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The determinants of decision time in an ambiguous context

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Abstract

This paper builds on the data from a published paper on behaviour under ambiguity (Conte & Hey, 2013)—henceforth C&H—to explore the determinants of *decision time*. C&H categorized individual subjects as being of one of four types (of decision-maker)—Expected Utility, Smooth Ambiguity, Rank Dependent and Alpha Expected Utility—by using the *decisions* of the subjects, but did not look at the *decision times* of the different types. We take as given the categorization identified by C&H, and explore whether the classification can explain the decision times of the subjects. We investigate whether and why different types take a different amount of time to decide. We explore the effects of various features related to (mainly psychological) theories of the *process* of decision-making—i.e., experience with the task, complexity, closeness to indifference and similarity of the options. Our results show that different types take a similar time to make their decisions on average, but decision times of different types are explained by different features of the decision task. This paper is the first investigating the heterogeneity of decision times based on a classification of subjects into different types in an ambiguous (rather than risky) decision context.

Keywords Decision time · Choice under uncertainty · Panel data · Cross-validation

JEL Classification C23 · C24 · C91 · D81

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1 Introduction

This paper uses data from C&H that has hitherto been ignored. That paper, using an experiment in which subjects faced a decision problem under ambiguity, classified each subject into one of four *types*. These types depended upon the preference functional that was apparently used by the subject in reaching their decisions. The types were the Expected Utility Model, the Smooth Ambiguity Model, the Rank Dependent Expected Utility Model and the Alpha Expected Utility Model. We will summarise these different models shortly.

The data from the experiment on the decisions that subjects took was used in this classification; data on the time that each subject took in coming to each decision *was not* used. This paper rectifies this omission, and shows that the analysis of the decision time data yields useful information about the cognitive processes of the subjects. In particular, we show here that decision time is closely related to the type of the subject. We present a *post hoc* analysis of the reasons why this might be so.

We start with a brief summary of the various identified types. As we say above, the identification depends upon the apparent preference functional used by each subject. One type that we consider is the type that apparently uses the *Expected Utility* Model (*EU*) (von Neumann et al., 1944), which states that agents compute the Expected Utility associated with each choice as the sum of the utility of the payoffs weighted by their associated probabilities (where these are necessarily subjective), and then chooses the option that gives the highest Expected Utility. A second type to explain choices under ambiguity is the *Smooth Ambiguity* Model (*SM*) proposed by Klibanoff et al. (2005). This can be interpreted as a *multiple prior* model which assumes that the decision-maker has a set of possible probabilities, and attaches a probability to each member of this set. The decision-maker then bases her[1] decision on the expected value of some function of expected utility for each member of this set. Another type is those that use the *Rank Dependent Expected Utility* Model (*RD*) of Quiggin (1982) which assumes that the decision-maker works with subjective probabilities and weights them in a way that depends on the rank of the possible outcomes. The final type is the *Alpha Expected Utility* Model (*AM*) of Ghirardato et al. (2004) which also posits a set of possible probabilities and the decision-maker decides on the basis of a weighted average of the lowest and highest expected utilities over this set.

We focus our analysis on the four types described above. As shown by C&H, these four types are the best in describing and predicting behaviour of subjects in their experiment. We will assume from now on that this classification is correct.[2]

[1] For 'her' read 'his or her'. And the same, *mutatis mutandis*, for 'she'.

[2] Later, we will provide econometric evidence relating to this. Of course, it can always be said that there could be other types (including a catch-all of 'any other behaviour') but this does not detract from our analysis (though, of course, we cannot say anything about the decision times of these other types). It may be useful at this stage to point out that our classification exercise used 19,668 observations.

Our analysis in this current paper concerns decision times. Psychologists have been incorporating decision times in the study of decision processes for a long time, while it is only recently that this tool has started to be used in the context of economic choices. We will provide more details on this in Section 3.

The innovative approach of our work relies on the heterogeneity in terms of the decision-making types identified in the data from C&H. We study how different types differ in decision times. We then probe deeper: we analyse which features of the choice problem influence the decision times for each type of subject. In particular, for each type, we analyse which aspects of the decision problem are the most salient. To the best of our knowledge, no study has yet investigated the factors described below in the context of decision-making under ambiguity.[3]

- The characteristics of a choice that influences (lengthening, shortening), or not influences decision times. It is well established that decision times are positively correlated with cognitive effort, but usually we can only observe (and record) the amount of time taken by subjects to reach a decision. We do not observe how much time is allocated or spent among the different attributes of the problem, and different features of the same problem might lengthen or shorten decision times.
- The effect these characteristics have on subjects of different types. A particular feature of a choice might be relevant for an *EU* type, extending the time and cognitive effort necessary to make a decision, but the same feature of the same choice might be irrelevant for an *AM* type, allowing her to choose in a way that is less costly in terms of time and effort.
- Deepening the relationships between the characteristics of a choice problem and agents' choices is a key step in developing a theory that unites cognitive and decision-making processes and that is in line with empirical evidence. In addition, fully understanding the influence these characteristics have on decision times ensures that policymakers can develop increasingly effective welfare improvement tools.[4]

The rest of the paper is organised as follows. In Section 2, we describe the experiment. Section 3 contains an overview of the literature about decision times and a description of the peculiarities of our data set. In Section 4, we present the econometric model. Section 5 describes the variables included in our model as controls and the estimation results. Section 6 shows a validation exercise that tests which of the estimated models (one per type) predicts decision time for each of the types of decision-maker considered best. Section 7 concludes.

[3] Our analysis may also be useful for analysing behaviour in other contexts.

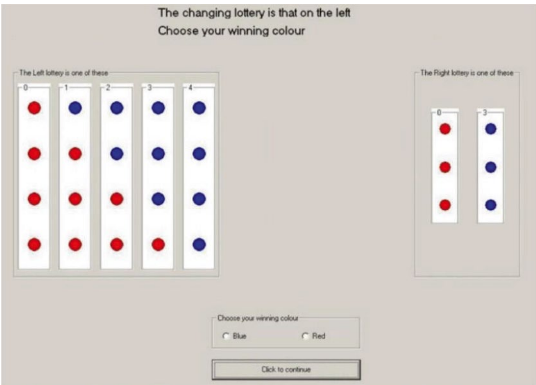
[4] Psychology, behavioural and experimental economics, through the study of behaviours and decisions, have repeatedly provided suggestions/insights that have resulted in improved collective welfare. Examples include the identification of so-called cognitive bias or nudge theory.

2 The experiment

The data analysed in this paper is based on the experiment reported in C&H. Here we recall only those features that are essential for understanding the analysis described in the rest of the present paper.

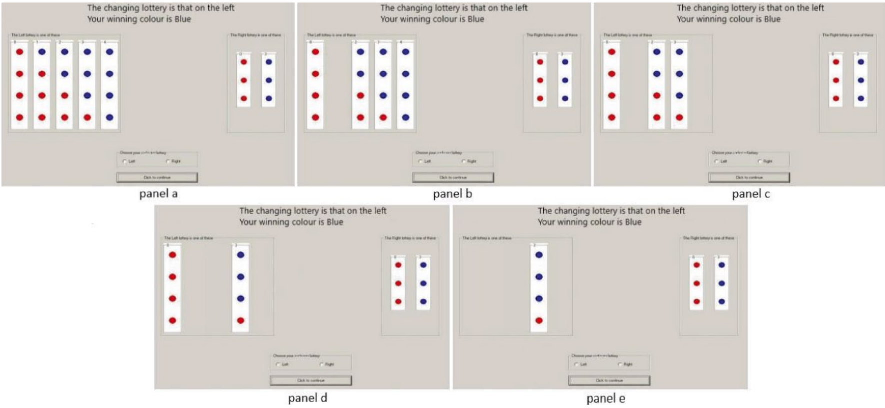
Subjects faced 49 tasks, each involving several rounds of decisions, and each decision involved a choice between two two-stage lotteries. Each two-stage lottery is made of several one-stage lotteries. A one-stage lottery is composed of a certain number of red balls and a certain number of blue balls. One of the two-stage lotteries displayed on the screen is labelled "unchanging lottery" and the other "changing lottery". As the denominations suggest, the difference between the lotteries is that the changing one varies across rounds, the unchanging one staying the same until the end of the task. Before subjects are asked to make any choices, the computer randomly selects a one-stage lottery both for the changing and unchanging lottery, *but does not give this information to the subjects*. We refer to this as the "actual lottery" which is the one-stage lottery that will be played out for real at the end of the experiment if that task is selected.

A task proceeds as follows. First thing, subjects are asked to choose a "winning colour" (blue or red). This is the colour that they want to bet on, and once chosen, it cannot be changed for that specific task. A visual example of a task and colour choice is shown in Fig. 1. A series of rounds starts at this point and subjects are required to make a choice between the two lotteries in each of them (an example is given in Fig. 2). In the first round, (Fig. 2, panel a) they face the same two options as for the colour choice but now they have to select which of the two-stage lotteries they prefer, the "changing" or the "unchanging" one. Then the second round starts (Fig. 2, panel b) and one of the one-stage lotteries (crucially not the "actual lottery") is selected at random by the computer and eliminated from the "changing lottery", leaving a visual gap. Subjects are asked once again which is their preferred two-stage lottery, and then a new round starts. The process goes on until the "actual



Note: The "changing" and "unchanging" lotteries are displayed on the left- and right-hand side of the screen, respectively. Here, subjects have to choose their winning colour.

Fig. 1 Example of a decision task used in the experiment



Note: In each round of the task, the computer randomly eliminates a one-stage lottery from the "changing lottery". After every elimination, subjects are asked to choose the lottery they prefer. This process repeats for a number of rounds, i.e. until only one lottery (the "actual lottery") in the "changing lottery" is left.

Fig. 2 Example of a sequence of rounds (eliminations and choices) for the decision task in Fig. 1

lottery" is the only one left in the "changing lottery" (Fig. 2, panel e). This concludes the task and the subjects move on to the next one. Note that, in each task, the number of rounds in which the subject is required to make a choice between the two two-stage lotteries equals the number of one-stage lotteries in the changing lottery. In Fig. 2, for example, having the changing lottery five one-stage lotteries, there are five rounds of choice between the changing and unchanging lotteries.

For each choice made, we collect decision times, which are the focus of the present work. We note that subjects are forced to wait for 5 seconds before they can make a decision between the two lotteries.

The experiment was conducted in the experimental lab of the Max Planck Institute of Economics, Jena, Germany, directed by Professor Werner Güth, with subjects recruited using the ORSEE system (Greiner, 2015). There was a total of 149 subjects.

The natural incentive mechanism was used. At the end of the experiment, for each subject, one of the 49 tasks was selected. The "actual lottery" chosen by the computer for that task was recalled. Then, the "actual lottery" for that task was revealed to the subject and the subject's stated preference on that task was checked. According to the preference expressed, the corresponding "actual lottery" was played for real. If the extracted ball was of the "winning colour", then the subject was paid an extra €40 in addition to the €7.5 of the show-up fee. If it was not, then there was no extra payment. The entire procedure was repeated for each subject. The order in which the 49 tasks were presented was randomised by the computer for each subject, in each task. As we have already noted, the "actual lottery" for each of the two two-stage lotteries at stake was selected at random by the computer, individually for each subject, and the elimination sequence was randomised as well. The consequences of this procedure are that a particular task was faced by each subject at a different point in her sequence of tasks. The outcome of each of the two two-stage lotteries in that task could differ and the elimination sequence could also differ across subjects.

Table 1 Description of notation

Notation	Meaning
$O(r_k, r)$	one-stage lottery
r	number of balls in a one-stage lottery
r_k	number of winning balls in a one-stage lottery
r_k/r	winning probability in a one-stage lottery
$\mathbf{R}(r_1,\ldots, r_k,\ldots, r_R; r)$	two-stage lottery
R	number of one-stage lotteries in a two-stage lottery
$V_{PF}\,[O(r_k, r)]$	valuation for preference functional of generic one-stage lottery
$V_{PF}\,[\mathbf{R}(r_1,\ldots, r_k,\ldots, r_R; r)]$	valuation for preference functional of generic two-stage lottery

The first column contains the name of the object. The second column gives a description of the object

Here we provide some notational definitions whose summary is provided in Table 1. Let us denote a generic one-stage lottery by $O(r_k, r)$, where r is the number of balls and r_k ($0 \leq r_k \leq r$) is the number of winning balls (the winning colour is chosen at the outset of a task by the subject). Each of these r_k balls has an equal probability of being drawn so the probability of drawing a winning ball is r_k/r. Let us denote a two-stage lottery by $\mathbf{R}(r_1, \ldots, r_k, \ldots, r_R; r)$, where r_k represents the number of winning balls in the k'th one-stage lottery ($O(r_k, r)$) and R is the number of one-stage lotteries (priors) comprised in the two-stage lottery, so that $k = 1, \ldots, R$. The probability of drawing each of the one-stage lotteries is $1/R$ and the probability of drawing a winning ball from the k'th one-stage lottery is r_k/r. Finally, let $V_{PF}\left[O\left(r_k, r\right)\right]$ and $V_{PF}\,[\mathbf{R}(r_1, \ldots, r_k, \ldots, r_R; r)]$ denote the valuations for preference functional PF of the generic one- and two-stage lotteries, respectively.[5]

3 Related literature and data description

In Section 3 we present the main findings of the literature on decision times to give the reader a dimension of what has been done so far both in psychology and economics. Then, considering the peculiarity of our data, we include info on the classification of subjects to types by the mixture estimation and summary statistics of decision times per type.

3.1 Literature review

In psychology, decision times are commonly used as a tool for understanding cognitive processes. The idea of using decision times, or reaction times in general, to infer cognitive processes dates back to Donders (1868), who is known as the father of mental chronometry. Only recently has the literature on economic decision-making under

[5] Details of the 49 tasks are given in Table A.1. Up to this point, only the first three columns of which should be read—the remaining columns will be explained later.

risk begun to incorporate decision times to study the choices of agents. Observing individuals' choices is the most direct way to gain information about their preferences and allows us to build behavioural models to predict how they might act under different circumstances and with different preferences.

Using decision times in addition to decision data is an efficient way to gain information about preferences, without necessarily having to use strong and often unrealistic assumptions. Alós-Ferrer et al. (2021) show that it is possible to learn the preferences of subjects within the sample using data on choices and decision times, without the need for any assumptions while it is also possible to obtain information on pairs of options outside the dataset when using the symmetric noise assumption. The advantage of using decision times is that these are observable (as opposed to error terms) and can be collected easily in laboratory, field or natural experiments. Since the collection of decision times is costless, provides further information improving model selection, and can be done without disrupting or influencing the decision-makers, we do not see any reason why behavioural scientists should not collect such data during experiments. For a comprehensive analysis of the advantages and disadvantages of using decision times we refer the reader to the work of Spiliopoulos and Ortmann (2018). The current work contributes to the emerging literature studying decision timing in relation to economic choices under ambiguity. Although economic decisions incorporate a stochastic component, regularities in decision-making processes have been identified. Our work builds on some of this empirical evidence and specifically on what is known in psychology as the *chronometric function*, which describes the relationship between choice complexity and decision times. An individual will take longer to make a choice when the problem is more difficult, and the decision will be faster if the difficulty of the problem is low. This regularity was observed in psychophysics (Cattell, 1902; Moyer & Landauer, 1967) but it is only recently that its implications for economic choices have begun to be studied. Chabris et al. (2009) studied this relationship for intertemporal choices showing that decision times decrease when the difference in utility between the two options widens. This implies that decision times can also be used as a proxy for the strength of preference for an option. Moffatt (2005) studied decision times for choices under risk and, by analysing data obtained from an experiment in which subjects had to choose between pairs of lotteries, found that decision times lengthen in the presence of more complex pairs of lotteries.

In our work, we study this relationship of choices to decision times under ambiguity but we probe deeper, distinguishing which features increase or decrease the difficulty of the problem and identify the ones that are most relevant for the choice, that is, the attributes that influence subjects' decision time the most. Moreover, the innovativeness of our work lies in the study of decision times for types of individuals that use different decision rules. Our aim is to identify which aspects of a decision problem are considered by each type, and how those aspects influence the actual choice.

Another observed regularity is what is commonly called the *psychometric function*, which links the difficulty of the problem and the probability of answering it correctly. In this case, easier problems are more likely to be answered correctly than more difficult problems. This relationship was also initially identified in the field of psychophysics, specifically in experiments in which there is a correct answer to the problem (Cattell, 1893; Klein, 2001; Laming, 1985; Wichmann & Hill, 2001), but the results have also been extended to subjective choices Dashiell (1937) and, in economics, to choices with risk. Mosteller and Nogee (1951) showed in their poker dice experiment that the percentage of subjects who chose the offer that guaranteed the highest expected utility increased as the difference in expected utilities between the two offers increased.

We show in the current work how these features of the psychometric function are also found in choices under ambiguity and how this is reflected in decision times. In our case, the objective similarity between options is the variable that influences the probability of choosing correctly, where the correct choice is understood as the choice that provides the greatest utility. The idea is that as the similarity between the available options increases, the probability of choosing correctly decreases and vice versa. Obviously, objective similarity also influences decision times so that options that are very similar to each other require longer decision times than choosing between options that are at first sight different. This relationship was also found by Alós-Ferrer and Garagnani (2022): in their experiment, decision times are longer when subjects face choices in which the difference in utility is smaller. As for the previous hypothesis, we are going to look at the factors that influence decision times and cognitive effort of our different types of subjects.

An additional aspect that we examine in our paper concerns the relationship between decision times and cognitive effort. Although the idea that decisions entail a cost to economic agents is well established (Marschak, 1968; Selten, 1978; Simon, 1955), the literature on decision times has only recently explored the connection between the two. Wilcox (1993) uses decision time to measure the cost of making decisions under risk. In addition to using decision time as a proxy for effort, the author shows how an increase in incentives leads to an increase in subjects' willingness to incur higher decision costs. Similar results were found by Alós-Ferrer and Buckenmaier (2021) in their laboratory experiment whose design involved participants playing the beauty contest game (Nagel, 1995) and the 11-20 money request game (Arad & Rubinstein, 2012). The authors confirmed that a higher observed depth of reasoning leads to longer deliberation times and that cognitive depth reacts to monetary incentives.

3.2 Data description

As explained, we use decision times and classification into decision types from C&H. They collected observations from 149 subjects, each facing 49 tasks consisting of several rounds, resulting in a total of 256 observations per subject. Each subject's observations were divided into two roughly equal halves: one part was used for estimating the mixture models, the "estimation sample"; while the other part was employed for predicting types based on the mixture model estimation results, the "validation sample".[6]

[6] In C&H, the validation sample is referred to as the "prediction sample". Here, we have preferred a different label for the sake of clarity.

We relegate to Appendix B a detailed description of how C&H used the data to fit the different preference functionals. We summarise the classification that C&H reached in Table 2, which gives results based both on the basis of individual likelihoods and through the mixture estimation. They are reassuringly close. In what follows, we will use the classification by mixture estimation.

In this paper, we follow a conceptually similar approach to C&H. We use decision times from the estimation sample to estimate a model of decision times for each of the four types. Subsequently, we use these estimated models to predict data in the validation sample.

Here we describe the experimental data set that we use in this paper, and highlight differences and similarities between types.

The experimental design sets a 5-second waiting period before subjects could make a choice. This was meant to force subjects to think seriously about the decision problem and avoid them choosing at random. However, especially in later periods, the waiting time set could be binding. In effect, we have realised that, after approximately the 30th task, the variability in decision times dramatically reduces. Essentially, while the decision times associated with early tasks are informative of the decision process, in later tasks subjects are quicker and their decision process mostly resolves in the 5-second waiting time. Therefore, we acknowledge that a 5-second waiting time may be excessive for later rounds and limit the estimation and validation to the first 30 tasks. It should be noted, however, that the main thrust of our results still holds if we use all the tasks in the estimation and validation. Moreover, we call for evidence that could demonstrate whether the sort of exercise described in this paper could yield different results if the 5-second waiting period were reduced or even eliminated.

Table 3 displays the summary statistics of decision time per type, distinguished between estimation and validation samples. On average, decision times range between about 10.5 and 13 seconds. Table 4 shows the p-value of pairwise variance-comparison (left panels) and mean-comparison tests of decision times (right panels). They suggest that there are significant pairwise differences in the variances but not in the means of the decision times for *EU*, *SM* and *RD* subjects. This holds for both the estimation and the validation samples. However,

Table 2 Assignment of subjects to types from C&H

Type	Number of subjects classified by individual likelihoods	% of subjects classified by individual likelihoods	Number of subjects classified by mixture estimation	% of subjects classified by mixture estimation
EU	36	24.2	38	26.0
SM	83	55.7	74	49.2
RD	17	11.4	30	19.8
AM	13	8.7	7	4.9
Total	149	100.0	149	100.0

Table 3 Summary statistics of decision time per type

		Estimation sample				Validation sample			
type	subjects	observations	mean	std. dev	median	observations	mean	std. dev	median
EU	38	3,180	12.62	14.65	7.57	2,783	12.10	12.42	7.25
SM	74	5,902	12.18	11.62	7.62	5,694	12.42	13.59	7.56
RD	30	2,466	12.76	11.40	8.00	2,254	12.84	11.64	8.09
AM	7	546	10.70	11.61	6.82	543	11.02	11.12	6.89

we cannot reject the hypothesis that the variance and the mean decision time of *AM* subjects do not differ from those of the other three types, except for the *AM*-*RD* pair whose mean decision time seems to differ in the validation sample.

These results do not seem to point out striking differences in decision time between types. The following econometric analysis will attempt to uncover which aspects of the decision problem influence the decision times of each type, if any. It may well be that decision times, while similar across types on average, are the result of different factors that each type of decision-maker weighs in their decision process.

Table 4 Decision time variance-comparison (left panels) and mean-comparison test (right panels) per pair of types

Estimation sample

	EU	SM	RD	AM
EU				
SM	0.000			
RD	0.001	0.000		
AM	0.295	0.284	0.262	

	EU	SM	RD	AM
EU				
SM	0.489			
RD	0.849	0.251		
AM	0.076	0.126	0.060	

Validation sample

	EU	SM	RD	AM
EU				
SM	0.000			
RD	0.000	0.000		
AM	0.094	0.115	0.097	

	EU	SM	RD	AM
EU				
SM	0.605			
RD	0.249	0.449		
AM	0.198	0.067	0.031	

Left panels: in each cell, the *p*-value of the test on the equality of the standard deviations of decision times per pair of types, against the bilateral alternative, is reported. Under the null hypothesis, the test statistic distributes according to the *F*-distribution with degrees of freedom equal to the number of observations in each of the two samples (one per type) minus 1. Right panels: in each cell, the *p*-value of the two-sample *t*-test on the equality of the means of decision times for pair of types, against the bilateral alternative, is reported. In all the cases, except for when type *AM* is involved, unequal variance is assumed. The tests are bootstrapped by clustering at the individual level (1000 replications).

4 The econometric model of decision time

Let y_{it} be subject i's decision time in choice problem t and let x_{it} be a vector of characteristics of the two two-stage lotteries involved in the choice problem.[7] Consider the log-linear regression model with individual-specific random effects under the hypothesis that subject i is of type $\tau \in \{EU, SM, RD, AM\}$

$$\ln(y_{it}) = \gamma^{\tau} + x'_{it}\beta^{\tau} + \alpha_i^{\tau} + \epsilon_{it}^{\tau}, \tag{1}$$

for $i = 1,\ldots, 149$ and $t = 1,\ldots, 256$. Here, γ^{τ} is an intercept, β^{τ} is a vector of coefficients, α_i^{τ} is the individual-specific random effect $NID(0, \sigma_{\alpha}^{\tau 2})$, and ϵ_{it}^{τ} is an idiosyncratic error term $NID(0, \sigma_{\epsilon}^{\tau 2})$, independent of α_i^{τ} and of anything else in the model.[8,9]

This is a standard random-effects linear model, where subject i's likelihood contribution is given by

$$l_i^{\tau} = \Pr\left(y_{i1}, \ldots y_{i256} | x_{i1}, \ldots, x_{i256}, \gamma^{\tau}, \beta^{\tau}\right) = \int_{-\infty}^{\infty} \prod_t f\left(y_{it} | x_{it}, \alpha_i^{\tau}, \gamma^{\tau}, \beta^{\tau}\right) g\left(\alpha_i^{\tau}\right) d\alpha_i^{\tau},$$

where $g\,(\alpha_i^{\tau})$ is the normal density function with mean 0 and variance $\sigma_{\alpha}^{\tau 2}$ evaluated at α_i^{τ} and $f\left(y_{it} | x_{it}, \alpha_i^{\tau}, \gamma^{\tau}, \beta^{\tau}\right)$ is given by

$$f\left(y_{it} | x_{it}, \alpha_i^{\tau}, \gamma^{\tau}, \beta^{\tau}\right) = \frac{1}{y_{it}\sqrt{2\pi\sigma_{\epsilon}^{\tau 2}}} \exp\left\{ -\frac{1}{2} \frac{\left[\ln\left(y_{it}\right) - \gamma^{\tau} - x'_{it}\beta^{\tau} - \alpha_i^{\tau}\right]^2}{\sigma_{\epsilon}^{\tau 2}} \right\}.$$

The total log-likelihood that we maximise to estimate the parameters of interest is

$$\text{Log} - \text{likelihood}^{\tau} = \sum_{i=1}^{149} \mathrm{d}_i^{\tau} \ln\left(l_i^{\tau}\right), \qquad \tau \in \{EU, SM, RD, AM\}. \tag{2}$$

The dummy d_i^{τ} taking the value 1 if subject i's posterior probability of type τ is the highest, 0 otherwise, enables us to estimate each type's decision time only from

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[7] It is worth recalling here that the vector of characteristics of the two lotteries is individual-specific, because not only the order of the tasks, but also the "actual lotteries" and the elimination sequence of each task were randomised across subjects, and the "winning colour" chosen for that task by the subject. We also use some regressors that depend on the subject's characteristics whose evaluation is based on the estimation results in C&H. We will expand on this in the next session.

[8] In previous versions of this paper, as participants were forced to wait for at least 5 seconds before reporting their preferred lottery, we assumed that those who took less than 5.5 seconds (inclusive of the 5 waiting seconds) to make a decision would have possibly been able to do it within the 5 waiting seconds. Essentially, we conceded 0.5 seconds of reaction time to those subjects who have already made up their mind during the waiting time. However, since the results change only marginally with respect to a model which assumes no censoring of decision time, we discuss the latter here.

[9] We note that the experimental software rounded to the nearest integer the decision time at which each decision problem is shown to participants. This results in an approximation in the range of (−0.5, 0.5) seconds. Since subjects could not control the time in which the decision was made and they were not even aware that their decision times were recorded, we can consider this rounding completely random, which is a minor and unsystematic measurement error that cannot bias the results in any way.

those subjects who are assigned to that type according to the majority rule explained above.

In the next sections, we will present the estimation results.

5 Estimation

5.1 Description of variables

As previously noted, we obtain our estimates by separately maximising the total log-likelihood in Eq. (2) for each type of the decision-maker $\tau \in \{EU, SM, RD, AM\}$ as classified in C&H. The variables we use are described in what follows and listed in Table 5. The unchanging and changing lotteries are indicated by **M** and **N**, respectively.

Definition 1 "Experience" with the decision problem both across tasks and within each task can instigate "learning" phenomena. Boredom and fatigue can manifest themselves throughout the experiment as well, impinging on and even overcoming the effect of learning.

Since the task position and the elimination process are randomly determined subject by subject, we include both the variable T, $T = 1, \ldots, 49$, which indicates the position in the sequence of tasks in which a particular task is presented to the subject, and the variable *round*, which represents the position in the elimination sequence of the task in which a particular choice problem is encountered (we note that the number of rounds varies from task to task—Table A.1 gives the detail). The use of these variables is aimed at modelling two effects: learning and fatigue.

We should make clear that, in each task, subjects first choose the winning colour and then start with the first round of choices between the two two-stage lotteries. To control for the peculiarity of the decision in the first round of a task, where the decision time is inflated by the time spent to choose the winning colour and to familiarise ourselves with the lotteries, we use an indicator for the first round of each decision task, $\mathbf{1}(round = 1)$.[10] Using this dummy, we purge the variable *round* of the extra time taken to carry out the first-round decision.

Following the literature on experience and decision time, we expect that all types of subjects will become faster in making choices as the experiment proceeds. This means that, overall, experience should reduce decision time regardless of the process implemented by subjects. Moreover, we expect to find a significant reduction of response time both between (quicker decisions for tasks presented later) and within tasks (faster choices in the final part of the elimination sequence of one-stage lotteries).

Along similar lines to Moffatt (2005), we add variables to control for: (i) complexity; (ii) objective similarity; and (iii) closeness to indifference. In the following, we define and discuss in detail the variables in each category.

[10] Here, $\mathbf{1}(.)$ has the standard meaning of an indicator function that takes the value 1 when the statement in the brackets is true; it is 0 otherwise.

Table 5 Detailed definition of the variables used in the estimation of the decision time equations

Variable	Formula	Category	Description
T		E	The position in the sequence of tasks in which a particular task is presented to the participant
round		E	The position in the elimination sequence of a particular task is presented to the participant The position in the elimination sequence of the task in which the choice problem is encountered
$\mathbf{1}(round=1)$		E	The indicator for the first round in each decision task
#balls	$m \times M + n \times N$	C	The total number of balls in the two lotteries
dimension-1sl	$m+n$	C	The number of balls in each of the one-stage lotteries in the unchanging two-stage lottery plus the number of balls in each of the one-stage lotteries in the changing two-stage lottery
#priors	$M+N$	C	The number of one-stage lotteries in the unchanging two-stage lottery plus the number of one-stage lotteries in the changing two-stage lottery
1(**M** is symmetric)		C	1 if unchanging lottery is symmetric (if interchanging the winning and losing balls does not change the lottery)
1(**N** is symmetric)		C	1 if changing lottery is symmetric (if interchanging the winning and losing balls does not change the lottery)
1(both **M** and **N** are symmetric)		S	1 if both unchanging and changing lotteries are symmetric
1(both **M** and **N** are symmetric) × $\mathbf{1}(round=1)$		S	1 if both unchanging and changing lotteries are symmetric and it is round 1
1(**M** and **N** are identical)		S	1 if the unchanging and changing lottery are identical
$\lvert\Delta(\#priors)\rvert$	$\lvert M-N\rvert$	S	The absolute difference between the number of one-stage lotteries in the unchanging twostage lottery and the number of one-stage lotteries in the changing two-stage lottery
$\lvert\Delta(\#win\text{-}balls)\rvert$	$\left\lvert\sum_{k=1}^{M} m_k - \sum_{j=1}^{N} n_j\right\rvert$	S	The absolute difference between the number of winning balls in the unchanging two-stage lottery and the number of winning balls in the changing two-stage lottery
$\lvert\hat{\Delta}\rvert$	$\lvert V_{i\tau}(A) - V_{i\tau}(B)\rvert$	I	The difference in utility evaluation between the two lotteries that depends on the type at the individual-specific parameter for subject i for that type

The first column contains the name of the variable. The second column shows the formula, if any, that computes it. The third column indicates what the variable is meant to control for: experience (*E*), complexity (*C*), objective similarity (*S*), closeness to indifference (*I*). The fourth column gives a description of the variable

Definition 2 The "complexity" level of a choice problem is related to the load of information the subject faces in a particular choice problem.

The complexity of a decision problem is modelled by five variables: *#balls*, *dimension-1sl*, *#priors* and two indicators taking the value 1 if either the unchanging or the changing lottery is symmetric. The first represents the dimension of the decision problem. For example, if one of the two-stage lotteries consists of M one-stage lotteries, each containing m balls and the other consists of N one-stage lotteries, each containing n balls, then the variable *#balls* is equal to $m \times M + n \times N$. The second variable captures the dimension of the one-stage lotteries ($m + n$). The third measures the number of one-stage lotteries in a decision problem, that is the number of priors involved in the decision ($M + N$). Essentially, these indicators account for the dimensionality of the decision problem from three different points of view, each of these might play a particular role in the evaluation of the preferred two-stage lottery by the different types of the decision-maker. It is worth noting that, while *dimension-1sl* is constant within a specific task, *#priors* changes with the round number of the task sequence. In particular, N decreases by one at each elimination of a one-stage lottery. The two dummy variables **1**(**M** is symmetric) and **1**(**N** is symmetric) also gauge complexity, in that a symmetric lottery is visually easy to detect and does not require to count the number of balls it contains. A two-stage lottery is symmetric if it does not change when exchanging the winning colour with the losing colour.

Intuitively, we might expect that higher complexity embedded in a decision would increase the time required to make a choice, as prescribed by the chronometric function. This might be true on average, but we expect to find different reactions to complexity from different types of subjects. Furthermore, subjects employing different decision processes might focus on different aspects of complexity of a choice. Subjects that employ simpler decision processes, like *AM*, might be less concerned by complexity variables. Finally, symmetry of one of the two lotteries should reduce the time it takes for a subject to make a decision, overall.

Definition 3 The "objective similarity" refers to the similarity in the physical appearance of the two lotteries in a particular decision problem.

The five variables included in category S in Table 5 are intended to measure objective similarity between two lotteries.

The dummy variable **1**(both **M** and **N** are symmetric), with and without controlling for the peculiarity of the first round choice, captures similarity because, besides the number of balls being different, subjects can perceive two lotteries as being similar because they share the characteristic of being symmetric. The variable $|\Delta(\#priors)|$ indicates the absolute difference in the number of priors involved in the decision choice ($|M - N|$). The variable $|\Delta(\#win\text{-}balls)|$ gives the absolute difference between the number of winning balls of the two two-stage lotteries. The variables described so far decrease when the two-stage lotteries become similar in their physical appearance. However, when all these variables equal 0 simultaneously, it does not necessarily mean that the two-stage lotteries are the same.[11] For this reason, we add the dummy variable **1**(**M** and **N** are identical), which takes the value 1 if the two lotteries are exactly the same and 0 otherwise.[12]

A possible way in which symmetry can influence decision time is given by the idea that the more similar the two lotteries in physical appearance, the longer the time required for a subject to make a decision. If this is the case, we should observe a strong, positive significant effect, especially from variables like $|\Delta(\#win\text{-}balls)|$ and **1**(both **M** and **N** are symmetric) since it should be easier to detect symmetry through them. Nevertheless, as for the case of complexity, we expect symmetry variables to have a different impact depending on the decision process of subjects.

Definition 4 "Closeness to indifference" refers to the difference in terms of valuation between the two lotteries. It measures the extent to which the two lotteries are close to being indifferent (the difference in valuation can be interpreted as a measure of subjective similarity).

To capture the difference in terms of utility between the two lotteries, we follow the approach described in Moffatt (2015, page 375, Equation 12) using the estimation results in C&H. Roughly speaking, this approach uses the estimated parameters of the preference functional to obtain individual-wise valuations of the lotteries and compute an estimate of their absolute difference, $\left|\widehat{\Delta}\right|$. The intuition behind the use of such a measure is that being close to indifference may either have a positive or negative impact on the decision time. A subject that is close to being indifferent between the two lotteries, on the one hand, may need more time to accurately assess which is the preferred one and, on the other hand, may choose faster since the two lotteries provide about the same value.

In contrast to Moffatt (2005), C&H estimate a 4-type mixture model, where three out of four models are characterised by a single parameter, except for the *EU* type whose functional has no parameters at all. Hence, for each subject *i* conditional on being of a particular type, we calculate the posterior expectation of the parameter of interest which characterises the functional for that type. Finally, we use the parameters so obtained to calculate the absolute valuation differential for each subject's decision problem, conditional on being of a certain type. We will refer to such absolute valuation differential as $\left|\widehat{\Delta}_{EU}\right|$, $\left|\widehat{\Delta}_{SM}\right|$, $\left|\widehat{\Delta}_{RD}\right|$ and $\left|\widehat{\Delta}_{AM}\right|$ in the Expected Utility Model, Smooth Ambiguity model, Rank Dependent Model and Alpha Expected Utility Model case, respectively.[13]

[11] Consider, for example, the two lotteries **R**(1, 2; 3) and **R**(0, 3; 3). These share the number of one-stage lotteries, the number of balls in the one-stage lotteries and the number of winning balls, but they are different, nevertheless.

[12] As a measure of objective similarity, Moffatt (2005) calculates the Euclidean distance between the probability vectors of the two lotteries. Here, we are unable to do the same because the structure of our two-stage lottery is such that it cannot be represented by a single probability vector.

[13] The hats over the Δ's indicate that the valuations are obtained by using the Maximum Simulated Likelihood estimates of the parameter from Table 3 in C&H. The procedure is explained in Moffatt (2005).

5.2 Regression results

The regression results are reported in Table 6. Results are organised in four columns, one for each type. Note that not all variables enter into all four columns; the order of powers for T, *round* and $\left|\widehat{\Delta}_\tau\right|$ were decided by using likelihood-ratio tests at a 5% significance level for each type separately. Powers are added to capture non-linearities.

Two important observations are in order. First, the significance and magnitude of the individual random effects, σ_α, testify that there is a large heterogeneity in the population whatever the type. Second, due to the log-linear specification of our model, the effects of the regressors on the decision time have to be interpreted in percentage terms.[14] For example, a unit increase in the total number of balls produces an expected increase in the decision time in seconds of 1.2% for *RD*, of 0.2% for *AM* and no significant effect for *EU* and *SM*.

In what follows, we examine the impact of the experience, complexity, similarity and closeness to indifference factors on decision time and itemise our findings.

Result 1 Experience significantly reduces decision time both across and within tasks for all the types. This result is in line with our expectations and with the sequential effect described by Drift Diffusion Models.

The position of a particular task, T, is clearly highly significant, not only in and of itself but also in its powers. Figure 3 displays the expected decision time against task position per type based on the estimation results in Table 6. The formula used is $E\left(\widehat{y}_t^{\tau*}\right) = \exp\left(\widehat{\gamma}^\tau + x_t'\widehat{\beta}^\tau + \widehat{\sigma}_\alpha^{\tau 2}/2\right)$, with all the regressors other than T and its relevant powers set to 0. Hence, what is seen in Fig. 3 is the effect of *only T* on decision times. The slope and the level of the curves change with the characteristics of the lotteries under examination since they have different effects and importance for different types.

Figure 3 shows that decision time curves are all downward-sloping and generally convex. There is a similar pattern for all the types: decision time sharply declines in the first 10 tasks; furthermore, it shows that *RD* types are expected to spend the highest amount of time whatever the order of the task in the sequence. Instead, *EU*, *SM* and *AM* types seem to behave similarly in terms of their expected decision times and spend less time than *RD* types. Experience has a highly significant impact on decision time for all types in that it reduces the time spent to make a decision but becomes marginally weaker with accumulated experience. We can appreciate the marginal effect of experience, task by task, throughout the experiment for all the types in Fig. 4. Such a marginal effect is strong for the *EU*, *SM* and *AM* subjects in the first 10 tasks, then the curve flattens and the effect is not anymore significantly

[14] This implies that each 1-unit increase in the explanatory variable, say x, multiplies the expected value of the decision time, y, by $\exp(\hat{c})$, where $\hat{c}$ is the estimated coefficient on x. Given that for small values of $\hat{c}$ (which is almost always our case), $\exp(\hat{c}) \approx 1 + \hat{c}$, we can loosely interpret $100 \times \hat{c}$ as the expected percentage change in y of a 1-unit increase in x.

Table 6 Estimation results of the random effects Linear regression model per type

variable	category	*EU*	*SM*	*RD*	*AM*
$\gamma\tau$	E	2.9052***	2.9322***	3.6738***	2.8289***
		(0.0502)	(0.0603)	(0.0971)	(0.1377)
T	E	-0.1875***	-0.1321***	-0.0434***	-0.1457***
		(0.0147)	(0.0107)	(0.0033)	(0.0331)
$T^2/10$	E	0.1565***	0.0980***	0.0085***	0.1285**
		(0.0190)	(0.0138)	(0.0010)	(0.0440)
$T^3/100$	E	-0.0579***	-0.0335***		-0.0511*
		(0.0092)	(0.0066)		(0.0217)
$T^4/1000$	E	0.0077***	0.0041***		0.0074*
		(0.0015)	(0.0011)		(0.0035)
$\mathbf{1}(round=1)$	E	1.1859***	1.0805***	1.1645***	0.9275***
		(0.0171)	(0.0385)	(0.0767)	(0.0617)
round	E		-0.0832***	-0.1875***	-0.1469**
			(0.0176)	(0.0318)	(0.0540)
$round^2$	E		0.0075***	0.0160***	0.0158*
			(0.0021)	(0.0037)	(0.0066)
#balls	C			0.0120***	0.0026*
				(0.0020)	(0.0012)
dimension-1sl	C		0.0116**		
			(0.0033)		
#priors	C			-0.0565***	
				(0.0129)	
1(**M** is symmetric)	C	-0.0570**	-0.0392*	-0.1918***	
		(0.0216)	(0.0165)	(0.0291)	
1(**N** is symmetric)	C			-0.3234***	
				(0.0333)	
1(both **M** and **N** are symmetric)	S		0.0391**		
			(0.0141)		
1(both **M** and **N** are symmetric) $\times$ $\mathbf{1}(round=1)$	S		-0.1185**	-0.5022***	
			(0.0379)	(0.0674)	
1(**M** and **N** are identical)	S		-0.1156*		
			(0.0448)		
$\lvert\Delta(\#priors)\rvert$	S		-0.0073*		
			(0.0028)		
$\lvert\Delta(\#win\text{-}balls)\rvert$	S			-0.0058***	
				(0.0017)	
$\left\lvert\widehat{\Delta}_\tau\right\rvert$	I	-0.3820**	-1.2525***	-6.5295***	
		(0.1278)	(0.1586)	(0.4283)	
$\left\lvert\widehat{\Delta}_\tau\right\rvert^2$	I	0.6118*	2.5019***	19.4074***	
		(0.2459)	(0.6005)	(1.7403)	

Table 6 (continued)

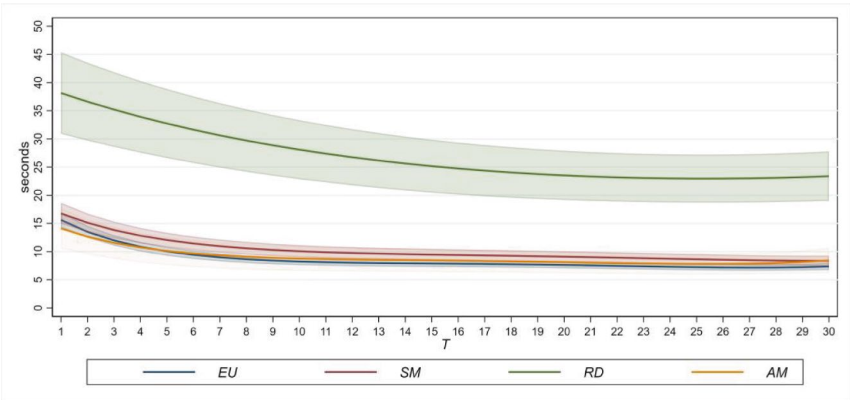
variable	category	*EU*	*SM*	*RD*	*AM*
$\left\|\widehat{\Delta}_{\tau}\right\|^{3}$	I		-1.6703**	-17.5590***	
			(0.6251)	(2.0239)	
σ_{α}		0.1755***	0.1510***	0.1402***	0.1153***
		(0.0209)	(0.0130)	(0.0194)	(0.0331)
σ_{ϵ}		0.3133***	0.3019***	0.3358***	0.2784***
		(0.0039)	(0.0028)	(0.0048)	(0.0084)
R^2		0.6764	0.6873	0.6497	0.7106
observations		3,180	5,902	2,466	546
subjects		38	74	30	7

Standard error in parentheses. ***, ** and * denote p-values < 0.001, < 0.01 and < 0.05, respectively

different from 0, showing no further reduction of decision times due to experience. The *RD* subject behaves differently, demonstrating a monotonic and steady reduction with each task.

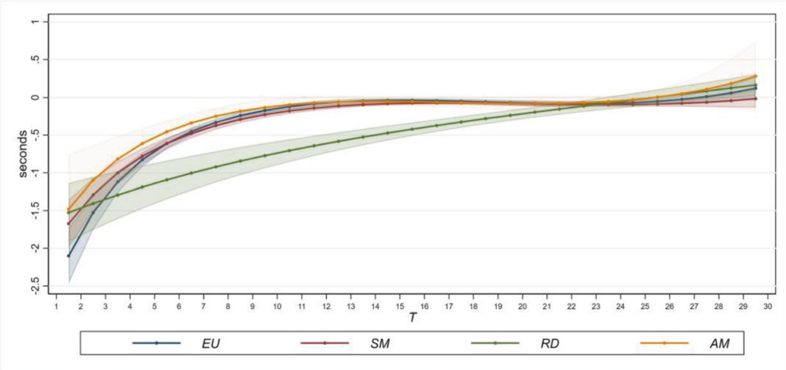
Figure 3 constitutes our baseline for the interpretation of the estimation results. Figuratively, in what follows, we will turn on and off the various regressors to analyse their effect on such curves.

Two other regressors used to examine the impact of experience on decision time are the dummy $\mathbf{1}(round = 1)$ and the variable *round*. Table 6 reveals that the former has a strong impact on decision times for every type of subject while the latter is relevant for all the types except for *EU*.



Note: Shaded areas represent 95% confidence intervals.

Fig. 3 Expected decision time (in seconds) against task position per type



Note: Dots represent marginal effects between T and $(T-1)$. Shaded areas represent 95% confidence intervals.

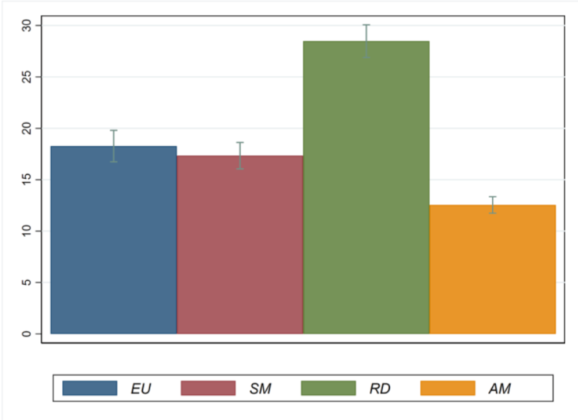
Fig. 4 Marginal effects of expected decision time (in seconds) against task position per type

Figure 5 shows the effect of the first round of a task.[15] For the *EU* and *SM* subjects the additional time due to the first round effect is very similar (approximately 18 additional seconds). The *RD* subject who seems to take the longest time throughout the experiment takes also the longest additional time in the very first round of each task (28 additional seconds). Not surprisingly, this effect is less pronounced for the *AM* subject with an additional time of about 12 seconds spent on the first round.

Now we consider how the decision time is affected across the rounds of a task. In order to purge the decision time from the time spent to decide on the winning colour in the very first round, we turn the dummy $\mathbf{1}(round = 1)$ off and the variable *round* on. That is, now all the variables except for γ, T and *round* are set to 0. Figure 6 shows how much, on average, the curves in Fig. 3 shift when a certain round of a task is played.

As we have seen in Fig. 5, an *RD* subject spends more time with respect to other types in the very first round, but Fig. 6 shows that this subject decreases the time spent by more than 5 seconds, on average, in the first 5 rounds. The decision time decreases also for the *AM* and *SM* subjects up to rounds 4, the effects for the following rounds are not statistically significant. However, the average reduction is much less than that of an *RD* subject: 1.8 seconds for an *AM* and 1 second for an *SM* subject. The marginal effect of *round* for the *EU* subject is absent from the figure, because her decision time does not seem to change within a task.

[15] Remember that the first round of each task is very special because it involves not only choosing between the two lotteries, but also deciding which colour the subjects want to bet on. An important remark that has to be done is that we cannot record the time spent choosing the colour separately from the time spent on the choice of the lottery. Subjects are probably developing a preference for one lottery over the other since the first part of the first round, when they are required to choose between blue and red balls. Separating the two response times would be pointless because we would have a bias in the time taken to choose between the lotteries, as the subjects would already be familiar with them. We decided to include the dummy $\mathbf{1}(round = 1)$ exactly to capture this feature of our experiment.



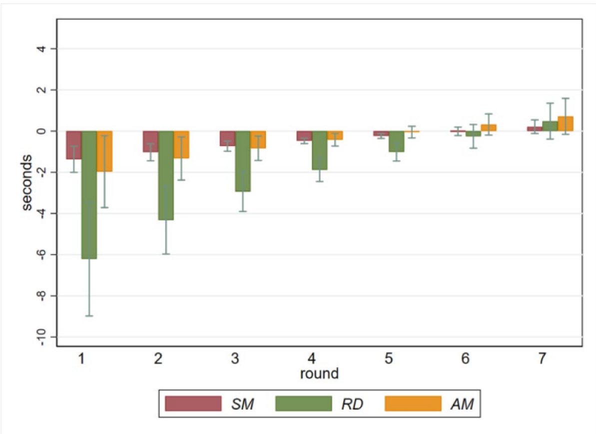
Note: Capped spikes represent 95% confidence intervals.

Fig. 5 Marginal effect of **1**(*round* = 1) on expected decision time (in seconds)

Our next result concerns how complexity, defined as the load of information of a lottery, and represented by the variables *#balls*, *dimension-1sl*, *#priors*, **1**(**M** is symmetric) and **1**(**N** is symmetric) affects the decision time of each type.

Result 2 A task appears more complex, so requiring a higher cognitive effort and increasing decision time, the higher the number of balls it contains. The same holds for the dimensionality of the first-stage lotteries. Rather surprisingly, the number of priors reduces the cognitive load for *RD* types, and consequently their decision times. The symmetry of the two-stage lotteries expedites decisions.

According to the pattern described by the *chronometric function*, there exists a positive correlation between the complexity of a choice problem and decision time,



Note: Capped spikes represent 95% confidence intervals.

Fig. 6 Marginal effect of *round* on expected decision time (in seconds) meaning that an individual will take longer to make a decision when the difficulty of the problem is greater. When analysing the estimation results of the variables which control for complexity on decision time, we apparently get mixed results. In fact, we find that decision time increases with the total number of balls in a decision task (*#balls*), making the decision of the *RD* and *AM* subjects slower; the dimension of the first-stage lotteries, *dimension-1sl*, increases decision time for the *SM* subjects; while it decrease decreases when *#priors* increases for an *RD* subject. We estimate a significant and negative effect for subjects of type *EU*, *SM* and *RD*, when the unchanging lottery is symmetric. Instead, the symmetry of the changing lottery is relevant only for the *RD* subject.

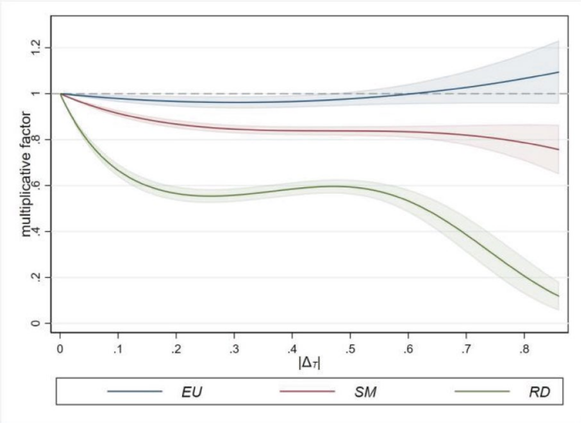
Some of these results may appear counterintuitive. A possible explanation could be the existence of a "complexity threshold" meaning that, when a choice becomes increasingly difficult, subjects cannot handle nor elaborate all the information provided and prefer to switch to simpler decision-making processes (heuristics) that are not vexing in terms of cognitive effort and make them save time. Another possible explanation, which, in some sense, relates to the previous one, is that the variables that control for complexity can be strictly positively or negatively correlated. Therefore, the opposite effect of two variables may simply be capturing non-linearities in the effect of complexity on decision time.

Our next result concerns how objective similarity, defined as the similarity in the physical appearance of choice alternatives and represented by the variables **1**(both **M** and **N** are symmetric), **1**(both **M** and **N** are symmetric)×**1**(*round* = 1), |Δ(*dimension-1sl*)|, |Δ(*#priors*)|, |Δ(*#win-balls*)| and **1**(**M** and **N** are identical), affect the decision time of each type.

Result 3 Objective similarity is extremely relevant for *SM* subjects while *RD* subjects are affected only partially. It has no significant effects on decision time for *EU* and *AM* types.

Among the variables that capture the *objective similarity* of the lotteries in a decision task, **1**(both **M** and **N** are symmetric), which turns on when both lotteries in a decision problem are symmetric, seems to be relevant only for an *SM* decision-maker, increasing her decision time. **1**(both **M** and **N** are symmetric) × **1**(*round* = 1) considers the effect of the symmetry of the two-stage lotteries in the first round of the task only. It has a negative, significant effect for the *SM* and *RD* subjects.[16] Two identical lotteries decrease the decision time of an *SM* decision-maker. The absolute difference in the number of priors, |Δ(*#priors*)|, has an effect for the *SM* subject only, which is estimated to be negative. The same goes for the absolute difference in the number of winning balls, which has a negative effect for *RD* subjects only.

[16] Changing lotteries are always symmetric in the first round (because they contain all the possible priors) and may be symmetric or not in the following rounds, depending on the random elimination sequence explained in Section 2. Hence, the effect of the symmetry of the changing lottery can be partially captured by the first-round dummy. For this reason we have added the interaction effect **1**(both **M** and **N** are symmetric)×**1**(*round* = 1).



Note: Shaded areas represent 95% confidence intervals.

Fig. 7 Fractional change in expected decision time with respect to utility gap

Summarising, objective similarity seems to be very relevant in speeding up decisions for *SM* decision-makers. An *RD* type halves her decision time on average when the two two-stage lotteries are symmetric in the first round. Moreover, as the difference in the number of winning balls grows larger, the decision time of an *RD* subject decrease. Finally, similarity in physical appearance of the two lotteries is irrelevant for *EU* and *AM* decision-makers.

Now, we turn our attention to the last factor: closeness to indifference. This variable is defined by the absolute difference between choice alternatives' utility evaluations according to each theory, hence it is type specific. We discuss the importance of closeness to indifference separately for each type because we cannot compare it across types, as they miss a common metric.[17]

Result 4 The utility gap between the two two-stage lotteries in a decision problem and its powers, representing closeness to indifference, have a statistically significant effect on the decision times of *EU*, *SM* and *RD* subjects. As the utility gap grows, these subjects tend to reduce their decision times, although the effect is very strong for *RD* and mild for *EU* subjects. Closeness to indifference seems to be irrelevant for *AM* subjects.

Figure 7 shows how the expected decision time changes (in proportion) with the utility gap. On the vertical axis, the level of the multiplicative factor of decision time is reported: the distance between 1 (dashed horizontal line) and the curves indicate the fraction of time saved by subjects when the difference in utilities between the two lotteries widens. Looking at Fig. 7, we can see that for a utility gap, $\left|\widehat{\Delta}_{RD}\right|$, equal to 0.05 the *RD* subject is expected to save 20% of the time she would spend when making a decision between two lotteries whose utility gap is 0, i.e., when she is indifferent between the two. A utility gap of approximately 0.65, instead, halves the

[17] The $|\hat{\Delta}|$'s vary from 0 to 0.67, 0.86, 0.81 and 0.87 for *EU*, *SM*, *RD* and *AM*, respectively.

expected decision time (i.e., the multiplicative factor is 0.5). The multiplicative effect is marginal for an *EU* subject, while somewhat in the middle between the two for an *SM* subject. Moving from the indifference point until $\left|\hat{\Delta}\right| = 0.25$, *SM* and *EU* subjects monotonically reduce their expected decision time by about 17% and 4%, respectively. The effect of closeness to indifference is, in general, non-linear. Overall we can nearly confirm the results obtained by Moffatt (2005) as far as the *RD* and to a lesser extent the *SM* subjects are concerned: when people are evaluating lotteries that have almost the same value they take more time. However, this is only marginally true for *EU* and not at all true for *AM* subjects.

We note that, when subjects classified as being non-*EU* types, as *RD* or *SM*, for example, are incorrectly treated *as if* they were *EU*, their pronounced reaction to the utility gap that we uncover for these types may average somewhat with the meagre reaction to utility gap of the true *EU* subjects. Therefore, the more marked reaction to utility gap of *EU* decision-makers reported in the literature may appear somewhat different from ours simply because it includes the presence of other types of decision-makers that C&H have successfully identified and separated.

In summary, *experience* significantly reduces decision time both across and within tasks for all the types. Rather surprisingly, the *number of priors* reduces the cognitive load for *RD* types, and consequently their decision times; this seems contrary to intuition. *Objective similarity* is extremely relevant for *SM* subjects (while *RD* subjects are affected only partially and it has no significant effects on *EU* and *AM* types. The *utility gap* between the two two-stage lotteries in a decision problem and its powers, representing *closeness to indifference*, have a statistically significant effect on the decision times of *EU*, *SM* and *RD* subjects. As the utility gap grows, these subjects tend to reduce their decision times, although the effect is very strong for *RD* and mild for *EU* subjects. Closeness to indifference seems to be irrelevant for *AM* subjects. It is clear from this that different factors influence different types in different ways.

6 Validation of decision times across types

Here we describe a validation exercise that enables us to test whether the decision times of a certain type of decision-maker under ambiguity are best predicted by the model of decision time estimated for that type with respect to that estimated for the other types.

We proceed as follows. We use the models estimated from the estimation sample in Section 5 (and displayed in Table 6) to predict data in the validation sample for each of the four types. We then regress the validation data of a certain type against the predictions from each of the estimated models, one at a time, and an intercept. For example, to test the performance of the various types in predicting *EU* 's response times in the validation sample, we regress data from the *EU* validation sample against "decision time predicted by τ " and an "intercept", with τ being one of the four types at a time.

For a type $\tau \in \{EU, SM, RD, AM\}$ to be a good predictor, it must hold that the coefficient on the predicted decision time equals 1 and the intercept equals 0 jointly. This joint hypothesis is assessed for each model via and *F*-test, which under the null hypothesis is distributed according to an F distribution with degrees of freedom equal to the number of estimated parameters (which is always 2) and the number of subjects in the validation sample minus 1, $F(2, \text{subjects} - 1)$. When this joint null cannot be rejected for more than one model, we look at the R^2, preferring the model with the higher measure. Another alternative way in which we assess the goodness-of-prediction of each model is the root-mean-square error (RMSE) between the validation sample and the prediction. According to this criterion, the "best" predictor is the model with the smallest RMSE.

Table 7 displays the results of this exercise. For the validation sample of each of the types, the aforementioned joint null hypothesis cannot be rejected always when the predicting model is that of the corresponding type, which is also the case with the highest R^2 when the joint null cannot be rejected for more than one predicting model (namely, predictors *EU* and *SM* for the *EU* validation data and *RD* and *AM* for the *RD* validation data). The RMSE criterion always indicates the best predictor of decision time for a particular type is the model specific to that type.

Table 7 Validation of decision times

EU **validation sample**		τ		
	EU	SM	RD	AM
decision time predicted by τ	0.9833	0.9912	0.9224***	1.0900***
	(0.0179)	(0.0163)	(0.0151)	(0.0182)
intercept	0.0387	0.0103	0.1467***	-0.2024***
	(0.0384)	(0.0354)	(0.0315)	(0.0408)
observations		4,712		
subjects		38		
$F(2, 37)$	0.56	0.81	13.91	12.34
p-value	**0.5774**	**0.4514**	0.0000	0.0001
R^2	**0.7509**	0.7341	0.6293	0.7451
RMSE	**0.3252**	0.3342	0.3982	0.3288

SM **validation sample**		τ		
	EU	SM	RD	AM
decision time predicted by τ	0.9562**	0.9951	0.9598**	1.0749***
	(0.0143)	(0.0153)	(0.0142)	(0.0162)
intercept	0.1069**	0.0128	0.0840**	-0.1574***
	(0.0301)	(0.0326)	(0.0295)	(0.0348)
observations		5,694		
subjects		74		
$F(2, 73)$	9.88	0.18	4.07	10.73
p-value	0.0002	**0.8391**	0.0212	0.0001
R^2	0.7306	**0.7496**	0.7078	0.7358
RMSE	0.3354	**0.3241**	0.3527	0.3340

RD **validation sample**		τ		
	EU	SM	RD	AM
decision time predicted by τ	0.9058***	0.9538*	0.9961	1.0236
	(0.0168)	(0.0177)	(0.0186)	(0.0194)
intercept	0.2188***	0.1017*	-0.0022	-0.0522
	(0.0363)	(0.0385)	(0.0403)	(0.0420)
observations		2,254		
subjects		30		
$F(2, 29)$	18.69	3.50	0.79	0.77
p-value	0.0000	0.0435	**0.4618**	**0.4703**
R^2	0.6306	0.6796	**0.7241**	0.6459
RMSE	0.3769	0.3500	**0.3288**	0.3647

AM **validation sample**		τ		
	EU	SM	RD	AM
decision time predicted by τ	0.8908**	0.9085**	0.8739**	1.0007
	(0.0257)	(0.0247)	(0.0255)	(0.0268)
intercept	0.2465**	0.1960**	0.2541**	0.0096
	(0.0408)	(0.0422)	(0.0393)	(0.0432)
observations		543		
subjects		7		
$F(2, 29)$	45.10	13.87	65.98	0.46
p-value	0.0002	0.0056	0.0001	**0.6496**
R^2	0.7420	0.7434	0.6733	**0.7468**
RMSE	0.3194	0.3214	0.3603	**0.3147**

Each panel corresponds to the validation of data for the specified type. Data from the validation sample of the specified type is regressed against predictions made using the estimated models of decision time in Table 6, along with a constant. The models are estimated via OLS with clustered standard errors at the individual level. ***, ** and * denote *p*-values <0.001, <0.01 and <0.05, respectively. The stars refer to the test of the null hypothesis that the coefficient on *decision time predicted by* τ is equal to 1 and the *constant* equal to 0, against a bilateral alternative. The F tests (and the corresponding *p*-values reported below) pertain to the null hypothesis that the two aforementioned hypotheses hold jointly. The *p*-values in bold indicate that the joint null hypothesis cannot be rejected (which is what we want for a model to be a good predictor). When such a joint null hypothesis cannot be rejected for two or more predicting models, the one with the highest R^2 value is remarked in bold. Root-mean-squared errors (RMSE) are calculated between the validation data and their predictors. The smallest RMSE is highlighted in bold.

7 Conclusions

This work extends the findings of C&H on decision under ambiguity by examining the factors that influence decision time. They classified individuals into four types of decision-makers (Expected Utility Model, Smooth Ambiguity Model, Rank Dependent Expected Utility Model and Alpha Expected Utility Model) based on their decisions, analysed via a finite mixture model. However, they did not investigate the decision times corresponding to the choices. In this paper, we analyse the decision times of those experimental subjects with the purpose of exploring how decision-makers allocate their time when choosing between ambiguous lotteries, discriminating between decision types. We consider various factors that can slow down or expedite decision time suggested by psychological theories of the decision-making process, and which have been largely overlooked by economists.

To our knowledge, this study is the first to investigate decision times based on a classification of subjects into different types, specifically in the context of ambiguity rather than risk.

Building on the assignment to types established by C&H, we employ the same methodology to explain decision times. The data is divided into two parts: an "estimation sample" and a "validation sample". The estimation sample is used to estimate the relationship between decision time and potential explanatory variables for each decision-maker type. Then, the validation sample is used to assess the accuracy of the predictions of decision time based on the model of decision time estimated for each type. We consistently find that the model that best predicts decision times for a particular type is that specific to that type. Consequently, we provide potential explanations for decision times for each decision-maker type.

At first sight, decision times seem not to differ between types greatly, as shown by the summary statistics in Section 3.2. However, when considering the various factors that could influence decision time, following the econometric analysis by Moffatt (2015), we discover significant differences between decision types. These differences are evident in how decision types allocate time to factors such as experience, complexity, similarity and closeness to indifference.

Rather obviously, decision time decreases, at a decreasing rate, throughout the experiment for all types of subjects. What is interesting is that not only do different types have different preference functionals, but they also seem to be processing the problems differently. Indeed there seems to be a connection between the type of the subjects and the way they allocate their decision time.

In Appendix C, we replicate the analysis in this paper using a popular data set of choice under risk (Hey, 2001) while assuming two types of decision-makers. Our findings closely mirror the results reported in this paper. Specifically, we observe that types differ in how they allocate their time to different characteristics of the decision problem, and the most effective predictors of decision time for a particular type are those specifically tailored to that type. These additional results not only reinforce those presented in this work but also enhance their significance by demonstrating that their relevance extends beyond the realm of choice under ambiguity.

Our analysis hints that theorists should investigate the *process* of pairwise choice, rather than seeing the decision as being arrived at by simply calculating the value of each lottery and comparing them discriminating by decision-making types. Crucially, our results strengthen the effectiveness of the mixture model in C&H in assigning subjects to types. Had the assignment been more casual, we would have not observed differences in decision times and, more importantly, in decision processes as marked as those uncovered by our analysis. All in all, decision time emerges as a great predictor of decision types and should be analysed along with choice data in order to develop theories of decision-making under ambiguity that encompass both decision and cognitive processes.

In conclusion, the main contribution of this paper lies in its expansion of the literature on decision times, particularly in terms of understanding the decision-making processes of different types of decision-makers under ambiguity. By incorporating psychological theories regarding the cognitive processes underlying different decision rules, we delve deeper into the subject matter.

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Data availability Data is available from the authors on request.

Declarations

Conflict of interest The authors declare that they have nothing to disclose.

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