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Mathematical and Statistical Methods for Actuarial Sciences and Finance

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Stochastic Dominance in the Outer Distributions of the α-Efficiency Domain

Sergio Bianchi, Augusto Pianese, Massimiliano Frezza, and Anna Maria Palazzo

Abstract The departures from market efficiency are used to provide evidence of overreaction and underreaction in two main stock indexes. Specifically, using the notion of α-efficiency, we document the presence of stochastic dominance in the conditional distributions of mean log-price variations.

Keywords Market Efficiency · Over/Underreaction · Pointwise Regularity

1 Introduction

The celebrated study of De Bondt and Thaler (see [5]) provided evidence that abnormal profits are achievable in the long-run, simply going short a portfolio of "winner stocks" (i.e., stocks with good performances in the past) and going long a portfolio of "loser stocks" (i.e., stocks that performed badly in the past). The authors ascribe these contrarian profits to the investors' excess of optimism and pessimism, the so called *overreaction* to information. Since then many studies documented contrarian abnormal profits in international markets and/or for short time horizons. Other results document the opposite phenomenon of *underreaction*: security prices can also underreact to news (they can trend up after an initial positive reaction to a good news and, samely, they can keep trending down after an initial negative reaction to a bad news). This reaction generates what is called the *momentum profit*; the trading

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strategy in this case consists in going long a portfolio of extremely winner stocks and going short a portfolio of extremely loser stocks (for a survey on overreaction and underreaction, see e.g. [1, 12]). Obviously, both overreaction and underreaction have much to do with the notion of informational (in)efficiency; once accepted the idea that financial markets can indeed be inefficient, one main issue becomes to seize the times and/or the markets (or even the individual stocks) susceptible to over or under-react. This is precisely the purpose of this paper: we exploit the characterization of semimartingales (Fama's definition of efficiency [6]) in terms of pointwise regularity exponent of their trajectories and the definition of α-efficiency introduced by [4] to provide evidence that the distributions of mean returns in case of negative inefficiency stochastically dominate the corresponding distributions originated by positive inefficiencies. Thus, every expected utility maximizer with an increasing utility function should prefer to go long on the market when it experiences negative inefficiencies. We document these results for the U.S. and the U.K. markets.

2 Efficiency, Pointwise Regularity and α-Efficiency

As well known, EMH requires at any time t the price of any stock to fully reflect all available information $\mathscr{F}_t$. This implies that prices only change as a reaction to new information or to (predictable or unpredictable) changes in stochastic discount factors. Therefore, departures from the "fair" value would be immediately arbitraged away by traders, who would outperform the market on a risk-adjusted basis. Since its adoption, a number of contributions questioned the validity of this assumption and, in this direction, one strand of research relies on the relation linking semimartingales to the value of the pointwise regularity exponent $H(t)$ of the price process at time t [4, 7], see Table 1 for its financial interpretation.[1] Several methods have been proposed in literature to estimate $H(t)$ (see, e.g., [8, 10]) and here we will refer to [11], who merge the unbiased, large-variance estimator $\hat{H}_{\nu,n}(t,a)$ introduced by [2, 9] with the biased, low-variance estimator $\hat{H}_{\nu,q,n,K^*}(t)$ deduced in [3]. In this way, they obtain the unbiased, low-variance estimator

[1] Given the stochastic process $X(t,\omega)$ with a.s. continuous and not differentiable trajectories over the real line $\mathbb{R}$, the local Hölder regularity of the trajectory $t \mapsto X(t,\omega)$ with respect to some fixed point t can be measured through the *pointwise* Hölder exponent, defined as $\alpha_X(t,\omega) = \sup\left\{\alpha \geq 0 : \limsup_{h\to 0} \frac{|X(t+h,\omega)-X(t,\omega)|}{|h|^\alpha} = 0\right\}$. For Gaussian processes, by virtue of zero-one law, there exists a non random quantity $a_X(t)$ such that $\mathbb{P}(a_X(t) = \alpha_X(t,\omega)) = 1$. In addition, when $X(t,\omega)$ is a semimartingale (e.g. Brownian motion), $\alpha_X = \frac{1}{2}$; values different from $\frac{1}{2}$ describe non-Markovian processes, whose smoothness is too high, when $\alpha_X \in \left(\frac{1}{2}, 1\right)$, or too low, when $\alpha_X \in \left(0, \frac{1}{2}\right)$, to satisfy the martingale property. In particular, the quadratic variation of the process can be proven to be zero, if $\alpha_X > \frac{1}{2}$ and infinite, if $\alpha_X < \frac{1}{2}$.

Table 1 Financial interpretation of $H(t)$

$H(t)$	Stochastic consequence	Investors' belief	Market consequence
$> \frac{1}{2}$	Persistence Low variance	Future information will confirm past positions	Low volatility/Underreaction Overconfidence/Positive inefficiency
$= \frac{1}{2}$	Independence Martingale	Past information fully discounted by prices	Efficiency
$< \frac{1}{2}$	Mean-reversion High variance	Future information will contradict past positions	High volatility/Overreaction Negative inefficiency

$$\hat{H}_{\nu,q,n}(t,a) = \hat{H}_{\nu,q,n,K^*}(t) + \frac{1}{n}\sum_{t=1}^{n}\left(\hat{H}_{\nu,n}(t,a) - \hat{H}_{\nu,q,n,K^*}(t)\right), \tag{1}$$

where ν is the size of the estimation window, q is the lag set for the increment process (usually 1), n is the length of the sample, K^* is an arbitrary scale parameter of the process and a is a discrete differencing operator acting to make the sequence locally stationary and to weaken the dependence between the observations. Since the martingale condition holds when $H(t) = \frac{1}{2}$, it comes natural to compare $\hat{H}(t)$, estimated through $\hat{H}_{\nu,q,n}(t,a)$, with this value. This can be done because $\hat{H}_{\nu,q,n}(t,a)$ is normally distributed around the true value with known variance, when $H(t) = \frac{1}{2}$. Thus, one has

$$\Phi(z) := \Phi_{\hat{H}_{\nu,q,n}(t,a)|H(t)=\frac{1}{2}}(z) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{(x-1/2)^2}{2\sigma^2}}dx \tag{2}$$

where $\sigma = \left(\frac{\sqrt{\pi}\,\Gamma\left(\frac{2k+1}{2}\right)-\Gamma^2\left(\frac{k+1}{2}\right)}{\nu k^2 \log^2(n-1)\Gamma^2\left(\frac{k+1}{2}\right)}\right)^{1/2}$.

Therefore, once the significant level α has been fixed, setting $(x)^+ = \max\{x, 0\}$, the contractive map of $\hat{H}_{\nu,q,n}(t,a)$ (see [4])

$$\gamma^{\alpha}(t) = \left(\hat{H}_{\nu,q,n}(t,a) - \Phi^{-1}(1-\alpha/2)\right)^{+} - \left(\Phi^{-1}(\alpha/2) - \hat{H}_{\nu,q,n}(t,a)\right)^{+} \tag{3}$$

filters out the values $\hat{H}_{\nu,q,n}(t,a)$ lying outside the confidence interval $[\Phi^{-1}(\alpha/2), \Phi^{-1}(1-\alpha/2)]$. In this framework, the following definition of α-efficiency can be given

Definition 1 A market is α-efficient at time t if and only if $\gamma^{\alpha}(t) = 0$. Functions

$$\gamma^{\alpha}_{+}(t) = \left(\hat{H}_{\nu,q,n}(t,a) - \Phi^{-1}(1-\alpha/2)\right)^{+} \tag{4}$$

$$\gamma_{-}^{\alpha}(t) = -\left(\Phi^{-1}(\alpha/2) - \hat{H}_{\nu,q,n}(t,a)\right)^{+} \tag{5}$$

filter out only the one-directional exceedances, respectively above and below the thresholds provided by the confidence interval. They characterize the positive (γ_{+}^{α}) and negative (γ_{-}^{α}) inefficiencies that, according to the interpretation of Table 1, trigger underreaction and overreaction, respectively.

3 Conditional Distributions

The setting introduced in the previous Section was used to deduce the conditional distributions of the time average log-price variations. They are calculated as follows:

- Build the sets $T_{\gamma_{-}^{\alpha}} = \{t : \gamma_{-}^{\alpha}(t) < 0\}$ and $T_{\gamma_{+}^{\alpha}} = \{t : \gamma_{+}^{\alpha}(t) > 0\}$ collecting all the epochs of negative (respectively, positive) inefficiencies;
- For each $t \in T_{\gamma_{-}^{\alpha}}$ ($t \in T_{\gamma_{+}^{\alpha}}$), the price $X(t+h)$ is collected, for a set of h trading days ahead with respect to time t;
- Vectors $\bar{Y}_{\gamma_{-}^{\alpha}}(h) = \frac{1}{\#(T_{\gamma_{-}^{\alpha}})} \sum_{t \in T_{\gamma_{-}^{\alpha}}} \left(\ln \frac{X(t+h)}{X(t)}\right)$ and $\bar{Y}_{\gamma_{+}^{\alpha}}(h) = \frac{1}{\#(T_{\gamma_{+}^{\alpha}})} \sum_{t \in T_{\gamma_{+}^{\alpha}}} \left(\ln \frac{X(t+h)}{X(t)}\right)$ are calculated;
- the conditional distributions of the average log-price variations $N(y) := F_{\bar{Y}_{\gamma_{-}^{\alpha}}(1,\ldots,h_{\max})}(y)$ and $P(y) := F_{\bar{Y}_{\gamma_{+}^{\alpha}}(1,\ldots,h_{\max})}(y)$ are estimated for some relevant $h_{\max}$.

The procedure ensures that the effects revealed by the conditional distributions do not depend on any specific event. Indeed, since the epochs in $T_{\gamma_{-}^{\alpha}}$ (as well as in $T_{\gamma_{+}^{\alpha}}$) can be very far one from each other, the consequent prices, the number of traded stocks, the market phases or even the economic cycle can greatly differ. Given the interpretation provided by Table 1, we expect to observe two effects:

- $N(y) \leq P(y)$ for all y, with strict inequality at some y (first-order stochastic dominance);
- $\int_{-\infty}^{\infty} (y - \mathbb{E}(y))^2 \, dN(y) > \int_{-\infty}^{\infty} (y - \mathbb{E}(y))^2 \, dG(y)$ (larger variance for negative inefficiency).

4 Application and Discussion of Results

The procedure described in Sect. 3 was applied to the analysis of two stock indexes: the U.S. Dow Jones Industrial Average (DJIA), and the U.K. Footsie 100 (FTSE100), both referred to a period of 35 years (from January 29, 1985 to December 31, 2019), resulting in 8802 observations for the DJIA and 8824 observations for the FTSE100.

The analysis was performed by setting h from 1 up to 250 trading days and $h_{\max}$ to 1, 3, 6 trading months and 1 trading year. The significance level to test for inefficiency

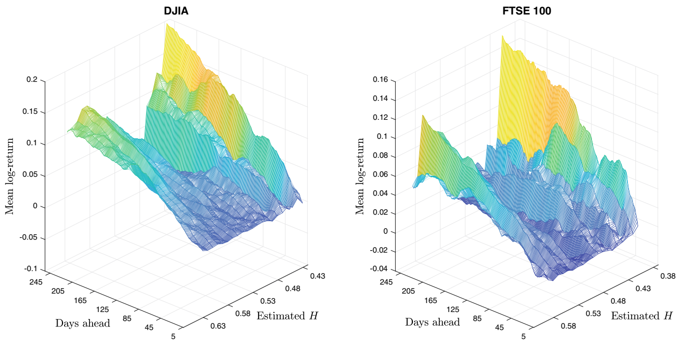

Fig. 1 Time average returns with respect to H and number of days ahead, with $h_{\max} = 1$ trading year

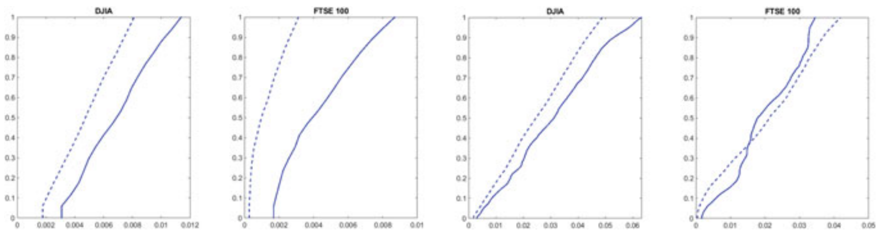

Fig. 2 Two left panels: Conditional distributions of the averaged log-price variations for positive (dotted line) and negative (continuous line) inefficiency, with $h_{\max} = 1$ trading month. Two right panels: Conditional distributions of the averaged log-price variations for positive (dotted line) and negative (continuous line) inefficiency, with $h_{\max} = 6$ trading months

was set at $\alpha = 0.05$, corresponding to $\Phi^{-1}(\alpha/2) \simeq 0.47$ and $\Phi^{-1}(1-\alpha/2) \simeq 0.53$. The results, reproduced in Figs. 1 and 2, show that in the short term the returns behave as expected. In detail, Fig. 1 displays that up to one trading year both the indexes have conditional mean variations generally higher for negative inefficiency than those of the positive inefficiency case. The pattern is even more evident if one looks at the extremal values of the estimated pointwise regularity eponents. An element of deep distinction between the two indexes can be observed as H approaches to $\frac{1}{2}$: the conditional mean variations continue to be largely positive for the DJIA whereas they incur in a significant downward correction for the FTSE100. A possible explanation for this effect is constituted by the injections of liquidity that the Federal Reserve provided to the U.S. market during the last global financial crisis. As well documented, this caused U.S. financial market to raise.

Figure 2 confirms the findings above in terms of (at least) first-order stochastic dominance between the distributions of the conditional mean variations up to one trading month ($h_{\max} = 21$ days). Interestingly, as $h_{\max}$ increases up to six trading months, the evidence becomes more questionable: whereas negative inefficiency continues to generate (moderate) overreaction for the DJIA (with $N(y)$ still domi-

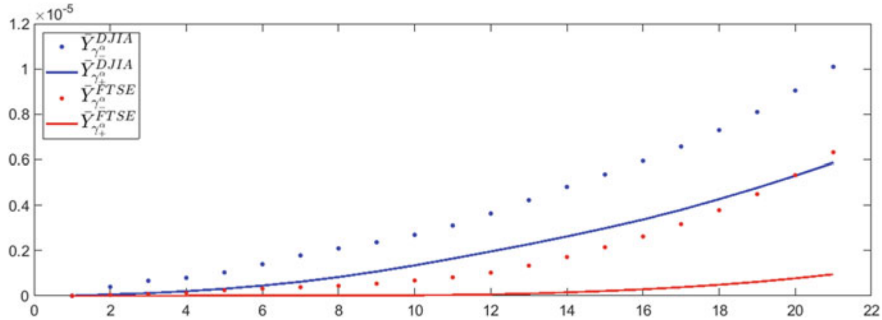

Fig. 3 Variance of the distributions of the averaged log-price variations, $h_{\max} = 21$ days

nating $P(y)$), for the FTSE100, $N(y)$ can be almost overimposed to $P(y)$. Again, this can be a symptom of the different level of liquidity provided to the two markets. Finally, Fig. 3 displays the behaviour of the variances of negative and positive inefficiency, up to $h_{\max} = 21$ days, which is the larger time horizon for which we find evidence of first-order stochastic dominance for both the indices. Data confirm that, at least in the two samples, larger variance occurs for negative inefficiency.

5 Conclusions and Further Developments

We used the notion of α-efficiency to characterize a well-documented behaviour of stock markets: the under/overreaction. Evidence is provided that negative inefficiency generates overreaction, opposed to positive inefficiency which generally triggers underreaction. More extensive analyses can be made on individual stocks.

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