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Maximizing the stable throughput of heterogeneous nodes under airtime fairness in a CSMA environment

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ABSTRACT

The stability region of non-persistent CSMA is analyzed in a general heterogeneous network, where stations have different mean packet arrival rates, packet transmission times probability distributions and transmission probabilities. The considered model of CSMA captures the behavior of the well known CSMA/CA, at least as far as stability and throughput evaluation are concerned. The analysis is done both with and without collision detection. Given the characterization of the stability region, throughput-optimal transmission probabilities are identified under airtime fairness, establishing asymptotic upper and lower bounds of the maximum achievable stable throughput. The bounds turn out to be insensitive to the probability distribution of packet transmission times. Numerical results highlight that the obtained bounds are tight not only asymptotically, but also for essentially all values of the number of stations. The insight gained leads to the definition of a distributed adaptive algorithm to adjust the transmission probabilities of stations so as to attain the maximum stable throughput.

1. Introduction

Carrier Sense Multiple Access (CSMA) has dominated the stage of random multiple access techniques, boosted by the impressive success of Wi-Fi networks, whose core MAC protocol algorithm, the so called CSMA/CA, conforms to non-persistent CSMA. CSMA/CA is the foundation of all amendments of IEEE 802.11, from high speed local wireless networks (IEEE 802.11b/a/g/n/ac, partly also IEEE 802.11ax), to vehicular networks (IEEE 802.11p/bd) to sensor networks (IEEE 802.11ah, IEEE 802.15.4). CSMA is found also in RFID networks [1].

This paper addresses the stability conditions for a heterogeneous CSMA network, where stations have different mean packet arrival rates, different and arbitrary packet transmission time probability distributions[1] and different back-off probabilities (also referred to as transmission probabilities). Different packet transmission times may arise due to application traffic characteristics, e.g., voice, short messages, app notifications are carried by short packets, versus, e.g., web browsing or file download traffic, that use systematically the maximum packet length allowed by the network. Different transmission probabilities can be used to achieve fairness among stations, as discussed in Section 5. The CSMA model considered in this paper is able to capture the behavior of the well known CSMA/CA, at least as far as stability and throughput evaluation are concerned.

The analysis of CSMA is done both without and with collision detection capability. Collision detection can be implemented by taking advantage of full-duplex hardware also in a wireless channel, e.g., see [2–4]. [2] proposes a CSMA protocol enhancement, where feedback control messages are introduced to support Collision Detection (CD). [3] exploits special physical layer symbols to re-design the RTS/CTS mode in a more efficient way. [4] exploits the Full Duplex capability of physical layer to provide simultaneous carrier sensing and transmission in WiFi.

The main contributions of this paper are as follows.

1. The stability region of the mean arrival rates at stations is assessed for a general heterogeneous CSMA environment, where packet transmission time Probability Density Functions (PDFs), mean arrival rates and transmission probabilities may be different for different stations.
2. Based on the result of the previous point, throughput-optimal transmission probabilities are identified, under a fairness constraint, referred to as airtime fairness [5,6], and bounds on the achievable stable throughput are established. The result takes an intriguing simple form in the limit for the number of stations tending to infinity. Numerical results show that the asymptotic bounds turn out to be tight for essentially all values of the number of stations, giving it practical relevance.

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[1] By packet transmission time it is intended here the time that a station holds the channel, once it wins the contention. This time can encompass the transmission of possibly multiple protocol data units, and includes also any additional overhead, e.g., preambles, inter-frame spaces, acknowledgments (if required).

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3. Leveraging on maximum throughput analysis, a distributed algorithm is defined to drive the heterogeneous stations towards the maximum achievable throughput (throughput optimality), by adjusting the transmission probabilities under airtime fairness.

The bounds on the maximum stable throughput under airtime fairness and the adaptive transmission algorithm depend only on the *mean* packet transmission times, i.e., they turn out to be insensitive to the probability distributions of the random variables that define the duration of packet transmission times. This insensitivity result makes the obtained bounds and the adaptive transmission algorithm robust and useful for practical applications.

The rest of the paper is so organized. An account of related works is given in Section 2. The CSMA algorithm and system model is introduced in Section 3. The stability region is assessed in Section 4. In Section 5 the maximum achievable stable throughput is investigated, under airtime fairness constraint. An adaptive transmission algorithm to achieve the maximum throughput is defined and evaluated by mean of simulations in Section 6. A discussion of potential usage and applications of the presented results is given in Section 7. Concluding remarks are given in Section 8. The Appendices contains the proofs of the main theorems and of other results mentioned in the paper.

2. Related works

Stability of CSMA is a well known issue, since the landmark work by Tobagi and Kleinrock [7]. A general analysis of the capacity region of CSMA networks, including the hidden station case, is offered in [8], with reference to an idealized case where the sensing time (i.e., the size of the back-off slot time) shrinks to zero. The sustainable offered rate region is studied in [9] for a model that can be cast into the RTS/CTS mode of CSMA/CA. As a matter of fact, a key detail of the model in [9] is that the collision time T_c is a constant, no matter which station is involved in the collision. Another restrictive assumption is that channel usage times are multiples of T_c, when a successful transmission takes place. The purpose of [9] is to characterize the stable rate region and its maximal convex subsets. The stability region of CSMA for a single-hop network with no hidden stations is characterized also in [10–12], assuming that transmission and collision times are the same for all stations. A general framework to assess stability of a multi-node system where nodes transmit to a central hub by using an ALOHA-type multiple access scheme in slotted time, is presented in [13]. Stable throughput for high priority traffic in WiFi is analyzed in [14]. This work focuses on the connection between saturation throughput and stable throughput limit. It also gives an adaptive algorithm to guarantee that high-priority traffic can achieve the maximum stable throughput. The analysis is developed for a homogeneous system, where all high priority stations use the same transmission probability and packet transmission times. Stable throughput optimization is pursued in a game-theoretic context in [15]. The stability region is given in terms of feasible schedules, i.e., schedules where interfering link operate in mutual exclusion. This idealized model does not account for collisions, even if the impact of back-off time is taken care of. The main contribution of [15] lies in the design of new penalty functions in the definition of utility functions, so that a fully distributed game-driven dynamics is obtained, without the need of message passing among stations.

Heterogeneous CSMA study is motivated by the mix of traffic flows appearing in most wireless networks [16]. It stems also from device evolution, i.e., multi-sensor platforms in wireless sensor networks produce a variety of packet sizes originating from a same node [17]. Heterogeneous CSMA performance evaluation and optimization has been considered extensively, since the key contribution [18], that introduced a capacity-achieving "adaptive CSMA" algorithm, e.g., see [15,19–27]. Some of those works consider fixed transmission times [20,21] or assume zero sensing time [15,18,22,24,27] and hence disregard collision events (idealized CSMA, where it is also usually assumed that packet transmission times are exponentially distributed). All of those works, even in case they account for collision events, assume a *fixed* collision time, irrespective of which stations are involved, i.e., they assume "probe" packets are used similarly to RTS/CTS mode in IEEE 802.11 MAC protocol. On the contrary, in this paper we stick to the original CSMA model of [28], where sensing requires a finite time, hence collisions are taken into account and their duration depends on the time on air of the involved stations.[2]

The model used in this paper is the classic CSMA model since the seminal paper of Tobagi and Kleinrock [28]. This kind of CSMA model is also useful in the analysis of CSMA/CA used in WiFi, as shown since Bianchi's work [29] and discussed later in [30]. The saturation throughput is predicted accurately by means of a model depending only on the transmission probabilities of stations. The same kind of model has been used to study the optimization of throughput of IEEE 802.11, e.g., in [31,32]. More in depth, the CSMA model used in this paper is consistent with the approach that has been shown to lead to throughput optimization of CSMA/CA, e.g., see the work of Cali et al. [31] as an early contribution in this sense. In that work, the uniform probability distribution of back-off of 802.11, with binary exponential back-off to adapt the contention window size in case of re-transmissions, is replaced with a p-persistent backoff protocol. The transmission probability p is set so as to maximize the saturation throughput, based on estimation of the current number of contending stations. Cali et al. show that 802.11 yields suboptimal throughput, and that their algorithm can approach optimal throughput under the same conditions. This motivates considering a p-persistent CSMA protocol also in the heterogeneous environment considered in this work.

Finally, it is worth mentioning the queue-based CSMA approach [33], which aims at defining a distributed algorithm to achieve throughput optimality in a random access network characterized by a conflict graph and an access protocol inspired to the RTC/CTS mode CSMA. The network model is time-slotted, i.e., packet transmission fits a fixed time slot for all stations. Exchange of control messages in dedicated mini-slots is also part of the proposed design. The purpose is to define a link scheduling algorithm to achieve throughput optimality. In a subsequent work [34], an efficient algorithm is defined to compute optimal scheduling in the same modeling framework as in [33]. The considered model is still time-slotted with fixed transmission and collision times. Optimization of throughput of a p-persistent CSMA model, similar to the CSMA model considered in this paper, was considered in early contributions of Bruno et al. [35,36].

Summing up, current literature addresses heterogeneous CSMA networks under some restrictive assumptions, e.g., (i) same packet transmission times for all nodes; (ii) fixed collision time, irrespective of which station is involved; (iii) negative exponential channel holding times, as well as back-off times. In the sequel, we address a general heterogeneous CSMA model, where packet transmission times of stations can have general probability distributions.

3. System model

The main notation used in the paper is listed in Table 1.

We consider a set of n stations sharing a broadcast communication channel. Each station is equipped with an infinite buffer, where packets arrive from the upper layer.

Channel access is operated according to non-persistent CSMA. Algorithm 1 lists the pseudo-code of the channel contention algorithm run by a station, whenever it has a packet ready to be (re)transmitted. In the CSMA algorithm, CCA stands for Clear Channel Assessment, i.e., the function that polls the physical layer to assess whether the

[2] This is particularly important in the heterogeneous case, where stations with long channel holding times mix up with stations requiring shorter channel holding times.

Table 1
Main notation of CSMA model.

Symbol	Meaning
n	Number of stations.
δ	Back-off slot time, same as the channel sensing time,
λ_i	Mean arrival rate of new packets at station i.
τ_i	Transmission probability of station i, when it is backlogged.
θ_i	Transmission probability of station i.
Y_i	Packet transmission time of station i.[a]
T_i	Mean packet transmission time of station i, $T_i = \mathrm{E}[Y_i]$.
$T_{\min}$	Minimum packet transmission time.
$T_{\max}$	Maximum packet transmission time.
T_c	Collision resolution time in case of CSMA with collision detection.
S_i	Success probability of transmission attempt of station i.
V	Virtual slot time.
$\overline{V}$	Mean virtual slot time, $\overline{V} = \mathrm{E}[V]$.
$Q_i(t)$	Number of packets queue at station i at the beginning of virtual slot time t.
$A_i(t)$	Number of newly arriving packets during virtual slot time t.
$U_i(t)$	Number of packets successfully delivered by station i during the virtual slot time t.
$\mathcal{S}$	State space of the joint process $\mathbf{Q}(t) = [Q_1(t), \dots, Q_n(t)]$, consisting of all n-tuples of non-negative integers.
$\mathcal{S}_{0,i}$	Set of all states where queue at station i is empty.
T_A	Time variable used to optimize the stable throughput under airtime fairness (see Section 5).
μ_n	Average of reciprocals of mean packet transmission times, equal to $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{T_i}$.
μ	Limit of μ_n for $n \to \infty$.
α	Non-dimensional parameter defined as $T_A\,\mu$.
β	Non-dimensional parameter defined as $\delta\,\mu$.
ξ	Non-dimensional parameter defined as $T_{\max}\,\mu$.
ψ	Non-dimensional parameter defined as $T_c\,\mu$.

[a]In this work "packet transmission time" is meant to be synonym of "channel holding time" of a station. It encompasses any overhead required to transmit a packet, along with the packet transmission time itself.

channel is idle or busy. This operation is performed by calling the function `channel_state(`δ`)`, which takes a time δ to return the state of the channel. The fixed time δ is the channel sensing time, which is assumed to be also the back-off slot time, as customary in CSMA. If the channel is busy, the station repeats the channel assessment. The WHILE cycle of Algorithm 1 is equivalent to sampling the channel status with a time period δ, as long as it stays busy. Once the channel is found idle, the station starts transmitting with probability τ. This is obtained by comparing the outcome of the function `rand()` with τ. Here `rand()` returns a random number uniformly distributed in [0,1]. With probability $1-\tau$, the station skips the current idle back-off slot time and repeats the channel assessment in the next back-off slot time. When station i eventually starts transmitting, it keeps the channel busy for a time Y_i, which includes any overhead (preamble, header, inter-frame spaces, acknowledgment, if required). The mean packet transmission time of station i is denoted with $T_i = \mathrm{E}[Y_i]$. Stations are labeled in the decreasing order of their mean packet transmission times, i.e., $i < j \Rightarrow T_i \geq T_j$.

If CD is available, once the transmission starts, the station monitors the channel. If a collision is detected, involved stations abort their on-going transmissions. When enabled, CD takes a fixed time T_c. For CD to be functional, it is required that packet transmission times be shorter than the time required to detect a collision, i.e., it must be $T_c < Y_i$, $\forall i$. As a matter of example, in Ethernet there is a minimum packet length requirement, to guarantee that transmissions last enough time to detect a collision. In a wireless channel, like WiFi, CD can be supported by means of the RTS/CTS mode, where T_c amounts to the sum of RTS packet transmission time plus CTS timeout This time is necessarily shorter than a regular transmission, which encompasses RTS and CTS control frames, a MAC data frame and the relevant acknowledgment. Alternative implementations of the CD functionality in WiFi are given in [2–4]. In all of these proposals, the time required to detect a collision is less than the full packet transmission time.

Algorithm 1 CSMA contention algorithm for one packet transmission attempt.

```
1: CCA ← channel_state(δ)
2: while CCA == BUSY do
3:   CCA ← channel_state(δ)
4: end while
5: if rand() > τ then
6:   Go to 1
7: end if
8: // The station has won the contention
9: send(packet)
10: if CD_enabled then
11:   collision ← check_tx(T_c)
12:   if collision then
13:     abort_tx
14:   else
15:     wait_for_tx_complete
16:   end if
17: else
18:   wait_for_tx_complete
19: end if
```

We do not consider Binary Exponential Backoff, which is a rough way of adapting the transmission probability in the face of repeated collisions. Hence, the value of transmission probability τ_i is not updated by station i because of packet retransmissions. In fact, a smarter way of adapting the transmission probability of each station is defined in Section 6, based on the insight gained from the stability analysis and the maximization of the stable throughput under fairness constraints. Transmission probability adaptation defined in Section 6 leads to throughput optimality.

It is assumed that stations can sense each other's transmissions, i.e., there are no hidden stations. Therefore stations are "synchronized", i.e., the time axis can be thought as split into *virtual slots*. A virtual slot consists of one back-off slot, possibly followed by a transmission. The transmission probability of station i in a virtual slot time is denoted with θ_i. Given that station i is backlogged, the transmission probability is $\theta_i = \tau_i$. If instead station i is idle (not backlogged), it is $\theta_i = 0$.

Either without or with CD, a successful transmission of station i in a virtual slot occurs if station i attempts a transmission and no other station does. The corresponding probability is

$$S_i(\boldsymbol{\theta}) = \theta_i \prod_{j=1, j\neq i}^{n} (1-\theta_j) \tag{1}$$

As for the mean virtual slot time, let us consider first the case with no CD and let us define the following random variables:

$$Z_i = \begin{cases} 0 & \text{w.p. } 1-\theta_i, \\ Y_i & \text{w.p. } \theta_i., \end{cases} \qquad i = 1, \dots, n. \tag{2}$$

The virtual slot time reduces to a back-off slot time, if no station transmits, otherwise it is the sum of the back-off time and of the longest packet transmission time. Formally,

$$V = \delta + \max\{Z_1, \dots, Z_n\} = \delta + V' \tag{3}$$

where we introduce the random variable $V' = \max\{Z_1, \dots, Z_n\}$, referred to as reduced virtual slot time.

Assuming the Z_i's are independent random variables, the Cumulative Distribution Function (CDF) of V' is given by

$$F_{V'}(x) = \mathcal{P}(V' \leq x) = \prod_{i=1}^{n} F_{Z_i}(x) = \prod_{i=1}^{n}\Big(1-\theta_i+\theta_i F_{Y_i}(x)\Big) \tag{4}$$

where we have used the identity $F_{Z_i}(x) = 1-\theta_i+\theta_i F_{Y_i}(x)$ that is a consequence of the definition of Z_i in Eq. (2), for $i = 1, \dots, n$. In the following, the Complementary CDF (CCDF) will be used, which follows immediately from Eq. (4)

$$G_{V'}(x) = \mathcal{P}(V' > x) = 1 - F_{V'}(x) = 1 - \prod_{i=1}^{n} \left(1 - \theta_i G_{Y_i}(x)\right) \tag{5}$$

According to Eq. (5), the mean virtual slot time $\overline{V} = \mathrm{E}[V]$ is given by

$$\overline{V}(\boldsymbol{\theta}) = \delta + \mathrm{E}[V'] = \delta + \int_0^{\infty} \left[1 - \prod_{i=1}^{n} \left(1 - \theta_i G_{Y_i}(x)\right)\right] dx \tag{6}$$

where $\boldsymbol{\theta} = [\theta_1, \dots, \theta_n]$.

This general expression can be simplified in special cases. A case of fairly wide interest is when packet transmission times take discrete values. Let all possible values of packet transmission time be integer multiples of a common (arbitrary) quantum Δx. Packet transmission times belong to the set $\{\Delta x, 2\Delta x, \dots, \ell\Delta x\}$, for a positive integer ℓ. Let $G_{Y_i}(h) = \mathcal{P}(Y_i \geq h\Delta x)$, $h = 1, \dots, \ell$ denote the discrete CCDF of the packet transmission time. Then, Eq. (6) becomes:

$$\overline{V}(\boldsymbol{\theta}) = \delta + \Delta x \sum_{h=1}^{\ell} \left[1 - \prod_{i=1}^{n} \left(1 - \theta_i G_{Y_i}(h)\right)\right] \tag{7}$$

If packet transmission times are constant, i.e., $Y_i = T_i$, $\forall i$, then it can be verified that:

$$\overline{V}(\boldsymbol{\theta}) = \delta + T_1\theta_1 + \sum_{i=2}^{n} T_i\theta_i \prod_{j=1}^{i-1} [1 - \theta_j] \tag{8}$$

The above expression stems from the assumed decreasing ordering of the T_i's.

With CD, since $T_c \leq Y_i$, $\forall i$, the virtual slot time is:

$$V_{CD} = \begin{cases} \delta & \text{if no station transmits,} \\ \delta + Y_i & \text{if station } i \text{ transmits}, i = 1, \dots, n, \\ \delta + T_c & \text{if more than one station transmit.} \end{cases} \tag{9}$$

The mean virtual slot time is derived from Eq. (9), yielding

$$\overline{V}_{CD}(\boldsymbol{\theta}) = \delta + \sum_{i=1}^{n} T_i S_i(\boldsymbol{\theta}) + T_c \left(1 - P_e(\boldsymbol{\theta}) - \sum_{i=1}^{n} S_i(\boldsymbol{\theta})\right) \tag{10}$$

where $P_e(\boldsymbol{\theta}) = \prod_{i=1}^{n}(1 - \theta_i)$ is the probability that no station transmits, and $S_i(\boldsymbol{\theta})$ is given in Eq. (1).

4. Stability

Let $Q_i(t)$ denote the length of the queue at station i, sampled at the beginning of virtual slot t. It evolves according to:

$$Q_i(t+1) = Q_i(t) - U_i(t) + A_i(t) \tag{11}$$

where $U_i(t) = 1$ if station i makes a successful transmission, while $U_i(t) = 0$ otherwise, and $A_i(t)$ is the number of new packets arriving during the tth virtual slot time. The array $\mathbf{Q}(t) = [Q_1(t), \dots, Q_n(t)]$ represents the state of the stations at the beginning of virtual slot t. The state space of the process $\mathbf{Q}(t)$ is made of all n-tuples of non-negative integers: $\mathcal{S} = \{\mathbf{q} = [q_1, \dots, q_n] : q_i \in \mathbb{Z}^+ \ \forall i\}$. If steady-state exists, $\mathbf{Q} = \mathbf{Q}(\infty)$ denotes the queue status random variable.

The notion of stability considered here is based on the boundedness of mean queue lengths. Formally, if $Q(t)$ denotes the length of a queue at the beginning of slot t, the queue is said to be (strongly) stable if (e.g., see [37, Ch.2]):

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathrm{E}[Q(t)] < \infty \tag{12}$$

Algorithm 1 depends on the transmission probability τ. Let us assume that a set of transmission probabilities τ_i, $i = 1, \dots, n$ are assigned to stations. We can arrange these probabilities in a row vector $\boldsymbol{\tau} = [\tau_1, \dots, \tau_n]$ belonging to $[0,1]^n$. The following result holds.

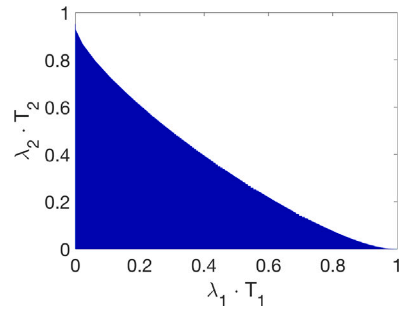


Fig. 1. Example of stability region for two stations, one with $Y_1 = T_1 = 100 \cdot \delta$, the other one with $Y_2 = T_2 = 20 \cdot \delta$. It can be noted that the stability region is not convex.

Theorem 1. *Let n stations share a channel according to the CSMA procedure in Algorithm 1 (without CD), with transmission probability τ_i and packet transmission times Y_i, with mean $T_i = \mathrm{E}[Y_i]$, for station $i = 1, \dots, n$. Station queues are stable if*

$$\lambda_i < \lambda_{i,sup} = \frac{S_i(\boldsymbol{\tau})}{\overline{V}(\boldsymbol{\tau})}, \quad i = 1, \dots, n, \tag{13}$$

where $\boldsymbol{\tau} = [\tau_1, \dots, \tau_n]$, $S_i(\cdot)$ is defined in Eq. (1) and $\overline{V}(\cdot)$ is defined in Eq. (6).

Proof. See Appendix A. □

If the transmission probability in Algorithm 1 can be adjusted, it is possible to stabilize the network for a whole set of arrival rates. In other words, given a row vector of mean arrival rates $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_n]$, applying these input vector to the CSMA network leads to stable queues, if there exists a vector of transmission probabilities such that the inequalities in Eq. (13) are met. We can therefore define a stability region as follows

$$\mathcal{L} = \left\{ \boldsymbol{\lambda} \in \mathbb{R}^n \mid \exists \boldsymbol{\tau} \in [0,1]^n : 0 \leq \lambda_i < \lambda_{i,\text{sup}} = S_i(\boldsymbol{\tau})/\overline{V}(\boldsymbol{\tau}),\ i = 1, \dots, n \right\} \tag{14}$$

where $\overline{V}(\cdot)$ and $S_i(\cdot)$ are the functions defined in Eqs. (1) and (6). As long as $\boldsymbol{\lambda} \in \mathcal{L}$, the considered CSMA algorithm can be stabilized by a suitable choice of the transmission probabilities.

For a given $\boldsymbol{\tau}$, the set of points $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_n]$ such that $0 \leq \lambda_i < \lambda_{i,\text{sup}} = S_i(\boldsymbol{\tau})/\overline{V}(\boldsymbol{\tau})$, $i = 1, \dots, n$, is an interval in $\mathbb{R}^n$. The set $\mathcal{L}$ is the union of all such intervals as $\boldsymbol{\tau}$ spans $[0,1]^n$. An example of stability region for two stations is given in Fig. 1. Transmission times for the two stations are deterministic and equal to $Y_1 = 100 \cdot \delta$ and $Y_2 = 20 \cdot \delta$.

The set $\mathcal{L}$ defines the stability region of the heterogeneous CSMA network, as stated in the following theorem.

Theorem 2. *Let n stations share a channel according to the CSMA procedure in Algorithm 1 (without CD), with packet transmission times Y_i, with mean $T_i = \mathrm{E}[Y_i]$, $i = 1, \dots, n$. If $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_n] \in \mathcal{L}$, then there exist transmission probabilities τ_i, $i = 1, \dots, n$, such that station queues can be stabilized by setting the parameter τ appearing in Algorithm 1 equal to τ_i for station i ($i = 1, \dots, n$). Conversely, if station queues can be stabilized by setting the transmission probability of station i to some value τ_i for $i = 1, \dots, n$, then the arrival rate vector $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_n]$ must belong to the region $\mathcal{L}$.*

Proof. The first part follows from the definition of $\mathcal{L}$. Namely, if $\boldsymbol{\lambda} \in \mathcal{L}$, there must exists probabilities τ_i such that $\lambda_i < \lambda_{i,\text{sup}} = S_i(\boldsymbol{\tau})/\overline{V}(\boldsymbol{\tau})$ for $i = 1, \dots, n$. Then, according to Theorem 1, station queues are stable.

The second part is proved in Appendix B. □

The definition of the stability region in Eq. (14) implies that it is possible to set the transmission probabilities of stations according to the specific input rates, with no constraints. Given the vector $\lambda \in \mathcal{L}$, a set of transmission probabilities satisfying the inequalities in Eq. (13) must be found and assigned to stations. There must exist at least one such vector, since λ belongs to the stability region $\mathcal{L}$. A practical implementation of queue stabilization consists of a distributed adaptive algorithm to properly set the transmission probabilities. An example of such an algorithm is defined in Section 6.1

The stability region definition is easily modified to account for restrictions in the choice of transmission probabilities. Assume transmission probabilities can only takes values in the set $\mathcal{T} \subseteq [0,1]^n$. Then, the stability region becomes:

$$\mathcal{L} = \left\{ \lambda \in \mathbb{R}^n \mid \exists \boldsymbol{\tau} \in \mathcal{T} \, : \, 0 \le \lambda_i < \lambda_{i,\sup} = S_i(\boldsymbol{\tau})/\overline{V}(\boldsymbol{\tau}) \, , \, i = 1, \dots, n \right\} \quad (15)$$

As a special case, the set $\mathcal{T}$ can reduce to just one point. This is what happens with CSMA/CA, as modeled according to the classic approach in [29]. In that case, the behavior of each station can be accurately represented by Algorithm 1, where the transmission probability of all stations is the same, equal to τ_0, and τ_0 is computed by solving the following non linear equation system:

$$\tau_0 = \frac{\sum_{k=0}^{\infty} p^k}{\sum_{k=0}^{\infty} p^k \frac{2^k W_{\min}+1}{2}} = \frac{2}{1 + W_{\min} \frac{1-p}{1-2p}} \quad (16)$$

$$p = 1 - (1-\tau_0)^{n-1} \quad (17)$$

where p is the collision probability and $W_{\min}$ is the minimum contention window size of the CSMA/CA protocol. The algorithm defined for CSMA/CA in WiFi, under the model defined in [29], leads to a constrained region $\mathcal{T}$ reduced to the point $[\tau_0, \dots, \tau_0]$. Hence the stability region corresponding to a rectangle, defined by $0 \le \lambda_i \le S_0/\overline{V}_0$, $i = 1, \dots, n$, where $S_0 = \tau_0(1-\tau_0)^{n-1}$ and

$$\overline{V}_0 = \delta + \int_0^{\infty} \left[1 - \prod_{i=1}^{n} \left(1 - \tau_0 G_{Y_i}(x) \right) \right] dx \quad (18)$$

In case $Y_i = T_0$, $\forall i$, we find $\overline{V}_0 = \delta + T_0 - T_0(1-\tau_0)^n$. Then, the stability limit on the arrival rate becomes $S_o/\overline{V}_0 = \frac{\tau_0(1-\tau_0)^{n-1}}{\delta + T_0 - T_0(1-\tau_0)^n}$, which is but the well known expression of the saturation throughput of one station in a homogeneous (symmetric) model (e.g., see [29]).

In case of CD, collisions last a fixed time T_c and it must be $T_c \le Y_i$, $\forall i$. We can repeat the steps in the proof of Theorem 1, to prove the following.

Theorem 3. *Let n stations share a channel according to the CSMA procedure in Algorithm 1 (with CD), with transmission probabilities τ_i and packet transmission times Y_i, with mean $T_i = \mathrm{E}[Y_i]$, for station $i = 1, \dots, n$. Let T_c be the time required to detect a collision and assume $T_c \le Y_j$ with probability 1, $j = 1, \dots, n$. Station queues are stable if*

$$\lambda_i < \lambda_{i,sup}^{CD} = \frac{S_i(\boldsymbol{\tau})}{\overline{V}_{CD}(\boldsymbol{\tau})} \, , \quad i = 1, \dots, n, \quad (19)$$

where $\boldsymbol{\tau} = [\tau_1, \dots, \tau_n]$, $S_i(\cdot)$ and $\overline{V}_{CD}(\cdot)$ are given by Eqs. (1) and (10).

Proof. The proof follows the same steps as in Theorem 1. □

Analogous to the case where CD is not allowed, if the transmission probability in Algorithm 1 can be adjusted, we can define a stability region as follows:

$$\mathcal{L}_{\mathrm{CD}} = \left\{ \lambda \in \mathbb{R}^n \mid \exists \boldsymbol{\tau} \in [0,1]^n \, : \, 0 \le \lambda_i < \lambda_{i,\sup}^{\mathrm{CD}} = S_i(\boldsymbol{\tau})/\overline{V}_{\mathrm{CD}}(\boldsymbol{\tau}) \, , \, i = 1, \dots, n \right\} \quad (20)$$

A property analogous to the one stated in Theorem 2 can be shown in case of CD, with reference to the stability region defined above.

5. Stable throughput maximization under fairness constraint

The theorems in Section 4 identify the stability region of the considered heterogeneous CSMA, as given in Eqs. (14) and (20). Those regions are referred to packet rates λ_i, $i = 1, \dots, n$. It is also possible to introduce a notion of throughput.

Let L_i denote the average length of packets transmitted by the ith station. This is related to the packet transmission time T_i through the average bit rate of station i, R_i, namely $L_i = T_i R_i$. Since λ_i represents the mean arrival rate of packets at station i (measured in packets per unit time), the bit rate sustained by station i is given by $\lambda_i L_i = \lambda_i T_i R_i$. Then, the overall network throughput (delivered bit per unit time) is $\Lambda = \sum_{i=1}^{n} \lambda_i T_i R_i$. Requiring stability, we get the following upper bound of the throughput, conditional on given transmission probabilities $\tau_i, i = 1, \dots, n$:

$$\Lambda_{\sup}(\boldsymbol{\tau}) = \sum_{i=1}^{n} \lambda_{i,\sup} T_i R_i = \sum_{i=1}^{n} \frac{S_i(\boldsymbol{\tau})}{\overline{V}(\boldsymbol{\tau})} T_i R_i \quad (21)$$

If $\Lambda_{\sup}(\boldsymbol{\tau})$ is maximized with unconstrained transmission probabilities, i.e., for $\boldsymbol{\tau} \in [0,1]^n$, it turns out that the best we can do is setting $\tau_{i^*} = 1$ and $\tau_i = 0$, $i \ne i^*$, with $i^* = \mathrm{argmax}_{1 \le i \le n} \frac{T_i R_i}{\delta + T_i}$. Only a single station transmits, an utmost unfair outcome. Hence $\Lambda_{\sup}(\boldsymbol{\tau})$ is to be maximized under a fairness criterion. Given the mean packet transmission times T_i, $i = 1, \dots, n$, the transmission probabilities τ_i, $i = 1, \dots, n$, can be set so as to achieve airtime fairness, i.e., equalizing the average fraction of time that the channel is used successfully by each station.[3] That fraction is proportional to $T_i S_i(\boldsymbol{\tau})$, which in turn is proportional to $T_i \tau_i/(1-\tau_i)$. Airtime fairness leads therefore to requiring that $T_i \tau_i/(1-\tau_i)$ be a constant independent of i, hence $T_i \tau_i/(1-\tau_i) = T_A/n$, for $i = 1, \dots, n$, with T_A a time, to be determined. The factor $1/n$ is introduced to prepare for the asymptotic analysis that will be carried out in this Section. Hence, airtime fairness implies that the transmission probabilities take the following form:

$$\tau_i = \frac{1}{1 + nT_i/T_A} \, , \quad i = 1, \dots, n. \quad (22)$$

The stable throughput bound $\Lambda_{\sup}(\boldsymbol{\tau})$ obtained by setting the transmission probabilities according to Eq. (22) is denoted with $\Lambda_{\sup}^{AF}$.

5.1. CSMA with no collision detection

Setting the transmission probabilities as prescribed in Eq. (22) and using Eqs. (1), (6), (13) and (21) for the stability bound $\lambda_{i,\sup}$, we find

$$\Lambda_{\sup}^{AF} = \sum_{i=1}^{n} R_i \cdot \frac{\frac{\tau_i T_i}{1-\tau_i} \prod_{j=1}^{n} (1-\tau_j)}{\delta + \overline{V}'} = \sum_{i=1}^{n} R_i \cdot \frac{\frac{T_A}{n} \prod_{j=1}^{n} \left(1 - \frac{1}{1+nT_j/T_A} \right)}{\delta + \overline{V}'} = \overline{R} \cdot \rho \quad (23)$$

with $\overline{R} = \frac{1}{n} \sum_{i=1}^{n} R_i$,

$$\rho(n, T_A) = \frac{T_A \prod_{j=1}^{n} \left(1 - \frac{1}{1+nT_j/T_A} \right)}{\delta + \overline{V}'} \quad (24)$$

and

$$\overline{V}' = \int_0^{\infty} \left[1 - \prod_{j=1}^{n} \left(1 - \frac{1}{1+nT_j/T_A} G_{Y_j}(x) \right) \right] dx \quad (25)$$

Given the mean air bit rate $\overline{R}$, maximizing the throughput upper bound $\Lambda_{\sup}^{AF}$ is the same as maximizing the normalized throughput $\rho(n, T_A)$ in Eq. (24) as a function of T_A, with $T_A > 0$.

[3] An entirely similar analysis would be obtained, if the definition of airtime fairness were changed slightly to requiring the same average fraction of time used by each station, irrespective of whether it was successful or not.

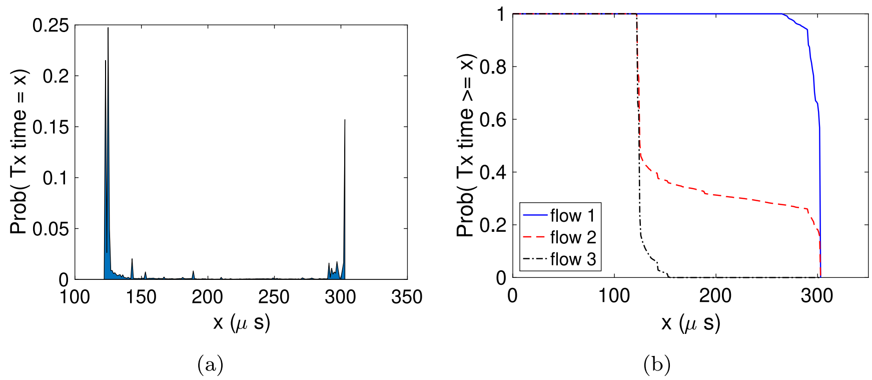


Fig. 2. (a) Empirical PDF of packet transmission times. (b) Empirical CCDFs of packet transmission times of the three considered traffic classes. Packet lengths are obtained from an IP traffic trace measured on an Internet link. Bit rate and overhead are set according to values drawn from the IEEE 802.11ac standard.

The optimal T_A, i.e., the value T_A^* that maximizes the normalized throughput ρ, depends on the number n of stations, that is, $T_A^* = T_A^*(n)$. There is no conceptual difficulty in finding $T_A^*(n)$ for an assigned value of n, given the packet transmission times PDFs of the stations. Then, the corresponding throughput-optimal fair transmission probabilities τ_i^* are found as $\tau_i^* = \frac{1}{1+nT_i/T_A^*(n)}$, $i = 1, \dots, n$.

It turns out that $T_A^*(n)$ is weakly dependent on n and we can effectively approximate $T_A^*(n)$ with $T_A^*(\infty)$. This motivates considering the asymptotic regime for $n \to \infty$, which is especially helpful to gain insight into the choice of the optimal value of T_A.[4]

We let $\mu_n = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{T_i}$ and assume that there exist a proper limit $\mu = \lim_{n\to\infty} \mu_n$. Moreover, we assume that Y_j have a finite support for all j and let $T_{\max}$ be such that $Y_j \le T_{\max}$ w.p. 1, $\forall j$. Then, it is shown in Appendix C that the following bounds hold for the asymptotic normalized throughput:

$$\frac{\alpha e^{-\alpha}}{\beta + \xi(1 - e^{-\alpha/\xi})} \le \rho_\infty(T_A) = \lim_{n\to\infty} \rho(n, T_A) \le \frac{\alpha e^{-\alpha}}{\beta + 1 - e^{-\alpha}} \tag{26}$$

where $\alpha = T_A\,\mu$, $\beta = \delta\,\mu$ and $\xi = T_{\max}\,\mu$.

It is to be emphasized that the bounds on the normalized throughput in Eq. (26) depend only on *mean* packet transmission times $T_i = \mathrm{E}[Y_i]$, i.e., they are *insensitive* to the probability distributions of the random variables Y_i. Strictly speaking, this holds asymptotically for $n \to \infty$, but numerical evidence points out that the obtained bounds are tight for essentially all values of n.

Once given δ and $T_{\max}$ (hence β and ξ), the bounds of ρ_∞ depend only on T_A through α. The upper bound in the rightmost side of Eq. (26) is maximized by setting $\alpha = \alpha^*$, where α^* is the unique solution of the equation $e^{-\alpha} = (\beta+1)(1-\alpha)$ for $\alpha \in (0, 1)$. Casting this equation into the form $(1-\alpha)e^{-(1-\alpha)} = \frac{1}{e(1+\beta)}$, we can give an expression of the solution α^* as a function of β, by exploiting the solution of the equation $ye^{-y} = u$, $u \in [0, 1/e]$ and $y \in [0, 1]$, which is known to be [38] $y = \sum_{n=1}^{\infty} \frac{n^{n-1}}{n!} u^n$. Then, we have

$$\rho_\infty^* = \max_{\alpha>0} \frac{\alpha e^{-\alpha}}{\beta + 1 - e^{-\alpha}} = 1 - \alpha^* = \sum_{n=1}^{\infty} \frac{n^{n-1}e^{-n}}{n!} \frac{1}{(1+\beta)^n} \tag{27}$$

Given α^*, the optimal value of T_A is $T_A^* = T_A^*(\infty) = \alpha^*/\mu$. The value $\rho_\infty^* = \frac{\alpha^* e^{-\alpha^*}}{\beta+1-e^{-\alpha^*}} = 1 - \alpha^*$ is the maximum achievable stable throughput under airtime fairness, obtained by setting the transmission probabilities as in Eq. (22) with $T_A = T_A^*$.

To give a numerical example, we consider a CSMA network with heterogeneous variable packet transmission times. Packet flows offered by stations belong to three different classes:

- long packet flows, e.g., bulk data traffic, web browsing.
- short packet flows, e.g., control traffic, real time audio, chatting, notifications, short messages.
- mixed packet flows, i.e., a mix of the two above classes.

To be consistent with the notation of the paper, the long packet flow type is labeled with index 1 (also referred to as traffic class 1), mixed packet lengths flows correspond to type 2 (traffic class 2), and short packet flows belong to type 3 (traffic class 3).

Packet length have been set based on Internet traffic measurements, i.e., IP packet traffic traces captured on a backbone link in San Jose, CA, USA, made available by CAIDA[5] Packet transmission times in the CSMA network are computed by using numerical values of parameters drawn from the IEEE 802.11ac standard. The backoff slot time is $\delta = 9$ μs. Considering unicast transmissions, the overall overhead, including Inter-Frame Spaces, physical layer preamble, MAC overhead and the acknowledgment, amounts to $T_{\mathrm{oh}} = 117.85$ μs. The air bit rate is fixed to $R = 65$ Mbit/s. Let L denote the IP packet length. Then, packet transmission time is $Y = T_{\mathrm{oh}} + L/R$. The empirical PDF of Y is shown in Fig. 2(a).

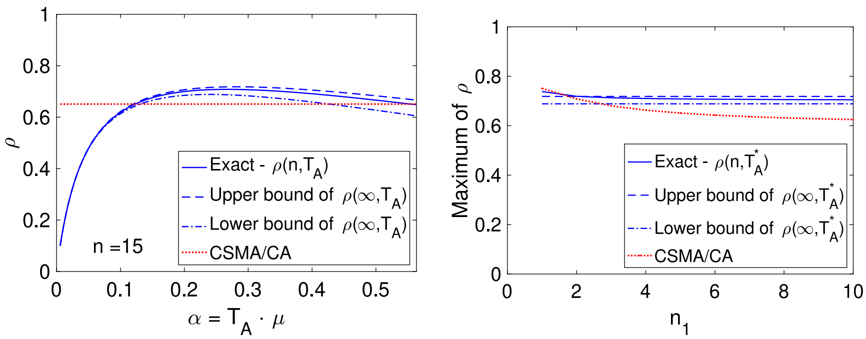
To identify the three traffic flow types listed above, we consider packet length samples not larger than 300 bytes for type 3 (short packets), packet lengths no smaller than 1200 bytes for type 1 (long packets) and all packet lengths for type 2 (mixed traffic). The resulting mean packet transmission times are $T_1 = 299$ μs, $T_2 = 180.33$ μs, and $T_3 = 126.34$ μs. The empirical CCDFs of packet transmission times of the three types of packet flows are illustrated in Fig. 2(b).

Let n_i denote the number of stations belonging to traffic class i, for $i = 1, 2, 3$. The expectation μ is evaluated as $\mu = \frac{n_1/n}{T_1} + \frac{n_2/n}{T_2} + \frac{n_3/n}{T_3}$, where $n = n_1 + n_2 + n_3$. Assuming $n_1 = n_2 = n_3$, it is $\mu = 0.0056$ μs^{-1}.

Given the values of δ and $T_{\max} = 303$ μs, we have in this example $\xi = 1.7$ and $\beta = 0.05$. The normalized throughput $\rho(n, T_A)$ is evaluated for $n_1 = n_2 = n_3 = 5$ as a function of $\alpha = T_A\mu$ in Fig. 3(a) (solid line), along with the bounds established in Eq. (26) (dashed lines). The normalized throughput achieved by CSMA/CA of WiFi, as evaluated according to Bianchi's model (see Section 4), is also depicted (dotted line). In that case, the transmission probability equals τ_0 for all nodes and no airtime fairness can be maintained. Repeating the derivation in Eq. (23) in case of WiFi CSMA/CA model, considering a fixed air bit

[4] In practice, it is not expected that an extremely large number of stations be contending in a CSMA network. The asymptotic analysis is instrumental in finding a simple analytical bound on the achievable stable throughput, that turns out to give an accurate estimate for essentially all values of the number of stations.

[5] https://www.caida.org/catalog/datasets/monitors/passive-equinix-sanjose/



(a) (b)

Fig. 3. Heterogeneous CSMA network with three types of sources, whose packet transmission times are built out of a measured IP traffic trace. (a) Normalized throughput $\rho(n, T_A)$ as a function of $\alpha = T_A\mu$; exact values are plotted as a solid line, lower and upper bounds as dashed lines. (b) Optimized throughput $\rho(n, T_A^*(n))$ as a function of the number of stations, along with lower and upper bounds evaluated for $\alpha = \alpha^* = T_A^*(\infty)/\mu$. The normalized throughput achieved by CSMA/CA of WiFi, as evaluated according to Bianchi's model, are shown (dotted line).

rate for all stations, we find $\Lambda_{\max}^{(\mathrm{CSMA/CA})} = \overline{R} \cdot \rho^{(\mathrm{CSMA/CA})}$, where

$$\rho^{(\mathrm{CSMA/CA})} = \left(\frac{1}{n}\sum_{i=1}^{n} T_i\right)\frac{n\tau_0(1-\tau_0)^{n-1}}{\overline{V}_0} \tag{28}$$

where $\overline{V}_0$ is given in Eq. (18).

The asymptotic bounds turn out to be very accurate, especially around the maximum achievable normalized throughput. They are therefore definitely useful to identify accurately the optimal value of T_A. The normalized throughput achievable under airtime fairness constraint outperforms the ordinary CSMA/CA, if T_A is suitably set. It is to be noted that CSMA/CA does *not* achieve airtime fairness. In the present case, the mean fraction of airtime used by class 2 stations (those with short packets) and class 3 stations (those with mixed length packets) are respectively 42% and 60% relative to class 1 stations (those with long packets), thus operating the channel quite far from a fair airtime sharing.

The remarkable accuracy of the asymptotic bounds for essentially all values of n is evident also from Fig. 3(b), where the maximum achieved normalized throughput $\rho(n, T_A^*(n))$ is plotted as a function of the number of stations $n_1 = n_2 = n_3$ (hence, it is $n = n_1 + n_2 + n_3 = 3n_1$), along with the values obtained by maximizing the upper and lower bounds in Eq. (26) (dashed lines). Except of the case $n_1 = 1$, the bounds predicts a correct and quite narrow interval where the exact value of the maximum stable normalized throughput falls within. More in depth, the maximized upper bound $\rho_\infty^* = 1 - \alpha^*$ provides an accurate estimate of the maximum achievable stable throughput, holding for virtually all number of stations, even if it has been established based on an asymptotic analysis for $n \to \infty$. The normalized throughput achieved by CSMA/CA of WiFi is shown as well (dotted line). In spite of not having to meet the airtime fairness constraint (thus providing only 42% and 60% airtime of class 3 stations to stations of class 1 and 2, respectively), CSMA/CA attains a lower throughput level as compared to the (fair) optimized setting, except of the case $n_1 = 1$. Moreover, while the optimized throughput level is insensitive to the number of contending stations, CSMA/CA throughput decays as n grows.

Two main results stem from these plots.

First, the bounds in Eq. (26) are very accurate for all values of the number of stations n, their accuracy improving as n grows.

A second point is that the maximum achievable normalized throughput $\rho(n, T_A^*(n))$ is only weakly dependent on n. This is the key result that backs up the use of the asymptotic analysis to build an adaptive algorithm that drives the system towards throughput optimality, under airtime fairness, for any value of n. This issue is dealt with of Section 6.

5.2. CSMA with collision detection

In case of CSMA with CD, the throughput upper bound under stability and airtime fairness constraints is $\Lambda_{\sup}^{\mathrm{CD}} = \overline{R} \cdot \rho_{\mathrm{CD}}$, where

$$\rho_{\mathrm{CD}}(n, T_A) = \frac{T_A P_e(n, T_A)}{\delta + T_A P_e(n, T_A) + T_c(1 - P_e(n, T_A) - \mu_n T_A P_e(n, T_A))} \tag{29}$$

with $\mu_n = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{T_i}$ and

$$P_e(n, T_A) = \prod_{i=1}^{n}\left(1 - \frac{1}{1 + nT_i/T_A}\right) \tag{30}$$

Assume μ_n has a proper limit as $n \to \infty$, namely $\mu_n \to \mu$. It is also easy to check that $P_e(n, T_A) \to e^{-T_A\mu}$ as $n \to \infty$. Then we have

$$\rho_{\infty,\mathrm{CD}}(T_A) = \lim_{n\to\infty}\rho_{\mathrm{CD}}(n, T_A) = \frac{\alpha e^{-\alpha}}{\beta + \alpha e^{-\alpha} + \psi\,(1 - e^{-\alpha} - \alpha e^{-\alpha})} \tag{31}$$

where $\alpha = T_A\mu$, $\beta = \delta\mu$, and $\psi = T_c\mu$.

The asymptotic throughput $\rho_{\infty,\mathrm{CD}}$ depends on T_A only through the non-dimensional parameter α. The maximum of $\rho_{\infty,\mathrm{CD}}$ is attained for $\alpha = \alpha_{\mathrm{CD}}^*$, where α_{CD}^* is the unique solution in $(0, 1)$ of the equation $e^{-\alpha} = (\beta/\psi + 1)(1 - \alpha)$. Note that $\beta/\psi = \delta/T_c$ does not depend on packet transmission times, only on (fixed) standard parameters. Hence the optimal choice of T_A in case of CD does not depend on packet transmission times. The maximum achievable asymptotic normalized throughput can be expressed as

$$\rho_{\infty,\mathrm{CD}}^* = \max_{T_A>0}\rho_{\infty,\mathrm{CD}}(T_A) = \frac{1 - \alpha_{\mathrm{CD}}^*}{1 - \alpha_{\mathrm{CD}}^* + \psi\alpha_{\mathrm{CD}}^*} \tag{32}$$

For $\psi = 1$, the maximum throughput is the same as in case of no CD. For $\psi < 1$, the optimized throughput bound of CSMA with CD becomes larger than in case of no CD and can get closer and closer to 1 as ψ becomes smaller and smaller. This matches with intuition, given that $\psi = T_c\mu \sim \frac{1}{n}\sum_{i=1}^{n} T_c/T_i$ can be viewed as the average ratio between the time required to detect a collision and the packet transmission time.

Fig. 4 shows numerical examples of the maximum achievable normalized throughput $\rho_{\mathrm{CD}}(n, T_A^*(n))$ as a function of the number of stations n, where $T_A^*(n)$ is the value of T_A that maximizes $\rho_{\mathrm{CD}}(n, T_A)$ for each given n. The dashed line is the asymptotic value of the maximum achievable normalized throughput $\rho_{\infty,\mathrm{CD}}^*$ given in Eq. (32). The back-off slot time is set to $\delta = 1$ and the collision resolution time is $T_c = 5$. Fig. 4(a) is obtained by considering mean packet transmission times uniformly distributed between $T_{\min} = 20$ and $T_{\max} = 100$. Bimodal mean packet transmission times are used in Fig. 4(b), with half of stations having mean packet transmission time $T_{\min} = 20$, the other half having mean packet transmission time $T_{\max} = 100$.

Similar comments apply here as those made in case of no CD. We notice also that the asymptotic bound of the normalized throughput,

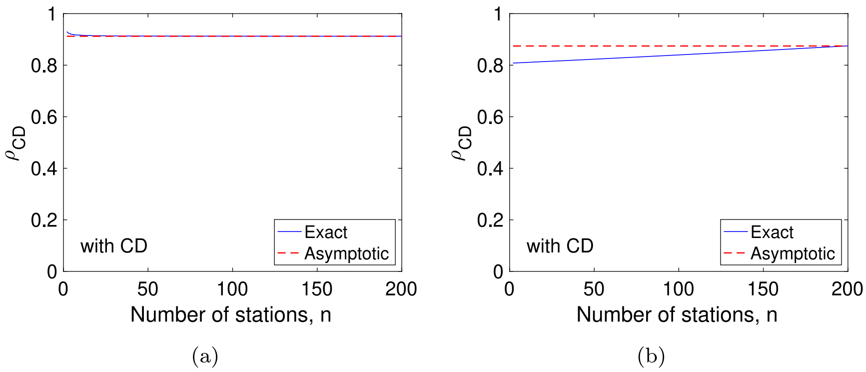


Fig. 4. Maximum achievable normalized throughput with CD as a function of the number of contending stations, n. Solid line represents $\rho_{CD}(n, T_A(n))$. Dashed line corresponds to the asymptotic value in Eq. (32). Back-off slot time is set to $\delta = 1$ and collision detection time is $T_c = 5$. (a) Mean packet transmission time uniformly distributed between $T_{min} = 20$ and $T_{max} = 100$. (b) Bimodal mean packet transmission times: odd-numbered stations have $T = T_{min} = 20$, even-numbered ones have $T = T_{max} = 100$.

i.e., $\rho^*_{\infty,CD}$ in Eq. (32), offers an excellent approximation of $\rho_{CD}(n, T^*_A(n))$ for virtually all values of n in case of uniformly distributed mean packet transmission times. In case of bimodal mean packet transmission times, the maximum error is 7.6% and it is attained for $n = 2$.

5.3. Variants on the throughput optimization problem statement

The airtime fairness concept used in this Section is not weighted. It is possible to consider a more general setting, where we require that the average fraction of time used successfully by station i be proportional to a weight $\phi_i > 0$, with $\sum_{i=1}^n \phi_i = 1$. Formally, we can require that $T_i S_i(\boldsymbol{\tau}) = \phi_i T_A/n$. The whole analysis developed in this Section can be repeated, modifying only the definition of μ_n and $G_n(x)$. More in depth, we replace the definitions in Eq. (C.1) with the following ones:

$$\mu_n^{(\phi)} = \frac{1}{n}\sum_{i=1}^n \frac{\phi_i}{T_i} \qquad G_n^{(\phi)}(x) = \frac{\frac{1}{n}\sum_{j=1}^n \frac{\phi_i G_{Y_j}(x)}{T_j}}{\frac{1}{n}\sum_{j=1}^n \frac{\phi_i}{T_j}} \tag{33}$$

Everything else stays the same, including the main result in Eq. (26).

Another variant stems from requiring that the achieved carried data rates be proportional to a given weight ϕ_i, rather than the throughput. The fairness requirement is then stated as follows: $\lambda_i L_i \propto \phi_i L/n, \forall i$, where L is a positive parameter that plays the same role as T_A in the baseline approach. Since stability entails that $\lambda_i < \lambda_{i,\sup}$, this new fairness statement leads to $\frac{\tau_i L_i}{1-\tau_i} = \phi_i \frac{L}{n}$, hence $\tau_i = \frac{1}{1+nL_i/(\phi_i L)}$. The derivations in Section 5.1 can be tracked down step by step, only replacing $A^{AF}_{\sup} = \rho(n, T_A) \cdot \frac{1}{n}\sum_{i=1}^n R_i$ in Eq. (23) with

$$A^{AF}_{\sup} = \frac{1}{\frac{1}{n}\sum_{i=1}^n \frac{\phi_i}{R_i}} \cdot \rho(n, T_L) \tag{34}$$

where $T_L = L\frac{1}{n}\sum_{i=1}^n \frac{\phi_i}{R_i}$. The throughput expression is still the product of the normalized throughput ρ (this time as a function of the parameter T_L, rather than T_A) and the average bit rate (this time the weighted harmonic average, rather that the weighted arithmetic average). The bounds in Eq. (26) still hold, provided that we replace μ with $\mu_L = \frac{\sum_{i=1}^n \phi_i/(R_i T_i)}{\sum_{i=1}^n \phi_i/R_i}$.

6. Adaptive transmission algorithm

Under airtime fairness constraints, it is shown that the stable throughput limit is maximized by setting the transmission probability of station i to $\tau_i = 1/(1+nT_i/T^*_A)$, where T_i is the mean packet transmission time of station i and $T^*_A = T^*_A(n) \approx T^*_A(\infty) = \alpha^*/\mu$ (in case of CD, α^* is replaced with α^*_{CD}).

The quantity α^* can be computed by solving a non-linear equation, while μ can be estimated by each station, observing channel busy times. The number n of contending stations can be estimated by the classic pseudo-Bayes algorithm proposed in [39] for general non-persistent CSMA algorithms, or by the Kalman filter approach studied in [40] for the basic IEEE 802.11 MAC protocol and in [41] for the IEEE 802.11e MAC protocol with priority handling.

Alternatively, an adaptive algorithm to estimate n is derived by exploiting IdleSense algorithm [32], adapted to the present context. The adaptive algorithm to adjust transmission probabilities of heterogeneous stations is defined in Section 6.1. Numerical examples of the time dynamics generated by the algorithm are presented in Section 6.2.

6.1. Algorithm for the adaptation of transmission probabilities

IdleSense adjusts the contention window of IEEE 802.11 CSMA/CA based on the observation of the estimated mean number of idle back-off slots occurring between consecutive transmissions. The optimal value of the contention window targeted by IdleSense is $W^* = 2n/\alpha^* - 1$. IdleSense algorithm adapts the contention window size of each station so as to converge to W^*. So long as the contention window W_i of station i is close to $W^* = 2n/\alpha^* - 1$ we can get an estimate of n from the contention window value as $n \approx \alpha^*(W_i + 1)/2$.

An interesting feature of IdleSense is to focus on idle back-off slots, whose number depends only on the contention mechanism. Since transmission attempts are frozen during packet transmission time, the number of idle back-off slots between successive transmissions does not depend on packet transmission times. This ensures that we can use IdleSense effectively also in the considered CSMA environment with heterogeneous packet transmission times.

In this paper, the contention window is not actually used by stations to rule access to the channel, rather the transmission probability τ_i is used by station i. Nonetheless, it is always possible to reproduce the IdleSense algorithm adapting the quantity W_i, just for the purpose of getting an estimate of n. Then the estimate of n is used in the expression of the optimized transmission probability, under airtime fairness, i.e. $\tau_i = 1/(1 + nT_i/T^*_A) = 1/(1 + nT_i\mu/\alpha^*)$, where n is estimated by means of IdleSense algorithm, namely $n \approx \alpha^*(W_i + 1)/2$.

More in depth, the probability of an idle back-off slot with n active stations is $P_e = \prod_{i=1}^n (1 - \tau_i)$. Using the expression of τ_i in Eq. (22), we have $P_e = P_e(n, T_A) = \prod_{i=1}^n \left(1 - \frac{1}{1+nT_i/T_A}\right)$. At the optimized working point, i.e., for $T_A = T^*_A(n)$, we have

$$P^*_e = P_e(n, T^*_A(n)) \approx P_e(\infty, T^*_A(\infty)) = e^{-T^*_A(\infty)\mu} = e^{-\alpha^*} \tag{35}$$

The mean number of idle back-off slots between two consecutive transmissions N_I is geometrically distributed with ratio P_e, hence its mean is $\overline{N}_I = E[N_I] = 1/(1 - P_e)$. At the optimal working point it is $\overline{N}^*_I = 1/(1 - P^*_e) \approx 1/(1 - e^{-\alpha^*})$. The contention window size W_i, hence the transmission probability τ_i, is adapted so that the estimated mean of N_I converges to $\overline{N}^*_I$.

Summing up, station i optimizes its own transmission probability τ_i, under fairness airtime constraint, by running the following steps:

1. Estimate μ by observing the channel. Let $\hat{\mu}$ denote the current estimate.
2. Estimate the mean number of idle back-off slots between two successive transmissions. Let $\hat{N}_I$ be the current estimate and $\overline{N}_I^* = 1/(1 - e^{-\alpha^*})$ be its target value.
3. Let W_i denote the contention window provided by IdleSense algorithm fed with $\hat{N}_I$ and $\overline{N}_I^*$.
4. Set $\tau_i = 1/\left(1 + \frac{W_i+1}{2} T_i \hat{\mu}\right)$.

The detailed pseudo-code of this procedure is given in Algorithm 2. Part of it is patterned after IdleSense. The innovation consists of lines 19–24. The parameter values suggested in [32] are used: $\lambda = 6$, $\mu = 1/1.0666$, $\Delta = 0.75$, $\gamma = 4$, maxtx $= 5$. Moreover, it is $g = 0.05$. The quantity $\hat{\mu}$ bears the estimate of μ It is obtained as $1/A$, where A is the average packet transmission time measured on the channel by means of an Exponentially Weighted Moving Average algorithm. Estimating $\hat{\mu}$ as $1/A$ is motivated by the bias induced by airtime fairness, i.e., the fact that stations with large packet transmission times transmit less frequently (see Appendix D for a detailed derivation).

Algorithm 2 Dynamic adaptation of transmission probability of station i.

Initialization

1: sum $\leftarrow 0$
2: ntx $\leftarrow 0$
3: $CW \leftarrow 16$
4: $A \leftarrow T_i$

After each transmission on the channel do

1: $N_I \leftarrow$ number of idle slot preceding tx
2: sumtx $\leftarrow$ sumtx+N_I
3: ntx $\leftarrow$ ntx+1
4: **if** ntx >= maxtx **then**
5: $\quad \hat{N}_I \leftarrow$ sum/ntx
6: $\quad$ sum $\leftarrow 0$
7: $\quad$ ntx $\leftarrow 0$
8: $\quad$ **if** $\hat{N}_I < \overline{N}_I^*$ **then**
9: $\qquad CW \leftarrow \lceil CW + \lambda \rceil$
10: $\quad$ **else**
11: $\qquad CW \leftarrow \lceil \mu \cdot CW \rceil$
12: $\quad$ **end if**
13: $\quad$ **if** $|\hat{N}_I - \overline{N}_I^*| < \Delta$ **then**
14: $\qquad$ maxtx $\leftarrow CW/\gamma$;
15: $\quad$ **else**
16: $\qquad$ maxtx $\leftarrow 5$;
17: $\quad$ **end if**
18: **end if**
19: **if** transmission is successfully decoded **then**
20: $\quad T_{tx} \leftarrow$ duration of transmission
21: $\quad A \leftarrow (1-g)A + gT_{tx}$
22: **end if**
23: $\hat{\mu} = 1/A$
24: $\tau_i \leftarrow 1/\left(1 + \frac{CW+1}{2} T_i \hat{\mu}\right)$

6.2. Numerical examples

Examples of the time behavior of Algorithm 2 under transients are illustrated in Figs. 5 and 6. These figures show the time behavior of transmission probabilities and cumulative airtime of stations.

A MATLAB based simulation software has been realized. It tracks the time evolution of a set of stations that share a common channel according to Algorithm 1. Stations may be either active or idle An idle station has no packet to send. An active station is backlogged continuously, as long as it remains active. Switching times between idle and active states are configured in each simulation experiment. Each

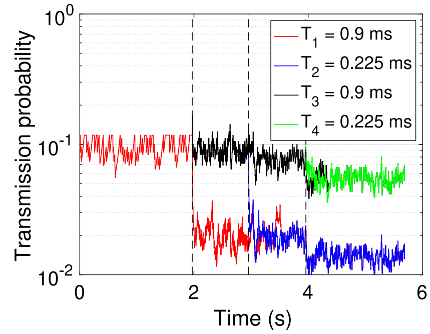


Fig. 5. Transmission probabilities versus time under Algorithm 2; the back-off slot time is set to $\delta = 9$ μs, packet transmission times are $T_1 = 0.9$ ms, $T_2 = 0.225$ ms. Four stations start transmitting at different times. The transmission probability of each station is depicted with a different color. Vertical dashed lines mark the start of transmission of each subsequent station after the first one.

station runs also the transmission probability adaptation algorithm described in Algorithm 2.

In simulations shown in Figs. 5 and 6, stations are assumed to belong to two classes, defined according to the value of the packet transmission time, either $Y_1 = T_1$ or $Y_2 = T_2$. Simulations implement the adaptive transmission algorithm, with $\delta = 9$ μs, $T_1 = 100 \cdot \delta = 0.9$ ms, $T_2 = 25 \cdot \delta = 0.225$ ms.

Fig. 5 represents transient evolution of transmission probabilities versus time for four stations, joining the network at different times. The start time of the second and subsequent stations are marked by vertical dashed lines. A station with packet transmission time T_1 starts transmitting at time 0 (red line). Then a second station with packet transmission time T_2 joins (black line). A third station adds up (blue line) with packet transmission time T_1, same as the first one. Finally a fourth station joins (green line), with packet transmission time T_2, same as the second station.

Another example of convergence and adaptation is given in Fig. 6. Here we consider one station with packet transmission time T_1 that starts transmitting at time 0. After about 0.5 s, another station with packet transmission time T_2 joins. After sharing the channel for about 1 s, station 2 leaves the channel. The plot on the left shows the behavior of the transmission probabilities of station 1 (red line) and station 2 (black line) respectively. The plot on the right illustrates the cumulative airtime obtained by the two stations versus time (only for the time interval when both stations are active on the channel).

From plots in Figs. 5 and 6, it appears that airtime fairness is achieved quite accurately and transmission probabilities are adapted quite fast to changes in the sharing pattern of the channel.

Numerical examples of the achieved normalized throughput are shown in Table 2. We consider two groups of stations: n_1 stations having packet transmission time $T_1 = 100 \cdot \delta$ and n_2 stations having packet transmission time $T_2 = 25 \cdot \delta$, with the back-off slot size δ taken as the time unit. Simulations are run to estimate the average achieved normalized throughput with the adaptive transmission algorithm. Confidence intervals at 95% level are checked for estimated throughput values to be below 0.01 of the estimated mean values.

It is apparent that airtime fairness is correctly maintained by the adaptive algorithm, namely it is $\rho_1 \approx \rho_2$. The normalized throughput gets close to the theoretical maximum value ρ^*_∞ (rightmost column of the Table), irrespective of $n = n_1 + n_2$. Note that ρ^*_∞ does not change with n (it is obtained by means of an asymptotic analysis for $n \to \infty$). Besides the theoretical asymptotic level, the actually achieved optimized throughput level appears to be almost independent of the number of contending stations.

Further simulation experiments have been run to obtain the results shown in Table 3. Here we consider three groups of stations, having

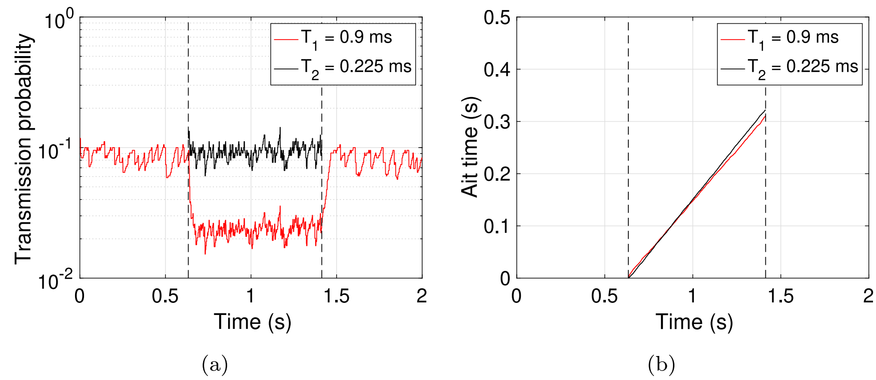


Fig. 6. Algorithm 2 with two stations. Station 1 (red line) has $T_1 = 100 \cdot \delta$, station 2 (black line) has $T_2 = 25 \cdot \delta$. The back-off slot time is set to $\delta = 9$ μs. (a) Transmission probabilities versus time. (b) Cumulative air time versus time.

Table 2
Achieved normalized throughput with Algorithm 2 in case of $\delta = 1$, n_1 stations with $T_1 = 100$, and n_2 stations with $T_2 = 25$. The last two columns are defined as follows: (i) $\rho = n_1\rho_1 + n_2\rho_2$; (ii) ρ^*_∞ is the maximum of the upper bound of the normalized throughput under airtime fairness, given in Eq. (27).

$n_1 = n_2$	ρ_1	ρ_2	ρ	ρ^*_∞
1	0.3773	0.4019	0.77921	0.79392
5	0.07644	0.07823	0.77335	0.79392
10	0.03847	0.03899	0.77454	0.79392

Table 3
Achieved normalized throughput with Algorithm 2 in case of n_i stations belonging to traffic class i, $i = 1, 2, 3$, according to the traffic model introduced in Section 5.1. The last two columns are defined as follows: (i) $\rho = n_1\rho_1 + n_2\rho_2 + n_3\rho_3$; (ii) ρ^*_∞ is the maximum of the upper bound of the normalized throughput under airtime fairness given in Eq. (27).

$n_1 = n_2 = n_3$	ρ_1	ρ_2	ρ_3	ρ	ρ^*_∞
1	0.2338	0.2441	0.2388	0.7166	0.7183
5	0.0467	0.0472	0.0471	0 .7051	0.7183
10	0.0233	0.0234	0.0234	0.7004	0.7183

variable packet transmission times with different PDFs. To this end, we consider the packet transmission time distributions estimated from IP traffic measurements introduced in Section 5.1. The adaptive transmission algorithm has been simulated with three groups of stations, having the same cardinality (hence $n_1 = n_2 = n_3$). Stations belonging to group i have packet transmission times distributed according to flow type i, $i = 1, 2, 3$, as defined in Section 5.1. The normalized throughput obtained on average by stations belonging to each class are shown in Table 3, along with the overall normalized throughput $\rho = n_1\rho_1 + n_2\rho_2 + n_3\rho_3$ and with the maximum ρ^*_∞ of the upper bound of the normalized throughput in Eq. (26).

It is apparent that also in this case with three heterogeneous traffic classes airtime fairness is achieved, since $\rho_1 \approx \rho_2 \approx \rho_3$. Also, the attained average normalized throughput is very close to the theoretical upper bound ρ^*_∞, shown in the rightmost column of the Table, thus confirming the throughput optimality of the adaptive transmission algorithm.

7. Applications

Applications of the technical contribution of this paper in practical system are discussed in this section.

Main current use cases where variants of non persistent CSMA play a major role are sensor networks, massive IoT, vehicular networking. Variants of CSMA/CA are used in IEEE 802.11ah, for sensor networks, IEEE 802.15.4, for low-rate wireless personal area networks, in IEEE 802.11p and IEEE 802.11bd, for vehicular networks. The insight gained in this paper and the adaptive algorithm presented in Section 6 can be adopted in any of the application scenarios and systems based on the standards recalled above.

As a matter of example, cooperative awareness in vehicular networks requires continuous messaging among vehicles (broadcast one-hop messages). Dedicated Short Range Communications (DSRC) [42] and ETSI ITS-G5 [43] provide standardized frameworks to support cooperative awareness and collective perception. Both are based on a shared communication channel ruled according to a variant of CSMA/CA as defined in IEEE 802.11p and IEEE 802.11bd [44]. As shown by experimental measurements on real On-Board Unit traffic, message lengths are not constant and they might differ significantly, depending also on optional fields and other features that can be vehicle specific or application specific, e.g., Cooperative Awareness Message (CAM) [45] and Decentralized Environmental Notification Message (DENM) [46]. The models defined in this paper can be applied to such a context. The presented stability analysis can help identify the load limits of reliable messaging on such vehicular networks, addressing heterogeneous environments where message generation rates are different from node to node (e.g., because they might depend on characteristics and kinematics specific to each vehicle node) and transmission times are as well different, because of different message lengths. The adaptive algorithm presented in Section 6 offers an easily implementable approach to size the transmission probability (directly tied to the contention window) of contending nodes, so as to achieve high throughput and fairness, in such heterogeneous environments.

Massive IoT is another context where the results of this paper can be applied. The use case of random access protocols in massive IoT arises when a large population of nodes is concentrated in a relatively restricted area. Data messages are sent by those nodes at random times, e.g., according to environmental events. Scheduling multiple access via signaling leads typically to excessive overhead in such contexts, given the large number of nodes to coordinate and their sporadic and hardly predictable message generation pattern. The overall amount of produced traffic might be enough to congest the shared channel, if access parameters are not properly tuned. In this context, CSMA and its variants are a popular and evergreen solution to manage the concurrent access to the shared communication channel. Applications are found in IoT scenarios [47–49], in WiFi based sensor networks as provided by IEEE 802.11ah [50], even in a 6G perspective [51] or to improve performance of LoRaWAN [52].

The analysis and results of this paper might help understanding the limits of such networks, in case they adopt non persistent CSMA (e.g., with IEEE 802.11ah or IEEE 802.15.4). They also point out the way to achieve maximum throughput under fairness constraints, which is typically desired, possibly in a weighted fairness setting, as described in Section 5.3. Thanks to the algorithm presented in Section 6, throughput maximization under fairness constraints can be achieved in a distributed way, even in heterogeneous environments, where traffic flows generated by applications residing on different nodes give rise to service times having different probability distributions in the contending nodes.

The throughput maximization results proved in Section 5 (see Eq. (26)) gives another valuable result. It states the bounds (at least asymptotically as $n \to \infty$) on the maximum achievable normalized throughput under fairness constraints. Those bounds prove to be *insensitive* to transmission times probability distributions, being only dependent on the mean transmission times. This is an interesting result in applications, where it could be difficult to assess, estimate or measure the actual probability distribution of transmission time. The result shown in this paper proves that it is enough to estimate the *mean* transmission time to drive the system towards its maximum throughput under fairness constraints.

From a more theoretical point of view, the results on stability in this paper close a gap in available knowledge on the stability of non persistent CSMA, encompassing the most general heterogeneous setting (different arrival rates of messages and different transmission times at nodes)

8. Conclusions

A non persistent model of CSMA is considered that can be applied to a general heterogeneous CSMA network, where stations are allowed to have different mean packet arrival rates, different packet transmission times with general probability distributions, and different transmission probabilities.

The stability region of the considered CSMA algorithm is identified, both with and without the collision detection function. The maximum achievable throughput under airtime fairness is investigated and tight upper and lower bounds are established. The bounds depend only on mean packet transmission times. They are therefore insensitive to packet transmission time probability distributions.

An adaptive transmission algorithm is presented that allows each station to adjust its transmission probability, so as to maximize the achievable throughput, under airtime fairness constraint, provided offered packet arrival rates are within the stability region. In other words, the proposed adaptive algorithm is throughput optimal.

The presented model can be extended by introducing a general network utility maximization framework instead of airtime fairness. A rate assignment that maximizes the social utility of the contending stations is said to be fair. Maximization is constrained by the stability requirement, that confines the arrival rate vector within the stability region identified in Theorems 1 and 2 in Section 4.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request

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Appendix A. Proof of Theorem 1

Let $\mathcal{S}$ denote the set of all states, i.e., the set of all n-tuples of non-negative integers. Let also $\mathcal{S}_{0,i} = \{\mathbf{q} \in \mathcal{S} \mid q_i = 0\}$, i.e., the set of all states such that queue i is empty.

By assumption, transmission probabilities are set so that the inequalities in Eq. (13) are met. We have to prove that, as a consequence, queues are stable.

Let us consider the following Lyapunov function

$$L(\mathbf{q}) = \frac{1}{2}\sum_{i=1}^{n} q_i^2 \tag{A.1}$$

where $\mathbf{q} = [q_1, \dots, q_n]$. The drift of the system for $t \ge 0$ is defined as

$$D(t, \mathbf{q}) = \mathrm{E}[L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t))|\mathbf{Q}(t) = \mathbf{q}] \tag{A.2}$$

Using the inequality $(\max\{0, Q - U\} + A)^2 \le Q^2+U^2+A^2+2Q(A-U)$, we derive:

$$D(t, \mathbf{q}) \le c + \sum_{i=1}^{n} q_i \mathrm{E}[A_i(t) - U_i(t)|\mathbf{Q}(t) = \mathbf{q}] \tag{A.3}$$

where c is a positive constant such that

$$\frac{1}{2}\sum_{i=1}^{n} \mathrm{E}[U_i(t)^2 + A_i(t)^2 \mid \mathbf{Q}(t) = \mathbf{q}] \le c \tag{A.4}$$

Since $U_i(t) \in \{0, 1\}$, it suffices to assume that arrival processes have finite second moments[6] for any state $\mathbf{q}$.

Let $\theta_i = \theta_i(q_i)$ be the transmission probability of station i in a virtual slot, when the backlog of station i is q_i packets. It is $\theta_i = \tau_i$ if $q_i > 0$, $\theta_i = 0$ otherwise. We define also the row vector $\boldsymbol{\theta}(\mathbf{q}) = [\theta_1(q_1), \dots, \theta_n(q_n)]$.

Given $\mathbf{Q}(t) = \mathbf{q}$, it is $U_i(t) = 1$ with probability $S_i(\boldsymbol{\theta}(\mathbf{q}))$ in Eq. (1). Then, it is

$$\mathrm{E}[U_i(t)|\mathbf{Q}(t) = \mathbf{q}] = S_i(\boldsymbol{\theta}(\mathbf{q})) \tag{A.5}$$

Further, it is

$$\mathrm{E}[A_i(t)|\mathbf{Q}(t) = \mathbf{q}] = \lambda_i \mathrm{E}[V(t)|\mathbf{Q}(t) = \mathbf{q}] = \lambda_i \overline{V}(\boldsymbol{\theta}(\mathbf{q})) \tag{A.6}$$

where $V(t)$ is the duration of the tth virtual slot time and $\overline{V}(\cdot)$ is given in Eq. (6). Then, Eq. (A.3) can be re-written as

$$D(t, \mathbf{q}) \le c + \sum_{i=1}^{n} q_i \left[\lambda_i \overline{V}(\boldsymbol{\theta}(\mathbf{q})) - S_i(\boldsymbol{\theta}(\mathbf{q}))\right] \tag{A.7}$$

It is easy to verify that

$$S_i(\boldsymbol{\theta}(\mathbf{q})) \ge S_i(\boldsymbol{\tau}) \quad \forall\, \mathbf{q} \in \mathcal{S} \setminus \mathcal{S}_{0,i} \tag{A.8}$$

$$\overline{V}(\boldsymbol{\theta}(\mathbf{q})) \le \overline{V}(\boldsymbol{\tau}) \quad \forall\, \mathbf{q} \in \mathcal{S} \tag{A.9}$$

Thanks to inequalities in Eqs. (A.8) and (A.9) and to the condition in Eq. (13), there exists $\epsilon > 0$ such that

$$q_i \left[\lambda_i \overline{V}(\boldsymbol{\theta}(\mathbf{q})) - S_i(\boldsymbol{\theta}(\mathbf{q}))\right] \le q_i \left[\lambda_i \overline{V}(\boldsymbol{\tau}) - S_i(\boldsymbol{\tau})\right] \le -q_i \epsilon \tag{A.10}$$

for all $\mathbf{q} \in \mathcal{S}$. In fact, if $q_i = 0$, the inequality in Eq. (A.10) reduces to $0 \le 0$. If instead, $q_i > 0$, since $\mathbf{q} \in \mathcal{S} \setminus \mathcal{S}_{0,i}$, we can use Eqs. (13), (A.8) and (A.9).

[6] This is certainly true if there is a finite bound to the arrival rate of new packets and a finite bound to the packet transmission time, as reasonable in a practical implementation of CSMA.

From Eqs. (A.7) and (A.10) we deduce

$$D(t,\mathbf{q}) = \mathrm{E}[L(\mathbf{Q}(t+1)) \mid \mathbf{Q}(t)=\mathbf{q}] - L(\mathbf{q}) \le c - \epsilon \sum_{i=1}^{n} q_i$$

for all $\mathbf{q} \in \mathcal{S}$ and $t \ge 0$. Removing the condition $\mathbf{Q}(t)=\mathbf{q}$, we have

$$\mathrm{E}[L(\mathbf{Q}(t+1))] - \mathrm{E}[L(\mathbf{Q}(t))] \le c - \epsilon \sum_{i=1}^{n} \mathrm{E}[Q_i(t)] \tag{A.11}$$

for $t \ge 0$. Summing over t between 0 and $T-1$ (telescoping sum) and rearranging, we find

$$\epsilon \sum_{t=0}^{T-1} \sum_{i=1}^{n} \mathrm{E}[Q_i(t)] \le cT + \mathrm{E}[L(\mathbf{Q}(0))] - \mathrm{E}[L(\mathbf{Q}(T))] \le cT + \mathrm{E}[L(\mathbf{Q}(0))]$$

where the last inequality is a consequence of the non-negativity of the Lyapunov function. Dividing by ϵT, taking the limit for $T \to \infty$ and observing that $\mathrm{E}[L(\mathbf{Q}(0))]$ is finite and independent of T, it is proved that Eq. (12) holds, which completes the proof of Theorem 1.

Appendix B. Proof of Theorem 2 , converse statement

Let us assume queues are stable with the transmission probabilities τ_i set at station i, respectively for $i=1,\dots,n$. To show that $\lambda \in \mathcal{L}$, it suffices to prove that there exists a vector of probabilities $\mathbf{u}$ such that $\lambda_i < \lambda_{i,\sup} = S_i(\mathbf{u})/\overline{V}(\mathbf{u})$ for $i=1,\dots,n$. Note that we do not claim that it must be $\tau_i = u_i$, $i=1,\dots,n$. Instead, it is proved that the possibility to realize a stable network with the given arrival rates and suitably set transmission probabilities implies that the arrival rates must belong to the stability region defined in Eq. (14), for which it suffices to show that there exists a probability vector that meets the inequalities in the definition of the stability region.

Since station queues are stable, a steady-state probability distribution of $\mathbf{Q}=[Q_1,\dots,Q_n]$ must exist. Let it be denoted with $f(\mathbf{q}) = \mathcal{P}(\mathbf{Q}=\mathbf{q})$, $\mathbf{q} \in \mathcal{S}$, where the state space $\mathcal{S}$ consists of all n-tuples of non-negative integers.

From Eq. (11), taking expectation on both sides and letting $t \to \infty$, since $\mathrm{E}[Q_i(t)]$ converges to a finite value, at equilibrium we have $\mathrm{E}[A_i] = \mathrm{E}[U_i]$, i.e.,

$$\lambda_i \sum_{\mathbf{q}\in\mathcal{S}} \overline{V}(\theta(\mathbf{q})) f(\mathbf{q}) = \sum_{\mathbf{q}\in\mathcal{S}} S_i(\theta(\mathbf{q})) f(\mathbf{q}) \tag{B.1}$$

for $i=1,\dots,n$, where $\theta_i(q_i)=0$, if $q_i=0$, $\theta_i(q_i)=\tau_i$, if $q_i>0$, and $\theta(\mathbf{q}) = [\theta_1(q_1),\dots,\theta_n(q_n)]$.

Let us split the summation over the disjoint state sets $\mathcal{S}_{0,i}$ and $\mathcal{S}\backslash\mathcal{S}_{0,i}$, i.e., the set of states where queue i is empty and those where queue i is backlogged, respectively. For each $\mathbf{q}\in\mathcal{S}_{0,i}$ we have $S_i(\theta(\mathbf{q}))=0$, while $\overline{V}(\theta(\mathbf{q}))>0$. Also, the probability that $\mathbf{q}\in\mathcal{S}_{0,i}$ is positive, since queue i being stable means that it shall attain state 0 with positive probability. Then, from Eq. (B.1) it is deduced that

$$\lambda_i \sum_{\mathbf{q}\in\mathcal{S}\backslash\mathcal{S}_{0,i}} \overline{V}(\theta(\mathbf{q})) f(\mathbf{q}) - \sum_{\mathbf{q}\in\mathcal{S}\backslash\mathcal{S}_{0,i}} S_i(\theta(\mathbf{q})) f(\mathbf{q}) < 0 \tag{B.2}$$

for $i=1,\dots,n$. Let us define the vector function $\mathbf{g}(\theta) : [0,1]^n \mapsto \mathbb{R}^n$ as $\mathbf{g}(\theta) = [g_1(\theta),\dots,g_n(\theta)]$, where

$$g_i(\theta) = \lambda_i \overline{V}(\theta) - S_i(\theta) \tag{B.3}$$

This is a continuous function of $\theta \in [0,1]^n$. Applying the theorem of the mean, it follows that there must exist a vector $\mathbf{u}\in[0,1]^n$ such that

$$g_i(\mathbf{u}) = \frac{\sum_{\mathbf{q}\in\mathcal{S}\backslash\mathcal{S}_{0,i}} g_i(\theta(\mathbf{q})) f(\mathbf{q})}{\sum_{\mathbf{q}\in\mathcal{S}\backslash\mathcal{S}_{0,i}} f(\mathbf{q})}, \quad i=1,\dots,n. \tag{B.4}$$

Then, substituting the expression of $g_i(\theta)$ into the right-hand side of Eq. (B.4), we get

$$g_i(\mathbf{u}) = \frac{\sum_{\mathbf{q}\in\mathcal{S}\backslash\mathcal{S}_{0,i}} \left[\lambda_i \overline{V}(\theta(\mathbf{q})) - S_i(\theta(\mathbf{q}))\right] f(\mathbf{q})}{\sum_{\mathbf{q}\in\mathcal{S}\backslash\mathcal{S}_{0,i}} f(\mathbf{q})} < 0 \tag{B.5}$$

where the last inequality stems from Eq. (B.2). From Eq. (B.5) we have $g_i(\mathbf{u}) < 0$, hence

$$\lambda_i \overline{V}(\mathbf{u}) - S_i(\mathbf{u}) < 0 \quad \Rightarrow \quad \lambda_i < \frac{S_i(\mathbf{u})}{\overline{V}(\mathbf{u})} \tag{B.6}$$

for $i=1,\dots,n$, which proves that the mean arrival rate vector $[\lambda_1,\dots,\lambda_n]$ belongs to the region defined in Eq. (14). This, completes the proof.

Appendix C. Asymptotic analysis of normalized throughput

Let us consider the asymptotic regime as $n\to\infty$. Assume $T_i \in [T_{\min}, T_{\max}]$, $\forall i$, with $0 < T_{\min} < T_{\max}$. We let

$$\mu_n = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{T_i} \qquad G_n(x) = \frac{\frac{1}{n}\sum_{j=1}^{n}\frac{G_{Y_j}(x)}{T_j}}{\frac{1}{n}\sum_{j=1}^{n}\frac{1}{T_j}} \tag{C.1}$$

for any positive n. We assume that $\mu = \lim_{n\to\infty}\mu_n$ and $G(x) = \lim_{n\to\infty} G_n(x)$ are well defined. A practically important case is when stations belong to M possible classes, each one characterized by its own PDF of channel holding time Y_j, $j=1,\dots,M$. If f_j denotes the fraction of stations belonging to class j, we have $\mu = \sum_{j=1}^{M} f_j \frac{1}{T_j}$ and $G(x) = \mu^{-1}\sum_{j=1}^{M} f_j \frac{1}{T_j} G_{Y_j}(x)$, with $f_1+\dots+f_M = 1.$. We aim at establishing the bounds in Eq. (26) on the limiting value of $\rho(n,T_A)$ as $n\to\infty$, for a fixed value of T_A. For large n, we have

$$\overline{V}' \sim \int_0^\infty \left[1 - \prod_{j=1}^{n}\left(1 - \frac{T_A}{nT_j}G_{Y_j}(x)\right)\right] dx \sim \int_0^\infty \left[1 - \prod_{j=1}^{n} e^{-T_A G_{Y_j}(x)/(nT_j)}\right] dx \sim \int_0^\infty \left[1 - e^{-T_A\mu_n G_n(x)}\right] dx \tag{C.2}$$

Then

$$\overline{V}'_\infty = \lim_{n\to n}\overline{V}' = \int_0^\infty \left[1 - e^{-\alpha G(x)}\right] dx \tag{C.3}$$

where $\alpha = T_A\mu$ and $G(x) = \lim_{n\to\infty} G_n(x)$. Note that $G_n(x)$ is a Complementary Cumulative Distribution Function (CCDF) $\forall n$, as well as $G(x)$. In fact, it is $G(0)=1$, $\lim_{x\to\infty}G(x)=0$, and $G(x)$ is monotonously decreasing. Moreover, since it is $\int_0^\infty G_{Y_j}(x)\,dx = T_j$, we have, taking the limit for $n\to\infty$:

$$\int_0^\infty G_n(x)\,dx = \frac{1}{\frac{1}{n}\sum_{j=1}^{n}\frac{1}{T_j}} \quad \Rightarrow \quad \int_0^\infty G(x)\,dx = \frac{1}{\mu} \tag{C.4}$$

It can be verified that $\frac{1-e^{-\alpha u}}{1-e^{-\alpha}} \ge u$ for $u\in[0,1]$ and for any positive α. Since $G(x)\in[0,1]$, applying this inequality to Eq. (C.3), it follows that

$$\overline{V}'_\infty = \int_0^\infty \left[1 - e^{-\alpha G(x)}\right] dx \ge (1-e^{-\alpha})\int_0^\infty G(x)\,dx = \frac{1-e^{-\alpha}}{\mu} \tag{C.5}$$

Assume now that $Y_j \le T_{\max}$ w.p. 1 and let $\xi = T_{\max}\mu$. Under this assumption, it is $G(x)=0$ for $x > T_{\max}$. We exploit the following inequality, holding for any positive ξ and α: $1-e^{-\alpha G(x)} = 1-e^{-(\alpha/\xi)\xi G(x)} = 1 - e^{-\alpha/\xi}e^{-(\alpha/\xi)[\xi G(x)-1]} \le 1 - e^{-\alpha/\xi} + e^{-\alpha/\xi}\frac{\alpha}{\xi}[\xi G(x)-1]$, where the last passage is a consequence of the inequality $e^{-y} \ge 1-y$. Integrating both sides of $1-e^{-\alpha G(x)} \le 1-e^{-\alpha/\xi} + (\alpha/\xi)e^{-\alpha/\xi}[\xi G(x)-1]$ over $[0,T_{\max}]$, we get

$$\overline{V}'_\infty = \int_0^{T_{\max}} \left[1 - e^{-\alpha G(x)}\right] dx \le T_{\max}(1-e^{-\alpha/\xi}) + (\alpha/\xi)e^{-\alpha/\xi}\int_0^{T_{\max}} [\xi G(x)-1]\,dx \tag{C.6}$$

The integral in the rightmost side is equal to 0, since $\int_0^{T_{\max}}[\xi G(x)-1]\,dx = \xi\int_0^{T_{\max}} G(x)\,dx - T_{\max} = \xi/\mu - T_{\max} = 0$. The last equality is a consequence of the definition of the parameter ξ.

Putting together the inequalities established in Eqs. (C.5) and (C.6), we have

$$\frac{1-e^{-\alpha}}{\mu} \le \overline{V}'_{\infty} \le T_{\max}(1-e^{-\alpha/\xi}) \tag{C.7}$$

Exploiting the relationship $\prod_{j=1}^{n}\left(1-\frac{1}{1+nT_j/T_A}\right) \sim e^{-\alpha}$ and the inequalities in Eq. (C.7), taking the limit for $n \to \infty$ in Eq. (24) leads to

$$\frac{T_A e^{-\alpha}}{\delta + T_{\max}(1-e^{-\alpha/\xi})} \le \rho_{\infty}(T_A) \le \frac{T_A e^{-\alpha}}{\delta + \frac{1-e^{-\alpha}}{\mu}} \tag{C.8}$$

where $\rho_{\infty}(T_A) = \lim_{n\to\infty} \rho(n, T_A)$.

Multiplying numerator and denominator of the left and right hand sides of inequalities in Eq. (C.8) by μ, reminding the definitions of α, ξ and $\beta = \delta\mu$, we get the bounds in Eq. (26).

Appendix D. Estimate of μ in Algorithm 2

The estimator of μ used by each station active on the channel in the adaptive transmission algorithm of Section 6 is defined as $\hat{\mu}_k = 1/A_k$, with

$$A_{k+1} = (1-g)A_k + gT_{\mathrm{tx},k} \tag{D.1}$$

where $T_{\mathrm{tx},k}$ is the packet transmission time observed on the channel and the index k refers to the kth observation. It is assumed that $\{T_{\mathrm{tx},k}\}_{k\ge 0}$ form a stationary sequence.

Given that the transmission is successful, the probability that station i did the successful transmission is

$$p_i = \frac{S_i(\boldsymbol{\tau})}{S_1(\boldsymbol{\tau}) + \cdots + S_n(\boldsymbol{\tau})} = \frac{\tau_i/(1-\tau_i)}{\sum_{j=1}^{n} \tau_j/(1-\tau_j)} \tag{D.2}$$

Under airtime fairness, it is $\tau_i/(1-\tau_i) = T_A/(nT_i)$. Hence

$$p_i = \frac{1/T_i}{1/T_1 + \cdots + 1/T_n} \tag{D.3}$$

Hence, we have

$$\begin{aligned} \mathrm{E}[T_{\mathrm{tx}}] &= \sum_{i=1}^{n} p_i \mathrm{E}[T_{\mathrm{tx}} \mid \text{station } i \text{ transmits}] \\ &= \sum_{i=1}^{n} p_i T_i = \frac{n}{1/T_1 + \cdots + 1/T_n} = \frac{1}{\mu_n} \end{aligned} \tag{D.4}$$

Taking expectation on both sides of Eq. (D.1) and solving the difference equation with initial condition A_0, we find

$$\mathrm{E}[A_k] = (1-q)^k A_0 + [1-(1-g)^k]\mathrm{E}[T_{\mathrm{tx}}], \quad k \ge 0. \tag{D.5}$$

As $k \to \infty$, we have $\mathrm{E}[A_k] \to \mathrm{E}[T_{\mathrm{tx}}] = 1/\mu_n$, where the last approximation stems from Eq. (D.4).

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