Discrete-time energy-balance passivity-based control

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Abstract

In this paper, new results for passivation and stabilization of discrete-time nonlinear systems via energy balancing are established. When specified on sampled-data systems, the approach is constructive for computing stabilizing digital controllers that assign, at all sampling instants, a target energy profile while stabilizing a target equilibrium. The class of mechanical systems is discussed as an example. Simulations are reported highlighting, for position regulation of a 2R robot, the effect of approximate solutions with respect to standard emulation.

Key words: passivity-based control; nonlinear discrete-time systems; digital implementation; asymptotic stabilization; sampled-data stabilization.

1 Introduction

Passivity-based control (PBC) via energy balancing (EB) represents the first step toward energy-based control at large. The aim stands in stabilizing a given passive system at a desired equilibrium $x_\star$ by assigning a target energy behavior possessing a minimum at $x_\star$. This method is remarkably appealing and applies to classes of dynamics including fully actuated mechanical ones. Starting from a passive system, for which the storage function represents the energy, the idea is to design a controller so as to modify the energy consumption of the system; more precisely, the feedback is designed so that the stored energy is equal to the difference between the stored and supplied energies (Astolfi et al., 2001; Hatanaka et al., 2015; Jeltsema et al., 2004; Ortega et al., 2001). This yields a simple but still elegant procedure to reshape the energy to achieve stabilization of a target equilibrium. The class of systems for which a solution exists is however quite restrictive because of the so called dissipation obstacle limiting the application to systems with no pervasive damping (i.e., with finite dissipation, Ortega and Mareels (2000)).

However, all of this makes reference to continuous-time dynamics with only a few results for discrete-time systems (Lopezlena and Scherpen, 2002), which are significant for a large variety of processes in the information industry and, most important, digital systems. As already mentioned, continuous-time EB-PBC applies to Lagrangian systems which are however controlled via digital devices; this justifies the need of a similar stabilizing procedure in the discrete-time framework. Besides the practical interest, the purely discrete-time case is always challenging even from a theoretical perspective. This is due to the typical technical issues to face as, for instance, the implicit characterization of the control law and the standard definition of passivity making sense only for dynamics with direct throughput (Byrnes and Lin, 1994; Laila and Nešić, 2003; Monaco and Normand-Cyrot, 1999; Navarro-López, 2005; Navarro-López and Fossas-Colet, 2004).

Part of those issues have been addressed in several of the authors' contributions describing in an algebraic-differential framework the basic control aspects, such as passivity (Monaco and Normand-Cyrot, 2011), feedback passivation (Mattioni et al., 2021) and, more recently, discrete-time Hamiltonian dynamics (Moreschini et al., 2021). Starting from those works, the objective of this

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paper is to extend EB-PBC design to discrete-time dynamics first and then specify the solution to dynamics issued from sampling. More in details, the contributions of this work are listed below.

- Discrete-time EB-PBC strategies are characterized in terms of the solution to an algebraic equality that is the counterpart of the partial derivative one typical in continuous time.
- When dealing with dynamics issued from sampling (i.e., continuous-time dynamics under piecewise constant control with state measures available only at the sampling instants), it is shown that the existence of a continuous-time solution implies the existence of a discrete-time solution for the equivalent sampled-data dynamics. Accordingly, the existence of a continuous-time EB-PBC implies the existence of a piecewise-constant EB-PBC feedback law with the same energy as in continuous time so that the sampled-data average dissipation obstacle is no more conservative than the continuous-time one.
- The proof is constructive. The piecewise constant EB-PBC admits the form of a series expansion in powers of the sampling period around the continuous-time solution. Accordingly, approximate solutions defined as truncation of the series solutions at a desired order are easily computable.
- Finally, the case of fully actuated mechanical systems is discussed and the corresponding EB-PBC stabilizer is explicitly constructed. Simulations highlight improvement with respect to standard emulation.

The paper is organized as follows. In Section 2, after a few recalls on continuous-time EB-PBC, the necessary background to handle discrete-time dynamics and passivity is given. The main result on discrete-time EB-PBC is provided and proved for purely discrete-time systems in Section 3. In Section 4, the result is specified for sampled-data dynamics proving that the existence of a continuous-time EB-PBC implies the existence of a sampled-data one. The proof is constructive for the digital controller. In Section 5, the result is applied to fully actuated mechanical dynamics and specified for position regulation of a 2R robot with illustrative simulations. Finally, Section 6 concludes the paper.

Notations. Throughout the paper all the functions and vector fields are assumed smooth and complete over the respective definition spaces. Given a real-valued function $V(\cdot):\mathbb{R}^n\to\mathbb{R}$ assumed differentiable, $\nabla V(\cdot)$ represents the gradient column-vector with $\nabla = col\{\frac{\partial}{\partial x_i}\}_{i=1,\dots,n}$ and $\nabla^2 V(\cdot)$ denotes its Hessian. For $v, w\in\mathbb{R}^n$, the discrete gradient is a vector-valued function of two variables, $\bar{\nabla} V|_v^w : \mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$, defined as

$$\bar{\nabla} V|_v^w = col\Big\{\bar{\nabla}_i V|_v^w\Big\}_{i=1,\dots,n} = \int_0^1 \nabla V(v+s(w-v))ds$$

satisfying $V(w)-V(v) = (w-v)^\top\bar{\nabla}V|_v^w$ with $\bar{\nabla}V|_v^v = \nabla V(v)$. Given a vector-valued function $F(x) = col(F_1(x),\dots,F_n(x))$, $J[F(x)] = \{\frac{\partial}{\partial x_j}F_i(x)\}_{i,j=1,\dots,n}$ denotes the Jacobian of F. I and I_d denote respectively the identity matrix and identity operator. 0 denotes the 0 vector (or matrix) of suitable dimension depending on the context. Given a smooth vector field $f(\cdot)$ over $\mathbb{R}^n$, $\mathrm{L}_f = \sum_{i=1}^n f_i(x)\frac{\partial}{\partial x_i}$ is the Lie operator with, recursively, $\mathrm{L}_f^i = \mathrm{L}_f\mathrm{L}_f^{i-1}$ and $\mathrm{L}_f^0 = I_d$. Accordingly, the exponential Lie series operator is defined as

$$e^{\mathrm{L}_f} = I_d + \sum_{i\geq 1}\frac{1}{i!}\mathrm{L}_f^i.$$

Given two vector fields f and g on $\mathbb{R}^n$, $ad_f g = (\mathrm{L}_f\mathrm{L}_g - \mathrm{L}_g\mathrm{L}_f)I_d$ denotes their Lie-bracket and, recursively, $ad_f^i g = ad_f\circ ad_f^{i-1}g$ and $ad_f^0 g = g$. A function $R(x,\delta) = O(\delta^p)$ is said of order δ^p, $p\geq 1$ if, whenever it is defined, it can be written as $R(x,\delta) = \delta^{p-1}\tilde{R}(x,\delta)$ and there exist a function $\theta\in\mathcal{K}_\infty$ and $\delta^*>0$ s. t. $\forall\delta\leq\delta^*$, $|\tilde{R}(x,\delta)|\leq\theta(\delta)$. The symbols "$>0$" and "$<0$" denote positive and negative definite functions whereas $\prec$ and $\succ$ ($\preceq$ and $\succeq$) positive and negative (semi) definite matrices.

2 Discrete-time feedback passivation via Energy Balance

2.1 EB-PBC in continuous time: recalls

Let an input-affine system

$$\dot{x} = f(x) + ug(x), \quad x\in\mathbb{R}^n, u\in\mathbb{R} \tag{1a}$$
$$y = h(x) = \mathrm{L}_g H(x) \tag{1b}$$

possess an equilibrium at $x=0$ with $u=0$ so that $h(0)=0$ and assume that it is passive with storage function $H(\cdot):\mathbb{R}^n\to\mathbb{R}_{\geq 0}$ such that $H(0)=0$. In the context of the present study, referring to a physically inspired vocabulary, the system (1) naturally satisfies the so-called energy-balancing (EB) equality

$$H(x(t)) - H(x(0)) = \int_0^t u^\top(s)y(s)\mathrm{d}s - d(x(t)) \tag{2}$$

splitting the stored energy into the energy supplied by the control and a natural dissipation term $d(x(t))\geq 0$. Note that, by virtue of the well-known Kalman-Yakubovitch-Popov (KYP) conditions (Byrnes et al., 1991), there is no loss of generality in assuming the passive output of the form in (1b).

The so-called first generation of PBC, for stabilization at the origin, is based on damping injection

$u(t) = -Ky(t), K > 0$, with the same energy function as in open loop.

The second generation of PBC relies on energy shaping (ES): one seeks for a feedback law $u = \beta(x) + v$ making the closed-loop system passive with new (target) storage function embedding the desired energy properties, say $H_d(\cdot)$ and with respect to a suitably defined output function, say z. More in detail, consider a possibly non-zero equilibrium to stabilize, say $x_\star$, verifying $H_d(x_\star) = 0$ and $\nabla H_d(x_\star) = 0$; passivation through energy shaping is guaranteed if one finds $\beta(\cdot) : \mathbb{R}^n \to \mathbb{R}$, such that setting $u = \beta(x) + v$, the closed-loop system verifies the new EB equality

$$H_d(x(t)) - H_d(x(0)) = \int_0^t v^\top(s)z(s)ds - d_d(x(t)) \quad (3)$$

with new damping $d_d(x) \geq 0$.

EB-PBC belongs to this class as specified in the following definition.

Definition 2.1 (EB-PBC) *Consider the system (1) that is assumed passive with positive definite storage function $H(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$. Let $x_\star \in \mathbb{R}^n$ be a desired equilibrium. A feedback $u = \beta(x) + v$, is said to be a continuous-time EB-PBC if there exists a function $H_a(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ so that $\nabla H_a(x_\star) = -\nabla H(x_\star)$ and the closed-loop energy is equal to the difference between the stored and supplied energies when setting $H_d(x) = H(x) + H_a(x)$, namely*

$$\begin{aligned} H_d(x(t)) - H_d(x(0)) &= H(x(t)) - H(x(0)) \\ &- \int_0^t \beta(x(s))g^\top(x(s))\nabla H(x(s))\mathrm{d}s. \end{aligned} \quad (4)$$

Paraphrasing the requirement, the closed-loop system is passive with respect to the new output $z = g^\top(x)\nabla H_d(x)$ with new storage function $H_d(x) = H(x) + H_a(x)$ and stabilization with additional damping to the target equilibrium $x_\star$ is achievable under new negative output feedback $v = -Kg^\top(x)\nabla H_d(x)$ whenever $H_d(\cdot)$ qualifies as a Lyapunov function (i.e., $H_d(x_\star) = 0$ and $H_d(x) > 0$ in a neighborhood of the equilibrium).

The EB-PBC $\beta(\cdot)$ exists if a suitably defined partial-differential equality (PDE) is solvable, as detailed in the following result (Ortega et al., 2001).

Proposition 2.1 (CT-EB-PBC) *Consider the passive system (1) with positive definite storage function $H(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ and let $x_\star \in \mathbb{R}^n$ be a desired equilibrium. Then $u = \beta(x) + v$ is an EB-PBC in the sense of Definition 2.1 if $\beta(x)$ solves the PDE*

$$-\beta(x)g^\top(x)\nabla H(x) = (f(x) + \beta(x)g(x))^\top \nabla H_a(x) \quad (5)$$

for some function $H_a(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that $\nabla H_a(x_\star) = -\nabla H(x_\star)$.

Paraphrasing the result, the existence of an EB-PBC requires finding a function $\beta(x)$ such that the energy supplied by the controller can be expressed as a function of the state. EB-PBC is appealing for its simplicity and its wide range of applicability as characterized by the condition below.

Remark 2.1 *We note that* (4)-(5) *are not fully equivalent to $-uy = \dot{H}_a$. The EB conditions must not hold for all u but for a specific $\beta(x)$ that, together with $H_a(x)$, provides the solution to the problem. Accordingly, when setting $u = \beta(x) + v$, the energy is correctly shaped, with the new input-output passive link $v \mapsto z$.*

Remark 2.2 (Dissipation obstacle) *A necessary condition for passivation through energy-balancing (5) is that for all $\bar{x}$ such that $f(\bar{x}) + \beta(\bar{x})g(\bar{x}) = 0$, then $\beta(\bar{x})\mathrm{L}_g H(\bar{x}) = 0$. This corresponds to requiring that the extracted power from the control (i.e., $\beta(x)\mathrm{L}_g H(x)$) is zero at all equilibria.*

2.2 Generalities on discrete-time systems

Consider now a generic discrete-time system over $\mathbb{R}^n$ described in the form of a map as

$$x^+(u) = x + F(x,u), \quad u \in \mathbb{R} \quad (6a)$$

$$y(x,u) = h(x,u) \quad (6b)$$

where the time dependencies are dropped out; namely, $x = x_k$, $u = u_k$ and $x^+(u) = x_k^+(u_k) = x_{k+1}$, at each time step $k \geq 0$. The map $x^+(u)$ describes any curve in $\mathbb{R}^n$ parameterized by $u \in \mathbb{R}$ and, for convenience (even if not necessary), the output map $h(x,u) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$, is set u-dependent to cope with a standard passivity inequality (direct input to output link). We denote by $x^+ = x^+(0) = x + F_0(x)$ with $F_0(x) = F(x,0)$ the drift term associated to (6a). In the following, it is assumed that (6) possesses an equilibrium at $x_\star = 0$ when $u = 0$, that is, $F(0,0) = 0$.

In Monaco and Normand-Cyrot (1999), an alternative representation to (6) has been proposed to split the control-dependent part as

$$g(x,u)u = F(x,u) - F_0(x) = \int_0^u G(x^+(w), w)\mathrm{d}w \quad (7)$$

with $F_0(x) = F(x,0)$ and control vector field $G(\cdot,\cdot)$ over $\mathbb{R}^n$ so to satisfy

$$G(x + F(x,u), u) = \frac{\partial F(x,u)}{\partial u}. \tag{8}$$

Accordingly, the control action in the state equation (6a) is specified by (7) that gets the form

$$g(x,u) = \int_0^1 G(x^+(su), su)\mathrm{d}s$$

with $g(x,0) = G(F_0(x),0)$. We note that $g(\cdot,u)$ admits formal series expansion in powers of u of the form

$$g(x,u) = g_0(x) + \sum_{i>0} \frac{1}{(i+1)!} g_i(x)u^i$$

with, in particular, $g_0(x) = g(x,0) = G(F_0(x),0)$.. Among many, the interest of this representations relies on the possibility of splitting the variation of any real-valued function $V(\cdot)$ on $\mathbb{R}^n$ along the dynamics (6a) into free and control dependent components as

$$\begin{aligned} V(x^+(u)) - V(x) =& V(x^+(0)) - V(x) \\ &+ \int_0^u \mathrm{L}_{G(\cdot,w)} V(x^+(w))\mathrm{d}w \end{aligned} \tag{9}$$

with

$$\mathrm{L}_{G(\cdot,w)} V(x^+(w)) = \Big(\frac{\partial V(x,w)}{\partial x} G(x,w)\Big)\Big|_{x=x^+(w)}$$

the Lie derivative with respect to the first argument evaluated at $x = x^+(w)$. This is at the basis of the definition of average passivity for discrete-time systems proposed by Monaco and Normand-Cyrot (2011) and recalled below.

Definition 2.2 (Average passivity) *The system (6) with storage function $H(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is said average passive if it is passive with respect to the average output*

$$Y(x,u) = \frac{1}{u}\int_0^u y(x^+(w), w)\mathrm{d}w; \tag{10}$$

i.e., for all $(x,u) \in \mathbb{R}^n \times \mathbb{R}$, the inequality below holds

$$H(x^+(u)) - H(x) \leq \int_0^u y(x^+(w), w)\mathrm{d}w = uY(x,u). \tag{11}$$

From the inequality (11), stabilization at the origin is achieved by the feedback solution of the implicit damping equality $u = -KY(x,u)$, $K > 0$. Solving such implicit equality with respect to u may be a difficult problem unavoidable in discrete time. Approximate bounded solutions efficient for preserving stabilization can be easily computed as proposed in Mattioni et al. (2019); Mazenc and Nijmeijer (1998).

Because KYP-like necessary and sufficient conditions for passivity do not hold in discrete time in general, the following result has been proved in Monaco and Normand-Cyrot (2011).

Proposition 2.2 *Let the system (6) be passive with storage function $H(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$. Then it is average passive with respect to $h(\cdot,u) = \mathrm{L}_{G(\cdot,u)}H(\cdot)$ and, equivalently, passive with respect to the average output*

$$Y(x,u) = \frac{1}{u}\int_0^u \mathrm{L}_{G(\cdot,w)} H(x^+(w))\mathrm{d}w \tag{12}$$

with $G(\cdot,u)$ verifying (8); namely, for all $(x,u) \in \mathbb{R}^n \times \mathbb{R}$

$$\begin{aligned} H(x^+(u)) - H(x) &= H(x^+(0)) - H(x) \\ &+ \int_0^u \mathrm{L}_{G(\cdot,w)} H(x^+(w))\mathrm{d}w \leq uY(x,u). \end{aligned} \tag{13}$$

When $H(x)$ is an energy function, the dissipation equality (13) can be seen as the one-step ahead EB equality below

$$H(x^+(u)) - H(x) = uY(x,u) - d(x) \tag{14}$$

with supplied energy $uY(x,u)$ and natural dissipation $d(x) \geq 0$. Iterating the reasoning over k-steps, one gets the discrete-time EB equality over $[0,k)$ as

$$\underbrace{H(x_k) - H(x_0)}_{\text{stored energy}} = \underbrace{\sum_{\ell=0}^{k-1} u_\ell Y(x_\ell, u_\ell)}_{\text{supplied}} - \underbrace{\sum_{\ell=0}^{k-1} d(x_\ell)}_{\text{dissipated}} \tag{15}$$

with dissipation $d(x_\ell)$ at time $\ell \in [0,k)$.

Remark 2.3 *In (14), we have referred to the term $uY(x,u)$ as supplied energy and not supplied power, as one might expect. This is motivated by the fact that it can be seen as the supplied energy over one time step (at $k+1$ starting from k). One might also refer to it as supplied power by implicitly relating the power to a one step increment which plays the role, in discrete time, of the differentiation in continuous time.*

Remark 2.4 *In the Hamiltonian framework, making reference to the recent description of discrete-time Hamiltonian structures in (Moreschini et al., 2019),* *the passive average output (12) rewrites in terms of the discrete gradient function as*

$$Y(x,u) = g^\top(x,u)\bar{\nabla}H|_{x^+}^{x^+(u)} \tag{16}$$

with $g(x,u)$ defined in (7), the drift $x^+ = x^+(0)$ and the supplied energy at time step $k > 0$ as

$$uY(x,u) = ug^\top(x,u)\bar{\nabla}H|_{x^+}^{x^+(u)}. \tag{17}$$

This highlights a straight analogy with the same concepts set in continuous time.

Remark 2.5 *In the multi-input case setting $u = (u^1 \dots u^m)^\top$, the extension of (6) is straightforward by defining*

$$F(x,u) = F_0(x) + \sum_{i=1}^{m} g^i(x,u)u^i$$

and $G^i(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ so that

$$G^i(x + F(x,u), u) = \frac{\partial F(x,u)}{\partial u^i}$$

so rewriting $g^i(x,u)u^i$ as

$$g^i(x,u)u^i = \int_0^{u^i} G^i(x^+(\bar{w}^i), \bar{w}^i)\mathrm{d}w^i \tag{18}$$

with $\bar{w}^i = (u^1, \cdots, u^{i-1}, w^i, 0, \cdots, 0)$ for $i = 1, \dots, m$. Identically, one sets for $i = 1, \dots, m$

$$Y^i(x,u) = (g^i(x,u))^\top \bar{\nabla}H|_{x^+}^{x^+(u)}. \tag{19}$$

3 EB-PBC in discrete time

From now on we consider a discrete dynamics (6), assumed average passive with respect to the output map $h(\cdot,u) = \mathrm{L}_{G(\cdot,u)}H(\cdot)$, equivalently passive with respect to the average output $Y(x,u)$ defined in (12) or, equivalently, (16).

As in continuous time, the origin is usually not the equilibrium of practical interest. It is thus essential to drive the trajectories to a new admissible equilibrium for (6), say $x_\star \in \mathbb{R}^n$. Mimicking the discussion in Section 2, one can formally define control by energy-shaping in discrete time as below.

Definition 3.1 (DT-ES-PBC) *Consider the discrete-time dynamics (6) and a desired equilibrium $x_\star \in \mathbb{R}^n$. The feedback $u = \beta(x) + v$ is said to be a Energy-Shaping (ES) PBC if, for some function $H_d(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ with isolated minimum at $x_\star \in \mathbb{R}^n$, the closed-loop system*

$$x^+(\beta(x) + v) = x + F(x, \beta(x) + v) \tag{20}$$

verifies the energy-balance equality

$$H_d(x_k) - H_d(x_0) = \sum_{\ell=0}^{k-1} v_\ell Z(x_\ell, v_\ell) - \sum_{\ell=0}^{k-1} d_d(x_\ell) \tag{21}$$

with new damping $d_d(x_\ell)$ and new passive output $Z(x,v) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$.

Remark 3.1 *In the general definition above, we are not apriori fixing the new passive output $Z(x,v)$. As it will be shown hereinafter in the context of energy-balancing, it can depend explicitly on the original output of the system, when suitably modified under feedback.*

Accordingly, one sets the definition of discrete-time energy-balance passivity-based controllers as follows.

Definition 3.2 (DT-EB-PBC) *Let the discrete-time dynamics (6) be average passive and, equivalently, passive with respect to the average output (12). A feedback $u = \beta(x) + v$ is said to be a discrete-time EB-PBC if there exists a function $H_a(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that $\nabla H_a(x_\star) = -\nabla H(x_\star)$ and the closed-loop energy is equal to the difference between the stored and supplied energies when setting $H_d(x) = H(x) + H_a(x)$, namely*

$$\begin{aligned} H_d(x_k) - H_d(x_0) =& H(x_k) - H(x_0) \\ & - \sum_{\ell=0}^{k-1} \beta(x_\ell)Y(x_\ell, \beta(x_\ell)). \end{aligned} \tag{22}$$

Accordingly, passivation through energy-balancing relies on the definition of a function $\beta(x) : \mathbb{R}^n \to \mathbb{R}$ making the energy supplied by the controller a function of the state that is

$$-\sum_{\ell=0}^{k-1} \beta(x_\ell)Y(x_\ell, \beta(x_\ell)) = H_a(x_k) + K. \tag{23}$$

From the energy-balancing equality (21), whenever $H_a(x_\star) = H(x_\star)$ and $H_d(x) > 0$ otherwise (at least in a neighborhood of $x_\star$), one concludes that the desired equilibrium $x_\star \in \mathbb{R}^n$, the point of minimum energy, is stable. The following result can be proved.

Proposition 3.1 *Let the discrete-time dynamics (6) be average passive, equivalently passive with output (12) and storage function $H(\cdot) : \mathbb{R}^n \to \mathbb{R}$; let $x_\star \in \mathbb{R}^n$ be an* *admissible equilibrium. If there exists a function $\beta(x)$ such that the algebraic equation*

$$-\int_0^{\beta(x)} \mathrm{L}_{G(\cdot,w)} H(x^+(w))\mathrm{d}w = H_a(x^+(\beta(x)) - H_a(x) \quad (24)$$

admits a solution for some $H_a(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, then the feedback $u = \beta(x) + v$ is a discrete-time EB-PBC in the sense of Definition 3.2; namely, the closed-loop system (20) is passive with new average output

$$Z(x,v) = \frac{1}{v}\int_0^v \mathrm{L}_{G(\cdot,\beta(x)+w)} H_d(x^+(\beta(x)+w))\mathrm{d}w \quad (25)$$

and storage function

$$H_d(x) = H(x) + H_a(x) \quad (26)$$

verifying the desired energy-balance equality (21). In addition, if $H_d(x_\star) = 0$, $\nabla H_d(x_\star) = 0$ and $H_d(x) > 0$ for $x \neq x_\star$, then $x_\star$ is a stable equilibrium of (20).

Proof: The proof follows noting that (24) rewrites as

$$H_a(x^+(\beta)) - H_a(x) = -\beta(x) Y(x, \beta(x)).$$

Accordingly, the feedback $u = \beta(x) + v$ makes the average map $v \to Z(x,v)$ with $Z(\cdot, v)$ as in (25) passive with the new energy function (26) because

$$H_d(x^+(\beta(x)+v)) - H_d(x) = H_d(x^+(\beta(x))) - H_d(x) + \int_0^v \mathrm{L}_{G(\cdot,\beta(x)+w)} H_d(x^+(\beta(x)+w))\mathrm{d}w \leq vZ(x,v).$$

Stability of $x_\star$ follows if it is a minimum of $H_d(x)$. ◁

From the result above, one immediately gets that the feedback v solution to the damping equality $v = -KZ(x,v)$ achieves asymptotic stabilization of $x_\star$ provided suitable technical properties are satisfied (see Monaco and Normand-Cyrot (2011) for further details).

Remark 3.2 (Discrete-time dissipation obstacle) *A necessary condition for solving (24) is that*

$$\int_0^{\beta(\bar{x})} \mathrm{L}_{G(\cdot,v)} H(x^+(v))dv = \beta(\bar{x}) Y(\bar{x}, \beta(\bar{x})) = 0$$

with $x^+(v) = \bar{x} + F(\bar{x}, v)$ at all equilibria $\bar{x} \in \mathbb{R}^n$ of the closed loop system, that is $F(\bar{x}, \beta(\bar{x})) = 0$. This implies that stabilization via EB-PBC admits a solution only for systems with finite dissipation as the extracted power $\beta(\bar{x}) Y(\bar{x}, \beta(\bar{x}))$ must be zero (and thus finite) at all equilibria of (20).

Remark 3.3 *In terms of the discrete gradient function, condition (24) rewrites as*

$$\begin{aligned} &-\beta(x) g^\top(x, \beta(x)) \bar{\nabla} H|_{x^+}^{x^+(\beta(x))} \\ &= \big(F(x, \beta(x))\big)^\top \bar{\nabla} H_a|_x^{x^+(\beta(x))}. \end{aligned} \quad (27)$$

It is important to note that the discrete-time problem requires solving the algebraic equation (24) that is the discrete-time counterpart of the PDE (5) to solve in continuous time. The computation of the EB-PBC feedback as the solution to the nonlinear implicit equality (24) may be hard in practice. We show in the next section that the problem greatly simplifies for discrete-time dynamics issued from sampling.

4 Energy Balance PBC under sampling

In this section, we show how the proposed DT-EB-PBC applies to dynamics issued from sampling. More in detail, we show that the existence of a solution to (5), implies the existence of a solution to its discrete-time counterpart (24), when specified on the sampled-data equivalent model (28). Moreover the proof is constructive for the digital feedback solution that is described through its series expansion in powers of the sampling period around the continuous-time solution. The following standing assumption is introduced.

Assumption 1 *Let the continuous-time nonlinear systems (1) be passive with energy function $H(\cdot) : \mathbb{R}^n \to \mathbb{R}$. There exists a EB-PBC feedback $u = \beta(x) + v$ solution to (5) for some function $H_a(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ making $x_\star \in \mathbb{R}$ a stable equilibrium in closed loop with $H_d(x) = H(x) + H_a(x) > 0$ for $x \neq x_\star$ and $H_d(x_\star) = 0$.*

Assume now that measures of the state of (1) are available at the sampling instants $t = k\delta$ only and that it is fed by piecewise constant input signals over sampling times of length $\delta \in]0, T^\star[$; i.e., $u(t) = u_k$ as $t \in [k\delta, (k+1)\delta[$ for all $k \geq 0$. As a consequence, at all sampling instants, (1) is described by the so-called *sampled-data equivalent model* in the form of a map (Monaco and Normand-Cyrot, 1985, 2007)

$$x^+(u) = x + \delta F^\delta(x,u) \quad (28)$$

parameterized by δ and that can be computed according to the exponential flow

$$\delta F^\delta(x,u) = e^{\delta(\mathrm{L}_f + u\mathrm{L}_g)}x - x = \delta F_0^\delta(x) + \delta g^\delta(x,u)u.$$

The sampled-data dynamics is described according to its series expansion in powers of δ; i.e.,

$$F^{\delta}(x,u)=\sum_{i>0}\frac{\delta^{i-1}}{i!}(\mathrm{L}_f+u\mathrm{L}_g)^i x$$

with $g^{\delta}(x,u)u = F^{\delta}(x,u) - F_0^{\delta}(x), F_0^{\delta}(x) = F^{\delta}(x,0)$. In what follows, the sampling δ is assumed to belong to a bounded interval $]0,T^{\star}[$, where $T^{\star}$ is chosen to ensure convergence of the corresponding series expansions.

As proved in Monaco and Normand-Cyrot (2011), passivity of (1) implies passivity of its discrete-time equivalent model (28) with respect to the output described in (12) which in this contexts takes the form

$$Y^{\delta}(x,u)=\frac{1}{\delta u}\int_0^u \mathrm{L}_{G^{\delta}(\cdot,w)}H(x^+(w))\mathrm{d}w \tag{29}$$

with by definition (see Monaco and Normand-Cyrot (1999) for details)

$$\begin{aligned}G^{\delta}(x,u)&=\int_0^{\delta} e^{-s\ ad_{f+ug}}g\mathrm{d}s\\ e^{-s\ ad_{f+ug}}g &=g+\sum_{i>0}\frac{(-s)^i}{i!}ad^i_{f+ug}g.\end{aligned}$$

Remark 4.1 *In terms of the discrete gradient function, the average output (29) rewrites as*

$$Y^{\delta}(x,u)=\left(g^{\delta}(x,u)\right)^{\top}\bar{\nabla}H|_{x^+}^{x^+(u)}. \tag{30}$$

The next result shows that the existence of a continuous-time EB-PBC for (1) (i.e., Assumption 1) implies the existence of a discrete-time EB-PBC for the sampled-data equivalent model (28) in the sense of Proposition 3.1, with the same function $H_a(x)$ as in continuous time. In addition, the sampled-data EB-PBC feedback can be explicitly constructed from the continuous-time one.

Theorem 4.1 (SD passivation through DT-EB) *Consider the continuous-time dynamics (1) under Assumption 1. Then, there exists a sampled-data EB-PBC $u=\beta^{\delta}(x)+v$ in the sense of Definition 3.2; namely, there exists $T^{\star}>0$, such that, for all $\delta\in]0,T^{\star}[$, the equality*

$$-\frac{1}{\delta}\int_0^{\beta^{\delta}(x)}\mathrm{L}_{G^{\delta}(\cdot,w)}H(x^+(w))\mathrm{d}w=\frac{H_a(x^+(\beta^{\delta}(x)))-H_a(x)}{\delta} \tag{31}$$

admits a unique solution $\beta^{\delta}(\cdot):\mathbb{R}^n\to\mathbb{R}$ of the form

$$\beta^{\delta}(x)=\beta(x)+\sum_{i>0}\frac{\delta^i}{(i+1)!}\beta_i(x). \tag{32}$$

Moreover, the closed-loop sampled-data model

$$x^+(\beta^{\delta}(x)+v)=x+\delta F^{\delta}(x,\beta^{\delta}(x)+v) \tag{33}$$

is passive with respect to the output

$$Z^{\delta}(x,v)=\frac{1}{\delta v}\int_0^v \mathrm{L}_{G^{\delta}(\cdot,\beta^{\delta}(x)+w)}H_d(x^+(\beta^{\delta}(x)+w))\mathrm{d}w \tag{34}$$

and the same energy function $H_d(x)=H(x)+H_a(x)$, as in continuous time.

Proof: The existence of a unique solution $\beta^{\delta}(x)$ to (31) in the form (32) follows from the Implicit Function Theorem (Rudin et al., 1976). To this end, let us first rewrite (31) as

$$\begin{aligned}\delta S(\delta,u,x):=&e^{\delta(\mathrm{L}_f+u\mathrm{L}_g)}H(x)-e^{\delta\mathrm{L}_f}H(x)\\ &+e^{\delta(\mathrm{L}_f+u\mathrm{L}_g)}H_a(x)-H_a(x).\end{aligned}$$

The equality above (equivalently, (31)) admits a solution u for all δ if and only if $S(\delta,u,x)=0$ does. In addition, by definition of the Lie exponential operator and smoothness of all vector fields and functions, one can rewrite $S(\delta,u,x)=0$ as a formal series equality in powers of δ; i.e., one gets

$$S(\delta,u,x)=S_0(u,x)+\sum_{i>0}\frac{\delta^i}{(i+1)!}S_i(u,x)=0$$

with

$$\begin{aligned}S_0(u,x)=&\beta(x)g^{\top}(x)\nabla H(x)\\ &+(f(x)+\beta(x)g(x))^{\top}\nabla H_a(x)\\ S_i(u,x)=&(\mathrm{L}_f+u\mathrm{L}_g)^iH(x)-\mathrm{L}_f^iH(x)\\ &+(\mathrm{L}_f+u\mathrm{L}_g)H_a(x),\quad i=1,2,\ldots.\end{aligned}$$

With this in mind, when $\delta\to 0$, $S(\delta,u,x)\to S_0(u,x)$ and the sampled-data equality $S(\delta,u,x)=0$, recovers the continuous-time one (5). Thus, as $\delta\to 0$, (31) is solved by the continuous-time solution $\beta^{\delta}(x)=\beta(x)$. Then, since $\left.\frac{\partial S(\delta,u,x)}{\partial u}\right|_{\delta\to 0,u=\beta(x)}=\mathrm{L}_gH_d(x)\neq 0$, one concludes the existence of a solution $u=\beta^{\delta}(x)$ to the equality $S(\delta,u,x)=0$ admitting an expansion in the form (32) around the continuous-time solution $\beta(x)$ for $\delta\in]0,T^{\star}[$ and $T^{\star}$ small enough. The condition $L_gH_d(x)\neq 0$, at least in a neighborhood of $x_{\star}$, follows from Assumption 1, KYP properties (Byrnes et al., 1991), and relative degree one. The rest of the proof follows from Proposition 3.1. ◁

Remark 4.2 *Along the lines of Remark 3.3, (31) rewrites, in terms of the discrete-gradient as*

$$
\begin{aligned}
&- \beta^\delta(x)\big(g^\delta(x,\beta^\delta(x))\big)^\top \bar{\nabla} H|_{x^+}^{x^+(\beta^\delta(x))} \\
&= \big(F^\delta(x,\beta^\delta(x))\big)^\top \bar{\nabla} H_a|_{x}^{x^+(\beta^\delta(x))}
\end{aligned}
$$

that is the sampled-data equivalent to (5).

The proof of Theorem 4.1 is constructive in the sense that by comparing the terms of the same power in δ in equality (31), one computes iteratively each additional term $\beta_i(x)$ (referred to as i^{th}*-order correcting term*) in the expansion (32) as solution to a linear equality. This is done substituting all terms of the corresponding series expansion (32) into the equality (31) (equivalently, setting $S(\delta,\beta^\delta(x),x)=0$) and equating the terms appearing with the same power of δ.
For the first terms, setting $f_d(x)=f(x)+\beta(x)g(x)$, the equalities to solve iteratively are the following

$$
\begin{aligned}
&\beta_0(x) = \beta(x)\\
&\beta_1(x)\mathrm{L}_g H_d(x) = \dot\beta(x)\mathrm{L}_g H_d(x) - \beta(x)\mathrm{L}_g\mathrm{L}_f H(x)\\
&\beta_2(x)\mathrm{L}_g H_d(x) = \ddot\beta(x)\mathrm{L}_g H_d(x)\\
&\quad -\frac{3}{2}\beta_1(x)\mathrm{L}_g(\mathrm{L}_f+\beta(x)\mathrm{L}_g)H_d(x)\\
&\quad +(2\dot\beta(x)-\frac{3}{2}\beta_1(x))(\mathrm{L}_f+\beta(x)\mathrm{L}_g)\mathrm{L}_g H_d(x)\\
&\quad -\Big((\mathrm{L}_f+\beta(x)\mathrm{L}_g)^2-\mathrm{L}_f^2\Big)\mathrm{L}_f H(x)+\beta(x)\dot\beta(x)\mathrm{L}_g^2 H_d(x)
\end{aligned}
$$

with $\dot\beta(x)=\mathrm{L}_{f_d}\beta(x)$ and $\ddot\beta(x)=L_{f_d}^2\beta(x)$. Further details on these computational aspects can be found in Tanasa (2012).

As in the discrete-time case, exact solutions are hard to be computed. As a consequence, only controllers computed as approximate solutions can be implemented in practice. We define the p^{th}-order approximate feedback solution to (31) as the truncation of (32), at any desired order $p\geq 0$, as

$$
\beta^{\delta,[p]}(x)=\beta(x)+\sum_{i=1}^{p}\frac{\delta^i}{(i+1)!}\beta_i(x). \tag{35}
$$

Remark 4.3 *Quantifying (also in a qualitative way) the Maximum Allowable Sampling Period (MASP) under approximate solutions of the form (35) is a tough problem that has been addressed in a wide literature when restricted to Euler or emulation-based approximation schemes (i.e., with* $p=0$*) in general. In this sense, one can refer to several tools available for investigating the problem: the concept of consistency (Nešić et al., 1999); set stabilization and practical stability within the hybrid framework (e.g., (Nesic et al., 2009)); modeling and analysis in the time-delay framework ((Mazenc et al., 2013; Pepe and Fridman, 2017)) and so on. The benefits of including correcting terms to emulation (i.e., setting* $p\geq 1$ *have been investigated in (Mattioni et al., 2017; Tanasa et al., 2015).*

5 Fully actuated mechanical systems

An interesting feature of continuous-time EB-PBC strategies is to be applicable to the large class of fully actuated mechanical systems. Let us specify the sampled-data solution in that case. Consider the position regulation of a fully actuated mechanical systems with generalized coordinates $q\in\mathbb{R}^n$, momentum $p=M(q)\dot q$, and total energy function

$$
H(q,p)=\frac{1}{2}p^\top M^{-1}(q)p+V(q). \tag{36}
$$

$M(q)=M^\top(q)$ is the generalized inertia matrix and $V(q):\mathbb{R}^n\to\mathbb{R}_{\geq 0}$ is the potential energy verifying $V(q)\geq c$ for $c\in\mathbb{R}$. Accordingly, the dynamics is

$$
\begin{pmatrix}\dot q\\ \dot p\end{pmatrix}=\begin{pmatrix}0 & I\\ -I & 0\end{pmatrix}\begin{pmatrix}\nabla_q H(q,p)\\ \nabla_p H(q,p)\end{pmatrix}+\begin{pmatrix}0\\ I\end{pmatrix}u \tag{37}
$$

where $u\in\mathbb{R}^n$ is the control torque and $y=M^{-1}(q)p=\dot q$, the passive output verifying the dissipation equality

$$
\dot H(q,p)=q^\top M^{-1}(q)u=y^\top u, \tag{38}
$$

and

$$
J=\begin{pmatrix}0 & I\\ -I & 0\end{pmatrix},\qquad B=\begin{pmatrix}0\\ I\end{pmatrix}.
$$

5.1 Continuous-time EB-PBC

As proved in Ortega et al. (2001), denoting $x=(q^\top\ p^\top)^\top$ and $x_\star=(q_\star^\top\ 0^\top)^\top$, (37) satisfies Assumption 1 with

$$
\begin{aligned}
H_a(q)&=-V(q)+V_d(q-q_\star) &\text{(39a)}\\
V_d(q)&=\frac{1}{2}q^\top K_p q &\text{(39b)}\\
H_d(q,p)&=\frac{1}{2}p^\top M^{-1}(q)p+V_d(q-q_\star) &\text{(39c)}\\
\beta(q)&=\nabla V(q)-K_p(q-q_\star) &\text{(39d)}
\end{aligned}
$$

for all $K_p=K_p^\top\succ 0$. Accordingly, the feedback $u=\beta(q)+v(q,p)$ with damping injection

$$
v(q,p)=-K_d M^{-1}(q)p \tag{40}
$$

makes the equilibrium $x_\star$ asymptotically stable for all $K_d \succ 0$.

5.2 Digital EB-PBC

When dealing with fully actuated mechanical systems, the passive output making the system (37) passive under sampling takes a simple, yet representative, structure as detailed in the result below specifying Proposition 2.2 to this context.

Proposition 5.1 *Consider the mechanical system (37) being passive (lossless) with output $y = M^{-1}(q)p$ and dissipation equality (38). Then the corresponding sampled-data equivalent model is passive with average output*

$$Y^\delta(q,p,u) = \frac{1}{\delta}(q^+(u) - q) \tag{41}$$

and dissipation equality

$$\begin{aligned} H(q^+(u), p^+(u)) - H(q,p) =& \delta u^\top Y^\delta(q,p,u) \\ =& u^\top (q^+(u) - q). \end{aligned} \tag{42}$$

Proof: The proof follows by integrating the continuous-time dissipation equality (38) over the sampling interval $[k\delta, (k+1)\delta[$; namely, one gets

$$\int_{k\delta}^{(k+1)\delta} \dot{H}(q(s), p(s)) \mathrm{d}s = u^\top \int_{k\delta}^{(k+1)\delta} M^{-1}(q(s))p(s)\mathrm{d}s.$$

Substituting in the equality above $\dot{q} = M^{-1}(q)p$ and recalling that $q^+(u) = q_{k+1} = q((k+1)\delta)$ and $p^+(u) = p_{k+1} = p((k+1)\delta)$, one gets (42) so that passivity with respect to the output (41) follows. The fact that (42) coincides with the average output (29) follows because the dynamics (37) is lossless (i.e., $\mathrm{L}_f H(q,p) \equiv 0$ with $f = J\nabla H(q,p)$) as proved in Monaco and Normand-Cyrot (2011). ◁

Remark 5.1 *For sampled-data mechanical systems the average output (42) recovers the passivating output typically used in the literature(e.g., Laila and Astolfi (2006a)).*

Remark 5.2 *In open loop the sampled-data supplied energy is given by the one-step increment of position over δ multiplied with torque; namely, from (42) one gets*

$$u^\top Y^\delta(q,p,u) = \frac{1}{\delta} u^\top (q^+(u) - q).$$

Remark 5.3 *In this case the discrete gradient associated to the Hamiltonian is given by*

$$\bar{\nabla} H|_x^{x^+} = \begin{pmatrix} \bar{\nabla} V|_q^{q^+} + \frac{1}{2}\int_0^1 \nabla_q [p^\top M(q))p]_{q=q+s(q^+-q)} \mathrm{d}s \\ \frac{1}{2} M(q)(p^+ + p) \end{pmatrix}$$

when setting, for the ease of notations, $x = (q^\top \; p^\top)^\top$.

At this point, from Theorem 4.1, the position regulation problem can be solved under digital control via discrete-time EB-PBC as summarized in the result below.

Proposition 5.2 *Consider the mechanical system (37) with energy function (36) and $x_\star = (q_\star^\top \; 0^\top)^\top$ an equilibrium to stabilize. Then there exists a digital EB-PBC feedback $\beta^\delta(q,p) : \mathbb{R}^{2n} \to \mathbb{R}^n$ solution to the EB equality*

$$\begin{aligned} (q^+(\beta^\delta(q,p)) - q)^\top \beta^\delta(q,p) = \big(\bar{\nabla} V|_q^{q^+(\beta^\delta(q,p))} \\ - \frac{1}{2} K_p (q^+(\beta^\delta(q,p))) + q - 2q_\star)\big)^\top \big(q^+(\beta^\delta(q,p)) - q^+\big) \end{aligned}$$

with $K_p = K_p^\top \succ 0$. Equivalently, the feedback $u = \beta^\delta(q,p) + v$ makes the sampled-data equivalent dynamics (28) passive with respect to the average output

$$Z^\delta(q,p,v) = q^+(\beta^\delta(q,p) + v) - q \tag{43}$$

and continuous-time target energy (39c). In addition, $x_\star = (q_\star^\top \; 0^\top)^\top$ is asymptotically stable with

$$v = v^\delta(q,p) = v_0(q,p) + \sum_{i>0} \frac{\delta^i}{(i+1)!} v_i(q,p)$$

defined as the unique solution to the damping equality

$$v = -K_d\big(q^+(\beta^\delta(q,p) + v) - q\big), \quad K_d \succ 0. \tag{44}$$

Proof: The proof follows from Theorem 4.1 when rewriting the energy-balance equality (31) as

$$\beta^{\delta\top}(x)\big(g^\delta(x, \beta^\delta(x))\big)^\top \bar{\nabla} H_d|_{x^+}^{x^+(\beta^\delta(x))} = \big(F_0^\delta(x)\big)^\top \nabla H_a|_x^{x^+}$$

with $F_0^\delta(x) = e^{\delta \mathrm{L}_f} x$ and $f(x) = J\nabla H(x)$,

$$\bar{\nabla} H_a(x)|_x^{x^+} = \begin{pmatrix} -\bar{\nabla} V|_q^{q^+} + \frac{1}{2} K_p (q^+ + q - 2q_\star) \\ 0 \end{pmatrix}$$

and $H_a(x) = H_a(q,p)$ as in (39a). At this point, the rest of the proof follows from Theorem 4.1. ◁

Denoting

$$\hat{K}_\ell = \begin{pmatrix} K_\ell & 0 \\ 0 & 0 \end{pmatrix}$$

for $\ell = \{d, p\}$, the final feedback is again in the form of a series expansion in powers of δ with, for the first terms

$$\begin{aligned}\beta_0(q,p) =& \beta(q) = \nabla V(q) - K_p(q-q_\star)\\ \beta_1(q,p) =& -(\nabla^2 V(q) - \hat{K}_p)J\nabla H_d(q,p)\\ \beta_2(q,p) =& J[\beta_1(q,p)]J\nabla H_d(q,p)\\ &- \Big(\nabla^\top H_d(q,p)B\Big)^\dagger \beta_1^\top(q,p)B^\top \times\\ &\Big(\nabla^2 H_d(q,p)J\nabla H(q,p)\\ &- \frac{3}{2}\nabla^2 H(q,p)J\nabla H_d(q,p)\Big)\end{aligned} \tag{45}$$

and for $S := J - BK_dB^\top$

$$\begin{aligned}v_0(q,p) =& v(q,p) = -K_dM^{-1}(q)p\\ v_1(q,p) =& -\hat{K}_dB^\top\nabla^2 H_d(q,p)S\nabla H_d(q,p)\\ &- \hat{K}_dB^\top\big(\nabla^2 H(q,p)J\nabla H_d(q,p)\\ &- \nabla^2 H_d(q,p)J\nabla H(q,p)\big)\\ v_2(q,p) =& J[v_1(q,p)]S\nabla H_d(q,p) - \hat{K}_dB^\top\nabla^2 H_dB\big(\beta_1(q,p)\\ &+ \frac{3}{2}v_1(q,p) - J[v_0(q,p)]S\nabla H_d(q,p)\big).\end{aligned} \tag{46}$$

Accordingly, approximate solutions can be defined as

$$u^{\delta,[i]}(q,p) = \sum_{\ell=0}^{i}\frac{\delta^\ell}{(\ell+1)!}(\beta_\ell(q,p) + v_\ell(q,p)), i \geq 0 \tag{47}$$

ensuring practical asymptotic stability of $(q_\star^\top\ 0^\top)^\top$ in closed loop (Mattioni et al., 2017, Proposition 4.2); i.e., trajectories of the closed-loop dynamics converge to a neighborhood of the desired equilibrium with radius δ^{i+1}.

5.3 A simulated example: the case of a 2R robot

We apply the results established for mechanical systems to the classical fully actuated 2R robot of the form (37) with in particular $(q,p) \in \mathbb{R}^4$

$$\begin{aligned}M(q) =& \begin{pmatrix} c_1 + c_2 + 2c_3\cos(q_2) & c_2 + c_3\cos(q_2)\\ c_2 + c_3\cos(q_2) & c_2\end{pmatrix}\\ V(q) =& -c_4 g\cos(q_1) - c_5 g\cos(q_1+q_2)\end{aligned}$$

for $c_1 = m_1 l_{c_1}^2 + m_2 l_1^2 + I_1$, $c_2 = m_2 l_{c_2}^2 + I_2$, $c_3 = m_2 l_1 l_{c_2}$, $c_4 = m_1 l_{c_1} + m_2 l_1$, $c_5 = m_2 l_{c_2}$ and $g = 9.81$, $m_1 = m_2 = 2$, $l_1 = 2$, $l_{c1} = 1$, $l_{c2} = 0.5$, $I_1 = 0.667$, $I_2 = 0.083$. The continuous-time control law $u(q) = \beta(q) + v(q,p)$ as in (39d)-(40) is compared with approximate sampled-data feedback laws of the form (47) with correcting terms as in (45)-(46). Moreover, we refer to the case $p = 0$ as the emulated control that is typically involved in the literature.

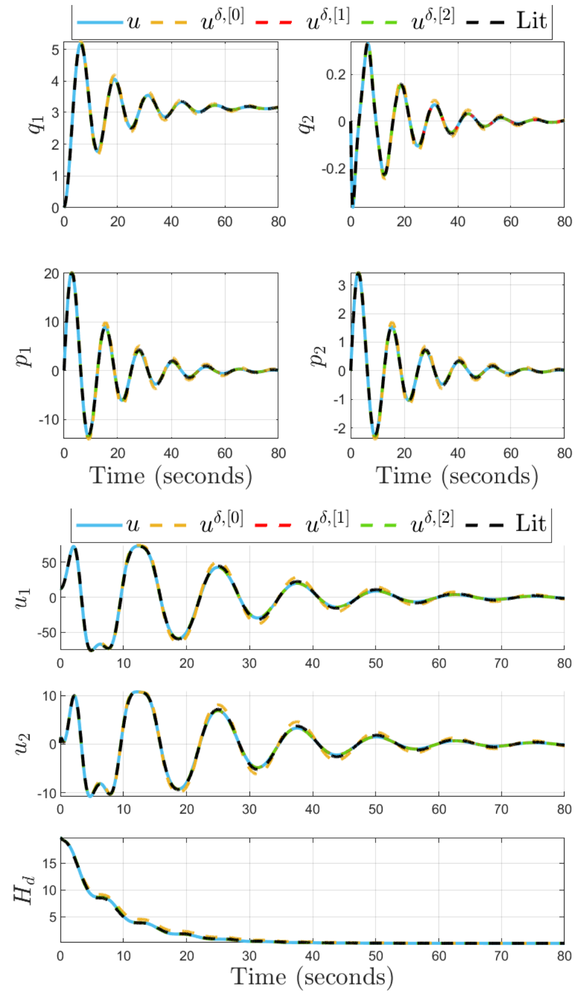

Fig. 1. Fully actuated 2R Robot with $\delta = 0.01$.

Simulations are reported in Figures 1-2 fixing the initial and desired configurations as $x_0 = 0$, $q_\star = (\pi, 0)$ with the gains $K_p = 4I$, $K_d = 2I$. The solid blue line represents the continuous-time controller, the dashed yellow the emulated controller, the dashed red the controller (47) truncated in $O(\delta^2)$, the dashed green the controller (47) truncated in $O(\delta^3)$, and the dashed black represents the stabilizing controller resulting from a sampled-data redesign proposed in Laila and Astolfi (2006b) adapted to this fully actuated case.

The results show that the emulated controller is prone to instability even under small sampling period ($\delta = 0.1$, Figure 2), so failing in assigning the desired closed-loop energy behavior. On the other hand, higher order approximate controllers (even with only one correcting term, i.e., with $i = 1$) achieve stabilization on the closed-loop system guaranteeing the desired energy profile at all sampling instants with remarkable performances. This testifies the striking improvement of approximate solutions (even with only one correcting term) in practice. As expected, for much smaller values of the sampling

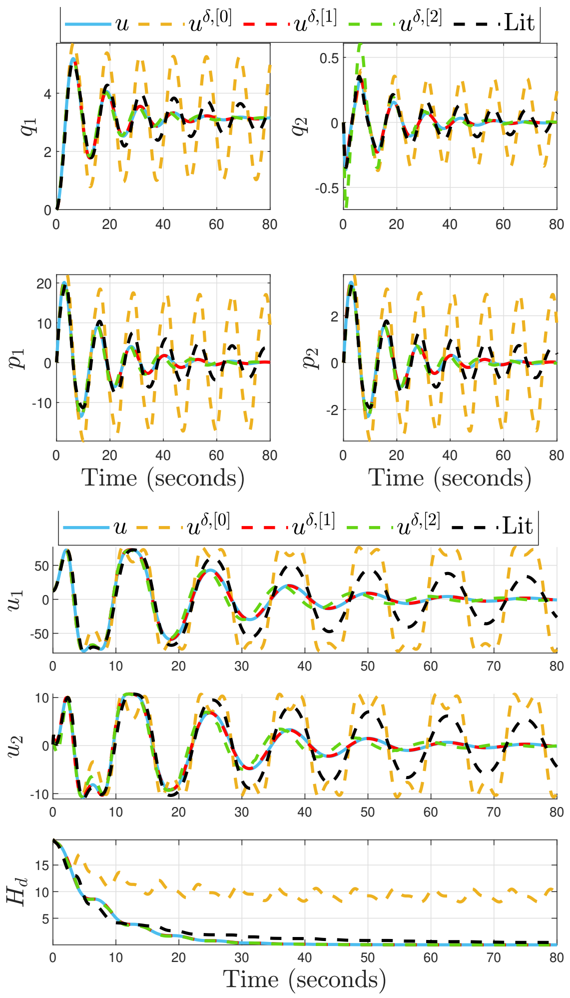

Fig. 2. Fully actuated 2R Robot with $\delta = 0.1$.

period ($\delta = 0.01$, Figure 1), even the emulation-based control guarantees stabilization in closed loop.

6 Conclusions

The problem of designing EB-PBCs for discrete-dynamics has been addressed providing conditions for its solvability; it is also shown that such solution naturally applies to discrete-time dynamics issued from sampling and that a constructive solution always exists whenever passivation via energy balancing holds in the continuous-time case. The case of fully actuated mechanical systems is also investigated enabling to get a characterization of the sampled-data passifying output as well as the energy balancing equation to solve under digital feedback. A simulated mechanical example illustrates the performances with respect to emulated strategies.

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