Integration of two theoretical lenses to analyse the potentialities of a practice-based task in fostering pre-service mathematics teacher specialized knowledge

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This paper presents an a priori analysis of a practice-based task designed to be implemented in the context of primary school pre-service mathematics teacher education. We integrate two different theoretical lenses aimed at describing teachers' practices and knowledge. By means of our analysis, we highlight the potentialities of the task to promote the goals of the educational program and to point out pre-service mathematics teacher specialised knowledge.

Keywords: Pre-service teacher, MTSK model, meta-didatical praxeologies, practice-based tasks, mathematics teacher specialised knowledge.

Background

This paper focuses on a specific task that was designed to be implemented in the context of pre-service mathematics teacher education. As Liljedahl et al. (2009) stressed, pre-service mathematics teachers (in the following, PMTs) live a unique experience within their educational paths, since

> they are both student and teacher, and through the constant shifting between student and teacher they are given the opportunity [...] to recast their initial (pre-conceived) beliefs about what it means to be a teacher, what it means to teach, what it means to learn, and even what it means for something to be mathematics (Liljedahl et al., 2009, p.29).

Researchers in the field of mathematics teacher education have stressed on the importance of focusing on practice-based approaches to mathematics teachers' professional development (Ball & Cohen, 1999), which aim to connect the ongoing professional development of teachers with the actual work of teaching. We think that these approaches could be particularly effective in the case of PMT education, since they could foster PMTs' reflections on teaching practice, filling the gap due to their lack of real classroom experience (Cusi & Morselli, 2018). In fact, by focusing on tasks aimed at fostering "activities that are situated in and organized around components and artifacts of instructional practice that replicate or resemble the work of teaching" (Silver, 2009, p.245) – the so called practice-based tasks. These approaches foster the development of a "useful and usable knowledge that builds mathematics teachers' capacity for the kinds of complex, nuanced judgments required in mathematics teaching" (Silver, 2009, p. 246). Therefore, teachers need to acquire and develop both subject matter knowledge and general pedagogical knowledge for teaching, but, as already highlighted in the 1980s by Shulman (1986), teachers' knowledge is characterised by the combination and amalgam of content knowledge and knowledge about teaching, students and curricula. This characteristic knowledge is defined by Shulman as "pedagogical content knowledge (PCK): "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986, p.9). Starting from Shulman's studies, different models to describe mathematics teachers' knowledge have been developed in the last decades: for example, the Mathematical Knowledge for Teaching - MKT model (Ball, et al. 2008), the Knowledge Quartet (Rowland, et al., 2005) and, more recently, the Mathematics Teacher's Specialised Knowledge-MTSK model (Carrillo-Yañez et al., 2018). In our study we referred to the MTSK model to describe some aspects of the specialized knowledge of PMTs. Teachers used/needed specialized knowledge to do their work and this specialized nature of their knowledge is linked to teaching:

> Our starting point is the assumption that in order to carry out their role (including lesson planning, liaising with colleagues, giving lessons and taking time to reflect on them afterwards) the teacher needs specific knowledge. We associate this specificity with mathematics teaching (Carrillo-Yañez et al., 2018, p. 239).

In this paper we present a specific type of practice-based task and we analyse the PMTs' specialised content knowledge that can emerge. The task combines the design of fictional classroom discussions representing virtual dialogues between a teacher and his/her students with the explicit request of justifying this design by means of specific theoretical lenses introduced during the teacher education courses. The activities fostered through this kind of task represent a fundamental component of a methodology for PMT education aimed at fostering PMTs' reflective practices (Jaworski, 2004), by actively involving them in the analysis of practice through the theoretical lenses provided by research (Cusi & Malara, 2016; Cusi & Morselli, 2018). To develop the *a priori* analysis of this type of task (in the following, FCD task, acronym for "fictional classroom discussions task"), we will integrate different theoretical lenses that are presented in the next section.

Analytical framework

The analytical framework is constituted by two main components. The first component is the Meta-didactical transposition (MDT) model (Arzarello et al., 2014). Based on Chevallard's Anthropological Theory of Didactics (Chevallard, 1985), this model was born to describe and analyze the evolution of mathematics teachers' and didacticians' practices within institutional contexts, when they are jointly engaged in professional development programmes or collaborative research projects (Arzarello et al., 2014). We use the term didacticians as "people from the university with knowledge of research and theory in the didactics of mathematics, interested to work with teachers to promote better opportunities for mathematics learning in classrooms." (Jaworski, 2012, p. 623).

In tune with Chevallard's framework, the MDT model focuses on the notion of praxeology, a tool to model the human activities developed within institutional contexts. A praxeology is structured in two main levels (García et al., 2006): the *praxis* or *know-how level*, which includes the task, or a family of tasks, and the techniques used to face the task; the *logos* or *knowledge level*, which includes the "discourses" developed to justify or frame the techniques for the task. The MDT model distinguished between: *didactical praxeologies*, which refer to tasks related to the knowledge to be taught and the technique being recognized and justified within a specific institution; and *meta-didactical praxeologies*, which focus on teachers' and didacticians' meta-level reflections on contents to be taught and corresponding didactical praxeologies (Arzarello et al., 2014). We chose to refer to the MDT model since: (a) it focuses on the role played by the meta-level reflective practices developed by communities of teachers and didacticians involved in professional development programs or collaborative research projects; and (b) it acknowledges teachers and didacticians' reciprocal influences when they work together in such contexts.

The second component of our analytical framework is aimed at describing PMTs' knowledge. We have chosen to use the interpretive lenses drawn from the MTSK model (Carrillo-Yañez et al., 2018) in order to describe the mathematics teacher specialized knowledge. This model is suitable for the analysis we want to develop since it focuses on the knowledge that teachers may use/need for the analysis and design of educational activities. We chose the interpretative tools provided by this model to understand and interpret teachers' praxeologies focusing on knowledge level. The starting assumption of the MTSK model is that teachers need specialized knowledge to fulfil their role. Therefore, knowledge developed and implemented for teaching is considered specialized. Inspired by Shulman's (1986) studies, the MTSK model distinguishes between mathematical knowledge (MK) and pedagogical content knowledge (PCK), both of which are considered as sub-domains of the teacher's specialized knowledge. The MTSK model describes three sub-domains of mathematical knowledge: Knowledge of Topic – KoT (e.g. knowledge of definitions, properties, procedures, representations, and applications of mathematics); Knowledge of the Structure of Mathematics – KSM (e.g. knowing how to connect activities in different domains of mathematics); and Knowledge of Practices in Mathematics – KPM (e.g. knowing how to prove, justify, define, make inferences and inductions, give examples and counterexamples). Pedagogical Content Knowledge is divided into three sub-domains: Knowledge of Mathematics Teaching – KMT (e.g. knowledge of theories of mathematics teaching or knowledge of teaching resources, materials and technologies, but also knowledge of strategies for introducing and representing contents and concepts, etc.); Knowledge of Features of Learning Mathematics – KFLM (e.g. knowledge of theories of mathematics learning or knowledge of the way in which pupils interact with mathematics); and Knowledge of Mathematics Learning Standards – KMLS (e.g. knowledge of expected learning outcomes and teaching goals in different school segments). The MTSK model in addition to detailing these subdomains of Mathematical Knowledge and PCK explicitly highlights the centrality of teachers' beliefs about mathematics and mathematics teaching-learning.

In the study presented in this paper, we integrate the theoretical lenses belonging to the MDT and MTSK models to develop an *a priori* analysis of a specific FCD task. The *a priori* analysis is performed in order to highlight the potentials of the FCD task in: (a) fostering the educational goals of the professional development programme within which the FCD task has been implemented; and (b) bringing out, consolidating and developing different aspects of PMTs' specialized content knowledge. As regards (a), we refer to the MDT model to frame the educational context in which this task has been implemented, characterizing, on one side, the praxeologies that guided the didacticians' design of the task and, on the other side, the praxeologies that PMTs have to activate in order to face the task. As regards (b), we use the MTSK model to characterize the different aspects of PMTs' specialized content knowledge that can arise when PMTs face the FCD task.

An example of FCD task

The example we present in this paper refers to the context of primary school PMT education. The FCD task on which we focus has been implemented within a 48 hours course for PMTs enrolled at the first year of the master-degree course “Primary education sciences” at Sapienza University of Rome. The course, named “From arithmetic to algebra. From algebra to arithmetic” was aimed, in tune with the studies presented in the background section, at making PMTs develop reflections, in an integrated way, on specific mathematical contents, mathematical processes and on specific pedagogical aspects of mathematics teaching-learning. The main mathematical contents, on which the course was focused, are: the use of algebraic language as a thinking tool; the different meanings of the equal sign; the construction and interpretation of mathematical representations and tools (such as tables, diagrams, graphs…); the study of sequences, relations and functions. As regards the mathematical processes, during the course PMTs had to opportunity to experience and reflect on generalization, argumentation, problem solving and posing. Finally, the main pedagogical aspects of mathematics teaching-learning that were discussed are: the possible approaches to early algebra; the use formative assessment in mathematics; the design of laboratorial activities; the role of the teacher in guiding classroom discussions. All the reflections, in tune with the MDT model, were always developed by referring to institutional aspects, such as the National Guidelines. PMTs faced different FCD tasks during the course together with other kinds of practice-based tasks that involved PMTs in the role of future teachers (classroom tasks, analysis of students’ written answers, and videos from real teaching experiments) and other laboratorial activities that involved PMTs in the main role of learners (activities focused on numerical explorations, conjecture and proof and on problem solving). The FDC tasks we are going to analyse was carried out at the end of this course. PMTs, at that time, had followed two other courses focused both on mathematics and mathematics education, for a total number of 100 hours.

The example of FCD task for PMTs analysed in this paper (Figure 3) requires to: (1) design an excerpt of a fictional classroom discussion, focused on a specific task for students (Figure 1), starting from a collection of six real students’ written answers (the translation of two of these answers is presented in Figure 2); (2) organize the discussion by selecting the students’ answers to be discussed, grouping them according to their characteristics and identifying the order in which to discuss them; (3) justify the choice made when designing the fictional classroom discussion, making explicit reference to the theoretical constructs introduced during the course.

Giovanni and Francesco have prepared a game for us, by constructing this three-floors pyramid. Complete the pyramid, explaining how you reasoned to identify the numbers to be put in the empty bricks.

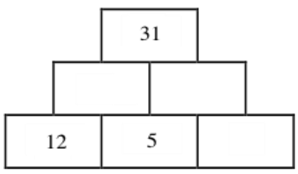


Figure 1: The task for students on which the FCD task is based

The task for students (Figure 1) is part of a sequence of tasks. During the work on the previous tasks, the students have already discovered the relationships between the numbers on the bricks that constitute a mini-pyramid (a pyramid of three bricks), that is “the number on the brick at the top of each mini-pyramid is the sum of the two numbers on the bricks at the base of the mini-pyramid”.

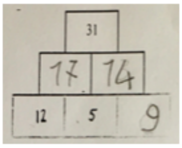
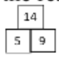
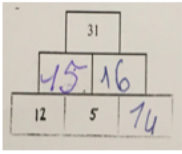
	We first made this pyramid:  Then we made the second pyramid. Since there were not two bricks [*they mean that they do not know one of the numbers at the base of this mini-pyramid*], we searched for the number to find out 31. We unified 14 with 17 and the result was 31. Then we made 14-5, which is 9; so, we finished also this pyramid: 
	In the first line we found out the number 31. Then we formed the same number in the other two [*lines*].

Figure 2: Translation of two of the real students' written answers on which the FCD task is based

The translation of the text of the FCD task (except the problem in Figure 1 and the real students' written answers in Figure 2) is presented in Figure 3.

Some students of a 2nd grade class have faced the following **problem** during a laboratorial activity. (*Here the text of problem in Figure 1 is inserted*) These are the **real answers** written by six of these students. (*Here the six students' written answers are inserted – two of them are in Figure 2*) **Design, by referring to the theoretical lenses introduced during the course, an excerpt of a fictional classroom discussion starting from these six students' answers.** Within the excerpt of the fictional classroom discussion, insert both the teacher's and his/her student' hypothetical interventions. The **aims of the discussion** are: (a) to compare the different argumentations proposed by the students; (b) to collectively reflect on strategies and mistakes; (c) to collectively construct representations aimed at making these strategies explicit. With these aims in mind, choose how to organize the phases of sharing and reflections on these written answers. For example, you can choose to discuss only some of them, to change the order in which to present the written answers, to group the answers according to the foci of the reflections you want to develop during the discussion.

Figure 3: Translation of the text of the FCD task

During the course different kind of data have been collected: the PMTs' written answers to the FCD task (that is the fictional classroom discussions they designed and the justifications of this design), videos of collective discussions between the didactician and the PMTs, PMTs' final reflections on their experience within the course (with a focus on each activity in which they were involved).

A priori analysis of the FCD task

The data collected during the course could be analysed at different levels: (1) the level of the *a priori* analysis of the FCD task as a tool for PMT education; (2) the level of the analysis of the excerpts of the fictional classroom discussions designed by PMTs and of the ways in which this design was justified referring to the theoretical tools introduced during the course; (3) the level of PMTs' reflections on the role played by their work on the FCD task in their professional development. In this paper we focus on the first level of analysis.

In this paper show an *a priori* analysis of the FCD task presented in the previous paragraph. We frame the activity on FCD tasks by using the theoretical lenses presented in the analytical framework. We refer to the MDT model to characterize the different practices, developed by the didacticians and the PMTs, in relation to this FCD task. First of all, we focus on the *praxeology of the didactician* who have conceived and designed the FCD task within an educational program for PMTs of primary school. This praxeology is related to the *task* of making PMTs experiment the activity of design of classroom discussions starting from students' answers. The adopted *technique* consists in the FCD task design itself. The *logos component* of this praxeology is constituted by the theoretical references that frame the educational program and support the justifying discourses behind the choice of focusing on methodologies for teacher education aimed at fostering PMTs' reflective practices. In relation to this, a fundamental element of the logos component is represented by the research studies focused on the role played by theory as a tool to support practice. As concerns the *praxeologies developed by the PMTs* in their role as authors of fictional classroom discussions, the *task* to which these praxeologies refer is that of designing a classroom discussion focused on a specific task for students and on specific students' written answers. The *technique* to face this task is characterized by different processes that have to be realised: the analysis of the task for students, the analysis of the students' written answers and the identification of the possible interventions that the teacher and her students could make during the classroom discussion. The *logos component* is constituted by the different theoretical lenses shared with PMTs during the whole course. It must be added that the didacticians' choice of the task for students on which the FCD task is focused (Figure 1) and the aims of the classroom discussion to be designed make PMTs direct their attention also on specific aspects related to different mathematical and pedagogical contents faced during the course (for instance, the role of argumentation in mathematics, early algebra, formative assessment…). Other elements could also be part of the logos components of PMTs' praxeologies, such as the PMTs' (mathematical and not mathematical) previous knowledge and their beliefs about teaching and about the mathematical content on which the task for students is focused. In order to analyse which aspect of specialized content knowledge might emerge when pre-service teachers face this task, we use the MTSK model. In the analysis of the task for students, the students' answers and the possible discussion about them, PMTs can use their KoT about: the properties of natural numbers and operations; the additive relation; the procedural/relational meaning of the equality symbol. With regard to KSM, their knowledge about relations among number sets and about pre-algebra should emerge. The role of examples and counterexamples in the production of hypotheses in arithmetic problems and the possibility of using different possible arguments are part of KPM. The task could make the PMTs deeply reflect on different and effective representations that can be used to work with students to explore numerical relations or artefacts and meaningful activities concerning problem solving (KMT). The different procedures that students might carry out or students' possible errors as well as the difficulties in producing arguments can be framed in the KFLM domain. The examples of students' answers (Figure 2) are useful for the development of shared reflections on students' argumentative processes and on the arithmetic relationships they can identify. It is important to be aware that the pyramid task aims not only at implementing computational schemes but, above all, at fostering problem solving and argumentation processes. These processes are key issues in the goals for the development of competences written in Italian National Guidelines (Standards) that PMTs have to know (KMLS). This detailed analysis of PMTs' mathematical knowledge and pedagogical content knowledge allows us to enhance the MDT model with regard to the description of the *logos component* of PMTs meta-didactical praxeologies.

Conclusion

In this paper we have presented a specific practice-based task – the FCD task – designed to be implemented within courses for PMTs. By integrating two different theoretical lenses, we developed an *a priori* analysis of the FCD task aimed at highlighting the potentialities of this kind of task in terms of promotion of both the educational goals of the course within which it was implemented, and development of PMTs' specialised content knowledge. In particular, the MDT model enabled us to reflect on the meta-didactical praxeologies activated by the protagonists of the educational process under scrutiny: on one side, the praxeologies that guided the design of the FCD task by the didactician; on the other side, the praxeologies activated by PMTs during their work on the FCD task. The MTSK model constituted the lenses through which we deepened the characterization of the logos component of the PMTs' praxeologies, by focusing on the specialised content knowledge that could emerge and be consolidated by means of the examined task. We detailed, for each of the subdomains of the MTSK model, what knowledge can emerge when PMTs face this task.

The results of our analysis could have both practical and theoretical implications. At the practical level, by highlighting the potentialities of FCD tasks, our analysis confirmed the effectiveness of the criteria that guided the design of this kind of tasks: focusing on meaningful mathematical problems (to foster the activation of mathematics teachers' specialized knowledge); asking to design fictional classroom discussions starting from students' real written answers (to promote PMTs' reflections on aspects related to different mathematical and pedagogical contents faced during the course); asking to justify the fictional classroom discussions' design by referring to specific theoretical lenses (to better trigger PMTs' reflective practices). At the theoretical level, our analysis shows that the integration of the MDT and the MTSK model was effective in highlighting both the educational aims connected to the design of FCD tasks and the possible results of the implementation of such tasks in terms of potential emergence of PMTs' specialised content knowledge.

In this paper we focused only on the *a priori* analysis of the FCD task. As a further step of our study, we will focus on the different data collected during and after the implementation of the FCD task to perform other levels of analysis in which we will continue to interweave theoretical lenses from the MDT and MTSK models.

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