Cooperative learning in game theory activities

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The purpose of our paper is to analyse the effectiveness of group work and the role of the emotions in game theory activities. Such a setting, unfamiliar to most of the students, allows to build activities with more than one solution and more than one solving strategy. In particular, the activity concerning a cooperative game is reported and, following Kagel and Cooper's studies, it is highlighted how students involving in groups activities have a more strategic and successful approach than those who tackle them individually. One of the aims of this paper is to study the change of the vision of the mathematics after these activities, using the lenses of the Di Martino and Zan framework. Furthermore, this research looks at the metacognitive activities that are stimulated during the resolution and how they lead to the construction of shared knowledge.

Keywords: Cooperative learning, metacognition, game theory.

Introduction and theoretical framework

Problem solving and argumentation are essential skills, and they are one of the objectives of many educational systems (i.e., OECD PISA, 2021). These processes can be supported by collective work, that if it is done correctly, it triggers the process of collective metacognition that leads to the construction of shared knowledge. This study was carried out on game theory activities because this context allows us to analyse the students' ability to apply the knowledge acquired during their studies in unfamiliar situations (Antonini, 2019). This choice is justified by the desire to create a conducive environment to the development of the problem solving and the argumentation processes (Cramer, 2014).

Schoenfeld's research shows that in order to solve a mathematical task correctly, there is a need to apply control processes. These processes involve understanding the text of the task, planning a strategy, controlling the situation, and managing one's resources. Schoenfeld highlights the importance to be aware of one's resources to be able to manage them. Furthermore, he analyses causes of failure or successful in activities of problem solving. The central elements are the beliefs and the individual's perception of themselves. The beliefs depend on previous experiences with mathematics, and they influence the individual's ability to use his or her knowledges (Schoenfeld, 1983). Beliefs, emotions, and attitude are very important elements in approaching mathematics; in fact, they are also essential in decision-making process. Emotional experiences are given by the combination of a cognitive and a psychological insight: the interruption of actions or a difference between facts and expectations can lead to an arousal (a general state of activation of the nervous system in response to internal or external stimuli). The emotional experience of the individual is given by the arousal and the formative evaluation of the experience. It is therefore clear that it is not the experience itself that arouses emotions, but the subject's interpretation of the situation, which depends on the individual's beliefs.

Di Martino and Zan highlight the importance of emotional component of the beliefs, in fact, the same belief can arouse different emotions in different individuals. They underline the need to use tools able to investigate the structure of the beliefs' system and to bring out the link between cognitive and emotional component. To analyse the data that emerged from their research, a model consisting of three dimensions was used (Di Martino & Zan, 2011): emotional disposition towards mathematics, perception of one's abilities and vision of mathematics.

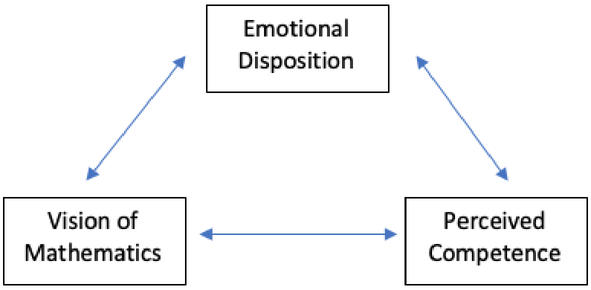

Figure 1: Di Martino-Zan three dimensional model for attitude (Di Martino & Zan, 2011)

It is possible to focus on the link between beliefs and emotions, in particular, negative emotional disposition, due to a negative approach with mathematics. A negative emotional disposition may be due to many characteristics that students assign to mathematics. This implies that there is a link between emotional disposition and vision of mathematics. The perception of one's abilities in mathematics influences the emotional disposition: in fact, the idea of successful in mathematics affects emotional disposition and thus the perception of one's competences.

There are many conceptions about idea of successful:

- successful identified with perception of the knowledge of rules and their correct application;
- successful identified with perceived knowledge of the meaning of the rules and their connection;
- successful identified with scholar successful.

The failure is often due to the link between negative emotional disposition and low perceived competence. Weiner classified the causes of the failure into (Weiner, 1986):

- local inside/local external: inside local causes depend on the subject, while external local causes depend on exterior features;
- stable/instable: whether or not it is possible to change them over time;
- controllable/uncontrollable: whether depend on the beliefs of subject.

Negative emotional disposition is a cause of failure in mathematics for some students. Low perceived competence and negative emotional disposition may be two independent dimensions: some students have a high perceived competence, but a negative emotional disposition and vice versa.

The development of activities in the game theory makes it possible to highlight how a collective performance promotes the activation of metacognitive processes to a greater extent. According to Kagel and Cooper (2005), those who tackle tasks in groups act more strategically than those who carry out them individually. Kagel and Cooper (2005) state that communication between individuals is a possible explanation for the more strategic approach taken by groups. Therefore, through confrontation with others, more ways to resolution were explored.

Cooperative learning is an approach in which small groups are created with students of different ability levels. The members of each group work together to achieve a common goal. The positive aspects of cooperative learning are group goals or positive interdependence, in which students must work together to achieve group success, and individual accountability. In other words, we mean the responsibility of everyone for the success of the group, which involves the motivation of each student to help to achieve the success of the whole group (Slavin, 1990). Efficient group work maximises the learning of each student.

Furthermore, organisational, cognitive, and metacognitive factors must be present for a group to successfully tackle problem solving activities. (Chalmers, 2009). Tuckman and Jensen's model of organisational factors consists of five phases that should be developed in the group activity: forming, storming, norming, performing and adjourning (Tuckman & Jensen, 1977). The first phase involves the setting up of the group work; in the second phase there is a comparison between the group members and the third phase is reached when a meeting point is found between the various ideas proposed. In the fourth phase the group members work together and in the last phase the work is reviewed. On a cognitive level, students need to develop shared knowledge in order to successfully complete the group work. Metacognition is defined as the student's awareness of her or his learning process; in the case of collective metacognition, each participant must be aware of his or her own and other group members mental processes through discussion and comparison. Collective metacognition requires that strategies of orientation, planning, monitoring, evaluation, reflection and elaboration are developed. Orientation is carried out before the problem occurs. Planning, monitoring and evaluation take place during the resolution of the task, while reflection and elaboration take place at the end of the task. These actions allow students to structure a solution strategy, monitor the processes involved, evaluate and interpret the results obtained and express the solution (Van der Stel et al., 2010). Comparison, along with the presentation of different points of view and questions from the group members help the construction of shared knowledge, allowing any individual difficulties to be identified and overcome. Each group member has their own solution strategy before determining the joint one, but previous studies have shown that groups perform better than they would have done individually (Frith, 2012).

By comparing with other group members, it is possible to produce different solution strategies to reach the final goal. Cooperative work, therefore, makes it possible to develop interaction between students, integration and to improve self-esteem (Slavin, 1990). Collective metacognition, moreover, helps to reduce the individual anxiety of failure by distributing it among all group members. The existence of a shared solution increases motivation to carry out such activities. Difficulties that may be encountered during a group activity are communication between individuals and cultural differences, aspects that may compromise collective metacognition (Chiu & Kuo, 2009).

Furthermore, one purpose of this research is to analyse, through two questionnaires, students' emotions and beliefs regarding mathematics and their vision about this discipline. It is important to understand the presence of a change about the student's vision on mathematics after carrying out activities.

Methodology

This experiment consists of two activities carried out within the game theory. One of the objectives of this research is to stimulate problem solving and argumentation processes, to make people take on different points of view, and to promote group activity and comparison between peers. At the end of each activity, an interview was conducted in order to capture the difficulties encountered during the task and some impressions regarding the group work. In addition, two questionnaires were proposed to the students: one at the beginning of the first task and one at the end of the second task. One of the purposes of the two questionnaires was to investigate the students' emotions and their vision on mathematics, and the second questionnaire also asked questions focused on the tasks carried out. The second questionnaire, moreover, analyses the change about student's vision on mathematics. Two questionnaires were created following Di Martino and Zan's researches (Di Martino & Zan, 2011): there were open questions to leave students free to express their emotions and closed ones to analyse specific aspects. Two questionnaires were compilated individually, while the interview was conducted during the final discussion about the activities' results.

Participants who took part in the experiment were 81 secondary school students, grade 9, 11 and 13, of the Italian education system (14-, 16- and 18-years old students). Three classrooms participated to experiment:

- classroom of grade 13 is composed by 29 students. In this classroom half of the students solved tasks individually;
- classroom of grade 11 is composed by 25 students;
- classroom of grade 9 is composed by 27 students.

In addition to these, the two activities were submitted to a "control group" consisting of mathematicians and non-mathematicians, to investigate the similarities and differences between the approaches adopted by high school students, mathematicians, and those with no specific mathematical skills. The control group was composed by people with different ages. In the control group there were 16 individuals, 10 students of master's degree in mathematics and 6 students of no mathematical course.

The experiment was carried out entirely remotely using platforms such as Zoom and Google Meet due to Covid-19 pandemic. Thanks to these, it was possible to create virtual rooms in which students could work in groups. The class of grade 13, due to classroom attendance regulations, was half in the classroom and half at home, so the task was carried out individually by those in the classroom and in groups by those remotely located. These roles were reversed in the second task. The control group took the task individually.

Each task took about two hours to complete. One hour was left for the students to solve the task in groups or individually. The remaining time was devoted to a collective discussion in which the students were able to compare and share the solutions they had arrived at and the reasoning for applying a particular strategy. During the discussion we noted students' statements to analyse their process of problem solving. In the following section we report some students' statements taken by interview's notes and answers to the questionnaires.

Analysis of the collected data

In this paper we focus on the data collected on the first task. The text is the following:

"In a shopping centre there are three shops, AltaModa, BlueJeans and CookLover, which need a new lighting contract. They have been offered several alternatives: if they take out the contract individually, they will pay €250, €200 and €350 per month respectively; if they decide to take out an overall contract, they will pay €600; alternatively, if they agree in pairs, the prices will be €350 per month for AltaModa and BlueJeans, €450 per month for AltaModa and CookLover and €420 per month for BlueJeans and CookLover.

Try to explain the offer and how the three shops could agree on the best offer. Give reasons for your answers."

The table below shows the solutions proposed by the test takers.

Table 1: Proposed solutions. A, B and C indicate expenses for A, expenses for B and expenses for C. % groups and % individuals represent the percentual of students' solutions.

Proposed solutions	**A**	**B**	**C**	**% groups**	**% individuals**
Equal division of the total contract	200	200	200	47%	65,5%
Proportional division of the total contract	187,50	150	262,50	31,5%	24%
Other solutions				21,5%	10,5%

The data comparison of this experiment shows, as argued by Kagel and Cooper (2005), that groups act more strategically than individuals. Students who took the test individually were unlikely to go beyond the first interpretation, namely the equal division of the overall contract. Analysing the response frequencies of both the students and the "control group", it emerges that less than 50% of the groups gave as their solution the equal division of the overall contract. On the contrary, among those who took the test individually, more than 60% supported this solution.

Equal division of the total contract is the most intuitive answer, to which everyone approaches at first stage; some individual students and most groups manage to move away from this first idea, planning other solution strategies.

Some students believed that the equal division was fairer because everyone pays less than or equal to the single contract, while others wondered if there was a division that provided savings for everyone, and this led to the formulation of new hypotheses for solving this task. In this way, those who reflected on these aspects showed that they implemented metacognitive actions such as monitoring and evaluation, as they were able to analyse and interpret the proposed strategies in the context of the problem. These considerations were more frequent in those who worked in groups. Groups that worked efficiently went through all the stages of Tuckman and Jensen model. The groups also took longer to reach a conclusion due to a more thorough analysis of the proposed strategies.

Kagel and Cooper research suggest that the groups should be able to arrive at an appropriate solution if it is proposed by at least one member of the group. The analysis of the group discussions shows that most of the times when a member proposed a strategy different from the first interpretation, the group was able to come up with a solution that deviated from the instinctive response, i.e., dividing the total contract equally. From the discussions in the groups, it is evident that the confrontation with others allowed the emergence of different points of view that led to various solution strategies. Sometimes some members of the group disagreed with the solution proposed by the team, on which occasions the students defended their ideas. At the end of the time allotted for solving the activity, most groups proposed an agreed solution.

Thanks to the interviews and questionnaires, it was possible to ask the students about their impressions of the interaction between groups. In the class in which the two activities were carried out by both individuals and groups, it was also possible to compare the two ways of carrying them out. For example, one request of questionnaires regarded the students' vision on mathematics. This question is proposed before and after the tasks. The why of this choice is justified by the needful to investigate the change of mathematics vision after carrying out the tasks. By students' responses emerged that their mathematics vision is changed thanks to this experience.

Some students stated:

Student 1: “I think my approach is changed: greater depth and focus on different strategies, without stopping at the first insights”;
Student 2: “We often think that mathematics is a strict science with certain rules, but in this activity, we were able to observe the existence of different points of view and different solutions”.

Moreover, in questionnaires, we asked them their impressions about interaction between groups.

Some students stated:

Student 3: “Group work helps, by exchanging ideas and thoughts you eventually reach the choice that is closest to the correct one”;
Student 4: “Every idea was made explicit to the whole group”;
Student 5: “initially I did not understand the problem very well, then the other members helped me to understand it”;
Student 6: “some ideas we had thought individually were changed by the analysis of the whole group, and others emerged thanks to the collective activity”;
Student 7: “it was a good confrontation, it was a ‘thinking together’ rather than a union of individual ideas: we perfected each other intuitions by helping each other.”

From the students own words, the comparison with others was useful for a better understanding of the problem: “...they helped me to understand it”. It also emerges that working together helped them to consider more strategies “...more emerged from the collective activity”. As Frith reports in his research (Frith, 2012), each student hypothesises his or her own solving strategy, but thanks to the collective activity, a better performance is achieved, by placing each proposed strategy under collective analysis. In some groups, the pupils advised their peers to identify with the situations or to consider the activities in a real context. This approach made it possible to concretely analyse the planned strategies and consider their possible implementation in real life.

Conclusion

In this paper we analysed capacity of students to act exploration of the problem, planning of strategies to be implemented, identifying objectives to be achieved. In particular, we focused the attention on the problem solving and the argumentation process and their implementation in group activity. In most cases, the collective performance of such tests ensured, the planning of a greater number of solution strategies and a more in-depth analysis of the procedures implemented and the results obtained. Each participant observed the work of the others, which made it possible to assess the effectiveness of the strategy. Through the exposition of different points of view and the analysis carried out by each component, almost all the groups managed to build a shared knowledge. Difficult students put their doubts to their peers and thanks to the explanations, sometimes repeated, of the other group members, they overcame their critical points. From the previous statements, moreover, we can see the implementation of metacognitive activities that led to the construction of shared knowledge. In fact, thanks to the support and the analysis of each group member, they were able to refine their initial ideas and make their peers understand things that were not clear.

The students themselves acknowledged that working efficiently in a group is better than working individually:

> Student 8: "I think that by working well in groups you can do more than you would do alone".

Those who had the opportunity to work on the activities once individually and once in a group stated that they enjoyed better working collectively. The development of individual metacognition, therefore, was supported by collective metacognition. In this way, thanks the collective work to achieve a shared solution, the motivation to carry out tasks was increased. In a few groups, the work was not conducted fruitfully, with little participation by some members. In such situations, however, collective metacognition was compromised, as students did not compare, preventing the construction of shared knowledge. The data collected through questionnaires allow to analyse subjects' emotional approach in these activities. From responses to the first questionnaire, some students stated that they had a good emotion regarding mathematics, but more than 60% of the students stated: "I like mathematics because I obtain good results". This highlights a strong link between emotional disposition and the idea of successful on mathematics (Di Martino & Zan, 2011). An interesting fact, obtained by responses to the second questionnaire, is the change of the students regarding the vision on mathematics. After these activities the students have more perception about the usefulness of the mathematics in the real life and they have had the opportunity to do mathematics in a different way.

As the tasks were solved, many groups stated that they were satisfied with the work done. Collective activity, in fact, helps to reduce the individual anxiety and fear of failure, distributing it among all the components. In fact, the answers given in the first questionnaire show that many students have contrasting emotions regard mathematical activities, but these students stated, in the second questionnaire, that they valued the group work positively. These results underline how cooperative learning is a positive element for a better approach with mathematics, but also for improving the emotional disposition towards mathematics. Observing the work and discussions of the groups, it emerged that in the classes where students are used to working in groups, there was more interaction and better organisation.

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