

# Spatio-temporal behaviour of femtosecond solitons in graded-index multimode fibers

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**Abstract**—We present a new class of spatiotemporal walk-off solitons in graded-index multimode optical fibers, characterized by unique properties. Their energies and pulsewidths depend only on chromatic and modal dispersion of the fiber, and on the input laser beam coupling conditions. Whereas the soliton properties are independent of the input pulse duration. The generated multimode solitons are composed by non-degenerate modes. However, upon propagation over distances comprising several hundreds of meters, the multimode solitons naturally evolve into singlemode pulses. For this reason, walk-off solitons are metastable objects, which are irreversibly attracted by the fundamental mode of the fiber.

**Index Terms**—Solitons, multimode grin fibers.

## I. INTRODUCTION

In recent years, the study of nonlinear pulse propagation in multimode optical fibers (MMFs) has attracted a growing interest, thanks to their capability of parallel information transmission via mode-division-multiplexing (MDM). MMFs can also deliver light beams with higher peak powers than singlemode fibers, whose core small size leads to high intensities, that may approach the damage threshold. For this reason, fiber lasers based on MMFs permits a significant power up-scaling, which is important for several applications. In this work we study the nonlinear propagation in the anomalous dispersion regime of femtosecond pulses in MMFs with a graded refractive index profile (GRIN), which support multimode (MM) solitons [1]–[4].

We reveal that the generation of femtosecond multimode solitons in GRIN fibers exhibits a previously unforeseen complex dynamics, which make them very different from their well-known singlemode counterparts. In singlemode fibers, solitons are formed when the broadening due to the chromatic dispersion is compensated by self-phase modulation. Thus, singlemode solitons can exhibit an arbitrary temporal duration,

which can be tuned by acting on the soliton peak power. On the other hand, in addition to chromatic dispersion, modal dispersion must also be compensated for, when generating MM solitons. This extra requirement reduces the degrees of freedom of multimode solitons. Specifically, it turns out that the pulsewidth and energy of MM solitons composed of non-degenerate modes are fixed, independently of the input pulse duration. Moreover, we observed that the modal content of multimode solitons irreversibly evolves during their propagation. Indeed, at distances encompassing several hundreds of dispersion distances, multimode solitons evolve into singlemode solitons carried by the fundamental mode. This results from the combined action of stimulated Raman scattering (SRS) and intermodal four-wave mixing (FWM) [11]. We will refer to this phenomenon as MM soliton modal attraction. These results are of particular importance for different applications of nonlinear fiber optics. We name these pulses as walk-off solitons: they are promising for high-bit rate MMF networks, as well as for high-power spatiotemporal mode-locked MMF lasers.

As a side note, we would like to point out that theoretical studies of MM solitons often rely on the so-called variational approach (VA) [5], [6]. This method is largely used because of its simplicity, and its capability of taking into account, at the same time, the effect of spatial self-imaging and the Kerr effect [7]–[10]. The main assumption of VA is that the initial ansatz for the injected beam is maintained unchanged upon propagation in the fiber, except for the periodic variation of beam width and amplitude. Our experiments invalidate theoretical predictions based on the variational method, since both the beam profile and its mode composition undergo dramatic changes with respect to the input beam.

## II. METHODS

### A. Numerical simulations

In our studies, we have supported the experimental results by numerical simulations. In particular, we numerically solved a model based on coupled-mode equations [14], [15]. This method allows for describing the propagation of multimode pulses over long spans of GRIN fibers. The coupling of the various propagating modes is described by FWM term in the propagation equations, which have the form  $Q_{plmn}A_lA_mA_n^*$ . Here,  $Q_{plmn}$  represents coupling coefficients, which are proportional to the overlap integrals of the transverse mode field distributions. In simulations, we used the following values of the nonlinear and dispersion fiber parameters:  $\beta_2 = -28.8$  ps<sup>2</sup>/km at 1550 nm,  $\beta_3 = 0.142$  ps<sup>3</sup>/km, and  $n_2 = 2.7 \times 10^{-27}$  m<sup>2</sup>/W. Whereas the Raman response temporal parameters are  $\tau_1 = 12.2$  fs and  $\tau_2 = 32$  fs [12], [13]. Finally, in order to describe optical linear losses in our model, we took into account the presence of a wavelength-dependent imaginary part of the fiber refractive index.

### B. Experimental setup

The experimental setup included an ultra-short pulsed laser system, pumped by a femtosecond Yb-based laser (Light Conversion PHAROS-SP-HP). The system generates light pulses at 100 kHz repetition rate with Gaussian spatial shape; the central wavelength was tuned between 1300 nm and 1700 nm by a hybrid optical parametric amplifier (Light Conversion ORPHEUS-F). Varying the wavelength of the laser system affects the source pulse duration, which had its minimum value of 60 fs at 1550 nm. In our experiment, we further tuned the input pulse duration by inserting a pass-band filter (PBF), so that this could reach a value of 240 fs. The laser beam was focused by a 50 mm uncoated lens into the fiber, with a  $1/e^2$  input diameter of approximately 30  $\mu$ m (15  $\mu$ m beam waist). The laser pulse energy was controlled by means of an external attenuator, and it was varied between 0.1 nJ and 20 nJ. The alignment between the laser and the fiber was optimized by obtaining an output near-field that was only composed by axial modes at low input powers (linear regime). We used a span (from 1 m to 850 m) of GRIN fiber, with cladding radius of 62.5  $\mu$ m, core radius  $r_{core} = 25$   $\mu$ m, cladding index  $n_{clad} = 1.444$  at 1550 nm, and relative refractive index difference  $\Delta = 0.01027$ . By means of a cascade of beam splitter cubes (BCS), the beam outgoing from the fiber was sent to several instruments for its full characterization (see Fig.1). Specifically, the beam spectral features were analyzed by means of an optical spectrum analyzer (Yokogawa AQ6370D, 600-1700 nm) and a real-time multiple octave spectrum analyzer (Fastlite Mozza, 1100-5000 nm). Whereas the beam intensity profile was detected by an InGaAs camera (Hamamatsu C12741-03). Finally, the output pulse temporal shape was measured by an intensity autocorrelator (APE pulseCheck 50) with femtosecond resolution, and by an infrared fast photodiode (Teledyne Lecroy WavePro 804HD) with 30 ps overall time response.

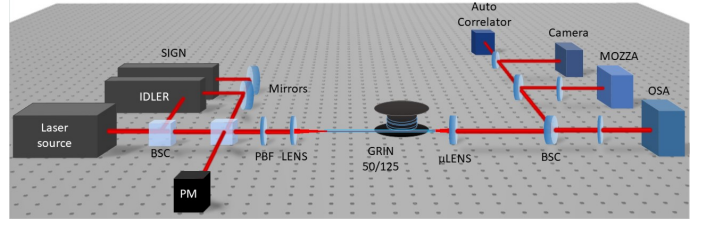


Fig. 1. Depiction of the experimental setup.

## III. RESULTS

### A. MM soliton modal attraction

The lack of symmetry between Raman gain and coupling coefficients is responsible for a slow, but irreversible transfer of high-order mode energy towards the fundamental mode. Such a energy transfer is reached when pulses carried by different modes are both spatially and temporally overlapping. At first, let us report numerical results showing MM soliton modal attraction in Fig.2. Specifically, Fig.2.a shows the simulated energy evolution of the three propagating modes (1, 0), (2, 0), and (3, 0) with axial symmetry, when a Gaussian pulse with 70 fs pulsewidth, 15  $\mu$ m radius, 1550 nm central wavelength, and optimal peak power of 28 kW is injected into the GRIN fiber. Whereas Fig.2.b shows the input temporal profile of the power in the three modes. Finally, in Fig.2.c we report the corresponding output mode power. Further simulations (see Fig.2.d) show that nonlinear coupling among the three non-degenerate modes initially leads to the generation of a MM soliton. Specifically, because of cross-phase modulation, the modes remain temporally trapped with each other, and propagate with common speed, in spite of their linear group-velocity walk-off. Therefore, we may name the obtained pulses as walk-off solitons.

However, as it can be seen by Fig.2.a, after approximately 120 m of propagation the fundamental mode acts as a dynamical attractor: it acquires the energy carried by higher-order modes. Furthermore, Fig.2.a shows that, at longer distances, a soliton with essentially singlemode nature propagates. This soliton experiences a wavelength shift caused by the Raman soliton self-frequency shift (SSFS). As the soliton wavelength increases above 1700 nm, it starts losing power by linear attenuation, and broadens temporally in order to conserve the singlemode soliton condition (see Fig.2.c).

We compared simulation results with experimental observations. Aiming at investigating the generation of MM soliton generation as well as their dynamics, we injected femtosecond pulses in multimode GRIN MMF spans whose length was varied up to 850 m. In Fig.3, we show the dependence of the measured output soliton pulsewidth on input pulse energy, after 850 m of GRIN fiber. In the simulations of Fig.2 we only show a single Raman soliton. However, a fission of the input pulse into multiple solitons was experimentally observed. This led to the generation of a train of solitons that we dub first, second and third Raman solitons in the caption of Fig.3, by following their order of appearance when increasing the input

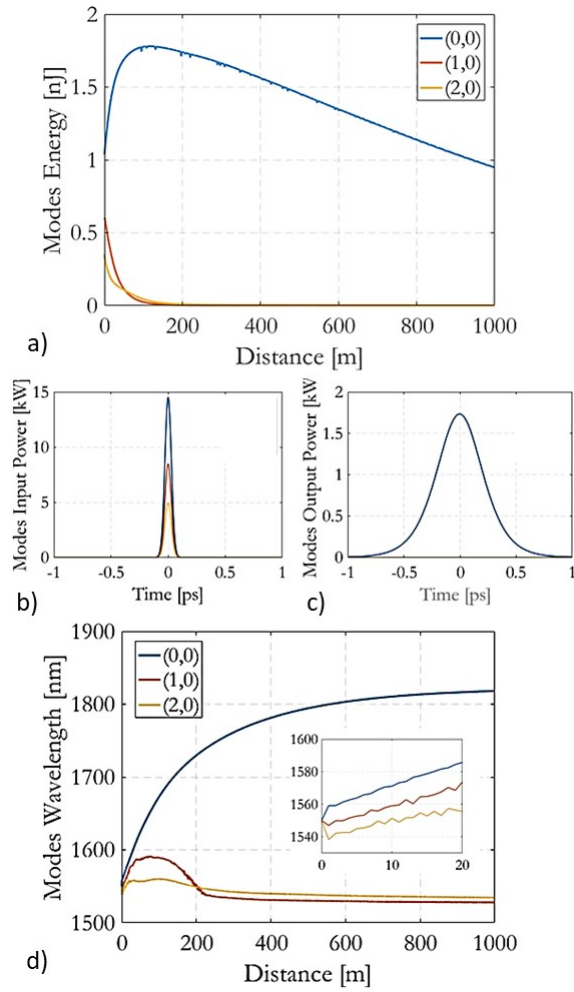


Fig. 2. Numerical simulation results of a) 3 axial modes propagating over 1,000 m of MMF fiber. b-c) Modal distribution of pulse powers at the fiber input facet b) and after 1,000 m transmission c). d) Numerical simulation results of the wavelength evolution of 3 axial modes over 1,000 m of MMF fiber.

pulse energy. As it can be seen from the red solid curve of Fig.3, there is an optimal input energy of 1.75-2.25 nJ, which provides the minimum soliton pulsewidth, being 450 fs for an 850 m long fiber. For comparison, we plot in the same graph the simulation results for the first Raman soliton as a red dashed line. Analogously to the first, even the second and the third Raman soliton experience pulsewidth compression and stretching when increasing the input energy above the threshold for their generation.

When carrying out the experiments of Fig.3, we simultaneously measured the output beam spatial profile, whose analysis is reported in Fig.4. As it can be seen, the curves shown in Fig.3 and Fig.4 have rather similar trends. Indeed, we observed that the beam waist progressively shrinks when increasing the input energy, until it reaches its minimum value at the same input energy which leads to the minimum pulsewidth.

Furthermore, the minimum waist value, that we experimentally estimate to be 8  $\mu\text{m}$ , is remarkably close to the value of

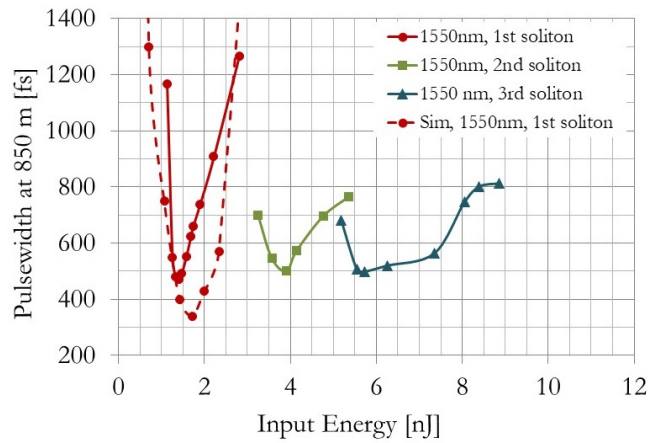


Fig. 3. Measured soliton pulsewidth for the first three Raman solitons at 1550 nm, after 850 m of GRIN fiber.

the fundamental mode waist (shown in Fig.4 by an horizontal line). This result confirms that the soliton which has survived at the GRIN MMF output is carried by the fundamental mode, as predicted by the simulation of Fig.2.

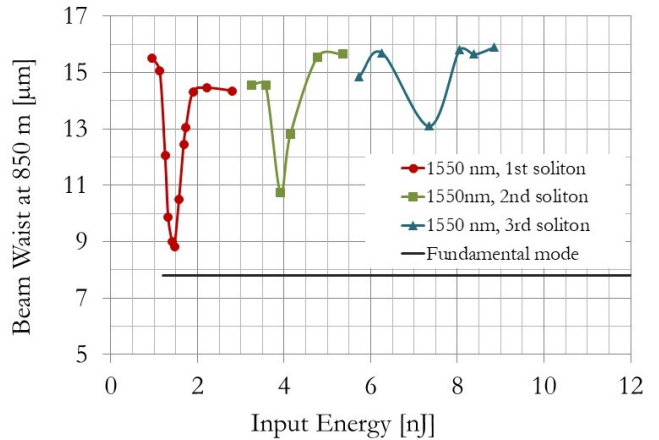


Fig. 4. Numerical and experimental measured output beam waist for the first three Raman solitons after 850 m of GRIN fiber.

### B. MM soliton pulsewidth dynamics

In order to analyze the details of the femtosecond MM soliton generation process, we carried out extensive simulations for a series of fiber lengths (i.e., 1, 2, 6, 10, 20, 120, and 850 m), whose results were compared with a cutback experiment. We also varied the input pulse wavelengths (i.e., 1300, 1350, 1420, 1550, and 1680 nm) and the input pulsewidth from 60 fs up to 240 fs in both simulations and experiments. In all cases, the input beam excitation consisted of the first three non-degenerate modes (i.e., (1, 0), (2, 0), and (3, 0)), which are axial symmetric. Fig.5 collects the results of numerical simulations. It illustrates the evolution of the generated MM soliton pulsewidth during propagation. Here we considered three different input wavelengths (1350, 1550, and 1680 nm), and two values of input pulsewidth (70 and 240 fs). On the

other hand, for each pair of wavelengths and pulse durations, we selected the optimal input pulse energy which provides a stable long-range first Raman soliton (i.e., over 120 m and more). In other words, we chose as working point the input conditions that correspond to the minimum pulsewidth of the red curve in Fig.3. Simulations show that, after just a few meters of propagation, the first Raman soliton has a fixed pulsewidth, independently of the input pulse duration. On the other hand, we observed that the soliton pulsewidth grows larger when increasing the input pulse wavelength. It was also found that the optimal input energy barely oscillates between 2 and 3 nJ, for all input pulsewidths. In simulations, we only observed a weak dependence on the input wavelength and coupling conditions.

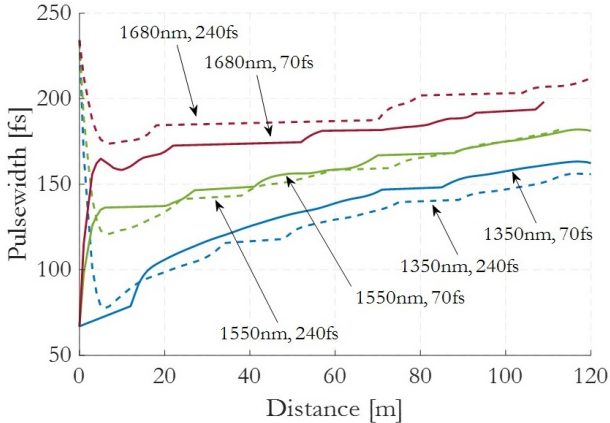


Fig. 5. Simulated MM soliton pulsewidth for three different wavelength input pulses (1350 nm, 1550 nm, and 1680 nm) and for two pulse duration 70 fs and 240 fs.

Our experimental results confirmed well the numerical predictions. We operated at the optimal input energy value, which was between 2 and 3 nJ, as predicted by simulations. The soliton dynamics was characterized for two relatively long spans of GRIN fiber, 120 m and 850 m long, respectively. We repeated our experiments with shorter fibers and the same value of optimal energy, and with input pulsewidths ranging between 60 and 100 fs, depending on the input wavelength. Short fiber spans were useful to determine the soliton generation conditions. At a given fiber length and for the optimal input energy, we tuned the input wavelength in order to make the autocorrelator able to measure the soliton pulsewidth. In this way, we could find out the duration of the soliton, right when it is generated. For example, we found that a soliton is generated at 2 m for an input 1300 nm wavelength, or at 6 m for other input wavelengths. These sets of experimental results are shown as green circles in Fig.6. In the same figure, we also plot the corresponding numerical simulations for same input energy, and input pulsewidths of either 70 fs (blue saltires) or 240 fs (red crosses). Fig.6 reveals the peculiar properties of femtosecond walk-off MM solitons, which are composed by non-degenerate modes. As it can be seen, the soliton pulsewidth remains nearly independent of the temporal duration of the input pulse, and it only depends on

the soliton wavelength. Whereas all MM solitons are generated for similar values of the input pulse energy (not shown).

As mentioned above, soliton formation in MMFs requires not only that chromatic dispersion and self-phase-modulation induced chirp balance each other, but also that cross-phase modulation compensates for modal dispersion. As a result, it is necessary that both dispersion and nonlinearity lengths are nearly equal to the fiber walk-off length [16]:  $L_D = L_{NL} = kL_W$ , being  $k$  an adjustment constant. From this condition, it is possible to calculate the theoretical black curve of Fig.6, which only depends on the fiber dispersion parameters, and as a consequence on the input wavelength. For femtosecond MM solitons, the nonlinear length is shorter than random mode coupling and birefringence lengths, i.e.,  $L_{NL} < L_{cm}, L_{cp}$ . Under these conditions, MM solitons irreversibly evolve into a fundamental-mode solitons at long distances.

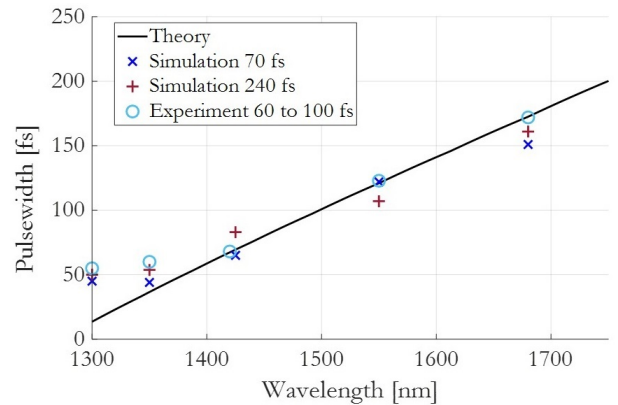


Fig. 6. Theoretical, numerical, and experimental pulsewidth of forming MM soliton vs. wavelength, at the optimal input energy (28 nJ).

#### IV. CONCLUSIONS

We discovered that femtosecond MM solitons composed by non-degenerate modes are characterized by unique properties, which turn out to be quite different from singlemode solitons. The main difference involves the soliton pulsewidth and energy, which are nearly independent of the input pulse duration. As a matter of fact, the generation mechanism of MM walk-off solitons is a trade-off between chromatic dispersion, modal walk-off, and nonlinearity. Therefore, the MM soliton temporal duration must only depend on the input source wavelength and on the fiber dispersion. Moreover, it is not possible to generate non-degenerate MM solitons with a pulsewidth longer than a few hundreds of fs. In addition, intermodal SRS and FWM lead to a progressive and irreversible transfer of energy from higher-order modes into the fundamental mode, which acts as an attractor for the modal composition of MM walk-off solitons.

#### ACKNOWLEDGMENT

We acknowledge the financial support from the European Research Council Advanced Grants No. 874596 and No. 740355 (STEMS), the Italian Ministry of University and

Research (R18SPB8227), and the Russian Ministry of Science and Education Grant No. 14.Y26.31.0017.

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