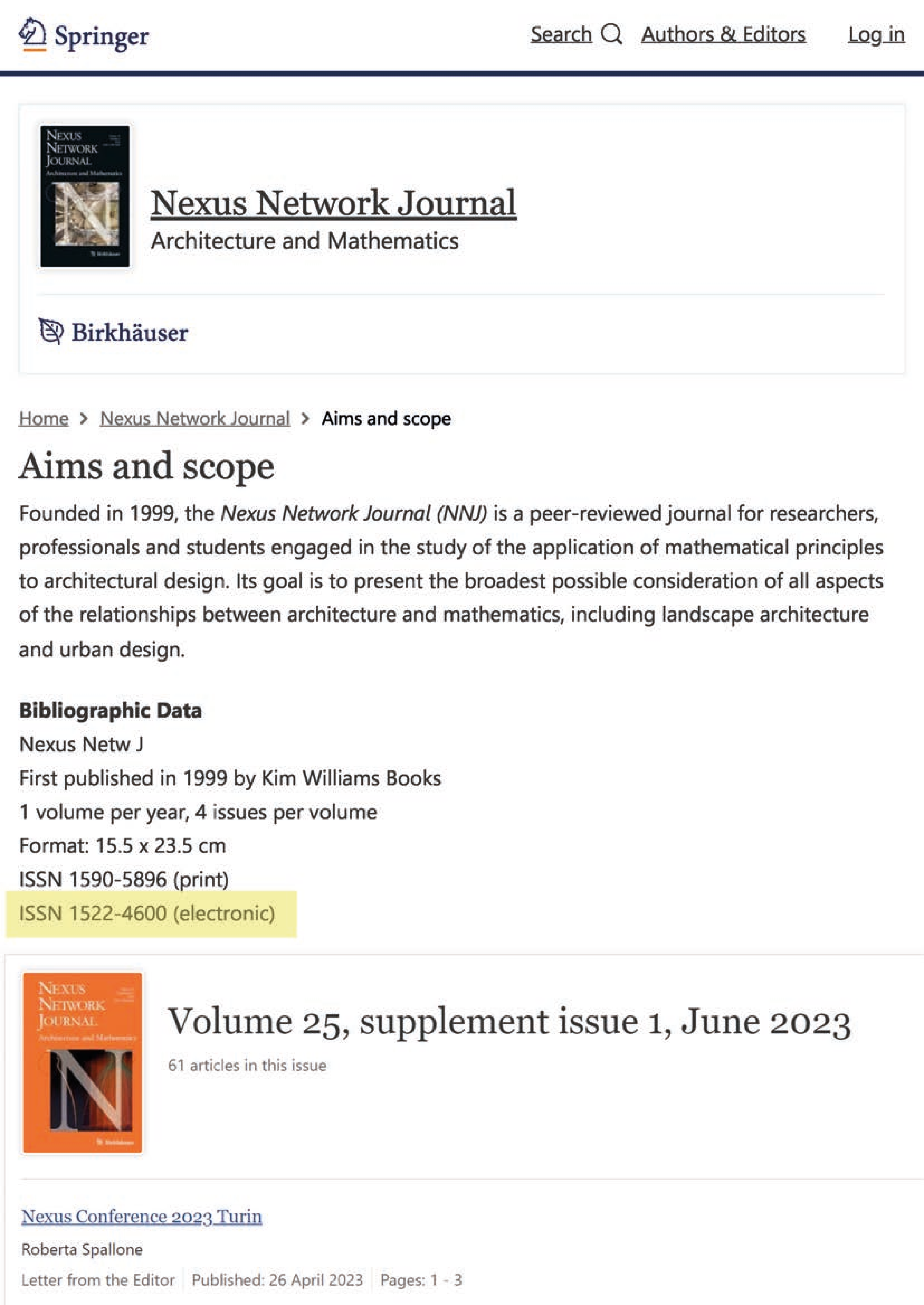

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Architecture and Mathematics

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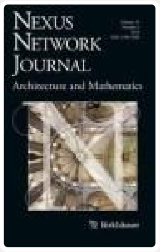

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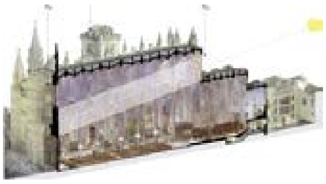

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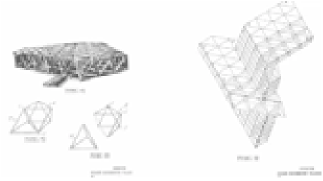

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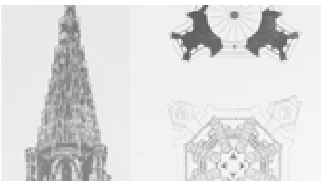

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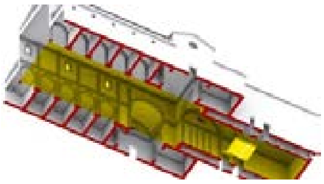

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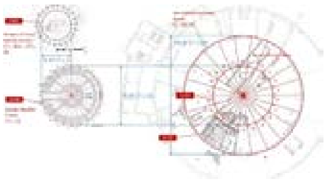

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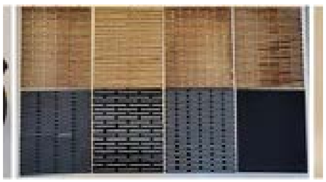

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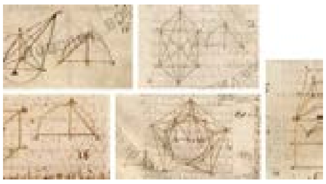

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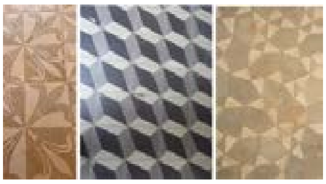

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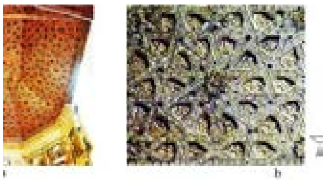

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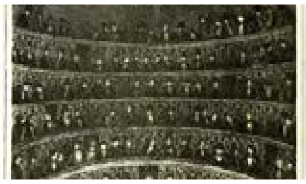

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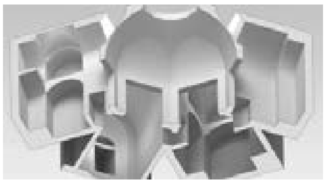

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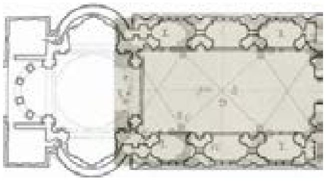

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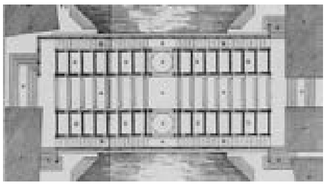

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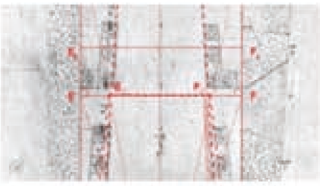

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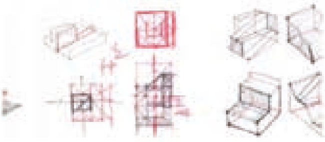

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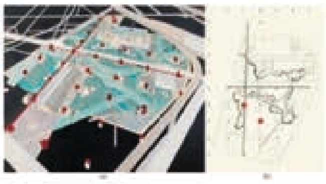

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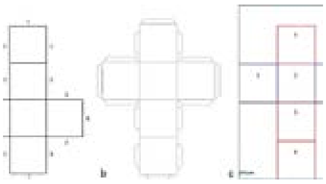

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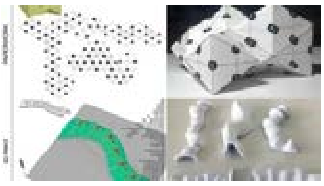

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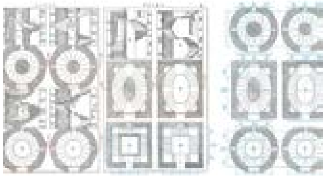

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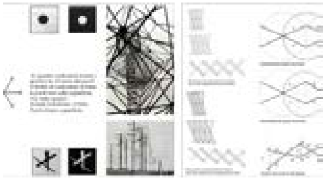

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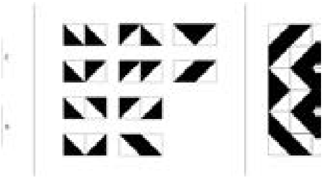

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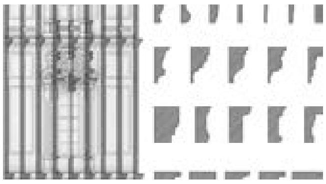

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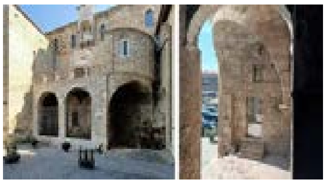

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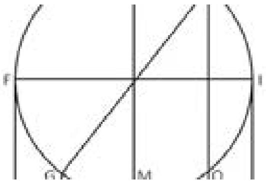

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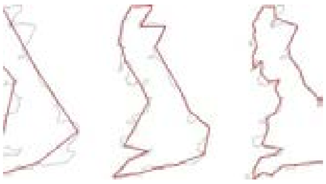

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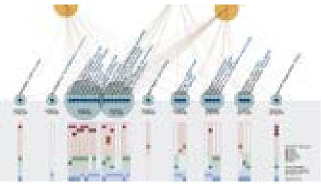

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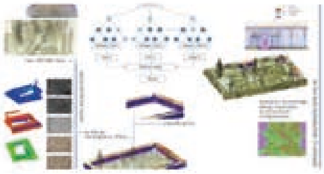

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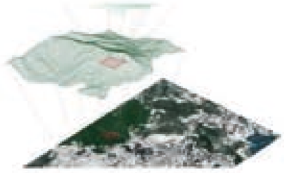

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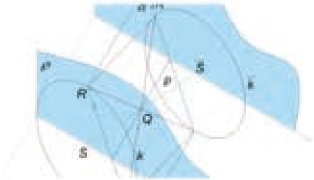

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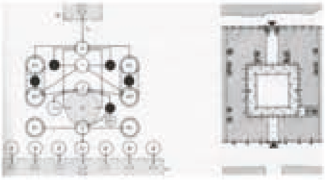

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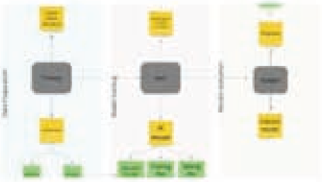

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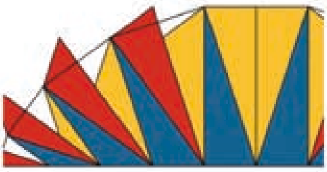

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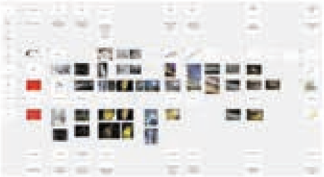

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CONFERENCE PAPER

Optical Effects by Escher Which Anticipate Digital Rendering

Luca Martelli[1] · **Laura Carnevali**[1] · **Elena Ippoliti**[1] · **Fabio Lanfranchi**[1]

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Abstract
Escher's depiction of transparency, refraction, and reflection overlaps geometric coding of chiaroscuro from the first half of the twentieth century and anticipates several eidomatic effects. After studying some digital rendering processes, we propose a method similar to Escher's and apply it to a drawing of Mies van der Rohe's Glashochhaus.

Keywords Optics · Representations of architecture (drawings and prints) · Geometry · Linear algebra · Modeling

Introduction

While scholars of representation science were still committed to geometric and mathematical codification of chiaroscuro, Escher applied his own method of depicting transparency, refraction, and reflection in some prints. The artist had used effects that could only be achieved with lithography and mezzotint (Griffiths 1996: 9–12). No matter how much empirical his approach, the aesthetic results achieved by Escher did not originate just from *mimesis* of scenes featuring material models in his studio, but also from application of geometric rules. He was able to anticipate several methods of digital rendering.

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We aim to formalise a procedure comparable to Escher's for representing transparency, refraction, and reflection in analogue geometric drawing for Architecture.

So we set ourselves the following objectives: to review mathematical modelling of chiaroscuro and some digital rendering processes; to analyse four peculiar prints by Escher; and to produce, on the basis of our studies, a watercolour manual perspective of Mies van der Rohe's Modell Glashochhaus.

The Research

As far as manual drawing is concerned, assuming that the luminous flux propagates as rectilinear rays, we can argue that chiaroscuro is a geometric construction conceived in space and transposed onto our image plane. To solve otherwise very complex problems, this method takes geometrical optics (essentially Lambert's cosine law) as its foundation and requires approximations dictated by the draftsman's visual experiences.

Firstly, chiaroscuro involves setting a shade scale corresponding to different degrees of illumination and shadow (Fig. 1a).

Lambert's cosine law also gives origin to a procedure for tracing isophotes, whose potential in manual geometric drawing goes beyond the execution of chiaroscuro (Fig. 1b).

In manual geometric drawing, chiaroscuro also covers reflections, albeit with certain limitations. Light rays hitting a body behave according to its physical characteristics. Rays can either bounce on the body (reflection) or penetrate it and, in most cases, pass through it (refraction).

Specular reflection is described by the eponymous law: *the angle of incidence is equal to the angle of reflection* (Fig. 1c, d).

In chiaroscuro, refraction of opaque bodies is regarded as negligible, while reflection and aerial perspective are addressed with considerable schematisations (Docci and Migliari 1992: 578–607). This limitation is a consequence of the formulation of shadow representation theory which, by its very premises, does not take transparent and translucent bodies into consideration.

With CAD achieving a certain level of reiterability and computing power as well as finesse in graphic resolution, it was possible not only to take advantage of isophotes construction more effectively, but also to explore the problems of transparency and refraction. Computational algorithms dedicated to virtual representation of chiaroscuro are structured along three main lines of development, respectively based on geometrical optics, physical optics, and quantum optics.

We have focused our attention on digital modelling derived from geometrical optics, since its computational theory is mainly based on the laws of Lambert, Snell, and Fresnel (Valenti 2012: 284–296). It therefore shows great synergistic potential with analogue modelling inherent in manual geometric drawing.

As in manual geometric drawing, digital shadow rendering is posed as a problem of projection and intersection (Fuchs et al. 1986: 111–116). With regard

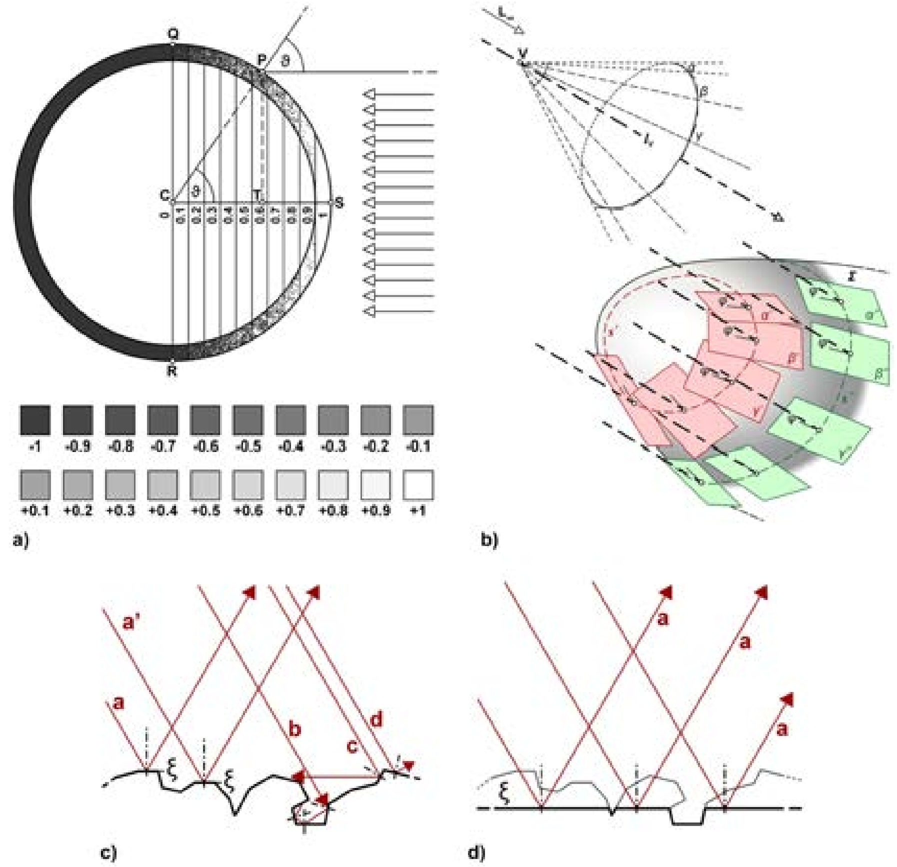

Fig. 1 Chiaroscuro rules for manual geometric drawing. **a** For each point *P* on a spherical surface, it results from Lambert's cosine law: $i = cos\vartheta = CT/CS$. The greater the number *n* of parts into which we will divide segment *CS* without taking value *0* into account, the greater the number *2n* of shades on the scale. **b** The isophote curve is the geometric locus on a surface where the value $i = sin\varphi = cos\vartheta$ is constant in each point. **c** With a surface consisting of innumerable perfectly reflecting and differently oriented faces, a small number of light rays will be reflected (a, a'). Others (b) will be scattered in cavities. Still others (c, d) will be deviated. **d** If we smooth out the surface, the reflective faces having inclination ξ will increase significantly and, consequently, the surface will appear shinier

to polygons, the construction of cast shadows can be approached by defining their shadow *frusta* as logical intersections of half-spaces (Fig. 2a, b).

In digital rendering, the problems of transparency and refraction involve all surfaces in the modelled scene during rasterization of each scan segment. In order to obtain a realistic representation of a translucent object, we must consider two issues: (1) the luminous intensity transmitted across objects is proportional to the amount of material through which it must pass; (2) the image we see across objects must be distorted as it would naturally occur when light is refracted as it passes through media of different densities.

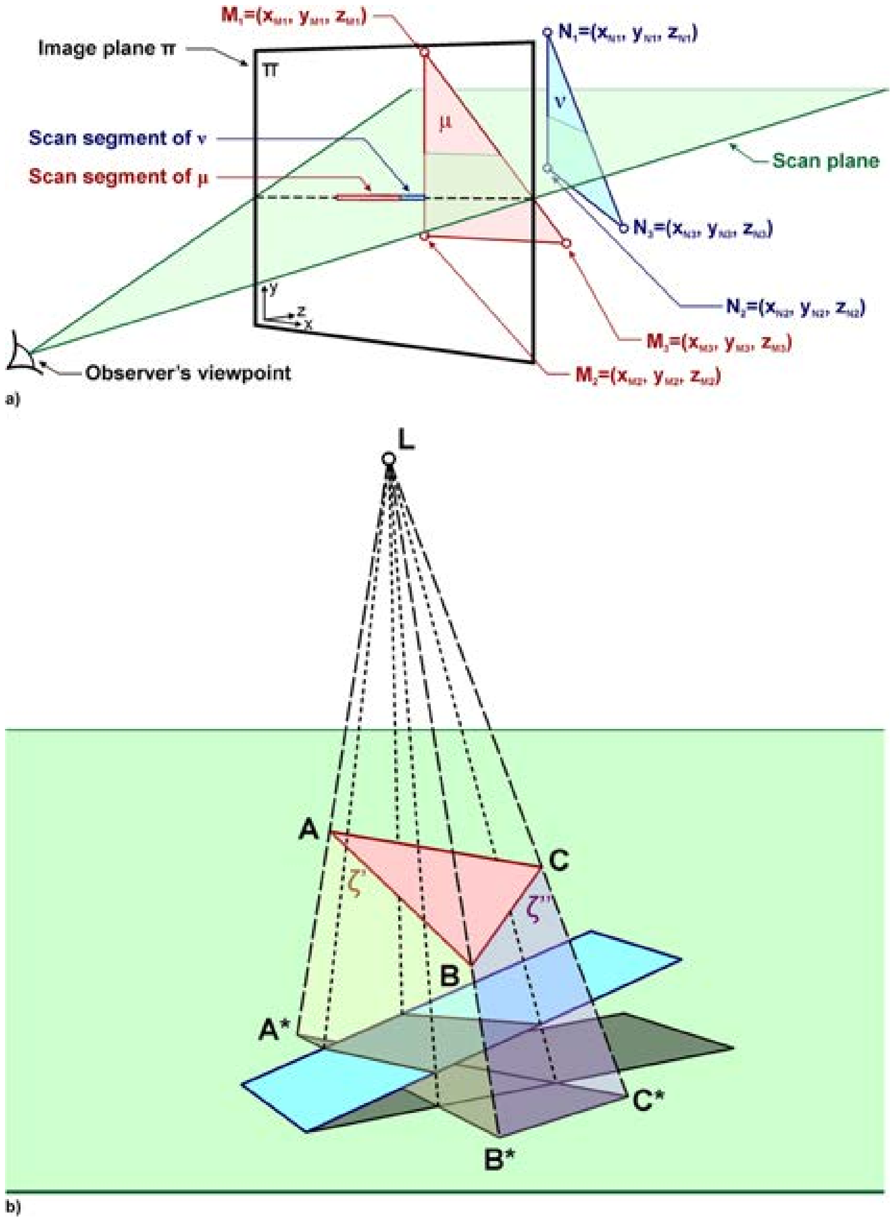

Fig. 2 Operation of rendering algorithms based on optical geometry. **a** Special translators rasterize just the portion of model included in the field of view, considering only polygons facing the observer and not covered by other objects. They then generate, for each polygon, a plane equation of type $z'=f(x'_e, y'_e)=a'x_e+b'y_e+c'$, i.e. a projection of the same polygon onto the observer's image plane. **b** For each edge of the polygon, a plane ζ is determined by the edge itself and the light source. We then consider two half-spaces with reference to ζ and exclude from computation those pixels that do not belong to the half-space in which the polygon casting the shadow is located. Iterating this process for each edge of the polygon, its shadow *frustum* is thus constructed

A simple algorithm for simulating transparency includes just a linear intensity function, but this is very inaccurate for curved surfaces (Fig. 3a). Ideally, transparency should depend on how much material light has to pass through (Crow 1978; Enderton et al. 2011; Kay and Greenberg 1979). Preliminary approximation for curved surfaces can be achieved by ensuring that transparency decreases near the edges (Fig. 3b).

Realism of simulated images can be improved by modelling refraction effects. An acceptable approximation can be obtained by making two simplifying assumptions: (1) when an object is hit by visual rays refracted by a material, its image is projected onto the surface of the material following the visual rays backwards, towards the observer. Then, the refracted projection of the object undergoes a homographic transformation as it passes from one surface to another; (2) all surfaces are considered to be directly exposed to light sources, even though they may be shadowed by other translucent surfaces.

When a visual ray strikes a surface at a given point, both unit vectors describing the direction of the surface normal and the direction vector of the ray itself are known. Snell's law describes the change in direction of a ray when it passes through the boundary between two media of different densities (Fig. 4a).

By approximating the index of refraction of air to the same degree as that of a vacuum space, finding the image of a body projected onto a refracting material becomes a problem of projection and intersection. Then, to trace the path of a ray through a medium, we must define the thickness of each surface. Kay and Greenberg's approach assumes that thickness is constant along the normal direction, at every point on a surface, and measures it by taking as positive orientation the versor of an outgoing vector from the observer (Kay and Greenberg 1979; Osman et al. 2003).

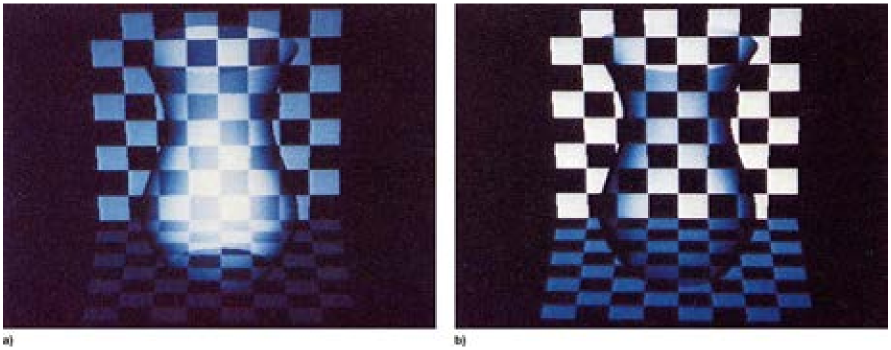

Fig. 3 Comparison of the effects of a linear and a non-linear algorithm for digital transparency rendering. **a** Rendering of a 50% transparent vase using a linear transparency function (image from Kay and Greenberg 1979: 159). **b** Rendering of a vase with a transparency power factor unity. Regardless of environment and scene configuration, transparency ultimately depends on local orientation of the surface. Different exponents can be used to simulate transparencies typical of several materials (image from Kay and Greenberg 1979: 159)

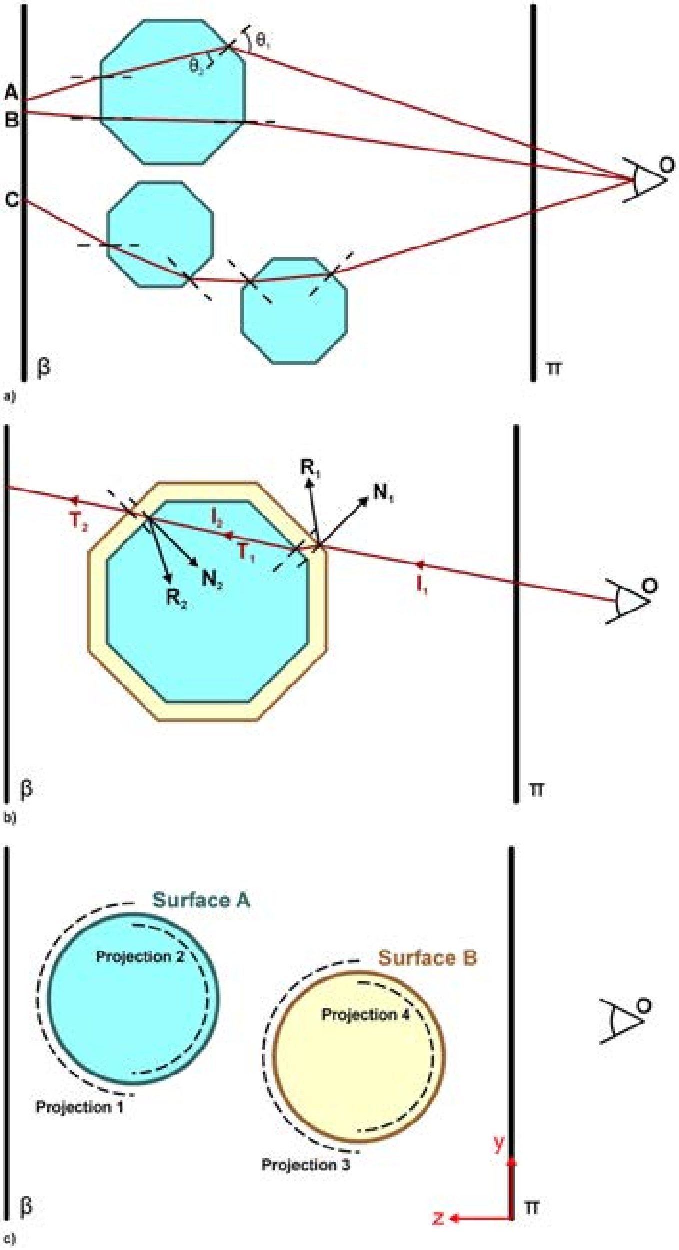

Fig. 4 Basic rules for ray tracing operations. **a** Even the simplest refraction algorithm must follow Snell's law. Sight rays from observer *O* pass through image plane π, refract through the object and strike the background β at points *A*, *B*, and *C*. **b** In order to find the new direction of the refracted ray in space, both vectors are rotated so that they lie in a plane, in order to change the angle between them according to Snell's law. The refraction vector $\underline{R}$ will be given by: $\underline{R}=\underline{S}+(1-S^2)^{1/2}\underline{N}$. **c** Kay and Greenberg's algorithm defines additional surfaces in the direction away from the observer. The processing of the scene between one surface and another must be carried out from background to foreground, going towards the observer (in the opposite direction to the *z*-axis)

The units of length relative to thickness are arbitrary but, once defined, we will have to refer to these units of measurement to express the material transparency (Fig. 4b, c).

Some of Escher's prints feature not just full manual control of chiaroscuro, but also a simplified yet rigorous application of the above-mentioned rules regarding transparency and refraction. This anticipation was made possible by his typical order and consistency, as well as by a search for appropriate grey tones by taking advantage of lithographic ink dilution and mezzotint range of nuances (Griffiths 1996: 83–108).

In a significant part of Escher's drawings and prints, three-dimensional geometric representations are often associated with inanimate objects such structures, artefacts, and crystals; freehand techniques are instead employed for animated or otherwise organic entities. We can correlate this dualism with one of the artist's key themes: searching for a fabric of mathematical and geometric perfection in an apparently chaotic universe (Bool et al. 1982; Bussagli 2004).

The *Synthesis* lithograph from 1947 depicts an alembic still in isometric orthogonal axonometry, placed on a central platform between four other smaller platforms. We find a reflection of a window on the still, probably a window of Escher's atelier. It is evident that the author used a material reference during execution of optical effects given by the alembic, including distortions of the platform beneath the layers of glass (Fig. 5a).

The *Crystal* mezzotint, also from 1947, depicts a crystalline polyhedron in perspective. The most striking features of this artwork are the graphic rendering of various levels of transparency and the refraction effects, despite the fact that the latter are limited to illumination of the crystal faces and do not imply any distortion of the background (Fig. 5b).

In the 1950 lithograph *Contrast (Order and Chaos)*, which is an axonometry, the idea of order is represented as a translucent stellated dodecahedron enclosed by an even more transparent sphere, while the concept of chaos is depicted by useless, broken objects reflected in the central solid. In addition to a complex treatment of transparency levels, refraction effects this time not just affect illumination of the solid faces, but also distortion of the images of the surrounding bodies, including the window of Escher's atelier (Fig. 5c).

In the lithograph *Order and Chaos (Compass Rose)*, dated 1955 and executed in orthogonal projection, a translucent dodecahedron is characterised by transparency and light refraction, but its surface is free of shadows, reflections, or refraction effects that should be caused by surrounding bodies (Fig. 5d).

In our opinion, the four prints by Escher discussed above demonstrate the possibility of applying the effects of transparency, refraction, and reflection with mathematical rigour even in manual geometric drawing for Architecture. To test our hypothesis, we compared a photomontage by Curt Rehbein of Mies van der Rohe's Modell Glashochhaus, a 1922 project that was never realised (Fig. 6a), with a perspective drawn manually by us (Fig. 6b) and to be treated in watercolour to achieve the optical effects of glass. We derived this process from Escher's techniques and the graphic algorithms that we reviewed. As with chiaroscuro, the

Fig. 5 Four prints by M. C. Escher characterised by remarkable effects of transparency, reflection, and refraction. **a** M.C. Escher's *"Synthesis"* © 2023 The M.C. Escher Company-The Netherlands. All rights reserved. www.mcescher.com. **b** M.C. Escher's *"Crystal"* © 2023 The M.C. Escher Company-The Netherlands. All rights reserved. www.mcescher.com. **c** M.C. Escher's *"Contrast (Order and Chaos)"* © 2023 The M.C. Escher Company-The Netherlands. All rights reserved. www.mcescher.com. **d** M.C. Escher's *"Order and Chaos (Compass Rose)"* © 2023 The M.C. Escher Company-The Netherlands. All rights reserved. www.mcescher.com

achievable realism depends on the fineness of our discretization and the number of iterations performed.

Conclusion

Geometrical optics, especially through Lambert's cosine law, guided the methodological development of chiaroscuro for manual geometrical drawing. Even procedures for rendering digital three-dimensional models are derived from geometrical optics. We focused mainly on methods for eidomatic modelling of

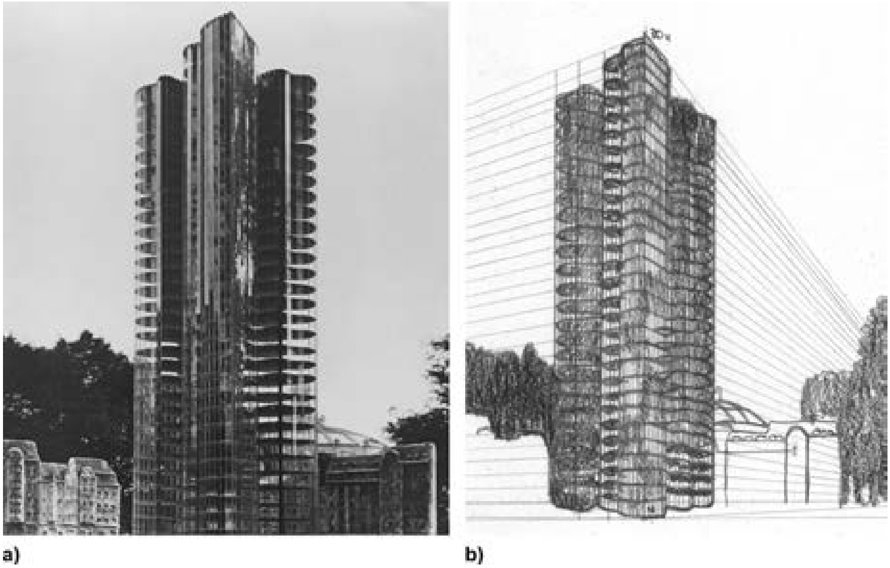

Fig. 6 Two different representations of L. Mies van der Rohe's Modell Glashochhaus (1922). **a** Photomontage of L. Mies van der Rohe's architectural model by Curt Rehbein. Photo kept at the Berlinische Galerie Museum of Modern Art (repro: Anja Elisabeth Witte/Berlinische Galerie). **b** Preliminary sketch of the manually executed perspective of L. Mies van der Rohe's architectural model

transparency and refraction effects, correlating them with notions of analogue chiaroscuro. Then, we have critically discussed four prints by Escher which demonstrate the possibility of applying, in a simplified manner, the procedures for representation of transparency, reflection, and refraction previously reviewed. We then reconstructed Escher's method by adapting it to watercolours and applied it to a manual perspective drawing, using a photo of a material architectural model as a reference. The realism of optical effects, dependent on our schematisation fineness and the number of iterations undertaken, can be assessed by comparing the drawing with the original photo.

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Data availability Data sharing is not applicable: we do not analyse or generate any datasets, because our work proceeds within a theoretical and mathematical approach.

Declarations

Conflict of Interest The authors declare no conflict of interest.

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