

MAESTRO AND HIS PUPILS: HISTORY OF A SCIENTIFIC PRODUCTION

ALBERTO M. BERSANI, ANTONIO CAZZANI, IVAN GIORGIO, AND MARIO SPAGNUOLO

Francesco dell'Isola trained generations of scholars in the many subsets of Continuum Mechanics in which he himself fine-tuned his knowledge. In his scientific production, one can recognize various periods on the basis of the main interest emerging from his publications of the period in question.

THE EARLY PRODUCTION

The first problems that dell'Isola tackled date back to the late 1980s, when he pursued a master's degree in Physics under the mentorship of Antonio Romano. With Romano he studies interface and phase transition problems (1; 2).

This work formed the basis for the subsequent development of studies, carried out independently or together with his early colleagues, still concerning interface (3) or phase transition problems (4) in fluids. It is interesting to note how the young dell'Isola approached the main theme of his scientific production even today, the materials described by means of second gradient theories, through the study of fluids (5), finding himself at the very roots of second gradient theories, which took their origin from Pierre Casal's studies on capillarity (6).

These initial studies are mostly of a theoretical nature, but soon dell'Isola realizes how important it is to have practical feedback and it is to this need that the early work with Ugo Andreus and others including numerics can be ascribed (7; 8).

Francesco dell'Isola finds in his fellow student from university days, Luigi Rosa, the closest collaborator of this period. With Rosa, he approaches the study of Saint-Venant cylinders (9; 10). He soon also extends his collaboration to other researchers, Giuseppe Ruta (Roma) and Romesh Batra (Rolla, Missouri), with whom he deals with topics still related to the Saint-Venant cylinders, but with peculiar characteristics (11; 12).

During this period, dell'Isola tackles various lines of research in parallel: with Rosa, Ruta and Batra Saint-Venant cylinders again, where he also introduces aspects of piezoelectricity (16; 13; 14; 15); with Czesław Woźniak (Warsaw) composite and microstructured materials, for which he refers to his early studies on interfaces (17; 18; 19; 20); with Kolumban Hutter (Darmstadt) application studies for different soil types (21; 22; 23; 24; 25). In addition, it is also during this same period that he begins his collaboration with Pierre Seppecher (Toulon) and the first germs of studies on the second gradient, contained in their work on edge contact forces (26).

THE "PIEZOELECTRIC YEARS"

During these years, dell'Isola, who in the meantime has obtained a Ph.D. in Mathematical Physics (1992) and the qualification for Maitre de Conférences (1995), supervises the research of several Ph.D. students: Stefano Vidoli, Giulio Sciarra, Maurizio Porfiri and Corrado Maurini.

The core of the research conducted by dell'Isola and his Ph.D. students during this period concerns piezoelectric materials. Returning to the study of the Saint-Venant cylinder, he addresses the problem of a piezoelectric circular bar with Vidoli (27).

In addition to some works of various kinds with some colleagues, as his fellow student Ramiro Dell'Erba, (28; 29), in this period, as mentioned, dell'Isola focuses mostly on the study of piezoelectric materials and their use in problems of relevance in mechanics. With Vidoli he faces one-dimensional problems (35; 30; 31; 32; 33; 34); he also studies piezoelectro-mechanical control system for bidimensional problems (36; 38; 37); with Porfiri, on the other hand, the attention is placed on the problems of circuit synthesis and on the study of analogous circuits for Mechanics (39; 40; 41; 48; 47; 42; 43; 44; 45; 49; 46); finally, with Maurini different problems related to piezoelectro-mechanical systems are studied (50; 51; 52; 54; 53).

An interesting aspect of dell'Isola's scientific production consists in the fact that periodically the lines of research that he conducts intersect and influence each other: it is remarkable that, parallel to the study of the problem of the synthesis of circuits with Porfiri, dell'Isola approaches with Pierre Seppecher and Jean-Jaques Alibert (Toulon) the problem of synthesis in the mechanical field (55). The result of this collaboration is a work that still today opens new horizons in the synthesis of metamaterials.

During this period, dell'Isola also strengthens his familiarity with Gerard Maugin, from whom he was invited to Jussieu (Paris) in 1994 and through whom he met Paul Germain (see the Preface). As a result of his connection with Maugin and his group, in addition to the co-direction of Maurini, a Ph.D. conducted with Joel Pouget, there are some works in the field of poroelasticity (56; 57). It is from these last works that the studies with Sciarra on poroelasticity develop (58; 59; 60; 61).

After a brief revival of the Saint-Venant problem (62), the piezoelectric years ideally conclude with three works concerning vibration control (63) and suppression (64) and damage detection (65).

THE "VARIATIONAL" AND "SECOND GRADIENT" YEARS

As it is clear from scrolling through the list of his works, Francesco dell'Isola has always been a "variational man". Descending ideally from the school of Lagrange, as we mentioned in the Preface, dell'Isola could be nothing but variational. So the title of this section must be put into context: by the *variational years* we mean that during this period dell'Isola explicitly adopts the variational approach to the study of the problems he faces. This period logically precedes and intersects the development of the main theme of his research activity: second gradient theories.

During this period, a second generation of doctoral students replaces the first one, now autonomous. These new Ph.D. students are: Luca Placidi, Angela Madeo and Giuseppe Rosi. Like many of dell'Isola's pupils, Madeo and Rosi, once they have obtained their Ph.D., will also be employed in French universities.

The variational period is strictly linked to the previous one. In fact, as we have mentioned, with Sciarra dell'Isola has started a study on poromechanics. This study, to whom also Olivier Coussy (Paris) collaborates, finally produces some results, through variational approach, in the second gradient theory field (66; 69; 68; 67; 70; 71).

During this period one of the current main dell'Isola's collaborators joins the group: he is Ivan Giorgio (Roma), and he is involved in the studies about vibration control, as Rosi (73; 72).

The variational approach can be fruitfully used also in biological field. This is the case of bone remodeling. In these years dell'Isola starts a deep collaboration with Tomasz Lekszycki, who is responsible for a biomechanical laboratory in Warsaw University. With Lekszycki and Madeo first, Giorgio later, dell'Isola studies bone mechanics (74; 75; 76; 77; 78).

With Placidi and Madeo, dell'Isola explores different properties of higher gradient continua, both from a practical point of view (through numerics) (79; 80; 81), and from a theoretical point of view, also in collaboration with Ali Javili (Ankara), Paul Steinmann (Erlangen), Felix Darve (Grenoble), Nicholas Auffray (Paris), Raffaele Esposito (Roma) and Mario Pulvirenti (Roma) (89; 90; 91; 82; 83; 84; 85; 86; 87; 88). We want to stress that Ref. (84) represents the first collaboration with Victor A. Eremeyev (Gdansk-Cagliari), who will become one of the principal scholars in dell'Isola's group.

A fundamental aspect in Francesco dell'Isola's personality is represented by his attitude to the search for the sources of mathematical and, in general, scientific theories. An important example is represented by Peridynamics, which only recently has become of main interest, although its germinal ideas can be found in Gabrio Piola's works. This has been stressed in Ref. (92). We cite the words of the authors (dell'Isola, Andreaus and Placidi), who well explain the problem of approaching history of Science:

The authors question the concept of a 'historical method' especially when applied to the history of science and history of mathematics. We claim that there is not any peculiar 'historical method' to be distinguished from the generic 'scientific method' which has to be applied to describe any other kind of phenomenon, although the subject of the investigation is as complex as those involved in the transmission, storage and advancement of scientific knowledge.

The interest for Piola's work has a very important influence on dell'Isola subsequent investigation. The study of edge problems using variational approaches had previously led dell'Isola to the development of second gradient theories (26). At this point in his academic career, he enters into the fullness of his scientific maturity and is able to approach this topic in a comprehensive manner (93). A fundamental inspiration comes from reading the works of Gabrio Piola, which dell'Isola, assisted by a number of scientists, translates, in two volumes, completely into English for the first time (94; 95; 96; 97; 98; 99; 100; 101).

THE "PANTOGRAPHIC" YEARS

In this period, the whole knowledge produced along the past years is employed for producing a practical realization of a second gradient material. In fact, one of the more frequent criticisms that dell'Isola receives when presents his ideas about second gradient materials at conferences and workshops is: "it's a good mathematical toy, but no practical applications are available because no second gradient material exists". His main purpose is then to realize a practical example of second gradient material. To this aim dell'Isola refers back to his paper with Alibert and Seppecher (55) and introduces the pantographic (micro)structure, which allows the realization, after a proper homogenization procedure, of the pantographic metamaterial. The first attempt to this purpose can be found in Ref. (102). In this paper numerical simulation are provided and compared to experiments. After it, many other works study the pantographic metamaterial and, more in general, fibrous second gradient metamaterials.

These years are characterized by many collaboration in experimental fields: again with Tomasz Lekszycki (Warsaw), Georg Gatzmüller (Freiburg) and Patrice Peyre (Paris), who

offer the possibility to print samples via 3D printer in their laboratories using different materials, polymers and metals. This will be crucial for the subsequent developments. Also from the theoretical and numerical point of view, one can recognize different collaborations: David Steigmann (Berkeley), whose expertise in differential geometry helps in generalizing the bidimensional model for pantographic structure to 3D space (103; 104; 105; 106; 107); Victor Eremeyev (Gdansk) (108; 109; 110), Claude Boutin (Lion) (111), Pierre Seppecher (Toulon) (88), who analyze pure mathematical aspects at the foundations of pantographic metamaterial; Massimo Cuomo and Leopoldo Greco (Catania), whose *know-how* in numerics is crucial for calibrating the models (112; 113; 114; 115); Anil Misra (Lawrence, Kansas), who studies exotic phenomena emerging in this metamaterial (116; 117); Ugo Andreaus (Roma) and Nicola Rizzi (Roma), who introduce the pantographic metamaterial into their Engineering background (118); Antonio Cazzani (Cagliari) and Emilio Turco (Sassari), who propose different numerical approaches to discrete models (119; 120; 121); Wolfgang Müller (Berlin) (122), Jean-François Ganghoffer (Nancy) (123), Philippe Boisse (Lion) (124; 125; 126), who study different aspects relevant for possible generalizations of pantographic structure; Ivan Giorgio, who deals with different aspects of the pantographic substructure, such as dynamics, identification of constitutive parameters for second gradient models, three-dimensional deformations (127; 128; 129; 130); François Hild (Paris), who introduces the powerful tools of Digital Image Correlation in the analysis of metamaterials (131).

An already remarked aspect of dell’Isola’s research style is represented by the interest in the source of scientific theories. Also the *pantographic years* are characterized by this style: with Simon Eugster (Stuttgart) an “exegesis” on the Hellinger’s work is approached and presented in Refs. (132; 133; 134); other works in the field of History of Science are (135; 136; 137).

This very large amount of collaborations fully emerges in two papers published in 2019 (138; 139): on the one hand, in Ref. (138) all the obtained results on pantographic metamaterial are recalled within the main contributions given by a large part of “old” dell’Isola’s collaborations; Ref. (139), on the other hand, presents the first results of the new lines of research on the field of this metamaterial. Most part of the themes referred in this last paper will be the main topics of the subsequent years and still constitute a map for future developments.

The mentioned collaborations make a trend topic the pantographic metamaterial: a huge amount of publications concerning this topic and resulting from such joint efforts can be found in the literature (140; 141; 142; 144; 145; 143; 146; 147; 148; 149; 150; 151; 152)

The third generation of Ph.D. students is composed by: Alessandro Della Corte, Antonio Battista, Mario Spagnuolo, Emilio Barchiesi and Michele De Angelo.

With Alessandro Della Corte the main contributions concern the nonlinear beam theory (153; 154; 155). Antonio Battista is guided to the study of swarm systems in describing Continuum Mechanical problems (156; 158; 157). With Mario Spagnuolo (159; 160), Emilio Barchiesi (162; 161) and Michele De Angelo (163) the main efforts are in the field of pantographic metamaterial.

CURRENT TIMES: DEEP INVESTIGATIONS IN GENERALIZED CONTINUA

Francesco dell’Isola is currently engaged in the study of the many aspects related to Generalized Continua. Specifically, with a large number of the collaborators mentioned in the previous sections, he is working on:

- i. pantographic metamaterial (164; 165; 166; 167; 168; 169; 170)

- ii. micromorphic materials (171; 172; 173; 174; 175; 176; 177)
- iii. granular materials and metamaterials based on this concept (178; 183; 180; 181; 182; 179)
- iv. fiber-reinforced materials (185; 186; 184)
- v. Digital Image Correlation applied to the study of metamaterials (187; 188; 189)
- vi. Dissipation in Continuum Mechanics (190)
- vii. Piezoelectric materials (a revival of) (191)
- viii. History of Science Investigations (a revival of) (192; 193; 194)

REFERENCES

- [1] F. dell’Isola and A. Romano. On the derivation of thermomechanical balance equations for continuous systems with a nonmaterial interface. *International Journal of Engineering Science*, 25(11–12):1459–1468, 1987.
- [2] F. dell’Isola and A. Romano. A phenomenological approach to phase transition in classical field theory. *International Journal of Engineering Science*, 25(11–12):1469–1475, 1987.
- [3] F. dell’Isola. Linear growth of a liquid droplet divided from its vapour by a “soap bubble”-like fluid interface. *International Journal of Engineering Science*, 27(9):1053–1067, 1989.
- [4] F. dell’Isola and D. Iannece. On phase transition in classical fluid mixtures with surface adsorption. *International Journal of Engineering Science*, 27(9):1069–1078, 1989.
- [5] F. dell’Isola and G. Rotoli. Validity of Laplace formula and dependence of surface tension on curvature in second gradient fluids. *Mechanics Research Communications*, 22(5):485–490, 1995.
- [6] P. Casal. Theory of second gradient and capillarity. *Comptes Rendus Hebdomadaires des Seances de l’Academie des Sciences Serie A*, 274(22):1571, 1961.
- [7] U. Andreaus and F. dell’Isola. On thermokinematic analysis of pipe shaping in cast ingots: A numerical simulation via FDM. *International Journal of Engineering Science*, 34(12):1349–1367, 1996.
- [8] F. dell’Isola, H. Gouin, and G. Rotoli. Nucleation of spherical shell-like interfaces by second gradient theory: Numerical simulations. *European Journal of Mechanics, B/Fluids*, 15(4):545–568, 1996.
- [9] F. dell’Isola and L. Rosa. An extension of Kelvin and Bredt formulas. *Mathematics and Mechanics of Solids*, 1(2):243–250, 1996.
- [10] F. dell’Isola and L. Rosa. Perturbation methods in torsion of thin hollow Saint-Venant cylinders. *Mechanics Research Communications*, 23(2):145–150, 1996.
- [11] F. dell’Isola and G.C. Ruta. Perturbation series for shear stress in flexure of Saint-Venant cylinders with Bredt-like sections. *Mechanics Research Communications*, 23(5):557–564, 1996.
- [12] F. dell’Isola and R.C. Batra. Saint-Venant’s problem for porous linear elastic materials. *Journal of Elasticity*, 47(1):73–81, 1997.
- [13] F. dell’Isola and L. Rosa. Almansi-type boundary conditions for electric potential inducing flexure in linear piezoelectric beams. *Continuum Mechanics and Thermodynamics*, 9(2):115–125, 1997.
- [14] F. dell’Isola and G.C. Ruta. Generalizing Jouravski formulas by techniques from differential geometry. *Mathematics and Mechanics of Solids*, 2(3):307–319, 1997.

- [15] F. dell'Isola, G.C. Ruta, and R.C. Batra. Second-order solution of Saint-Venant's problem for an elastic pretwisted bar using Signorini's perturbation method. *Journal of Elasticity*, 49(2):113–127, 1997.
- [16] F. dell'Isola and L. Rosa. St. Venant problem in linear piezoelectricity. *Proceedings of SPIE - The International Society for Optical Engineering*, 2715:399–409, 1996.
- [17] F. dell'Isola, L. Rosa, and C. Woźniak. Dynamics of solids with micro periodic nonconnected fluid inclusions. *Archive of Applied Mechanics*, 67(4):215–228, 1997.
- [18] F. dell'Isola and C. Woźniak. On continuum modelling the interphase layers in certain two-phase elastic solids. *ZAMM Zeitschrift fur Angewandte Mathematik und Mechanik*, 77(7):519–526, 1997.
- [19] F. dell'Isola and C. Woźniak. On phase transition layers in certain micro-damaged two-phase solids. *International Journal of Fracture*, 83(2):175–189, 1997.
- [20] F. dell'Isola, L. Rosa, and C. Woźniak. A micro-structured continuum modelling compacting fluid-saturated grounds: The effects of pore-size scale parameter. *Acta Mechanica*, 127(1–4):165–182, 1998.
- [21] F. dell'Isola and K. Hutter. Continuum mechanical modelling of the dissipative processes in the sediment-water layer below glaciers [Modélisation en mécanique des milieux continus des phénomènes de dissipation dans la couche sédimentaire saturée d'eau au-dessous des glaciers]. *Comptes Rendus de l'Academie de Sciences - Serie IIB: Mecanique, Physique, Chimie, Astronomie*, 325(8):449–456, 1997.
- [22] F. dell'Isola and K. Hutter. A qualitative analysis of the dynamics of a sheared and pressurized layer of saturated soil. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 454(1980):3105–3120, 1998.
- [23] F. dell'Isola and K. Hutter. What are the dominant thermomechanical processes in the basal sediment layer of large ice sheets? *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 454(1972):1169–1195, 1998.
- [24] F. dell'Isola and K. Hutter. Variations of porosity in a sheared pressurized layer of saturated soil induced by vertical drainage of water. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 455(1988):2841–2860, 1999.
- [25] F. dell'Isola, M. Guarascio, and K. Hutter. A variational approach for the deformation of a saturated porous solid. A second-gradient theory extending Terzaghi's effective stress principle. *Archive of Applied Mechanics*, 70(5):323–337, 2000.
- [26] F. dell'Isola and P. Seppecher. Edge contact forces and quasi-balanced power. *Meccanica*, 32(1):33–52, 1997.
- [27] R.C. Batra, F. dell'Isola, and S. Vidoli. A second-order solution of Saint-Venant's problem for a piezoelectric circular bar using Signorini's perturbation method. *Journal of Elasticity*, 52(1):75–90, 1998.
- [28] F. dell'Isola, G.C. Ruta, and R.C. Batra. Generalized Poynting effects in predeformed prismatic bars. *Journal of Elasticity*, 50(2):181–196, 1998.
- [29] R. Dell'Erba, F. dell'Isola, and G. Rotoli. The influence of the curvature dependence of the surface tension on the geometry of electrically charged menisci. *Continuum Mechanics and Thermodynamics*, 11(2):89–105, 1999.
- [30] F. dell'Isola and S. Vidoli. Continuum modelling of piezoelectromechanical truss beams: An application to vibration damping. *Archive of Applied Mechanics*, 68(1):1–19, 1998.
- [31] F. dell'Isola and S. Vidoli. Damping of bending waves in truss beams by electrical transmission lines with PZT actuators. *Archive of Applied Mechanics*, 68(9):626–636, 1998.

- [32] S. Vidoli, R.C. Batra, and F. dell'Isola. Saint-Venant's problem for a second-order piezoelectric prismatic bar. *International Journal of Engineering Science*, 38(1):21–45, 2000.
- [33] S. Vidoli and F. dell'Isola. Modal coupling in one-dimensional electromechanical structured continua. *Acta Mechanica*, 141(1):37–50, 2000.
- [34] S. Vidoli and F. dell'Isola. Vibration control in plates by uniformly distributed PZT actuators interconnected via electric networks. *European Journal of Mechanics, A/Solids*, 20(3):435–456, 2001.
- [35] F. dell'Isola and S. Vidoli. Distributed control of beams by electric transmission lines with PZT actuators. *Proceedings of SPIE - The International Society for Optical Engineering*, 3241:312–321, 1997.
- [36] S. Alessandrini, F. dell'Isola, and F. Frezza. Optimal piezo-electro-mechanical coupling to control plate vibrations. *International Journal of Applied Electromagnetics and Mechanics*, 13(1–4):113–120, 2001.
- [37] F. dell'Isola, E. Santini, and D. Vigilante. Purely electrical damping of vibrations in arbitrary PEM plates: A mixed non-conforming FEM-Runge-Kutta time evolution analysis. *Archive of Applied Mechanics*, 73(1–2):26–48, 2003.
- [38] S. Alessandrini, U. Andreaus, and F. dell'Isola. A novel passive electric network analog to Kirchhoff-Love plate designed to efficiently damp forced vibrations by distributed piezoelectric transducers. *Proceedings of SPIE - The International Society for Optical Engineering*, 5052:380–391, 2003.
- [39] F. dell'Isola, E.G. Henneke, and M. Porfiri. Synthesis of electrical networks interconnecting PZT actuators to damp mechanical vibrations. *International Journal of Applied Electromagnetics and Mechanics*, 14(1–4):417–424, 2001.
- [40] S. Alessandrini, F. dell'Isola, and M. Porfiri. A revival of electric analogs for vibrating mechanical systems aimed to their efficient control by PZT actuators. *International Journal of Solids and Structures*, 39(20):5295–5324, 2002.
- [41] F. dell'Isola, M. Porfiri, and S. Vidoli. Piezo-electromechanical (PEM) structures: Passive vibration control using distributed piezoelectric transducers. *Comptes Rendus - Mecanique*, 331(1):69–76, 2003.
- [42] S. Alessandrini, U. Andreaus, F. dell'Isola, and M. Porfiri. Piezo-ElectroMechanical (PEM) Kirchhoff-Love plates. *European Journal of Mechanics, A/Solids*, 23(4):689–702, 2004.
- [43] U. Andreaus, F. dell'Isola, and M. Porfiri. Piezoelectric passive distributed controllers for beam flexural vibrations. *JVC/Journal of Vibration and Control*, 10(5):625–659, 2004.
- [44] F. dell'Isola, C. Maurini, and M. Porfiri. Passive damping of beam vibrations through distributed electric networks and piezoelectric transducers: Prototype design and experimental validation. *Smart Materials and Structures*, 13(2):299–308, 2004.
- [45] M. Porfiri, F. dell'Isola, and F.M.F. Mascioli. Circuit analog of a beam and its application to multimodal vibration damping, using piezoelectric transducers. *International Journal of Circuit Theory and Applications*, 32(4):167–198, 2004.
- [46] S. Alessandrini, U. Andreaus, F. dell'Isola, and M. Porfiri. A passive electric controller for multimodal vibrations of thin plates. *Computers and Structures*, 83(15–16):1236–1250, 2005.
- [47] F. dell'Isola, E.G. Henneke, and M. Porfiri. Piezoelectromechanical structures: A survey of basic concepts and methodologies. *Proceedings of SPIE - The International Society for Optical Engineering*, 5056:574–582, 2003.

- [48] F. dell'Isola, E.G. Henneke, and M. Porfiri. Piezoelectromechanical structures: New trends towards the multimodal passive vibration control. *Proceedings of SPIE - The International Society for Optical Engineering*, 5052:392–402, 2003.
- [49] M. Porfiri and F. dell'Isola. Multimodal beam vibration damping exploiting PZT transducers and passive distributed circuits. *Journal De Physique. IV : JP*, 115:323–330, 2004.
- [50] F. dell'Isola, D. Del Vescovo, and C. Maurini. Distributed electric absorbers of beam vibrations. *Proceedings of SPIE - The International Society for Optical Engineering*, 5052:230–241, 2003.
- [51] C. Maurini, F. dell'Isola, and D. Del Vescovo. Comparison of piezoelectronic networks acting as distributed vibration absorbers. *Mechanical Systems and Signal Processing*, 18(5):1243–1271, 2004.
- [52] C. Maurini, J. Pouget, and F. dell'Isola. On a model of layered piezoelectric beams including transverse stress effect. *International Journal of Solids and Structures*, 41(16–17):4473–4502, 2004.
- [53] C. Maurini, J. Pouget, and F. dell'Isola. Extension of the Euler-Bernoulli model of piezoelectric laminates to include 3D effects via a mixed approach. *Computers and Structures*, 84(22–23):1438–1458, 2006.
- [54] C. Maurini, F. dell'Isola, and J. Pouget. On models of layered piezoelectric beams for passive vibration control. *Journal De Physique. IV : JP*, 115:307–316, 2004.
- [55] J.-J. Alibert, P. Seppecher, and F. dell'Isola. Truss modular beams with deformation energy depending on higher displacement gradients. *Mathematics and Mechanics of Solids*, 8(1):51–73, 2003.
- [56] S. Quiligotti, G.A. Maugin, and F. dell'Isola. Wave motions in unbounded poroelastic solids infused with compressible fluids. *Zeitschrift fur Angewandte Mathematik und Physik*, 53(6):1110–1138, 2002.
- [57] S. Quiligotti, G.A. Maugin, and F. dell'Isola. An Eshelbian approach to the nonlinear mechanics of constrained solid-fluid mixtures. *Acta Mechanica*, 160(1–2):45–60, 2003.
- [58] G. Sciarra, F. dell'Isola, and K. Hutter. A solid-fluid mixture model allowing for solid dilatation under external pressure. *Continuum Mechanics and Thermodynamics*, 13(5):287–306, 2001.
- [59] F. dell'Isola, G. Sciarra, and R.C. Batra. Static deformations of a linear elastic porous body filled with an inviscid fluid. *Journal of Elasticity*, 72(1–3):99–120, 2003.
- [60] F. dell'Isola, G. Sciarra, and R.C. Batra. A second gradient model for deformable porous matrices filled with an inviscid fluid. *Solid Mechanics and its Applications*, 125:221–229, 2005.
- [61] G. Sciarra, F. dell'Isola, and K. Hutter. Dilatational and compacting behavior around a cylindrical cavern leached out in a solid-fluid elastic rock salt. *International Journal of Geomechanics*, 5(3):233–243, 2005.
- [62] R.C. Batra, F. dell'Isola, and G.C. Ruta. Second-order solution of Saint-Venant's problem for an elastic bar predeformed in flexure. *International Journal of Non-Linear Mechanics*, 40(2–3):411–422, 2005.
- [63] M. Porfiri, F. dell'Isola, and E. Santini. Modeling and design of passive electric networks interconnecting piezoelectric transducers for distributed vibration control. *International Journal of Applied Electromagnetics and Mechanics*, 21(2):69–87, 2005.

- [64] R.C. Batra, F. dell'Isola, S. Vidoli, and D. Vigilante. Multimode vibration suppression with passive two-terminal distributed network incorporating piezoceramic transducers. *International Journal of Solids and Structures*, 42(11–12):3115–3132, 2005.
- [65] F. dell'Isola, F. Vestroni, and S. Vidoli. Structural-damage detection by distributed piezoelectric transducers and tuned electric circuits. *Research in Nondestructive Evaluation*, 16(3):101–118, 2005.
- [66] G. Sciarra, F. dell'Isola, and O. Coussy. Second gradient poromechanics. *International Journal of Solids and Structures*, 44(20):6607–6629, 2007.
- [67] G. Sciarra, F. dell'Isola, N. Ianiro, and A. Madeo. A variational deduction of second gradient poroelasticity part I: General theory. *Journal of Mechanics of Materials and Structures*, 3(3):507–526, 2008.
- [68] L. Placidi, F. dell'Isola, N. Ianiro, and G. Sciarra. Variational formulation of prestressed solid-fluid mixture theory, with an application to wave phenomena. *European Journal of Mechanics, A/Solids*, 27(4):582–606, 2008.
- [69] A. Madeo, F. dell'Isola, N. Ianiro, and G. Sciarra. A variational deduction of second gradient poroelasticity II: An application to the consolidation problem. *Journal of Mechanics of Materials and Structures*, 3(4):607–625, 2008.
- [70] F. dell'Isola, A. Madeo, and P. Seppacher. Boundary conditions at fluid-permeable interfaces in porous media: A variational approach. *International Journal of Solids and Structures*, 46(17):3150–3164, 2009.
- [71] F. dell'Isola, G. Sciarra, and S. Vidoli. Generalized Hooke's law for isotropic second gradient materials. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 465(2107):2177–2196, 2009.
- [72] G. Rosi, J. Pouget, and F. dell'Isola. Control of sound radiation and transmission by a piezoelectric plate with an optimized resistive electrode. *European Journal of Mechanics, A/Solids*, 29(5):859–870, 2010.
- [73] H. Shen, J. Qiu, H. Ji, K. Zhu, M. Balsi, I. Giorgio, and F. dell'Isola. A low-power circuit for piezoelectric vibration control by synchronized switching on voltage sources. *Sensors and Actuators, A: Physical*, 161(1–2):245–255, 2010.
- [74] A. Madeo, T. Lekszycki, and F. dell'Isola. A continuum model for the bio-mechanical interactions between living tissue and bio-resorbable graft after bone reconstructive surgery. *Comptes Rendus - Mecanique*, 339(10):625–640, 2011.
- [75] T. Lekszycki and F. dell'Isola. A mixture model with evolving mass densities for describing synthesis and resorption phenomena in bones reconstructed with bio-resorbable materials. *ZAMM Zeitschrift für Angewandte Mathematik und Mechanik*, 92(6):426–444, 2012.
- [76] I. Giorgio, U. Andreaus, D. Scerrato, and F. dell'Isola. A visco-poroelastic model of functional adaptation in bones reconstructed with bio-resorbable materials. *Biomechanics and Modeling in Mechanobiology*, 15(5):1325–1343, 2016.
- [77] I. Giorgio, U. Andreaus, F. dell'Isola, and T. Lekszycki. Viscous second gradient porous materials for bones reconstructed with bio-resorbable grafts. *Extreme Mechanics Letters*, 13:141–147, 2017.
- [78] I. Giorgio, F. dell'Isola, U. Andreaus, F. Alzahrani, T. Hayat, and T. Lekszycki. On mechanically driven biological stimulus for bone remodeling as a diffusive phenomenon. *Biomechanics and Modeling in Mechanobiology*, 18(6):1639–1663, 2019.

- [79] F. dell’Isola, A. Madeo, and L. Placidi. Linear plane wave propagation and normal transmission and reflection at discontinuity surfaces in second gradient 3D continua. *ZAMM Zeitschrift für Angewandte Mathematik und Mechanik*, 92(1):52–71, 2012.
- [80] A. Madeo, F. dell’Isola, and F. Darve. A continuum model for deformable, second gradient porous media partially saturated with compressible fluids. *Journal of the Mechanics and Physics of Solids*, 61(11):2196–2211, 2013.
- [81] M. Ferretti, A. Madeo, F. dell’Isola, and P. Boisse. Modeling the onset of shear boundary layers in fibrous composite reinforcements by second-gradient theory. *Zeitschrift für Angewandte Mathematik und Physik*, 65(3):587–612, 2014.
- [82] F. dell’Isola, P. Seppecher, and A. Madeo. How contact interactions may depend on the shape of Cauchy cuts in Nth gradient continua: Approach “à la D’Alembert”. *Zeitschrift für Angewandte Mathematik und Physik*, 63(6):1119–1141, 2012.
- [83] A. Javili, F. dell’Isola, and P. Steinmann. Geometrically nonlinear higher-gradient elasticity with energetic boundaries. *Journal of the Mechanics and Physics of Solids*, 61(12):2381–2401, 2013.
- [84] N. Auffray, F. dell’Isola, V.A. Eremeyev, A. Madeo, and G. Rosi. Analytical continuum mechanics à la Hamilton-Piola least action principle for second gradient continua and capillary fluids. *Mathematics and Mechanics of Solids*, 20(4):375–417, 2015.
- [85] A. Carcaterra, F. dell’Isola, R. Esposito, and M. Pulvirenti. Macroscopic description of microscopically strongly inhomogenous systems: A mathematical basis for the synthesis of higher gradients metamaterials. *Archive for Rational Mechanics and Analysis*, 218(3):1239–1262, 2015.
- [86] F. dell’Isola. Models to detect scientific creativity: Why something simpler than Fréchet Metric Manifolds? *Mathematics and Mechanics of Solids*, 20(9):1146–1149, 2015.
- [87] F. dell’Isola, P. Seppecher, and A. Della Corte. The postulations à la D’Alembert and à la Cauchy for higher gradient continuum theories are equivalent: A review of existing results. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 471(2183):20150415–1–25, 2015.
- [88] F. dell’Isola, A. Madeo, and P. Seppecher. Cauchy tetrahedron argument applied to higher contact interactions. *Archive for Rational Mechanics and Analysis*, 219(3):1305–1341, 2016.
- [89] F. dell’Isola and L. Placidi. Variational principles are a powerful tool also for formulating field theories. In S. Gavriluk, F. dell’Isola (Eds.), *Variational Models and Methods in Solid and Fluid Mechanics*, 1–15. Springer, Vienna, 2011.
- [90] F. dell’Isola, P. Seppecher, and A. Madeo. Beyond Euler-Cauchy continua: The structure of contact actions in N-th gradient generalized continua: a generalization of the Cauchy tetrahedron argument. In S. Gavriluk, F. dell’Isola (Eds.), *Variational Models and Methods in Solid and Fluid Mechanics*, 17–106. Springer, Vienna, 2011.
- [91] F. dell’Isola, P. Seppecher, and A. Madeo. Fluid shock wave generation at solid-material discontinuity surfaces in porous media. In S. Gavriluk, F. dell’Isola (Eds.), *Variational Models and Methods in Solid and Fluid Mechanics*, 315–358. Springer, Vienna, 2011.
- [92] F. dell’Isola, U. Andreaus, and L. Placidi. At the origins and in the vanguard of peridynamics, non-local and higher-gradient continuum mechanics: An underestimated and still topical contribution of Gabrio Piola. *Mathematics and Mechanics of Solids*, 20(8):887–928, 2015.

- [93] P. Seppecher, J.-J. Alibert, and F.D. Isola. Linear elastic trusses leading to continua with exotic mechanical interactions. *Journal of Physics: Conference Series*, 319:012018–1–13, 2011.
- [94] N. Auffray, F. dell’Isola, V. Eremeyev, A. Madeo, L. Placidi, and G. Rosi. Least action principle for second gradient continua and capillary fluids: A Lagrangian approach following Piola’s point of view. In G. Maier, U. Perego, U. Andreaus, R. Esposito, S. Forest, F. dell’Isola (Eds.), *The complete works of Gabrio Piola: Volume I*, 606–694. Springer, Cham, 2014.
- [95] F. dell’Isola, U. Andreaus, A. Cazzani, U. Perego, L. Placidi, G. Ruta, and D. Scerrato. On a debated principle of Lagrange’s analytical mechanics and on its multiple applications [Di un principio controverso della meccanica analitica di Lagrange e delle molteplici sue applicazioni]. In G. Maier, U. Perego, U. Andreaus, R. Esposito, S. Forest, F. dell’Isola (Eds.), *The complete works of Gabrio Piola: Volume I*, 371–590. Springer, Cham, 2014.
- [96] F. dell’Isola, U. Andreaus, A. Cazzani, and L. Placidi. Gabrio Piola’s Works translated into English: A tribute to a great mathematical-physicist who participated to Italian Risorgimento (Resurgence). In G. Maier, U. Perego, U. Andreaus, R. Esposito, S. Forest, F. dell’Isola (Eds.), *The complete works of Gabrio Piola: Volume I*, viii–x. Springer, Cham, 2014.
- [97] F. dell’Isola, U. Andreaus, and L. Placidi. A still topical contribution of Gabrio Piola to Continuum Mechanics: The creation of peri-dynamics, non-local and higher gradient continuum mechanics. In G. Maier, U. Perego, U. Andreaus, R. Esposito, S. Forest, F. dell’Isola (Eds.), *The complete works of Gabrio Piola: Volume I*, 696–750. Springer, Cham, 2014.
- [98] F. dell’Isola, U. Andreaus, L. Placidi, and D. Scerrato. About the fundamental equations of the motion of bodies whatsoever, as considered following the natural their form and constitution. MEMOIR of Sir Doctor Gabrio Piola [Intorno alle equazioni fondamentali del movimento di corpi qualsivogliono, considerati secondo la naturale loro forma e costituzione. MEMORIA DEL SIG. DOTTOR GABRIO PIOLA]. In G. Maier, U. Perego, U. Andreaus, R. Esposito, S. Forest, F. dell’Isola (Eds.), *The complete works of Gabrio Piola: Volume I*, 1–185. Springer, Cham, 2014.
- [99] F. dell’Isola, U. Andreaus, A. Cazzani, and E. Barchiesi. Introductory remarks about the Volume II of the complete works of Gabrio Piola. In F. dell’Isola, U. Andreaus, A. Cazzani, R. Esposito, L. Placidi, U. Perego, G. Maier, and P. Seppecher (Eds.), *The Complete Works of Gabrio Piola: Volume II*, 1–22. Springer, Cham, 2019.
- [100] F. dell’Isola, U. Andreaus, A. Cazzani, and L. Placidi. Preface. In F. dell’Isola, U. Andreaus, A. Cazzani, R. Esposito, L. Placidi, U. Perego, G. Maier, and P. Seppecher (Eds.), *The Complete Works of Gabrio Piola: Volume II. Advanced Structured Materials*, v. Springer, Cham, 2019.
- [101] F. dell’Isola, M. Laudato, and D. Scerrato. On the applications of principles of analytical mechanics by Lagrange to the principal problems. In F. dell’Isola, U. Andreaus, A. Cazzani, R. Esposito, L. Placidi, U. Perego, G. Maier, and P. Seppecher (Eds.), *The Complete Works of Gabrio Piola: Volume II. Advanced Structured Materials*, 23–579. Springer, Cham, 2019.
- [102] F. dell’Isola, I. Giorgio, M. Pawlikowski, and N.L. Rizzi. Large deformations of planar extensible beams and pantographic lattices: Heuristic homogenization, experimental and numerical examples of equilibrium. *Proceedings of the Royal*

- Society A: Mathematical, Physical and Engineering Sciences*, 472(2185):20150790–1–23, 2016.
- [103] F. dell’Isola and D. Steigmann. A two-dimensional gradient-elasticity theory for woven fabrics. *Journal of Elasticity*, 118(1):113–125, 2015.
- [104] F. dell’Isola, D. Steigmann, and A. Della Corte. Synthesis of fibrous complex structures: Designing microstructure to deliver targeted macroscale response. *Applied Mechanics Reviews*, 67(6):060804–1–21, 2015.
- [105] I. Giorgio, R. Grygoruk, F. dell’Isola, and D.J. Steigmann. Pattern formation in the three-dimensional deformations of fibered sheets. *Mechanics Research Communications*, 69:164–171, 2015.
- [106] D.J. Steigmann and F. dell’Isola. Mechanical response of fabric sheets to three-dimensional bending, twisting, and stretching. *Acta Mechanica Sinica*, 31(3):373–382, 2015.
- [107] I. Giorgio, A. Della Corte, F. dell’Isola, and D.J. Steigmann. Buckling modes in pantographic lattices. *Comptes Rendus – Mecanique*, 344(7):487–501, 2016.
- [108] V.A. Eremeyev, F. dell’Isola, C. Boutin, and D. Steigmann. Linear pantographic sheets: Existence and uniqueness of weak solutions. *Journal of Elasticity*, 132(2):175–196, 2018.
- [109] V.A. Eremeyev and F. dell’Isola. A note on reduced strain gradient elasticity. In J. Pouget, M. Rousseau, B. Collet, T. Michelitsch, H. Altenbach (Eds.), *Generalized Models and Non-classical Approaches in Complex Materials 1*, 301–310. Springer, Cham, 2018.
- [110] V.A. Eremeyev, F.S. Alzahrani, A. Cazzani, F. dell’Isola, T. Hayat, E. Turco, and V. Konopińska-Zmysłowska. On existence and uniqueness of weak solutions for linear pantographic beam lattices models. *Continuum Mechanics and Thermodynamics*, 31(6):1843–1861, 2019.
- [111] C. Boutin, F. dell’Isola, I. Giorgio, and L. Placidi. Linear pantographic sheets: Asymptotic micro-macro models identification. *Mathematics and Mechanics of Complex Systems*, 5(2):127–162, 2017.
- [112] M. Cuomo, F. dell’Isola, and L. Greco. Simplified analysis of a generalized bias test for fabrics with two families of inextensible fibres. *Zeitschrift für Angewandte Mathematik und Physik*, 67(3):61–1–23, 2016.
- [113] F. dell’Isola, A. Della Corte, L. Greco, and A. Luongo. Plane bias extension test for a continuum with two inextensible families of fibers: A variational treatment with Lagrange multipliers and a perturbation solution. *International Journal of Solids and Structures*, 81:1–12, 2016.
- [114] M. Cuomo, F. dell’Isola, L. Greco, and N.L. Rizzi. First versus second gradient energies for planar sheets with two families of inextensible fibres: Investigation on deformation boundary layers, discontinuities and geometrical instabilities. *Composites Part B: Engineering*, 115:423–448, 2017.
- [115] F. dell’Isola, M. Cuomo, L. Greco, and A. Della Corte. Bias extension test for pantographic sheets: numerical simulations based on second gradient shear energies. *Journal of Engineering Mathematics*, 103(1):127–157, 2017.
- [116] A. Misra, T. Lekszycki, I. Giorgio, G. Ganzosch, W.H. Müller, and F. dell’Isola. Pantographic metamaterials show atypical Poynting effect reversal. *Mechanics Research Communications*, 89:6–10, 2018.
- [117] E. Turco, F. dell’Isola, and A. Misra. A nonlinear Lagrangian particle model for grains assemblies including grain relative rotations. *International Journal for Numerical*

- and Analytical Methods in Geomechanics*, 43(5):1051–1079, 2019.
- [118] U. Andreaus, F. dell’Isola, I. Giorgio, L. Placidi, T. Lekszycki, and N.L. Rizzi. Numerical simulations of classical problems in two-dimensional (non) linear second gradient elasticity. *International Journal of Engineering Science*, 108:34–50, 2016.
- [119] E. Turco, F. dell’Isola, A. Cazzani, and N.L. Rizzi. Hencky-type discrete model for pantographic structures: numerical comparison with second gradient continuum models. *Zeitschrift für Angewandte Mathematik und Physik*, 67(4):85–1–28, 2016.
- [120] E. Turco, F. dell’Isola, N.L. Rizzi, R. Grygoruk, W.H. Müller, and C. Liebold. Fiber rupture in sheared planar pantographic sheets: Numerical and experimental evidence. *Mechanics Research Communications*, 76:86–90, 2016.
- [121] E. Turco, I. Giorgio, A. Misra, and F. dell’Isola. King post truss as a motif for internal structure of (meta)material with controlled elastic properties. *Royal Society Open Science*, 4(10), 2017.
- [122] B.E. Abali, W.H. Müller, and F. dell’Isola. Theory and computation of higher gradient elasticity theories based on action principles. *Archive of Applied Mechanics*, 87(9):1495–1510, 2017.
- [123] Y. Rahali, I. Giorgio, J.F. Ganghoffer, and F. dell’Isola. Homogenization à la Piola produces second gradient continuum models for linear pantographic lattices. *International Journal of Engineering Science*, 97:148–172, 2015.
- [124] A. Madeo, M. Ferretti, F. dell’Isola, and P. Boisse. Thick fibrous composite reinforcements behave as special second-gradient materials: three-point bending of 3D interlocks. *Zeitschrift für Angewandte Mathematik und Physik*, 66(4):2041–2060, 2015.
- [125] F. dell’Isola, M.V. d’Agostino, A. Madeo, P. Boisse, and D. Steigmann. Minimization of shear energy in two dimensional continua with two orthogonal families of inextensible fibers: The case of standard bias extension test. *Journal of Elasticity*, 122(2):131–155, 2016.
- [126] P. Boisse, N. Hamila, E. Guzman-Maldonado, A. Madeo, G. Hivet, and F. dell’Isola. The bias-extension test for the analysis of in-plane shear properties of textile composite reinforcements and preregs: a review. *International Journal of Material Forming*, 10(4):473–492, 2017.
- [127] I. Giorgio, A. Della Corte, and F. dell’Isola. Dynamics of 1D nonlinear pantographic continua. *Nonlinear Dynamics*, 88(1):21–31, 2017.
- [128] I. Giorgio, F. dell’Isola, and D.J. Steigmann. Axisymmetric deformations of a 2nd grade elastic cylinder. *Mechanics Research Communications*, 94:45–48, 2018.
- [129] I. Giorgio, P. Harrison, F. dell’Isola, J. Alsayednoor, and E. Turco. Wrinkling in engineering fabrics: A comparison between two different comprehensive modelling approaches. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 474(2216):20180063–1–20, 2018.
- [130] I. Giorgio, F. dell’Isola, and D.J. Steigmann. Edge effects in Hypar nets [Effets de bord dans les réseaux paraboloidaux hyperboliques]. *Comptes Rendus – Mécanique*, 347(2):114–123, 2019.
- [131] E. Turco, A. Misra, M. Pawlikowski, F. dell’Isola, and F. Hild. Enhanced Piola-Hencky discrete models for pantographic sheets with pivots without deformation energy: Numerics and experiments. *International Journal of Solids and Structures*, 147:94–109, 2018.
- [132] S.R. Eugster and F. dell’Isola. Exegesis of the introduction and sect. I from “Fundamentals of the mechanics of continua” by E. Hellinger. *ZAMM Zeitschrift für*

- Angewandte Mathematik und Mechanik*, 97(4):477–506, 2017.
- [133] S.R. Eugster and F. dell’Isola. Exegesis of sect. II and III.a from “Fundamentals of the mechanics of continua” by E. Hellinger. *ZAMM Zeitschrift für Angewandte Mathematik und Mechanik*, 98(1):31–68, 2018.
- [134] S.R. Eugster and F. dell’Isola. Exegesis of sect. III.b from “Fundamentals of the mechanics of continua” by E. Hellinger. *ZAMM Zeitschrift für Angewandte Mathematik und Mechanik*, 98(1):69–105, 2018.
- [135] F. dell’Isola, S. Bucci, and A. Battista. Against the fragmentation of knowledge: The power of multidisciplinary research for the design of metamaterials. In M. Aßmus, H. Naumenko (Eds.), *Advanced Methods of Continuum Mechanics for Materials and Structures*, 523–545. Springer, Singapore, 2016.
- [136] F. dell’Isola, A. Della Corte, R. Esposito, and L. Russo. Some cases of unrecognized transmission of scientific knowledge: From antiquity to Gabrio Piola’s peridynamics and generalized continuum theories. In S. Forest, H. Altenbach (Eds.), *Generalized Continua as Models for Classical and Advanced Materials*, 77–128. Springer, Cham, 2016.
- [137] F. dell’Isola and V.A. Eremeyev. Some introductory and historical remarks on mechanics of microstructured materials. In V. Eremeyev, A. Porubov, F. dell’Isola (Eds.), *Advances in Mechanics of Microstructured Media and Structures*, 1–20. Springer, Cham, 2018.
- [138] F. dell’Isola, P. Seppecher, J.-J. Alibert, T. Lekszycki, R. Grygoruk, M. Pawlikowski, D. Steigmann, I. Giorgio, U. Andreaus, E. Turco, M. Gołaszewski, N.L. Rizzi, C. Boutin, V.A. Eremeyev, A. Misra, L. Placidi, E. Barchiesi, L. Greco, M. Cuomo, A. Cazzani, A. Della Corte, A. Battista, D. Scerrato, I.Z. Eremeeva, Y. Rahali, J.-F. Ganghoffer, W. Müller, G. Ganzosch, M. Spagnuolo, A. Pfaff, K. Barcz, K. Hoschke, J. Neggers, and F. Hild. Pantographic metamaterials: an example of mathematically driven design and of its technological challenges. *Continuum Mechanics and Thermodynamics*, 31(4):851–884, 2019.
- [139] F. dell’Isola, P. Seppecher, M. Spagnuolo, E. Barchiesi, F. Hild, T. Lekszycki, I. Giorgio, L. Placidi, U. Andreaus, M. Cuomo, S.R. Eugster, A. Pfaff, K. Hoschke, R. Langkemper, E. Turco, R. Sarikaya, A. Misra, M. De Angelo, F. D’Annibale, A. Bouterf, X. Pinelli, A. Misra, B. Desmorat, M. Pawlikowski, C. Dupuy, D. Scerrato, P. Peyre, M. Laudato, L. Manzari, P. G’oransson, C. Hesch, S. Hesch, P. Franciosi, J. Dirrenberger, F. Maurin, Z. Vangelatos, C. Grigoropoulos, V. Melissinaki, M. Farsari, W. Müller, B.E. Abali, C. Liebold, G. Ganzosch, P. Harrison, R. Drobnicki, L. Igumnov, F. Alzahrani, and T. Hayat. Advances in pantographic structures: design, manufacturing, models, experiments and image analyses. *Continuum Mechanics and Thermodynamics*, 31(4):1231–1282, 2019.
- [140] F. dell’Isola, I. Giorgio, and U. Andreaus. Elastic pantographic 2D lattices: A numerical analysis on the static response and wave propagation. *Proceedings of the Estonian Academy of Sciences*, 64(3):219–225, 2015.
- [141] F. dell’Isola, T. Lekszycki, M. Pawlikowski, R. Grygoruk, and L. Greco. Designing a light fabric metamaterial being highly macroscopically tough under directional extension: first experimental evidence. *Zeitschrift für Angewandte Mathematik und Physik*, 66(6):3473–3498, 2015.
- [142] F. dell’Isola, A. Della Corte, I. Giorgio, and D. Scerrato. Pantographic 2D sheets: Discussion of some numerical investigations and potential applications. *International Journal of Non-Linear Mechanics*, 80:200–208, 2016.

- [143] F. dell’Isola, A. Della Corte, and I. Giorgio. Higher-gradient continua: The legacy of Piola, Mindlin, Sedov and Toupin and some future research perspectives. *Mathematics and Mechanics of Solids*, 22(4):852–872, 2017.
- [144] G. Rosi, L. Placidi, and F. dell’Isola. “Fast” and “slow” pressure waves electrically induced by nonlinear coupling in Biot-type porous medium saturated by a nematic liquid crystal. *Zeitschrift für Angewandte Mathematik und Physik*, 68(2):51–1–14, 2017.
- [145] Y. Wang, F. dell’Isola, T. Liu, and C. Yang. Metamaterials and smart structures in a big data era. *Advances in Materials Science and Engineering*, 2017:6358497–1, 2017.
- [146] F. dell’Isola, E. Turco, A. Misra, Z. Vangelatos, C. Grigoropoulos, V. Melissinaki, and M. Farsari. Force-Äisplacement relationship in micro-metric pantographs: Experiments and numerical simulations. *Comptes Rendus – Mecanique*, 347(5):397–405, 2019.
- [147] S.R. Eugster, F. dell’Isola, and D.J. Steigmann. Continuum theory for mechanical metamaterials with a cubic lattice substructure. *Mathematics and Mechanics of Complex Systems*, 7(1):75–98, 2019.
- [148] M. Ferretti, G. Piccardo, F. dell’Isola, and A. Luongo. Dynamics of taut strings undergoing large changes of tension caused by a force-driven traveling mass. *Journal of Sound and Vibration*, 458:320–333, 2019.
- [149] M.E. Yildizdag, C.A. Tran, E. Barchiesi, M. Spagnuolo, F. dell’Isola, and F. Hild. A multi-disciplinary approach for mechanical metamaterial synthesis: a hierarchical modular multiscale cellular structure paradigm. In A. ‘Ochsner, H. Altenbach (Eds.), *State of the Art and Future Trends in Material Modeling*, 485–505. Springer, Cham, 2019.
- [150] F. dell’Isola, A.M. Bragov, L.A. Igumnov, B.E. Abali, A.K. Lomunov, D.A. Lamzin, and A.Y. Konstantinov. Mechanical response change in fine grain concrete under high strain and stress rates. In H. Altenbach, F. dell’Isola, V. Eremeyev, A. ‘Ochsner, B. Abali (Eds.), *New Achievements in Continuum Mechanics and Thermodynamics*, 71–80. Springer, Cham, 2019.
- [151] F. dell’Isola, L.A. Igumnov, S.Y. Litvinchuk, A.A. Ipatov, A.N. Petrov, and I.A. Modin. Surface waves in dissipative poroviscoelastic layered half space: Boundary element analyses. In A. Belyaev, V. Eremeyev, A. Krivtsov, A. Porubov, H. Altenbach (Eds.), *Dynamical Processes in Generalized Continua and Structures*, 305–319. Springer, Cham, 2019.
- [152] F. dell’Isola, I.A. Volkov, L.A. Igumnov, S.R. Eugster, S.Y. Litvinchuk, D.A. Kazakov, V.A. Gorohov, and B.E. Abali. Estimating fatigue related damage in alloys under block-type non-symmetrical low-cycle loading. In H. Altenbach, F. dell’Isola, V. Eremeyev, A. ‘Ochsner, B. Abali (Eds.), *New Achievements in Continuum Mechanics and Thermodynamics*, 81–92. Springer, Cham, 2019.
- [153] A. Della Corte, F. dell’Isola, R. Esposito, and M. Pulvirenti. Equilibria of a clamped euler beam (Elastica) with distributed load: Large deformations. *Mathematical Models and Methods in Applied Sciences*, 27(8):1391–1421, 2017.
- [154] A. Della Corte, A. Battista, F. dell’Isola, and P. Seppecher. Large deformations of Timoshenko and Euler beams under distributed load. *Zeitschrift für Angewandte Mathematik und Physik*, 70(2):52–1–19, 2019.
- [155] F. dell’Isola, A. Della Corte, A. Battista, and E. Barchiesi. Extensible beam models in large deformation under distributed loading: A numerical study on multiplicity of

- solutions. In W. Müller, B. Abali, H. Altenbach (Eds.), *Higher Gradient Materials and Related Generalized Continua*, 19–41. Springer, Cham, 2019.
- [156] A. Della Corte, A. Battista, and F. dell’Isola. Referential description of the evolution of a 2D swarm of robots interacting with the closer neighbors: Perspectives of continuum modeling via higher gradient continua. *International Journal of Non-Linear Mechanics*, 80:209–220, 2016.
- [157] A. Battista, A. Della Corte, F. dell’Isola, and P. Seppecher. Large deformations of 1D microstructured systems modeled as generalized Timoshenko beams. *Zeitschrift für Angewandte Mathematik und Physik*, 69(3):52–1–22, 2018.
- [158] A. Della Corte, A. Battista, F. dell’Isola, and I. Giorgio. Modeling deformable bodies using discrete systems with centroid-based propagating interaction: Fracture and crack evolution. In F. dell’Isola, M. Sofonea, and D. Steigmann (Eds.), *Mathematical Modelling in Solid Mechanics*, 59–88. Springer, Singapore, 2017.
- [159] M. Spagnuolo, K. Barcz, A. Pfaff, F. dell’Isola, and P. Franciosi. Qualitative pivot damage analysis in aluminum printed pantographic sheets: Numerics and experiments. *Mechanics Research Communications*, 83:47–52, 2017.
- [160] F. dell’Isola, I. Giorgio, L. Placidi, M. Spagnuolo, P. Peyre, C. Dupuy, J. Dirrenberger, M. Pawlikowski, and L. Igumnov. Pantographic metamaterials: A view towards applications. *Materials Physics and Mechanics*, 42(5):637–645, 2019.
- [161] E. Barchiesi, S.R. Eugster, L. Placidi, and F. dell’Isola. Pantographic beam: a complete second gradient 1D-continuum in plane. *Zeitschrift für Angewandte Mathematik und Physik*, 70(5):135–1–24, 2019.
- [162] E. Barchiesi, F. dell’Isola, M. Laudato, L. Placidi, and P. Seppecher. A 1D continuum model for beams with pantographic microstructure: asymptotic micro-macro identification and numerical results. In Eremeyev V. Porubov A. dell’Isola, F. (Eds.), *Advances in Mechanics of Microstructured Media and Structures*, 43–74. Springer, Cham, 2018.
- [163] N. NejadSadeghi, M. De Angelo, R. Drobnicki, T. Lekszycki, F. dell’Isola, and A. Misra. Parametric experimentation on pantographic unit cells reveals local extremum configuration. *Experimental Mechanics*, 59(6):927–939, 2019.
- [164] L. Placidi, F. dell’Isola, and E. Barchiesi. Heuristic homogenization of Euler and pantographic beams. In J.-F. Ganghoffer, C. Picu (Eds.), *Mechanics of Fibrous Materials and Applications*, 123–155. Springer, Cham, 2019.
- [165] E. Barchiesi, F. dell’Isola, F. Hild, and P. Seppecher. Two-dimensional continua capable of large elastic extension in two independent directions: Asymptotic homogenization, numerical simulations and experimental evidence. *Mechanics Research Communications*, 103:103466–1–5, 2020.
- [166] E. Barchiesi, S.R. Eugster, F. dell’Isola, and F. Hild. Large in-plane elastic deformations of bi-pantographic fabrics: asymptotic homogenization and experimental validation. *Mathematics and Mechanics of Solids*, 25(3):739–767, 2020.
- [167] M.E. Yildizdag, E. Barchiesi, and F. dell’Isola. Three-point bending test of pantographic blocks: numerical and experimental investigation. *Mathematics and Mechanics of Solids*, 25(10):1965–1978, 2020.
- [168] I. Giorgio, V. Varano, F. dell’Isola, and N.L. Rizzi. Two layers pantographs: A 2D continuum model accounting for the beams’ offset and relative rotations as averages in $SO(3)$ Lie groups. *International Journal of Solids and Structures*, 216:43–58, 2021.

- [169] Z. Vangelatos, M.E. Yildizdag, I. Giorgio, F. dell'Isola, and C. Grigoropoulos. Investigating the mechanical response of microscale pantographic structures fabricated by multiphoton lithography. *Extreme Mechanics Letters*, 43:101202–1–8, 2021.
- [170] E. Turco, E. Barchiesi, and F. dell'Isola. A numerical investigation on impulse-induced nonlinear longitudinal waves in pantographic beams. *Mathematics and Mechanics of Solids*, 27(1):22–48, 2022.
- [171] J. Chróścielewski, F. dell'Isola, V.A. Eremeyev, and A. Sabik. On rotational instability within the nonlinear six-parameter shell theory. *International Journal of Solids and Structures*, 196–197:179–189, 2020.
- [172] V.A. Eremeyev and F. dell'Isola. Weak solutions within the gradient-incomplete strain-gradient elasticity. *Lobachevskii Journal of Mathematics*, 41(10):1992–1998, 2020.
- [173] V.A. Eremeyev, S.A. Lurie, Y.O. Solyaev, and F. dell'Isola. On the well posedness of static boundary value problem within the linear dilatational strain gradient elasticity. *Zeitschrift für Angewandte Mathematik und Physik*, 71(6):182–1–16, 2020.
- [174] E.F. Grekova, A.V. Porubov, and F. dell'Isola. Reduced linear constrained elastic and viscoelastic homogeneous Cosserat media as acoustic metamaterials. *Symmetry*, 12(4):521–1–22, 2020.
- [175] I.A. Volkov, L.A. Igumnov, F. dell'Isola, S.Y. Litvinchuk, and V.A. Eremeyev. A continual model of a damaged medium used for analyzing fatigue life of polycrystalline structural alloys under thermal, ãmechanical loading. *Continuum Mechanics and Thermodynamics*, 32(1):229–245, 2020.
- [176] V.A. Eremeyev, A. Cazzani, and F. dell'Isola. On nonlinear dilatational strain gradient elasticity. *Continuum Mechanics and Thermodynamics*, 33(4):1429–1463, 2021.
- [177] V.A. Eremeyev and F. dell'Isola. On weak solutions of the boundary value problem within linear dilatational strain gradient elasticity for polyhedral Lipschitz domains. *Mathematics and Mechanics of Solids*, 27(3):433–445, 2022.
- [178] I. Giorgio, F. dell'Isola, and A. Misra. Chirality in 2D Cosserat media related to stretch-micro-rotation coupling with links to granular micromechanics. *International Journal of Solids and Structures*, 202:28–38, 2020.
- [179] E. Turco, E. Barchiesi, I. Giorgio, and F. dell'Isola. A Lagrangian Hencky-type non-linear model suitable for metamaterials design of shearable and extensible slender deformable bodies alternative to Timoshenko theory. *International Journal of Non-Linear Mechanics*, 123:103481–1–19, 2020.
- [180] A. Misra, L. Placidi, F. dell'Isola, and E. Barchiesi. Identification of a geometrically nonlinear micromorphic continuum via granular micromechanics. *Zeitschrift für Angewandte Mathematik und Physik*, 72(4):157–1–21, 2021.
- [181] E. Turco, E. Barchiesi, and F. dell'Isola. In-plane dynamic buckling of duoskelion beam-like structures: discrete modeling and numerical results. *Mathematics and Mechanics of Solids*, 27(7):1164–1184, 2021.
- [182] Y. Solyaev, S. Lurie, H. Altenbach, and F. dell'Isola. On the elastic wedge problem within simplified and incomplete strain gradient elasticity theories. *International Journal of Solids and Structures*, 239–240:111433–1–13, 2022.
- [183] E. Barchiesi, F. dell'Isola, A.M. Bersani, and E. Turco. Equilibria determination of elastic articulated duoskelion beams in 2D via a Riks-type algorithm. *International Journal of Non-Linear Mechanics*, 128, 2021.

- [184] J. Schulte, M. Dittmann, S.R. Eugster, S. Hesch, T. Reinicke, F. dell’Isola, and C. Hesch. Isogeometric analysis of fiber reinforced composites using Kirchhoff, Love shell elements. *Computer Methods in Applied Mechanics and Engineering*, 362:112845–1–34, 2020.
- [185] M. Spagnuolo, P. Franciosi, and F. dell’Isola. A Green operator-based elastic modeling for two-phase pantographic-inspired bi-continuous materials. *International Journal of Solids and Structures*, 188–189:282–308, 2020.
- [186] C. Boutin and F. dell’Isola. Green’s functions and integral representation of generalized continua: the case of orthogonal pantographic lattices. *Zeitschrift für Angewandte Mathematik und Physik*, 72(2):58–1–26, 2021.
- [187] P. Auger, T. Lavigne, B. Smaniotto, M. Spagnuolo, F. dell’Isola, and F. Hild. Poynting effects in pantographic metamaterial captured via multiscale DVC. *Journal of Strain Analysis for Engineering Design*, 56(7):462–477, 2021.
- [188] E. Barchiesi, F. dell’Isola, and F. Hild. On the validation of homogenized modeling for bi-pantographic metamaterials via Digital Image Correlation. *International Journal of Solids and Structures*, 208–209:49–62, 2021.
- [189] F. Hild, A. Misra, and F. dell’Isola. Multiscale DIC applied to pantographic structures. *Experimental Mechanics*, 61(2):431–443, 2021.
- [190] A.M. Bersani, P. Caressa, and F. dell’Isola. Approximation of dissipative systems by elastic chains: Numerical evidence. *Mathematics and Mechanics of Solids*, doi:10.1177/10812865221081851, 2022.
- [191] R. Darleux, B. Lossouarn, I. Giorgio, F. dell’Isola, and J.-F. De´u. Electrical analogs of curved beams and application to piezoelectric network damping. *Mathematics and Mechanics of Solids*, 27(4):578–601, 2022.
- [192] U. M´uhlich, B.E. Abali, and F. dell’Isola. Commented translation of Erwin Schr´odinger’s paper ‘On the dynamics of elastically coupled point systems’ (Z´ur Dynamik elastisch gekoppelter Punktsysteme). *Mathematics and Mechanics of Solids*, 26(1):133–147, 2021.
- [193] M. Spagnuolo, F. dell’Isola, and A. Cazzani. The study of the genesis of novel mathematical and mechanical theories provides an inspiration for future original research. In S.R. Eugster, M. Spagnuolo, E. Barchiesi, F. dell’Isola (Eds.), *Evaluation of Scientific Sources in Mechanics*, 1–73. Springer, Cham, 2022.
- [194] M. Spagnuolo, F. dell’Isola, B. Gerber, and A.M. Cazzani. Translation of Heiberg’s prolegomena. In S.R. Eugster, M. Spagnuolo, E. Barchiesi, F. dell’Isola (Eds.), *Evaluation of Scientific Sources in Mechanics*, 75–97. Springer, Cham, 2022.

DIPARTIMENTO DI INGEGNERIA MECCANICA E AEROSPAZIALE, SAPIENZA UNIVERSITY OF ROME, ROME, ITALY

DICAAR, UNIVERSITY OF CAGLIARI, CAGLIARI, ITALY

DICEA, UNIVERSITY OF L’AQUILA, L’AQUILA, ITALY

DICAAR, UNIVERSITY OF CAGLIARI, CAGLIARI, ITALY