

Fifth conference of the  
International Network for  
Didactic Research in University Mathematics

**INDRUM** 2024  
June  
10-14

**BARCELONA**

Centre de Recerca Matemàtica

**PROCEEDINGS**

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European Society for Research in Mathematics Education



# INDRUM2024 PROCEEDINGS

## Fifth conference of the International Network for Didactic Research in University Mathematics

10-14 June 2024

Barcelona

Spain

**Editors:** Alejandro S. González-Martín, Ghislaine Gueudet, Ignasi Florensa, Nathan Lombard

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## INDRUM2024 editorial

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INDRUM2024 (<https://indrum2024.sciencesconf.org/>) was the fifth conference of the International Network for Didactic Research in University Mathematics (<https://hal.science/INDRUM>). Since its creation in 2014, INDRUM has aimed to contribute to the development of research in didactics of mathematics at all levels of postsecondary and tertiary education, with a particular focus on supporting emerging researchers in the field and fostering dialogue with mathematicians. Following four highly successful conferences—in 2016 (Montpellier, France), 2018 (Kristiansand, Norway), 2020 (Bizerte, Tunisia, held online), and 2022 (Hannover, Germany)—the INDRUM Network Scientific Committee (INSC) decided to continue the biennial conference cycle. The fifth INDRUM conference was held in Barcelona, Spain, from June 10-14, 2024, hosted at the Centre de Recerca Matemàtica (CRM, see more details at <https://www.crm.cat/indrum-2024>).

The INSC nominated the INDRUM2024 International Programme Committee (IPC) and the Local Organising Committee (LOC), with some common members to facilitate the coordination of both committees. The IPC was composed of:

Alejandro S. González-Martín (University of Montreal, Canada), IPC Chair,

Ghislaine Gueudet (Université Paris Saclay, France), IPC Co-chair,

Ignasi Florensa (EUSS – Univ. Autònoma de Barcelona, Spain), Berta Barquero (University of Barcelona, Spain), Matija Bašić (University of Zagreb, Croatia), Laura Branchetti (University of Milan, Italy), Erik Hanke (University of Hannover, Germany), Thomas Hausberger (University of Montpellier, France), Mitsuru Kawazoe (Osaka Metropolitan University, Japan), Elena Nardi (University of East Anglia, UK), Frode Rønning (Norwegian University of Science and Technology, Norway), Heidi Strømskag (Norwegian University of Science and Technology, Norway), María Trigueros (Benemérita Universidad Autónoma de Puebla, Mexico), Olov Viirman (Uppsala University, Sweden).

The LOC was composed of:

Ignasi Florensa (EUSS – Univ. Autònoma de Barcelona, Spain), LOC Chair,

Jordi Cuadros (IQS – Univ. Ramon Llull, Spain), Laura Fernández-Ruano (IQS – Univ. Ramon Llull, Spain), Iria Fraga (EUSS – Univ. Autònoma de Barcelona, Spain), María Josep Freixanet (Universitat Politècnica de Catalunya, Spain), Nathan Lombard (EUSS – Univ. Autònoma de Barcelona, Spain), Kristina Markulin (EUSS – Univ. Autònoma de Barcelona, Spain), Víctor Martínez-Junza (EUSS – Univ. Autònoma de Barcelona, Spain), Noemí Ruiz-Munzón (EUSS – Univ. Autònoma de Barcelona, Spain), Lúdia Serrano (Universidad de Deusto, Spain).

The first announcement, published in June 2023, outlined the structure of the conference. Similar to the two previous INDRUM conferences, INDRUM2024 addressed themes related to teacher and student practices, the teaching and learning of specific mathematical topics at undergraduate and postgraduate levels, as well as across disciplines. Accepted scientific contributions were to be organised into thematic working groups, determined based on the submissions accepted. The programme also featured a workshop for early-career researchers, a plenary lecture by Marianna Bosch (University of Barcelona, Spain) with a reaction by Yves Chevallard (University of Aix-Marseille, France), an expert panel discussion on the theme “Mathematics for non-specialists”, and a poster exhibition.

Although English was the primary language of the conference, the linguistic context of the host country was acknowledged, as in previous INDRUM conferences. Authors were therefore allowed to write and present their paper or poster in Spanish or Catalan, provided the presenter ensured accessibility for the linguistically diverse audience through English-language slides or handouts. Additionally, INDRUM2024 was the fifth INDRUM conference to be recognised as a Topic Conference by the European Society for Research in Mathematics Education (ERME).

In response to the call, 109 papers and 27 posters were submitted. The review process, coordinated by the chair and co-chair, followed principles discussed among the IPC. Each paper was reviewed by a member of the INSC or another expert researcher, as well as by an author or co-author of another paper. Posters were reviewed by the chair, the co-chair, and one of the INDRUM Co-coordinators (Elena Nardi). For submissions with divergent reviewer opinions, final decisions were made following IPC discussions. At the conclusion of the review process, 77 papers and 25 posters were accepted, with 32 additional papers reassigned as posters. Some authors withdrew their contribution, resulting in 76 papers and 39 posters being included in the pre-proceedings.

Given the number of accepted contributions and the keywords provided by the authors, the IPC decided to organise five balanced thematic working groups (TWG). The chair and co-chair proposed the allocation of papers and posters, which was approved by the IPC. TWG co-leaders were appointed from among INSC members and paper authors, with attention to geographical diversity, gender balance, and the inclusion of colleagues who had not recently or previously served as leaders. We were grateful that all invited TWG leaders were able to accept our invitation. The second announcement, published in May 2024, included the following list of TWGs and their co-leaders:

**TWG1: Teaching and learning of analysis and calculus**

*Chairs: Erik Hanke (Germany), Rafael Martínez-Planell (Puerto Rico)*

**TWG2: Teaching and learning of linear and abstract algebra**

*Chairs: Yael Fleischmann (Norway), Megan Wawro (United States)*

### **TWG3: Mathematics and other disciplines**

*Chairs: Matija Bašić (Croatia), Ida Maria Landgårds-Tarvoll (Norway)*

### **TWG4: Teacher education and knowledge**

*Chairs: Imène Ghedamsi (France), Pedro Nicolás (Spain)*

### **TWG5: Teachers' and students' practices and experience**

*Chairs: Athina Thoma (United Kingdom), Olov Viirman (Sweden)*

The second announcement also included the conference timetable, which allocated 14 hours for work within the TWGs. It provided the title of the plenary, announcing it was followed by a reaction, as well as the title of the plenary panel and the names of the panel chair and panelists. The four panelists were selected by the IPC, which chose to invite two mathematics education researchers—one specialised in mathematical modelling and the other in mathematics for economics—alongside a researcher in chemistry education and a researcher in physics education. The details are outlined below:

*Plenary talk: Teaching inquiry at the university: in need of mathematical infrastructures:* Marianna Bosch (University of Barcelona, Spain).

*Reaction to the plenary talk:* Yves Chevillard (University of Aix-Marseille, France).

*Plenary Panel: Mathematics and other disciplines: epistemological issues and their impact on teaching practices at tertiary level*

Chair: Laura Branchetti (University of Milan “La Statale”, Italy)

Frank Feudel (Humboldt-Universität zu Berlin, Germany)

Felix Ho (Uppsala University, Sweden)

Ricardo Karam (University of Copenhagen, Denmark)

Noemí Ruiz Munzón (Escola Universitària Salesiana de Sarrià, Barcelona, Spain)

Elena Nardi (University of East Anglia, United Kingdom) and Megan Wawro (Virginia Tech, USA) conducted a workshop for INDRUM early-career researchers titled “Starting to write journal articles”. Drawing on two of their published papers, the workshop aimed to share experiences and foster discussion on the challenges of preparing a manuscript for submission to a mathematics education research journal, particularly in the area of university mathematics education. Participants explored strategies to address these challenges effectively.

The conference pre-proceedings were prepared by Nathan Lombard and Ignasi Florensa, and were made available on the conference website by the end of May. Meanwhile, the LOC finalised preparations to welcome delegates to Barcelona, Spain. A total of 160 participants from 25 countries registered for the INDRUM2024 conference (see Table 1). For the first time, 10 grants covering registration and accommodation were offered to attendees, thanks to support from the CRM.

Algeria	1	Germany	12	Norway	13
Australia	1	Greece	4	Puerto Rico	1
Austria	2	Ireland	3	Spain	22
Belgium	1	Israel	2	Sweden	4
Canada	4	Italy	8	Switzerland	1
Croatia	5	Japan	6	United Kingdom	9
Czech Republic	1	Mexico	21	United States	14
Denmark	2	Netherlands	1		
France	20	New Zealand	2		

**Table 1: Participants and their countries, INDRUM 2024**

INDRUM2024 marked several milestones and set new records for the INDRUM conferences. It received the highest number of submissions to date, resulting in the largest number of accepted presentations. Consequently, INDRUM2024 also had the highest number of registered participants. Additionally, INDRUM2024 became the longest INDRUM conference, spanning five days, and featured the most extensive allocation of hours dedicated to work within TWGs. Furthermore, as noted earlier, it was the first INDRUM conference to offer grants to participants, fostering greater inclusivity and accessibility.

The opening and closing sessions were dynamic and engaging, thanks to the efforts of the LOC. The work within the TWGs was equally lively, with stimulating exchanges and interaction among delegates. The TWG leaders actively encouraged meaningful engagement and collaboration among participants. They also prepared summaries of their group’s work, which were presented during the closing session. The proceedings include the full papers (10 pages) and poster presentations (2 pages) contributed to the conference.

We would also like to express our heartfelt gratitude to the IPC and LOC, as well as the CRM support team (with special thanks to Núria Hernández Martín!), for their dedicated efforts over many months in organising the conference and ensuring its smooth execution. We extend our appreciation to all TWG co-leaders for their invaluable contributions in making INDRUM2024 a productive and enjoyable experience for all participants. A special thanks also goes to Nathan Lombard for his outstanding work on both the pre-proceedings and these proceedings.

The INDRUM2024 closing ceremony included the announcement of some exciting news. We are pleased to share that the Chair, the co-Chair and LOC Chair of the conference are forming a team of potential guest editors who will propose a special issue for an international journal. Authors of accepted contributions (papers or posters) from INDRUM2024 will have the opportunity to submit an expanded, updated, or

reworked version of their work to align with the theme of this special issue. Submissions will also be open to contributors outside the INDRUM2024 conference, marking a significant milestone for INDRUM. Unlike previous proceedings-focused special issues, this initiative will centre on a thematic special issue. Further details will be shared soon.

Finally, we are also delighted to announce that INDRUM2026 will be held in Dubrovnic, Croatia, from June 15-19, 2026. The Local Chair will be Matija Bašić, with Chris Rasmussen (USA) serving as IPC Chair and Imène Ghedamsi (Tunisia and France) as co-Chair. The INDRUM2026 website (<https://indrum2026.sciencesconf.org/>), currently under construction, will be updated with information as soon as it becomes available.

We now invite you to explore this volume and hope its contents inspire you to join or continue your engagement with the ambitious and dynamic INDRUM network.

# Interdisciplinarity and second transition: novices' attention towards examples in mathematics and physics

Lorenzo Pollani<sup>1</sup>, Laura Branchetti<sup>2</sup>, and Francesca Morselli<sup>3</sup>

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*This study concerns pre-service teacher education at university level, with a special focus on interdisciplinarity. The process of becoming teachers tackles the issue of the second transition from university to teaching at school, where a discontinuity can occur. Such discontinuity usually concerns university students who cannot connect university knowledge of mathematics to knowledge useful to teach. We reflect on the second transition for interdisciplinary teachers starting from the analysis of a discussion with pre-service mathematics and physics teachers about two physics textbooks during a university course in mathematics education. The analysis is centred on their structure of attention with respect to examples and exemplification.*

*Keywords: Transition to, across and from university mathematics, Teaching and learning of mathematics in other disciplines, teacher education, mathematics and physics, structure of attention.*

## INTRODUCTION

This contribution is framed in a wider study concerning pre-service teacher education, with a focus on interdisciplinarity between mathematics and physics (Branchetti et al., 2023). Reflecting on the interplay between mathematics and other disciplines from a didactical point of view is relevant for university mathematics education since one of main challenges at tertiary level is teaching non-mathematicians in such a way that mathematics is perceived as meaningful to other disciplines and professional fields and that the constraints and needs of other disciplines are considered (Gueudet et al., 2023).

The main issue to face is that people with a strong preparation in mathematics might not be aware of the nature of this interplay and not to be able to identify the critical issues that characterize it. University students usually have no opportunities to reflect deeply on these aspects. Here we focus on pre-service teacher education in university contexts and to second transition, from university into upper secondary school teaching (Gueudet et al., 2016). At its core, the second discontinuity concerns the detachment between the university mathematics and teaching mathematics at school. The issue is particularly critical when teacher educators face the challenge to train pre-service mathematics teachers to deal with interdisciplinarity between mathematics and physics. Indeed, the corresponding academic knowledge is expected to be developed both in advanced mathematics, physics or even mathematical physics courses, but to show the connections between different disciplines is not usually within the scopes of such courses. In the Italian context, this issue is very relevant since a master's degree in mathematics allows university students to become secondary mathematics and

physics teachers, and the national curriculum requires explicitly to show the interdisciplinary connections between mathematics and physics (Bagolini et al., 2021).

In this paper, we interpret the second transition as part of the long-term process of (mathematics and physics) teacher education. We focus on examples and exemplification in textbooks since dealing with them is crucial both in mathematics (Watson & Chick, 2011) and physics (Kuo, 2023), thus this is a good example of critical interdisciplinary issue from a didactical point of view at secondary level that is not addressed in an explicit way in university courses.

## **THEORETICAL FRAMEWORK**

### **Mathematics pre-service teacher education**

By teacher education, we mean the process of promoting the shift from novice to expert (Mason, 1998), where “The expert differs from the novice in the form and structure of their attention” (p. 243). According to Mason, the structure of attention “encompasses the locus, focus and form of attention moment by moment” (p. 250). We operationalize this structure as follows: the *locus of the attention* is a specific semiotic element in the textbook (for example, a word, a sentence, a figure, or a table). It is studied by examining what semiotic elements are referred to in the comments. Concerning the *form*, the attention towards a locus may be focused or diffuse, “centered on a single domain or else either flit between or simultaneously locate you in different worlds” (ibid.). Concerning the *focus*, it is the comment performed around the locus, including the reason why the locus is addressed. It is studied by examining the comments performed on specific aspects of the textbooks, and the extent to which these reasons are explicitly stated.

According to Viennot and Décamp (2018), a significant feature of expert teachers is the *critical attitude*, which means engaging in intellectual dynamics in first person and being able to discuss critically the knowledge at stake. They also state the worthiness of examining this faculty when pre-service teachers are confronted by any textual resource designed by a person whose aim is to explain something. Moreover, Viennot (2001) introduces the notion of *critical detail* to identify those aspects that have the potential to trigger, and deserve, a critical discussion. From Viennot and Décamp (2018)’s perspective, the recognition and properly articulated analysis of such critical details in textual resources might support the enactment of a *critical attitude*. Furthermore, we claim that enacting a critical attitude can be of support for the second transition, since critical discussing the knowledge at stake, from a teaching point of view, includes the ability of discerning connections between university and school knowledge. From Mason (1998)’s perspective, this coherently suggests examining the structure of attention while teachers comment on textbooks and interact with teacher educators.

## **Interdisciplinarity between mathematics and physics**

We adopt a theoretical perspective on interdisciplinarity between mathematics and physics which derives from the European Erasmus+ project IDENTITIES (n. 2019-1-IT02-KA203-063184). Two concepts are crucial: *boundary* (Akkerman & Bakker, 2011) and *epistemic core of a discipline* (Erduran & Dagher, 2014).

We describe *discipline-based communities* as communities characterized by expertise in a discipline (e.g., scientists or mathematics teacher educators), which constitutes the *boundary of a community* (Akkerman & Bakker, 2011). Specifically, “a boundary can be seen as a sociocultural difference leading to discontinuity in action or interaction” (p. 133). Simultaneously belonging to both worlds and neither, boundaries embody not only ambiguities and risks but also opportunities. The two worlds can be bridged by artifacts, called *boundary objects*. Furthermore, *boundary people* are those subjects that are recognized and recognize themselves as members of a community but whose knowledge, practices, and interests do not belong only to that community. They indeed can move outside and bring into the community new languages and challenges (ibid.). Coherently, the *boundary between disciplines* is a metaphorical space of encounter and interaction where members of a discipline-based community deal with (boundary) objects produced by members of another (e.g., physics textbooks for mathematics pre-service teachers, as in Pollani & Branchetti, 2022).

Even with interdisciplinarity, it is still important to reflect on the single disciplines. Nevertheless, separating what belongs to one discipline and what does not is considered a problematic approach (Erduran & Dagher, 2014; Pollani & Branchetti, 2022; Satanassi et al., 2023). As proposed in Satanassi et al. (2023), we adopt the Family Resemblance Approach (FRA), as reconceptualized in Erduran and Dagher (2014). In the FRA, the *epistemic core of a discipline* consists of *aims and values* (like abstraction, objectivity, consistency, etc.), *practices* (like collecting data, making experiments, proving, etc.), *methods and methodological rules* (like inductive/deductive reasoning), and *knowledge* (like the laws of dynamics, the theorem of mean value, etc.) (ibid.).

### **The desired expertise concerning examples and exemplification**

We claim that interdisciplinary teachers should become able to interact successfully with boundary objects at the boundary between disciplines. Expert interdisciplinary teachers should be able to enact a *critical attitude* by structuring their attention to identify, and critically discuss, critical details in textbooks that are relevant from an interdisciplinary point of view. Consistently, their structure of attention should be localized, and focused, on details that mirror the relationship between disciplines, respect their epistemic cores, reveal the resemblances between disciplines or the specificities with respect to their values and methods (Erduran & Dagher, 2014).

In this contribution, we focus on the critical attitude related to examples and exemplification. Bills et al. (2006) argue that analyzing examples offers “both a practically useful and an important theoretical perspective [...] on the professional

development of mathematics teachers” (p. 126). Watson and Chick (2011) argue that examples may act as *examples-of*, and the learner must be able to recognize variation of dimensions and at the same retain the essential properties. Examples may also act as *examples-for*, promoting the understanding of a new concept by means of generalization. The authors describe the following actions on examples: *analysis* (searching for relation between elements of an example), *generalization* (finding and describing similarities among examples), and *abstraction* (classifying similar examples and identifying them as a concept). We claim that it is important to move from considering the example per se to finding relations between the elements of the example (*analysis*), finding and describing similarities between examples (*generalizing*), classifying similar examples (*abstracting*). Bills and colleagues also refer to *generic example* as “something specific is being offered to represent a general class” (2006, p. 127). In physics education, worked examples are of interest for their use in the processes of problem-solving and self-explanation (Kuo, 2023). Another topic of interest is the notion of *real-world example*, that is used to pursue the goal to show the relevance of physics in everyday life. This aim might become controversial if we consider naïve epistemological positions directly connecting the real-world examples and theoretical laws, not deepening the role of models and experiments (Tasquier et al., 2016). The ambiguity of examples, which can play a role of *examples-for* and *examples-of*, *worked example*, *generic example*, *real-world example* can be relevant for interdisciplinary teaching, in the sense that they might function as (sources of) critical details that attract the attention of the reader. Indeed, they allow to deepen the issue of bridging the concrete and the ideal and conceptual world, that is at the core of interdisciplinarity between mathematics and physics.

### **Research question**

From the perspective of the second transition and of the design of a teacher education activity that aim to smooth the second discontinuity, we aim to investigate what teacher education activities might trigger the enactment of the *critical attitude* in the sense of Viennot and Décamp (2018). In this paper, we start addressing this research issue by analyzing novice pre-service mathematics and physics teachers’ structure of attention emerged during a teaching activity based on secondary physics textbooks analysis. This analysis concerns the attention posed to interdisciplinary aspects related to exemplification. We ask ourselves: *what structure of attention can pre-service teachers have as novice interdisciplinary teachers commenting on examples in secondary school physics textbooks?*

### **METHODS**

The data collection was carried out during a two-hour lesson of a university course in Mathematics Education at the Department of Mathematics of the University of Genoa in the academic year 2022-23. It involved five first-year master students who attended the course, three in presence and two remotely. All of them hold a bachelor’s degree in mathematics that encompasses compulsory courses in physics (about classical mechanics, thermodynamics, electromagnetism, and modern physics), and chose a

curriculum for a master's degree that encompasses courses in Mathematics Education. In this paper, we will refer to them as pre-service teachers since they are following a curriculum focused on teaching, their project is to become (secondary) teachers, and they have little or no previous teaching experience. The author FM was the teacher educator of the course, and the authors LB and LP attended the session and acted as teacher educators for the specific topic. All the authors were present in person. Participants were presented two excerpts from secondary school physics textbooks about the motion of a projectile (Ruffo, 2014, pp. 96–97; Walker, 2010, pp. 79–80). The topic was chosen for its relevance both for the national curriculum and for the co-evolution of mathematics and physics (Branchetti et al., 2022). Walker (2010) was chosen for its epistemological richness (Bagaglioni et al., 2021), while Ruffo (2014) was chosen to push details forward by creating a background and a foreground. Pre-service teachers were asked to read the excerpts and to answer the following questions, first individually, then in small groups: “Is there any aspect that stands out for you? Why? Which one of the texts would you use in your class? Why? Which one of the texts do you feel most comfortable with? Why?”. The choice of presenting secondary textbooks at university can create an opportunity for reflecting on second transition, fostered by these couples of questions: indeed, the first and the third can elicit pre-service teachers' mention of past experiences as students (or teachers, if any), while the second explicitly recalls their (future) teaching practice. After the small groups work, there was a discussion led by the three authors. Later, participants could make comments relying on the construct of Habermas' rationality (Habermas, 2003; for more details, see Branchetti et al., 2023) and on a third extract from Amaldi (2011, p. 298). The whole collective discussion was video recorded and transcribed. Pre-service teachers were pseudonymized. We carried out a qualitative analysis by coding the transcript with the above operationalization of the structure of attention (namely, locus, form – diffused or focused –, and focus). We coded separately and then confronted our analyses to reach a mutual agreement. We selected all the excerpts where pre-service teachers explicitly mention examples. The extracts from the textbooks are provided as supplementary materials.

## ANALYSIS OF SELECTED EXCERPTS

As a premise, we briefly sketch some relevant aspects of the desired expertise concerning examples and exemplification in relation to the chosen excerpts. Text1, Text2 and Text 3 stand respectively for Walker (2010, p. 79–80), Ruffo (2014, p. 96–97) and Amaldi (2011, p. 298). The term “example” appears explicitly five times: once in both Text2 and Text3, and three times in Text1. In Text2 it stands for a *worked example*, showing a substitution of quantities in the formulas, while the others can be framed as *generic examples* (Bills et al., 2006) with respect to the sentence preceding the one containing “example”. From an expert perspective, we note an epistemological ambiguity (are they *example-of*, *example-for*, *worked example*, *generic examples*?) that could foster reflections on crucial interdisciplinary issues such as the relation between observations, experiments and measures, and different levels of modeling (also in

relation to provided representations). We now analyze a selection of excerpts from the discussion that contain pre-service teachers' references to examples revealing their structure of attention. First, we observe that what they refer to as "example" includes what textbooks present as such, but they used this term also to refer to physical phenomena or models (e.g., the motion of a ball falling from a table). They did not provide any explicit characterization of their meaning of the term "example".

### **One case and *the case*: the place of examples**

Amelia: The second page [*of Text1*] [...] immediately brings an example [...] in our opinion a little bit thrown in there, while [...], when it [*Text2*] starts with the case of horizontal velocity, first it explains it generically, and then with... anyway the case of the ball but keeping, let's say, the letters... and then, right at the last, it says okay let's substitute two numbers, gives the little, tiny example and shows two things, however it stands very much on the generic [...]. We [*Alice and herself*] like it anyway: instead of dwelling so much on one case, as in the first book, here it talks just about the case of horizontal velocity.

The locus of Amelia's attention is the example of the turtle and of the ball. Her form of attention is focused on the comparison between these two examples, contained in the two textbooks (Text1 and Text2, respectively). She observes that both are shown at the beginning ("*immediately*", "*starts*") and she reflects on their nature: the example of the turtle is perceived as very specific, while the example of the ball is appreciated for being more general ("*keeping... the letters*"). Only at the end Text2 "*gives the tiny example*", referring to the worked example where it considers and substitutes two numbers. Amelia criticizes the initial example of the turtle, for it does not fit in, it has numbers, and it comes first. For Amelia, numbers are discriminant between Text2's "*little, tiny example*", *example-of* (Watson & Chick, 2011), coming "*at the last*", and Text2's initial example of the ball, "*the case*", *example-for* (Watson & Chick, 2011), coming first. Amelia appreciates the approach of Text2 which seems to be frameable as *generic example* (Bills et al., 2006).

### **The difference in physicality of examples**

Olivia: It [*Text3*] presents three examples of... always of the motion of projectiles, but it presents a different physicality [...] one is a motion that is vertic... that I see it going up and down [...] they are all motion of projectiles, but they are a little different... I don't know how to say, how to explain myself better: it is true that the model to be used for all is that of the motion of projectiles and the equations are the same, but they may seem to be three different motions, and they are indeed in the reality! [...]

LB: I was interested in the sense of physicality that you mentioned earlier [...] You see it physical, don't you? That kind of incipit gives you this sense.

Olivia: Yes... In the sense of reality [...] recalling something you can do, I mean, I can pick up a ball and decide to throw it, in that sense.

- LB: And there is an evaluation of what is the phenomenon before systematizing with trajectory, vectors, etc. [...]
- Olivia: Then whatever, I would put a blowgun in the hands of the students in the classroom.

The locus of Olivia's attention is the three *examples-of* from Text3. Her form of attention is focused on the comparison between these examples of the motion of projectiles. On the one hand, she points out their mutual diversity: "*they may seem to be three different motions and they indeed are*", they have a "*different physicality*", "*in the sense of reality*". This can be explained as a possible (inter)action, "I can *pick up a ball* and decide *to throw it*", and as a different look, "*one [...]* I see *it going up and down*". On the other hand, Olivia recognizes that one similarity is that "*the model to be used for all is that of the motion of projectiles*". This sentence can be further explored for reflecting on the interesting relation between the physics phenomenon and a model, and the role of contextual factors. Finally, it is remarkable her mention of how she would act in classroom putting "*a blowgun in the hands of the students*".

### **Examples in physics, and the mathematization of examples**

- Albert: Deciding to start with an example, at least in physics, can be in my opinion a winning strategy, which you must distinguish it from mathematics by the fact that in physics... what is explained and illustrated has been proved by practical experience. So, I am pro-examples at the level of exposition [...].
- Alice: But the second text also starts with a real situation like the ball falling off the table, I find very similar such example to the ones in the third book [...]
- LB: Do you see any similarity and difference between the two types of examples [*in Text2 and Text3*] [...]?
- Olivia: To me it seems different because these ones here [*Text1 and Text2*] [...] are very mathematized already, they are not real, they have been modeled... while instead those of the Text3's [...] keep having [...] a much more physical dimension, not in the sense of matter [...], much more real than the former, although it is true that they are all examples.
- LB: That is, you're saying that they're not already [...] neither modeled nor even described by perhaps a language of the discipline, not even of physics. [...]
- Emily: Even here [*Text2*] it is said "it [*the ball*] leaves it [*the table*] with a horizontal velocity" and not "it falls", that is closer to one could say. It is already being framed in a more... mathematical context.

Albert's first sentence is not localized in a specific element of textbooks but rather is on the general habit of starting with an example in physics. Here it emerges an epistemic and epistemological position of the role of "*practical experience*" in validating knowledge. Nevertheless, his appreciation of the strategy is on a communicative level, rather than an epistemological one. Afterwards, Alice localizes

her attention on the beginning of the second text, with a focus on the similarity between Text2 and Text3 in the use of a real situation. The locus of Olivia's attention is the example from all the three texts. Her form of attention is focused on their comparison, and she stresses that in Text1 and Text2 examples have been "*very mathematized*", so partially losing a "*physical dimension*" and reality. After the intervention of LB, Emily strengthens Olivia's comment by localizing her attention on a sentence of Text2, "*it leaves it with a horizontal velocity*". Her focus is on the comparison between this expression and the alternative, "*it falls*", which is not already in a mathematical context but rather is closer to the natural language.

### **Examples in mathematics, in physics and past experiences**

Albert: In mathematics textbooks you hardly ever see examples, also because finding in mathematics examples related to everyday life maybe is even a little bit more difficult? But for domain like probability [...] it is easier to introduce the chapter with an example. Physics, which is based on empirical laws, in my opinion requires this kind of approach. Otherwise, it could be interpreted by students as the bad relative of mathematics [...] I was lucky because I liked physics, in high school they introduced it to me using Amaldi [...] I am more satisfied what I did in physics than what I did in mathematics. But there it's the fault of the school reforms... and not professor's fault.

Albert's sentence is not localized in a specific element of textbooks but rather is on the general occurrence of examples in mathematical textbooks. His focus is again on how "*to introduce the chapter*", assuming a didactical perspective. First, he compares mathematics domains, mentioning probability as an easier domain to find everyday-life examples. Then, he compares physics and mathematics with a focus on making students not interpreting physics as "*the bad relative of mathematics*". Again, it emerges the epistemological position of physics as empirical. Albert mentions his experience of learning physics and mathematics at school (but not at university), and the impact of school reforms.

### **DISCUSSION AND CONCLUSIONS**

We addressed the following question: *what structure of attention can pre-service mathematics teachers have as novice interdisciplinary teachers commenting on examples in secondary school physics textbooks?* Concerning the *foci* of their attention, several lines of reflection emerged: the relation between examples and generality (the presence of numbers, the place of examples), examples in mathematics and in physics, the "mathematization" and the "physicality". These aspects could be further developed to promote the enactment of a critical attitude. We observed that pre-service teachers, although not providing any explicit characterization of the term "example", labelled as "examples" the situations presented in the excerpts. We may note that in literature on physics education the term "example" is much little frequent, apart for referring to worked example. The frequent use of the label "example" by pre-service teachers can be related to the phenomenon of "disciplinary capture" (Frodeman et al., 2012), which

means not considering disciplines other than that of one own's. Concerning the *loci*, we observed that detaching from textbooks here occurs with stereotypical conceptions of physics as empirical and mathematics as less related to everyday life. Moreover, pre-service teachers do not mention university knowledge. All this suggests a reflection on the second transition with respect to interdisciplinarity between mathematics and physics. During their university instruction, pre-service teachers attended courses both in physics and in mathematics. Nevertheless, they seem not able to reflect and relate the epistemologies of the two disciplines. The case of examples paves the way for reflecting on university teaching in a second transition perspective: can we say that pre-service did not acquire an “academic knowledge” of physics and of relating mathematics and physics epistemologies during their university studies? Or should we say they acquired it but were not able to use it when commenting on physics textbooks? The present study suggests that university courses about physics contents alone did not promote pre-service teachers awareness of interdisciplinary issues. The answers to the previous questions are worth exploring in future research to inform a design of teacher education activities.

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