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Myopic oligopoly pricing ☆

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ABSTRACT

This paper examines capacity-constrained oligopoly pricing with sellers who seek myopic improvements. We employ the Myopic Stable Set solution concept and establish the existence of a unique pure-strategy price solution for any given level of capacity. This solution is shown to coincide with the set of pure-strategy Nash equilibria when capacities are large or small. For an intermediate range of capacities, it predicts a price interval that includes the mixed-strategy support. This stability concept thus encompasses all Nash equilibria and offers a pure-strategy solution when there is none in Nash terms. It particularly provides a behavioral rationale for different pricing patterns, including Edgeworth price cycles and states of hyper-competition with supply shortages. We also analyze the impact of a change in firm size distribution. A merger among the biggest firms may lead to more price dispersion as it increases the maximum and decreases the minimum myopically stable price.

1. Introduction

A common assumption in the literature on oligopoly pricing is that firms aim to maximize their profits.[1] In game-theoretic terms, players are presumed to select *best-responses* to each other's choices. Although it may be reasonable to assume such maximizing behavior, there are compelling arguments for why sellers sometimes make suboptimal decisions. They simply need not be fully rational, for instance, or make mistakes. Also, they might lack the information to identify their most preferred alternative. For example, a firm may not be able to precisely determine its profit-maximizing price *ex ante* and regret its decision *ex post*, i.e., after it has observed the actual choices of its competitors.

With this in mind, this paper offers a novel perspective on oligopoly pricing by postulating that sellers are *myopic* and simply aim to improve upon their current situation. Specifically, we analyze a model of price competition with capacity constraints under the assumption that firms choose better- rather than best-responses. This in particular means that they can, but may not, behave like the neoclassical profit-maximizing firm.

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[1] An in-depth discussion of classical models of oligopoly pricing is provided by Vives (1999).

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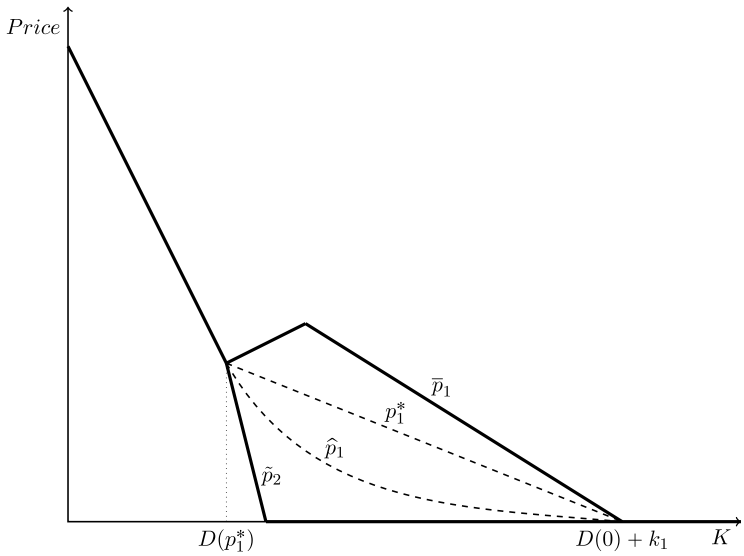

Fig. 1. An illustration of the Myopic Stable Set with two firms and $k_2 = \frac{9}{10}k_1$.

Under this assumption, we employ the solution concept of *Myopic Stable Set* (MSS), which was recently introduced by Demuynck et al. (2019a). A set of price profiles is *myopically stable* when it satisfies three conditions: deterrence of external deviations, asymptotic external stability and minimality. The 'deterrence of external deviations' requirement holds when none of the sellers gains by moving from a price profile in the MSS to a price profile outside the MSS. 'Asymptotic external stability' ensures that it is possible to get from any price profile outside the set (arbitrarily close) to a price profile inside the MSS through a sequence of domination steps. Finally, 'minimality' holds when the MSS is minimal with respect to set inclusion.

We establish the existence of a unique MSS for any given level of capacities. Since firms' profit functions are discontinuous in our setting, this finding extends Demuynck et al. (2019a) who study strategic (normal) form games with continuous payoff functions. In terms of characterization, we show that if the set of pure-strategy Nash equilibria is nonempty, then it coincides with the MSS. A corollary to this result is that the MSS reduces to the pure-strategy Nash solutions that exist for sufficiently large or small production capacities. If capacities are in an intermediate range, then there typically is no pure-strategy Nash equilibrium. In these cases, there is a mixed-strategy Nash equilibrium, the support of which is shown to be contained in the MSS. The MSS therefore also permits price dispersion, but the range of possible 'sales' is wider than in a mixed-strategy Nash equilibrium.

The perspective taken in this paper has several advantages over the standard Nash approach to oligopoly pricing and to capacity-constrained price competition in particular. For example, the MSS solution concept rests on a less-stringent behavioral assumption since sellers are supposed to behave myopically and choose better- rather than best-responses to rivals' prices. Yet, they nevertheless charge precisely the same prices as they would have in a pure-strategy Nash equilibrium. Moreover, the MSS offers a solution in pure strategies when there is none in Nash terms. In these cases, and similar to the mixed-strategy solution, the MSS comprises a range of prices. In fact, we find that it permits larger price fluctuations than in a mixed-strategy Nash equilibrium. Yet, this price dispersion results from sellers following pure rather than random strategies.[2] This set solution is therefore a unifying concept in that it encompasses all the existing pure- and mixed-strategy Nash equilibria in an intuitive and natural fashion.

To illustrate, consider Fig. 1, which depicts a lean version of the detailed example that is presented in Section 6. The graph relates total industry capacity $K = k_1 + k_2$ (x-axis) to price (y-axis) for a linear demand duopoly. Firm 1 is the bigger firm (i.e., $k_1 > k_2$) and it is assumed that both grow in proportion with an increase in industry capacity. For a sufficiently small level of total capacity (i.e., $K < D(p_1^*)$), there is pure-strategy Nash equilibrium with both firms charging the market-clearing price and selling at capacity. $D(\cdot)$ is the market demand function and we refer to p_i^* as the *residual profit-maximizing price*, i.e., the optimal price of a highest-priced firm. This price is *ceteris paribus* increasing in a firm's capacity so that $p_1^* > p_2^*$. The downward-sloping line between the origin and $K = D(p_1^*)$ shows how the market-clearing equilibrium price gradually decreases with an increase in industry capacity. If capacities are in an intermediate range (i.e., $D(p_1^*) < K < D(0) + k_1$), then there is no pure-strategy Nash equilibrium. For these cases, the support of the mixed-strategy equilibrium (i.e., $[\hat{p}_1, p_1^*]$) is indicated by the dashed lines. Finally, when industry capacity is sufficiently large (i.e., $K > D(0) + k_1$), there again is a Nash equilibrium in pure strategies. This 'Bertrand-type equilibrium' has both firms pricing at marginal cost, which is normalized to zero.

In Fig. 1, the MSS is marked by the bold lines. Observe that it coincides with the pure-strategy Nash equilibrium when industry capacity is small (i.e., $K < D(p_1^*)$) or large (i.e., $K > D(0) + k_1$). For intermediate capacities, however, the MSS strictly includes the

[2] Several authors have argued that mixed strategies might be implausible in the context of oligopoly pricing games. See Friedman (1988) and, more recently, Edwards and Routledge (2019).

mixed-strategy support. In this case, the maximum of the MSS is given by firm 1's so-called *iso-profit price* ($\overline{p}_1$). Simply put, a firm's iso-profit price is the lowest price above the market-clearing price for which it obtains the same profit as at the market-clearing level (cf. Definition 5). Note that firm 1's iso-profit price $\overline{p}_1$ lies strictly above the support's maximum p_1^*. The lowest myopically stable price is $\tilde{p}_2$, which we refer to as firm 2's *hyper-competitive price* (cf. Definition 6) as it is below the market-clearing price. Note that the infimum of the mixed-strategy support, $\widehat{p}_1$, exceeds the minimum of the MSS for the entire range of intermediate capacities. Taken together, Fig. 1 then visualizes how the MSS encompasses all Nash equilibria.

By offering a behavioral foundation for oligopoly pricing, this research contributes to the emerging literature on behavioral industrial organization, as recently reviewed by Heidhues and Kőszegi (2018). Thus far, research in this field has mainly focused on psychological factors on the demand side. As indicated by Tremblay and Xiao (2020), however, there is increasing attention for analyzing behavioral aspects on the supply side. The application of the MSS stability concept to oligopoly pricing contributes to this research agenda. In particular, it helps shedding light on some real-world economic phenomena such as price dispersion, supply shortages and Edgeworthian price cycles. We, for example, examine how the MSS would be affected by a change in firm size distribution (e.g., through a merger). The maximum myopically stable price is shown to increase only when the merger becomes the new industry leader in terms of production capacity. This may additionally lead to a decline in the minimum myopically stable price, and therefore to a larger price dispersion, when the smallest firm does not take part in the merger. By contrast, a merger among the smallest firms may reduce the range of myopically stable prices, as it may not affect the maximum and increase the minimum values of the MSS.[3]

The MSS also provides a rationale for different types of pricing patterns, including the following two interesting possibilities: (1) A state of *hyper-competition* with corresponding supply shortages, and (2) Edgeworth-like price cycles.

Myopic oligopoly pricing can lead to a state of hyper-competition in which sellers collectively price below the market-clearing price. The logic is roughly as follows. Starting from a market-clearing situation, the biggest market player may have an incentive to hike its price and operate as a monopolist on its contingent demand curve. This creates an incentive for smaller producers to hike their own prices and (approximately) match the price of the largest firm. The biggest supplier can now improve its situation by shaving its price below the prices of its smaller-sized rivals, leaving the latter worse off than in the initial market-clearing situation. This, in turn, makes prices below the market-clearing price a better-response. Myopic oligopoly pricing may therefore induce a (temporary) state of hyper-competition in which myopic sellers end up setting a price below market-clearing levels. The MSS consequently provides a rationale for rationing, i.e., a situation in which demand exceeds supply.

The MSS moreover offers an explanation for the emergence and magnitude of Edgeworth-like price cycles. Edgeworth (1925) pointed out the possibility of producers not being able to meet their demand.[4] If so, prices may never stabilize, but instead oscillate indefinitely between some upper and lower bound. More specifically, his analysis hints at the emergence of asymmetric price cycles that essentially consist of two parts. If prices are relatively high, then sellers have an incentive to slightly undercut each other. This leads prices to decrease gradually until a floor is reached (the *price war phase*). At that point, firms have an incentive to hike their price and act as a monopolist on their residual demand curve (the *relenting phase*). This latter incentive comes from (i) the fact that cheaper suppliers will not meet all demand forthcoming to them, and (ii) part of these unserved customers still prefers to buy at the higher price. This in turn provides an incentive for low-priced sellers to hike their price, which induces a new cycle.[5]

Several empirical studies have documented the existence of Edgeworth-like price cycles in practice. Eckert (2003) and Noel (2007a,b), for example, provide evidence of such 'sawtooth shape' price patterns in Canadian retail gasoline markets. Among other things, they show that large firms are likely to initiate the relenting phase through a price hike, whereas small firms take the lead in the price war phase. Wang (2008) reports on collusive price cycles in an Australian retail gasoline market.[6] More recently, Zhang and Feng (2011) and Hauschultz and Munk-Nielsen (2020) have shown the presence of Edgeworth-like price patterns in online search-engine advertising and pharmaceutical markets, respectively.

This paper is naturally related to the rich body of theoretical work on capacity-constrained price competition, a literature that basically can be divided in two parts. One focuses on the existence and characterization of the mixed-strategy Nash equilibrium. Such mixed-strategy solutions have been provided by Beckmann (1965), Levitan and Shubik (1972), Osborne and Pitchik (1986), Allen and Hellwig (1986), and Deneckere and Kovenock (1992), amongst others. Another part aims to restore the existence of a pure-strategy Nash equilibrium by rationalizing why residual demand for a high-priced seller would be significantly reduced or even eliminated. For example, Dixon (1990) shows that producers may no longer have an incentive to act as a monopolist on their contingent demand curve when there are cost to turning customers away. Other solutions along this line include Dixon (1992), Tasnádi (1999) and, more

[3] The MSS stability concept thus provides a pure-strategy rationale for price dispersion. See also the recent work by Myatt and Ronayne (2019), who offer a theory of stable price dispersion with pure strategies.

[4] Edgeworth (1925) examines price competition under capacity constraints. Edgeworth (1922) considers the equivalent case in which suppliers are not *willing* to meet the demand forthcoming to them. This may occur when the production technology exhibits decreasing returns to scale, for example. Note also that, since the MSS is a static solution concept, it essentially provides an intuitive explanation for particular price patterns following myopic *better-responses*. For an analysis and discussion of Edgeworthian price cycles based on myopic *best-responses*, see De Roos (2012).

[5] Absent capacity constraints, Maskin and Tirole (1988) show how asymmetric price cycles may emerge in equilibrium when firms pick prices sequentially from a grid.

[6] De Roos and Smirnov (2021) analyze theoretically the pricing behavior of a less than all-inclusive price cartel and provide a rationale for collusive Edgeworth-like price cycles. There is also evidence for asymmetric price cycles in European retail gasoline markets. See, e.g., Foros and Steen (2013) and Linder (2018).

recently, Edwards and Routledge (2019). All this work concentrates on Nash solutions and is consequently based on best- rather than better-responses, which is the focus of our analysis.[7]

Capacity-constrained price competition has also been studied in controlled experimental laboratory settings. Kruse et al. (1994), for example, conducted a series of twenty experiments to test the wide variety of theoretical pricing predictions. Among other things, they find a general price decline during the first periods. Towards the middle or the end, however, they observe patterns of upward and downward price swings. This is confirmed by Fonseca and Normann (2013) who also find prices to move up and down for a wide range of capacities. Interestingly, they conclude that[8]:

> ...the data are better explained by Edgeworth-cycle behavior. Not only are average prices closer to the predicted Edgeworth-cycle prices, *but we cannot reject the hypothesis that firms are engaging in some form of myopic price adjustment.*

The behavioral foundation that we present in this paper advances our understanding of such complex pricing dynamics by providing a simple and relatively uncontroversial rationale for the observed pricing patterns.

The remainder of the paper is organized as follows. The next section presents the model. Section 3 offers a detailed description of the MSS solution concept. Section 4 and Section 5 contain our main findings. These findings are illustrated by means of a linear demand example in Section 6. Section 7 concludes. The proofs are relegated to Appendix A. In Appendix B, we explore the relation between the MSS and better-reply dynamics in a dynamic stochastic setting.

2. Model

Consider a homogeneous-good price-setting oligopoly with a finite set of firms: $N = \{1, \ldots, n\}$, with $n \geqslant 2$. Each firm $i \in N$ has a production capacity $k_i > 0$ and produces to order at constant marginal cost, which we normalize to zero. Without loss of generality, we assume that $k_1 \geqslant k_2 \geqslant \ldots \geqslant k_n > 0$ so that firm 1 is the (weakly) largest and firm n is the (weakly) smallest firm in the market. Total industry capacity is given by $K = \sum_{i \in N} k_i$ and $K_{-i} = \sum_{j \in N \setminus \{i\}} k_j$ is the combined production capacity of all firms other than i.

Let market demand be given by the function $D : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$. We make the standard assumptions that $D(\cdot)$ has a finite upper bound ($D(0)$) and is twice continuously differentiable, with $D'(\cdot) < 0$. There is a choke price $\alpha > 0$ and therefore $D(\cdot) = 0$ at prices larger or equal than α. Sellers pick prices simultaneously, and we denote supplier i's strategy space by $P_i = [0, \infty)$ so that $P = \prod_{i \in N} P_i$ is the set of all possible strategy profiles.

Since products are homogeneous, consumers prefer to buy from a supplier setting the lowest price. As firms may face capacity constraints, however, it is possible that only part of them will be served, in which case higher-priced sellers might still receive demand. To specify individual (residual) demand, let $\Omega(p_i, p_{-i}) = \{j \in N | p_j = p_i\}$ and $\Delta(p_i, p_{-i}) = \{j \in N | p_j < p_i\}$ denote the set of firms that price at and below some given price p_i, respectively, where $p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n)$ indicates the prices of all firms other than i. Demand for firm i's products is then given by $D_i(p_i, p_{-i}) = D(p_i)$ when all its competitors charge a strictly higher price. If there is at least one other seller setting the same price, then its demand is:

$$D_i(p_i, p_{-i}) = \max\left\{ \frac{k_i}{\sum_{j \in \Omega(p_i, p_{-i})} k_j} \left(D(p_i) - \sum_{j \in \Delta(p_i, p_{-i})} k_j \right), 0 \right\}.$$

Finally, if firm i sets the strictly highest price in the industry, its demand is:

$$D_i(p_i, p_{-i}) = \max\left\{ D(p_i) - K_{-i}, 0 \right\}.$$

Thus, (i) customers first visit the lowest-priced seller(s) at the set prices, (ii) at equal prices, demand is allocated in proportion to production capacity, and (iii) rationing is efficient.[9]

Profits are then given by:

$$\pi_i(p_i, p_{-i}) = p_i \cdot \min\{k_i, D_i(p_i, p_{-i})\}, \text{ for all } i \in N.$$

To facilitate the ensuing analysis, we denote firm i's profit by $\pi_i^{\ell}(p_i)$ when p_i is the strictly lowest price and by $\pi_i^h(p_i)$ when p_i is the strictly highest price in the industry. Furthermore, we assume that $\pi_i^h(p_i) = p_i(D(p_i) - K_{-i})$ is strictly concave when there is residual demand for the highest-priced firm (i.e., when $D(p_i) > K_{-i}$) and we write $p_i^* = \arg\max_{p_i} \pi_i^h(p_i)$ to indicate the corresponding **residual profit-maximizing price.**[10] Also, assuming that $K < D(0)$, let $\underline{p}$ be the price for which market demand equals total production

[7] It is worth noting that both better- and best-response dynamics are well-known concepts in the game-theoretic literature on learning. A central issue in this work is whether, and under what conditions, better- and best-response adjustments lead to convergence to an equilibrium. See, for example, Milgrom and Roberts (1990), Monderer and Shapley (1996), Friedman and Mezzetti (2001) and Arieli and Young (2016). Moreover, some papers have studied the link between firms' pricing and competition taking an evolutionary perspective. Examples include Alos-Ferrer et al. (1999); Alos-Ferrer et al. (2000); Alos-Ferrer and Ania (2005); Alos-Ferrer and Kirchsteiger (2010). A recent survey of this literature is provided by Newton (2018). We explore the relation between the MSS and better-reply dynamics in Appendix B.

[8] Fonseca and Normann (2013, p. 201), italics is ours.

[9] Such a surplus maximizing scheme is also used by Levitan and Shubik (1972), Kreps and Scheinkman (1983), Osborne and Pitchik (1986) and Edwards and Routledge (2019), amongst others.

[10] We assume strict concavity for analytical convenience. Strictly speaking, it would be sufficient to impose a weaker requirement such as single-peakedness.

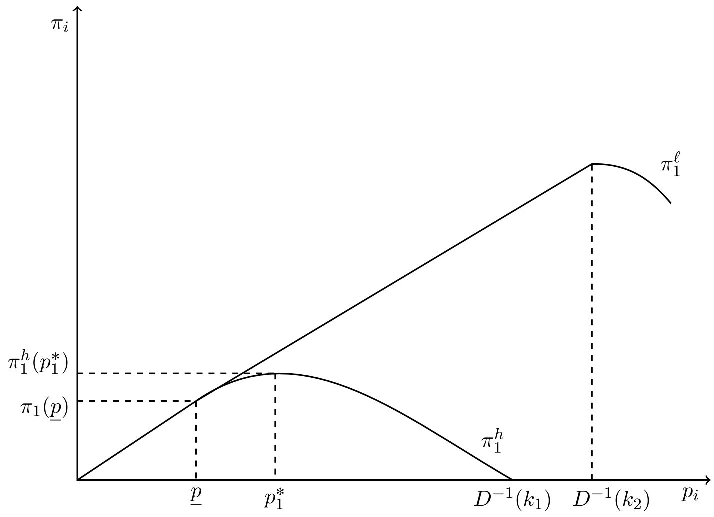

Fig. 2. An illustration of firm 1's profit function when $n = 2$.

capacity ($D(\underline{p}) = K$) and let $\underline{p} = 0$ when $K \geqslant D(0)$. We refer to $\underline{p}$ as the **market-clearing price**.[11] Finally, it is worth noting that a firm produces at capacity whenever there is (weakly) excess demand at the set price (i.e., if $D(p_i) \geqslant K$, then $\pi_i(p_i, p_{-i}) = p_i k_i$).[12] Fig. 2 provides a graphical illustration for the two-firm case.

As is well-known, existence of a pure-strategy Nash equilibrium in capacity-constrained pricing games critically depends on available production capacities.[13] If capacities are large enough in relation to market demand, then there is a symmetric 'Bertrand-type' pure-strategy solution in which all sellers price at cost. A pure-strategy equilibrium also exists when capacities are sufficiently small in the sense that market demand is elastic at all prices above $\underline{p}$.[14] In that situation, all suppliers charge the same market-clearing price and produce at capacity. Finally, for an intermediate range of production capacities, there is no pure-strategy Nash solution. There does exist a mixed-strategy equilibrium, however, which will be elaborated on in Section 5 below.

3. Solution concept

In the following, we do not take the standard Nash approach. Instead, we employ the solution concept of *Myopic Stable Set* which is based on the idea that sellers may simply aim to improve upon their situation and not necessarily maximize their profits. In this section, we introduce this equilibrium concept in detail.

Consider a price profile $p = (p_1, \ldots, p_n) \in P$. The following definition specifies what it means for a price profile to dominate another.

Definition 1. Let $p, p' \in P$ be two price profiles. The price profile p' *dominates* p, $p' \succ p$, if there exists a firm $i \in N$ such that $\pi_i(p') > \pi_i(p)$ and $p'_{-i} = p_{-i}$.

An alternative price profile $p' \in P$ thus dominates p when there is a firm that can unilaterally deviate to p' and earn a higher profit under p' than under p. That is, given the strategy profile p, this firm has a better-reply since it can myopically improve itself by inducing price profile p'.

Next, given some price profile $p \in P$, we write $f(p)$ to describe the subset of P consisting of all dominating price profiles in conjunction with p:

$$f(p) = \{p\} \cup \{p' \in P | p' \succ p\}.$$

Given f, let the set of pure-strategy Nash equilibria be denoted by:

$$\text{NE} = \{p \in P | f(p) = p\}.$$

[11] Note that, since production is to order, there are in fact many market-clearing prices in this model. Moreover, to economize on notation, we refer to $\underline{p}$ sometimes as a price and sometimes as a price profile with all firms pricing at $\underline{p}$. It will be clear from the context what is meant.

[12] See Appendix A, Lemma 1.

[13] See, for example, Chapter 5 of Vives (1999).

[14] For a detailed analysis of this possibility, see Tasnádi (1999).

To capture the better-reply dynamics that can be generated by the firms, we define the κ-fold iteration $f^{\kappa}(p)$ as the subset of P that contains all the price profiles obtained by a composition of dominance correspondences of length $\kappa \in \mathbb{N}$. Thus, $p' \in f^{\kappa}(p)$ when there is a $p'' \in P$ such that $p' \in f(p'')$ and $p'' \in f^{\kappa-1}(p)$. Note further that if $\kappa \leqslant t$, then $f^{\kappa}(p) \subseteq f^{t}(p)$, for all $\kappa, t \in \mathbb{N}$. We indicate the set of price profiles that can be reached from p by a finite number of dominations by $f^{\mathbb{N}}(p)$:

$$f^{\mathbb{N}}(p) = \bigcup_{\kappa \in \mathbb{N}} f^{\kappa}(p).$$

Given $p', p \in P$, we say that a price profile p' asymptotically dominates p when, starting from p, it is possible to get arbitrarily close to p' through a finite number of myopic improvements.

Definition 2. A price profile $p' \in P$ *asymptotically dominates* $p \in P$ if, for all $\epsilon > 0$, there exists a number $\kappa \in \mathbb{N}$ and a price profile $p'' \in f^{\kappa}(p)$ such that $||p' - p''|| < \epsilon$.

We denote by $f^{\infty}(p)$ the set of all strategy profiles in P that asymptotically dominate p. Formally,

$$f^{\infty}(p) = \{p' \in P | \forall \epsilon > 0,\ \exists \kappa \in \mathbb{N}, \exists p'' \in f^{\kappa}(p) : ||p' - p''|| < \epsilon\}.$$

Notice that the set $f^{\infty}(p)$ coincides with the closure of the set $f^{\mathbb{N}}(p)$.

We now have all the ingredients available to define the Myopic Stable Set (MSS) for the capacity-constrained pricing game:

Definition 3. Let $G = \{N, (P_i, \pi_i)_{i \in N}\}$ be a capacity-constrained pricing game as specified in Section 2. The set $M \subseteq P$ is a *Myopic Stable Set* when it is closed and satisfies the following three conditions:

i. **Deterrence of External Deviations:** For all $p \in M$, $f(p) \subseteq M$.
ii. **Asymptotic External Stability:** For all $p \notin M$, $f^{\infty}(p) \cap M \neq \emptyset$.
iii. **Minimality:** There is no closed set $M' \subsetneq M$ that satisfies conditions i and ii.

Suppose there is a set M of myopically stable price profiles. 'Deterrence of External Deviations' means that no firm can profitably deviate to a price profile outside M. 'Asymptotic External Stability' requires that any price profile outside M is asymptotically dominated by a price profile in M. Hence, from any price profile outside M it is possible to get arbitrarily close to one in M by a finite number of myopic improvements. Finally, 'Minimality' means that there is no smaller (closed) set for which the first two conditions are met. Roughly speaking, the MSS can thus be pictured as a set of price profiles that, once entered through the dominance dynamics, is never left.

4. Results

For normal form games, Demuynck et al. (2019a) show the existence of a unique MSS when the strategy space is compact and the payoff functions are continuous. These assumptions are not satisfied in the capacity-constrained pricing model, however. In this section, we prove that this game also possesses a unique MSS for any given level of capacities. Moreover, we characterize this solution and compare it to the set of pure-strategy Nash equilibria as well as to the support of the mixed-strategy Nash equilibrium. Among other things, we establish that the MSS encompasses all existing Nash equilibrium solutions.

4.1. Pricing equilibria with large or small capacities

We begin by exploring the relationship between the MSS and the set of pure-strategy Nash equilibria. Toward that end, denote a subset of sellers $S \subseteq N$ *minimal* when $\sum_{j \in S \setminus \{i\}} k_j \geqslant D(0)$, for all $i \in S$. That is, each combination S has sufficient capacity to meet market demand at a zero price when a member leaves the coalition.

To facilitate the analysis, let us first characterize the set of pure-strategy Nash equilibria NE for the capacity-constrained pricing game as specified in Section 2.[15]

- If $K_{-1} \geqslant D(0)$, then
  $\text{NE} = \left\{ p \in P | p \in \prod_{i \in S} \{0\} \times \prod_{i \in N \setminus S} [0, \infty), \forall S \subseteq N \right\}$.
- If $K \leqslant D(p_1^*)$, then
  $\text{NE} = \left\{ p \in P | p_i = \underline{p} > 0, \text{ for all } i \in N \right\}$
- If $K > D(p_1^*)$ and $K_{-1} < D(0)$, then
  $\text{NE} = \{\emptyset\}$.

[15] Vives (1986) provides conditions for non-emptiness of the set of symmetric pure-strategy Nash equilibria under a surplus maximizing scheme. In Proposition 1, we additionally admit asymmetric pure-strategy Nash equilibria.

Proposition 1. *Let G be a capacity-constrained pricing game as specified in Section 2. The set NE is the set of pure-strategy Nash equilibria of G.*

Simply put, there are two types of pure-strategy Nash equilibria. If industry capacity is sufficiently large, then there is a set of pure-strategy solutions, all of which have firms making zero economic profit. One solution in this case is the symmetric 'Bertrand-type' pure-strategy equilibrium in which all firms price at cost. There are also many asymmetric equilibria in which part of the firms price above cost and have no demand. If aggregate capacity is sufficiently small, then there is a symmetric pure-strategy Nash equilibrium in which each firm sets its price equal to the market-clearing price.

Before establishing the relation between the MSS and the set of pure-strategy Nash equilibria, it is useful to first introduce the so-called *weak improvement property*.

Definition 4. A strategic form game satisfies the weak improvement property when $\mathrm{NE} \neq \emptyset$ and $f^{\infty}(p) \cap \mathrm{NE} \neq \emptyset$ for each price profile $p \notin \mathrm{NE}$.

A normal form game possesses the *weak improvement property* when any non-Nash equilibrium strategy profile converges to a Nash equilibrium through a finite sequence of myopic improvements. Demuynck et al. (2019a) extend previous results by Monderer and Shapley (1996), Friedman and Mezzetti (2001) and Dindõs and Mezzetti (2006) by showing that supermodular games (Friedman and Mezzetti, 2001) and pseudo-potential games (Dubey et al., 2006), including games of strategic complements or substitutes with aggregation (e.g., Cournot oligopolies), exhibit the weak improvement property. The capacity-constrained pricing model does not belong to any of the aforementioned game classes, however. Nevertheless, we establish with the next proposition that this type of games also exhibits the weak improvement property.

Proposition 2. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is nonempty, then this game exhibits the weak improvement property.*

This result states that any price profile that is not a pure-strategy Nash equilibrium is asymptotically dominated by the pure-strategy solution(s). That is, from any price profile not in NE it is possible to get arbitrarily close to a pure-strategy equilibrium by a finite number of myopic improvements.[16]

Using the preceding results, we now show that the set of pure-strategy equilibria coincides with the MSS whenever the former is nonempty.

Theorem 1. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is nonempty, then NE is the unique Myopic Stable Set.*

To illustrate the logic behind this finding, consider a duopoly with $K \leqslant D(p_1^*)$ so that the pure-strategy equilibrium has both firms setting the market-clearing price $\underline{p}$. Suppose, however, that the current price profile is non-Nash and has at least one firm pricing below $\underline{p}$. Since $D(\underline{p}) = K$, this implies that the lower-priced firm(s) is (are) capacity-constrained. A producer pricing below $\underline{p}$ can then raise its price to $\underline{p}$ and still sell at capacity. There thus is a price path of myopic improvements from a profile with prices below the market-clearing price to a profile with all prices weakly above $\underline{p}$.

Suppose then that all prices are weakly above the market-clearing price. When prices differ, the higher-priced firm either faces demand or it does not. If it faces no demand, then it can improve itself by matching or slightly undercutting the price of the cheaper supplier. If it does face residual demand, then the cheaper supplier is capacity-constrained. This lower-priced rival can then increase its profits by raising its price arbitrarily close to the highest price, while still selling at capacity. This enables the more expensive seller to myopically improve by slightly undercutting the lower-priced firm and sell at capacity. In turn, this can be mimicked by the rival, *et cetera*. The resulting downward price spiral stops at the market-clearing price. At equal prices above the market-clearing price, both can increase their profit by (slightly) undercutting their rival's price since that would yield a discrete increase in sales. Taken together, this means that there is also a path of myopic improvements from a profile with prices above the market-clearing price to the pure-strategy equilibrium.

A similar logic applies to all other pure-strategy equilibria. Once the pricing dynamics enters the set NE, however, there is no way out. Indeed, by the very nature of a Nash equilibrium, none of the sellers can profitably deviate to a price profile outside NE. Combining these forces yields the result of Theorem 1; that is, there is a unique MSS that coincides with the set of pure-strategy Nash equilibria NE.

[16] It is noteworthy that Proposition 2 is consistent with Bőrgers (1992) who shows that the market-clearing price survives iterated elimination of strictly dominated strategies when the $n-1$ firms have sufficient capacity to meet demand when pricing at marginal cost.

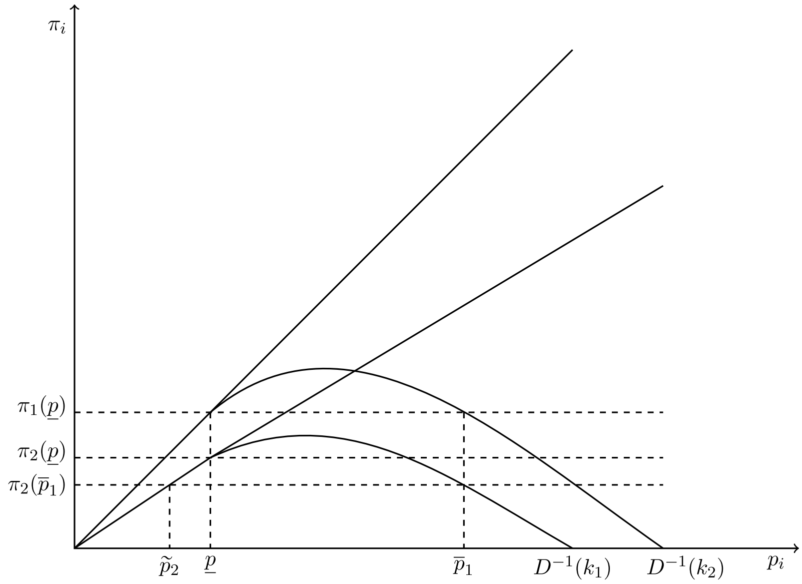

Fig. 3. An illustration of the iso-profit price $\overline{p}_1$ and the hyper-competitive price $\widetilde{p}_2$.

4.2. Myopic stability with intermediate capacities

As indicated above, the set of pure-strategy Nash equilibria is empty in the capacity-constrained pricing model when capacities are in an intermediate range (i.e., when $D(p_1^*) < K < D(0) + k_1$). We now proceed with analyzing the MSS for these types of cases. To that end, we introduce two concepts that are useful in the ensuing analysis; the **iso-profit price** and the **hyper-competitive price**.

Definition 5. For each firm $i \in N$, the **iso-profit price** is:

$$\overline{p}_i = \begin{cases} \min \left\{ p_i \in P_i | \pi_i^h(p_i) = \underline{p} \cdot k_i \text{ with } p_i \neq \underline{p} \right\} & \text{if } D(0) > K_{-i} > D(p_i^*) - k_i, \\ \underline{p} & \text{otherwise.} \end{cases}$$

Given that all its competitors charge a lower price, the iso-profit price of firm i is the lowest price above the market-clearing price $\underline{p}$ for which it receives the same profit as when it would price at $\underline{p}$. It should be emphasized that the iso-profit price differs from the market-clearing price only when the following two conditions hold. First, firm i must face residual demand for some prices (i.e., $D(0) > K_{-i}$). Second, its residual profit-maximizing price must exceed the market-clearing price, which requires a sufficiently large capacity (i.e., $k_i > D(p_i^*) - K_{-i}$). If either of the two conditions is violated, then the iso-profit price coincides with the market-clearing price.

Let us now introduce the hyper-competitive price.

Definition 6. For each firm $i \in N$, the **hyper-competitive price** is:

$$\widetilde{p}_i = \min \left\{ p_i \in P_i | \pi_i^h(\overline{p}_1) = p_i \cdot k_i \right\}.$$

In words, the hyper-competitive price is the lowest price for which a firm obtains the same profit as when it sets the iso-profit price of the largest seller, given that this iso-profit price is the strictly highest price in the market. Note that the min operator in Definition 6 is needed only when $\underline{p} = 0$.

Appendix A presents some properties of $\overline{p}_1$ (Lemma 2) and $\tilde{p}_2$ (Lemma 3) and how they relate (Lemma 4). A graphical illustration of both concepts is provided in Fig. 3.

Now that we have introduced the iso-profit and hyper-competitive prices, we can analyze the MSS when capacities are in an intermediate range. Specifically, we show in the following that the MSS is given by[17]:

$$M = \left\{ p \in P \;\middle|\; \tilde{p}_i \leqslant p_i \leqslant \overline{p}_1, \;\; \forall i \in N \right\}. \tag{1}$$

[17] The existence proof in Demuynck et al. (2019a) (Theorem 3.1) builds on Zorn's lemma, whereas the proof of Theorem 2 is constructive in nature. Moreover, their uniqueness result (Theorem 3.4) does not apply since it assumes the dominance correspondence to be lower hemi-continuous, whereas it is discontinuous in our setting.

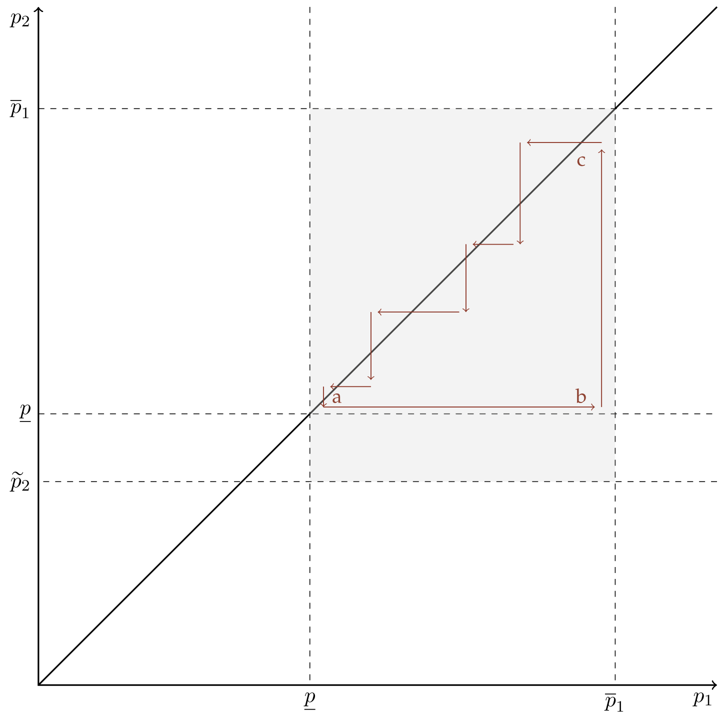

Fig. 4. An illustration of the Myopic Stable Set when $n = 2$ and $k_1 > k_2$.

Theorem 2. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is empty, then M as given in (1) is the unique Myopic Stable Set.*

To see the logic of this Theorem, recall that each firm's iso-profit price exceeds its residual profit-maximizing price, i.e., it is on the downward-sloping portion of the residual profit curve as illustrated in Fig. 3. Moreover, the maximum of the MSS (i.e., $\overline{p}_1$) is the highest iso-profit price and above the market-clearing price (Appendix A, Lemma 2). Since a highest-priced firm is not capacity-constrained when pricing above $\overline{p}_1$, it can improve itself by setting a price below $\overline{p}_1$. For example, it can make more profit by reducing its price to the residual-profit maximizing or market-clearing level. Regarding the lower bound, note that a firm that prices below its hyper-competitive price is capacity-constrained. It can gain more profit by raising its price to its hyper-competitive price since this still allows it to sell at capacity.

Next, let us consider price profiles in M and suppose, for simplicity, there are two firms. Suppose further that the largest seller (firm 1) currently sets the market-clearing price. Since it sells at capacity, lowering its price implies a profit reduction. By definition of the iso-profit price and concavity of the residual profit function, raising its price above the market-clearing level constitutes a myopic improvement as long as it does not exceed $\overline{p}_1$ (see Fig. 3 for a graphical illustration). Notice that when the smaller firm prices weakly above the market-clearing price, it cannot improve itself by pricing above $\overline{p}_1$ since $\overline{p}_1$ exceeds $\overline{p}_2$.

Suppose then that firm 1 hikes its price to the maximum $\overline{p}_1$ and firm 2 myopically improves by raising its price (arbitrarily close) to $\overline{p}_1$ and sell at capacity. In turn, this allows firm 1 to myopically improve by undercutting firm 2's price slightly. Since firm 2 prices above its own iso-profit price, this implies that its profit in the new situation is below the market-clearing level. Consequently, it can myopically improve by setting a price below the market-clearing price, but above its hyper-competitive price (see Fig. 3 and Fig. 5 for a graphical illustration).[18] Note that by charging the hyper-competitive price, a firm earns the same as when it would be the highest-priced firm pricing at $\overline{p}_1$. Also in this case, therefore, concavity of the residual profit function implies that it cannot improve by raising its price above $\overline{p}_1$. Taken together, this characterizes the MSS as specified in (1).

In the following, we give some more intuition for this solution set by discussing it in relation to three economic phenomena: (1) Edgeworth Price Cycles, (2) Hyper-competition, and (3) Mergers.

4.2.1. *Edgeworth price cycles*

In Fig. 4, the MSS is given by the shaded area. In principle, this area admits different types of pricing patterns. The red arrows represent one particular better-response price path. Starting at point 'c' firms are undercutting each other's prices until point 'a'. At 'a', firm 1 hikes its price to $\overline{p}_1$ at point 'b'. This, in turn, makes it a better-response for firm 2 to price slightly below $\overline{p}_1$. As this

[18] Note that the biggest firm's hyper-competitive price coincides with the market-clearing price (Appendix A, Lemma 3).

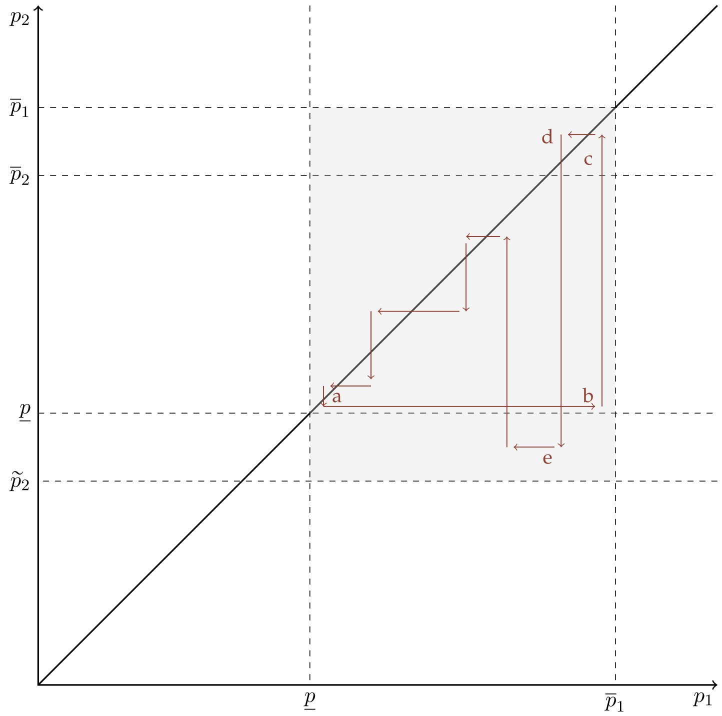

Fig. 5. An illustration of the Myopic Stable Set when $n = 2$ and $k_1 > k_2$.

example illustrates, the MSS naturally captures Edgeworth-like price cycles and consequently provides a clear rationale for such 'sawtooth shape' price patterns.

4.2.2. Hyper-competition

Another striking possibility is that a smaller supplier may find it in its interest to set a price below the market-clearing price. Such a scenario is depicted in Fig. 5. As before, suppose that there are two sellers that price close to the market-clearing price at 'a'. By hiking its price, firm 1 may then induce a price profile at 'b'. This, in turn, may trigger firm 2 to slightly undercut firm 1's price, which leads to a price profile around 'c'. In particular, firm 2 may set a price $p_2' \in (\overline{p}_2, \overline{p}_1]$. Slightly undercutting p_2' may then constitute a better-response for firm 1, which results in a price profile at, say, 'd'. Yet, in that case it is profitable for firm 2 to reduce its price from p_2' to some price $p_2'' \in (\tilde{p}_2, \underline{p})$ below the market-clearing price. This would result in a price profile around 'e'. Firms can leave such a hyper-competitive state by raising their price to the market-clearing level, which in turn may induce similar pricing dynamics. Note that this possibility strongly relies on the assumption of firms following myopic better responses since a hyper-competitive price like at 'e' does not constitute a best-response to 'd'.

The MSS therefore provides a rationale for 'price wars' where all but the strictly largest seller(s) set a price below the market-clearing price.

Corollary 1. *If each firm $i \in N\backslash\{1\}$ is strictly smaller than firm 1, then there exists a price profile $p \in M$ with $p_1 = \underline{p}$ and $p_i \in [\tilde{p}_i, \underline{p})$.*

Note that such hyper-competitive price profiles are Pareto-dominated in that all producers would be better-off (and no one worse off) when pricing weakly above the market-clearing price. Moreover, this result highlights the possibility of an equilibrium shortage, i.e., a situation in which market demand exceeds aggregate supply.

4.2.3. Mergers

How is the MSS affected when there is a change in firm size distribution? We address this question by analyzing the impact of a merger. In the context of our model, a merger is taken to mean a transformation of two or more firms into a single, larger, entity. In the following, let $k_S = \sum_{i \in S} k_i$ denote the joint capacity of a merger between a strict subset of firms $S \subset N$. An $|N|$ dimensional pre-merger price profile thus transforms into an $|N| - |S| + 1$ dimensional price profile post-merger.

By Theorem 2, recall that the pre-merger MSS with intermediate capacities is given by:

$$M = \left\{ p \in P \;\middle|\; \tilde{p}_i \leqslant p_i \leqslant \overline{p}_1, \;\; \forall i \in N \right\}.$$

We consider two cases: (i) merging parties establish a firm with a production capacity that exceeds the capacity of the biggest pre-merger supplier (Proposition 3), and (ii) merging parties form a firm with less production capacity than the largest pre-merger supplier (Proposition 4).

We first consider scenario (i) in which the merger becomes the new industry leader in terms of production capacity. The next proposition shows that this induces a wider range of myopically stable prices when the smallest firm is not taking part in the merger. Specifically, it raises the maximum myopically stable price since there is a new largest firm post-merger that has a higher iso-profit price (*i.e.*, $\overline{p}_1$ increases). This, in turn, creates a downward pressure on the hyper-competitive prices of the non-merging parties (*i.e.*, $\tilde{p}_i$ for all $i \in N \backslash S$ weakly decrease). All else equal, this effect is more pronounced the larger the merger.

Proposition 3. *Consider a merger between a subset of firms $S \subset N$ and suppose that $k_S > k_1$.*

(i) *The merger increases the maximum of the Myopic Stable Set M. Moreover, this maximum value is rising with the size of the merger.*
(ii) *If $D(0) < K < D(0) + k_1$, then the merger has no effect on the minimum myopically stable prices.*
(iii) *If $D(p_1^*) < K < D(0)$, then the merger leads to a decrease of the non-merging parties' hyper-competitive prices.*

Let us now turn to scenario (ii), which is the possibility that the merger does not affect the size of the largest firm in the industry. In this case, there is a contraction of the MSS. Specifically, since the iso-profit price of the biggest supplier is unaffected, the maximum of the MSS does not change (*i.e.*, $\overline{p}_1$ remains the same). By contrast, the merger's hyper-competitive price weakly exceeds the pre-merger hyper-competitive prices of the parties involved. In particular, the merger may lead to an increase of the minimum myopically stable price when it includes the smallest supplier.

Proposition 4. *Consider a merger between a subset $S \subset N$ of firms and suppose that $k_S < k_1$.*

(i) *The merger has no effect on the maximum of the Myopic Stable Set M.*
(ii) *If $D(0) < K < D(0) + k_1$, then the merger has no effect on the minimum myopically stable prices.*
(iii) *If $D(p_1^*) < K < D(0)$, then the merger's hyper-competitive price exceeds the pre-merger hyper-competitive prices of the merging parties.*

In sum, any merger that excludes the smallest firm and becomes the industry leader in terms of production capacity induces a broader range of myopically stable prices. In particular, the maximum of the MSS increases, whereas the hyper-competitive prices of the non-merging parties decrease. By contrast, there is a (weakly) narrower range of myopically stable prices when the merger does not become the biggest industry player. In that case, the maximum of the MSS is not affected, whereas the hyper-competitive prices of the merging parties are replaced by a single higher hyper-competitive price of the merger. Although the analysis is conducted assuming a single merger, results would be similar in case of multiple mergers. This is because the critical driver of the above findings is the impact on the maximum and minimum values of the MSS. The key question is therefore whether or not there is a merger that takes over the leading position in the industry.

5. Equilibria in pure and mixed strategies

Theorem 1 and Theorem 2 establish the existence and characterization of a unique Myopic Stable Set for any given level of capacities. In particular, they offer a pure-strategy solution when there is none in Nash terms. Let us now conclude by relating the MSS as derived above to the mixed-strategy Nash equilibrium solution concept. The existing literature has repeatedly shown that there is a Nash equilibrium in mixed strategies under fairly weak assumptions.[19] Specifically, since in the above capacity-constrained pricing model demand is continuously decreasing and residual profit functions are continuous and strictly concave, there exists an equilibrium in mixed strategies without 'holes' when capacities are within an intermediate range, i.e., when $D(p_1^*) < K < D(0) + k_1$.[20]

To specify the mixed-strategy support, recall that there exists a unique residual profit-maximizing price p_i^* for each firm $i \in N$. By construction of the contingent demand functions $D(p_i) - K_{-i}$, it then follows that $p^m \geqslant p_1^* \geqslant p_2^* \geqslant \ldots \geqslant p_n^*$, where p^m is the monopoly price.[21] Note that p^m approaches p_1^* when K_{-1} approaches zero. Since none of the sellers has an interest in charging a price in excess of p_1^*, this price constitutes the maximum of the mixed-strategy support. Let the minimum values be indicated by $\widehat{p}_i$, where $\widehat{p}_i \neq p_i^*$ is the price solving

$$p_i^* \left[D(p_i^*) - K_{-i} \right] = \min \left\{ \widehat{p}_i k_i, \widehat{p}_i \cdot D\left(\widehat{p}_i\right) \right\}.$$

The mixed-strategy support is therefore given by:

[19] Maskin (1986) provides a general analysis and discussion of these existence conditions.

[20] See, e.g., Deneckere and Kovenock (1992) and, more recently, Tasnádi (2004) and Tasnádi (2020).

[21] Since for any $i, j \in N$, p_i^* and p_j^* are such that:

$$D(p_i^*) + p_i^* D'(p_i^*) = K_{-i} \text{ and } D(p_j^*) + p_j^* D'(p_j^*) = K_{-j},$$

and, therefore, $K_{-i} < K_{-j} \Leftrightarrow p_i^* > p_j^*$ by concavity of firms' (residual) profit functions.

$$\mathcal{K} = \prod_{i \in N} \left[\hat{p}_i, p_1^*\right] \subset P.$$

Let us now relate $\mathcal{K}$ to the MSS. Recall that in this case there is a unique MSS given by the set M (Theorem 2). The next result shows that the MSS permits larger price fluctuations in comparison to the mixed-strategy equilibrium.

Theorem 3. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is empty, then $\mathcal{K} \subset M$.*

The intuition underlying this finding is as follows. Regarding the maximum, with random strategies no firm puts mass on prices above the maximizer of its (residual) profit function. By contrast, the MSS permits such prices since prices in excess of the maximizer may still constitute a better-response. The story is to some extent similar for the minimum values. To see this, notice that in a mixed-strategy equilibrium none of the sellers prices below $\underline{p}$ because either it can sell its entire capacity at $\underline{p}$ or $\underline{p} = 0$. As there are no pure-strategy Nash equilibria in this case, there is at least one firm that would be willing to hike its price when all firms price at $\underline{p}$. It can be easily verified that this holds for the largest firm. Since the higher-priced firm has residual demand, the lower-priced firms are capacity-constrained. This provides an incentive to also raise their prices, which in turn implies that no seller puts mass on prices weakly below $\underline{p}$. By contrast, following the definition of the MSS maximum $\overline{p}_1$, $\underline{p}$ must be part of the MSS since profits are the same at both prices. In fact, and as illustrated in Figs. 3-5 above, it is quite possible that one or more sellers have a better-response below $\underline{p}$.

In sum, the above analysis shows that myopic sellers set the same price as their profit-maximizing counterparts when production capacities are either 'large' or 'small'. For an intermediate range of capacities, the set of mixed-strategy profiles is a subset of the MSS. The MSS thus encompasses all Nash solutions. The next section provides an example illustrating these findings.

6. Example

Let us examine a Bertrand-Edgeworth duopoly with linear market demand: $D(p) = 1 - p$. Demand for the products of firm i, $i = 1, 2$ and $i \neq j$, is then described by the following demand structure:

$$D_i(p_i, p_j) = \begin{cases} 1 - p_i & \text{if } p_i < p_j, \\ \frac{k_i}{k_i + k_j}(1 - p_i) & \text{if } p_i = p_j, \\ \max\{0, 1 - p_i - k_j\} & \text{if } p_i > p_j. \end{cases} \tag{2}$$

It is assumed that $k_1 > k_2$ so that firm 1 is strictly larger in terms of capacity. Below, we derive the MSS for the entire range of production capacities and compare it to the standard Nash solution.

6.1. Nash equilibria

Let us begin with the situation where capacities are 'large' so that a pure-strategy Nash equilibrium exists. Specifically, this is the case when each seller can serve the whole market at the competitive price, i.e., when $k_1 > k_2 \geqslant 1$. The Nash equilibrium is then such that both firms charge a price equal to marginal cost and therefore (by Theorem 1):

$$M = NE = \{(0, 0)\}.$$

A pure-strategy Nash equilibrium also exists when capacities are sufficiently small. Specifically, this is true when $k_1 \leqslant \underline{k}_1$, where $\underline{k}_1$ solves the following equality:

$$\underline{k}_1 = D(p_1^*) - k_2. \tag{3}$$

In our linear example, firm 1's residual profit-maximizing price is:

$$p_1^* = \frac{1}{2}\left(1 - k_2\right).$$

Substituting in (3) and rearranging gives the threshold value $\underline{k}_1 = \left(1 - k_2\right)/2$. Thus, for $k_1 \leqslant \underline{k}_1$ (or, equivalently, $k_2 \leqslant 1 - 2k_1$), there is a pure-strategy solution for which the market clears. Moreover, by Theorem 1, this pure-strategy Nash equilibrium coincides with the MSS:

$$M = NE = \{\left(1 - k_1 - k_2, 1 - k_1 - k_2\right)\}.$$

For the capacity ranges specified above there exists no nondegenerate mixed-strategy Nash equilibrium. Let us now turn to the possibility where there is a nondegenerate Nash equilibrium in mixed strategies. This is the case when $k_1 > \underline{k}_1$ or, equivalently, when

$1 > k_2 > 1 - 2k_1$. To determine the minimum of the mixed-strategy support, notice that firm 1 is indifferent between being the high- and the low-priced firm when[22]:

$$\pi_1^h\left(p_1^*\right) = p_1^* \cdot (1 - p_1^* - k_2) = \frac{1}{4}\left(1 - k_2\right)^2 = \pi_1^\ell\left(\widehat{p}_1\right) = \widehat{p}_1 \cdot \min\{k_1, 1 - \widehat{p}_1\},$$

so that

$$\widehat{p}_1 = \frac{1}{4k_1}\left(1 - k_2\right)^2 \text{ when } k_2 \leqslant 1 - \sqrt{1 - (2k_1 - 1)^2},$$

and

$$\widehat{p}_1 = \frac{1}{2} - \frac{1}{2}\sqrt{2k_2 - k_2^2} \text{ when } 1 - \sqrt{1 - (2k_1 - 1)^2} < k_2 \leqslant 1.$$

The mixed-strategy support of this Bertrand-Edgeworth game is, therefore, given by:

$$\left[\widehat{p}_1, p_1^*\right] = \left[\frac{\left(1 - k_2\right)^2}{4k_1}, \frac{1 - k_2}{2}\right] \text{ or } \left[\widehat{p}_1, p_1^*\right] = \left[\frac{1}{2} - \frac{1}{2}\sqrt{2k_2 - k_2^2}, \frac{1 - k_2}{2}\right],$$

depending on the capacity levels.

6.2. Myopic stable set

Let us now derive the MSS for this intermediate range of capacities. By Theorem 2, we know that the maximum of the MSS is the price $\overline{p}_1$ solving

$$\pi_1^h\left(\overline{p}_1\right) = \overline{p}_1\left[D(\overline{p}_1) - k_2\right] = \pi_1\left(\underline{p}\right),$$

whereas the minimum value for the largest firm (firm 1) is given by:

$$\underline{p} = \max\left\{0, D^{-1}(k_1 + k_2)\right\}. \tag{4}$$

We therefore need to distinguish two cases.

The first is where combined production capacity is sufficiently large to serve the whole market at the competitive price: $k_1 + k_2 \geqslant 1$. In this case, $\underline{p} = 0$ and $\overline{p}_1$ is the price that solves

$$\pi_1^h\left(\overline{p}_1\right) = \overline{p}_1\left[D(\overline{p}_1) - k_2\right] = \pi_1\left(\underline{p}\right) = 0,$$

which implies

$$D(\overline{p}_1) = k_2, \text{ and therefore } \overline{p}_1 = D^{-1}(k_2) = 1 - k_2.$$

In this case, $\underline{p} = 0$ is also the MSS lower bound for firm 2. Thus, when $1 > k_1 \geqslant 1 - k_2$, the MSS is symmetric and given by:

$$M = \left[0, D^{-1}(k_2)\right] \times \left[0, D^{-1}(k_2)\right] = \left[0, 1 - k_2\right] \times \left[0, 1 - k_2\right].$$

Observe that an increase in k_2 lowers the maximum value of the MSS since it reduces the residual demand for firm 1's products when it is the high-priced seller. This, in turn, makes that the 'optimal high price' is lower leaving fewer prices that qualify as a better-response.

Now consider the other possibility where $k_1 + k_2 < 1$ so that $\underline{p} = 1 - k_1 - k_2 > 0$. In this case, the maximum of the MSS is obtained by solving

$$\underline{p} \cdot (1 - \underline{p} - k_2) = \overline{p}_1 \cdot (1 - \overline{p}_1 - k_2),$$

which is equivalent to

$$k_1 \cdot \left(1 - k_1 - k_2\right) = \overline{p}_1 \cdot (1 - \overline{p}_1 - k_2),$$

and therefore $\overline{p}_1 = k_1$. In contrast, the minimal myopically stable price differs across firms. For firm 1 this minimum is the same as in the previous case. For firm 2, however, it is the price $\widetilde{p}_2$ given by:

$$\pi_2^h\left(\overline{p}_1\right) = \overline{p}_1\left[D\left(\overline{p}_1\right) - k_1\right] = \widetilde{p}_2 \cdot k_2$$

[22] Note that in this duopoly example $\widehat{p}_1$ exceeds $\widehat{p}_2$ and firm 2 has no incentive to put mass on prices below $\widehat{p}_1$. For a detailed analysis, see Deneckere and Kovenock (1992).

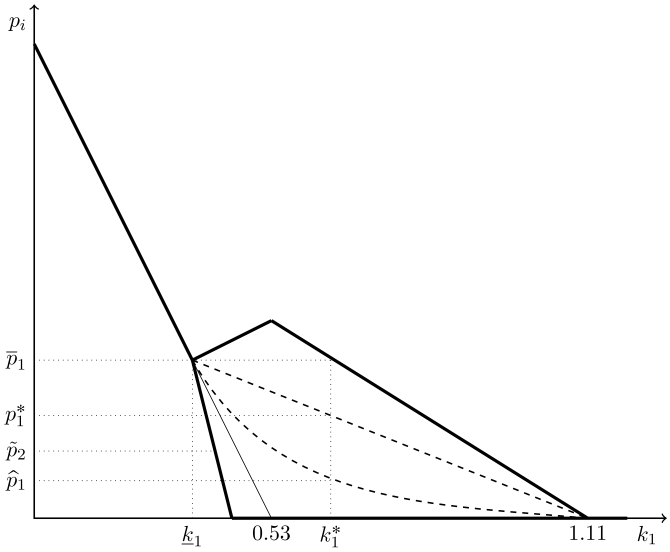

Fig. 6. Myopic Stable Set in a linear demand duopoly with $k_2 = \frac{9}{10}k_1$.

with $\widetilde{p}_2 \neq \overline{p}_1$. This price solves $k_1\left(1-2k_1\right) = \widetilde{p}_2 k_2$, which gives $\widetilde{p}_2 = k_1\left(1-2k_1\right)/k_2 < \underline{p}$. Thus, when $1-k_2 > k_1 > (1-k_2)/2$, the MSS is asymmetric and given by:

$$M = \left[\underline{p}, \overline{p}_1\right] \times \left[\widetilde{p}_2, \overline{p}_1\right] = \left[1-k_1-k_2, k_1\right] \times \left[k_1\left(1-2k_1\right)/k_2, k_1\right].$$

Notice that k_1 has a positive impact on the size of the MSS since it reduces the minimal myopically stable prices while increasing the maximum value (Proposition 3).[23] Also, and in contrast to the previous case, k_2 has a positive impact on the size of the MSS by reducing the minimum values.

6.3. Comparison

To conclude, let us now compare the range of the mixed-strategy support $\left[\widehat{p}_1, p_1^*\right]$ with the price range of the MSS for all (relevant) capacity levels. Fig. 6 provides a graphical illustration.

Fig. 6 is a more detailed variation of Fig. 1. The MSS is depicted by the solid (thick) black line for every level of k_1 and k_2 (expressed as a function of k_1, which in this figure is $k_2(k_1) = \frac{9}{10} \cdot k_1$). Starting from the left, for sufficiently small capacities there is a pure-strategy Nash equilibrium that coincides with the MSS (Theorem 1). For this specific example, this is true as long as $k_1 \leqslant \underline{k}_1 \approx 0.345$.[24] At that point, the market-clearing price $\underline{p}$ (indicated by the thin solid line) starts to fall below the maximizer of π_1^h (indicated by the straight dashed line). This provides an incentive for firms to hike their price and become the high-priced firm.

The increase in capacities not only undermines the existence of a pure-strategy Nash solution; it also widens the range of better-responses. To see this, recall that $\overline{p}_1$ is the lowest price above $\underline{p}$ for which firm 1 obtains the same profit. Therefore, and due to the fact that the profit function is strictly concave and has a unique maximum (see Fig. 2 and Fig. 3), the MSS maximum value ($\overline{p}_1$) is increasing and the MSS minimum value ($\widetilde{p}_2$) is decreasing in the gap between p_1^* and $\underline{p}$. This range of better-response prices is rising until $k_1 \approx 0.53$, the capacity level at which $\underline{p}$ becomes zero.[25] Beyond that point, $\underline{p}$ remains zero. Since profits at $\underline{p}$ are zero, profits at $\overline{p}_1$ must be zero too. Note that residual demand for the high-priced seller gradually decreases when capacities grow further. This implies that the residual demand choke price is declining and therefore $\overline{p}_1$ declines as well. The range of better-response prices is narrowing until $k_1 \geqslant \frac{10}{9}$ and $k_2 \geqslant 1$. At that point, the MSS coincides with the pure-strategy Nash equilibrium in which both firms charge a price of zero.

The MSS can be compared to the mixed-strategy support $\left[\widehat{p}_1, p_1^*\right]$. A non-degenerate Nash equilibrium in mixed strategies exists when capacities are within the intermediate range $0.345 \approx \underline{k}_1 \leqslant k_1 < \frac{10}{9}$. In Fig. 6, the mixed-strategy support is the vertical distance between the dashed lines. The upper bound of the support is given by the maximizer of the residual profit function π_1^h, which is linearly decreasing in k_1. Notice that for this range of capacities, the maximum of the MSS, $\overline{p}_1$, is higher than p_1^*, because there are prices in excess of this maximizer that still constitute a better-response to the market-clearing price. Note further that the mixed-strategy support depends quadratically on k_1 and reaches its maximum at $k_1^* \approx 0.618$. Finally, observe that the minimum

[23] Note that $\tilde{p}_2$ is decreasing in k_1 for $k_1 > 1/4$, which holds true in this case.

[24] This threshold value can be computed by using $k_1 = \underline{k}_1 = \frac{(1-k_2)}{2}$ and $k_2 = \frac{9}{10}k_1$. Combining gives $k_1 \approx 0.345$ and $k_2 \approx 0.31$.

[25] This maximum MSS price interval is reached at $k_1 = 1 - k_2$. Using $k_2 = \frac{9}{10}k_1$, this gives $k_1 \approx 0.53$.

of the mixed-strategy support, $\widehat{p}_1$, exceeds both $\underline{p}$ and $\widetilde{p}_2$. Fig. 6 thus visualizes how the MSS strictly includes the support of the mixed-strategy Nash equilibrium (Theorem 3).

6.4. Merger in a symmetric oligopoly

Let us conclude this section by illustrating the impact of a merger on the MSS. To that end, we consider a simple linear demand oligopoly with n identical firms. Each firm has a production capacity $k > 0$ so that total industry capacity is given by $K = nk$. The demand structure is the n-firm version of the duopoly demand specification presented above. In what follows, our focus is on the situation where there is no pure-strategy Nash equilibrium. That is, capacities are in an intermediate range: $D(p_1^*) < K < D(0) + k$, which implies $1/(n+1) < k < 1/(n-1)$.

6.4.1. Pre-merger

To specify the MSS, recall that its maximum is given by the iso-profit price (Theorem 2), which is the price $\overline{p}$ that solves:

$$\pi_i^h\left(\overline{p}\right) = \overline{p}\left[D(\overline{p}) - (n-1)k\right] = \pi_i\left(\underline{p}\right).$$

The minimum of the MSS is given by:

$$\underline{p} = \max\left\{0, D^{-1}(nk)\right\}. \tag{5}$$

As before, we need to distinguish two cases. The first is when total industry capacity is sufficiently large to meet market demand at the competitive price: $n \cdot k \geqslant 1$ or $k \geqslant 1/n$. In this case, it holds that $1/(n-1) > k \geqslant 1/n$ and $\underline{p} = 0$ so that the iso-profit price is determined by:

$$\pi_i^h\left(\overline{p}\right) = \overline{p}\left[D(\overline{p}) - (n-1)k\right] = \pi_i\left(\underline{p}\right) = 0,$$

which implies

$$D(\overline{p}) - (n-1)k = 0 \Longrightarrow \overline{p} = D^{-1}((n-1)k) = 1 - (n-1)k.$$

Thus, in this case the MSS is symmetric and given by:

$$M = [0, 1-(n-1)k]^n.$$

Now consider the second possibility where $1/(n+1) < k < 1/n$ and $\underline{p} > 0$. In this case, it holds that $D(\underline{p}) = 1 - \underline{p} = nk$ so that $\underline{p} = 1 - nk > 0$. The iso-profit price is then obtained by solving:

$$\overline{p} \cdot (1 - \overline{p} - (n-1)k) = \underline{p} \cdot k = (1 - nk) \cdot k,$$

which gives $\overline{p} = k$. Moreover, each firm's hyper-competitive price coincides with the market-clearing price pre-merger. Taken together, the MSS is therefore given by:

$$M = \left[\underline{p}, \overline{p}\right]^n = [1-nk, k]^n.$$

6.4.2. Post-merger

We now consider the impact of a merger between a subset of firms $S \subset N$ on the MSS M, where $1 < s < n$ is the number of firms involved in the merger. Since the merger becomes the largest industry player, the post-merger maximum of the MSS is given by $\overline{p}_s$, where the subscript 's' indicates the merged entity. This iso-profit price is the price solving:

$$\pi_s^h\left(\overline{p}\right) = \overline{p}_s\left[D(\overline{p}_s) - (n-s)k\right] = \pi_s\left(\underline{p}\right).$$

Like in the pre-merger situation, we need to distinguish two cases. One is where total production capacity is sufficient to serve the market at the competitive price: $(n-s) \cdot k + (s \cdot k) \geqslant 1$ or $k \geqslant 1/n$. In this case, $\underline{p} = 0$ and the iso-profit price of the merger ($\overline{p}_s$) is the price that solves:

$$\pi_s^h\left(\overline{p}\right) = \overline{p}_s\left[D(\overline{p}_s) - (n-s)k\right] = \pi_s\left(\underline{p}\right) = 0,$$

which implies

$$D(\overline{p}_s) - (n-s)k = 0, \;\; \overline{p}_s = D^{-1}((n-s)k) = 1 - (n-s)k.$$

The minimum remains $\underline{p} = 0$ so that the MSS post-merger (M') is given by:

$$M' = [0, 1-(n-s)k]^{(n-s+1)}.$$

Since the minimum remains unaffected and the maximum increases with the size of the merger, there is a wider range of myopically stable prices in this case.

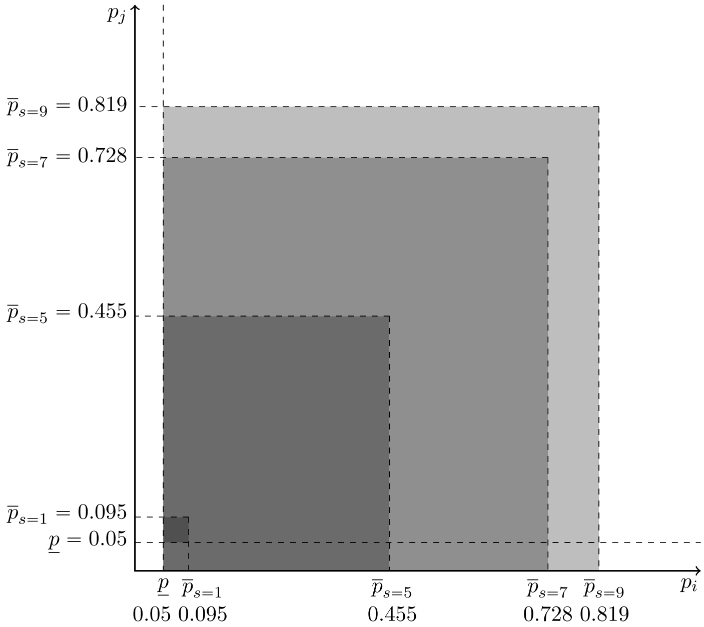

Fig. 7. Effect of mergers on MSS coordinates $\{p_i, p_j\}_{i\in S,\ j\in N\setminus S}$ for $n = 10$, $s = 1$, $s = 5$, $s = 7$, $s = 9$ and $k = 0.091$.

The second situation is one with a strictly positive market-clearing price: $\underline{p} = 1 - nk > 0$. The iso-profit price of the merged entity is then given by:

$$\pi_s^h\left(\overline{p}_s\right) = \overline{p}_s\left[D(\overline{p}_s) - (n-s)k\right] = \pi_s\left(\underline{p}\right) = (1-nk)\,sk,$$

which implies $\overline{p}_s = sk$.

Regarding the minimum myopically stable prices, recall that the hyper-competitive price of the largest seller coincides with the market-clearing price. Hence, for the merging firms, the hyper-competitive price remains the market-clearing price in this case. Next, let us determine the hyper-competitive price of the non-merging parties, $\widetilde{p}_j$ for $j \in N\setminus S$, which are the smallest industry members post-merger:

$$\widetilde{p}_j \cdot k = \pi_j^h\left(\overline{p}_s\right) = \overline{p}_s\left[D\left(\overline{p}_s\right) - (n-1)k\right],$$

which implies

$$\widetilde{p}_j = s\left[1 - sk - (n-1)k\right].$$

Taken together, the MSS post-merger is therefore:

$$M' = \left[\underline{p}, sk\right] \times \left[s\left(1-(n+s-1)k\right), sk\right]^{(n-s)}.$$

Notice that $\overline{p}_s = sk > \overline{p} = k$ for any $s > 1$ so that the maximum of the MSS monotonically increases in the size of the merger. As to the minimum values, the hyper-competitive prices are concave in s and reach there maximum at $(1-k(n-1))/2k$, which is smaller than 1. Hence, the minimum myopically stable prices are decreasing in merger size until they reach a value of zero. Fig. 7 provides a graphical illustration of how the MSS expands with the size of the merger.

In sum, this example shows how a merger can (weakly) reduce the minimum and strictly increase the maximum of the MSS, thereby inducing a wider range of myopically stable prices.[26]

7. Concluding remarks

Within the growing body of work on behavioral industrial organization, there is an increasing focus on behavioral aspects of the firm. In this paper, we have relaxed the common assumption that firms are pure profit-maximizers and supposed that sellers seek myopic improvements instead. Under this assumption, we addressed a classic and persistent question in economics: How are prices determined in industries with a few powerful firms? To analyze this oligopoly pricing problem, we employed the Myopic Stable Set

[26] See Bos and Marini (2023) for a more extended analysis of the impact of firm size and number on oligopoly pricing in a homogeneous-good Bertrand-Edgeworth model.

solution concept within the context of a capacity-constrained pricing game and established the existence of a unique MSS for any given level of capacities. This result was then compared with the standard Nash solution.

A main takeaway from our analysis is that the less demanding behavioral assumption of firms choosing myopic better-responses does not affect existing pure-strategy Nash price predictions. If the set of pure-strategy Nash equilibria is nonempty, like when capacities are sufficiently large or small, it coincides with the MSS. With moderate-sized capacities, the Nash equilibrium is in mixed strategies. For these cases, all prices in the mixed-strategy support are part of the MSS. This set solution therefore offers an alternative foundation for oligopoly pricing. Moreover, we have shown that the MSS provides a rationale for different types of pricing patterns. In particular, it gives an explanation for the emergence and magnitude of Edgeworth-like price cycles as well as states of hyper-competition in which supply falls short of market demand. We have also shown how the MSS can be affected by changes in market structures.

We see several avenues for future research. One is to use the notion of MSS within the context of other oligopoly models.[27] A potentially interesting variation on this paper's capacity-constrained pricing model would be to assume that production precedes sales. Another avenue is to analyze oligopoly pricing under different behavioral assumptions such as heterogeneity in rationality or competition among quasi-myopic agents.[28] Finally, and especially because the MSS is rich enough to permit heterogeneous pricing, we can imagine it to serve as a foundation for further empirical and experimental work.

Declaration of competing interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Data availability

No data was used for the research described in the article.

Appendix A. Formal statements and proofs

This appendix contains all proofs and some supportive statements that are used in establishing the main results. It is organized by (sub-)section.

A.1. Section 2

Lemma 1. *Fix p_{-i}. If $K < D(0)$ and $0 \leqslant p_i \leqslant \underline{p}$, then $\pi_i(p_i, p_{-i}) = p_i k_i$, for all $i \in N$.*

Proof of Lemma 1. Suppose that $D(0) > K$ so that $\underline{p} > 0$. Since $D(\underline{p}) = K$ and demand is decreasing in price, it holds that $D(p_i) \geqslant K$ when $p_i \leqslant \underline{p}$. This means that $D(p_i) \geqslant k_i$ and $D(p_i) - K_{-i} \geqslant k_i$. It also implies that $\frac{k_i}{\sum_{j\in\Omega(p_i,p_{-i})} k_j}\left(D(p_i) - \sum_{j\in\Delta(p_i,p_{-i})} k_j\right) \geqslant k_i$, because $D(p_i) \geqslant K \geqslant \sum_{j\in\Omega(p_i,p_{-i})} k_j + \sum_{j\in\Delta(p_i,p_{-i})} k_j$. Hence, if $0 \leqslant p_i \leqslant \underline{p}$, then firm $i \in N$ always produces at capacity and, therefore, $\pi_i = p_i k_i$. □

A.2. Section 4.1

Proposition 1. *Let G be a capacity-constrained pricing game as specified in Section 2. The set NE is the set of pure-strategy Nash equilibria of G.*

Proof of Proposition 1. If $K_{-1} \geqslant D(0)$, then $\underline{p} = 0$. To begin, suppose all firms set the same price. If all price at some $p' > \underline{p} = 0$, then none of them is capacity-constrained since $K > K_{-1} \geqslant D(0) > D(p')$. Hence, each seller has an incentive to (marginally) undercut its rivals. If all price at $\underline{p} = 0$, then firm 1 has no incentive to deviate since $K_{-1} \geqslant D(0)$. It would therefore face no residual demand at a price above zero. As the largest firm has no incentive to deviate, none of the firms has an incentive to deviate. We conclude that, when $K_{-1} \geqslant D(0)$, there is a symmetric pure-strategy Nash equilibrium with all firms pricing at $\underline{p} = 0$.

In addition, there are many asymmetric pure-strategy Nash equilibria, which have in common that there is a subset of sellers that price at zero. To see this, suppose that, by contrast, all set a price strictly above zero. Suppose further there is one firm charging the strictly highest price. In that case, this firm faces no demand since $K_{-1} \geqslant D(0)$. Hence, it would be better off by charging a lower price, e.g., match the price of the lowest-priced firm(s).

Suppose then that there are two or more sellers that set the strictly highest price. If they face no residual demand, there is again an incentive to deviate, e.g., they would be better off by matching the lowest price in the industry. If they do face residual demand,

[27] For example, Demuynck et al. (2019b) have characterized the MSS for a homogeneous-good Bertrand duopoly with asymmetric costs.

[28] See Dixon (2020) for a recent study of strategic firm behavior under the assumption of almost-maximization, i.e., competition among almost-rational sellers who choose almost best-responses.

then they are not capacity-constrained. To see this, consider a highest-priced seller i that sets $p_i' > 0$, whereas its rivals set a weakly lower price. This seller is not capacity-constrained when:

$$\frac{k_i}{\sum_{j\in\Omega(p_i',p_{-i}')} k_j}\left(D(p_i') - \sum_{j\in\Delta(p_i',p_{-i}')} k_j\right) < k_i \Longleftrightarrow D(p_i') < \sum_{j\in\Omega(p_i',p_{-i}')} k_j + \sum_{j\in\Delta(p_i',p_{-i}')} k_j = K,$$

which holds since $K > D(0) > D(p_i')$.

Next, note that in this case firm i finds it beneficial to slightly reduce p_i' when:

$$(p_i' - \epsilon)\cdot \min\left\{k_i, \frac{k_i}{\sum_{j\in\Omega(p_i'-\epsilon,p_{-i}')} k_j}\cdot\left(D(p_i'-\epsilon) - \sum_{j\in\Delta(p_i'-\epsilon,p_{-i}')} k_j\right)\right\} >$$
$$p_i'\cdot\frac{k_i}{\sum_{j\in\Omega(p_i',p_{-i}')} k_j}\left(D(p_i') - \sum_{j\in\Delta(p_i',p_{-i}')} k_j\right),$$

with $\epsilon > 0$ and sufficiently small. Suppose that firm i is capacity-constrained when cutting price to $p_i' - \epsilon$ so that the preceding inequality reduces to:

$$(p_i' - \epsilon)k_i > p_i'\cdot\frac{k_i}{\sum_{j\in\Omega(p_i',p_{-i}')} k_j}\left(D(p_i') - \sum_{j\in\Delta(p_i',p_{-i}')} k_j\right)\Longleftrightarrow$$
$$p_i' - \epsilon > p_i'\cdot\frac{1}{\sum_{j\in\Omega(p_i',p_{-i}')} k_j}\left(D(p_i') - \sum_{j\in\Delta(p_i',p_{-i}')} k_j\right)\Longleftrightarrow$$
$$p_i'\left[1 - \frac{1}{\sum_{j\in\Omega(p_i',p_{-i}')} k_j}\left(D(p_i') - \sum_{j\in\Delta(p_i',p_{-i}')} k_j\right)\right] > \epsilon.$$

Note that the LHS is positive when the term inside the square brackets is positive. That is,

$$1 > \frac{1}{\sum_{j\in\Omega(p_i',p_{-i}')} k_j}\left(D(p_i') - \sum_{j\in\Delta(p_i',p_{-i}')} k_j\right)\Longleftrightarrow K = \sum_{j\in\Omega(p_i',p_{-i}')} k_j + \sum_{j\in\Delta(p_i',p_{-i}')} k_j > D(p_i'),$$

which holds. Hence, in this case, firm i finds it profitable to reduce its price to $p_i' - \epsilon$.

Now suppose that firm i is not capacity-constrained when reducing its price to $p_i' - \epsilon$. By continuity, and assuming a sufficiently small ϵ, the above inequality then reduces to:

$$(p_i' - \epsilon)\cdot\left(D(p_i'-\epsilon) - \sum_{j\in\Delta(p_i'-\epsilon,p_{-i}')} k_j\right) > p_i'\cdot\frac{k_i}{\sum_{j\in\Omega(p_i',p_{-i}')} k_j}\left(D(p_i') - \sum_{j\in\Delta(p_i',p_{-i}')} k_j\right),$$

which holds since firm i faces more (residual) demand at $p_i' - \epsilon$. Hence, also in this case, each of the highest-priced sellers would have an incentive to (marginally) lower its price. We conclude that in equilibrium there is a subset of firms pricing at zero.

For such a subset S to emerge in equilibrium it must hold that none of the firms $i \in S$ has an incentive to hike its price. Note that this is true for each subset that is *minimal*, i.e., each coalition S for which it holds that $\sum_{j\in S\setminus\{i\}} k_j \geqslant D(0)$, for all $i \in S$. Moreover, all sellers who are not part of such a minimal subset can charge any price since all prices give zero profits.

Taken together, therefore, the set of (a)symmetric pure-strategy Nash equilibria when $K_{-1} \geqslant D(0)$ is given by:

$$\mathrm{NE} = \left\{ p\in P \,\middle|\, p \in \prod_{i\in S}\{0\} \bigtimes \prod_{i\in N\setminus S} [0,\infty), \forall S\subseteq N\right\}.$$

Now suppose that $K_{-1} < D(0)$. We can distinguish two cases: (1) $K \geqslant D(0)$ so that $\underline{p} = 0$, and (2) $K < D(0)$ so that $\underline{p} > 0$.

Case (1): If $K \geqslant D(0) > K_{-1}$, then $\underline{p} = 0$. We argue that there can be no pure-strategy Nash equilibrium in this case. To begin, note that there is no symmetric pure-strategy Nash equilibrium. If all firms would price at 0, then firm 1 would have an incentive to hike its price since $K_{-1} < D(0)$. If all sellers would price at some $p' > \underline{p} = 0$, then none of them is capacity-constrained since $K \geqslant D(0) > D(p')$. Consequently, each supplier has an incentive to (marginally) undercut its rivals. We conclude that there is no symmetric pure-strategy Nash equilibrium in this case.

Let us now argue that there does not exist an asymmetric pure-strategy Nash equilibrium either. First, notice that in such an equilibrium no firm can set a price at 0. Since $K_{-1} < D(0)$, this clearly holds for firm 1 as this firm always has a price above zero for which its (residual) demand is positive. However, given that firm 1 prices above zero, there is no combination of firms (other than firm 1) with a combined capacity sufficient to meet market demand at a price of zero. This implies that each firm that prices at zero has an incentive to hike its price. We conclude that there is no asymmetric pure-strategy Nash equilibrium in which one or more firms price at zero.

Suppose then that each firm prices above zero. In this case, there are one or more firms setting the strictly highest price. A highest-priced seller then either faces no demand in which case it has an incentive to match the lowest price in the industry or it does face residual demand. In case it does face residual demand, all lower-priced sellers are capacity-constrained and can improve themselves by raising their price arbitrarily close to the highest price. In that case, however, a highest-priced seller has an incentive to slightly undercut its lower-priced rivals since this leads to a discrete increase in demand, which it can meet at approximately the same price. We conclude that there is no (a)symmetric pure-strategy Nash equilibrium when $K \geqslant D(0) > K_{-1}$.

Case (2): If $K_{-1} < K < D(0)$, then $\underline{p} > 0$. We first argue that in this case there cannot be an asymmetric pure-strategy Nash equilibrium. If there was, then one or more firms would be charging the strictly highest price, say p'. If $0 < p' \leqslant \underline{p}$, then lower-priced sellers could profitably raise their price till p', because in this case it holds that $K \leqslant D\left(p'\right)$ (Lemma 1). Suppose then that $p' > \underline{p}$ and that there are two or more firms charging the strictly highest price. As shown in the first part of this proof, these sellers are not capacity-constrained since $K > D(p')$ when $p' > \underline{p}$ and each has an incentive to (marginally) lower its price.

This leaves the possibility of a single highest-priced firm. Following a similar logic as above, this firm is not capacity-constrained and cannot have zero demand in equilibrium. Suppose, therefore, that there is a single highest-priced seller pricing at $p' > \underline{p}$ with positive residual demand. This implies that lower-priced suppliers are capacity-constrained and therefore could raise their prices below, but arbitrarily close to p'. However, in that case, the highest-priced seller has an incentive to match the price of its rivals, because:

$$p'\frac{k_i}{K}D\left(p'\right) > p'\left(D\left(p'\right) - K_{-i}\right) \iff \frac{k_i}{K}D\left(p'\right) > D\left(p'\right) - K_{-i} \iff$$

$$K_{-i} > D\left(p'\right)\left[1 - \frac{k_i}{K}\right] \iff K_{-i} > D\left(p'\right)\left[\frac{K_{-i}}{K}\right] \iff K > D\left(p'\right),$$

which holds. We conclude that if there is a pure-strategy Nash equilibrium in this case, then it must be symmetric.

Suppose therefore that all firms charge the same price. If all price at $p' < \underline{p}$, then each firm can profitably deviate to a higher price (Lemma 1). In a similar vein, if all price at $p' > \underline{p}$, then $K > D\left(p'\right)$ so that each has an incentive to (marginally) undercut its rivals.

This leaves all firms pricing at $\underline{p}$ as the candidate equilibrium. Clearly, since all sellers produce at capacity in this case, none has an incentive to cut price. Moreover, if $K > D(p_1^*)$, then firm 1 has an incentive to hike its price so that all firms pricing at the market-clearing level is no equilibrium either. Yet, if $K \leqslant D(p_1^*)$, then none of the firms has an incentive to hike its price. To see this, suppose that $K = D(p_1^*)$ so that $\underline{p} = p_1^*$. Since its first-order condition for a maximum is satisfied at $\underline{p} = p_1^*$, firm 1 does not have an incentive to hike its price:

$$D(\underline{p}) - K_{-1} + \underline{p}D'(\underline{p}) = D(p_1^*) - K_{-1} + p_1^* D'(p_1^*) = 0.$$

As for all other firms, $i \in N\setminus\{1\}$, note that $p_1^* \geqslant p_2^* \geqslant \ldots \geqslant p_n^*$ by strict concavity of the residual profit functions so that their marginal profit at $\underline{p}$ is negative:

$$\overline{p} = D(\underline{p}) - K_{-i} + \underline{p}D'(\underline{p}) < 0.$$

Consequently, none of the firms has an incentive to raise price. Finally, note that if $K < D(p_1^*)$, then $\underline{p} > p_1^*$ so that firm 1's marginal profit is negative at $\underline{p}$ and therefore also for its smaller capacity rivals. Hence, also in this case none of the firms has an incentive to hike its price. We conclude that there is a unique symmetric pure-strategy Nash equilibrium when $K \leqslant D(p_1^*)$ and it has all firms pricing at $\underline{p} > 0$. □

Proposition 2. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is nonempty, then this game exhibits the weak improvement property.*

Proof of Proposition 2. The proof is by construction. Following the definition of NE as provided in Section 4.1 and the proof of Proposition 1, we distinguish two cases and consider each case in turn.

Case (1): Suppose that $K_{-1} \geqslant D(0)$. To show that $f^{\infty}(p) \cap \mathrm{NE} \neq \emptyset$ for each $p \notin NE$, we proceed in four steps.

Step 1: Following the proof of Proposition 1, if $K_{-1} \geqslant D(0)$, then there are two types of non-Nash price profiles: (i) a price profile with some firms pricing at zero, or (ii) a price profile with all firms pricing above zero. In case of (ii), move to Step 2. In case of (i), let S be the set of sellers who price at zero and let $p \in P\setminus NE$ be the corresponding price profile. Since the price profile p is not a Nash equilibrium, the largest member of S can profitably raise its price. Note that the resulting price profile is also not a Nash equilibrium so that we can repeat the argument. We conclude that there exists a sequence, which results in all firms charging a strictly positive price.

Step 2: Following Step 1, there exists a profitable price path from a non-Nash price profile with some firms pricing at zero to a non-Nash price profile with all firms pricing strictly above zero. Let $p' \in P\backslash NE$ be a non-Nash price profile with all firms charging a strictly positive price. We can again distinguish two cases: (i) all sellers set the same strictly positive price, or (ii) there are two or more firms charging a different strictly positive price. In case of (ii), move to Step 3. In case of (i), note that since $K > D(0)$ it is profitable for each firm to (marginally) undercut the price of its competitors. Hence, there is a profitable deviation in this case resulting in a price profile consisting of two or more different prices.

Step 3: Let $p'' \in P\backslash NE$ be a price profile resulting from Step 1 and Step 2. That is, p'' exclusively consists of prices above zero and contains at least two different prices. We can again distinguish two cases: (i) there are two or more firms charging the strictly highest price, or (ii) there is a single seller setting the strictly highest price. In case of (ii), move to Step 4. In case of (i), and following the proof of Proposition 1, the highest-priced firms are not capacity-constrained and can profitably undercut their highest-priced rivals.

Step 4: By steps 1,2 and 3, there is a sequence of myopic improvements from any non-Nash price profile to a non-Nash price profile with (i) strictly positive prices only, and (ii) a single strictly highest price. Let p be a given non-Nash price profile with these two characteristics and let the single highest-priced seller be denoted by h. Note that, since $K_{-1} \geqslant D(0)$, the highest-priced seller has no residual demand. Consequently, this firm can profitably deviate to a price p_h° lower than the lowest price in p and arbitrarily close to zero: $||p_h^\circ - 0|| < \epsilon$, for all $\epsilon > 0$. This would create a situation with a new highest-priced firm (perhaps via Step 3) and the argument can be repeated. This implies that there is a sequence of myopic improvements from the price profile p to a price profile with a sufficient number of sellers pricing arbitrarily close to zero. That is, there is a $\kappa > 0$ such that the κ-fold iteration of Step 4 generates a sequence:

$$p = p^0, p^1 \in f(p^0), p^2 \in f(p^1), ..., p^\kappa \in f(p^{\kappa-1}),$$

where p^κ is arbitrarily close to some $p' \in NE$: $||p^\kappa - p'|| < \epsilon$, for all $\epsilon > 0$. Then, by definition, $p' \in f^\infty(p)$ so that $f^\infty(p) \cap \text{NE} \neq \emptyset$.

Case (2): Now suppose that $K \leqslant D(p_1^*)$ so that all firms pricing at $\underline{p} > 0$ is the unique pure-strategy Nash equilibrium (Proposition 1). To begin, note that each seller who prices below $\underline{p}$ can profitably raise its price to $\underline{p}$ (Lemma 1). The remaining type of non-Nash price profile to consider is therefore one with all prices weakly above and at least one strictly above $\underline{p}$.

If there are two or more firms charging the strictly highest price (above $\underline{p}$), then the situation is comparable to Step 2 and Step 3 of Case (1) above. That is, a highest-priced seller can profitably deviate to a price (marginally) below the highest price. The resulting price profile is then one with a single strictly highest price. If this firm faces no residual demand, then it can improve itself by setting the market-clearing price. If it does face residual demand, then its lower-priced rivals are capacity-constrained and, therefore, can myopically improve by raising their price below, but arbitrarily close to the highest price. In this case, and following the same logic as in the proof of Proposition 1, the highest-priced firm can increase its profits by matching the price of its lower-priced competitors. This creates a similar situation as above in which more than one seller sets the highest price. Taken together, we conclude that if $K \leqslant D(p_1^*)$, then $f^\infty(p) \cap \text{NE} \neq \emptyset$. □

Theorem 1. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is nonempty, then NE is the unique Myopic Stable Set.*

Proof of Theorem 1. The set NE as defined in Section 4.1 is a MSS when it is *closed* and satisfies *deterrence of external deviations*, *asymptotic external stability*, and *minimality*.

Closedness: If $K \leqslant D(p_1^*)$, then the pure-strategy Nash equilibrium is a singleton, which is closed. If $K_{-1} \geqslant D(0)$, then the set NE is given by:

$$\text{NE} = \left\{ p \in P \middle| p \in \prod_{i \in S} \{0\} \bigtimes \prod_{i \in N \backslash S} [0, \infty), \forall S \subseteq N \right\}.$$

Hence, it is effectively the product of a finite number of closed sets, which is closed.

Deterrence of External Deviations: Notice that the set of pure-strategy Nash equilibria is effectively the set of undominated strategy profiles: $\text{NE} = \{p \in P | f(p) = p\}$, which implies that no firm can profitably deviate to a price profile outside NE.

Asymptotic External Stability: This condition holds by Proposition 2, which establishes that the capacity-constrained pricing game exhibits the weak improvement property. Hence, from any price profile not in NE it is possible to get arbitrarily close to a pure-strategy equilibrium by a finite number of myopic improvements.

Minimality: Since the set NE is closed and the previous two conditions hold, minimality follows directly from Corollary 3.11 in Demuynck et al. (2019a); a mirror result which effectively shows that MSS ⊇ NE when the set of pure-strategy Nash equilibria is nonempty.

Combining the above, we conclude that the set NE is a MSS. It remains to be shown that it is also the unique MSS.

Uniqueness: Suppose there would be another MSS, M, different from NE. As NE is a MSS, first note that neither $M \supset$ NE, nor NE $\supset M$, because otherwise the minimality requirement would be violated for either M or NE. Moreover, note that neither $M \cap \text{NE} = \emptyset$, nor $M \cap \text{NE} \neq \emptyset$ with $M \neq$ NE. If so, then there would be a price profile in NE that is not in M. Yet, for each price profile in NE it holds that no firm has a profitable deviation to a price profile outside NE, which implies that the Asymptotic External Stability condition would be violated for M. We conclude that $M =$ NE and therefore that NE is the unique MSS. □

A.3. Section 4.2

In the following, we present three Lemmas that highlight some useful properties of the iso-profit price, the hyper-competitive price and the relation between both.

Part *(i)* of Lemma 2 gives conditions under which the iso-profit price exceeds the market-clearing price and shows that the iso-profit price is increasing in capacity. Part *(ii)* and *(iii)* describe when the iso-profit price coincides with the market-clearing price, which is the case when the firm is sufficiently small. Part *(ii)* captures the possibility that a firm faces residual demand, but where its residual profit-maximizing price is lower than the market-clearing price. Part *(iii)* shows the possibility that a firm faces no residual demand at any price. In all cases, it holds that the iso-profit price of the largest firm is strictly positive and above the market-clearing price.

Lemma 2. *Suppose there is no pure-strategy Nash equilibrium.*

(i) *If $D(0) > K_{-i} > D(p_i^*) - k_i$, for all $i \in N$, and $k_i > k_j$, for any $i, j \in N$, then $\overline{p}_i > \overline{p}_j > \underline{p} \geqslant 0$.*
(ii) *For all $i \in N \setminus \{1\}$, if $k_i \leqslant D(p_i^*) - K_{-i}$, then $\overline{p}_i = \underline{p}$. For firm 1, $\overline{p}_1 > \underline{p}$.*
(iii) *If $D(0) \leqslant K_{-m}$ with $n \geqslant m > 1$, then $\overline{p}_i = \underline{p} = 0$ for each firm $i = m, m+1, m+2, ..., n$ weakly smaller than m.*

Proof of Lemma 2. Let us prove each of the three cases in turn.

(i) For each $i \in N$, if $D(0) > K_{-i} > D(p_i^*) - k_i$, then $D(\underline{p}) > D(p_i^*)$ because either (1) $K < D(0)$ and $\underline{p} > 0$ so that $D(\underline{p}) = K > D(p_i^*)$, or (2) $K \geqslant D(0)$ and $\underline{p} = 0$ so that $D(\underline{p}) = D(0) > D(p_i^*)$. Hence, in this case, $p_i^* > \underline{p} \geqslant 0$. Since the residual profit functions are strictly concave and have a unique maximizer at p_i^*, it follows that the iso-profit price is at the decreasing part of the residual profit function: $\overline{p}_i > p_i^* > 0$, for all $i \in N$.
Let us now show that the iso-profit price is increasing with firm capacity. To that end, consider two firms i and j, with $k_i > k_j$, and suppose that $\underline{p} > 0$. Suppose further that they both pick a price p from $\left(0, D^{-1}(K_{-i})\right)$. Comparing their residual profits gives:

$$\begin{aligned}\pi_i^h(p) - \pi_j^h(p) &= p\left(D(p) - K_{-i}\right) - p\left(D(p) - K_{-j}\right) \\ &= p\left(K_{-j} - K_{-i}\right) = p\left(k_i - k_j\right) > 0.\end{aligned} \tag{6}$$

Moreover, for $\underline{p} > 0$ and following Definition 5 of the iso-profit price, it must hold that:

$$\pi_i^h\left(\overline{p}_i\right) - \pi_j^h\left(\overline{p}_j\right) = \underline{p}\left(k_i - k_j\right) > 0. \tag{7}$$

To show that $\overline{p}_i > \overline{p}_j > 0$, suppose the opposite, i.e., $\overline{p}_j > \overline{p}_i$, in view of a contradiction. As established above, firms' residual profits are decreasing at their iso-profit price so that:

$$\pi_i^h\left(\overline{p}_j\right) < \pi_i^h\left(\overline{p}_i\right) \text{ for } \overline{p}_j > \overline{p}_i,$$

and, therefore:

$$\pi_i^h\left(\overline{p}_j\right) - \pi_j^h\left(\overline{p}_j\right) < \pi_i^h\left(\overline{p}_i\right) - \pi_j^h\left(\overline{p}_j\right) = \underline{p}\left(k_i - k_j\right).$$

Notice, however, that:

$$\begin{aligned}\pi_i^h\left(\overline{p}_j\right) - \pi_j^h\left(\overline{p}_j\right) &= \overline{p}_j\left(D(\overline{p}_j) - K_{-i}\right) - \overline{p}_j\left(D(\overline{p}_j) - K_{-j}\right) \\ &= -\overline{p}_j\left(K - k_i\right) + \overline{p}_j\left(K - k_j\right) = \overline{p}_j\left(k_i - k_j\right) > \underline{p}\left(k_i - k_j\right),\end{aligned}$$

which contradicts the previous result that $\pi_i^h\left(\overline{p}_j\right) - \pi_j^h\left(\overline{p}_j\right) < \underline{p}\left(k_i - k_j\right)$.
Now suppose that $\underline{p} = 0$. In this case, and again following Definition 5, it holds that:

$$\pi_i^h\left(\overline{p}_i\right) = \overline{p}_i\left(D(\overline{p}_i) - K_{-i}\right) = \underline{p} \cdot k_i = 0,$$

which implies that the iso-profit price of firm i is given by:

$$\overline{p}_i = D^{-1}(K_{-i}) > 0,$$

and, therefore:

$$D^{-1}(K_{-i}) > D^{-1}(K_{-j}) > 0,$$

for every $i, j \in N$ with $k_i > k_j$. We conclude that if $k_i > k_j$, then $\overline{p}_i > \overline{p}_j > 0$, for all $i, j \in N$.

(ii) Suppose now that $D(p_i^*) - K_{-i} \geqslant k_i$, for all $i \in N \setminus \{1\}$. Since in this case $D(p_i^*) \geqslant K$, it follows that $D(p_i^*) \geqslant K \equiv D(\underline{p})$ for every $i \neq 1$, which implies that $\underline{p} \geqslant p_i^*$. Since in this case every firm i (except firm 1) sells at capacity (see Lemma 1), its profit line $\underline{p}k_i$ intersects the residual profit π_i^h either at the maximum or at the decreasing part. Consequently, the only price for which $\underline{p} \cdot k_i = \pi_i^h(p_i)$ is the market-clearing price $\underline{p}$ at which the residual profit of firm i reaches its peak and declines afterward at any higher price $p_i > \underline{p}$. Finally, recall that in case of intermediate capacities: $D(p_1^*) < K$ and $D(0) > K_{-1}$. Hence, for firm 1 it always holds that $\overline{p}_1 > p_1^* > \underline{p}$.

(iii) The third situation to consider is when $D(0) \leqslant K_{-m}$, with $n \geqslant m > 1$. In this case, it holds that $D(0) < K$ and therefore $\underline{p} = 0$. Notice that residual demand for firm m is zero at all prices, which implies that all firms weakly smaller than firm m also face no residual demand. We conclude that $\overline{p}_i = \underline{p} = 0$ for each firm $i = m, m+1, m+2, \ldots, n$ with capacity $k_i \leqslant k_m$. □

Part *(i)* of Lemma 3 shows that hyper-competitive prices are (weakly) below the market-clearing price whenever the latter is strictly positive. Part *(ii)* establishes a positive relationship between the hyper-competitive price and firm size. Part *(iii)* states that the hyper-competitive price is zero for a highest-priced firm not facing residual demand.

Lemma 3. *Suppose there is no pure-strategy Nash equilibrium.*

(i) *If $K < D(0)$ and each firm $i \in N \setminus \{1\}$ is strictly smaller than firm 1, then $0 < \widetilde{p}_i < \underline{p}$ and $\widetilde{p}_1 = \underline{p}$.*
(ii) *If $K < D(0)$ and $k_i > k_j$, then $\widetilde{p}_i > \widetilde{p}_j$, for all $i, j \in N$.*
(iii) *If $D(0) \leqslant K_{-i}$, then $\widetilde{p}_i = \underline{p} = 0$, for all $i \in N$.*

Proof of Lemma 3. Let us discuss the three cases in turn.

(i) Suppose that $K < D(0)$ and that firm 1 is the strictly largest seller. In this case, $\underline{p} > 0$ and the iso-profit price $\overline{p}_1$ is the price solving the following equation:

$$\pi_1^h(\overline{p}_1) = \overline{p}_1 \left(D(\overline{p}_1) - K_{-1} \right) = \underline{p} \cdot k_1.$$

The hyper-competitive prices are given by:

$$\pi_i^h(\overline{p}_1) = \overline{p}_1 \left(D(\overline{p}_1) - K_{-i} \right) = \tilde{p}_i \cdot k_i.$$

Hence, it immediately follows that $\widetilde{p}_1 = \underline{p}$. As to a firm $i \in N \setminus \{1\}$, note that

$$\underline{p} = \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-1} \right)}{k_1} \text{ and } \widetilde{p}_i = \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-i} \right)}{k_i}.$$

Comparing and rearranging gives:

$$\underline{p} - \widetilde{p}_i = \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-1} \right)}{k_1} - \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-i} \right)}{k_i} = \overline{p}_1 \cdot \frac{\left(k_1 - k_i \right) \left[K - D(\overline{p}_1) \right]}{k_1 k_i} > 0,$$

which holds because $\overline{p}_1 > \underline{p}$ (Lemma 2) and firm 1 is the strictly largest seller. We conclude that if $K < D(0)$ and each firm $i \in N \setminus \{1\}$ is strictly smaller than firm 1, then $0 < \widetilde{p}_i < \underline{p}$ and $\widetilde{p}_1 = \underline{p}$.

(ii) Given that $D(0) > K$ and following the same steps as under *(i)* above, it holds that

$$\widetilde{p}_i - \widetilde{p}_j = \overline{p}_1 \frac{\left(k_i - k_j \right) \left[K - D(\overline{p}_1) \right]}{k_i k_j} > 0,$$

for any two firms $i, j \in N$ with $k_i > k_j$.

(iii) If $D(0) \leqslant K_{-i}$, then $D(0) \leqslant K$ and therefore $\underline{p} = 0$. Moreover, the residual demand of firm $i \in N \setminus \{1\}$ at $\overline{p}_1$ is $D(\overline{p}_1) - K_{-i} \leqslant 0$, which implies $\pi_i^h(\overline{p}_1) = 0$ and therefore $\widetilde{p}_i = \underline{p} = 0$. □

Lemma 4 relates the iso-profit and hyper-competitive prices to a firm's profit. Part *(i)* of Lemma 4 states that, given that the largest supplier sets its iso-profit price, all its rivals can profitably raise their price from the market-clearing price $\underline{p}$ to some higher price weakly below firm 1's iso-profit price. Part *(ii)* complements this by showing that if firm 1 prices below $\overline{p}_1$ and any highest priced seller i other than firm 1 prices at $\overline{p}_1$, then the latter can myopically improve by reducing its price from $\overline{p}_1$ to a price below the market-clearing price.

Lemma 4. *Suppose there is no pure-strategy Nash equilibrium. For all $i \in N \backslash \{1\}$:*

(i) *If $p_1 = \overline{p}_1$ and $p_i \in (\underline{p}, \overline{p}_1)$, then $\pi_i(p_i) = p_i k_i > \pi_i(\underline{p})$.*
(ii) *If $\underline{p} > 0$ and $p_i \in (\widetilde{p}_i, \underline{p}]$, then $\pi_i(p_i) = p_i k_i > \pi_i^h(\overline{p}_1)$.*

Proof of Lemma 4.

(i): Suppose that $p_1 = \overline{p}_1$. We can distinguish two cases: either $\underline{p} = 0$ or $\underline{p} > 0$.
If $\underline{p} = 0$, then firm 1 makes zero profit at $\overline{p}_1 > 0$ and, by definition of the iso-profit price, it holds that $D(\overline{p}_1) - K_{-1} = 0$. Now consider some firm $i \in N \backslash \{1\}$ and suppose that it is pricing at $p_i < \overline{p}_1$. This firm faces the following demand:

$$D_i(p_i, p_{-i}) = \frac{k_i}{\sum_{j \in \Omega(p_i, p_{-i})} k_j} \left(D(p_i) - \sum_{j \in \Delta(p_i, p_{-i})} k_j \right),$$

which is larger than k_i as:

$$\frac{k_i}{\sum_{j \in \Omega(p_i, p_{-i})} k_j} \left(D(p_i) - \sum_{j \in \Delta(p_i, p_{-i})} k_j \right) > k_i \iff D(p_i) > \sum_{j \in \Omega(p_i, p_{-i})} k_j + \sum_{j \in \Delta(p_i, p_{-i})} k_j,$$

which holds since $D(p_i) > D(\overline{p}_1)$ and $\sum_{j \in \Omega(p_i, p_{-i})} k_j + \sum_{j \in \Delta(p_i, p_{-i})} k_j \leqslant K_{-1}$. Hence, a firm pricing below $\overline{p}_1$ is capacity-constrained, which implies that its profits monotonically increase over the range $(\underline{p}, \overline{p}_1)$.
If $\underline{p} > 0$, then firm 1 makes positive profits at $\overline{p}_1$ and, therefore, it holds that $D(\overline{p}_1) - K_{-1} > 0$. Since firm 1 faces residual demand at its iso-profit price, all lower-priced firms are capacity-constrained. Hence, for each firm other than 1, profits monotonically increase over the range $(\underline{p}, \overline{p}_1)$.

(ii): By definition, $\widetilde{p}_i = \min\{p_i \in P_i | \pi_i(p_i) = \pi_i^h(\overline{p}_1)\}$. By Lemma 1, we know that if $0 < p_i \leqslant \underline{p}$, then $\pi_i = p_i k_i$. Consequently, since π_i is strictly increasing in p_i over the range $(\tilde{p}_i, \underline{p}]$ it holds that $\pi_i(p_i) > \pi_i^h(\overline{p}_1)$ for $p_i \in (\tilde{p}_i, \underline{p}]$. □

The next two results, Lemma 5 and Lemma 6, are used in the proof of Theorem 2.

Lemma 5. *If $p \in M$ and $p' \in f^\infty(p)$, then $p' \in M$.*

Proof of Lemma 5. Toward a contradiction, suppose that $p' \notin M$ when $p'^\infty(p)$ and $p \in M$. Since M is closed, there is a $\delta > 0$ such that $\mathcal{B}_\delta(p') \cap M = \emptyset$, where $\mathcal{B}_\delta$ is the open ball with radius δ. Furthermore, by the definition of f^∞, there is a $\kappa \in \mathbb{N}$ and a $p^\kappa \in f^\kappa(p)$ such that $p^\kappa \in \mathcal{B}_\delta(p')$, which means $p^\kappa \notin M$. Since $p^\kappa \in f^\kappa(p)$, there is a sequence $p^0, p^1, ..., p^\kappa$ of length κ such that

$$p^0 = p, p^1 \in f(p^0), ..., p^\kappa \in f(p^{\kappa-1}).$$

Let $\kappa' \in \{1, ..., \kappa\}$ be such that $p^{\kappa'}$ is the first element in this sequence with the property that $p^{\kappa'} \notin M$. Hence, $p^{\kappa'-1} \in M$ and $p^{\kappa'} \in f(p^{\kappa'-1})$, which violates deterrence of external deviations of M. Consequently, $p' \in M$. □

Lemma 6. *Let $\pi_i^s(p_i)$ denote the profit of firm $i \in N$ when at least one other firm $j \neq i$ is charging the same price p_i. Then, it holds that $\pi_i^s(p_i) > \pi_i^h(p_i)$ and $\pi_i^\ell(p_i) > \pi_i^h(p_i)$ for any $p_i \in \left(\underline{p}, \alpha\right)$.*

Proof of Lemma 6. Let us start by showing that:

$$\pi_i^s(p_i) > \pi_i^h(p_i).$$

We can distinguish two cases: (1) the firms charging the same price are capacity-constrained, or (2) the firms charging the same price are not capacity-constrained. Note that in either case a single highest-priced firm is not capacity-constrained, because with $p_i \in \left(\underline{p}, \alpha\right)$ it holds that $K > D(p_i)$. As to (1), we have that

$$\pi_i^s(p_i) > \pi_i^h(p_i) \iff p_i k_i > p_i \left[D(p_i) - \sum_{j \in \Delta(p_i, p_{-i})} k_j \right] \iff K > D(p_i),$$

which holds. As to (2), we have that

$$\pi_i^s(p_i) > \pi_i^h(p_i) \iff p_i \frac{k_i}{\sum_{j \in \Omega(p_i, p_{-i})} k_j} \left[D(p_i) - \sum_{j \in \Delta(p_i, p_{-i})} k_j \right] > p_i \left[D(p_i) - \sum_{j \in \Delta'(p_i, p_{-i})} k_j \right],$$

with $\Delta'(p_i, p_{-i}) \neq \Delta(p_i, p_{-i})$. This is equivalent to

$$\frac{k_i}{\sum_{j\in\Omega(p_i,p_{-i})}k_j}\left[D(p_i)-\sum_{j\in\Delta(p_i,p_{-i})}k_j-\sum_{j\in\Omega(p_i,p_{-i})}k_j+\sum_{j\in\Omega(p_i,p_{-i})}k_j\right]>D(p_i)-\sum_{j\in\Delta'(p_i,p_{-i})}k_j,$$

or

$$\frac{k_i}{\sum_{j\in\Omega(p_i,p_{-i})}k_j}\left[D(p_i)-K+\sum_{j\in\Omega(p_i,p_{-i})}k_j\right]>D(p_i)-\sum_{j\in\Delta'(p_i,p_{-i})}k_j.$$

Rearranging gives:

$$k_i\left[\frac{D(p_i)-K}{\sum_{j\in\Omega(p_i,p_{-i})}k_j}\right]>D(p_i)-\sum_{j\in\Delta'(p_i,p_{-i})}k_j-k_i\Longleftrightarrow k_i\left[\frac{D(p_i)-K}{\sum_{j\in\Omega(p_i,p_{-i})}k_j}\right]>D(p_i)-K.$$

Since $D(p_i)<K$, the above simplifies to

$$\frac{k_i}{\sum_{j\in\Omega(p_i,p_{-i})}k_j}<1,$$

which holds. We conclude that $\pi_i^s\left(p_i\right)>\pi_i^h\left(p_i\right)$ when $p_i\in\left(\underline{p},\alpha\right)$.

Let us now show that:

$$\pi_i^{\ell}\left(p_i\right)>\pi_i^h\left(p_i\right).$$

We can again distinguish two cases: (1) the firm charging the lowest price is capacity-constrained, or (2) the firm charging the lowest price is not capacity-constrained. As to (1), the story is the same as above. As to (2), we have that

$$\pi_i^{\ell}\left(p_i\right)>\pi_i^h\left(p_i\right)\Longleftrightarrow p_iD\left(p_i\right)>p_i\left[D(p_i)-\sum_{j\in\Delta(p_i,p_{-i})}k_j\right],$$

which holds because $\sum_{j\in\Delta(p_i,p_{-i})}k_j>0$. We conclude that $\pi_i^{\ell}\left(p_i\right)>\pi_i^h\left(p_i\right)$ when $p_i\in\left(\underline{p},\alpha\right)$. □

Theorem 2. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is empty, then M as given in (1) is the unique Myopic Stable Set.*

Proof of Theorem 2. First note that the set M is closed by definition. In the following, we show that the set M also satisfies *deterrence of external deviations*, *asymptotic external stability* and *minimality*. Finally, we prove that M is unique.

Deterrence of External Deviations: Let $p\in M$ be some price profile in M. We show that there is no profitable deviation to a price profile in $P\backslash M$. Take any firm $i\in N$ and suppose that it sets the strictly highest price. We can distinguish two cases: $D(0)>K_{-i}$ or $D(0)\leqslant K_{-i}$. In case of $D(0)>K_{-i}$, firm i faces residual demand or it does not. If it does not, then it also faces no demand at higher prices. Hence, $\pi_i^h(\overline{p}_1)=0$ and, therefore, $\tilde{p}_i=0$. Moreover, since $\pi_i^h(\overline{p}_1)=0$, its profits are zero at all higher prices too. In this case, therefore, it has no profitable deviation to a price profile not in M. Now suppose it does face residual demand. Then, by strict concavity of π_i^h, its profit is lowest in M at $\overline{p}_1$. Indeed, by Lemma 1 and Lemma 6, since $\pi_i^{\ell}(p_i)\geqslant\pi_i^h(p_i)$ and $\pi_i^s(p_i)\geqslant\pi_i^h(p_i)$, for all $p_i\in[\widetilde{p}_i,\overline{p}_1]$, there is no other price in the set M giving a lower profit. By definition of $\widetilde{p}_i$, $\pi_i^h(\overline{p}_1)$ is the same as when it would charge $\widetilde{p}_i$. Thus, whenever $D(0)>K_{-i}$ and firm i faces residual demand, the lowest profit is obtained either at $\widetilde{p}_i$ or at $\overline{p}_1$ given that it is the highest-priced firm in the market. Note that if a firm i unilaterally deviates to a $p'\in P\backslash M$, it must be either that (a) $p_i'<\widetilde{p}_i$, or (b) $p_i'>\overline{p}_1$. Consider first case (a). Then, by Lemma 1, this firm will obtain $\pi_i(p_i')\equiv p_i'\cdot k_i<\widetilde{p}_i\cdot k_i\equiv\pi_i(\widetilde{p}_i)$ and, therefore, reducing its price below $\widetilde{p}_i$ is not an improvement. Now consider case (b). If firm i deviates with a $p_i'>\overline{p}_1$, it holds that $\pi_i^h(p_i')<\pi_i^h(\overline{p}_1)$ and, therefore, for any price $p_i'>\overline{p}_1$ such a deviation is not an improvement either. Finally, consider the case $D(0)\leqslant K_{-i}$. By Lemma 3, we have that $\widetilde{p}_i=0$. Then, the only possible deviation to $P\backslash M$ for firm i is to some $p_i'>\overline{p}_1$. However, profits are zero at all prices in excess of $\overline{p}_1$ so that firm i has no profitable deviation to such a price.

Note that all other firms earn weakly higher profits. This trivially holds if the highest-priced firm faces no residual demand. If it does face residual demand, then all lower-priced firms are capacity-constrained. The minimum profit of a lower-priced firm is then obtained when it prices at its hyper-competitive price. This, by definition, gives it the same profit as when it would be the strictly highest-priced seller pricing at $\overline{p}_1$. By the preceding logic, such a firm can therefore not improve itself by raising its price above $\overline{p}_1$. Finally, note that a firm's profit does not decrease when its competitors match its price or when it becomes the lowest-priced firm in the industry (Lemma 6). Consequently, none of the firms has an incentive to induce a price profile outside M.

Asymptotic External Stability: Consider a price profile $p\in P\backslash M$. We show that there exists a $p'\in M$ such that $p'\in f^{\mathbb{N}}(p)$. To begin, notice that if $p\in P\backslash M$, then there is at least one firm pricing below its hyper-competitive price $\widetilde{p}_i$ or above firm 1's iso-profit price $\overline{p}_1$.

Let $L(\widetilde{p}_i) = \{i \in N | p_i < \widetilde{p}_i\}$ be the set of sellers who are pricing below their hyper-competitive price and let $H(\overline{p}_1) = \{i \in N | p_i > \overline{p}_1\}$ be the set of sellers who price above firm 1's iso-profit price. Moreover, let $\lambda \geqslant 0$ and $\eta \geqslant 0$ denote the cardinality of $L(\widetilde{p}_i)$ and $H(\overline{p}_1)$, respectively.

Step 1: If $L(\widetilde{p}_i) = \emptyset$, then $H(\overline{p}_1) \neq \emptyset$; In this case, proceed with Step 2. If $L(\widetilde{p}_i) \neq \emptyset$ then, for each firm $i \in L(\widetilde{p}_i)$, $p_i < \widetilde{p}_i \leqslant \underline{p}$, so that $\pi_i(p_i, p_{-i}) = p_i k_i$ by Lemma 1. This implies that each firm $i \in L(\widetilde{p}_i)$ can profitably deviate to the market-clearing price $\underline{p} \in M$, which induces a sequence of price profiles:

$$p = p^0,\ p^1 \in f\left(p^0\right),\ p^2 \in f\left(p^1\right), \ldots, p^\lambda \in f\left(p^{\lambda-1}\right).$$

If $p^\lambda \in M$, then the asymptotic external stability condition is met. If $p^\lambda \notin M$, then proceed with Step 2.

Step 2: Let p^λ, with $\lambda \geqslant 0$, be the price profile resulting from Step 1. By construction, it holds that $L(\widetilde{p}_i) = \emptyset$ and $H(\overline{p}_1) \neq \emptyset$. Suppose $D(0) > K$ which implies that $\underline{p} > 0$. First, recall that $\widetilde{p}_i$ is the price solving $\pi_i(\widetilde{p}_i) = \pi_i^h(\overline{p}_1)$, for all $i \in N$. This implies $\pi_i^h(p_i) \leqslant \pi_i^h(\overline{p}_1)$ when $p_i \geqslant \overline{p}_1$, because $\pi_i^h(p_i)$ is strictly concave. Next, denote by $h(p^\lambda) \in H(\overline{p}_1)$ the firm charging the highest price at p^λ. Since $p_h^\lambda > \overline{p}_1$ by construction and following the previous observations in combination with Lemma 1, firm $h(p^\lambda)$ has a profitable deviation to $\underline{p}$. This induces a new price profile $p^{\lambda+1}$ in which case either all firms price weakly below $\overline{p}_1$, or there are still one or more firms pricing above $\overline{p}_1$. In case of the former, the asymptotic external stability condition is met, whereas in case of the latter we can repeat the argument. Denote by $h(p^{\lambda+1})$ the firm charging the highest price at $p^{\lambda+1}$. This firm has a profitable deviation to $\underline{p} > \widetilde{p}_{h(p^{\lambda+1})}$. Extending the above logic delivers a sequence:

$$p^{\lambda+1} \in f\left(p^\lambda\right),\ p^{\lambda+2} \in f\left(p^{\lambda+1}\right), \ldots, p^{\lambda+\eta} \in f\left(p^{\lambda+\eta-1}\right).$$

Hence, by construction, $p^{\lambda+\eta} = p' \in M$, and, therefore, the asymptotic external stability condition is met.

Finally, suppose that $D(0) \leqslant K$ so that $\underline{p} = 0$. Consequently, $K_{-1} < D(0)$ for otherwise all firms pricing at zero would constitute a pure-strategy Nash equilibrium. Now consider some price profile $p \notin M$ with at least one firm pricing above $\overline{p}_1$. If p is such that firm 1 prices at zero, then let this firm raise its price to p_1^*, which is profitable and in the set M, because $p_1^* < \overline{p}_1$. Note that since $K_{-1} - k_i < D(0)$ for any firm i other than firm 1, each firm pricing at zero can profitably raise its price to p_1^*. This results in a price profile with all firms strictly pricing above zero.

Next, consider the highest price in the market. If there are two or more firms charging the highest price, say p_i, then they are not capacity-constrained since $D\left(p_i\right) - \sum_{j \in \Delta\left(p_i, p_{-i}\right)} k_j < \sum_{j \in \Omega\left(p_i, p_{-i}\right)} k_j$, which is equivalent to $K > D\left(p_i\right)$ and this holds, because $K \geqslant D(0)$. If their profits are positive, then there is a myopic improvement by cutting their price slightly since this gives a discrete increase in demand. This yields a situation in which one firm charges the strictly highest price. As $D\left(\overline{p}_1\right) - K_{-1} = 0$ in this case, we have that $D\left(\overline{p}_1\right) - K_{-i} \leqslant 0$ for each firm i other than 1 and, therefore, $D\left(p_i'\right) - K_{-i} < 0$ for any $p_i' > \overline{p}_1$ and $i \in N$. Hence, this single highest-priced firm receives zero profits and, hence, has a profitable deviation to p_1^*.

Minimality: Toward a contradiction, suppose that there exists a closed set $M' \subsetneq M$ satisfying *deterrence of external deviations* and *asymptotic external stability*. We distinguish two cases: either the market-clearing price profile $\underline{p} \in M'$, or the market-clearing price profile $\underline{p} \notin M'$.

Case 1: Suppose that $\underline{p} \in M'$. Note that at $\underline{p}$, if a firm i has a positive residual demand, then, by concavity of the residual profit functions, it has a profitable deviation to any p_i with $\underline{p} < p_i < \overline{p}_i$. Note that this always holds for firm 1 (Lemma 2). Recall also that, by Lemma 2 we have that $\overline{p}_1 \geqslant \ldots \geqslant \overline{p}_n$. It follows that, from $\underline{p}$, the largest price interval is effectively determined by firm 1. Thus, by the property of *deterrence of external deviations* of M' and the fact that $\underline{p} \in M'$, the following price profiles are contained in M':

$$M_1' = \{p \in P | \underline{p} \leqslant p_1 < \overline{p}_1, p_j = \underline{p}, \forall j \neq 1\} \subseteq M'.$$

Moreover, as firm 1 can charge a price in M' arbitrarily close to $\overline{p}_1$, then, by Lemma 5, the following price profiles are also contained in M':

$$M_1' \subset M_2' = \{p \in P | \underline{p} \leqslant p_1 \leqslant \overline{p}_1, p_j = \underline{p}, \forall j \neq 1\} \subseteq M'.$$

Now fix $p_1 = \overline{p}_1$ in M_2'. Then, by Lemma 4, each firm i other than firm 1 has a profitable deviation to any p_i between the market-clearing price and firm 1's iso-profit price. Moreover, such a firm i can charge a price in M' arbitrarily close to firm 1's iso-profit price. It then follows from the *deterrence of external deviations* of M' that:

$$M_3' = M_2' \cup \{p \in P | p_1 = \overline{p}_1,\ \ \underline{p} \leqslant p_i \leqslant \overline{p}_1, \forall i \neq 1\} \subseteq M'.$$

Next, fix some $p \in M_3'$ with $p_1 = \overline{p}_1$ and all firms other than firm 1 pricing below it. Then, by concavity of the residual profit function and Lemma 6, firm 1 has a profitable deviation to any price $p_1' \in (\underline{p}, \overline{p}_1)$. It then follows from the *deterrence of external deviations* of M', and the fact that firm 1 can pick a price arbitrarily close to $\underline{p}$, that the following price profiles are also contained in M'.

$$M_3' \subset M_4' = \{p \in P | \underline{p} \leqslant p_i \leqslant \overline{p}_1, \forall i \in N\} \subseteq M'.$$

Next, fix some $p \in M_4'$ with $p_i = \overline{p}_1$ for some $i \neq 1$ and $p_1 < \overline{p}_1$. In this case, this firm i has a profitable deviation to some price $p_i' \in (\tilde{p}_i, \underline{p}]$ (Lemma 4). Since the choice of firm i is arbitrary and M' satisfies *deterrence of external deviations*, it follows that:

$$M_4' \subset M_5' = \{p \in P | \tilde{p}_i < p_i \leqslant \overline{p}_1, \forall i \in N\} \subseteq M'.$$

Finally, note that since firm i can charge a price in M' arbitrarily close to $\widetilde{p}_i$, it follows from Lemma 5 that:

$$M_5' \subset M_6' = \{p \in P | \tilde{p}_i \leqslant p_i \leqslant \overline{p}_1, \forall i \in N\} \subseteq M',$$

and, therefore, $M_6' = M \subseteq M'$, a contradiction.

Case 2: Suppose now that the price profile $\underline{p} \notin M'$. Fix some $p \in M'$. We show that $\underline{p} \in f^{\infty}(p)$. It then follows from Lemma 5 that $\underline{p} \in M'$, a contradiction.

Step 1: Let $L(\underline{p}) = \{i \in N | p_i < \underline{p}\}$ be the set of sellers who are pricing below the market clearing price. Either $L(\underline{p})$ is empty or not. In case of the former proceed with Step 2. In case of the latter, note that each firm pricing below the market-clearing price has a profitable deviation to $\underline{p}$ (Lemma 1). By letting each firm to deviate we induce a new price profile $p'^{\lambda}(p)$ for some $\lambda > 0$, in which each firm prices weakly above the market-clearing price. Notice that, since M' satisfies *deterrence of external deviations*, it must hold that p''. If $p' = \underline{p}$, then we have a contradiction concluding the proof. Otherwise proceed with *Step 2*.

Step 2: Let p be the price profile resulting from Step 1. Thus p is such that all firms weakly price above $\underline{p}$ with at least one seller pricing strictly above $\underline{p}$. In this second step, we show there exists a path of myopic improvements such that the resulting price profile consists of two prices. Specifically, this price profile has one or more firms charging the highest price p_h and one or more firms charging the lowest price $p_h - \epsilon$, with ϵ positive and sufficiently small.
To begin, consider a lowest priced firm l. If this firm is not capacity-constrained, then all higher priced sellers face zero residual demand. If $p_l = \underline{p}$, then all higher priced sellers can profitably match p_l in turn. This, induces a new price profile $\underline{p} \in f^{\mu}(p)$ for some $\mu > 0$, By the μ-fold iteration of *deterrence of external deviations* we have that $\underline{p} \in M'$, a contradiction concluding the proof. If $p_l > \underline{p}$, then each higher priced firm can profitably deviate to a price $p_l - \epsilon \geqslant \underline{p}$ for some $\epsilon > 0$. This results in a price profile with two prices: p_l and $p_l - \epsilon$.
Next, suppose that there is a single lowest priced seller who is capacity-constrained. In this case, we can distinguish two scenarios. Either, (i) it raises its price till p_l' for which $k_l = D\left(p_l'\right)$. That is, to the lowest price for which its capacity is non-binding, while remaining the lowest priced firm in the market, or (ii) it raises its price till it matches the price of the second lowest priced firm(s). We study the two cases separately.

Case (i): In this case, we are back in the first situation where none of the higher priced firms faces residual demand. Hence, they can profitably lower their price to $p_l' - \epsilon$. Again, the result is a price profile with two prices: p_l' and $p_l' - \epsilon$.

Case (ii): Let firm i be one of the lowest priced firms. Either, (iia) firm i raises its price until it is no longer capacity-constrained, or (iib) firm i matches the next lowest prices firm. We consider the two possibilities separately.

(iia): Firm i raises its price till p_l'' for which it holds that $D\left(p_l''\right) - \sum_{j \in \Delta\left(p_l'', p_{-l}''\right)} k_j = k_i$. This implies that all firms pricing below p_l'' can also profitably raise their price till p_l'', whereas all firms pricing above p_l'' face no residual demand. Hence, these higher priced firms can myopically improve by lowering their price to $p_l'' - \epsilon$, which again results in a price profile with two prices: p_l'' and $p_l'' - \epsilon$.

(iib): Firm i raises its price to the price of the next lowest priced firm(s) in which case all lower priced firms can do the same since they are capacity-constrained. This brings us back either to the situation described under (iia) above or iterate (iib) until the highest priced firms still face residual demand, in which case lower priced firms can raise their price till $p_h - \epsilon$, where p_h is the highest price in the market.

Step 3: Let $p \in M'$ be the price profile resulting from the previous steps. Note that, by construction, at p there are two groups of firms: the highest priced firms $H(p)$ charging p_h and the lowest priced firms $L(p)$ charging $p_l = p_h - \epsilon$. According to Step 2 we have two cases: either (i) the lowest priced firms are not capacity-constrained or (ii) the lowest priced firms are capacity-constrained. Let us consider the two cases separately.

Case (i): Since we are in the case such that the lowest priced firms are not capacity-constrained then the residual demand of the highest priced firm(s) is zero and so its profit. Therefore, each highest priced firm has a profitable deviation to a price $p_l - \epsilon$ for an arbitrarily $\epsilon > 0$. Such a deviation is profitable by the fact that $\pi_i(p_h) = 0 < \pi_i(p_i, p_{-i})$ for any $p_i \in (\underline{p}, p_l)$, which is the case.
By letting each highest priced firm $h \in H(p)$ to deviate we induce a new price profile $p'^{q}(p)$ for some $q > 0$. Such a price profile is also characterized by two groups of firms: the highest priced firms $H(p')$ charging p_h' and the lowest priced firms $L(p')$ charging p_l'. If the lowest priced firms are capacity-constrained, then move to Case (ii), otherwise case (i) applies again.

Case (ii): Since we are in the case such that the lowest priced firms are capacity-constrained then the residual demand of the highest priced firm(s) is positive and so its profit.
At p_h, an highest priced firm h has a profitable deviation undercutting $p_l = p_h - \epsilon$ for some $\epsilon > 0$ when

$$\pi_h(p_h - 2\epsilon) > \pi_h(p_h) \Longleftrightarrow$$

$$k_h(p_h - 2\epsilon) > \frac{k_h}{\sum_{j\in\Omega(p_h,p_{-h})} k_j}\left[D(p_h) - \sum_{j\in\Delta(p_h,p_{-h})} k_j\right] p_n \Longleftrightarrow$$

$$k_h p_h - \frac{k_h}{\sum_{j\in\Omega(p_h,p_{-h})} k_j}\left[D(p_h) - \sum_{j\in\Delta(p_h,p_{-h})} k_j\right] p_n > 2k_h\epsilon \Longleftrightarrow$$

$$p_h\left[1 - \frac{D(p_h) - \sum_{j\in\Delta(p_h)} k_j}{\sum_{j\in\Omega(p_h,p_{-h})} k_j}\right]\frac{1}{2} > \epsilon \Longleftrightarrow$$

Let denote by $\mathbb{A}$ the LHS of the above inequality, i.e.

$$\mathbb{A} \equiv p_h\left[1 - \frac{D(p_h) - \sum_{j\in\Delta(p_h,p_{-h})} k_j}{\sum_{j\in\Omega(p_h,p_{-h})} k_j}\right]\frac{1}{2}.$$

Note that $\mathbb{A} > 0$ when

$$1 - \frac{D(p_h) > \sum_{j\in\Delta(p_h,p_{-h})} k_j}{\sum_{j\in\Omega(p_h,p_{-h})} k_j} \Longleftrightarrow \sum_{j\in\Omega(p_h,p_{-h})} k_j > D(p_h) - \sum_{j\in\Delta(p_h,p_{-h})} k_j \Longleftrightarrow$$

$$\sum_{j\in\Omega(p_h,p_{-h})} k_j + \sum_{j\in\Delta(p_h,p_{-h})} k_j = K > D(p_h),$$

which holds since $p_h > \underline{p}$ by Step 1.

Since the choice of ϵ in Step 2 and Step 3 was arbitrary, we conveniently fix $\epsilon \in (0, \mathbb{A})$ such that each highest priced firm in $H(p)$ has a profitable deviation to $p_h - 2\epsilon < p_h - \epsilon$. Note we can let each highest priced firm deviates by the same ϵ. Indeed, every round a highest priced firm deviates and then $\mathbb{A}$ increases implying that ϵ is well defined.

The transition of each highest priced firm induces a new price profile $p'^{\nu}(p)$ for some $\nu > 0$, characterized by two groups of firms: the highest priced firms charging p'_h and the lowest priced firms charging p'_l. Since a lowest priced firm is capacity-constrained, Case (ii) applies again.

Step 4: The iteration of previous steps constitutes a procedure which generates a sequence

$$p = p^1 \in f(p^0), p^2 \in f(p^1), ..., p^\kappa \in f(p^{\kappa-1}).$$

By construction, for all $\epsilon > 0$ there exists a $\kappa > 0$ such that $||p^\kappa - \underline{p}|| < \epsilon$. Then, by definition of f^∞, it holds that $\underline{p} \in f^\infty(p)$. By Lemma 5, it follows that $\underline{p} \in M'$, a contradiction.

Uniqueness: Finally, we show that M is the unique MSS. By contrast, let us assume that there is another MSS M'. First, we show that $M \cap M' \neq \emptyset$. Towards a contradiction, suppose that $M \cap M' = \emptyset$. Then, by asymptotic external stability of M', for all $p \in M$ there is $p' \in M'$ such $p' \in f^\infty(p)$. Then, by closedness of M the intersection between the open ball around p' with radius ϵ and M is empty, i.e. $B_\epsilon(p') \cap M = \emptyset$. By definition of f^∞, there is $\kappa \in \mathbb{N}$ and a $p'' \in P$ such that $p'' \in f^\kappa(p)$ and $p'' \in B_\epsilon(p')$. By κ-fold application of *deterrence of external deviations*, it holds that $p'' \in M$, but $p'' \in B_\epsilon(p')$, a contradiction. Thus $M \cap M' \neq \emptyset$. In what follows we prove that $M \subseteq M'$. Equality follows from the minimality of M'. As before, we have that either the market-clearing price belong to the set M' or not.

(1): $\underline{p} \in M'$. Then by Case 1 of the minimality proof, M' contains also $M\backslash\{\underline{p}\}$.
Hence, $M \subseteq M'$.

(2): $\underline{p} \notin M'$. This possibility is ruled out by Case 2 of the minimality proof. □

Proposition 3. *Consider a merger between a subset of firms $S \subset N$ and suppose that $k_S > k_1$.*

(i) *The merger increases the maximum of the Myopic Stable Set M. Moreover, this maximum value is rising with the size of the merger.*
(ii) *If $D(0) < K < D(0) + k_1$, then the merger has no effect on the minimum myopically stable prices.*
(iii) *If $D(p_1^*) < K < D(0)$, then the merger leads to a decrease of the non-merging parties' hyper-competitive prices.*

Proof of Proposition 3. (i) The merger is assumed to become the new industry leader in terms of production capacity. This implies an increase of the highest iso-profit price post-merger. To see this, note that the iso-profit price of the largest firm is given by $\overline{p}_1 = D^{-1}(K_{-1})$. Since total industry capacity K is constant, an increase in k_1 implies an increase in the iso-profit price of the largest seller. We conclude that the merger leads to an increase of the MSS maximum and that this effect is stronger the larger the merger, all else equal.

(ii) If $D(0) < K < D(0) + k_1$, then $\underline{p} = 0$. This also is the minimum of the MSS in this case and, since total industry capacity is unaffected by the merger, remains the minimum.

(iii) If $D(p_1^*) < K < D(0)$, then $\underline{p} > 0$. In this case, the minimum of the MSS is given by $\underline{p}$ for the largest seller(s) and by the hyper-competitive price $\widetilde{p}_j$ for each firm j with strictly less production capacity. Since the merger is assumed to be the single largest supplier, suppose that there is one firm (firm 1) with strictly more capacity than each of its rivals. Using the definitions of iso-profit, hyper-competitive, and market-clearing prices, we can write:

$$\widetilde{p}_i = \frac{\overline{p}_1\left(D(\overline{p}_1) - K_{-i}\right)}{k_i}, \underline{p} = \frac{\overline{p}_1\left(D(\overline{p}_1) - K_{-1}\right)}{k_1} \Longrightarrow \underline{p} \cdot k_1 + \overline{p}_1 K_{-1} = \overline{p}_1 D(\overline{p}_1).$$

Combining gives:

$$\widetilde{p}_i = \frac{\overline{p}_1 D(\overline{p}_1) - \overline{p}_1 K_{-i}}{k_i} = \frac{\underline{p} \cdot k_1 + \overline{p}_1\left(K_{-1} - K_{-i}\right)}{k_i},$$

and, therefore,

$$\widetilde{p}_i(k_1, k_i) = \frac{1}{k_i} \cdot \underline{p} k_1 - \frac{1}{k_i}\overline{p}_1\left(k_1 - k_i\right), \tag{8}$$

for each $i \in N \setminus \{1\}$.

Now consider a merger with joint capacity $k_S > k_1$. Note that this is equivalent to an increase of k_1 assuming that total industry capacity K remains constant. Consequently, one can evaluate the impact of the merger on the MSS by assessing the effect of an increase of k_1. Total differentiation of (8) with respect to k_1 yields:

$$\frac{d\widetilde{p}_i}{dk_1} = \frac{1}{k_i} \cdot \left[\frac{d\underline{p}}{dk_1}k_1 - \frac{d\overline{p}_1}{dk_1}\left(k_1 - k_i\right) - \left(\overline{p}_1 - \underline{p}\right)\right], \tag{9}$$

where, the first term $d\underline{p}/dk_1 = 0$ (since K is constant) while the third is negative since $\underline{p} < \overline{p}_1$. Regarding the second term, note that a small change in k_1 affects the iso-profit price $\overline{p}_1 = \underline{p} \cdot k_1 / \left(D(\overline{p}_1) - K_{-1}\right)$ as follows:

$$\frac{d\overline{p}_1}{dk_1} = \frac{d\left(\frac{\underline{p}\cdot k_1}{D(\overline{p}_1) - K_{-1}}\right)}{dk_1} = \frac{\underline{p}\left[D(\overline{p}_1) - K_{-1}\right] - \left(\frac{dD(\overline{p}_1)}{d\overline{p}_1}\frac{d\overline{p}_1}{dk_1} - \frac{dK_{-1}}{dk_1}\right)\underline{p} \cdot k_1}{\left[D(\overline{p}_1) - K_{-1}\right]^2}.$$

Since $\frac{dK_{-1}}{dk_1} = -1$, this simplifies to

$$\frac{d\overline{p}_1}{dk_1} = \frac{\underline{p}\left[D(\overline{p}_1) - K\right]}{\left[D(\overline{p}_1) - K_{-1}\right]^2 + \frac{dD(\overline{p}_1)}{d\overline{p}_1}\underline{p} \cdot k_1} > 0.$$

To see that it is positive, note that the numerator is negative. The denominator is also negative when

$$\left[D(\overline{p}_1) - K_{-1}\right]^2 < -\frac{dD(\overline{p}_1)}{dp_1}\underline{p} \cdot k_1, \tag{10}$$

which is equivalent to

$$\frac{d\left[D(\overline{p}_1) - K_{-1}\right]}{dp_1}\frac{\overline{p}_1}{D(\overline{p}_1) - K_{-1}} < -1, \tag{11}$$

and this inequality holds because residual demand is elastic at $\overline{p}_1$. Hence, the second term is also negative so that $\frac{d\widetilde{p}_i}{dk_1} < 0$. That is, the merger leads to a decrease of the hyper-competitive prices of non-merging parties. □

Proposition 4. *Consider a merger between a subset $S \subset N$ of firms and suppose that $k_S < k_1$.*

(i) *The merger has no effect on the maximum of the Myopic Stable Set M.*
(ii) *If $D(0) < K < D(0) + k_1$, then the merger has no effect on the minimum myopically stable prices.*
(iii) *If $D(p_1^*) < K < D(0)$, then the merger's hyper-competitive price exceeds the pre-merger hyper-competitive prices of the merging parties.*

Proof of Proposition 4. (i) By assumption, the merger does not become the largest seller in the industry. Hence, the maximum of the MSS remains unaffected.

(ii) If $D(0) < K < D(0) + k_1$, then $\underline{p} = 0$. This also is the minimum of the MSS in this case and, since total industry capacity is unaffected by the merger, remains the minimum.

(iii) If $D(p_1^*) < K < D(0)$, then $\underline{p} > 0$. In this case, all myopic stable prices are in the interval: $\max\left\{\widetilde{p}_i, 0\right\} \leqslant p_i \leqslant \overline{p}_1$. The hyper-competitive prices of the merging parties are replaced by a single, higher, hyper-competitive price post-merger. To see this, consider the effect of an increase in capacity of a merging firm i (leaving K constant):

$$\frac{d\widetilde{p}_i}{dk_i} = \frac{k_1}{k_i}\left(\frac{d\underline{p}}{dk_i} - \frac{d\overline{p}_1}{dk_i} + \frac{\overline{p}_1 - \underline{p}}{k_i}\right) + \frac{d\overline{p}_1}{dk_i} > 0.$$

In this case, it holds that $d\underline{p}/dk_i = 0$, $d\overline{p}_1/dk_i = 0$, $\overline{p}_1 > \underline{p} \geqslant 0$, and, therefore, $d\widetilde{p}_i/dk_i > 0$. □

A.4. Section 5

Proof of Theorem 3. To prove that $\mathcal{K} \subset M$ when the set of pure-strategy Nash equilibria is empty, we show that: *(i)* $\overline{p}_1 > p_i^*$, and *(ii)* $\widehat{p}_i > \tilde{p}_i$, for all $i \in N$.

Case (i): To begin, let us establish that $\overline{p}_1 > p_i^*$. We can distinguish two cases: (1) $\underline{p} > 0$, and (2) $\underline{p} = 0$.

(1) Suppose that $\underline{p} > 0$. If all price at $\underline{p}$, then all produce at capacity. Hence, there is no incentive to cut price. Since the set of pure-strategy Nash equilibria is empty, it then must hold that at least one firm is willing to hike its price. It can be easily verified that when the largest firm does not want to raise its price, none of the firms has an incentive to raise price. Hence, firm 1 has an incentive to hike its price, which implies $\underline{p} < p_1^*$. Moreover, by strict concavity of π_1^h, firm 1's residual profit function is increasing up to the (unique) profit-maximizing price and decreasing at prices in excess of p_1^* until its contingent demand is zero. Since firm 1's profits are positive at $\underline{p} > 0$, this implies that $\overline{p}_1$ is on the decreasing part of the residual profit function and therefore that $\overline{p}_1 > p_1^*$. Finally, by strict concavity of π_i^h, it holds that $p_1^* \geqslant p_i^*$, for all $i \in N \setminus \{1\}$, so that $\overline{p}_1 > p_i^*$.

(2) Now suppose that $\underline{p} = 0$. Since the set of pure-strategy Nash equilibria is empty it holds that $K_{-1} < D(0)$. Hence, firm 1 faces a strictly concave residual profit function with a unique maximizer p_1^*. Because it receives zero profits at $\underline{p} = 0$, it follows that $\overline{p}_1 = D^{-1}(K_{-1}) > p_1^*$. Moreover, following the same logic as under (1) above, $p_1^* \geqslant p_i^*$ and therefore $\overline{p}_1 > p_i^*$, for all $i \in N \setminus \{1\}$.

Case (ii): Let us now turn to the lower bound and show that $\widehat{p}_i > \tilde{p}_i$, for all $i \in N$. We again distinguish two cases: (1) $\underline{p} > 0$, and (2) $\underline{p} = 0$.

(1) Suppose that $\underline{p} > 0$. Since each firm can sell its entire capacity at $\underline{p}$, all prices below $\underline{p}$ are strictly dominated. As $\tilde{p}_i < \underline{p}$ for all firms strictly smaller than firm 1, none of these firms puts mass on its hyper-competitive price (or any lower price). Regarding the largest firm(s), recall that they have an incentive to raise their price above $\underline{p}$ in this case, which means $\pi_i^h(p_i^*) > \pi_i^h(\underline{p})$. Consequently, none of the largest sellers puts mass on prices weakly below $\underline{p}$, which is their hyper-competitive price.

(2) Now suppose that $\underline{p} = 0$. Since $K_{-1} < D(0)$, firm 1 can guarantee itself a strictly positive profit independent of the prices set by its competitors. Hence, firm 1 puts zero mass on $\underline{p} = 0$ in equilibrium. Yet, given that firm 1 prices strictly above 0, the same logic applies to firm 2. That is, this firm can guarantee itself a strictly positive profit by pricing below but arbitrarily close to the minimum of firm 1's mixed-strategy support. Consequently, firm 2 also puts zero mass on $\underline{p} = 0$ in equilibrium. This iterative domination argument can be repeated till firm n. We conclude that when $\underline{p} = 0$, none of the firms puts mass on 0 in a mixed-strategy Nash equilibrium.

Taken together, $\widehat{p}_i > \tilde{p}_i$, for all $i \in N$, and therefore $\mathcal{K} \subset M$. □

Appendix B. Myopic stable set and better-reply dynamics

In this appendix, we explore the relation between the MSS in the capacity-constrained price-setting game G and *better-reply dynamics* as analyzed by Friedman and Mezzetti (2001) and Dindõs and Mezzetti (2006). In these papers, the better-reply dynamics results from a dynamic stochastic process in which better responses are selected with positive probability. The ensuing analysis builds on Theorem 3.16 of Demuynck et al. (2019a), which establishes the equivalence between the MSS and the set of all recurrent classes of a dynamic process under the assumption that the state space is finite. For this reason, we here consider a discretized version of the capacity-constrained pricing game G, denoted G^ϵ. To facilitate the comparison, we furthermore compactify the firms' strategy sets by imposing a common maximum price $\alpha > 0$, *i.e.*, $P_i = [0, \alpha], \forall i \in N$. We show that when the set of pure-strategy Nash equilibria of the game G^ϵ (denoted NE^ϵ) is nonempty, then the MSS in G can be interpreted as the asymptotic outcome of the better-reply dynamics in G^ϵ. Assuming identical firms, we show a similar result in case NE^ϵ is empty.

Let us start with specifying the discretized version of our game. Every firm $i \in N$ simultaneously selects a price from its strategy set P_i^ϵ, which is an ϵ-fine partition of P_i, with $0 = p_{i,1} < p_{i,2} < \ldots < p_{i,H} = \alpha$ and $|p_{i,h} - p_{i,h-1}| < \epsilon$ for $\epsilon > 0$ and $h = 2, \ldots, H$. The set of all price profiles is thus given by $P^\epsilon = \bigtimes_{i \in N} P_i^\epsilon$. The discretized version of our game is then $G^\epsilon = \langle N, (P_i^\epsilon, \pi_i^\epsilon)_{i \in N} \rangle$, where π_i^ϵ is a restriction of the profit function π_i to P_i^ϵ. It is assumed that all firms can set the market-clearing price and firm 1's iso-profit price, *i.e.*, $\underline{p}, \overline{p}_1 \in P_i^\epsilon$, $\forall i \in N$. The results are derived for ϵ (sufficiently) small.

We use the notion of better-reply dynamics as introduced by Friedman and Mezzetti (2001). Specifically, in every period t, $t = 0, 1, 2, \ldots$, there is a *status quo* price profile $p \in P^\epsilon$. A firm $i \in N$ is then randomly selected with positive probability and samples a price $p_i^E \in P_i^\epsilon \setminus \{p_i\}$. The available $|P_i^\epsilon| - 1$ samples are drawn independently with probability Q. Firm i changes the status quo to $p^E = (p_i^E, p_{-i})$ if, and only if, $\pi_i(p^E) > \pi_i(p)$.

Note that this better-reply dynamics effectively generates a Markov process on P^ϵ. Let $\mathbb{P}(p, p^E; Q)$ be the transition probability from state p to state p^E, and let $\mathbb{P}(Q)$ be the associated Markov transition matrix. A state is said to be *recurrent* when, any time the industry leaves this state, it re-emerges with probability one. A set $\mathbb{C} \subseteq P^\epsilon$ is a *recurrent class* of $\mathbb{P}(Q)$ when all states in the class are recurrent. Let $\mathbb{CC}$ be the union of all recurrent classes. A singleton recurrent class is called an *absorbing state*. Let $\mathbb{A}$ be the union of all absorbing states. A Markov process is *absorbing* when an absorbing state emerges from each non-absorbing state in which case $\mathbb{CC} = \mathbb{A}$.

In the following, let f_ϵ be the restriction of f to P^ϵ, which implies $f_\epsilon(p) \subseteq f(p)$, for all $p \in P^\epsilon$. Using the terminology in Demuynck et al. (2019a), notice that f_ϵ is *consistent* with $\mathbb{P}(Q)$ since, by construction, it holds that for every p, p', if $p' \in f(p) \setminus \{p\}$, then $\mathbb{P}(p, p'; Q) > 0$ and if $p' \notin f(p) \setminus \{p\}$, then $\mathbb{P}(p, p'; Q) = 0$. Together with the fact that the strategy spaces of G^ϵ are finite, this implies that the premises of Theorem 3.16 in Demuynck et al. (2019a) hold. This yields the following conclusion:

Result: The game G^ϵ has a unique MSS that is given by the union of all recurrent classes that emerge from the better-reply dynamics.

In light of this result, we now proceed with exploring the relation between the MSS in the game G^ϵ and the MSS in the original game G. As a starter, note that when $p \in P$ is a Nash equilibrium of G and $p \in P^\epsilon$, then p is a Nash equilibrium of G^ϵ. The opposite, however, does not generally hold. The next result characterizes the set of pure-strategy Nash equilibria NE^ϵ of G^ϵ. In stating this result, recall that $S \subseteq N$ is a minimal coalition, *i.e.*, $\sum_{j \in S \setminus \{i\}} k_j \geqslant D(0)$, for all $i \in S$.

Proposition 5. *Assume a sufficiently fine price grid. The set of pure-strategy Nash equilibria NE^ϵ of the game G^ϵ coincides with its MSS and it is given by:*

1. $NE^\epsilon = \left\{ p \in P^\epsilon \mid p_i = \underline{p}, \forall i \in N \right\}$ *when* $K \leqslant D(p_1^*)$.
2. $NE^\epsilon = \left\{ p \in P^\epsilon \mid p_i = \epsilon, \forall i \in N \right\}$ *when* $D(0) > K_{-1} \geqslant D(2\epsilon)$.
3. $NE^\epsilon = \left\{ p \in P^\epsilon | p \in \prod_{i \in S} \{0\} \bigtimes \prod_{i \in N \setminus S} P_i^\epsilon, \ \forall S \subseteq N \right\} \cup \left\{ p \in P^\epsilon \mid p_i = \epsilon, \forall i \in N \right\}$ *when* $K_{-1} \geqslant D(0)$.
4. $NE^\epsilon = \emptyset$ *when* $K > D(p_1^*)$ *and* $K_{-1} < D(2\epsilon)$.

Proof of Proposition 5. Part (i) considers the case where capacities are 'small', whereas parts (ii) and (iii) cover the case where capacities are 'large'. Finally, part (iv) shows that there is no pure-strategy equilibrium in the remaining cases. We prove each part in turn:

1. Suppose that $K \leqslant D(p_1^*)$, which implies $\underline{p} \geqslant p_1^* > 0$. We argue that $NE^\epsilon = \left\{ p \in P^\epsilon \mid p_i = \underline{p}, \forall i \in N \right\}$. To begin, suppose that one or more firms price below $\underline{p}$. As $K = D(\underline{p})$ by definition, these firms are capacity-constrained. Each of them can then profitably deviate to $\underline{p}$ as they can still sell at capacity at the higher, market-clearing price. We conclude that in equilibrium there is no firm pricing below $\underline{p}$.
   Next, suppose that one or more firms price above $\underline{p}$ and let the highest price in the industry be given by $p' > \underline{p} + \epsilon$. A highest-priced seller then either faces residual demand or it does not. If it does not, then it can improve itself by cutting price to $\underline{p}$ and sell at capacity. Alternatively, it faces demand in which case all lower-priced firms are capacity-constrained. Note that a highest-priced firm is not capacity-constrained since it prices above the market-clearing price. All lower-priced firms then either price at $p' - \epsilon$ or can improve themselves by doing so since pricing at $p' - \epsilon$ still allows them to sell at capacity. Yet, in that case, a highest-priced seller can profitably undercut its rivals since:
   $$p'(D(p') - K_{-i}) < (p' - 2\epsilon)k_i, \forall i \in N,$$
   for ϵ sufficiently small. Following a similar logic, a highest-priced firm that sets $p' = \underline{p} + \epsilon$ has an incentive to cut its price to the market-clearing level, because:
   $$p'(D(p') - K_{-i}) < (p' - \epsilon)k_i, \forall i \in N,$$
   for ϵ sufficiently small. Finally, in case all firms set the highest price, none of them is capacity-constrained and each of them can improve by undercutting its rivals by the smallest possible amount. We conclude that in equilibrium there is no firm charging a price in excess of the market-clearing price.
   This leaves all firms pricing at $\underline{p}$ as the equilibrium candidate. To see that this is indeed an equilibrium, note that all sell at capacity so that no firm can gain by lowering its price. Moreover, the largest seller has no incentive to hike its price since $\underline{p} \geqslant p_1^*$. As firm 1 has no incentive to deviate, all smaller firms have no incentive to deviate either. We conclude that $NE^\epsilon = \left\{ p \in P^\epsilon \mid p_i = \underline{p}, \forall i \in N \right\}$ when $K \leqslant D(p_1^*)$.
2. Suppose that $D(0) > K_{-1} \geqslant D(2\epsilon)$. We argue that there is a unique pure-strategy equilibrium with all firms pricing at ϵ. First, suppose that all firms price at zero. Since $K_{-1} < D(0)$, firm 1 can set a price above zero and make strictly positive profits. If it does hike its price, then firm 2 can hike its price too and make a positive profit since $K_{-1} - k_2 < D(0)$. Following this logic, this unravels until all firms set their price above zero. We conclude that there is no equilibrium in which a firm prices at zero.
   Next, suppose that all firms price weakly above 2ϵ. Since $K_{-1} \geqslant D(2\epsilon)$ it holds that $K > D(2\epsilon)$ and, therefore, $\underline{p} < 2\epsilon$. Consequently, a highest-priced firm is not capacity-constrained. If a highest-priced firm faces no residual demand, then it can improve itself by cutting price to ϵ and sell at capacity. Suppose, then, that a highest-priced firm sets p' and faces residual demand. In that case, all lower-priced firms are capacity-constrained and either sell at capacity at $p' - \epsilon$ or can improve themselves by doing so. If $p' - \epsilon > \epsilon$, then a highest-priced firm can profitably undercut its rivals since:
   $$p' \frac{k_i}{\sum_{j \in \Omega} k_j}(D(p') - K_{-i}) < (p' - 2\epsilon)k_i, \forall i \in N,$$
   for ϵ sufficiently small. If $p' - \epsilon = \epsilon$, then a highest-priced firm can profitably deviate to ϵ, because:

$$p' \frac{k_i}{\sum_{j\in\Omega} k_j}(D(p') - K_{-i}) < (p' - \epsilon)k_i, \forall i \in N,$$

for ϵ sufficiently small. We conclude that in equilibrium there is no firm pricing at 2ϵ or higher.

This leaves all firms pricing at ϵ as the equilibrium candidate. Suppose all price at ϵ. Clearly, no firm individually gains by lowering price to zero. Moreover, firm 1 does not want to hike its price since $K_{-1} \geqslant D(2\epsilon)$, which implies it faces zero residual demand at prices weakly above 2ϵ. As firm 1 does not want to deviate, none of the firms wants to deviate. We conclude that $NE^\epsilon = \left\{ p \in P^\epsilon |\ p_i = \epsilon, \forall i \in N \right\}$ when $D(0) > K_{-1} \geqslant D(2\epsilon)$.

3. Suppose that $K_{-1} \geqslant D(0)$ so that $K > D(0)$ and, therefore, $\underline{p} = 0$. By the same logic as under (ii) above, there is no equilibrium with a firm pricing at 2ϵ or higher. All firms pricing at ϵ is an equilibrium since none of the firms is better off by cutting price to zero and $K_{-1} \geqslant D(0) > D(\epsilon)$. Hence, firm 1 has no residual demand when raising its price to 2ϵ or higher and, therefore, none of the firms has an incentive to raise price.

   Moreover, a price profile with a non-minimal subset of firms pricing at zero is no equilibrium since any firm in this set can improve itself by raising its price to ϵ. If the subset of sellers pricing at zero is minimal, then there are many asymmetric equilibria with all sellers not in the set pricing above zero and facing no demand. Finally, since $K_{-1} \geqslant D(0)$, all pricing at zero is an equilibrium too since firm 1 faces no residual demand at prices above zero, which implies that none of the firms can raise price without losing all demand. We conclude that

$$NE^\epsilon = \left\{ p \in P^\epsilon | p \in \prod_{i\in S}\{0\} \times \prod_{i\in N\setminus S} P_i^\epsilon,\ \forall S \subseteq N \right\} \cup \left\{ p \in P^\epsilon |\ p_i = \epsilon, \forall i \in N \right\}$$

   when $K_{-1} \geqslant D(0)$.

4. Let us conclude this proof by showing that the characterization (i)-(iii) is complete. That is, we show that there is no pure-strategy Nash equilibrium when $K > D(p_1^*)$ and $K_{-1} < D(2\epsilon)$.

   To begin, suppose that $\underline{p} > 0$. As $K_{-1} < D(2\epsilon) < D(0)$ and following the logic under (ii) above, no firm will price at zero in equilibrium. Next, suppose that all firms set a strictly positive price and consider a strict subset $S \subset N$ setting the highest price, say p', in the industry. If a highest-priced firm faces no demand, then it can improve itself by reducing its price to ϵ. Suppose, then, that there is a highest-priced firm that faces residual demand. In that case, lower-priced sellers set $p' - \epsilon$ or they can improve themselves by raising price to $p' - \epsilon$ and sell at capacity. If $p' > \underline{p}$, then by the logic under (i)-(ii) and for a sufficiently fine price grid, a highest-priced seller can improve itself by slightly undercutting or matching (in case $p' = \underline{p} + \epsilon$ the price of its lower-priced rivals. If $p' \leqslant \underline{p}$, then all firms pricing strictly below $\underline{p}$ can improve themselves by raising their price to the market-clearing level since at $\underline{p}$ they still sell at capacity. We conclude there is no pure-strategy equilibrium in which firms set different prices and $\underline{p} > 0$.

   Suppose, then, that all firms set the same price. If this price is below the market-clearing level, all can improve by raising price to $\underline{p}$. If it is above $\underline{p}$, then each can improve by slightly undercutting its rivals for ϵ sufficiently small. This leaves all firms pricing at the market-clearing level as the equilibrium candidate. Yet, at $\underline{p}$, firm 1 has an incentive to hike its price, because $K_{-1} < D(\underline{p})$ and $p_1^* > \underline{p}$. We conclude that there is no pure-strategy Nash equilibrium when $K > D(p_1^*)$, $K_{-1} < D(2\epsilon)$ and $\underline{p} > 0$.

   Finally, suppose that $\underline{p} = 0$ so that $K \geqslant D(0)$. All firms pricing at zero is no equilibrium, because firm 1 can do better by raising its price to ϵ, which gives a positive price-cost margin and residual demand since $D(\epsilon) - K_{-1} > 0$. Following the argument under (ii) above, no firm will therefore price at zero in equilibrium. Next, by the preceding logic, there is no equilibrium in which firms charge different prices. Also, all setting the same price at 2ϵ or higher is not an equilibrium since $K \geqslant D(0)$ so that none of the firms is capacity-constrained at such price levels. Hence, for a sufficiently fine price grid it pays for each firm to slightly undercut its rivals. This leaves all pricing at ϵ as the equilibrium candidate. This, however, is no equilibrium either because firm 1 has an incentive to raise price. To see this, note that $\underline{p} = 0$, which implies that firm 1 has zero profit when charging its iso-profit price. By concavity of the residual profit function and the fact that $K_{-1} < D(2\epsilon)$, firm 1's marginal profit at ϵ is strictly positive provided that the price grid is sufficiently fine. As a result, when all price at ϵ, firm 1 wants to raise its price. We conclude that there is no pure-strategy equilibrium when $K > D(p_1^*)$ and $K_{-1} < D(2\epsilon)$. □

Next, we prove that NE^ϵ equals the MSS of G^ϵ. From the proof of parts (i)-(iii) of the preceding Proposition 5, it follows immediately that the game G^ϵ exhibits the *weak finite improvement property*. That is, whenever the set of pure-strategy Nash equilibria is nonempty, there is a path of myopic improvements from any non-Nash price profile to a pure-strategy equilibrium price profile. Note also that NE^ϵ is a closed set. Then, our claim is implied by Theorem 3.13 in Demuynck et al. (2019a), which shows that if a game satisfies the weak finite improvement property and if the Nash equilibrium is closed then the latter coincides with the MSS of the game.

Given, then, that NE^ϵ is nonempty, the next result establishes that it is "close" to the MSS M of the original game G. As such, the MSS can be interpreted as the result of the better-reply dynamics as described above. To measure the distance between the two sets, we use the concept of *Hausdorff distance*. For any pair of subsets $A, B \subseteq P$, the Hausdorff distance is:

$$d_H(A, B) = \max\left\{ \sup_{p\in A} d(p, B), \sup_{p'\in B} d(p', A) \right\},$$

where $d(p, A) = \inf_{p'\in A} d(p, p')$ is the Euclidean distance between p and the set A.

Proposition 6. *Suppose that NE^ϵ is nonempty. If the price grid is sufficiently fine, then $d_H(M, NE^\epsilon) < r$, for any $r > 0$.*

Proof of Proposition 6. Fix an $r > 0$. We show that there exists an ϵ small enough such that $d_H(M, NE^\epsilon) < r$. We consider the three cases in which NE^ϵ is nonempty (Proposition 5) in turn.

1. If $K \leqslant D(p_1^*)$, then $M = NE^\epsilon$ by Proposition 5. Consequently, $d_H(M, NE^\epsilon) = 0$. In this case, the Markov process induced by the better reply dynamics on G^ϵ is absorbing: $\mathbb{CC} = \mathbb{A} = NE^\epsilon$.
2. If $D(0) > K_{-1} \geqslant D(2\epsilon)$, then by Proposition 5 above:
$$NE^\epsilon = \left\{p \in P^\epsilon \,|p_i = \epsilon \text{ , } \forall i \in N\right\}. \tag{12}$$
By Theorem 2, the MSS M in the original game G is given by:
$$M = \left\{p \in P \,|\widetilde{p}_i \leqslant p \leqslant \overline{p}_1 \text{ ,} \forall i \in N\right\}.$$
Note that:
$$M \subseteq \{p \in P \mid 0 \leqslant p \leqslant 2\epsilon \text{ ,} \forall i \in N\}.$$
That is, for this case, myopically stable prices are weakly above zero and do not exceed 2ϵ.
Since $NE^\epsilon \subseteq M$, $\sup_{p' \in NE^\epsilon} d(p', M) = 0$. Next, note that the difference between a firm's price in M and the Nash equilibrium price is no larger than ϵ, which implies $\sup_{p \in M} d(p, NE^\epsilon) \leqslant \sqrt{n\epsilon^2}$. Taken together, this means that $d_H(M, NE^\epsilon) \leqslant \sqrt{n\epsilon^2}$. Choosing $\epsilon < r/\sqrt{n}$ then implies $d_H(M, NE^\epsilon) < r$, for any $r > 0$.
3. If $K_{-1} \geqslant D(0)$, then by Proposition 5 above:
$$NE^\epsilon = \left\{p \in P^\epsilon | p \in_{i \in S} \{0\} \times_{i \in N \setminus S} P_i^\epsilon, \ \forall S \subseteq N\right\} \cup \left\{p \in P^\epsilon | \ p_i = \epsilon, \forall i \in N\right\}.$$
Let $s = |S|$ be the cardinality of a minimal coalition S and let $n - s = |N \setminus S|$. Since any equilibrium price of an outsider to S is feasible in the original game, it holds that:
$$\sup_{p' \in NE^\epsilon} d(p', M) = \sqrt{(n-s) \cdot (0)^2 + s \cdot \epsilon^2} = \sqrt{s\epsilon^2}. \tag{13}$$
Moreover,
$$\sup_{p \in M} d(p, NE^\epsilon) = \sqrt{s \cdot (0)^2 + (n-s)(\epsilon/2)^2}, \tag{14}$$
because the zero prices played by a minimal coalition are in both M and NE^ϵ, whereas for all outsiders the maximum distance between their Nash equilibrium price in M and the nearest equilibrium price in the grid is $\epsilon/2$. The Hausdorff distance is then:
$$d_H(M, NE^\epsilon) = \max\left\{\sqrt{s\epsilon^2}, \sqrt{(n-s)(\epsilon/2)^2}\right\}.$$
Hence, choosing $\epsilon < \min\left\{r/\sqrt{s}, 2r/\sqrt{n-s}\right\}$ then implies $d_H(M, NE^\epsilon) < r$, for any $r > 0$. □

The preceding proposition establishes that the MSS in the original game G approximates the result of better-reply dynamics in G^ϵ when NE^ϵ is nonempty. Let us now consider the case where capacities are in an intermediate range (when NE^ϵ is empty, that is).

To that end, let firm i's *quasi-iso-profit price* be given by:

$$\check{p}_i = \min\left\{p_i \in P_i^\epsilon | \pi_i^h(p_i) = (\underline{p} + \epsilon) \cdot \frac{k_i}{K} \cdot D(\underline{p} + \epsilon), \text{ with } p_i \neq \underline{p} + \epsilon\right\}.$$

Assuming identical firms and that $\check{p}_i \in P_i^\epsilon$ for all $i \in N$, the next result characterizes the MSS of G^ϵ when the set of pure-strategy Nash equilibria is empty.

Proposition 7. *Assume that $k_i = k > 0$ for all $i \in N$, and that $\check{p}_i \in P_i^\epsilon$ for all $i \in N$. If NE^ϵ is empty, then the unique MSS $M^\epsilon \subseteq P^\epsilon$ of G^ϵ is given by:*

$$M^\epsilon \equiv \left\{p \in P^\epsilon \;\middle|\; \begin{array}{ll} \underline{p} \leqslant p_i \leqslant \overline{p}_i - \epsilon, \ \ \forall i \in N, & \text{if } D(p_1^*) < K < D(0) \\ \epsilon \leqslant p_i \leqslant \check{p}_i - \epsilon, \ \ \forall i \in N, & \text{if } D(0) \leqslant K. \end{array}\right\}$$

Proof of Proposition 7. The proof is analogous to the proof of Theorem 2. To begin, note that M^ϵ is closed. In what follows, we consider the conditions *deterrence of external deviations*, *asymptotic external stability* and *minimality*. Note that the uniqueness of M^ϵ follows from the fact P^ϵ is discrete and the dominance correspondence $f_\epsilon : P^\epsilon \mapsto P^\epsilon$ is (lower-hemi) continuous. This enables the application of Theorem 3.4 in Demuynck et al. (2019a), which identifies lower-hemi continuity of f as a sufficient condition for uniqueness.

Deterrence of External Deviations: We omit this part of the proof since it closely follows its counterpart in the proof of Theorem 2.

Iterated External Stability[29]**:** The proof is similar to its counterpart in the proof of Theorem 2 where it is shown that, for a sufficiently small $\epsilon > 0$, there is a finite sequence of better replies from any price profile outside the MSS to either $\underline{p}$ or some other price profile in the MSS with $p_i = p_1^*$, for some $i \in N$.

Minimality: Toward a contradiction, suppose that there exists a set M' satisfying *deterrence of external deviations* and *iterated external stability*. First, we assume that $D(p_1^*) < K < D(0)$. We distinguish two cases: either $\underline{p} \in M'$, or $\underline{p} \notin M'$.

Case 1A: Suppose that $\underline{p} \in M'$. Note that at $\underline{p}$, firm 1 has a positive residual demand by concavity of the residual profit functions (Lemma 2). Since we are in the case such that $k_i = k_1 = k$, then each firm $i \in N$ has a profitable deviation to any p_i with $\underline{p} < p_i \leqslant \overline{p}_i - \epsilon$. Thus, by the property of *deterrence of external deviations* of M' and the fact that $\underline{p} \in M'$, the following price profiles are contained in M':

$$M_1' = \{p \in P^\epsilon | \underline{p} \leqslant p_i \leqslant \overline{p}_i - \epsilon,\ p_j = \underline{p}, \text{ for some } i \in N, \text{ and all } j \neq i\} \subseteq M'.$$

Now fix $p \in M_1'$ such that $p_i > \underline{p}$ for some firm $i \in N$, whereas, by the definition of M_1', $p_j = \underline{p}$, for all $j \neq i$. Next, notice that now any firm $j \neq i$ in M_1' can improve its profit by either matching firm'i price (since Lemma 6 implies that $\pi_j^s(p_j) > \underline{p}k$ for $p_j = p_i$) or by charging a price $\underline{p} < p_j < p_i$, using a similar argument as in Lemma 4. Since $p_i > \underline{p}$ was picked arbitrarily, it can also be as high as $\overline{p}_i - \epsilon$. It then follows from *deterrence of external deviations* of M' that:

$$M_2' = \{p \in P^\epsilon | \underline{p} \leqslant p_i \leqslant \overline{p}_i - \epsilon,\ \forall i \in N, \} \subseteq M'$$

Note that $M_2' = M^\epsilon$. Then, M_2' satisfies deterrence of external deviations and iterated external stability. This led to a contradiction since minimality of M' is violated.

Case 2A: Now suppose that $\underline{p} \notin M'$. Fix some $p \in M'$. We show that $\underline{p} \in f^{\mathbb{N}}(p)$. It then follows from Lemma 5 that $\underline{p} \in M'$, a contradiction. For an arbitrarily small $\epsilon > 0$ the proof proceeds in four steps as in Theorem 2, with the exception of Step 3 case (ii) and Step 4, that we consider below.

Step 3: Let $p \in M'$ be the price profile resulting from the preceding steps. Note that, by construction, at p there are two groups of firms: the highest priced firms $H(p)$ charging p_h and the lowest priced firms $L(p)$ charging $p_l = p_h - \epsilon$. According to previous steps, we have two cases: either (i) the lowest-priced firms are not capacity-constrained, or (ii) the lowest-priced firms are capacity-constrained. In case of (i), the highest-priced firms have zero profit so we only consider the latter.

Case (ii): In this case, the residual demand and profit of the highest priced firm(s) is positive. The proof of Theorem 2 shows that in this case there exists a sequence converging to the market-clearing price where firms undercut each other in turn by a small ϵ, that is $\underline{p} \in f^\infty(p)$. We show that in G^ϵ there exists a finite sequence that from p ends to the market clearing price $\underline{p}$, that is $\underline{p} \in f^{\mathbb{N}}(p)$. Given the sequence already constructed in the proof of Theorem 2, we only have to show that it actually ends at $\underline{p}$. Define $p_h = \underline{p} + \epsilon$ and $p_l = \underline{p}$. We show that, for any ϵ small enough, each highest-priced firm has an incentive to deviate to $p_l = \underline{p}$. Each firm $h \in H(p)$ has an incentive to reduce its price to the market-clearing level when:

$$\begin{aligned}
\pi_h(\underline{p}) > \pi_h(\underline{p} + \epsilon) \iff \pi_h(p_h - \epsilon) > \pi_h(p_h) \iff \\
k_h(p_h - \epsilon) > \frac{k_h}{\sum_{j \in \Omega(p_h, p_{-h})} k_j}\left[D(p_h) - \sum_{j \in \Delta(p_h, p_{-h})} k_j\right] p_h \iff \\
k_h p_h - \frac{k_h}{\sum_{j \in \Omega(p_h, p_{-h})} k_j}\left[D(p_h) - \sum_{j \in \Delta(p_h, p_{-h})} k_j\right] p_h > k_h \epsilon \iff \\
p_h\left[\frac{K - D(p_h)}{\sum_{j \in \Omega(p_h, p_{-h})} k_j}\right] > \epsilon,
\end{aligned}$$

which holds for ϵ sufficiently small because $\underline{p} > 0$ and $K > D(p_h)$.

Step 4: Iteration of the previous steps shows that from $p \in M'$ it is possible to construct a sequence such that $\underline{p} \in f^{\mathbb{N}}(p)$. Then, by Lemma 5, $\underline{p} \in M'$, a contradiction.

[29] Iterated external stability replaces f^∞ with $f^{\mathbb{N}}$ in the definition of asymptotic external stability. Asymptotic external stability reduces to iterated external stability when the strategy sets are finite as in G^ϵ.

Next, suppose that $D(0) \leqslant K$. As before, we distinguish two cases: either $\epsilon \in M'$, or $\epsilon \notin M'$.

Case 1B: Assume that $\epsilon \in M'$. In the same vein as Case 1A above, suppose that all firms charge ϵ. Then, for an arbitrarily small $\epsilon > 0$, by the concavity of the residual profit function of all firms (Lemma 2), each firm $i \in N$ has a profitable deviation to any p_i with $\epsilon < p_i \leqslant \check{p}_i - \epsilon$. Thus, by the property of *deterrence of external deviations* of M' and the fact that $\epsilon \in M'$, the following price profiles are contained in M':

$$M_1' = \{p \in P^\epsilon | \epsilon \leqslant p_i \leqslant \check{p}_i - \epsilon,\ p_j = \epsilon, \text{ for any } i \in N, \text{ and } j \neq i\} \subseteq M'.$$

Now fix $p \in M_1'$ such that $p_i > \epsilon$ for some $i \in N$, whereas $p_j = \epsilon$ for all $j \neq i$. It follows that any firm $j \neq i$ can now improve its profit by either matching or undercutting firm' i price. In the matching case, first note that any of firm i's rivals can improve by matching firm'i price when $p_i = \check{p}_i - \epsilon$. Indeed,

$$\epsilon k < (\check{p}_i - \epsilon)\frac{k}{\sum_{j\in\Omega(\check{p}_i-\epsilon)} k_j}(D(\check{p}_i - \epsilon) - \sum_{j\in\Delta(\check{p}_i-\epsilon)} k_j),$$

which, using the definition of $\check{p}_i$, is equivalent to:

$$\check{p}_i(D(\check{p}_i) - K_{-i}) < (\check{p}_i - \epsilon)\frac{k}{\sum_{j\in\Omega(\check{p}_i-\epsilon)} k_j}(D(\check{p}_i - \epsilon) - \sum_{j\in\Delta(\check{p}_i-\epsilon)} k_j).$$

This inequality holds for a sufficiently fine price grid when:

$$D(\check{p}_i) - K_{-i} < \frac{k}{\sum_{j\in\Omega(\check{p}_i-\epsilon)} k_j}(D(\check{p}_i - \epsilon) - \sum_{j\in\Delta(\check{p}_i-\epsilon)} k_j),$$

which can be written as:

$$K - D(\check{p}_i) > \frac{k}{\sum_{j\in\Omega(\check{p}_i-\epsilon)} k_j}(K - D(\check{p}_i - \epsilon)),$$

which holds since $D(\check{p}_i) < D(\check{p}_i - \epsilon) < K$.
Then, by concavity of the profit function π_j^s, firm j improves its profit by matching firm i for any firm'i price $\epsilon < p_i \leqslant \check{p}_i - \epsilon$.
In the undercutting case, using a similar argument as in Lemma 4, each firm j other than firm i has a profitable deviation to any p_j such that $\epsilon < p_j < p_i$ whenever $p_i > 2\epsilon$. Then, the property of *deterrence of external deviations* of M' together with the fact that the choice of $p_i > \epsilon$ was picked arbitrarily it can also be as high as $p_i - \epsilon$ implies that

$$M_2' = \{p \in P^\epsilon | \epsilon \leqslant p_i \leqslant \check{p}_i - \epsilon,\ \forall i \in N\} \subseteq M',$$

and, therefore, $M_2' = M^\epsilon \subseteq M'$, a contradiction.

Case 2B: Now suppose that $\epsilon \notin M'$. The proof proceeds as in Case 2A. Indeed, consider a price profile with all firms charging a strictly positive price with at least one firm pricing strictly above ϵ. If a highest-priced firm faces no demand, then it can improve itself by lowering its price to ϵ. Suppose that any highest-priced firm with no demand does precisely that. This leaves the possibility that a highest-priced firm faces residual demand. Note that, since prices are above the market-clearing level, such a firm is not capacity-constrained. All its lower-priced rivals are then capacity-constrained. These lower-priced firms then either undercut the highest price by the smallest possible amount or can improve themselves by doing so. Suppose they do precisely that. This yields a situation as described under Case 1B so that there is a path of myopic improvements to a price profile with each pricing at ϵ. Thus, $\epsilon \in M'$, a contradiction. □

Assuming identical firms, the next result shows that the MSS in the original game G also approximates the result of better-reply dynamics in G^ϵ when NE^ϵ is empty.

Proposition 8. *Assume that $k_i = k > 0$, for all $i \in N$, and suppose that NE^ϵ is empty. If the price grid is sufficiently fine, then $d_H(M, M^\epsilon) < r$, for any $r > 0$.*

Proof of Proposition 8. Fix an $r > 0$. We show that there exists an ϵ small enough such that $d_H(M, M^\epsilon) < r$. We consider the two cases in which NE^ϵ is empty (Proposition 7) in turn.

1. If $D(p_1^*) < K < D(0)$, then

$$M^\epsilon = \left\{ p \in P^\epsilon \mid \underline{p} \leqslant p_i \leqslant \overline{p}_i - \epsilon,\ \ \forall i \in N \right\}. \tag{15}$$

By Theorem 2, $M^\epsilon \subset M$ and therefore $\sup_{p'\in M^\epsilon} d(p', M) = 0$. Next, note that the maximum distance between every firm's price in M and M^ϵ is ϵ. Therefore,

$$d_H(M, M^\epsilon) = \max\left\{ \sup_{p' \in M^\epsilon} d(p', M), \sup_{p \in M} d(p, M^\epsilon), \right\} = \sqrt{n\epsilon^2}. \quad (16)$$

Choosing $\epsilon < r/\sqrt{n}$ then implies $d_H(M, M^\epsilon) < r$, for any $r > 0$.

2. If $D(0) \leqslant K$, then by Proposition 7:

$$M^\epsilon = \left\{ p \in P^\epsilon \mid \epsilon \leqslant p_i \leqslant \check{p}_i - \epsilon, \quad \forall i \in N \right\}.$$

By Theorem 2, $M^\epsilon \subset M$ and therefore $\sup_{p' \in M^\epsilon} d(p', M) = 0$. Next, note that $\sup_{p \in M} d(p, M^\epsilon) = \max\left\{\epsilon, \overline{p}_i - \check{p}_i + \epsilon\right\} = \overline{p}_i - \check{p}_i + \epsilon$ when $\overline{p}_i - \check{p}_i > 0$, which holds. Taken together, the Hausdorff distance is then given by:

$$d_H(M, M^\epsilon) = \sqrt{n\left(\overline{p}_i - \check{p}_i + \epsilon\right)^2}.$$

Note that the difference $\overline{p}_i - \check{p}_i$ is monotonically decreasing in ϵ and that $\overline{p}_i = \check{p}_i$ when $\epsilon = 0$. Choosing $\epsilon + \left(\overline{p}_i - \check{p}_i\right) < r/\sqrt{n}$ then implies $d_H(M, M^\epsilon) < r$, for any $r > 0$. □

A natural next question to ask is what would happen when production capacities are heterogeneous and the set of pure-strategy Nash equilibria NE^ϵ is empty. This is computationally much more involved than the symmetric situation, but the underlying logic remains the same. We therefore conjecture that the result will be similar in this case.

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