separation height $\eta_{\psi=0}/c$ (g,h), for flow cases $Re = 3 \times 10^5$ (left panels) and $Re = 6 \times 10^5$ (right panels). Panels c $\rightarrow$ h show distributions on the suction side only. Vertical dashed lines denote most upstream and most downstream shock positions; solid lines denote the mean separation point.

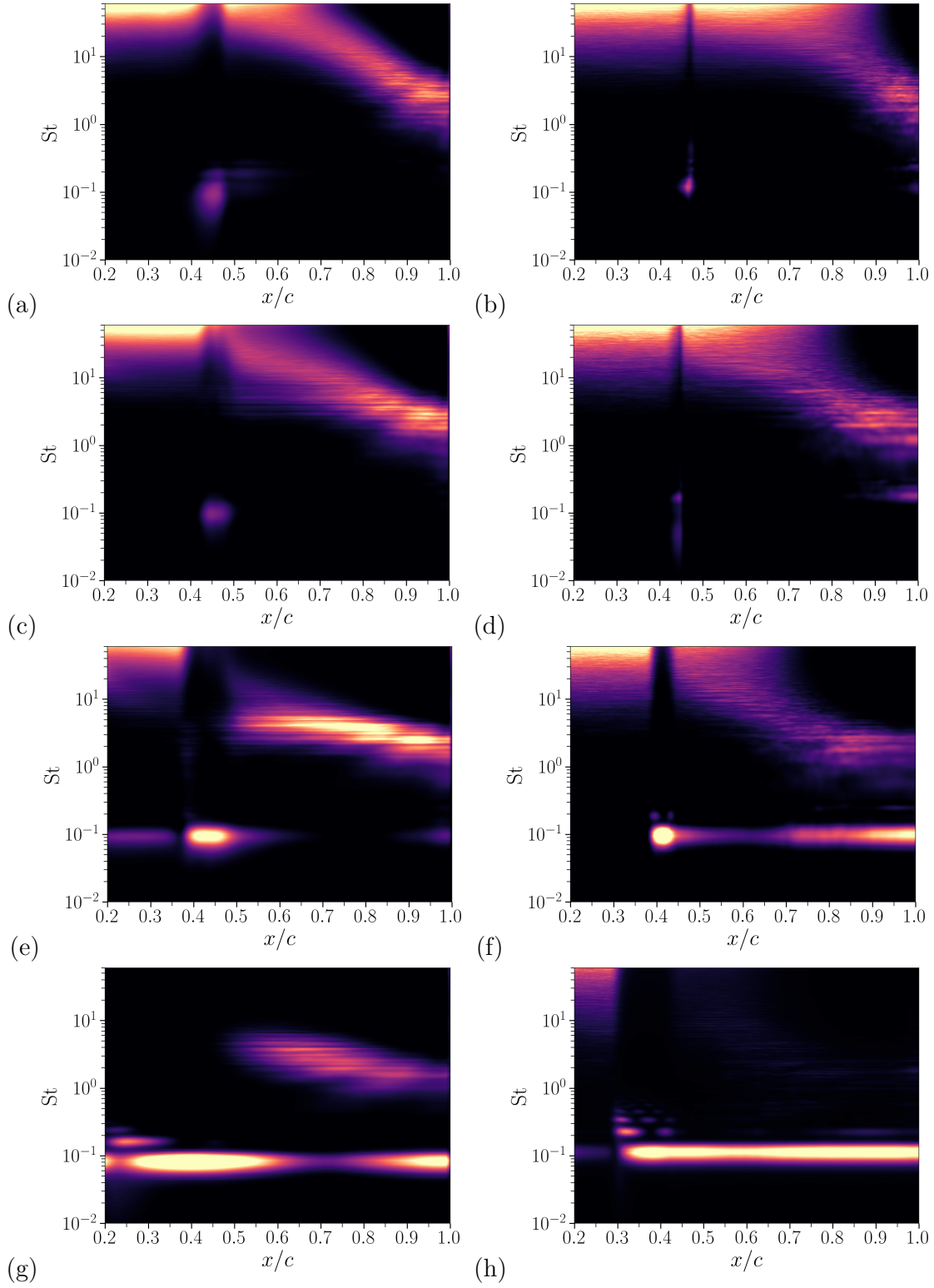


Figure 5.6: Normalized and frequency-premultiplied PSD of the wall pressure ($\hat{E}_{p,w}$) on the suction side as a function of the x coordinate and Strouhal number St for $\alpha = 4^\circ$ (a,b), $\alpha = 5^\circ$ (c,d), $\alpha = 6^\circ$ (e,f), and $\alpha = 7^\circ$ (g,h). Left panels refer to flow cases at $Re = 3 \times 10^5$, right panels to $Re = 6 \times 10^5$.

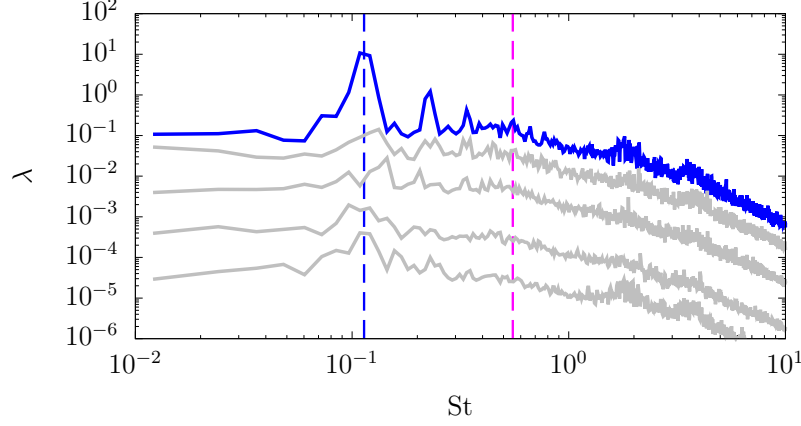


Figure 5.7: Frequency spectra of the first five SPOD eigenvalues obtained by pressure norm for the fully-buffet flow case at $\alpha = 7^\circ$ and $\text{Re} = 6 \times 10^5$. Blue and magenta dashed lines are at $\text{St} \approx 0.11$ (the buffet frequency) and $\text{St} \approx 0.55$.

$\hat{E}_{p,w}$, are shown in Fig. 5.6 for all considered flow cases, where the hat symbol denotes local normalization.

For the non-buffet cases at $\alpha = 4^\circ$ (a,b) and 5° (c,d), the spectra exhibit features consistent with canonical SBLI, as further discussed in the next chapter. The impact of the incoming turbulent boundary layer on the wall pressure is visible at high frequencies ($\text{St} \approx 50$), a value roughly five times higher than the characteristic wall-shear frequency (not shown). This reflects the different scales of convection velocities: wall-pressure disturbances are convected by motions in the buffer layer, whereas wall-shear fluctuations convect at friction-velocity scales. Near the mean shock location ($\bar{x}_s \approx 0.45c$), a distinct energy peak at the buffet frequency ($\text{St} \approx 0.1$) appears, corresponding to the low-amplitude, low-frequency oscillation of the shock foot. Downstream of the shock, the concentration of energy at intermediate frequencies ($1 < \text{St} < 4$) is associated with the wake mode (Moise et al., 2022), which includes the vortex-shedding dynamics.

The effects of increasing Reynolds number (right panels) include a sharper and narrower low-frequency peak, indicative of a stronger shock, as well as the emergence of a high-frequency spectral peak associated with post-shock turbulence, which is absent at low Re . The characteristic frequencies of both the buffet-mode and wake-mode remain consistent across the range of angles of attack and Reynolds numbers considered.

The incipient-buffet case at $\alpha = 6^\circ$ (e,f) displays spectral properties intermediate between non-buffet and fully-buffet flows, featuring a broadened peak at the buffet frequency. At $\text{Re} = 6 \times 10^5$ (Fig. 5.6f), an additional energy peak at $\text{St} \approx 0.1$ appears in the trailing-edge region. This ‘energy bridge’ between the shock location and the trailing edge becomes particularly pronounced at $\alpha = 7^\circ$ (g,h), where the low-frequency peak dominates over all other spectral contributions. Near the mean shock position, subharmonics of the buffet mode also appear at $\text{St} \approx 0.2$, $\text{St} \approx 0.3$, and $\text{St} \approx 0.4$.

An attempt to elucidate the connection between shock buffet and trailing-edge low-frequency unsteadiness is made via **SPOD** (Schmidt et al., 2020). Unlike standard Proper Orthogonal Decomposition (**POD**), which extracts spatially coherent structures only, **SPOD** yields modes that are coherent in both space and time (Towne et al., 2018). Here the analysis is performed using pressure as the energy norm, which is well suited to capture both the buffet and wake dynamics, and only focusing on the fully-buffet flow case at $\text{Re} = 6 \times 10^5$. The eigenvalue spectra of the pressure-based **SPOD** (Fig. 5.7) display pronounced energy peaks at $\text{St} \approx 0.11$ and $\text{St} \approx 2$; at each spectral peak the first eigenvalue is substantially larger than the remainder, indicating clear low-rank behaviour.

Two representative energetic **SPOD** modes are selected for detailed inspection at $\text{St} \approx 0.11$, the buffet frequency, and $\text{St} \approx 0.55$, its fifth subharmonic, marked by the blue and magenta dashed lines, respectively. The eigenvectors of the buffet mode are visualised in Fig. 5.8(a→d) for different phase angles, $\varphi = 2\pi \text{St} u_0 / c$. At $\varphi = \pi$ (c) the shock is in its most upstream position, and the buffet eigenmode concentrates strong pressure fluctuations (red contours) in the shock region. These disturbances are anti-correlated with negative pressure fluctuations (blue contours) in the trailing-edge and near-wake region, showing a phase opposition between the shock motion and the resulting shear layer unsteadiness. The reverse occurs when the shock attains its most downstream position, at $\varphi = 0$ (a). The remaining phase angles $\varphi = \pi/2$ (b) and $\varphi = 3\pi/2$ (d) capture the upstream and downstream shock motion, respectively. The upstream traveling shock (b) is stronger, as the relative upstream Mach number is increased, whereby positive pressure fluctuations dominate the post-shock region.

Fig. 5.8(e → h) shows the eigenvectors at $\text{St} \approx 0.55$ for the same phase angles. This frequency was selected as it remains associated with the shock buffet (it corresponds to a subharmonic of the buffet) while being sufficiently higher than the primary buffet frequency, to reveal additional flow structures. Acoustic waves originating at the trailing edge and propagating upstream along both surfaces of the aerofoil are clearly visible. These waves are consistent with the Kutta waves invoked in Lee’s feedback-loop model. In the initial portion of their path they travel at a speed approximately equal to $a_0 - u_0 \approx 0.3 a_0$, where a_0 denotes the freestream speed of sound and u_0 the freestream velocity. The subset of waves forming on the pressure side propagate upstream along the lower surface, and leave the airfoil unperturbed. Their role in sustaining buffet appears thus marginal. By contrast, waves on the suction side reach the shock, are deflected toward the surface, enter the supersonic pocket, and impinge on the shock from the upstream side.

The **SPOD** analysis is instrumental for estimating the time required for these waves to travel upstream from the trailing edge to the fore side of the shock. A pressure wave emitted at the trailing edge at phase $\varphi_0 = 0$ arrives at the interaction point at $\varphi_{\text{up}} \approx 5.5\pi$. Hence we estimate the upstream propagation time as

$$t_{\text{up}} = \frac{\varphi_{\text{up}}}{2\pi \text{St}} \frac{c}{u_0} \approx 5 \frac{c}{u_0},$$

where $\text{St} = 0.55$. This same upstream travel time was obtained by Memmolo et al. (2018) via a different approach: using geometrical acoustics they traced acoustic rays and estimated that the travel time of a ray striking the shock front at the midpoint of the sonic line is in fact $t_{\text{up}} \approx 5 c/u_0$.

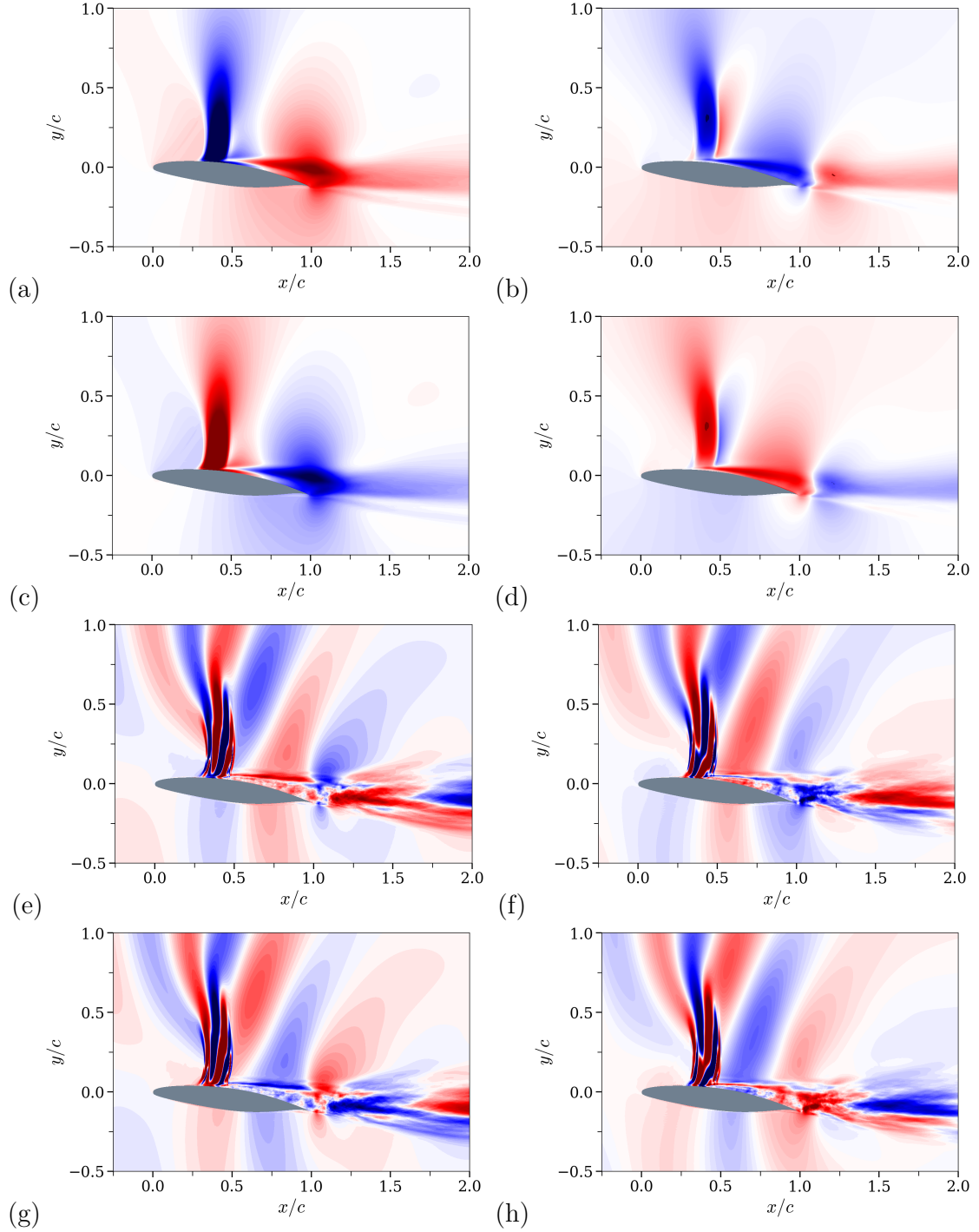


Figure 5.8: The first **SPOD** mode at $St \approx 0.11$ (a $\rightarrow$ d) and at $St \approx 0.55$ (e $\rightarrow$ h) obtained with pressure norm for the fully-buffet flow case at $\alpha = 7^\circ$ and $Re = 6 \times 10^5$. The selected phase angles are $\varphi = 0$ (a,e), $\pi/2$ (b,f), π (c,g), $3\pi/2$ (d,h).

5.5 BUFFET MECHANISM

As noted above, shock buffet is commonly interpreted as an acoustic feedback loop, whereby pressure disturbances generated at the shock foot are convected downstream along the suction surface toward the trailing edge, where scattering produces upstream-traveling waves that in turn perturb the shock and close the loop. The completion time of this closed loop must match the buffet period:

$$T_B = t_{\text{dn}} + t_{\text{up}},$$

where t_{dn} denotes the downstream travel time of disturbances from the shock foot to the trailing edge, and t_{up} denotes the upstream travel time of the scattered waves back to the shock. Although the model introduced by Lee (1990a) captures this feedback mechanism, its quantitative predictions for the buffet frequency have shown poor agreement with a number of experiments and numerical simulations (Jacquin et al., 2009; Moise et al., 2022). Two sources of uncertainty are particularly important. First, the convection speed of the downstream-traveling pressure disturbances is not trivial to determine: it depends on the local flow within the separated shear layer and may vary along the path, so direct measurement or reliable modelling is challenging. Second, the actual path followed by the upstream-traveling disturbances is debated in the literature, and different authors have proposed distinct characteristic length scales and propagation routes. These discrepancies have led some investigators to question the dynamical relevance of scattered Kutta waves for buffet.

In the previous section we used SPOD to extract t_{up} from the phase behaviour of the upstream-traveling acoustic signature. We now focus on t_{dn} : the buffet SPOD mode displayed in Fig. 5.8(a → d) exhibits coherent flow structures that originate at the shock foot and are convected downstream along the separated shear layer with a phase lag relative to the shock motion. To further examine this coupling, Fig. 5.9 presents the fluctuating pressure on a near-wall plane as a function of space and time. The spanwise-averaged pressure field has been phase-averaged, namely the time-average is taken over corresponding instants within the buffet cycle after alignment of signals from different cycles by their buffet phase. The resulting pressure signal is shown in two spectral bands: panel (a) reports the high-pass filtered signal and panel (b) the low-pass filtered signal, both using a cut-off at $\text{St} = 0.3$. This filtering isolates the buffet-related dynamics from flow phenomena occurring at faster time scales.

The spatiotemporal contours of the high-pass filtered pressure reveal two distinct types of coherent structures: 1) pressure disturbances that envelope the shock position (indicated by the black solid line) and move both upstream and downstream at very low speed ($\approx 0.05 u_0$), with motion locked to the shock oscillation; 2) faster pressure disturbances generated at the shock location and convected downstream with a high speed ($u_{\text{dn,fast}} \approx 0.66 u_0$). Using the mean shock position $\bar{x}_s \approx 0.4 c$ and the trailing-edge coordinate $x_{\text{TE}} = c$, the characteristic downstream travel time for the fast disturbances results $t_{\text{dn,fast}} \approx 0.9 c/u_0$, which yields clearly a much shorter period than the buffet period, and therefore is inconsistent with direct participation of these fast convected disturbances in closing the buffet feedback loop.

The fast convected disturbances are most intense during the mid-phase of the buffet cycle, corresponding to the upstream motion of the shock. This correlation arises because

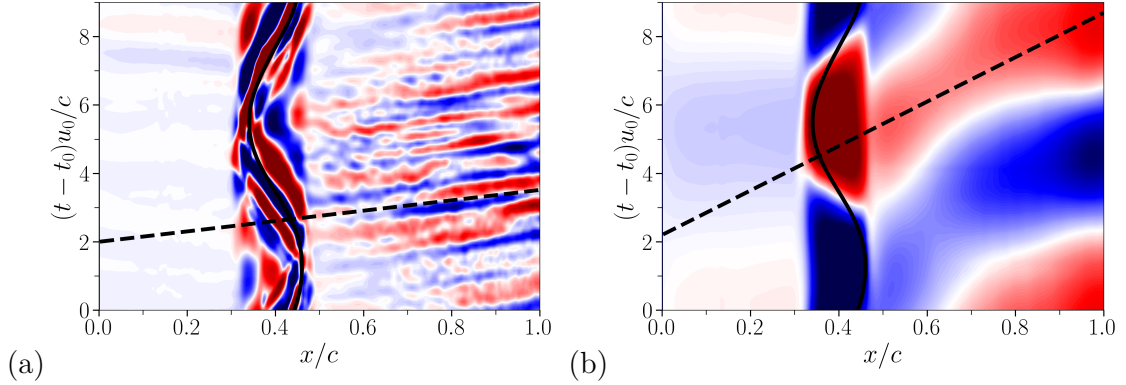


Figure 5.9: Spatiotemporal variation of fluctuating pressure in a wall-parallel plane at $\eta \approx 5 \times 10^{-3} c$ for the fully-buffet flow case at $\alpha = 7^\circ$ and $\text{Re} = 6 \times 10^5$. The pressure signal is spanwise- and phase-averaged using the buffet period of $T_B \approx 9 c/u_0$, corresponding to $\text{St} \approx 0.11$. The signal is high-pass filtered (a) and low-pass filtered (b) with cut-off frequency fixed at $\text{St} = 0.3$. Black solid lines mark the phase-averaged shock position; black dashed lines denote the estimated downstream convection speeds.

a stronger shock enlarges the separated region (see Fig. 5.3) and thereby amplifies unsteady vortex shedding associated with shock-induced separation, producing stronger downstream-travelling pressure signatures.

The low-pass filtered pressure in Fig. 5.9(b) clearly exposes the coupling between shock motion and shear-layer unsteadiness described in Fig. 5.8(a $\rightarrow$ d). At a representative instant of the buffet cycle (say, $t \approx 4.5 c/u_0$, corresponding to a phase angle $\varphi = \pi$) a strong positive pressure patch arises at the shock. Farther downstream along the upper surface one observes a sign reversal: the positive disturbance (red contours) is followed by a negative region (blue contours) associated with the unsteady shear layer generated by the shock interaction. This arrangement reflects the phase lag between shock displacement and the flow structures in the unsteady shear layer. The downstream signatures are generated by large-scale vortical structures that, due to their hydrodynamic nature, convect more slowly than the acoustic disturbances tied to the shock. This explains their delayed appearance at the trailing edge relative to the instantaneous shock motion.

A direct estimate of the downstream convection time can be obtained from the space-time map by combining the effective downstream path length, from the mean shock position $\bar{x}_s \approx 0.4 c$ to the trailing edge at $x_{\text{TE}} = c$, with the empirically measured convection speed of the large-scale structures, $u_{\text{dn}} = 0.15 u_0$. These values give

$$t_{\text{dn}} \approx \frac{x_{\text{TE}} - \bar{x}_s}{u_{\text{dn}}} \approx 4 \frac{c}{u_0}.$$

When this downstream time is combined with the upstream acoustic time previously estimated of $t_{\text{up}} \approx 5 c/u_0$, the resulting feedback-loop period is $T_B \approx 9 c/u_0$. Expressed as Strouhal number, this corresponds to $\text{St} \approx 0.11$, virtually matching the DNS result. This consistency supports the feedback-loop mechanism in which large-scale vortical structures set the downstream time scale, while upstream-travelling acoustic disturbances close the loop following the path outlined above.

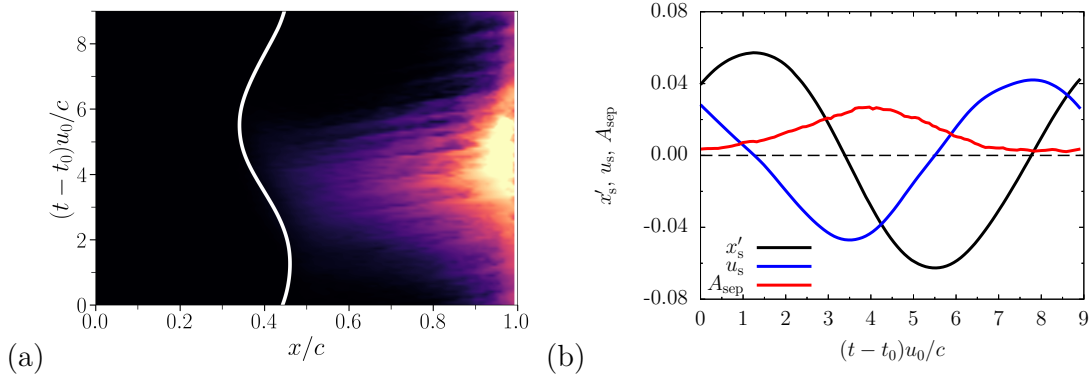


Figure 5.10: (a) Spatiotemporal variation of the separation height $\eta_{\psi=0}$ for the fully-buffet flow case at $\alpha = 7^\circ$ and $\text{Re} = 6 \times 10^5$. The separation height, defined as the wall-normal distance to the dividing streamline ($\psi = 0$), is spanwise- and phase-averaged over the buffet period $T_B \approx 9 c/u_0$ (i.e., cycles are time-aligned by buffet phase prior to averaging). (b) Temporal variation of the fluctuating shock position $x'_s = x_s - \bar{x}_s$, shock velocity $u_s = dx_s/dt$, and separated flow area $A_{\text{sep}} = \int_{\text{sep}} \eta_{\psi=0} d\xi$. All signals are spanwise- and phase-averaged.

The dynamics of shock-induced flow separation is elucidated in Fig. 5.10(a), which presents the space–time diagram of the separation height $\eta_{\psi=0}$ obtained as described in section 5.3. A more direct depiction is given in Fig. 5.10(b), which shows the time evolution within one buffet period of the fluctuating shock position $x'_s = x_s - \bar{x}_s$, the shock velocity $u_s = dx_s/dt$, and the separated-flow area A_{sep} . As in the previous analysis, all fields have been averaged in the spanwise direction and phase-averaged.

During the half-cycle in which the shock recedes upstream ($u_s < 0$ in Fig. 5.10b), the separated area grows markedly owing to the strengthened adverse pressure gradient. The maximum of A_{sep} occurs near $t \approx 4 c/u_0$, with an approximate delay of $0.6 c/u_0$ relative to the instant of maximum upstream shock velocity. This delay is explained by the finite propagation speed of information from the shock toward the trailing edge, as evidenced by the space-time traces in Fig. 5.10(a). The spatiotemporal diagram further shows that the receding-shock portion of the cycle is characterised by a fully separated boundary layer (black contours denote attached flow, and bright-yellow contours indicate the maximum separation height). This fully separated state explains the increased backflow probability in the region between the shock and the trailing edge (see Fig. 5.5).

These observations corroborate early experimental studies (Pearcey et al., 1962; Jacquin et al., 2009; D’Aguanno et al., 2021) that emphasised the central role of shock-induced separation in buffet onset. Specifically, complete boundary-layer separation promotes shear-layer roll-up and the formation of coherent vortical structures; at the same time, the larger shear associated with the receding shock strengthens these flow structures, which are convected to the trailing edge and increase acoustic scattering. A similar interpretation was reached by Hartmann et al. (2013).

5.6 CONCLUSIONS

In this chapter we presented an extensive dataset of transonic turbulent flow over the V2C supercritical aerofoil, spanning angles of attack from stable conditions to fully-developed buffet. The [DNS](#) approach allowed us to characterise, without modeling ambiguity, the changes in shock motion, shear-layer unsteadiness, separation dynamics and acoustic scattering across the buffet threshold.

A key finding concerns the evolution of shock motion with increasing angle of attack (α). The [DNS](#) confirmed the experimentally observed inversion of mean shock displacement: from $\alpha = 4^\circ$ to 5° the mean shock location moves downstream, whereas at $\alpha \approx 6^\circ$ it reverses direction and began to oscillate with a limited amplitude of excursions. In the fully-buffet regime ($\alpha = 7^\circ$) the oscillation amplitude increases by a factor of four, the shock trajectory became more regular, and the buffet mode dominates the wall-pressure spectrum, with observable subharmonics up to $St \approx 0.5$. A low-frequency spectral peak at $St \approx 0.09\text{--}0.11$ was also detected in non-buffet cases, albeit at reduced amplitude compared with buffeting flows, indicating that the buffet instability exists in a *latent* state.

We analysed the boundary-layer response to buffet. At $Re = 6 \times 10^5$, the backflow probability in the shock region remains below 50% (i.e. the flow is attached on average), showing that a steady, shock-induced separation bubble is *not* a necessary condition for buffet onset. Differently from non-buffet flows – where backflow is mostly confined beneath the shock and near the trailing edge – buffeting flows display a significant backflow probability in the region between the shock and the trailing edge. When buffet is fully developed, the receding shock (i.e. moving upstream) increases the relative upstream Mach number and strengthens the adverse pressure gradient, producing a fully separated boundary layer and enhancing the shedding of vortical structures. This behaviour is a plausible trigger for the buffet instability that leads to sustained shock oscillations.

Spectral analysis of wall pressure quantified the characteristic frequencies of the problem: incoming turbulence ($St \approx 50$), trailing-edge vortex shedding ($1 < St < 4$), and low-frequency unsteadiness ($St \approx 0.1$). The latter is confined to a narrow region across the shock in non-buffet flows, whereas it extends from the shock to the trailing-edge region in fully-buffet cases. The dynamical coupling between shock motion and separated shear-layer dynamics was elucidated through modal analysis of the fully-buffet flow field. The [SPOD](#) based on pressure norm revealed a dominant buffet mode at $St \approx 0.11$ that couples shock displacement to large-scale coherent structures, which are convected downstream within the separated shear layer.

From spatiotemporal diagrams of phase-averaged, low-pass-filtered pressure fluctuations we estimated the downstream convection speed as $u_{dn} \approx 0.15 u_0$, corresponding to a downstream travel time $t_{dn} \approx 4c/u_0$. The [SPOD](#) mode at $St \approx 0.55$ captures upstream-travelling acoustic waves, which are generated at the trailing edge. These waves propagate along the suction side, circumvent the shock tip and perturb the shock from its fore side, after an upstream travel time $t_{up} \approx 5c/u_0$. The total feedback-loop time, obtained by summing downstream and upstream propagation times, is $T_B \approx 9c/u_0$, in excellent agreement with the buffet period extracted from the [DNS](#). This analysis supports the interpretation that shock buffet is sustained by a mixed hydrodynamic–acoustic feedback loop, involving downstream convection of large-scale

vortical structures through the separated shear layer and upstream propagation of acoustic waves scattering from the trailing edge.

The present results provide a detailed characterisation of transonic buffet dynamics, aimed at clarifying several debated aspects of the buffet mechanism. Moreover, the [DNS](#) database generated in this work will serve as a reference for future stability analyses of shock buffet and for flow-control studies based on [URANS](#) simulations.

SUPERSONIC RAMPS

In this chapter we investigate shock–boundary layer interactions on supersonic compression ramps, a canonical framework for studying high-speed aerodynamics. Special attention is paid to the effects of crossflow, which acts as a sideways push on the mean flow, skewing it relative to the separation line. This configuration mirrors real-world applications and introduces three-dimensional effects that reshape the separated region and amplify flow unsteadiness. Understanding these mechanisms is essential for establishing scaling laws that capture the spatiotemporal dynamics of supersonic flows over swept ramps.

6.1 INTRODUCTION

SBLI are phenomena that occur on aerospace vehicles and play a key role in shaping their aerodynamic performance. These interactions often lead to flow distortion, turbulence amplification, localized heat transfer peaks, increased aerodynamic loads, and reversed flow, posing critical challenges for predictive modeling and flow control strategies. The onset and severity of flow separation in **SBLI** are governed by multiple factors, including the characteristics of the incoming boundary layer, shock strength, and surface geometry. In strong interactions – those involving mean flow separation – the recirculating bubble exhibits a highly unsteady dynamics, with dominant frequencies that in statistically two-dimensional, steady flows are far lower than the typical frequency scales of turbulence. These low-frequency oscillations are associated with an intermittent motion of the separation shock foot and with the emergence of large-amplitude, spatially-coherent pressure fluctuations, which can degrade aerodynamic performance and induce severe structural loading if not properly accounted for in the design process. An accurate analysis of the spectral characteristics and spatial pattern of wall pressure fluctuations is therefore essential to mitigate their detrimental effects and improve the aerodynamic performance of high-speed vehicles.

Extensive research has focused on 2D-**SBLI** due to their relative simplicity and reproducibility. Canonical test cases for understanding the underlying flow physics include (1) the oblique impinging shock set-up, where a boundary layer develops over a flat plate and interacts with an externally generated shock wave, which is relevant to supersonic inlets and air-breathing propulsion systems, and (2) the compression ramp set-up, where the shock is induced by flow deflection at the ramp corner, which models the flow over aircraft surfaces, engine inlet ducts, and control surfaces, and it is used in supersonic vehicle design to optimize aerodynamics and heat loads. Both configurations produce two-dimensional ‘closed’ interactions, namely where the freestream direction is orthogonal to the mean separation line and the separated fluid recirculates upstream trapped in a separation bubble. Dussauge et al. (2006) showed that the characteristic time scale in two-dimensional strong interactions can be obtained by defining a Strouhal

number ($St_{L,0}$) based on the separation length (L_s) and upstream velocity (u_0), which typically falls between 0.03 and 0.05. This scaling holds for different flow configurations, indicating that the low-frequency motion of the separation shock exhibits general properties independent of its generation mechanism.

However, most aerospace applications involve inherently three-dimensional flow fields, which motivates the investigation of 3D-SBLI. The breakdown of spanwise homogeneity can manifest in various ways, leading to a diverse range of interactions where the flow structure strongly depends on the specific geometry. For instance, the shock topology may vary from oblique to conical to detached, depending on whether the flow interacts with a sharp fin, a semi-cone, or a blunt fin, respectively (Gaitonde et al., 2023). A more general approach, which retains certain effects of flow three-dimensionality while suppressing others, involves simulating an ‘infinite’ swept interaction (Gross et al., 2018). In this configuration, a crossflow velocity component in the spanwise direction imposes a sweep angle between the freestream and the direction normal to the mean separation line. This idealized numerical setup produces SBLI with cylindrical symmetry, meaning that the mean separation and reattachment lines remain parallel. Such interactions have been observed experimentally in swept compression corners (Allen et al., 2023) and in impinging SBLI (Padmanabhan et al., 2021) for sweep angles below a threshold that depends on the inviscid shock strength. Beyond this maximum sweep angle, the symmetry of the separated region transitions from cylindrical to quasi-conical, with diverging separation and reattachment lines (Threadgill et al., 2020). Under the conditions investigated in this study, the limiting sweep angle was found to be approximately 20° (Settles et al., 1984).

Regardless of the separation bubble symmetry and shock generation mechanism, various experimental studies (Allen et al., 2023; Padmanabhan et al., 2021; Erengil et al., 1993; Doehrmann et al., 2018; Vanstone et al., 2019a) consistently reported that the unsteadiness of the separated zone shifts to higher frequencies as the sweep angle increases. This frequency modification has sometimes been attributed to coherent flow structures in the separated region, which in swept interactions are advected by the crossflow inducing corrugations along the separation line (Sabnis et al., 2023). In contrast, recent numerical investigations of strong three-dimensional interactions around sharp fins and swept compression ramps (Adler et al., 2020; Zhang et al., 2022) reported that the modified separated-flow topology tends to mute the low-frequency band characteristic of two-dimensional unsteadiness. Hence, it remains unclear whether the mechanism governing a fully three-dimensional interaction can be linked to the spanwise corrugations observed in the 2D case. Based on DNS of swept SBLI, Ceci et al. (2023a) derived a predictive model to characterize low-frequency unsteadiness in the presence of non-zero sweep angle (γ_0). Their DNS dataset revealed that the structure of wall pressure fluctuations on the mean separation location is dominated by traveling pressure waves, whose characteristic wavelength scales with L_s . The key prediction is that the peak Strouhal number should scale as

$$St_L = \left| St_{L,0} \pm \frac{\eta \tan \gamma_0}{\alpha} \right|, \quad (6.1)$$

where $St_L = fL/u_0$, $St_{L,0} \approx 0.04$ is the typical Strouhal number in two-dimensional interactions ($\gamma_0 = 0^\circ$), and $\alpha \approx 2$ and $\eta \approx 0.7$ are model coefficients based on

the characteristic length and velocity scales of the problem. These predictions were confirmed at higher Reynolds numbers by a [LES](#) study (Bergier et al., 2023), and in the hypersonic regime by a subsequent [DNS](#) study (Ceci et al., 2024).

Derivation and validation of this model are both based on numerical experiments performed in the impinging shock set-up. However, swept [SBLI](#) are more commonly observed in configurations akin to compression ramps, such as those found in control surfaces of high-speed aircraft (Sabnis et al., 2023). Although swept ramps represent more closely typical aerospace applications, their increased geometric complexity has led previous computational studies on [SBLI](#) to prioritize the impinging shock set-up. From a numerical standpoint, structured flow solvers based on Cartesian coordinates are inadequate for simulating compression ramp flows, which require more advanced methods to accurately handle the ramp geometry. An additional challenge in performing high-fidelity computations of swept [SBLI](#) is the dual requirement of sufficiently long simulation times, required to capture the low-frequency dynamics, and sufficiently large domains, needed to accommodate the spanwise rippling of the separation line and avoid spurious spanwise resonance (Ceci et al., 2023a; Larsson et al., 2022).

In this work, we generate a [DNS](#) dataset of supersonic flow over a compression ramp. By varying the inflow sweep angle, we assess the influence of crossflow on [SBLI](#) with cylindrical symmetry. After characterizing the pressure field through instantaneous visualizations and flow statistics, we focus on the spatio-temporal dynamics of wall pressure fluctuations in the separated zone. Spatial structures associated with low-frequency unsteadiness are identified via spectral analysis, which is used to determine the characteristic time and length scales of the flow and to identify possible causes of the shift in dominant frequencies observed in swept flow cases.

The main goal of this work is to verify the applicability of the model proposed by Ceci et al. (2023a) to swept interactions over compression ramps, with the analysis further supported by a comparison with experimental data available in the literature. A straightforward extension of the model might be expected given the success of the free-interaction concept in describing 2D-[SBLI](#) (Dussauge et al., 2006), for which scaling laws have been shown to be independent of the mechanism triggering flow separation – whether it is a solid obstacle or an incident shock wave. Otherwise, discrepancies would indicate a configuration-dependent behavior in swept interactions. The high fidelity of the present [DNS](#) data enables a precise determination of the model parameters, providing valuable insights into the universality of scaling laws for low-frequency unsteadiness in swept [SBLI](#).

6.2 METHODOLOGY

[DNS](#) of turbulent flow over a compression ramp are carried out using the solver FLEW (see chapter 2). As sketched in Fig. 6.1, the coordinate system is aligned such that x is normal to the corner edge, y is normal to the flat wall upstream of the corner, and z is parallel to the corner edge. In addition to the Cartesian system, the curvilinear system (ξ, η) is used to post-process the z -averaged fields, where ξ is tangent to the wall and η is normal to the wall.

A turbulent boundary layer with thickness δ_0 is injected at the left boundary skewed by an angle $\gamma_0 = \arctan(w_0/u_0)$ with respect to the positive x direction, where u_0 and

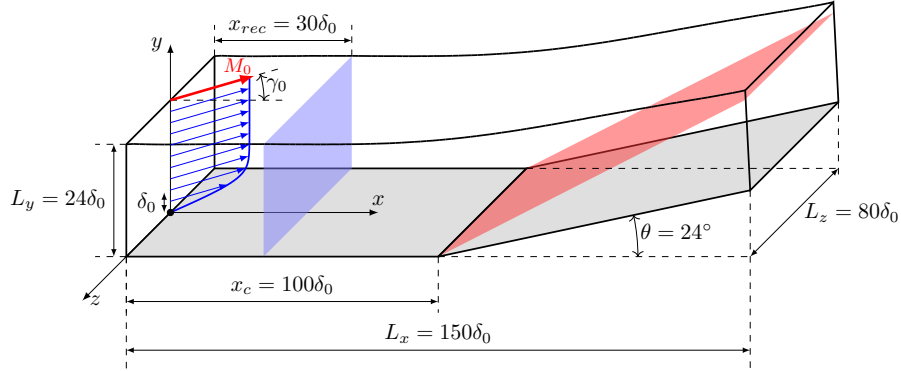


Figure 6.1: Schematic of computational set-up for supersonic compression ramp (not to scale). L_x , L_y , L_z are the domain sizes, M_0 , γ_0 , and δ_0 are the Mach number, sweep angle, and boundary layer thickness at the inflow, θ is the deflection angle, x_c and x_{rec} are the locations of the corner and recycling plane. The blue and red shaded area are the recycling plane and the inviscid shock representation.

w_0 are the inflow velocity components in the x and z directions. At the inlet, a mean turbulent profile is derived as the inverse Van Driest transform of the incompressible velocity distribution proposed by Musker (1979). At each time step, velocity fluctuations are evaluated in a recycling plane and added to the mean inlet profile upon proper rescaling and spanwise shifting (Ceci et al., 2022). The recycling plane is placed at $x_{rec} = 30\delta_0$. Temperature fluctuations are obtained from the fluctuating streamwise velocity using the Strong Reynolds Analogy. No-slip isothermal conditions are enforced at the bottom boundary, where the wall temperature is set to the nominal adiabatic value upstream of the interaction. Flow periodicity is assumed in the spanwise direction, and non-reflecting boundary conditions based on characteristic wave relations (Pirozzoli et al., 2013) are applied at top and outlet boundaries.

The domain dimensions are $L_x \times L_y \times L_z = (150 \times 24 \times 80)\delta_0$, much larger than in previous swept SBLI simulations (Adler et al., 2020; Larsson et al., 2022), to accommodate the largest flow structures without artificially constraining the flow physics due to the spanwise periodicity. The corner edge is placed $100\delta_0$ downstream of the inlet, a sufficient extent to ensure that the turbulent boundary layer is fully developed and free from any spurious correlation before interacting with the shock (Pirozzoli et al., 2011). The computational grid consists of $5632 \times 512 \times 6656$ points, and the resulting wall spacings in viscous units, evaluated both upstream and downstream of the interaction, are listed in Table 6.1. The two-dimensional grid in the x - y plane is generated based on the approach by Martín et al. (2003), with some modifications in both the x and y directions. The grid is piecewise uniform in the x direction, with a hyperbolic tangent blending at the ramp corner. The first section optimizes the number of points in the pre-interaction region, where grid requirements are less stringent, while the second section is more refined. As shown in Fig. 6.2(a), this strategy allows to achieve the DNS resolution requirement both upstream ($\Delta x^+ < 6$) and downstream ($\Delta x^+ < 10$) of the interaction. The wall-normal grid spacing is tuned so that it never exceeds twice the local value of the Kolmogorov length scale upstream of the interaction (Ceci et al., 2023b), and the grid spacing at the wall is $\Delta y_w^+ < 1$ in the entire computational domain. The grid points are uniformly distributed in the z direction,

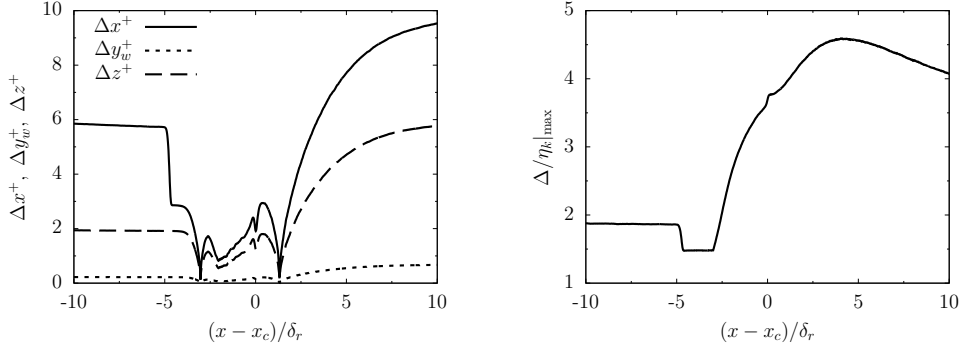


Figure 6.2: Distributions of grid spacings in wall units (a) and maximum value of Δ/η_k (b) for $\gamma_0 = 0^\circ$, where $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ and η_k is the local Kolmogorov scale.

γ_0 [°]	M_0	Re_{δ_r}	Re_{δ_2}	δ_r/δ_0	$\Delta x^+ \times \Delta y_w^+ \times \Delta z^+$	$(x_s - x_c)/\delta_r$	L_s/δ_r	$St_{L,pk}$
0	2.90	34370	1226	2.21	5.82/9.95 $\times$ 0.22/0.68 $\times$ 1.93/5.94	-3.03	4.78	0.049
15	3.00	35339	1191	2.26	5.47/9.73 $\times$ 0.21/0.65 $\times$ 1.82/5.68	-3.40	5.18	0.117
30	3.35	43789	1220	2.39	5.10/9.45 $\times$ 0.19/0.64 $\times$ 1.69/5.53	-4.50	7.31	0.336

Table 6.1: DNS details: inflow sweep angle (γ_0) and Mach number (M_0), Reynolds number based on boundary layer thickness (Re_{δ_r}) and momentum thickness (Re_{δ_2}) at the reference location, boundary layer thickness at the reference location (δ_r/δ_0), grid spacings in viscous units ($\Delta x^+ \times \Delta y_w^+ \times \Delta z^+$) upstream and downstream of the interaction, mean separation point $(x_s/\delta_r - x_c/\delta_r)$, separation length (L_s/δ_r), peak Strouhal number ($St_{L,pk}$).

resulting in a spanwise spacing of $\Delta z^+ < 2$ upstream of the interaction and $\Delta z^+ < 6$ downstream of it. The adequacy of the mesh is further highlighted in Fig. 6.2(b), which shows that the x -wise distribution of the grid spacing in Kolmogorov units never exceeds 2 in the upstream region, reaching a maximum value of approximately 4.5 past the corner location.

Three flow cases characterized by different inflow sweep angles are considered to investigate the effects of crossflow: (1) $\gamma_0 = 0^\circ$, (2) $\gamma_0 = 15^\circ$, and (3) $\gamma_0 = 30^\circ$. Higher values of the sweep angle are not increased as in real-life cases the flow would transition to a quasi-conical symmetry state (Padmanabhan et al., 2021; Settles et al., 1984). The inviscid parameters determining the shock strength are the x -projected inflow Mach number ($M_{0,x}$) and the deflection angle (θ). The values of $M_{0,x} = 2.9$ and $\theta = 24^\circ$ are selected to match the set-up investigated experimentally by Bookey et al. (2005) and numerically by Wu et al. (2007). Those 2D interactions were used to validate the present solver in Chapter 3. The x -projected Mach number is kept constant to ensure consistent shock strength across the three cases, resulting in different values of $M_0 = M_{0,x}/\cos \gamma_0$ (listed in Table 6.1). The varying Mach number corresponds to a varying Reynolds number based on boundary layer thickness,

$$\text{Re}_\delta = \rho_0 \delta_r \sqrt{u_0^2 + w_0^2} / \mu_w, \quad (6.2)$$

where the subscript w denotes properties evaluated at the wall, whereas r indicates fluid properties at the reference location $x_r = 80\delta_0$ (upstream of the interaction). To minimize the uncertainty related to variations in the incoming Reynolds numbers with γ_0 , we have ensured that all flow cases exhibit roughly the same value of the equivalent incompressible momentum thickness Reynolds number,

$$Re_{\delta_2} = \rho_0 \theta_r \sqrt{u_0^2 + w_0^2} / \mu_w \approx 1200, \quad (6.3)$$

where θ_r is the momentum thickness at the reference location. This value is sufficiently high to avoid any uncertainty due to low Reynolds number effects, as shown in previous studies (Larsson et al., 2022; Di Renzo et al., 2022). We are confident that flow modifications due to the sweep angle are properly isolated, as verified by the perfect collapse of the incoming boundary layer properties across the three cases.

Statistical quantities averaged in time and in z direction, denoted with the overline symbol, are obtained by averaging over a minimum interval of $1600 \delta_0/u_0$ after reaching a statistically steady state, determined by monitoring the spanwise-averaged position of the separation point. Spectral analysis of wall pressure fluctuations is carried out by collecting snapshots of the flow field at the constant sampling interval $\Delta t_s = 0.17 \delta_0/u_0$, and the energy spectra are computed by applying the Welch method with five time windows with 75% overlap.

6.3 FLOW ORGANIZATION

In Fig. 6.3 we show a side view in the x - y plane of the time and spanwise averaged pressure field, $\bar{p}(x, y)$, for $\gamma_0 = 0^\circ$ (a) and $\gamma_0 = 30^\circ$ (b). The magenta line marks the dividing streamline (ψ), determined by integrating $\partial\psi/\partial\eta = \overline{p u_\xi}$ in the η (wall-normal) direction, where u_ξ is the velocity component along the ξ (wall-parallel) direction. White and gray lines mark respectively the sonic line (along which $M = 1$) and the boundary layer thickness (δ_{99}), which is identified as the edge of the rotational viscous boundary layer, $y_\omega(x)$ (Spalart et al., 1993). Specifically, $\delta_{99}(x)$ is defined as the location where $\tilde{u}_s = \sqrt{\tilde{u}_\xi^2 + \tilde{w}^2} = 0.99\tilde{u}_s(y_\omega)$, where the subscript s denotes the streamwise (skewed) direction and the tilde symbol denotes the Favre average. In turn, $y_\omega(x)$ is defined as the location where $|\omega_z/\omega_{z,w}| < 10^{-4}$, where $\omega_z = \partial\bar{v}/\partial x - \partial\bar{u}/\partial y$.

By comparing the two panels, upstream displacement of the separation shock is evident in the swept flow case, associated with an enlargement of the separation bubble. A compression fan forms at the reattachment location (yellow contours), which results in localized thickening of the boundary layer. The compression fan merges with the separation shock and reinforces it, thus giving rise to a distinctive ‘lambda’ structure.

Instantaneous flow structures are examined through the fluctuating pressure field, $p'(x, y, z, t) = p(x, y, z, t) - \bar{p}(x, y)$, displayed in Fig. 6.3(c,d). The inset provides a close-up view of the dividing streamline. While the intensity of pressure fluctuations in the incoming boundary layer is relatively low, it is significantly amplified downstream of the interaction with the shock, whose location is distinctly marked by a thin, oblique flow region exhibiting intense pressure fluctuations. The simultaneous occurrence of positive and negative pressure fluctuations along the shock line reveals a deformation of the shock structure relative to its mean configuration. This deformation is attributed to

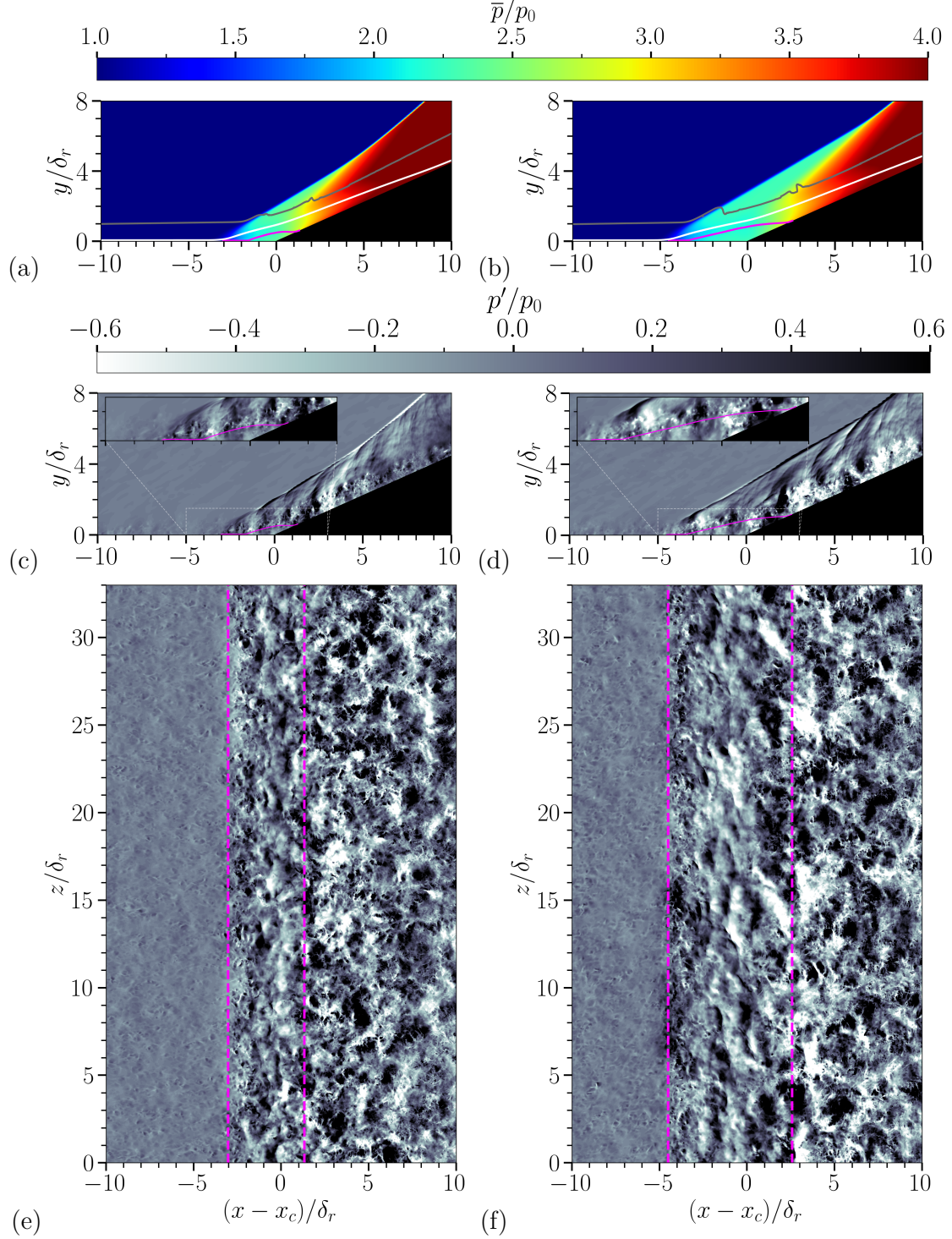


Figure 6.3: Flow field visualizations for $\gamma_0 = 0^\circ$ (left panels) and $\gamma_0 = 30^\circ$ (right panels): mean pressure ($\bar{p}/p_0$) in the x - y plane (a,b); instantaneous fluctuating pressure (p'/p_0) in the x - y plane (c,d) and at the wall (e,f). The magenta, white, and gray solid lines mark the dividing streamline (ψ), the sonic line ($M = 1$), and the boundary layer thickness (δ). Magenta dashed lines mark the mean separation and reattachment lines.

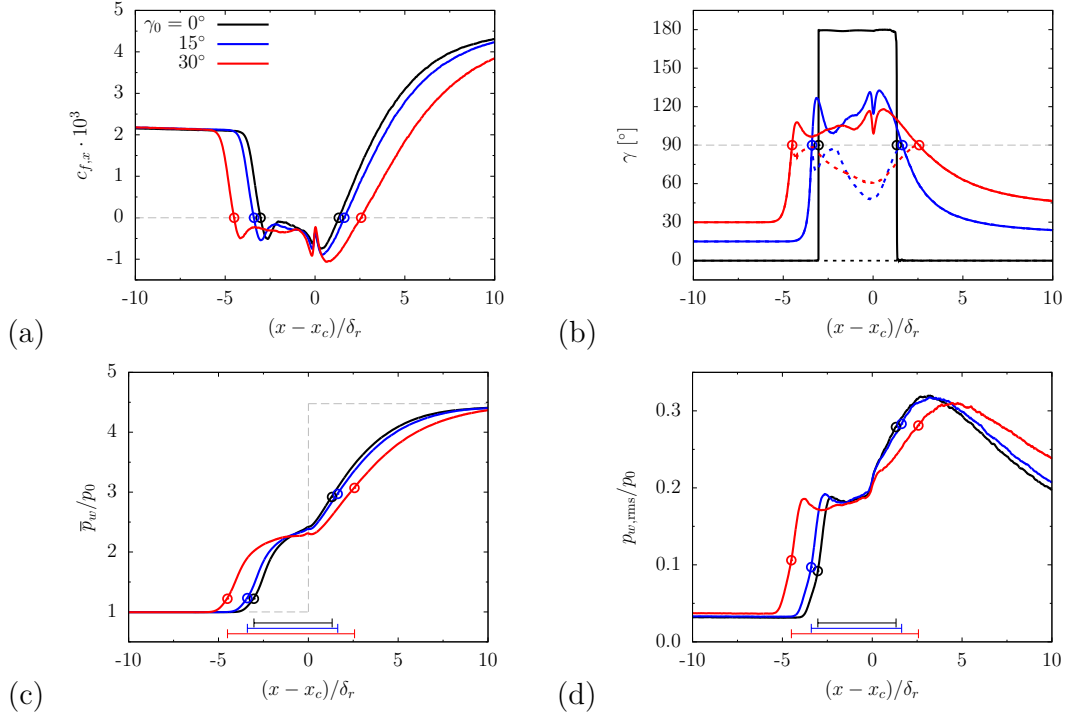


Figure 6.4: Mean distributions of: (a) x -projected friction coefficient, $c_{f,x}$; (b) flow direction angle, γ , evaluated at the wall (solid lines) and at the dividing streamline (dashed lines); (c) wall pressure, $\bar{p}_w/p_0$. (d) **RMS** profile of fluctuating wall pressure, $p_{w,rms}/p_0$. In each panel, mean separation and reattachment points are indicated by circles for each flow case; the separated flow region is indicated in (c,d) by segments (see Table 6.1 for the color code).

strong fluctuations of the compression waves originating at the flow reattachment point and coalescing on the separation shock. Small-scale pressure fluctuations along the dividing streamline indicate the presence of eddies shedding from the separated shear layer, which are pushed toward the ramp surface in the proximity of the reattachment point, and interact with the compression waves. In the unswept case, these eddies remain clearly visible downstream of reattachment, whereas large-scale coherent structures dominate the downstream region for $\gamma_0 = 30^\circ$.

Figure 6.3 presents visualizations of wall pressure fluctuations for $\gamma_0 = 0^\circ$ (e) and $\gamma_0 = 30^\circ$ (f), with magenta dashed lines indicating the mean separation and reattachment locations. Signatures of small-scale turbulent structures are observed at the wall within the incoming boundary layer. At the separation line, pressure fluctuations are significantly amplified, and the reversed flow region is characterized by large-scale, spanwise-elongated structures. In the swept case, these structures exhibit increased characteristic size and spanwise coherence, with large-scale spanwise ripples along the separation line also evident.

The separation length (L_s) is defined as the distance measured along the wall-aligned coordinate between the mean separation and reattachment points, marked by circles in

Fig. 6.4. These points are determined from the zero crossings of the streamwise friction coefficient:

$$c_{f,x} = \frac{2\mu_w}{\rho_0 u_{0,x}^2} \frac{\partial \tilde{u}_\xi}{\partial \eta} \bigg|_w.$$

The distributions of $c_{f,x}$ in Fig. 6.4(a) indicate that increasing the value of γ_0 results in upstream and downstream shifts of the separation and reattachment points, respectively. Upstream of the interaction, the friction coefficient exhibits the expected behavior for a zero-pressure-gradient boundary layer, with a mild decrease due to boundary layer thickening. In this region, $c_{f,x}$ is independent of γ_0 , further demonstrating that the state of the incoming boundary layer is virtually identical across all flow cases.

In addition to the extent of the separation bubble, valuable information on the flow organization is provided by the mean flow direction in the x - z plane. Fig. 6.4(b) shows the x -wise distribution of the local sweep angle, evaluated at the wall (γ_w , solid line) and along the dividing streamline (γ_d , dashed line), which are defined as

$$\gamma_w = \tan^{-1} \left(\frac{\partial \tilde{w} / \partial \eta}{\partial \tilde{u}_\xi / \partial \eta} \bigg|_w \right), \quad \gamma_d = \tan^{-1} \left(\frac{\tilde{w}}{\tilde{u}_\xi} \bigg|_d \right), \quad (6.4)$$

where the subscript d denotes the dividing streamline. The incoming flow is aligned with the nominal direction imposed by the inflow sweep angle (γ_0). Time-averaged reversed motion is evidenced by γ_w exceeding 90° in the region where $c_{f,x}$ is negative. In the unswept case, a step change in γ_w occurs from 0° to 180° at the separation point and resets to 0° at reattachment, indicating that the near-wall flow within the recirculation bubble is exactly opposite to the external flow. In the swept cases, γ_w begins to increase just upstream of the separation point due to a sharp reduction in the shock-normal velocity component ($\tilde{u}_\xi$), while the spanwise component ($\tilde{w}$) remains virtually unaffected. Within the separated zone, the distributions of γ_d and γ_w are approximately symmetrical relative to the 90° line, with an offset of $\pm 40^\circ$ for $\gamma_0 = 15^\circ$ and $\pm 25^\circ$ for $\gamma_0 = 30^\circ$. Downstream of the interaction, the flow tends to realign with the nominal inviscid direction, as predicted by the Rankine–Hugoniot relations.

The mean pressure distribution at the wall is displayed in Fig. 6.4(c). The pressure rise occurs slightly upstream of the separation point, indicating the presence of an upstream influence mechanism (Pirozzoli et al., 2009), which is anticipated by an increased inflow sweep angle. As expected in strong interactions (Burggraf, 1975), a wall pressure plateau occurs in the separated region, between the separation point and the ramp corner. This plateau is more pronounced at $\gamma_0 = 30^\circ$, consistent with the enlarged separated zone. Hence, the wall pressure distribution progressively deviates from the inviscid solution (indicated by the gray dashed line) as sweep increases. Further downstream, all curves tend to converge to the inviscid value, as a result of the boundary layer relaxation to an equilibrium state.

Figure 6.4(d) presents the RMS of the fluctuating wall pressure, revealing a characteristic double-peak pattern across all flow cases. In the upstream boundary layer, wall pressure fluctuations are relatively weak, followed by a sharp increase leading to the first peak. Being located in the proximity of the separation point, this peak is attributed to the motion of the separation shock foot. The RMS pressure is nearly constant between this peak and the ramp corner, past which pressure fluctuations rise again, driven by

oscillations in the separated shear layer including the structure shedding observed in Fig. 6.3(a,b). The maximum RMS value, reached downstream of reattachment, is likely related to the strongly fluctuating compression waves interacting with the residual shear-layer unsteadiness, as highlighted by flow visualizations in Fig. 6.3(a,b).

6.4 SPECTRAL ANALYSIS

The spectral characteristics of wall pressure fluctuations are analyzed based on frequency and wavenumber spectra, expressed in terms of the corresponding pre-multiplied PSD. The PSD at each spatial location are normalized by the corresponding wall pressure variance, enabling a direct comparison of dominant temporal and spatial scales. Wall pressure spectra ($\hat{E}_{p,w}$) are shown across the interaction region in Fig. 6.5 for $\gamma_0 = 0^\circ$ (a,b), $\gamma_0 = 15^\circ$ (c,d), and $\gamma_0 = 30^\circ$ (e,f), where the hat symbol denote the spatially local normalization. Fig. 6.5(a,c,e) report the frequency-multiplied spectra ($f\hat{E}_{p,w}$) as a function of the x coordinate and Strouhal number, while Fig. 6.5(b,d,f) display the wavenumber-multiplied spectra ($\kappa_z\hat{E}_{p,w}$) as a function of the x coordinate and spanwise wavelength.

The unswept case (Fig. 6.5a) exhibits frequency features typical of canonical SBLI (Touber et al., 2009; Priebe et al., 2016; Grilli et al., 2012). Upstream of the separation point, the influence of the undisturbed turbulent boundary layer on the wall pressure is evident at relatively high frequencies ($St_\delta \approx 1$). Near the separation location, significant low-frequency unsteadiness is signaled by an energy peak at $St_L \approx 0.05$, corresponding to large-scale oscillations of the shock foot. Within the separated region and downstream of the reattachment point, the energy density concentrated at intermediate frequencies ($St_L \approx 1$) is associated with large coherent fluctuations of the separated shear layer, including vortex shedding (Pasquariello et al., 2017) and flapping of the reflected shock (Gaitonde et al., 2023).

A similar pattern is observed in the wavenumber spectra (Fig. 6.5b), which show a clear energy shift from small scales ($\lambda_z \approx \delta_r$) in the incoming boundary layer to larger scales ($\lambda_z \approx L_s$) at the separation line. As will be shown later, these energetic large-scale modes are associated with a spanwise undulation of the separation line. Within the separation bubble, energetic structures with intermediate wavelengths dominate.

Fig. 6.5(c,e) illustrate the effect of increasing the sweep angle. At the separation line, the dominant frequencies shift to increasingly higher values compared to the unswept case (the peak Strouhal number is listed in Table 6.1 for each case). This trend underscores that the frequency of the shock foot motion increases for higher values of sweep, consistent with numerical findings for impinging SBLI (Ceci et al., 2023a; Bergier et al., 2023) and experimental results for swept ramp interactions (Allen et al., 2023; Erenkil et al., 1993).

Unlike the frequency spectra, the wavenumber spectra in Fig. 6.5(d,f) do not show significant changes in the dominant wavelength along the separation line, which is $\lambda_z \approx L_s$ across all cases. Hence, the characteristic size of the separation line corrugations is comparable to the separation length, regardless of the sweep angle. In swept cases, however, energetic modes at larger wavelengths are also evident, particularly at $\gamma_0 = 15^\circ$, where a sharp peak emerges at $\lambda_z \approx 2L_s$. This observation suggests the coexistence of

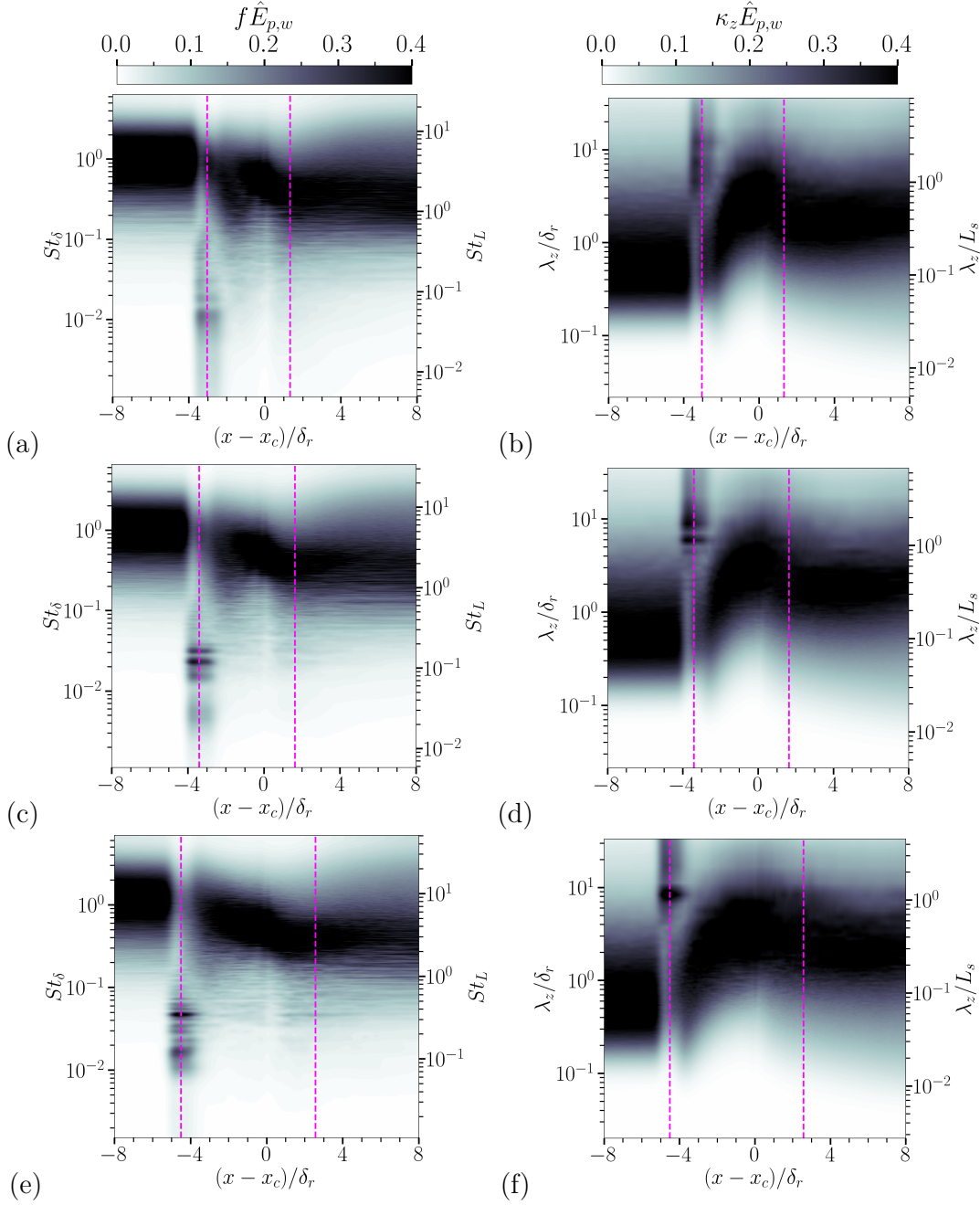


Figure 6.5: Normalized energy spectra of fluctuating wall pressure ($\hat{E}_{p,w}$) across the interaction region for $\gamma_0 = 0^\circ$ (a,b), $\gamma_0 = 15^\circ$ (c,d), and $\gamma_0 = 30^\circ$ (e,f). Left panels: frequency-premultiplied PSD ($f\hat{E}_{p,w}$) as a function of the x coordinate and Strouhal number ($St_\delta = f\delta_r/u_{0,x}$ on left axis, $St_L = fL_s/u_{0,x}$ on right axis); right panels: wavenumber-premultiplied PSD ($\kappa_z\hat{E}_{p,w}$) as a function of the x coordinate and spanwise wavelength (λ_z) scaled by δ_r (left axis) and L_s (right axis). Separation and reattachment locations are indicated by magenta dashed lines.

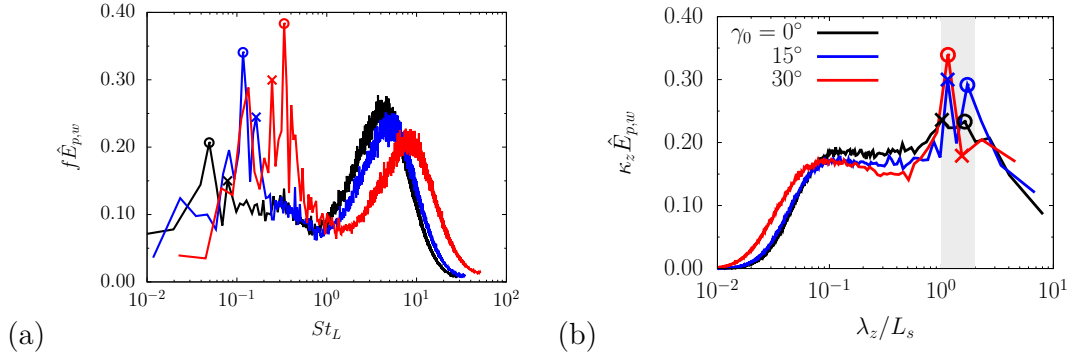


Figure 6.6: Pre-multiplied and normalized PSD of wall pressure fluctuations at the mean separation line for various sweep angles: (a) $f \hat{E}_{p,w}$ as a function of St_L , and (b) $\kappa_z \hat{E}_{p,w}$ as a function of λ_z/L_s . Circles and crosses mark the low-frequency peaks (a) and the corresponding large-wavenumber peaks (b).

spanwise corrugations on various scales, besides underscoring the need of sufficiently wide domains to capture these energetic modes at very large wavelengths.

This is more clearly shown in Fig. 6.6, in which the frequency and wavenumber spectra are shown at the mean separation line. Focusing on low and intermediate frequencies, the wall pressure spectra in Fig. 6.6(a) reveal multiple energy peaks. For each flow case, the Strouhal numbers corresponding to the primary and secondary peaks are marked by circles and crosses, respectively, and both will be considered in the subsequent analysis to characterize more comprehensively the low-frequency unsteadiness. These energy peaks become increasingly pronounced as the sweep angle increases, indicating a more intense low-frequency unsteadiness. In this respect, note that normalization of the spectra does not affect the comparison, since the RMS pressure at the separation point is approximately equal for all three cases, as seen in Fig. 6.4(d).

At higher frequencies ($St_L \approx 5$ or $St_\delta \approx 1$), an energy peak arises from the convection of turbulent fluctuations from the incoming boundary layer. These turbulent structures also leave their imprint on the wavenumber spectra shown in Fig. 6.6(b), where a bump centered at $\lambda_z \approx 0.1L_s \approx 0.5\delta_r$ indicates the presence of a small-scale undulation of the separation line. Similar ripples, with size comparable to the boundary layer thickness, have been observed in both unswept impinging SBLI (Pasquariello et al., 2017) and swept compression ramps (Vanstone et al., 2019b). However, the energy density is predominantly clustered in the range $L_s < \lambda_z < 2L_s$ (highlighted by the gray shaded area) for all flow cases. The energy peaks marked by circles and crosses correspond to large-scale corrugations of the separation line, with each of these modes oscillating at a characteristic frequency.

To establish a correlation between the time and length scales identified by the spectra, Fig. 6.7 examines the imaginary part of the Fourier modes of the fluctuating wall pressure, $\hat{p}_w(x, z)$, for the two Strouhal numbers corresponding to the most energetic low-frequency peaks, as highlighted in Fig. 6.6(a). The aim of this analysis is to elucidate the spatial pattern of the dominant flow structures in the vicinity of the separated zone.

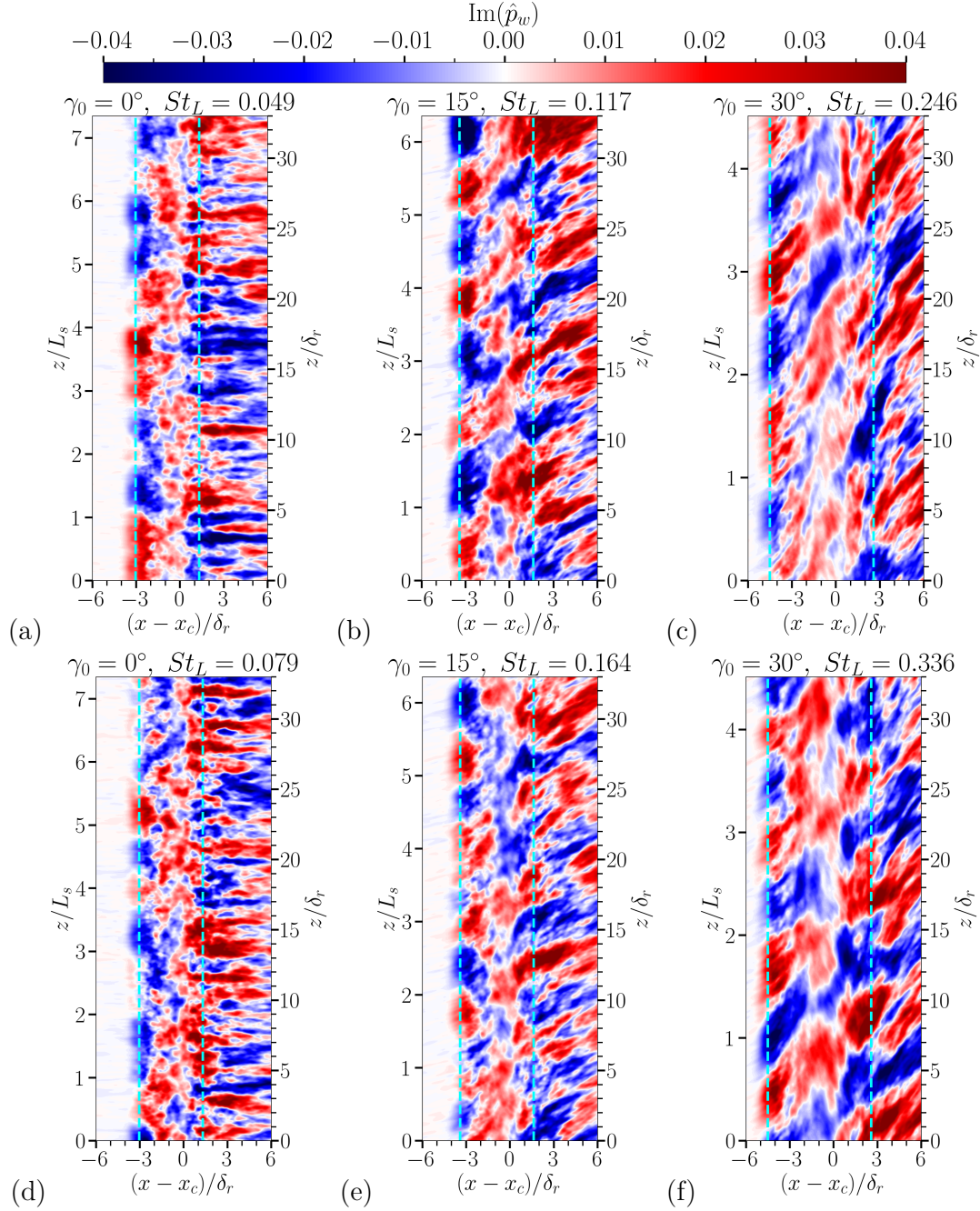


Figure 6.7: Spatial distribution of imaginary part of fluctuating wall pressure Fourier coefficients, $\text{Im}(\hat{p}_w)$, for $\gamma_0 = 0^\circ$ (a,d), $\gamma_0 = 15^\circ$ (b,e), and $\gamma_0 = 30^\circ$ (c,f). The Strouhal numbers under scrutiny correspond to the most energetic low-frequency peaks marked in Fig. 6.6(a).

In the unswept flow case, structures of different sizes appear to coexist, precluding precise association of a single wavelength with the two Strouhal numbers under investigation. Nevertheless, at $St_L = 0.049$ (Fig. 6.7a), separation-line corrugations appear as spanwise-coherent pressure fluctuations with a characteristic size comparable to the separation length ($\lambda_z \approx 2L_s$). Similar structures with a reduced wavelength ($\lambda_z \approx L_s$) are visible at $St_L = 0.079$ (Fig. 6.7d). Wall pressure corrugations are also observed at reattachment, exhibiting approximately the same size as those at separation. The perfect phase opposition between these corrugations can be interpreted as a signature of the two-dimensional ‘breathing’ of the separation bubble. Additionally, smaller-scale coherent structures are visible downstream of reattachment, which are likely associated with pressure disturbances originating from the mixing layers surrounding the separation bubble.

The swept flow cases exhibit a more organized pattern of pressure fluctuations, with large-scale corrugations emerging uniformly along the separation line. For both $\gamma_0 = 15^\circ$ (Fig. 6.7b,e) and $\gamma_0 = 30^\circ$ (Fig. 6.7c,f), these coherent ripples display characteristic wavelengths that closely match those identified in the one-dimensional wavenumber spectra. This correspondence provides clear frequency-wavelength associations between the energy peaks marked in Fig. 6.6(a,b), contrary to the results obtained for the two-dimensional interaction.

In addition, large-scale oblique patterns appear in the separated region of the swept flow cases. The associated peaks and troughs (red/blue contours) along the separation line align with the corresponding peaks and troughs along the reattachment line, at angles of 50° with respect to the positive x axis for $\gamma_0 = 15^\circ$, and of about 65° for $\gamma_0 = 30^\circ$. At the reattachment position, these coherent pressure fluctuations are partly advected by the backflow toward the separation line with a spanwise-symmetric orientation, and partly convected downstream deflecting toward the nominal inviscid sweep angle. The orientation of the oblique structures is consistent with the mean sweep angle in the separated zone, as shown in Fig. 6.4(b). Specifically, coherent structures moving from the separation toward the reattachment line tend to align with the sweep angle evaluated at the dividing streamline (dashed lines in Fig. 6.4b), whereas the backward-moving structures appear to follow the sweep angle distribution at the wall (solid lines in Fig. 6.4b).

Beyond providing correlations between the characteristic time and length scales of the problem, this analysis clarifies the dynamics of large-scale pressure perturbations within the separated zone, which is driven by the superposition of two effects: (1) the breathing motion of the separation bubble, typical of two-dimensional interactions and evidenced by the phase opposition between the spanwise corrugations at the separation and reattachment lines, and (2) the spanwise drift of coherent pressure disturbances at a speed proportional to the crossflow.

This observation is further substantiated in Fig. 6.8, which presents the frequency–wavenumber spectra of wall pressure fluctuations ($\hat{E}_{p,w}$) at the separation location. In this figure, logarithmic-scale contours of the normalized wall pressure PSD are plotted as functions of the Strouhal number (St_L) and the spanwise wavenumber based on the separation length (L_s/λ_z). Cyan circles and crosses mark the primary and secondary peak pairs ($St_L, \lambda_z/L_s$), as identified from the one-dimensional spectra in Fig. 6.6(a,b). In the unswept case (Fig. 6.8a), the highest energy density is clustered at low St_L and low L_s/λ_z , for both positive and negative wavelengths, indicating that the pressure fluc-

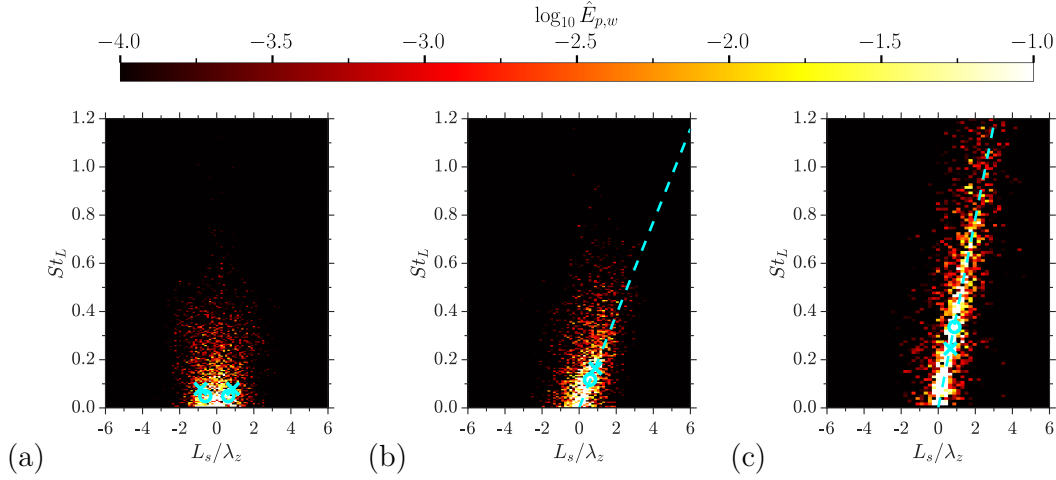


Figure 6.8: Logarithmic-scale contour plots of normalized PSD of wall pressure fluctuations ($\hat{E}_{p,w}$) at the mean separation location, as a function of separation length-based spanwise wavenumber (L_s/λ_z) and Strouhal number (St_L), for $\gamma_0 = 0^\circ$ (a), $\gamma_0 = 15^\circ$ (b), and $\gamma_0 = 30^\circ$ (c). Dashed lines denote the linear relationship $\omega = \kappa_z w_c$, where $\omega = 2\pi f$ and $w_c = 0.7u_{0,x} \tan \gamma_0$. For each case, circles and crosses mark the primary and secondary peak pairs (St_L , λ_z/L_s) consisting of the Strouhal numbers marked in Fig. 6.6(a) and the corresponding wavelengths marked in Fig. 6.6(b).

tuations exhibit no preferential spanwise direction. As expected, the peak Strouhal numbers and their corresponding peak wavenumbers lie within the high-energy-density region.

In contrast, for $\gamma_0 = 15^\circ$ (Fig. 6.8b) and even more markedly for $\gamma_0 = 30^\circ$ (Fig. 6.8c), the wall pressure PSD concentrates along a linear trend. This linear clustering is particularly pronounced for modes at low frequencies and low wavenumbers, implying that large-scale coherent disturbances travel at steady spanwise velocity without dispersion. The slope of the linear regression (dashed cyan line) provides an estimate for the intensity of the convection velocity (w_c), yielding $w_c \approx 0.7w_0 = 0.7u_0 \tan \gamma_0$. The positive angular coefficient of the data-fitted lines indicates that the convection velocity is directed toward positive z .

This same spanwise convection velocity for pressure disturbances has been observed in supersonic (Ceci et al., 2023a) and hypersonic (Ceci et al., 2024) impinging swept SBLI, and a similar value of $w_c/w_0 \approx 0.7 - 0.8$ has been reported for supersonic swept ramps (Vanstone et al., 2018). For the swept flow cases, the peak pairs (St_L , λ_z/L_s) are located along the straight line defined by the dispersion relation, further confirming that the dominant frequencies are associated with flow structures of specific sizes, drifting in the spanwise direction at constant speed.

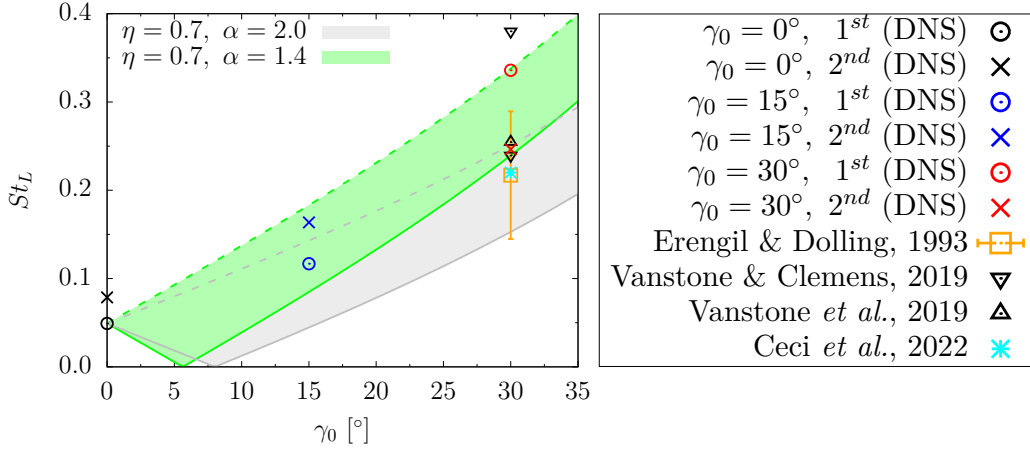


Figure 6.9: Dominant frequencies as a function of the inflow sweep angle (γ_0). Dashed and solid lines denote the predictions of (6.5), with gray and green shaded area corresponding to $\alpha = 2$ and $\alpha = 1.4$. All symbols are defined in the key.

6.5 MODELING THE LOW-FREQUENCY OSCILLATIONS

Ceci et al. (2023a) proposed that pressure fluctuations at the separation line can be modeled as sinusoidal waves oscillating in time and space. The underlying assumptions are that (1) the spanwise wavelength is proportional to the separation length, $\lambda_z = \alpha L_s$, and (2) the temporal oscillations arise from the combined contributions of a typical frequency associated with two-dimensional breathing, $St_{L,0}$, and the spanwise drift of pressure disturbances at speed $w_c \approx \eta u_0 \tan \gamma_0$, where α and η are model constants. Under these assumptions, they derived the following expression to estimate the peak Strouhal number associated with low-frequency unsteadiness:

$$St_L = \left| St_{L,0} \pm \frac{\eta \tan \gamma_0}{\alpha} \right|, \quad (6.5)$$

with the fitting parameters $St_{L,0} \approx 0.04$, $\alpha \approx 2$, and $\eta \approx 0.7$, as obtained from DNS results of impinging swept SBLI. The two distinct frequencies predicted by Equation (6.5) should be interpreted as representing broadband peaks, that could merge into one if the frequency range of the modeled phenomenon is sufficiently wide.

In Fig. 6.9 we assess the applicability of the proposed model for swept ramp interactions, by comparing the dominant Strouhal numbers predicted by the model with those obtained from DNS for various values of γ_0 . The shaded area corresponds to the ‘broadband peak’ located between the two solutions (dashed and solid lines) predicted by the model. Both the primary (circles) and secondary (crosses) peak Strouhal numbers from the DNS results are reported, as well as the dominant frequencies extrapolated from previous experiments on swept compression ramps.

The present DNS results consistently indicate the same convection velocity as reported by Ceci et al. (2023a), and therefore we retain the value $\eta = 0.7$. However, our results reveal a reduced wavelength for the separation line corrugations, suggesting

that adjustments of the fitting parameter α may be necessary. In fact, the gray shaded area corresponding to $\alpha = 2$ tends to underestimate the peak Strouhal number. Closer agreement between the model predictions and the DNS results is instead achieved using $\alpha = 1.4$ (green shaded area), which in fact more closely corresponds to the dominant spanwise wavelengths observed in Fig. 6.6(b). This improvement is particularly evident for $\gamma_0 = 30^\circ$, consistent with the stronger coherence of convecting pressure disturbances observed in Fig. 6.8.

For this specific sweep angle, previous experiments on swept compression ramps have reported the dominant frequencies of the wall pressure signal at the separation location. Under those conditions, conical symmetry is observed with increase of the separation length away from the virtual origin. The Strouhal numbers shown in Fig. 6.9 are extrapolated from (Erengil et al., 1993; Vanstone et al., 2019a,b), by multiplying the documented frequencies by the separation length measured at the spanwise location of the pressure transducers. The data show clear increase of the typical Strouhal number with respect to the case of two-dimensional interactions. The agreement with the present theory is rather good, on account of cylindrical symmetry assumed in the model, and/or experimental uncertainties. Agreement is especially good with the experiments of Vanstone et al. (2019b), who reported a peak Strouhal number $St_L \approx 0.255$, based on a separation length $L_s \approx 3.8\delta_{99}$.

6.6 CONCLUSIONS

We have investigated SBLI by performing DNS of supersonic flow over a compression ramp. Under the considered flow conditions – a deflection angle of $\theta = 24^\circ$, an x -projected Mach number of $M_{0,x} = 2.9$, and an upstream Reynolds number of $Re_{\delta_2} \approx 1200$ – a significant reversed flow region develops. Particular emphasis has been placed on the characterization of wall pressure fluctuations in the separated zone, as inaccurate predictions of these fluctuations may result in incorrect aerodynamic design, leading to reduced performance and compromised structural integrity of high-speed aircraft. Aiming to develop scaling laws applicable to engineering scenarios where the flow is inherently three-dimensional, this study focused on the effects produced by a misalignment between the incoming flow and the direction normal to the mean separation line, commonly referred to as the inflow sweep angle (γ_0). By introducing a crossflow velocity component in the spanwise direction, we simulated flow cases with $\gamma_0 = 0^\circ, 15^\circ$, and 30° . This idealized configuration produces SBLI with cylindrical symmetry, which is representative of swept interactions with small-to-moderate sweep angles (Threadgill et al., 2020; Settles et al., 1984; Erengil et al., 1993; Settles et al., 1980).

Snapshots of the flow field have shown qualitatively that the pressure fluctuations associated with turbulence structures are significantly amplified, both in intensity and size, when the boundary layer crosses through the shock wave. Swept flow cases are characterized by an upstream displacement of the separation shock foot, associated with an enlarged separation bubble, and enhanced spatial coherence of the pressure fluctuations.

Pre-multiplied PSD maps have revealed low-frequency unsteadiness at the separation line typical of two-dimensional SBLI, as indicated by a clear energy peak at a separation

length-based Strouhal number of $St_L \approx 0.05$. When sweep is present, this prominent peak shifts to higher frequencies, in agreement with previous studies on swept [SBLI](#) (Erengil et al., 1993; Doehrmann et al., 2018; Vanstone et al., 2019a; Ceci et al., 2023a; Bergier et al., 2023), and its amplitude increases, indicating more intense low-frequency unsteadiness.

Energetic large-scale modes have been observed at the separation line with spanwise wavelengths between L_s and $2L_s$ irrespective of the sweep angle, showing that the separation length is a suitable normalization scale also in swept interactions. To correlate the peak frequencies and wavelengths identified by the spectra, we examined the spatial distribution of wall pressure Fourier modes. For each flow case, the two dominant Strouhal numbers were selected, revealing that distinct flow patterns with a uniform size correspond to the peak frequencies. The typical low-frequency pressure disturbances manifest as spanwise corrugations along the separation line, with characteristic wavelengths consistent with the large-scale energetic modes observed in the wavenumber spectra.

The spatio-temporal dynamics of these large-scale pressure disturbances was further examined. In the swept flow cases, two driving mechanisms appear to coexist. The first has been identified as a breathing motion of the separation bubble, typical of 2D-[SBLI](#), while the second is related to a spanwise drift of pressure disturbances. This combined motion leaves a clear imprint on the Fourier modes, revealing a spatial pattern of pressure fluctuations forming a series of peaked structures. The alignment of these pressure waves is consistent with the local sweep angle, which exhibits a symmetric distribution when evaluated at the wall and at the dividing streamline.

By means of the frequency-wavenumber spectra, we have shown that the spanwise drift of large-scale pressure disturbances occurs at a speed aligned with the crossflow and with an intensity estimated as $w_c \approx 0.7w_0 = 0.7u_0 \tan \gamma_0$, consistent with previous findings in swept [SBLI](#) (Ceci et al., 2023a; Bergier et al., 2023; Ceci et al., 2024; Vanstone et al., 2018). Moreover, the linear dispersion relation in the low-frequency, low-wavenumber limit underscores a robust frequency-wavelength association, linking the characteristic time and length scales observed at the separation line.

A key outcome of this work is the evaluation of the predictive model proposed by Ceci et al. (2023a), which estimates the dominant Strouhal number in swept [SBLI](#). Previous numerical studies have shown that the predictions of the model remain consistent across variations in Reynolds number (Bergier et al., 2023) and Mach number (Ceci et al., 2024). However, those studies were based on the same impinging shock configuration, leaving open the question of whether the accuracy of the model extends to different flow geometries. Given the robustness of the free-interaction concept (Chapman et al., 1958) in establishing scaling laws for two-dimensional [SBLI](#) (Dussauge et al., 2006), one might expect such general applicability. The present results confirm that the model is applicable to compression ramp interactions, however only after the fitting parameters are appropriately calibrated. This adjustment accounts for the observed reduction in the characteristic wavelength of spanwise corrugations at the separation line.

Furthermore, the analysis presented in Sec. 6.4 demonstrates that the frequency scaling applies not only under cylindrical symmetry, but also provides accurate predictions for the large-scale dynamics of quasi-conical interactions. In fact, the ‘local’ peak frequency at a given distance from the virtual conical origin well agrees with the Strouhal number predicted by our model, provided the local separation length is used as

characteristic scale. The good agreement between the present scaling and experimental data for swept compression ramps (Vanstone et al., 2019b) at Reynolds numbers one order of magnitude higher than the present DNS and across Mach numbers up to 5 reinforces the reliability of the model, even in real-world scenarios involving conical interactions. These findings suggest that while low-frequency unsteadiness in swept SBLI exhibits universal qualitative characteristics, the specific time and length scales involved may depend quantitatively on the mechanism by which the shock is generated.

CONCLUSIONS

This thesis examines how wall curvature modifies the large-scale organisation and dynamics of turbulent flows across a range of geometries and Mach number regimes. We studied several configurations, including curved channels, supercritical aerofoils, and compression ramps. Although the individual cases differ in geometry and Mach number, they exhibit recurring large-scale structures that strongly influence the organization of the flow field and its interaction with turbulence.

A central contribution of this work is the development of the FLEW solver, a high-fidelity DNS code for compressible flows on curvilinear geometries. In Chapter 2 we describe the numerical algorithms that underpin FLEW, with emphasis on high-order FD schemes formulated in a generalized curvilinear-coordinate framework that extends conventional Cartesian approaches. We also document the porting of the code to multiple GPU architectures and its integration with the Cartesian STREAMS code, yielding STREAMS-2. The solver is validated against a set of canonical benchmarks in Chapter 3. The Taylor–Green vortex in a curvilinear, triply periodic domain demonstrates the solver energy-conserving behaviour in the inviscid, low-Mach-number limit. Preservation of kinetic energy is essential to suppress numerical diffusion and ensure that the finest turbulence scales are resolved. Additional validation cases, including curved channels, transonic aerofoils, and compression ramps, show excellent agreement with experimental and numerical references across a wide range of flow regimes.

The curved-channel flow presented in Chapter 4 was chosen as a model problem for more complex configurations such as spatially developing boundary layers over curved walls. By considering two curvature levels – mild and strong – we systematically examined Reynolds-number effects through a campaign of DNS that spans regimes from laminar to moderately high Reynolds number up to $Re = 87\,000$, based on bulk velocity and channel height. The analysis highlights the influence of curvature on the friction coefficient: concave curvature promotes earlier transition, whereas convex curvature delays transition. For mild curvature, the transitional regime exhibits a reduction in frictional drag compared with the plane channel. Spectral analysis shows that the near-wall turbulence regeneration cycle persists in mildly curved channels, whereas is absent or severely inhibited on the convex wall of strongly curved channels. Streamwise large-scale structures resembling Dean vortices are weakly dependent on Reynolds number but strongly affected by curvature: increasing curvature shifts these vortices toward the outer wall, and reduces their size and coherence, thereby limiting their contribution to streamwise velocity fluctuations and momentum transport. For strong curvature, spanwise large-scale structures are also observed. These structures are associated with large pressure fluctuations and suppression of turbulent stresses near the convex wall, where a region of negative turbulence production is detected and characterized using quadrant analysis.

In Chapter 5 we present DNS of transonic turbulent buffet over the V2C supercritical aerofoil, used as a simplified model for the more complex buffeting flow over aircraft wings. Simulations at chord-based Reynolds numbers $Re = 3 \times 10^5$ and 6×10^5 span

angles of attack from stable conditions to fully developed buffet. The DNS reproduces the experimentally observed inversion of mean shock displacement between $\alpha = 4^\circ$ – 6° , and shows that beyond $\alpha \approx 6^\circ$ the shock begins to oscillate. At $\alpha = 7^\circ$ the oscillation amplitude increases and a dominant buffet peak appears in the wall-pressure spectrum. Low-frequency energy at $St \approx 0.09$ – 0.11 is present even below the buffet onset, indicating that a ‘latent’ instability is already present below the critical angle of attack. Boundary-layer statistics show that a steady shock-induced separation bubble is not required for buffet onset, although the fully buffeting flow shows a significantly higher probability of backflow. Via SPOD we identify a dominant coherent buffet mode at $St \approx 0.1$ that couples the shock motion with large-scale shear-layer structures convected downstream at a speed $u_{dn} \approx 0.15 u_0$. Upstream-traveling acoustic waves are also identified: they originate near the trailing edge, propagate along the suction side, circumvent the shock tip, and impinge on the shock upstream face. The combined propagation time of downstream- and upstream-traveling disturbances matches the DNS buffet period, supporting a mixed hydrodynamic–acoustic feedback loop as the sustaining mechanism.

Chapter 6 focuses on the compression-ramp configuration, a canonical framework to study the interaction among shock waves, shock-induced separation bubbles, and turbulence. We performed DNS of fully turbulent flow at an x -projected Mach number of 2.9 over a 24° compression ramp to investigate SBLI with flow separation. By imposing a crossflow velocity corresponding to sweep angles of 0° , 15° , and 30° , we reproduced three-dimensional interactions under cylindrical symmetry. We concentrate on the wall-pressure structure within the separated region, which enlarges under swept conditions. Increasing sweep increases the characteristic length scales of pressure fluctuations and enhances their spatial coherence. Spectral analysis of the wall-pressure reveals large-scale disturbances with spanwise wavelengths comparable to the separation length; these appear as coherent corrugations along the separation line and are associated with distinct dominant frequencies. In the unswept case, a low-frequency peak is observed at a separation-length-based Strouhal number $St \approx 0.05$. This peak shifts to higher frequency and grows in amplitude with increasing sweep. Spatio-temporal diagnostics further reveal a dual mechanism driving pressure disturbances in the separated zone, comprising a two-dimensional *breathing* motion of the separation bubble coupled with a spanwise drift at approximately 70% of the crossflow velocity. Finally, we show that the scaling law for the characteristic frequency of shock motion proposed by Ceci et al. (2023a) for swept interactions also describes the compression-ramp flow after suitable recalibration of the coefficients, a conclusion supported by available experimental data.

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