Numerical investigation on the modal properties of slopes

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ABSTRACT

Earthquake-induced displacements of slopes are often evaluated through decoupled approaches combined with empirical relationships accounting for the characteristics of the input motion and the dynamic properties of the sliding mass. As per the latter point, recent studies highlighted the key role played by the system modal properties on the slope performance. Notwithstanding, the natural periods of the slope are commonly evaluated through over-simplified expressions relating to homogenous soil deposit with horizontal ground level. With the aim to provide a more realistic and comprehensive dynamic identification, this study presents a numerical investigation on the modal features of slopes with planar slip surface, implemented in the analysis framework OpenSees. The consideration of several slope geometries and mechanical properties point out i) the influence of the soil incompressibility, of particular interest under saturated conditions, and ii) effective ranges for the multiaxial modal periods and participation masses as a function of the soil stiffness. The results of the parametric study are finally used to derive a practice-oriented identification procedure of the modal features of slopes, as starting information for calibrating simplified numerical tools devoted to their large-scale assessment.

Keywords: slopes, modal analysis, multidirectional response, dynamic identification, OpenSees

1 INTRODUCTION

A realistic evaluation of the seismic displacements of natural slopes is of paramount importance for assessing damages induced in adjacent buildings and infrastructural systems. To this purpose, several simplified methods developed over the years allow to perform extensive studies of the seismic performance of slopes at a low computational cost.

The behaviour of the unstable mass of a slope, that is the soil region involved in a global plastic mechanism, is traditionally investigated through the scheme of a rigid sliding block according to the well-known Newmark method (Newmark, 1965). Notwithstanding, more recently this assumption has been often overcome to account for the deformability of the unstable mass. A possible strategy consists of adopting a discrete lumped-mass system along the line originally tracked by Kramer & Smith (1997), then further extended to account for the multidirectional shaking (Kayen, 2017) and the multi-modal response of the unstable soil region through the introduction of a series of additional masses into simplified interpretative models (e.g. Song et al., 2022; Tropeano et al, 2016). On the other hand, in the so-called *continuous dynamic systems* the dynamic response of the soil within the slip surface is described in a simplified manner through a combination of masses and stiffnesses along the height of the slope moving in the direction of sliding, calibrated to reproduce the fundamental modal shape of a one-dimensional soil deposit with horizontal ground level (e.g. Rathje & Bray, 1999; Katsenis et al., 2020).

In light of the above, there is the necessity of a more systematic investigation of the dynamic response of natural slopes. For instance, the aforementioned discrete lumped-mass approach is affected by an uncertain calibration of the masses participating in the dynamic slope response, while the continuous dynamic systems rely on oversimplified expressions of the vibration period of the unstable mass in the sole direction of sliding.

In this perspective, in the last few years modal analysis using finite element methods (FEM) was performed to analyse the dynamic characteristics of jointed rock slopes (e.g. Song et al., 2019, 2020, 2021). However, there is still the lack of standardised, practice-oriented procedures for the identification of the dynamic features of natural slopes.

This study aims at proposing a more general view to the modal features of natural slopes through an extensive use of modal analysis. The latter was carried out on simplified, bi-dimensional numerical layouts, considering a variety of slope conditions. The preliminary results discussed in the following are presented as a series of abacuses identifying the multiaxial modal characteristics of the unstable mass, of particular interest for calibrating simplified approaches for the regional seismic analysis of slopes.

2 NUMERICAL FRAMEWORK

Figure 1a depicts a typical, in-plane global plastic mechanism of a natural slope that can be triggered by the seismic shaking. In this study, it is regarded as a triangular-shaped unstable mass resting on an inclined, planar sliding surface. As a further simplifying assumption, the latter is assumed to exhibit a linear elastic behaviour, defined by a shear wave velocity, V_s, Poisson's ratio, ν, and mass density, ρ, constant in space. In particular, the approximation to linear behaviour is needed for investigating the modal features of the unstable mass. The elastic stiffness can be accordingly taken as the small-strain value or, in first approximation, can be considered in secant terms.

The modal properties of the sliding mass are explored through a parametric study considering different geometry and mechanical properties of the slope. This was accomplished by means of a series of bi-dimensional numerical layouts, implemented in the finite element analysis framework OpenSees (McKenna, 1997; McKenna et al., 2000). The mesh generation and visualisation of the results was performed through the pre/post-processor software GID (Diaz and Amat, 1999).

The domain discretisation was performed through a parametric mesher, implemented in MATLAB (2022). The generic mesh is shown in Figure 1c: it is composed of quadrilateral elements working under plane strain conditions. Since the focus is on the modal characteristics of the unstable mass, the nodes along the slip surface are fixed. The eventual occurrence of an undrained response of a saturated deposit is taken into account in a simplified manner by setting $\nu = 0.5$.

Assuming a flat upstream soil region, the geometry of the unstable mass is completely defined by the slope height, H, and the inclinations of the sliding surface and of the sloping ground, β and ε, respectively.

Referring to the model settings above, a parametric study was carried out on the modal features of triangular-shaped sliding masses, as described in the following section. For each slope configuration analysed, 300 vibration periods were computed to ensure the attainment of cumulative modal participation mass ratios, α_i, greater than 95% for the global degrees of freedom of the domain (two translations and one rotation).

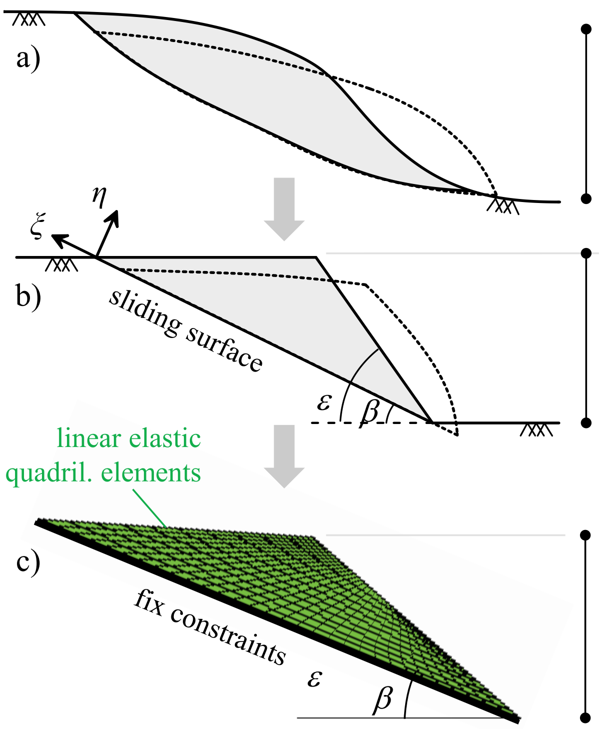

Fig. 1. a) Schematic layout of a slope with formation of a sliding mass under dynamic loading, b) geometry of the considered global plastic mechanism, c) numerical model of the sliding mass.

2.1 Parametric study

To explore the role of the geometry of the unstable mass on its dynamic properties, different inclinations of the sloping ground $\varepsilon = 10°, 15°, 20°$ and of the slip surface $\beta = 5°, 10°, 15°$ were analysed, considering only the realistic combinations of the angles corresponding to $\varepsilon > \beta$.

The soil is characterised by a mass density $\rho = 2$ Mg/m^3, while two values of the Poisson's ratio equal to 0.25 and 0.5 were adopted to reproduce, although in a simplified fashion, drained and undrained conditions (the latter of interest in the case of fully saturated domains), respectively.

As per the soil stiffness, seven values of V_s were used, ranging from 150 m/s to 800 m/s in accordance to the soil classification prescribed in Eurocode 8, Part 1 (CEN, 2003). As a result, a total of 84 modal analyses were carried out.

3 MODAL FEATURES

The modal features of the unstable slope mass are evaluated in terms of vibration periods, T_ξ and T_η, and participating mass ratios, α_ξ and α_η, along the ξ and η-directions, parallel and normal to the sliding surface, respectively.

For each case analysed, the translational vibration modes along the ξ and η-directions having participating mass ratio greater than 5% (significant modes) are always decoupled. Typically, the higher order modes in the η-direction present a minor coupling with the global rotational degree of freedom (combined normal translation-rotation of the unstable mass). The number of significant modes ranges between 7 and 10 when ν=0.25 and, in most of the cases, is not greater than 4 for the case of incompressible soil.

As an example, Figure 2 shows the modal shape, vibration periods and participation mass ratios of the fundamental mode (that is, the one characterised by the highest mass ratio) for an unstable mass with β = 15°, ε = 20°, V_s = 180 m/s, and ν = 0.25, 0.5. As expected, the period along the direction parallel to the sliding surface, T_ξ, is larger than that in the normal direction, T_η. Moreover, it is evident that only the modes along the η-direction are affected by Poisson's ratio: when the soil is incompressible the response becomes much stiffer in the normal direction, with a significant reduction of the vibration period.

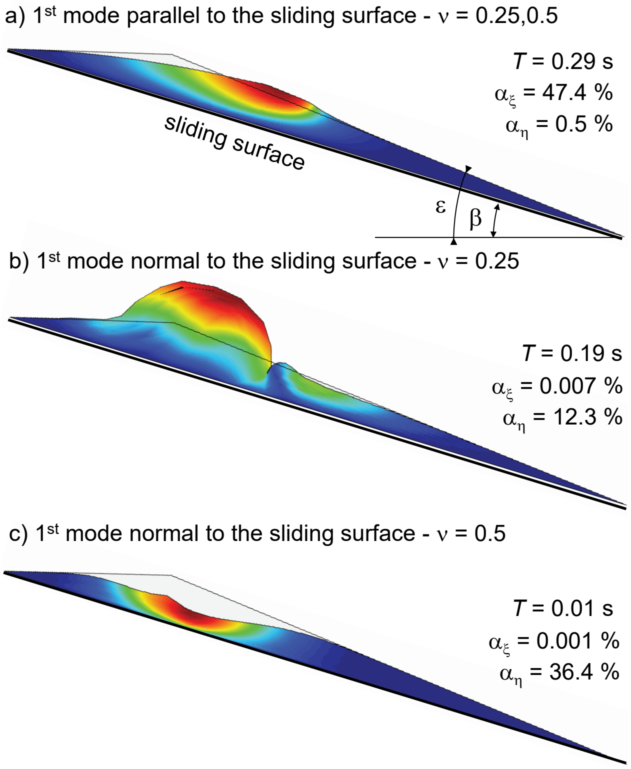

Fig. 2. Fundamental modal features (i.e., modal shape, vibration period T, modal participating mass ratios $\alpha_{\xi,\eta}$) of a triangular-shaped sliding mass characterized by sliding surface and slope inclinations, β = 15° and ε = 20°, shear wave velocity, V_s = 180 m/s, and Poisson's ratio, ν = 0.25, 0.5.

The vibration periods associated with the fundamental translational modes obtained for all the considered cases are plotted in Figure 3 as a function of the shear wave velocity, for compressible and incompressible soil. Consistently to the results of Figure 2, T_ξ is practically independent on Poisson's ratio, while T_η is remarkably lower for ν = 0.5 and not affected by the soil stiffness. The latter is however slightly greater than zero because of the development, though marginal, of shear strains caused by the peculiar geometry of the unstable mass.

The vibration periods visibly reduce as the shear wave velocity increases and are characterised by a progressively narrower band of variability, meaning that the scattering obtained for soil categories C and D is closely related to the geometric characteristics of the unstable mass.

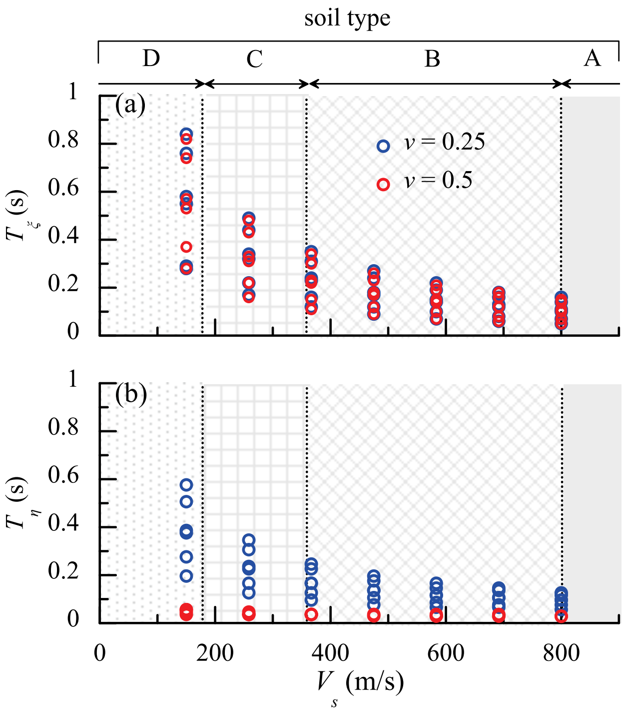

Fig. 3. Variability of the fundamental periods, T_ξ and T_η, in the direction a) parallel and b) normal to the sliding surface, respectively, with the shear wave velocity.

4 PREDICTIVE LAWS

The results of the parametric study are now used to derive a straightforward identification procedure of the modal features of a sliding sloping ground, as a key element to calibrate simplified approaches for the seismic assessment of slopes at the territorial scale (e.g. Chen et al., 2023).

As a first step, the fundamental period T_ξ of the unstable mass in the direction of sliding can be obtained by the fundamental period T_{1D}=4H/V_s of a homogeneous soil column with height H and flat ground level, using the correlation indicated in Figure 4. The former is always significantly lower than the latter, with a ratio T_ξ/T_{1D} not exceeding 0.65. However, for a given tan(β)/tan(ε), T_ξ/T_{1D} can be reasonably taken as a constant, independently from the volumetric stiffness of the soil, so that T_ξ can be evaluated without any numerical simulation.

Figure 5a shows the ratio between T_ξ and the fundamental period T_η in the direction normal to the sliding, as a function of the shear wave velocity. This ratio slightly increases with V_s. For the considered slope configurations, T_η can be taken as 0.6 T_ξ in the case of ν

= 0.25, whereas the soil deformability in the η-direction can be reasonably neglected in the case of incompressible soil conditions (e.g., fully saturated soils under dynamic loading).

From Figures 5b,c it can be inferred that also the modal participation mass ratios associated with the fundamental periods in directions ξ and η are marginally influenced by V_s. In the cases examined, α_ξ and α_η are never greater than 60%, as a demonstration that the dynamic response is controlled by a number of vibration modes. In the direction of sliding (Fig. 5b), the mass participation is slightly influenced by the volumetric stiffness of the soil, with values well concentrated in the range 40-60%. In the normal direction (Fig. 5c), the volumetric stiffness plays a more significant role: for ν=0.25, α_η ranges between 10-30%, whilst the soil incompressibility leads to greater values of the fundamental modal mass, within 30-45%. However, as noted in the discussion of Fig. 5a, when ν=0.5 the dynamic response of the system in the η-direction is activated for extremely low periods, beyond the typical frequency content of the seismic loading.

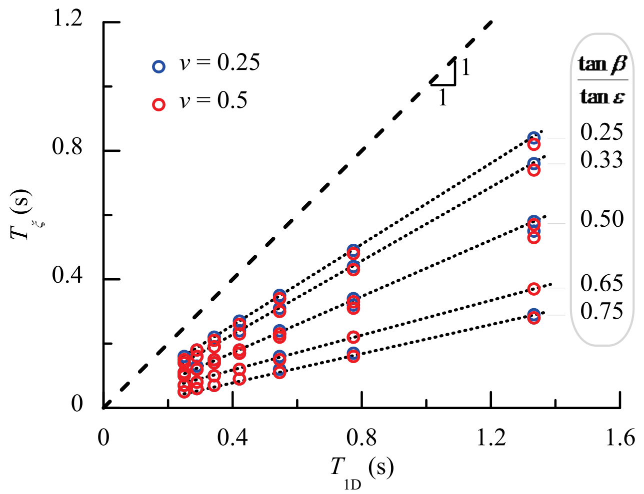

Fig. 4. Fundamental period, T_ξ, of the unstable mass in the direction of sliding plotted as a function of the 1D free-field fundamental period, T_{1D}.

5 CONCLUSIONS

This paper presents the results of a series of modal analyses of natural slopes. The eigenvalue analysis allowed for the identification of key elements governing the dynamic response of the unstable sloping ground, i.e., the multiaxial vibration periods and participating mass ratios. Different configurations of the slope in terms of geometry and mechanical properties were investigated. In this regard, it was shown the remarkable role played by the soil shear stiffness and the volumetric compressibility, the latter of particular interest in the case of saturated sloping grounds under seismic loading.

The proposed results, although preliminary and currently under extension to a broader range of cases, can be regarded as a practice-oriented tool for fast evaluations of the dynamic properties of slopes without the necessity of specific, demanding numerical simulations, of crucial interest for practice-oriented approaches for large-scale seismic assessment. The comparison between the vibration periods obtained through the proposed numerical framework and widely used solutions available in the literature pointed out the importance of the peculiar topographic conditions of slopes and their multiaxial dynamic response.

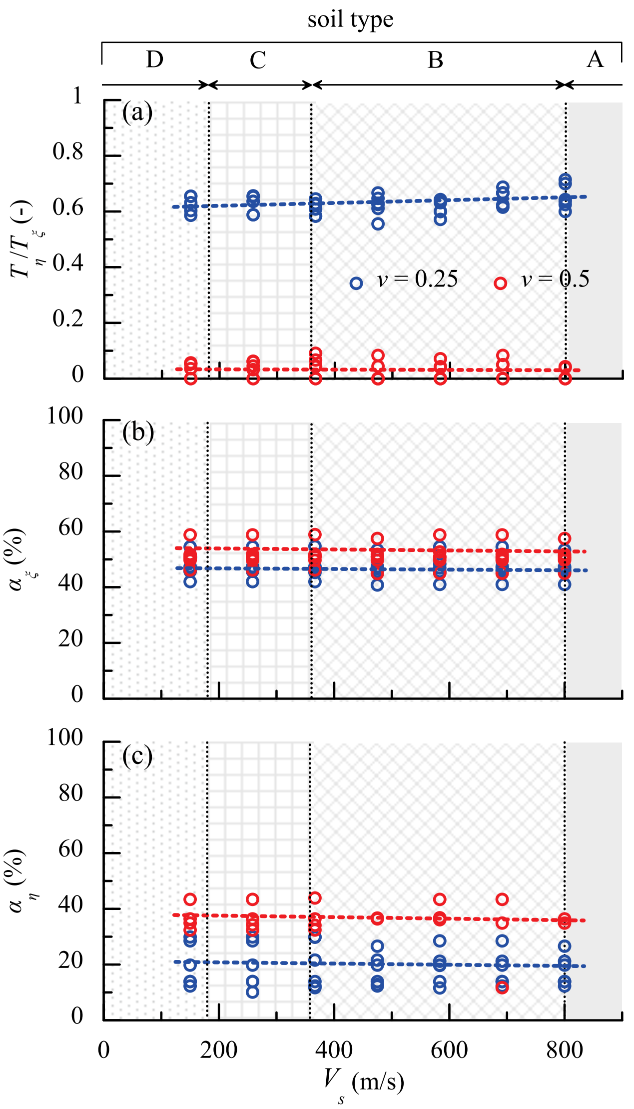

Fig. 5. a) Normal-to-parallel vibration period ratio, T_η/T_ξ, and b,c) modal participation mass ratios parallel and normal to the sliding surface, α_ξ and α_η, plotted as a function of the shear wave velocity.

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