Fourth conference of the
International Network for
Didactic Research in University Mathematics

INDRUM 2022

October
19-22

Hannover

Leibniz University Hannover

PROCEEDINGS

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INDRUM 2022 is an ERME topic conference

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ERME Topic Conferences (ETC) are conferences organised on a specific research theme or themes related to the work of ERME as presented in associated working groups at CERME conferences. Their aim is to extend the work of the group or groups in specific directions with clear value to the mathematics education research community.

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Table of contents

INDRUM2022 Editorial **1**

Plenary talk **5**

Developing digital networks for learning and teaching mathematics in introductory courses, Eichler Andreas [et al.] 5

Panel **23**

Innovations in university teaching based on mathematic education research, Florensa Ignasi [et al.] 23

TWG1: Transition to, across and from university mathematics **44**

TWG1: Transitions to, across and from University Mathematics, Hausberger Thomas [et al.] 44

Student Teachers' Knowledge of Congruence before a University Course on Geometry, Hoffmann Max [et al.] 50

Transitions in abstract algebra throughout the Bachelor: the concept of ideal in ring theory as a gateway to mathematical structuralism, Jovignot Julie [et al.] . . 60

Design and analysis of an unusual curve sketching exercise for first year teacher students, Khellaf Sarah [et al.] 70

On-campus vs distance learning in university preparatory courses: Which factors influence students' course decision?, Kirsten Katharina [et al.] 80

Insights into a reference model for school-based approaches to vectors, Mai Tobias [et al.] 90

The notion of a polynomial in the secondary-tertiary transition, Pleština Jelena [et al.] . . . 100

Difficulties and strategies of first-year students in formal reasoning using definitions - The borderline case of constant sequences, Schlüter Sarah [et al.] . . . 110

Didactic transposition of concavity of functions: From scholarly knowledge to mathematical knowledge to be taught in school, Strømskag Heidi [et al.] . . . 120

How to choose relevant mathematical content to address Klein's second transition? The case of the inner product, Grenier-Boley Nicolas . . . 130

Fostering Enculturation in the Transition to University Mathematics through Processes of Informal Learning, Günther Lukas . . . 132

Coherence of discourses of asymptote in the university transition, Katalenic Ana [et al.] . . . 134

CASPER: A framework to assess the difficulty of exercises in terms of semiotic registers, Seznec Julien [et al.] . . . 136

TWG2: Teaching and learning of analysis and calculus 138

TWG2: Teaching and learning of analysis and calculus, Hanke Erik [et al.] . . . 138

Personal Meanings versus Basic Mental Models for the Integral, Dreyfus Tommy [et al.] . . . 144

Returning to the classroom after taking online classes during the Covid-19 pandemic: A longitudinal study of student attainment, Griffiths Barry J. [et al.] . . . 154

Vertical coherence in the teaching of integrals? An example from complex analysis, Hanke Erik . . . 164

An epistemological gap between Analysis and Calculus: the case of Nathan, Kouropatov Anatoli [et al.] . . . 174

The Effect of Interactive Tasks in Instructional Videos on Students' Procedural Flexibility, Kraemer Sandra [et al.] . . . 183

Different interpretations of the total derivative and how they can be reconstructed in textbooks for Multivariable Real Analysis, Lankeit Elisa [et al.] . . . 193

Development of the differential calculus schema for two-variable functions, Trigueros María [et al.] . . . 203

Learning two-variable functions using 3D dynamic geometry, Trigueros María [et al.] . . . 213

Analysing the Quality of advanced Mathematics Lectures regarding the Presentation of Definitions – The Case of Real Analysis Lectures, Umgelter Karyna [et al.] . . . 223

An investigation of potential changes in students' images of derivative at the entrance of the university, Bouguerra Rahma [et al.] . . . 233

Investigation of metarules in lecturing for enabling students' participation, Karavi Thomais [et al.] . . . 235

TWG3: Teaching and learning of linear and abstract algebra, logic, reasoning and proof 237

TWG3: Teaching and learning of linear and abstract algebra, logic, reasoning and proof, Durand-Guerrier Viviane [et al.] . . . 237

Generic power of number theory proofs, Battie Véronique . . . 243

Proof assistants for undergraduate mathematics and computer science education: elements of a priori analysis, Bartzia Evmorfia Iro [et al.] . . . 253

A tale of four cities: reflections of master's students in mathematics on a visual word problem, Beccuti Francesco . . . 263

Mental constructions for the learning of the concept of vector space, Can Cabrera Angel [et al.] . . . 273

Analysing proofs of the Bolzano-Weierstrass theorem. A mean to improve proof skills and understanding of completeness, Durand-Guerrier Viviane . . . 283

Modes of description and indicators of formalism in students' descriptions of eigenvectors, Lyse-Olsen Emilie [et al.] . . . 293

Refutation feedback on student proofs beyond counter-examples, Pinto Alon [et al.] 303

Objectification processes in engineering freshmen while jointly learning eigentheory, Piroi Margherita . . . 313

Toward a framework for integrating computational thinking into teaching and learning linear algebra, Turgut Melih . . . 323

Die Konstruktion epistemologischer Dreiecke zur Analyse von Begriffsdeutungen bei mathematischen Lückentexten, Vo Thi My Hanh . . . 333

The characteristics of proof teaching in a first-year university mathematics course, Lipper Dimitri [et al.] 343

Digitale Brücke zwischen Schule und Hochschule? Ausgewählte Vorschläge für die Unterstützung von Beweisprozessen durch CAS in der Hochschulmathematik, Szűcs Kinga 345

Recognizing matrix equations as eigenequations or not: Examples of student reasoning in quantum mechanics, Wawro Megan [et al.] 347

TWG4: Teaching and learning of mathematics for engineers, and other disciplines 349

TWG4: Teaching and learning of mathematics for engineers, and other disciplines, Florensa Ignasi [et al.] 349

Modelling periodic phenomena through trigonometric polynomials using digital tools, Cabrera Alejandro [et al.] 355

Mathematics curriculum reforms in undergraduate engineering: An institutional analysis, Cuenca Luis [et al.] 365

Introducing a Study and Research Path in a course of Statistics for Engineering, Freixanet Maria-Josep [et al.] 375

Teaching mathematics to non-specialists: a praxeological approach, Gueudet Ghislaine [et al.] 385

A possible relationship between students' mathematical views and their evaluation of application examples, Hilger Susanne [et al.] 395

Questioning the quantum world? A priori analysis of an SRP at the interface between mathematics and quantum mechanics, Lombard Nathan 405

Cross disciplinary characteristics of study and research paths: statistics for business administration, Markulin Kristina [et al.] 415

An experience of exploring the boundary between mathematics and physics with preservice teachers, Pollani Lorenzo [et al.] 425

Potential conflict factors in learning exact differential equations: an impact of institutional practices, Rogovchenko Svitlana [et al.] 435

Mathematics students' self-reported learning experiences in a Challenge-based course, Salinas-Hernández Ulises [et al.] 445

Between mathematics and computer science? A study of research and teaching in numerical analysis, Burr Laura 455

Didactic proposal to re-signify the differential equation by modelling simple harmonic motion with the support of digital technology, Hernández-Méndez Ricardo [et al.] 457

We want news about Pingu: An explanatory action research case study in undergraduate service mathematics, Rizzo Ottavio 459

Contextual learning of mathematics for engineers, Rønning Frode 461

TWG5: Teacher education in university **463**

TWG5: Teacher education at the university, Bosch Marianna [et al.] 463

Resources to question mathematical knowledge: from teacher education to university mathematics, Barquero Berta [et al.] 468

Activity Theory as a Base for Course Design in Pre-Service Teacher Education: Design Principles and Their Application in Two Examples, Bauer Thomas [et al.] 478

Relevance attributions to contents in mathematics teacher education, Büdenbender-Kuklinski Christiane [et al.] 488

Promoting inquiry in mathematics: professional development of university lecturers, Gómez-Chacón Inés M [et al.] 498

Mathematicians and experienced teachers: crossing the boundary, Goor Myriam [et al.] 508

Statistical modelling in the Brousseaunian guessing game: A case of teacher education in Japan, Hakamata Ryoto [et al.] 518

Student Teachers' Development of Introductory ODE Learning Units - Subject-specific and further Challenges, Hochmuth Reinhard [et al.] 528

Mathematicians' Perspective on lectures in mathematics for secondary teacher candidates, Pustelnik Kolja 538

Teacher learning from team teaching an advanced course in mathematics and didactics for prospective upper secondary teachers, Viirman Olov [et al.] 548

Task design for Klein's second discontinuity, Winsløw Carl [et al.] 558

Coding in math learning: A 'triple instrumental genesis' approach to support the transition from university learner to school teacher, Broley Laura [et al.] 568

The reform of mathematics teacher education in Hungary, Csapodi Csaba 570

Design principles for intertwining local and nonlocal mathematics - The case of relating registers and representations in abstract algebra, Dellori Anna [et al.] . . 572

Promoting favourable beliefs of prospective math teachers concerning the nature of mathematics by using Interactive Mathematical Maps, Vinerean Mirela 574

TWG6: Students' practices and assessment **576**

TWG6: Students' practices and assessment, Grenier-Boley Nicolas [et al.] 576

Problem solving in the digital era: Examples from the work of mathematics students on a divisibility problem, Biza Irene 582

Effective orchestration features of a project-based approach to learning programming for math investigation, Broley Laura [et al.] 592

What are suitable positions for blanks in guided notes? – An investigation of students' perspectives, Feudel Frank [et al.] 602

Getting through the exam: A case study of four Finnish and German students' self-regulated learning of university mathematics, Göller Robin [et al.] 612

Informal learning situations in the context of mathematics studies - development of an analysis framework, Günther Lukas [et al.] 622

Self-regulated learning in a mathematics course for engineers in the first semester: insights into students' reported resource management and cognitive strategies, Kolbe Tim [et al.] 632

"exponential" as just "another word" to say "fast": When colloquial discourse tells only part of a story, Nardi Elena 642

Analyzing multilingual students' experiences in introductory college mathematics courses, Rios Jocelyn 652

To want and not want to learn, Ruge Johanna 662

University students' self-efficacy beliefs changes during first-year courses: A preliminary analysis, Gaio Aaron [et al.] 672

Analysis of informal learning situations at the Ostfalia University to support digital competences of first semester students, Marten Nico 674

Change of math anxiety in mathematics course for non-STEM university students in Japan, Oguro Takuo [et al.] 676

Emotions in Undergraduate Mathematics: A Distance Education Perspective, Pantelaki Sophia 678

Promoting co-operative learning in undergraduate mathematics via primetime instruction, Rämö Johanna [et al.] 680

Author Index **682**

INDRUM2022 Editorial

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INDRUM2022 was the fourth conference of the International Network for Didactic Research in University Mathematics. Initiated by an international team of researchers in didactics of mathematics in 2014, INDRUM aims at contributing to the development of research in didactics of mathematics at all levels of tertiary education, with a particular concern for the development of new researchers in the field and the dialogue with mathematicians. After three very successful conferences in 2016 (Montpellier, France), 2018 (Kristiansand, Norway), and 2020 (Bizerte, Tunisia, online) the INDRUM Network Scientific Committee (INSC) decided to continue the cycle of biennial conferences with a fourth INDRUM conference to be held in Hannover, Germany, on October 19th-22nd 2022.

The INSC nominated the INDRUM2022 International Programme Committee (IPC) and the Local Organising Committee (LOC), with an intersection to facilitate the coordination of both committees. The IPC was composed of María Trigueros (Ciudad de México, México) Chair; Berta Barquero (Barcelona, Spain) Co-chair; Rolf Biehler (Paderborn, Germany); Marianna Bosch (Barcelona, Spain); Laura Branchetti (Milan, Italy); Viviane Durand-Guerrier (Montpellier, France); Alejandro González-Martín (Montreal, Canada); Thomas Hausberger (Montpellier, France); Reinhard Hochmuth (Hannover, Germany); Barbara Jaworsky (Loughborough, United Kingdom); Rafael Martínez–Planell (Mayagüez, Puerto Rico); Chris Rasmussen (San Diego, United States). The LOC was composed of Reinhard Hochmuth (Hannover, Germany) Chair; Christine Bessenrodt (Hannover), Rolf Biehler (Paderborn), Andreas Eichler (Kassel), Sarah Khellaf (Hannover), Michael Liebendörfer (Paderborn), Jana Peters (Hannover), Johanna Ruge (Hannover), Hanh Vothi (Hannover) and Johannes Wildt (Bielefeld).

The first announcement, published in September 2021, communicated the structure of the conference. Similarly to the two previous INDRUM conferences, themes to be addressed at INDRUM2022 covered teacher and student practices and the teaching and learning of specific mathematical topics at undergraduate and postgraduate level, as well as across disciplines. Accepted scientific contributions were to be discussed in four to six thematic working groups (six hours each) after their presentation in two parallel sessions of short communications (two sessions of 1h 30min). The programme also included a poster exhibition and a workshop for early-career researchers. It was announced that Andreas Eichler (University of Kassel, Germany) had accepted to give

the plenary lecture and that an expert panel discussion on tertiary education on "Innovation in teaching at the university based on research in mathematics education" chaired by Rafael Martinez-Planell (Universidad de Puerto Rico at Mayagüez) was in preparation. Although the primary language of the conference was English, the linguistic characteristics of the host country were considered, as in previous INDRUM conferences. Therefore, authors were offered the opportunity to write and present a paper in German, provided that the presenter considered how to address the conference audience in its linguistic diversity through slides or a handout in English. Besides, INDRUM2022 was the fourth INDRUM conference to be accepted as a Topic Conference by the European Society for Research in Mathematics Education (ERME).

The second announcement was published in October 2021 with further details on the submission. Authors were provided with a list of 15 keywords or topics as a means to classify their submission (using two keywords from the list and three optional additional keywords) and also to help us in the subsequent process of allocating papers to different working groups after the review process.

In response to the call, 76 papers and 20 posters were received. The review process was organised by the chair and co-chair according to principles that were discussed among the IPC. Thus, each paper was reviewed by a member of the INSC and by an author of another paper; posters were reviewed by the chair or the co-chair and by an author of another poster. Final decisions, in cases where both reviewers had diverging opinions, were made after discussion among the IPC. At the end of the reviewing process, 56 papers and 20 posters were accepted for presentation at the conference. Authors of rejected papers that fell within the scope of the conference were offered the opportunity to resubmit their contribution as a poster. This last step increased the number of accepted posters to 39 in total.

Given the number of accepted contributions and the keywords provided by the authors, it was considered possible and appropriate to organise six balanced thematic working groups (TWG). The allocation of papers and posters was proposed by the chair and co-chair, and approved by the IPC. The appointment of TWG co-leaders from among the INSC members was made with a view to geographical diversity, gender balance, and the involvement of colleagues who had not previously or recently served as leaders. We were grateful that the appointed INSC members were able to accept our invitation.

The third announcement was published in September 2022 with the following list of groups (TWG) and names of co-leaders:

> TWG1: *Transition to, across and from university mathematics.* Chairs: Thomas Hausberger (France), Heidi Strømskag (Norway)
>
> TWG2: *Teaching and learning of analysis and calculus.* Chairs: Erik Hanke (Germany), Rafael Martínez-Planell (Puerto Rico)

TWG3: *Teaching and learning of linear and abstract algebra, logic, reasoning and proof.* Chairs: Viviane Durand-Guerrier (France), Melih Turgut (Turkey-Norway)

TWG4: *Teaching and learning of mathematics for engineers, and other disciplines.* Chairs: Ignasi Florensa (Spain), Ghislaine Gueudet (France),

TWG5: *Teacher education in university.* Chairs: Marianna Bosch (Spain), Carl Winsløw (Denmark)

TWG6: *Students' practices and assessment.* Chairs: Nicolas Grenier-Boley (France), Frank Feudel (Germany)

The third announcement also included the names of the panel chair and panelists, who were appointed by the IPC from among the conference participants on the basis of their expertise in the topic of the panel. Rafael Martinez-Planell (Universidad de Puerto Rico in Mayagüez, Puerto Rico) accepted to chair the panel on "Innovation in teaching at the university based on research in mathematics education" with Ignasi Florensa (Universidad Salesiana de Sarrià-Universitat de Barcelona, Spain), Max Hoffmann (Paderborn University, Germany), Avenilde Romo (CINVESTAV, México) and Michelle Zandieh (University of Arizona, USA) as speakers. Finally, Elena Nardi (University of East Anglia, United Kingdom) and Megan Wawro (Virginia Tech, USA) prepared a workshop for INDRUM early career researchers on "Starting to write journal articles" for INDRUM early career researchers based on two of their published papers. The purpose of the workshop was to share experiences and stimulate discussion on what constituted the challenges –and ways to overcome them– of preparing a manuscript for submission to a mathematics education research journal, with a particular focus on university mathematics education.

Table 1. Participants and countries

Country	Participants
Canada	4
Croatia	6
Denmark	2
Finland	3
France	12
Germany	39
Greece	2
Hungary	1
Italy	6
Israel	4
Japan	6
Mexico	3
Norway	6
Puerto Rico	1
Spain	7
Sweden	5
The Netherlands	4
Tunisia	1
United Kingdom	2
United States	4

The third announcement included the conference timetable and the conference pre-proceedings. In parallel, the LOC was getting ready to welcome delegates in Hannover, Germany.

118 participants from 20 countries registered for the INDRUM2022 conference (see Table 1). The opening and closing sessions were lively thanks to the work of the LOC. Work in the TWG was also lively with interesting exchanges and interaction between delegates. The TWG's leaders encouraged rich interaction among participants. They also prepared a summary which was presented at the closing session.

Papers and posters appear in these Proceedings in a version chosen by the participants, following the optional possibility to upload a final version of their paper.

Last but not least, we would like to thank the IPC, chaired by Maria Trigueros and Berta Barquero, and the LOC, chaired

by Reinhard Hochmuth and his entire team (especially Mrs. Krampe), for their tireless work over many months to organise the conference. We are also grateful for the support provided by Jana Peters for her work on both the pre-proceedings and these proceedings. Many thanks to all TWG co-leaders for your support and help in making INDRUM2022 a fruitful and enjoyable experience for the participants.

FOLLOW-UP

The INDRUM2022 closing ceremony was the occasion to announce some additional interesting news. We are delighted to announce that authors of an accepted contribution (paper or poster) in the INDRUM2022 proceedings will be offered the opportunity to publish an expanded, updated or reworked version of their contribution to match the requirements of a special issue in IJMEST (International Journal of Mathematical Education in Science and Technology) Special Issue guest-edited by the INDRUM2022 Chair, co-Chair and LOC Chair. We invite papers of 15-20 pages, written in English, with the aim of publishing approximately ten papers among the best research represented in the INDRUM2022 Proceedings. While we aim to reflect the thematic richness of the INDRUM2022 programme, we will not commit to a strict representation of the conference structure. We particularly welcome proposals that substantially elaborate and expand the content of the INDRUM2022 submissions.

The deadlines for this Special Issues have been fixed as follows: April 23rd, 2023: deadline to submit papers; July 28th, 2023: decision letters sent to authors; September 29th, 2023: deadline for revised manuscripts; December 5th, 2023: final decisions and February 2024 Publication. The official call for contributions has been sent to the authors of INDRUM2022 accepted contributions through the INDRUM mailing list. The Call for papers is available at the INDRUM2022 webpage (https://indrum2022.sciencesconf.org/); as well as in the IJMEST website: https://bit.ly/INDRUM2022_Conference.

Finally, we are delighted to spread the news that INDRUM2024 will be held in Barcelona, Spain in June 2024 from 10th to 14th June 2024. The final dates will be decided in September 2023. The local Chair is Ignasi Florensa, with Alejandro González Martín (Canada) as Chair of the IPC and Ghislaine Gueudet (France) as co-Chair. The INDRUM2024 website https://indrum2024.sciencesconf.org/, which is currently under construction, will open with updated information as soon as possible.

We now invite you to carry on reading this volume and we hope that the promise of its contents will encourage you to consider joining or continuing to be part of the ambitious and stimulating INDRUM network.