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Maximum load for medium voltage lines in N-1 conditions

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ABSTRACT

This paper proposes a methodology able to assess the maximum value of the load that can be connected to a medium voltage (MV) feeder in addition to the existing one without impairing the *N*-1 security criterion. A simplified approach is pursued, the original problem is split into several separate subproblems, and the maximum additional load is calculated solving multiple optimization problems. The proposed method is simple and practically applicable by distribution system operators (DSOs) that, after a request by a customer to connect a load, must know if an MV feeder is fit to host the additional demand or if a new infrastructure is needed. The application of the methodology to the whole MV distribution grid of Rome, in Italy, is presented; execution times show that the DSO can apply the model daily, to follow the load growth day by day.

1. Introduction

Distribution networks are currently subject to new challenges and changes, including increased load for electrification of end-use consumption, e-mobility and integration of distributed generators. For this reason, operators aim to acquire systems and tools that improve the evaluation of the maximum allowed load that a network could connect.

In general, the problem of the maximum amount of load that may be connected to a distribution network is a relevant topic in the literature. Within this topic, two main paths can be found; on the one hand, many studies are focused on loadability evaluated by considering the limit on the voltage stability index, on the other hand, less studies have evaluated the available supply capability (ASC) of a distribution network. ASC normally refers to the amount of apparent power that a distribution system can supply in addition to its present load. According to [1] and [2], ASC should be calculated as the difference between the Total Supply Capacity (TSC) and the present load. TSC is normally defined as the maximum load that a distribution system can supply under the *N*-1 security criterion [1,2]. Surveying the literature on this topic, the main issues are the development of methodologies for assessing load margin and the definition of the worst *N*-1 conditions for properly calculating such load margin. Some other studies are focused on optimizing network operation for improving the load margin and analysing the maximum additional load that could be connected to the network. Assessing the load margin during degraded operating conditions (i.e., *N*-1 conditions) is extremely relevant for the distribution system operator (DSO). Indeed, under fault conditions the DSO modifies the network topology by changing the status of the switches in order to isolate the faulted section of a feeder and to connect the safe sections to other feeders with the aim to supply as many loads as possible. Under these operating conditions, the capability to supply the largest possible number of customers depends on the adequacy of the feeders to supply an aggregate electrical load larger than the one in normal operating conditions. For this reason, DSOs must know the effects of the loss of a feeder in order to plan the new load connections to the grid. In addition, most of the DSOs in the world are subject to regulatory mechanisms that define rewards and penalties based on certain service quality standards; therefore, reducing the number of unsupplied users is crucial to pay less penalties. Reward-penalty mechanisms have been developed in many countries so that they are the main drivers for planning investments in the network and connecting new users to the network.

Among the studies that evaluate the loadability of distribution networks, it is worth mentioning reference [3] which aims to assess the limits to the loading at some buses of a radial grid considering power losses and voltage stability index within a desired limit. Similarly, in [4] the load supply capability is evaluated in terms of voltage stability. In [5] and [6], a fuzzy theory is used to overall evaluate the power supply capability considering several indicators, such as voltage and loading rates.

According to the surveyed literature, different types of events could be considered in order to define *N*-1 conditions in which ASC is calculated. As an example, reference [7] considers an *N*-1 contingency as the failure of a main-transformer and evaluates if the load normally supplied

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Nomenclature[1]

Sets

$\boldsymbol{B}$	Set of buses
$\boldsymbol{B'}$	Set to which belongs only the bus selected for the connection of additional load demand
$\boldsymbol{BR}$	Set of branches
$\boldsymbol{BR^1}$	Set of branches that are closed in the initial configuration
$\boldsymbol{BR^f}$	Set of first branches whose parent node is node “from”
$\boldsymbol{BR^t}$	Set of first branches whose parent node is node “to”
$\boldsymbol{BR^0}$	Set of branches that are open in the initial configuration
$\boldsymbol{BR'}$	Subset of branches whose “from” or “to” node belongs to one of the two feeders of the combination of examined MV lines
$\boldsymbol{G}$	Set of generators (i.e. the external grids connected to the first node of each MV feeder), subset of set $\boldsymbol{B}$

Parameters

$\overline{\mathbf{F}}$	Branch capacity (p.u.) [$\lvert\boldsymbol{BR}\rvert$ x 1]
$\overline{\mathbf{F}}^{\mathbf{F}}$	Out of bounds of each feeder [$\lvert\boldsymbol{G}\rvert$ x 1]
$\boldsymbol{L}$	Load request by each node (p.u.) [$\lvert\boldsymbol{B}\rvert$ x 1]
M	Large enough number
$\boldsymbol{s^0}$	Initial status of each branch (1 = closed, 0 = open) [$\lvert\boldsymbol{BR}\rvert$ x 1]
$\underline{V}, \overline{V}$	Minimum and maximum acceptable value for the node voltages (p.u.)
$\boldsymbol{Z}$	Absolute value of the branches’ impedance (p.u.) [$\lvert\boldsymbol{BR}\rvert$ x 1]

Real Variables

$\mathbf{F^{SCF}}$	Fictitious single commodity flow through each branch [$\lvert\boldsymbol{BR}\rvert$ x 1]
$\boldsymbol{I}$	Absolute value of the current flowing through each branch (p.u.) [$\lvert\boldsymbol{BR}\rvert$ x 1]
$\boldsymbol{I^G}$	Absolute value of the current injected by each external grid (p.u.) [$\lvert\boldsymbol{G}\rvert$ x 1]
L'	Maximum additional load that can be connected to the node in the $\boldsymbol{B'}$ set
$\boldsymbol{V}$	Absolute value of the node voltage (p.u.) [$\lvert\boldsymbol{B}\rvert$ x 1]

Binary Variables

$\boldsymbol{s}$	$\lvert\boldsymbol{BR}\rvert \times 1$ vector relative to branch status (1=closed, 0=open)
$\boldsymbol{y1}$	$\lvert\boldsymbol{BR}\rvert \times 1$ vector indicating if “from” node is parent of “to” node (1=yes, 0=no)
$\boldsymbol{y2}$	$\lvert\boldsymbol{BR}\rvert \times 1$ vector indicating if “to” node is parent of “from” node (1=yes, 0=no)

by the faulted transformer could be supplied by another transformer. Similarly, [1] determines the power supply capability when a transformer is disconnected applying the procedure to a real 44-bus distribution network; in addition, [8] reports an evaluation method in case of simultaneous disconnection of many transformers. Reference [9] considers both feeder and substation transformer *N*-1 contingencies applying a Generalized Reduced Gradient method for assessing the additional load that could be connected to a distribution network. In [10], the impact of transformers on TSC is investigated. Among the most recent contributions, reference [2] calculates ASC of a 43-bus distribution network assuming that one branch is disconnected and reconfiguration is needed.

As previously explained, ASC in *N*-1 conditions can be exploited for improving both network operation and planning. Regarding the network operation, reference [11] proposes a fuzzy adaptation of the evolutionary programming algorithm for optimal reconfiguration of distribution systems to maximize loadability. Reference [12] assesses the loadability of active distribution networks in the presence of direct current controllable links. Reference [13] includes loadability of the network in a stochastic framework for secure reconfiguration of active distribution networks. Reference presents an algorithm for network reconfiguration based on the maximization of system loadability. Finally, [14] proposes an approach for the real time update of the load margin of power systems related to voltage instabilities and small-signal instabilities considering variations in load growth. Regarding the planning activities, load margin calculation under *N*-1 conditions is fundamental for evaluating if new connection requests and load growth scenarios are allowed in the network. ASC has been frequently used for analizing the effects of load growth scenarios: in [2], several load growth patterns are assessed for verifying if a network can be still securely operated, in [15] the impact of load growth rate and reserve capacity level on system reliability is evaluated for a distribution system in India.

According to the surveyed contributions, investigations about ASC assessments are still needed; in particular, applications to large case studies with real data are often missing so that the applicability of these evaluation methods in a real industrial context could be not properly verified. Moreover, research papers have not investigated yet the potential benefits of their implementation for supporting the operators when they receive connection requests. In this respect, an up-to-date calculation of ASC (e.g., calculating ASC every day) enables operators estimating if a connection request is allowed by the network as is or if grid reinforcements are needed. To fill these gaps, the authors developed a methodology, which is presented in this paper, that aims to find the maximum additional load that can be connected to a single Medium Voltage (MV) feeder without impairing the possibility to supply the load by means of different MV feeders in case of fault (*N*-1 security criterion). Another goal of the developed methodology is its integration into the network operation system so that ASC can be daily updated in order to consider load growth and network topology changes. Differently from [4], the developed methodology considers *N*-1 conditions for each feeder of the network and calculates ASC for the most severe events that could happen in the grid and, in comparison with [5], it adopts a conservative approach, simulating the worst operating conditions when ASC is calculated; moreover, the proposed approach allows to disregard uncertainties of load profiles of the secondary substations. In the developed methodology, the worst-case scenario has been defined according to these three assumptions:

- Peak load demand scenario: the maximum historical load is connected to each bus.
- Absence of distributed generation: the presence of distributed generation along the feeder affected by the fault lowers the amount of load that must be restored, so requirements are more stringent if distributed generation is disregarded.
- Worst possible bus to connect the additional load to the MV feeder: for each feeder multiple optimization problems are solved, in each problem the additional load is connected to a different bus, and the most conservative result among the obtained ones is selected.

The worst-case analysis has been selected since it is quite common in case of robust optimization, that is a modelling framework able to handle several parameters, as explained in [16]. Indeed, according to [17], robust optimization often relies on worst-case analysis, namely, a

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[1] Quantities in bold characters refer to vectors and matrices.

solution is evaluated assuming the most unfavourable conditions, achieving a trade-off between system performance and protection against uncertainty (i.e., avoiding an overconservative approach). However, conservative approach is well suited in power system operation if the penalty associated with infeasible solutions is very high. Indeed, distribution network planning often considers conservative approaches as it surveyed in [18]. Moreover, ASC is frequently calculated analysing the worst-case scenario, as in [1].

Since a conservative approach has been adopted, it is worth mentioning that contributions from energy flexibility sources are neglected in the methodology. Indeed, even if the benefits from flexibility resources have been studied in many recent contributions, as in [19–25], their effects are still under investigation and the related market frameworks are still under development. As an example, in [23], the latest activities on Vehicle-to-everything shows that such schemas are still under investigation from a technical point of view and that are still not mature enough to be considered for a robust/conservative approach, as it is usually required by the distribution network operator.

The methodology developed in this paper has two main parts: on the one hand, it analyses the restorability of the load for each MV line; on the other hand, it analyses the maximum additional load that can be connected to each MV feeder. The restorability analysis assesses the ability of the grid to correctly operate in *N*-1 conditions. Through the solution of an optimization problem, for each MV feeder the restorability study determines if, in the case of a fault on the first section of the feeder, the load normally supplied by the feeder can be supplied by different feeders. Feeders whose load cannot be completely supplied are the ones that the DSO must invest in, in order to restore the *N*-1 security condition. A second step of the methodology analyses the maximum additional load that can be connected to each MV feeder. Indeed, for each feeder that meets the *N*-1 security criterion the analysis assesses the maximum additional load that can be connected still fulfilling the *N*-1 criterion. Multiple optimization problems are solved, each one yielding in output the maximum additional load that can be hosted by the feeder in a specific condition, and by selecting the most conservative result of all the different conditions.

The contributions of the paper can be summarized as follows:

- Development of a methodology capable of calculating the maximum additional load connectable to each MV feeder.
- The methodology can follow the daily load evolution on the network, yielding robust results within a reduced time frame even for real large size networks.
- The methodology has been tested on the real distribution network operated by Areti S.p.A., which is the DSO of Rome (Italy), and is daily applied by the DSO. Nevertheless, the methodology has a general purpose since results do not depend on economic parameters and other country-related constraints.

Whilst being conservative, the proposed method is realistic since the DSO, in compliance to the additional load requests, must necessarily connect the new load: the methodology allows the DSO to establish whether the MV feeder is fit to supply the additional load or if new infrastructure is needed. Its daily application mitigates the over-conservatism issues since the results of the methodology are exploited for the very short-term scenarios, notably for managing the requests of connections that the DSO receives every day.

The remainder of the paper is structured as follows. Section 2 presents the architecture of the whole study, which is subdivided into two main parts respectively detailed in Sections 3 and 4. Section 5 presents results obtained by the application of the methodology on an illustrative test network and on the whole MV distribution grid of Rome. Lastly, Section 6 concludes the paper.

2. Architecture of the methodology

One of the main features of MV distribution grids is having a meshed topology and a radial operation. On one hand such property allows the DSO to reconnect the load disconnected due to a fault in a considerably smaller amount of time than the one required with a radial grid, and on the other hand allows the use of much simpler protection devices than the ones required with a meshed operation (as in case of transmission networks).

The meshed topology enables the “reconnection” of the disconnected loads downstream of the fault: in case a fault occurs along an MV feeder, all the load nodes affected by the outage can be restored by means of another feeder, so that the duration of the power supply interruption is not related to the repair time of the component affected by the fault. To ensure that the loss of supply is as short as possible, the switches installed in the medium voltage / low voltage (MV/LV) substations that are normally open, often referred to as tie switches, are remotely controlled and manoeuvred in times of the order of seconds. In this paper, the terminal nodes of a tie switch are called “cutting” nodes.

The design criteria of the MV grid requires that the aggregate load supplied by each MV feeder can be completely transferred to another feeder: even in the worst-case scenario, i.e. a fault occurs at the first branch of an MV feeder (the first branch is the branch originating from the primary substation busbars and ending at the first bus of the feeder) and all the downstream MV nodes are consequently de-energised, all the load demand normally supplied by such feeder can be supplied by another feeder. This planning criterion of a distribution grid is referred to as the *N*-1 security criterion.

Thus, prior to studying the maximum amount of load that can be connected to a specific MV feeder, a service restoration study must be carried out. The objective of the study is to assess if the MV feeder satisfies the *N*-1 security criterion, i.e. if a set of manoeuvres can completely restore the load normally supplied by the feeder. If such set exists then the feeder is marked as “restorable”, and therefore able to supply a potentially larger load than the current one. If such set does not exist, the feeder is marked as “not restorable”, i.e. no additional load can be connected and line reconductoring should be foreseen. Aiming at results adherent as much as possible to the actual operation of the MV grid, the number of feeders at disposal for the service restoration is limited to one for each faulted feeder, since the operator of the control room of the DSO usually closes just one tie switch to re-energise the nodes affected by the fault.

After the service restoration study of the whole MV grid, each feeder is marked as either “restorable” or “not restorable”. The set of all the “restorable” feeders is then further studied to assess, for each feeder, the maximum additional load that can be connected to it without impairing neither its “restorability” neither the restorability of the feeders directly connected to it.

3. Service restoration of MV feeders

3.1. Assumptions

High voltage (HV) busbars and HV/MV transformers in the primary substations are not simulated, whereas 1 p.u. voltage is imposed at the MV busbars in the primary substations. The worst-case scenario is considered: all loads are set to the peak value recorded during the year and distributed generation is disregarded.

The proposed optimization model assumes that, in each node of the grid, the voltage is close to the nominal value and thus the per unit apparent power flowing through the branches is equal to the per unit current [26]. In addition to the rated capacity, for each feeder another limit is considered, referred in this paper as the “out of bounds” and expressing the maximum current that can flow through the circuit breaker at the beginning of each feeder. The limit is set according not only to the nominal rating of the first branch, but also to the experience of the control room operators that know the actual state of the feeder and are able to evaluate its capabilities. The “out of bounds” limit essentially downgrades the ampacity of an MV feeder considering the actual degradation.

Lastly, when the service restoration of a feeder is performed, the starting status of such feeder is open so that one normally open branch must be closed.

3.2. Optimization problem

- *Objective function*

$$f = \sum_{(i,j)\in \mathbf{BR}} \frac{I_{ij}^2}{\overline{F}_{ij}^2} \tag{1}$$

- *Non-negativity constraints*

$$V_i \geq 0, \forall i \in \mathbf{B} \tag{2}$$

$$I_i^G \geq 0, \forall i \in \mathbf{G} \tag{3}$$

- *Radiality constraints*

$$\sum_{j:(i,j)\in \mathbf{BR}} F_{ij}^{SCF} = -1,\ \forall j \in \mathbf{B}\backslash\mathbf{G},\ \forall i : (j,i) \in \mathbf{BR} \tag{4}$$

$$-s_{ij}\cdot|\mathbf{B}| \leq F_{ij}^{SCF} \leq s_{ij}\cdot|\mathbf{B}|,\ \forall(i,j) \in \mathbf{BR} \tag{5}$$

$$y1_{ij} + y2_{ij} = s_{ij},\ \forall(i,j) \in \mathbf{BR} \tag{6}$$

$$y1_{ij} = s_{ij},\ \forall(i,j) \in \mathbf{BR^f} \tag{7}$$

$$y2_{ij} = s_{ij},\ \forall(i,j) \in \mathbf{BR^t} \tag{8}$$

$$\sum_{j:(i,j)\in \mathbf{BR}} s_{ij} = |\mathbf{B}| - |\mathbf{G}| \tag{9}$$

- *Operational constraints of the grid components*

$$\underline{V} \leq V_i \leq \overline{V},\ \forall i \in \mathbf{B} \tag{10}$$

$$V_i = 1,\ \forall i \in \mathbf{G} \tag{11}$$

$$-s_{ij}\cdot\overline{F}_{ij} \leq I_{ij} \leq s_{ij}\cdot\overline{F}_{ij},\ \forall(i,j) \in \mathbf{BR} \tag{12}$$

$$I_i^G \leq \overline{F}_i^F,\ \forall i \in \mathbf{G} \tag{13}$$

- *Additional constraints*

$$\sum_{(i,j)\in \mathbf{BR}^1} \left(s_{ij}^0 - s_{ij}\right) + \sum_{(i,j)\in \mathbf{BR}^0} \left(s_{ij} - s_{ij}^0\right) \leq 1 \tag{14}$$

- *Kirchhoff’s laws*

$$\sum_{j:(i,j)\in \mathbf{BR}} I_{ji} = -L_j + I_j^G,\ \forall j \in \mathbf{B},\ \forall i : (j,i) \in \mathbf{BR} \tag{15}$$

$$-M\cdot\left(1 - s_{ij}\right) \leq Z_{ij}\cdot I_{ij} - \left(V_i - V_j\right) \leq M\cdot\left(1 - s_{ij}\right), \forall i \in \mathbf{B}, \forall j : (i,j) \in \mathbf{BR} \tag{16}$$

In the optimization problem, the objective function in (1) is the minimization of the sum of the squared branch loadings, each weighted by the squared branch rated capacity. Constraints (2) and (3) impose the non-negativity of bus voltages and of the current injected at the MV busbars of the primary substations, respectively. Constraints (4)-(9) are the radiality constraints, which guarantee the radial operation of the network when combined with the fictitious single commodity flow method, as explained in [27]. Constraint (4) expresses the Kirchhoff’s current law for the fictitious flow of each non-generation node, whereas (5) imposes a null fictitious flow on branch *(i, j)* if the branch is open, i.e. $s_{ij} = 0$, otherwise the fictitious flow can be either positive or negative and its maximum absolute value is equal to the number of nodes of the grid. According to (6), for each closed branch the power flow is either from the “from” node to the “to” node or from the “to” node to the “from” node, i.e. by means of the auxiliary binary variables $y1_{ij}$ and $y2_{ij}$ the constraint simply guarantees that if the branch *(i, j)* is connected, either node *i* is the parent node of node *j* or vice versa. Constraint (7) sets to one the variable $y1_{ij}$ of each closed first branch *(i, j)* of a feeder whose “from” node is the parent node, and correspondingly (8) sets to one the variable $y2_{ij}$ of each closed first branch *(i, j)* of a feeder whose “to” node is the parent node. Equation (9) imposes the condition that the number of closed branches is equal to the difference between the number of nodes and the number of generators.

Each bus voltage is constrained between the minimum and maximum allowable values by (10), and specifically is set to 1 p.u. at the first node of each feeder by (11). Constraint (12) limits the power flow through a close and an open branch to the branch capacity and to zero, respectively, whereas (13) limits the current injected by each generator to the “out of bounds” limit of the corresponding feeder. Constraint (14) imposes that only one branch can change its status from the initial configuration to the final one, whereas (15) and (16) impose the Kirchhoff’s current and voltage laws, respectively.

4. Maximum additional load connectible to each MV feeder

For each “restorable” feeder a further study is then conducted with the aim to assess the maximum additional load that can be connected to it without impairing neither its restorability nor the restorability of the feeders directly connected to it. For the sake of comprehension, at first the architecture of the study and then the proposed optimization model will be described.

4.1. Architecture of the study

The methodology adopted for the evaluation of the maximum additional load connectible to an MV feeder is schematized in the block diagram in Fig. 1.

In the external loop the methodology iterates over all the MV feeders in the network. In each iteration the maximum additional load connectible to a feeder of the network is calculated; hereafter, such feeder is named input feeder. For each input feeder a sub-grid is extracted from the original network, to minimize computational complexity and speed up the resolution of the optimization problems. The sub-grid consists of the input feeder and the “*k*” feeders directly interconnected with it. Each iteration of the loop solves an optimization problem on a network containing “*k*+1” feeders, representing a subset of the entire distribution network. Once the sub-grid is extracted, the first step of the problem begins, i.e. assessing for the input feeder the compliance with the *N*-1 criterion. The modelling of this optimization problem is detailed in Section 3.2 via the objective function (1) and constraints (2)-(16). If the feeder does not satisfy the *N*-1 criterion the methodology stops: an investment is necessary to restore the *N*-1 security condition. If the feeder satisfies the *N*-1 criterion, then step 1 of the methodology also indicates which feeder restores the input feeder and which feeder is restored by the input feeder. The feeder restoring another one is referred to as the “natural back-up feeder”.

The second step of the methodology is the calculation of the maximum additional load that can be connected to the input feeder. A nested “for” loop iterates *k*+2 times (being *k* the number of feeders directly interconnected to the input feeder) and in each iteration an

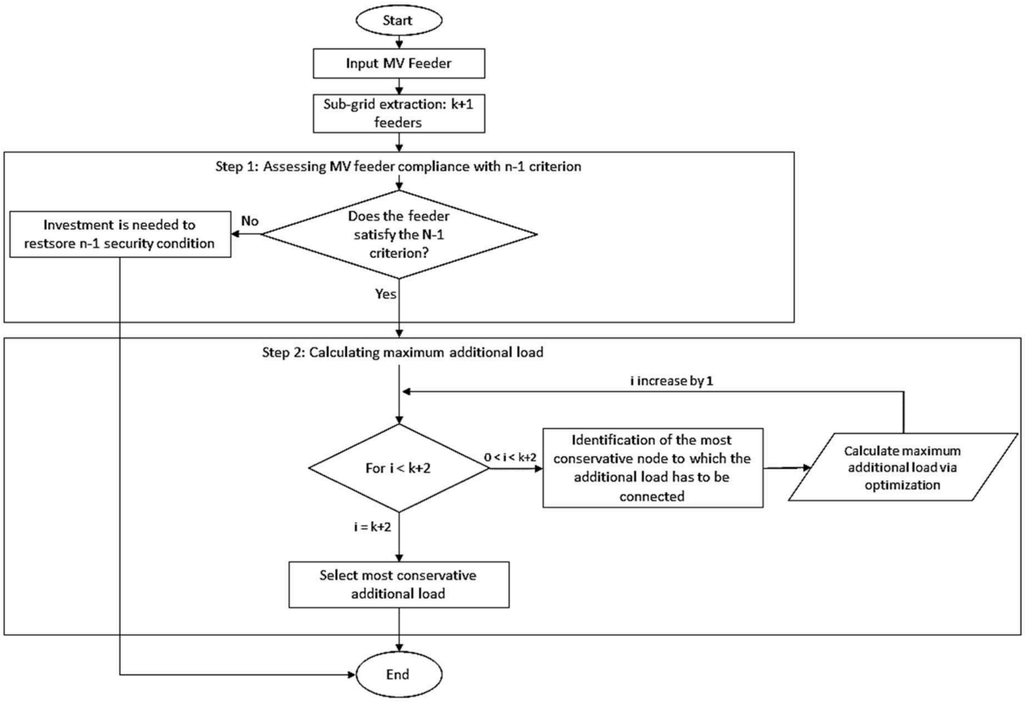

Fig. 1. Block diagram of the methodology.

optimization problem is solved, calculating the maximum load that can be connected to the input feeder considering a different network configuration. A total number of $k+2$ iterations is necessary because it is essential to select the maximum load connectible to the feeder in the following possible configurations:

1. Case 1: feeder without fault, the optimal solution of this problem is the maximum additional load of the feeder in normal operation.
2. Case 2: feeder affected by a fault, one of the directly interconnected feeders must restore the input feeder.
3. Case 3: feeder without fault, but a fault affects one of the feeders directly interconnected to it, which is restored by the input feeder. Such a case is simulated k times, once for each feeder directly interconnected to the input feeder.

For each network configuration where a fault is simulated, it is conservatively assumed that the fault is located on the first branch of the feeder, i.e. the branch originating from the primary substation busbars and ending at the first bus of the feeder. This is the worst fault, since the feeder restoring the faulted one must supply the whole load of the faulted feeder and not just a fraction, as would happen in case of a fault on another branch, where a part of the load would be supplied by the feeder affected by the fault and the remaining part by the interconnected feeder. From a practical standpoint, this means that:

- In case 1 there is no fault, and no branch of the network is open.
- In case 2 the first branch of the input feeder is open.
- In case 3 the first branch of the feeder directly interconnected with the input feeder is open.

This ensures that each feeder of the grid fulfills the N-1 security criterion in any case even with the connection of the additional load.

Before solving the optimization problem, it is determined the bus where the additional load is connected for each of the $k+2$ network configurations. Since the aim is to provide the DSO with a tool capable of identifying the maximum additional load that can be connected to each feeder regardless of its location, it is appropriate to select the bus representing the worst-case scenario from an operational standpoint, hereafter named “worst” bus. In doing so, it is certain that the maximum additional load calculated can be connected to any of the buses of the feeder, since it can be connected to the "worst" bus without violating any operational constraint. From a practical standpoint, this means that:

- In case 1, the “worst” bus is the one with the largest impedance from the primary substation.
- In case 2, the “worst” bus is the one with the smallest impedance from the primary substation. This implies the worst possible operational conditions for the feeder that restores the input feeder when it is affected by the fault.
- In case 3, the “worst” bus is the “cutting” node between the input feeder and the feeder affected by the fault. This implies the worst possible operational conditions for the input feeder, which has to restore the feeder affected by the fault.

In each iteration, once the network configuration and the “worst” bus have been identified, the optimization problem is solved and the value of the maximum additional load that can be connected to the feeder in the considered network configuration is calculated.

At the end of all the iterations, “k+2” additional load values are obtained. Some of them may be disregarded, specifically in configurations where the input feeder restores one of the feeders interconnected with it but step 1 of the methodology does not identify the input feeder as the “natural back-up feeder”. The smallest one among all the remaining additional load values is the maximum additional load that can be connected to the feeder without impairing the N-1 security criterion. With the aim to better explain the proposed methodology, consider the example reported in Fig. 2.

The proposed grid is made up of four feeders: “Feeder 1” is the input feeder, whose maximum additional load is the object of the study. The other three feeders (k=3) are directly interconnected to the input feeder by means of normally open tie switches, as shown in Fig. 2.

In the first step of the methodology the compliance of the N-1 security criterion for the input feeder is verified by solving the optimization problem illustrated in Section 3.2. Feeder 1 meets the N-1 criterion

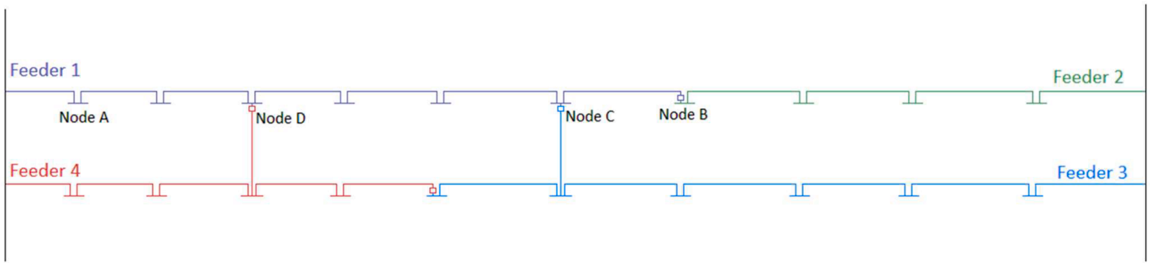

Fig. 2. One-line diagram of the illustrative network for the maximum additional load study. Feeders are differentiated by color and white squares represent normally open load switches.

and is identified as the "natural back-up-feeder" of Feeder 3, whereas other feeders are "natural back-up-feeder" of Feeder 2 and Feeder 4. The second step of the methodology then requires the solution of five different optimization problems, specifically:

1. Maximum load connectible to Feeder 1 in normal operation: the additional load is connected to node B, which is the node with the largest impedance from the primary substation.
2. Maximum load connectible to Feeder 1 while Feeder 1 is affected by a fault and thus its first branch is open: the additional load is connected to node A, which is the node with the smallest impedance from the primary substation. This ensures that all loads, included the additional load, can be restored by another feeder anywhere a fault occurs along Feeder 1.
3. Maximum load connectible to Feeder 1 while Feeder 2 is affected by a fault and thus its first branch is open: the additional load is connected to node B, which is the "cutting node" between Feeder 1 and Feeder 2. This ensures that, anywhere a fault occurs along Feeder 2, all loads connected to Feeder 2 can be restored despite the additional Feeder 1 load demand.
4. Maximum load connectible to Feeder 1 while Feeder 3 is affected by a fault and thus its first branch is open: the additional load is connected to node C, which is the "cutting node" between Feeder 1 and Feeder 3. This ensures that, anywhere a fault occurs along Feeder 3, all loads connected to Feeder 3 can be restored despite the additional Feeder 1 load demand.
5. Maximum load connectible to Feeder 1 while Feeder 4 is affected by a fault and thus its first branch is open: the additional load is connected to node D, which is the "cutting node" between Feeder 1 and Feeder 4. This ensures that, anywhere a fault occurs along Feeder 4, all loads connected to Feeder 4 can be restored despite the additional Feeder 1 load demand.

As a numerical example, consider that the maximum load values connectible to Feeder 1 in the five optimization problems are the one listed in Table 1.

Results of problems number 3 and number 5 can be disregarded, since Feeder 1 is the "natural back-up feeder" only of Feeder 3, whereas both Feeder 2 and Feeder 4 are restored by other feeders.

The smallest additional load value of the three remaining problems is 4 MW, which is the maximum additional load connectible to Feeder 1 without impairing Feeder 3 *N*-1 security criterion. This implies that the "worst case" scenario for Feeder 1 is the one represented by a fault occurring on Feeder 3, which is restored by Feeder 1.

Table 1
Results of the five optimization problems of the illustrative example shown in Fig. 2.

Problem No.	Input feeder	Feeder affected by a fault	Maximum additional load connectible to the Input Feeder (MW)
1	Feeder 1	None	6
2	Feeder 1	Feeder 1	5
3	Feeder 1	Feeder 2	2
4	Feeder 1	Feeder 3	4
5	Feeder 1	Feeder 4	3

The analysis is performed for all the feeders of the MV grid. If one of the optimization problems (considering only the combinations that are not disregarded) is infeasible, then the maximum additional load connectible to the feeder is zero. Since the optimization problem, which is described in detail in section 4.2, is based on simplifying assumptions, after each solution is found a full AC load-flow simulation is performed to check if operational limits of all the components are fulfilled. In case at least one operational limit is exceeded, the optimization problem is solved iteratively by tightening operational constraints in each iteration until no violation is found in the AC load-flow simulation. This procedure is illustrated by the block diagram in Fig. 3.

The architecture of the proposed methodology is such that the maximum connectible additional load can be assessed one MV feeder at a time, and not for multiple feeders at once. Such an architecture was chosen to be more adherent to the real-life applications: in fact, DSOs normally receive new connection requests in different moments and for each request must decide whether the existing grid is suitable to host the new load or a new MV line must be installed.

4.2. Optimization problem

The input grid is the same described for the "service restoration" optimization problem. Among the feeders of the grid, one is the input feeder, while another one (that can coincide or not with the previous one) is the one with its first branch open that has to be restored by another feeder.

If the two above mentioned lines are different from one another, to ensure that only the normally open switch between the two lines is manoeuvred the subset ***BR'*** is required. All the branches that do not belong to ***BR'*** cannot be manoeuvred. The complete optimization problem is reported below.

- *Objective function*

$$f = -L' \tag{17}$$

- *Non-negativity constraints*

Equations (2)-(3)

$$L' \geq 0 \tag{18}$$

- *Radiality constraints*

Equations (4)-(9)

- *Operational constraints of the grid components*

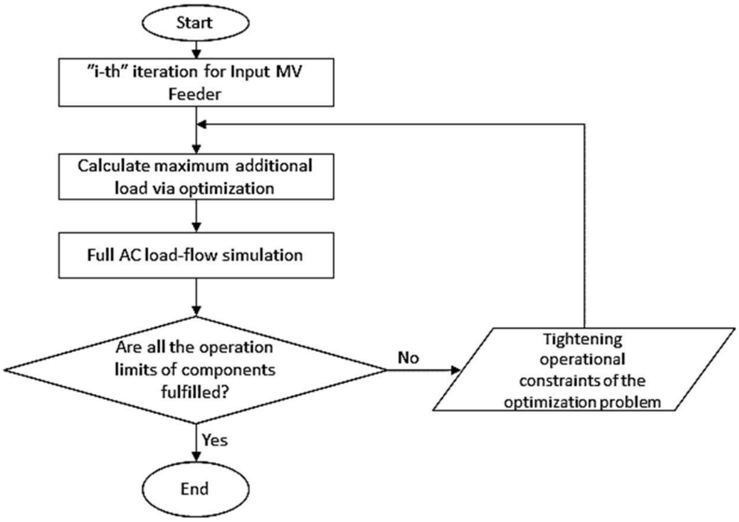

Fig. 3. Block diagram of the iterative procedure that calculates the maximum additional load for an MV feeder.

Equations (10)-(13)

- *Additional constraints*

Equation (14)

$$0 \leq \left(s_{ij}^{0} - s_{ij}\right) \leq 0, \forall (i,j) \notin \mathbf{BR}' \tag{19}$$

- *Kirchhoff's laws*

Equation (16)

$$\sum\nolimits_{j:(i,j)\in\mathbf{BR}} I_{ji} = -L_j - L' + I_j^G, \forall j \in \mathbf{B}', \forall i : (j,i) \in \mathbf{BR} \tag{20}$$

$$\sum\nolimits_{j:(i,j)\in\mathbf{BR}} I_{ji} = -L_j + I_j^G, \forall j \in \mathbf{B}\backslash\mathbf{B}', \forall i : (j,i) \in \mathbf{BR} \tag{21}$$

In the optimization problem, the objective function (17) maximizes the additional load demand *L*' connectible to the feeder under study. Constraint (18) imposes the non-negativity of the additional load demand, whereas (19) allows manoeuvring only tie-switches. Constraints (20) and (21) enforce the Kirchhoff's current law in the bus selected for the additional load demand and for all other buses, respectively.

5. Numerical results

The study presented in this paper was conducted on the whole MV distribution grid of Rome.

The model is implemented in MATLAB environment, interfaced with the CPLEX 12.9 solver by YALMIP [28] using an Intel Core i7 × 2GWh CPU with 32 GB RAM; the execution time of the optimization model is about 13 hours.

This section at first presents detailed results of the applications of the maximum additional load study on a 67-bus grid, then the results of the application on the whole MV distribution grid of Rome, Italy, operated by Areti S.p.A.

5.1. 67-buses 4-feeders MV grid

The one-line diagram of the grid is shown in Fig. 4.

The grid is made up of four 20 kV feeders; each feeder is directly interconnected with two other feeders as in the following:

- Feeder 1 is directly interconnected with Feeder 4 and Feeder 2
- Feeder 4 is directly interconnected with Feeder 1 and Feeder 3
- Feeder 2 is directly interconnected with Feeder 1 and Feeder 3
- Feeder 3 is directly interconnected with Feeder 4 and Feeder 2.

The overall branch length is 51 km; branches are 3 × 1 × 185mm^2 Aluminium conductor underground cables (40.7 km aggregate length) or 3 × 1 × 150mm^2 Copper conductor underground cables (10.3 km aggregate length).

The total load amounts to 8.2 MW, subdivided as follows:

- 2.3 MW connected to Feeder 1
- 2.2 MW connected to Feeder 4
- 1.8 MW connected to Feeder 2
- 1.9 MW connected to Feeder 3.

Results obtained by the "service restoration" problems (step 1 of the presented methodology) are:

- Feeder 1 is restored by Feeder 2
- Feeder 4 is restored by Feeder 3
- Feeder 2 is restored by Feeder 3

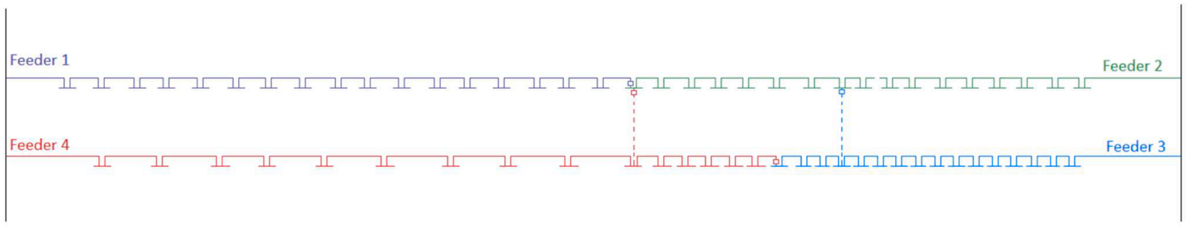

Fig. 4. One-line diagram of the 67-buses 4-feeders MV network: feeders are differentiated by color; white squares represent normally open load switches and dotted lines represent normally open bus tie breakers.

Table 2
Results of the 67-buses test network.

Problem No.	Input feeder	Feeder affected by a fault	Maximum additional load connectible to the Input Feeder (MW)	Combination to disregard
1	Feeder 1	Feeder 1	3.83	0
2	Feeder 1	Feeder 2	5.51	1
3	Feeder 1	Feeder 4	5.85	1
4	Feeder 1	None	7.69	0
5	Feeder 2	Feeder 1	4.96	0
6	Feeder 2	Feeder 2	2.52	0
7	Feeder 2	Feeder 3	4.48	1
8	Feeder 2	None	6.13	0
9	Feeder 3	Feeder 2	5.7	0
10	Feeder 3	Feeder 3	5.55	0
11	Feeder 3	Feeder 4	7.26	0
12	Feeder 3	None	9.1	0
13	Feeder 4	Feeder 1	6.77	1
14	Feeder 4	Feeder 3	7.26	0
15	Feeder 4	Feeder 4	5.55	0
16	Feeder 4	None	9.16	0

- Feeder 3 is restored by Feeder 4.

Once the preliminary “service restoration” study is completed, the maximum connectible additional load is then investigated: the study requires the solution of 16 optimization problems, four for each feeder (“$k+2$”, being “k” the number of lines directly interconnected with each feeder, equal to 2 for each one of the four feeders). Results of all the optimization problems are reported in Table 2.

Results of 4 out of 16 optimization problems have to be disregarded, since they represent combinations of lines that are normally used to restore one another (the input feeder is not the “natural back-up feeder” of the one affected by the fault).

After solving all the 16 optimization problems the maximum connectible load to each feeder in the second column of Table 2 is the minimum between results in column 4 referred to combinations not disregarded, namely:

- Feeder 1: 3.83 MW (the most restrictive scenario is the one where Feeder 1 is affected by a fault and has to be restored)
- Feeder 2: 2.52 MW (the most restrictive scenario is the one where Feeder 2 is affected by a fault and has to be restored)
- Feeder 3: 5.55 MW (the most restrictive scenario is the one where Feeder 3 is affected by a fault and has to be restored)
- Feeder 4: 5.55 MW (the most restrictive scenario is the one where Feeder 4 is affected by a fault and has to be restored).

For all the feeders the worst additional loading condition, from the perspective of component’s operating conditions relative to their operational limits, is the fault condition. If the additional load on the feeder under study exceeds the maximum additional load value resulting from the optimization problem, the feeder becomes no longer capable of being counter-supplied.

5.2. MV distribution grid of Rome

The whole MV distribution grid of Rome serves approximately 1600000 users located in an area of 31000 km^2. The MV grid is supplied by 70 HV/LV substations and is made up of 1500 MV feeders with an extension of 10600 km, mostly underground cables, supplying power to roughly 13300 MV/LV substations. Peak power demand amounts to 2.2 GW. The whole study involves the resolution of 6000 optimization problems:

- 1500 problems find the maximum additional load connectible to the feeders in normal operation
- 1500 problems find the maximum additional load connectible to the feeders when affected by a fault, i.e. their first branch is open
- 3000 problems find the maximum additional load connectible to the feeders while one of the interconnected feeders is affected by a fault, i.e. its first branch is open.

From the solution of all the optimization problems, the following conclusions emerge:

- For about half of the feeders the worst additional loading condition happens when a fault affects the feeder. If the additional load on the input feeder exceeds the maximum additional load value resulting from the optimization problem, the feeder cannot be restored by any other feeder interconnected to it.
- For about half of the feeders the worst additional loading condition happens in case a fault affects one of the interconnected feeders. If the additional load on the input feeder exceeds the maximum additional load value resulting from the optimization problem, the feeder cannot restore the other feeders of which it ss the “natural back-up feeder”.

Main results of the study are summarized in Fig. 5. In Fig. 5a MV feeders are clustered based on the maximum additional load that can be connected expressed in percent of their rated capacity. The obtained distribution shows that, on average, the MV feeders are individually able to host an additional load equal to 43% of their rated capacity, without impairing the N-1 security criterion. The standard deviation of the distribution is 25%. For the top 20% of the feeders, an additional load value greater than or equal to 60% of the feeder rated capacity can be connected, whereas the bottom 20% feeders are able to host an additional load less than or equal to 20% of their rated capacity. Clustering the MV feeders based on the absolute value of the maximum additional connectible load, it is obtained that about 25% of the MV feeders cannot host more than 1 MW additional load, whereas an additional load greater than 3 MW could be connected to approximatively 47% of feeders; lastly, about 12% of feeders are able to host more than 7 MW additional load.

6. Conclusions

The paper presents a methodology that can be used by DSOs to assess the maximum additional load that connectible to each feeder of a medium voltage distribution grid not impairing the N-1 security criterion.

To find such additional load a preliminary study must be carried out with the goal of assessing the “restorability” status of each MV feeder. After determining whether the feeder meets the N-1 security criterion or

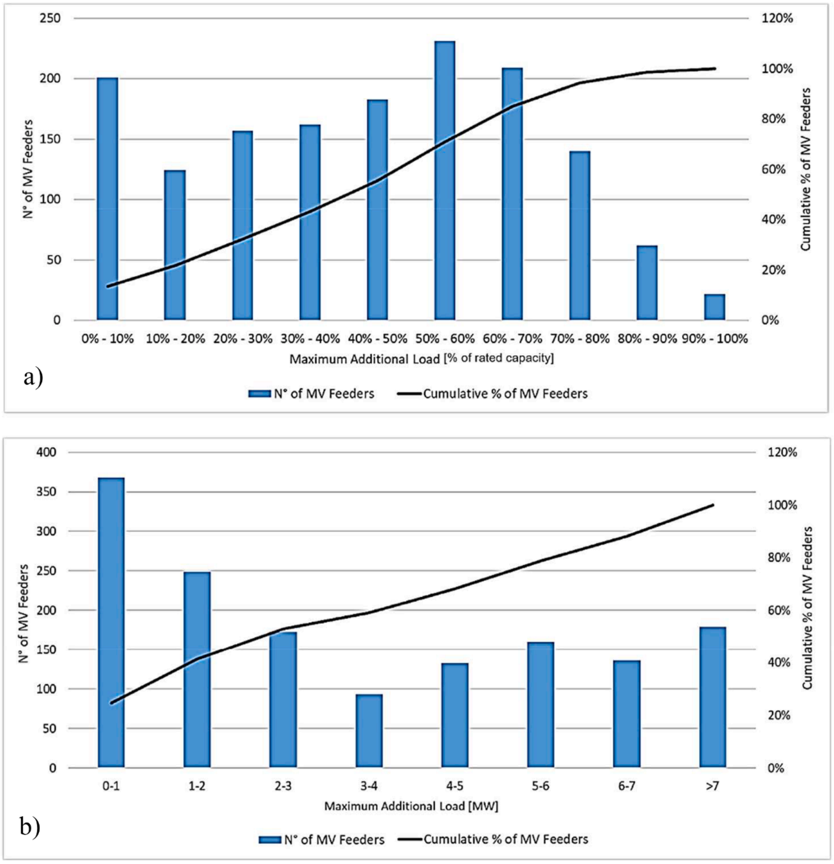

Fig. 5. Maximum additional load connectible to the MV feeders in the distribution grid of Rome: a) in % of the feeder rated capacity; b) in MW.

not, the maximum additional load that the feeder is able to host is found by the solution of multiple optimization problems. The combination of the two analyses allows the DSO to understand whether the grid is able as it is to host the additional load or if new infrastructures are needed. The methodology can be applied to any medium voltage distribution network provided that the peak demand of each MV/LV secondary substation is known.

The presented methodology is currently employed by Areti S.p.A, the DSO of Rome, Italy. Execution times allow the DSO to apply the model daily, even in case of a very large distribution grid, following the evolution of the load day by day and providing useful data on the ability of any MV feeder of the grid to host additional load. The effects of the load growth are therefore included in the methodology leveraging its day-by-day application; moreover, different load profiles may be provided in input of the methodology so that the foreseen load growth could be taken into account by modifying the input data according to the desired scenarios.

CRediT authorship contribution statement

T. Bragatto: Writing – original draft. **F.M. Gatta:** Validation, Supervision. **A. Geri:** Supervision, Formal analysis, Conceptualization. **M. Maccioni:** Writing – review & editing, Validation, Supervision, Methodology, Conceptualization. **A. Palazzoli:** Supervision, Data curation. **P. Sancioni:** Writing – original draft, Software, Methodology, Investigation, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available on request.

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