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Contents

Different Processes for Graphical Recognition of Derivative of a Function: An Eye-Tracker Analysis

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Abstract The goal of this research is to uncover the struggles that students face when working on Calculus tasks. This study uses a qualitative analysis method, with the help of an eye-tracking tool, to examine the variations and similarities in cognitive processes between experts and novices. Using an eye-tracking tool, we can observe an individual's ability to move between different forms of representation. The Calculus tasks focus on the concepts of derivatives, specifically, students were asked to identify the graph of a derivative function.

Keywords Eye-tracker · Semiotic registers · Mathematics education

1 Introduction and Literature Review

Eye tracking has been used in several research areas, such as psychology, neuroscience, linguistics, and creativity of cognitive processes [1]. This tool has been applied to analyze cognitive processes in areas such as geometry [2], algebra [3], and interpretation of motion graphs [4]. Recent research has used the eye-tracker tool to study cognitive processes in high schools ([5, 6]). A study by Just and Carpenter [7] showed that a person's gaze provides valuable information about his or her cognitive and learning processes, a hypothesis known as the eye-mind hypothesis. In addition, the eye-tracker tool allows analysis of the ability to switch between registers. Recently it has also been used as a tool in mathematics education. Using this tool,

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it is possible to collect data on eye movements and provide insights into cognitive processes. In the context of mathematics education research, by studying eye movements, it is possible to analyze the problem-solving process in a mathematical task.

Although the cognitive pathway followed in tackling a mathematical problem is certainly influenced by external factors (such as educational, socio-cultural, technological, etc.) that contribute to the strategy for arriving at the answer, the influence these factors have on the analysis carried out with the eye-tracker is, in our opinion, insignificant, as the eye-tracker tool is in fact unobtrusive: what a candidate has to do is simply observe a screen and this does not affect the ability to determine the answer to the question, the latter being the prerogative solely of the internal abilities of the candidate, whether novice or expert. On the other hand, eye-tracker technology offers many advantages: it reduces the influence of social expectations (e.g. the shyness of students who are afraid of making mistakes or who fear that their method of solving is inadequate), as well as some perceived rules in the context of educational contract theory during the data collection process (the student essentially assumes freer behaviour). In addition, eye-tracking data are less affected by problems related to memory, introspection, metacognitive reflection, language problems or verbalization of solutions found, all of which are related, for example, to students thinking aloud.

Students in mathematics and science faculties encounter many difficulties in the transition from secondary to tertiary education. These difficulties can be grouped into categories such as epistemological and cognitive, sociological, and cultural and educational. Students beginning a new course of study encounter new concepts and approaches that may not be directly related to their previous studies. For example, one epistemological and cognitive difficulty is the lack of in-depth understanding of a mathematical concept. High school students often rely on algorithms and procedures but may not fully understand mathematical objects. In addition, taking notes and reading texts can be a problem for high school students [8]. This aspect is relevant to our research because the way a student reads a problem can give us information about cognitive processes.

Several research studies have explored the differences in the presentation of mathematical concepts between secondary and tertiary education. One issue that has been identified regarding the transition between secondary and tertiary school is the different way in which mathematical objects are used. In high school, students tend to use mathematical objects in their calculations, while at university, they need to be able to handle a higher level of abstraction. Specifically, university requires students to be able to deal with theoretical aspects, whereas in secondary school this aspect is never considered [9]. Praslon [10] showed an example on the different tasks presented in university and high school on the concept of the derivative. Through comparing the tasks, he showed that the derivative is seen as a tool in high school, while university students study it as a mathematical object. Moreover, university requires students to be able to switch from a register to representations to another [11] and between frames [12]. The different approaches to the concept of the derivative are emphasized by these elements [13]. There is a lack of mastery of mathematical objects and difficulty in switching representation registers.

Our research aimed to study the processes involved for students in Calculus task related to the concept of derivative. Our work focuses on the ability to read a graph and be able to infer relevant information and find relationships at the level of syntax, discern its overall meaning, and position it correctly in relation to the phenomenon it is modelling.

Our research questions are: What information can be inferred by looking at the graph of the derivative and how can it be used? Are there significant differences between experts and novices in solving strategies based on the graphical properties of derivative functions?

2 Methodolody

The research participants were divided into novices and experts. Novices were composed by 13 university students: 6 students from engineering's faculty, 3 from physics, 2 from chemistry, 1 from mathematics, and 1 from statistics. The expert group was composed by 3 mathematics undergraduates, 2 doctoral students, and 2 high school teachers.

The analysis was focused on the study of visual behaviour through the eye-tracker tool. In particular, the elements that characterize eye movement were studied: fixation, saccade, and scan path. The first is a prolonged gazing at a specific point in the visual field, typically lasting for a few tenths of a second. Its key measures are the length of time it lasts and the number of fixations. In contrast, a saccade is a quick movement of the eyes between two fixations, and its key measures are the distance of the movement and the frequency of saccades. The last is the study of the pattern of fixations and saccades over time, and it can reveal information about where a person is directing their attention and what they are focusing on.

The instruments used to analyze the data collected with eye-tracker tool are heat maps and gaze plots. A heat map is a visual representation that shows where a person's attention is concentrated while viewing a specific image or scene. It uses colours to indicate the level of attention given to different areas, with warmer colours indicating areas where the person spent more time looking or looked more frequently. The gaze plot, on the other hand, is a tool that shows the order and timing of where a person is looking. It displays the duration of fixations on an object by using circles, with the size of the circle representing the amount of time spent looking at that object. Larger circles indicate longer fixations. For the test, we employed the Tobii Pro Nano® system, a screen-based eye-tracking device that uses pupillary and corneal reflections with both dark and bright pupillary lighting modes to gather data. The data obtained are then analyzed by a specialized processing unit that utilizes dedicated software to calculate gaze data, as well as to collect, observe and analyze both individual and aggregated data for comparisons, as used in this study.

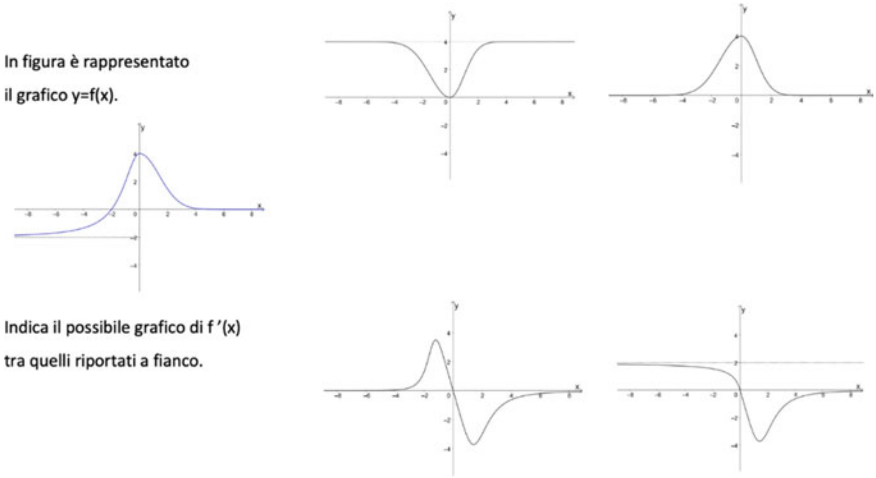

Fig. 1 Identifying the derivative function

The test was conducted using an image that was displayed on a monitor connected to the eye-tracking camera. The participant was given a task with no time limit, and their eye movements were recorded by the eye-tracker as they viewed the image. After selecting the correct answer, the participant was then interviewed and asked to explain their reasoning for their choice. During the interview, the task was shown again, and the audio recording was synced with the eye-tracking data, in order to compare the participant's eye movements with their cognitive motivations, as well as to analyze the participants' reasoning abilities.

The task proposed was the following:

The text of the question shown in Fig. 1 is as follows: *In the figure the graph* $y = f(x)$ *is represented. Select the possible graph of* $f'(x)$ *from those shown alongside.*

The different graphs were labelled with letters (a), (b), (c), and (d), respectively, from the top left to the bottom right. These labels were not included in the main question image to avoid adding any extraneous distractions.

3 Analysis and Discussion

From the heat maps of expert candidates, it can be seen that they tend to focus on the main image for a longer period of time, while they quickly scan through the alternative options. This suggests that their visual analysis of the main image is more detailed and local, while their analysis of the alternatives is more general and global. The approach that expert candidates seem to prefer is to first identify general characteristics of the main image, and then choose a characteristic that is unique to it, and compare it to the alternatives, paying more attention to the details of the chosen alternative. Once this unique characteristic is found, the question is considered solved and further verification is minimized. The choice of this unique characteristic is influenced by the individual candidate's preferences, experiences, and the type of main image. It also affects the cognitive approach used and the visual pattern recorded by the eye-tracker.

Figure 2 shows an example of a heat map of expert candidate that focused exclusively on the presence of a non-horizontal tangent inflection points contained within the interval (− 2, 0). We can see a clear focus of visual and cognitive attention on a specific point in the main image. The rest of the graph is glanced over quickly with many short saccades, following the overall trend of the entire domain, up to the horizontal asymptotes. It is worth noting that the candidate paid more attention to the single zero in the function, which was classified as a distinct point, than to the maximum point. It turns out that the zero was a red herring and the maximum point was not used since another unique feature was identified first.

Candidate stated: "initially I was struck by the point where the function changed sign" […] "immediately afterwards I noticed the point of inflection and therefore the derivative must have a maximum point because the function passes from convex to concave". From this statement, two observations are clear: on one hand, the left-handed inflection point was identified by chance as the closest element to the zero of the function, suggesting that the gaze guides the brain's processing of the information. On the other hand, the subsequent cognitive process, based on prior theoretical knowledge, allows for the deduction of a graphic property of the response even before it is observed. In this case, the process is reversed: the cognitive process guides the visual observation, which becomes a mere verification of what has already been hypothesized. The candidate then states "in fact, graph (c) is the only one that presents this characteristic". Some long saccades between the main image and the

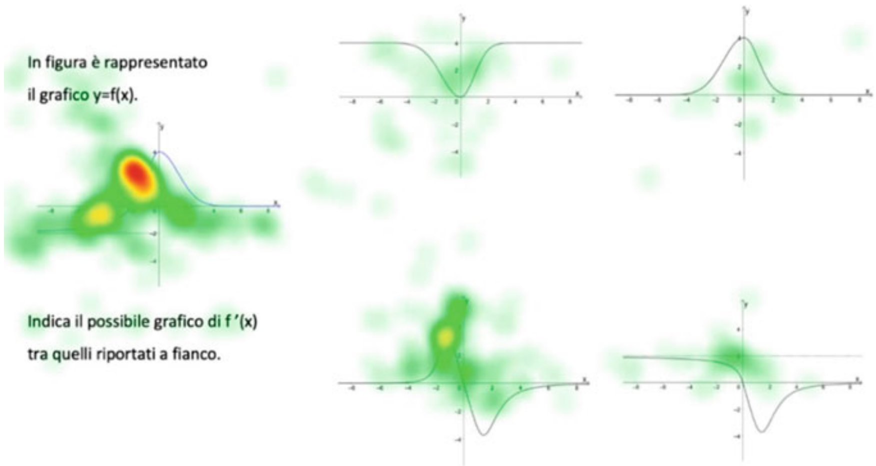

Fig. 2 Heat map of expert candidate

four alternatives help to identify the uniqueness of the chosen property, which is confirmed by some fixations on the maximum point of alternative (c). Finally, the candidate says “then I just quickly verified that (c) also met other criteria, such as the sign”. The latter property was used only as confirmation of his choice.

A typical non-expert candidate’s eye movement pattern is more intricate compared to that of an expert candidate. The first noticeable feature is that it is more disorganized, with a general tendency of many short saccades within a single graph, and also many long saccades between graphs. The long saccades indicate a high level of comparison activity between the main image and each alternative, as well as between the different alternatives. The distribution of fixations among the five graphs in the question is consistent among non-expert candidates. This can be attributed to two factors: lack of expertise in selecting specific elements, and the need for more verification once a perceived correct answer has been identified.

An example of a heat map of a non-expert candidate is shown in the Fig. 3. The heat map of this candidate clearly shows a focus on the central area of the main graph, specifically between the two inflection points and in the central area of the four alternatives. The textual section only draws attention when it contains the symbolic/algebraic notation ($y = f(x)$ and $f'(x)$). During the interview, the candidate explains: “I started by trying to understand how the function $f(x)$ was constructed”. This candidate seems to have a strong inclination for the elimination method rather than the construction method of finding the solution. In fact, he, on one hand, looks to confirm that the information obtained from the main image is present in one or more of the alternatives, and on the other hand, searches for elements that eliminate certain alternatives. This is especially evident in the comparison of the main image and alternatives (c) and (d) as indicated by the eye-tracker through the presence of long saccades between these graphs.

4 Conclusions

A key aspect of our study is the effort to discern unique similarities and disparities in the approach used by experienced and novice candidates. Regarding the second research question, we can observe that there is a different approach between experts and novices. In fact, the expert candidate identified an exclusive characteristic that allows him to find the correct alternative and used another property to confirm the choice. In contrast, the novice used a confused approach, and the correct answer was chosen by proceeding by exclusion. We can make the following observations: experts and novices had different behaviour during the visual acquisition of information in alternatives, in fact different points of interest and eye trajectories were recorded; need to acquire more information by novices than experts to solve the task. They had a similar behaviour in the visual acquisition of information in stimulus graph, they focused the attention on the similar points and areas of interest.

In general, the stage of collecting information is related to generating hypotheses, while the selection of decisive elements for solving the question is linked to the

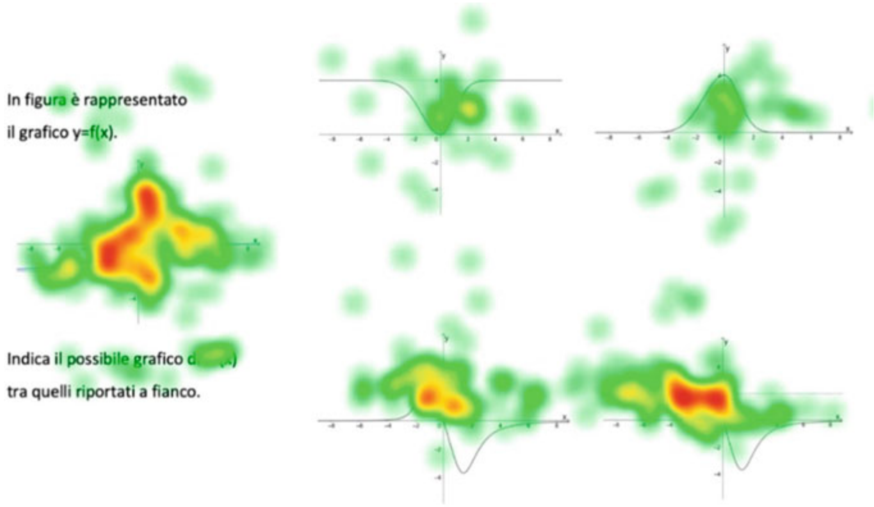

Fig. 3 Heat map of a non-expert candidate

ability to make analytical deductions and to verify them. This means that the novice's overabundance of gathered data hinders or diminishes their ability to generate valid hypotheses, resulting in an increase in difficulty in making deductions and verifying them. The presence of distracting elements both in strategy formation is an indication of this: the novice's ability to selectively process information is diminished as they have a harder time discarding previously evaluated hypotheses, and the abundance of information gathered poses a challenge as it complicates the decision-making process. Regarding the first research question, the information acquired from the graph is different and is used differently by an experienced candidate and a novice. Specifically, the expert candidate was able to infer certain exclusive properties that guided his or her solving process, while the novice gathered much data from the graph and they had difficulty classifying what was essential for solving the task.

In this work, we attempted to identify the cognitive processes involved in the candidates' approach to the given questions by analyzing specific eye movement patterns and to encode them, to try to obtain a more abstract analysis of the problem-solving strategies used. However, different eye movements also indicate how these mental processes are executed and failed attempts to extract useful information for solving the problem. The cognitive process underlying the resolution of such questions is extremely complex, involving simultaneous sensory (visual) processing of various stimuli, which are all different in nature. This is particularly true when transitioning from a graphical representation of information in the stimulus to another graphical representation in the alternatives, all mediated by the candidate's prior theoretical knowledge acquired during a course on mathematical analysis.

From the discussion above, certain challenges in the learning of mathematical analysis for students in their first year of a scientific academic programme are clear.

These difficulties may stem from the complicated transition from high school to the first year of university [14–16], where students not only experience changes in teaching methods but also a diversification of goals set by university mathematics courses. The findings presented in this paper provide, in our view, some suggestions for enhancing the teaching of mathematics at the university level. Generally, students need to learn how to approach a mathematical question and how to focus their attention to gather the necessary information to solve the problem at hand. Mason [17] argues that learning is an education in awareness, which is closely tied to attention and observation. Additionally, we think it is important to foster students' capacity to generate hypotheses and make analytical deductions, as this is a crucial step in any form of scientific reasoning, be it theoretical or practical.

The main limitation of the experiment we presented consists of the fact that the most delicate and complicated moment in a study based on the eye-tracker is precisely the analysis of the data obtained through the experiment and the reconstruction of the resolution strategy on the basis of the spontaneous eye movements recorded by the eye-tracker, due to the impressive amount of data at our disposal. For this reason, we repeated the experiment during the subsequent interview with the candidates by synchronizing the audio with the video in order to assess the candidate's awareness in the resolution of the question, measuring and subsequently interpreting the level of coherence between what was consciously stated and what was previously done during the actual resolution of the question. Not infrequently, especially for novice candidates, the results of the two phases show, in our opinion, significant differences. This work took a lot of time because we had to examine the recorded video of eye movements and listen to the audio of the interview several times, trying to eliminate non-essential and especially uncertain information. This time was a major limitation for our experiment because it reduced the number of candidates examined and only allowed us to make qualitative analyses.

Finally, we conclude by noting that socio-cultural aspects are undoubtedly a key element in the elaboration of solution strategies when approaching a mathematical question. It is in fact through our sensorimotor experiences with our surroundings that our cognitive attention is constantly captured. The eye is certainly one of our sensory organs that is most stressed and stimulated by the external environment. But visual perception is understood not only as a physiological substrate, but above all as a culturally and socially shaped sense that allows us to access mathematical representations in a certain structured way. From this point of view, therefore, the eye-tracker instrument could also be a useful investigative tool for future research, for example by administering the same test to groups of candidates from different social backgrounds or geographical areas, or with different prior education to see if the data obtained show significant differences in strategic choices. Of course, all this would be meaningful if a large amount of data could be made available (e.g. through international collaboration), a sufficiently robust quantity for significant statistical analysis.

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