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ON THE BETATRON RADIATION IN CYLINDRICALLY SYMMETRIC PLASMA-ION CHANNELS

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Abstract

The relativistic interaction of short pulsed lasers or electrons with plasma has recently led to the birth of a new generation of femtosecond X-ray sources. Radiations with properties similar to those that can be observed from a wiggler or undulator, can be generated by the oscillations induced in the exited plasma by electrons (PWFA) or by lasers (LWFA), making plasma an interesting medium both for the acceleration as well as for the radiation source, with properties of being compact, providing collimated, incoherent, femtosecond radiation. Thus a lot of effort is being made to understand and improve this new source to make it really competitive, This paper summarizes and shows some theoretical results and numerical simulations of a simplified model called plasma ion column, using as a starting point the parameters expected for the EuPRAXIA@SPARC_LAB facility, highlighting strengths, limitations and scaling laws, which allow for a comparison with other types of more consolidated sources of light as Compton, Synchrotron and Free Electron Lasers (FEL).

INTRODUCTION

X-ray radiation has been one of the most effective tools for exploring the properties of matter in a wide range of scientific research since its discovery over a century ago [1]. Over time, successive generations of radiation sources have been developed, producing radiation with increasing brightness, shorter wavelengths, and shorter pulse durations. Despite significant progress in x-ray generation methods, there is still a need for light sources that can deliver femtosecond pulses of bright, high-energy x-ray and gamma-ray radiation from compact sources, and this request is due to the innumerable applications in fundamental science, industry, and medicine. One of the promising answers to this request comes from plasma-based accelerators. During the acceleration process in plasma, electrons encounter a strong focusing force proportional to their radial displacement, thus electrons with an offset from the central axis of the accelerator, or with some initial transverse momentum, will thus oscillate around this axis as they propagate. The resulting motion leads to the emission, referred to as betatron radiation, similar to that which occurs in the classic insertion device but with one important difference: the compactness. Wigglers and undulators emitted radiation wavelength λ_X depends on the period of the magnetic field λ_B of the undulator (or wiggler) as $\lambda_X = \lambda_B/(2\gamma^2)(1+K)$, where K is the classical strength parameter, typically, λ_B is in the millimeter range. To achieve X-ray with wavelength $\lambda_X = 1$ nm, the electron energy must scale to as high as $\gamma \sim 10^3$ and this is still very challenging, otherwise the radiation wavelength in betatron radiation is determined by the plasma wavelength through $\lambda_X = \lambda_p/(2\gamma^2)$. For a typical value of the plasma density employed in LWFA, $n_p = 1 \times 10^{19}/\mathrm{cm}^3$, correspondingto a plasma wavelength of $\lambda_p \sim 10$ µm. In this case, to reach the X-ray domain of $\lambda_X = 1$nm, the electron energy could be as low as $\gamma \sim 100$. Such an electron energy is readily available from contemporary LWFAs. In first approximation, this form of oscillation may exhibit similarities to what happens in a wiggler or undulator, but we will discuss that there are important differences between radiation due to classical insertion devices (wigglers or undulators) and that due to the plasma.

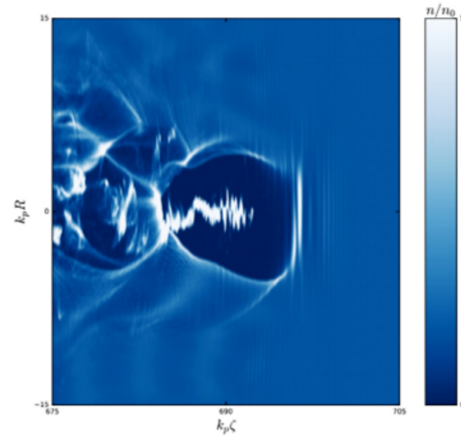

Figure 1: Results of PIC simulation showing electron bunch oscillation in blow-out regime.

BETATRON RADIATION

We will consider the betatron radiation of an ultrarelativistic electron that propagates in a plasma column, namely ion channel [2]. The plasma column is a cylindrical region free of electron, which is a good approximation of the bub-

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ble regime [3]. In much the same way as the accelerating forces in a plasma accelerator are greater than those in a conventional accelerator, so are the focussing forces also many orders of magnitude greater, these transverse, rerstoring forces (Eq. (1)) are created by the radial displacement and longitudinal motion of the plasma wave electrons.

$$F_{res} = -m\omega_p^2 r/2 \tag{1}$$

Where $\omega_p = \sqrt{e^2 n_e/m_e \epsilon_0}$ is the plasma frequency. In these fields, the electrons wiggle with a betatron frequency $\omega_\beta = \omega_p/\sqrt{2\gamma}$ and wavelength given by $\lambda_\beta = \sqrt{2\gamma}\lambda_p$.4

Electron Dynamics

Let's consider as we said before, the Ion Plasma Channel model, modelling the bubble regime, which we use to study the electron dynamics. In this region, we can assimilate the motion of electrons to that which occurs in a traditional wiggler. Assuming the electron orbit lies in the (x, z) plane, the orbit is given by

$$x = r\sin(k_\beta ct) \tag{2}$$

$$z = z_0 + \beta_z \left(1 - \frac{r^2 k_\beta^2}{4\beta_z^2}\right) ct - \frac{r^2 k_\beta^2}{8\beta_z^2}\cos(2k_\beta ct) \tag{3}$$

where r is the amplitude (assumed constant) of the betatron orbit, k_β is the betatron wavenumber [4] . The resulting motion is presented in a Fig. 1 where the plasma bubble formed by the laser pulse is shown, with oscillating electrons inside.

Betatron Strength Parameter

A fundamental parameters has been introduced, $K_\beta = \gamma r k_\beta$, defined as the betatron strength parameter in analogy with magnetic insertion devices strenght parameter. In theory, this parameter distinguishes two working regimes: undulator-like if $K_\beta << 1$, and wiggler-like if $K_\beta > 1$. However, this net differentiation is a strong approximation, as K_β is assumed to be constant, when actually it has several factors that vary. So at the end we will have different mechanisms that do not allow for a peaked spectral emission, but will contribute to a broadening of the spectrum. Firstly, it is important to consider that electrons in the blowout regime are accelerating, and therefore the Lorentz factor $\gamma(t)$ varies over time and influence $K_\beta(t) = \gamma(t) r_\beta(t) k_\beta(t)$. Furthermore, K_β also depends on the trajectory amplitude $r(t)$, which varies for each electron as the plasma focusing force is different for each particle. Although there may still be electrons undergoing small oscillations near the axis, resulting in line emissions, the number of photons emitted will be limited, as in the undulator regime the number of emitted photons N_{ph} are proportional to K_β^2, instead electrons with greater oscillation amplitude will contribute more $N_{ph} \sim K_\beta$. Therefore, we can conclude that there is no clear distinction between the two emission regimes, and the time dependence contributes to an inhomogeneous broadening of the spectral lines, leading the emission to be mostly wiggler like.

Radiation in a Plasma Column

By using the Lienard-Wiechert potentials [5], it is possible to calculate the energy spectrum of the radiation emitted by a single electron on any orbit $r(t)$ and $\beta(t)$.

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-T/2}^{T/2} n \times (n \times \beta) e^{i\omega(t - n\cdot r/c)}\, dt\right|^2 \tag{4}$$

The quantity $d^2I/d\omega d\Omega$ represents the energy radiated per frequency ω and per solid angle Ω during the interaction time T, while n is a unit vector pointing the direction of observation, θ is the observation angle with respect to the electron propagation axis and ϕ is the azimuthal observation angle, which was set to zero.

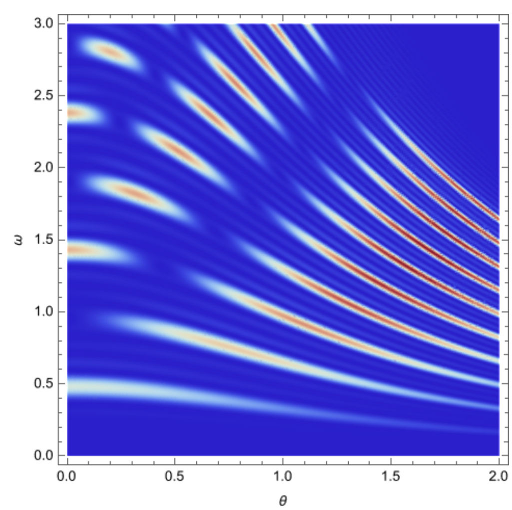

Figure 2: Normalized spectrum $d^2I/d\omega d\Omega$, versus normalized frequency ω and angle θ from single electron with $K_\beta = 2$. We can clearly see the odd harmonics on-axis $\theta = 0$, while off-axis even harmonics appear.

On-axis Radiation

Of particular interest is the radiation emitted along the axis, where only the odd harmonics are finite, i.e., the even harmonics vanish. Fig. 2 displays several spectra calculated with the above equations for EUPRAXIA SPARC_LAB expected parameter [6], where the linac will be capable of providing energy up to 600 MeV in a a capillary of 0.6 m and plasma density of $n_p = 9 \times 10^{16}\text{cm}^{-3}$. The electrons will therefore undergo oscillations with the amplitude strongly depends on α_n (Eq. (8)) which is further dominated by K_β. So we will examine how the radiation spectrum evolves with it. In the case of Fig. 3a, a small oscillation amplitude $r = 0.1\mu\text{m}$ is used, corresponding to $K_\beta = 0.13$. As seen, the spectrum is very pure with only the fundamental harmonic . The frequency width is determined through Eq. (9) to be $\Delta\omega/\omega_{n=1} = 1/N_\beta = 1/10$. This regime with $K_\beta << 1$

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is known as undulator regime. When K_β becomes of the order of unity for a larger oscillation amplitude $r = 0.8$ µm, higher order harmonics clearly appear, as shown in Figure 3b, where 4 harmonics are appreciable. With an even larger radius r= 1.5 µm in Fig. 3c, high order harmonics become stronger and numerous, and the harmonic amplitudes follow a synchrotron-like distribution, in this case, many harmonics are excited, closely spaced, and unresolved. Their envelope can be well described by synchrotron radiation. The most striking feature is that photon energies extend to as high as tens of KeV. This regime of $K_\beta >> 1$ is known as the wiggler regime [7]. Utilizing the analytical solution derived from Esarey [4], we can generate a spectral-angular distribution on axis $\theta = 0$ (Fig. 3) starting from Eq. (5).

$$\frac{d^2I}{d\omega\Omega}(\theta = 0) = \sum_{n=1}^{\infty} \frac{e^2}{\pi\epsilon_0 c}\frac{\omega}{\omega_n}\frac{\gamma^2 N_\beta^2 F_n R_n}{1 + K_\beta^2/2}, \qquad (5)$$

where n is odd, because the even harmonics vanish, ω_n is the nth harmonic frequency of radiation, given by

$$\omega_n = 2n\omega_\beta \frac{\gamma^2}{1 + K_\beta^2} \qquad (6)$$

and N_β is the number of oscillation periods that electron undergoes. F_n indicates the the nth harmonic amplitude

$$F_n = n\alpha_n[J_{(n-1)/2}(\alpha_n) - J_{(n+1)/2}(\alpha_n)]^2, \qquad (7)$$

where J_n is the first kind Bessel function of the nth order, and

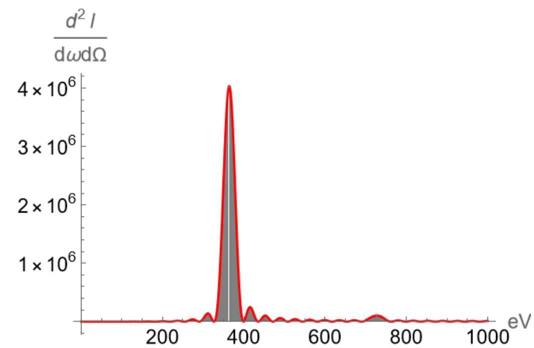
$$\alpha_n = \frac{n\left(\frac{\omega}{\omega_n}\right)K_\beta^2}{4\left(1 + K_\beta^2/2\right)} \qquad (8)$$

R_n is the spectrum function

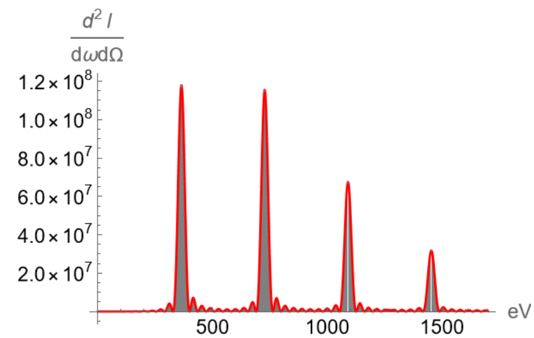
$$R_n = \frac{sin^2[n\pi N_\beta(\omega/\omega_n - 1)]}{[n\pi N_\beta(\omega/\omega_n - 1)]^2} \qquad (9)$$

CONCLUSION

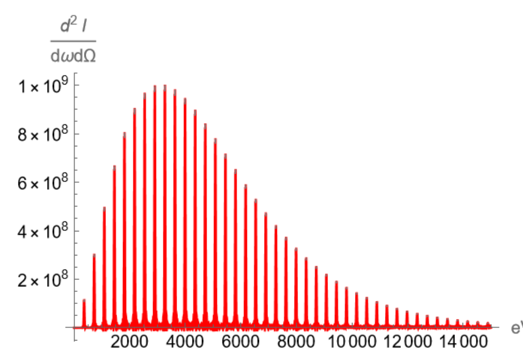
This paper briefly summarizes the fundamental aspects of betatron radiation, explores its parallels with radiation derived from undulators and wigglers, and emphasizes that these analogies result from several strong approximations made when considering particles with identical strength parameters. Additionally, preliminary studies on this radiation were presented based on the parameters predicted for EuPRAXIA@SPARC_LAB.



(a) *Single line emission as result of* $K_\beta << 1$, $\gamma = 1174$, $r = 0.1$ µm.



(b) *Multiple line emission as result of* $K_\beta \sim 1$, $\gamma = 1174$, $r = 0.8$ µm.



(c) *Synchrotron-like emission as result of* $K_\beta > 1$, $\gamma = 1174$, $r = 1.5$ µm.

Figure 3: Betatron radiation spectra calculated with density $n_p = 9 \times 10^{16}\text{cm}^{-3}$. Three cases are presented, $K_\beta << 1$, $K_\beta \sim 1$, $K_\beta > 1$.

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