Discussion on the paper by Feiyu Jiang, Zifeng Zhao, and Xiaofeng Shao

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We congratulate Feiyu Jiang, Zifeng Zhao, and Xiaofeng Shao on a thorough and interesting work. Their change-point detection procedure for the piecewise linear quantile trend model clearly represents a competitive, innovative, and flexible tool for several application areas. We would like to bring the authors' attention to an important issue concerning a potential disconnect between the proposed methods and the specific application discussed. More specifically, while their work is motivated by modelling COVID-19 log-counts $Y_t = \log(R_t + 1)$ (a discrete random variable), Assumption 1 in Section 3.1 requires the existence of a continuous density function that is bounded away from 0 (and ∞) at the quantile $Q_\varepsilon(\tau)$. This assumption is crucial for the Bahadur representation and asymptotic behaviour of $\hat{\beta}(\tau)$ (Koenker, 2005). The good performance of GOALS and M-GOALS in the simulation studies reported by the authors revolves around normally distributed responses. Thus, it is natural to wonder whether the performance deteriorates when the response is discrete, especially when counts are small or are irregularly spaced.

Quantile modelling of discrete distributions is notoriously troublesome because of the lack of consistency and asymptotic normality of L_1 estimators in the presence of ties (see, for example, Ma et al., 2011; Jentsch and Leucht, 2016). In the present context, COVID-19 counts could be easily modelled using a jittering approach (Machado and Santos Silva, 2005), which would involve adding artificial random noise repeatedly to the counts R_t. Owing to some limitations of jittering and other existing approaches to discrete quantile regression, Geraci and Farcomeni (2021) have introduced mid-quantile regression, a conditional extension of

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Parzen's (1993) marginal mid-quantiles. Not only do mid-quantiles bridge modelling of discrete and continuous distributions, but they also handle different types of discreteness. The conditional mid-quantile estimator is versatile and well-behaved asymptotically (Geraci and Farcomeni, 2021).

We thank the authors for their stimulating work and hope that further contributions to the development of piecewise linear regression will see *conditional mid-quantiles* as a viable solution in change-point estimation problems with discrete responses, such as COVID-19 daily new cases. Modelling and forecast of the latter would also benefit from adjusting models for important confounders like daily number of swabs (e.g., see Alaimo Di Loro et al., 2021; Bartolucci and Farcomeni, 2021; Farcomeni et al., 2021; Girardi et al., 2021), as the authors seem to allude to in their final remarks.

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