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Detailed study of HWP non-idealities and their impact on future measurements of CMB polarization anisotropies from space (*Corrigendum*)

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In Eq. (7) of the paper we introduce the substitution

$$\theta(t) = \omega t + \frac{\psi(t)}{2}, \tag{1}$$

where ω is the half-wave plate (HWP) rotation frequency and ψ is the orientation angle of the instrument boresight. This substitution is carried through all the time-ordered data (TOD) equations. However, it is strictly valid only in the case of an ideal HWP.

By computing the Mueller matrix of the full optical chain,

$$M_{\text{full},\xi}(t) = M_{pol}(\xi) M^T_{\text{rot}}(\omega t) M_{\text{HWP}} M_{\text{rot}}(\omega t) M_{\text{rot}}(\psi(t)), \tag{2}$$

where ξ is the polarizer orientation in the focal plane ($\xi = 0$ for a polarizer at the x-axis, $\xi = \pi/2$ for a polarizer at the y-axis), we get the correct expressions of the Mueller matrix elements expanded at first order in the non ideal parameters (equivalent to Eqs. (28)–(30) in the paper):

– $h_1, h_2 \neq 0$:

$$\begin{aligned}
M^{TT}_{\text{full},\xi} &\simeq \frac{1}{2}(1 + h_1 + h_2) + \frac{1}{2}(h_1 - h_2)\cos(2\omega t - 2\xi),\\
M^{TQ}_{\text{full},\xi} &\simeq \frac{1}{2}(h_1 - h_2)\cos(2\omega t + 2\psi) + \frac{1}{2}(1 + h_1 + h_2)\cos(4\omega t - 2\xi + 2\psi),\\
M^{TU}_{\text{full},\xi} &\simeq \frac{1}{2}(h_1 - h_2)\sin(2\omega t + 2\psi) + \frac{1}{2}(1 + h_1 + h_2)\sin(4\omega t - 2\xi + 2\psi),
\end{aligned} \tag{3}$$

– $\beta \neq 0$:

$$\begin{aligned}
M^{TT}_{\text{full},\xi} &= \frac{1}{2},\\
M^{TQ}_{\text{full},\xi} &\simeq \frac{1}{4}(1 - \cos\beta)\cos(2\psi + 2\xi) + \frac{1}{4}(1 + \cos\beta)\cos(4\omega t - 2\xi + 2\psi),\\
M^{TU}_{\text{full},\xi} &\simeq \frac{1}{4}(1 - \cos\beta)\sin(2\psi + 2\xi) + \frac{1}{4}(1 + \cos\beta)\sin(4\omega t - 2\xi + 2\psi),
\end{aligned} \tag{4}$$

$-\zeta_1, \zeta_2, \chi_1, \chi_2 \neq 0$:

$$\begin{aligned}
M^{TT}_{\mathrm{full},\xi} &\simeq \frac{1}{2} + \frac{1}{2}\left(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2\right)\sin(2\omega t - 2\xi),\\
M^{TQ}_{\mathrm{full},\xi} &\simeq \frac{1}{2}\cos(4\omega t - 2\xi + 2\psi) - \frac{1}{2}\left(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2\right)\sin(2\omega t + 2\psi) - \frac{1}{2}\left(\zeta_1\cos\chi_1 + \zeta_2\cos\chi_2\right)\sin(4\omega t - 2\xi + 2\psi),\\
M^{TU}_{\mathrm{full},\xi} &\simeq \frac{1}{2}\sin(4\omega t - 2\xi + 2\psi) + \frac{1}{2}\left(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2\right)\cos(2\omega t + 2\psi) + \frac{1}{2}\left(\zeta_1\cos\chi_1 + \zeta_2\cos\chi_2\right)\cos(4\omega t - 2\xi + 2\psi).
\end{aligned} \tag{5}$$

With a non-ideal HWP with $h_1 \neq h_2 \neq 0, \beta \neq 0$ and $\zeta_1\cos\chi_1 \neq \zeta_2\cos\chi_2 \neq 0$, the substitution

$\theta(t) \to \omega t + \psi(t)/2$

in

$M_{pol}(\xi)M^T_{\mathrm{rot}}(\theta(t))M_{\mathrm{HWP}}M_{\mathrm{rot}}(\theta(t))$

(the optical chain considered in the paper) leads to an expression of the Mueller matrix elements that is not equivalent to the correct result from Eq. (2).

Nevertheless, we expect the results in the paper not to be affected, since terms modulated at $2\omega t$ are averaged out in the map-making procedure (see Figs. 3 and 4 in the paper). In fact, the M^{TT} terms (obtained with the substitution in Eq. (1)) in the paper are more worrisome than the correct ones. Indeed, terms modulated as cos / sin$(2\omega t + \psi(t))$ instead of cos / sin$(2\omega t$ + constant angle) appear in the incorrect M^{TT} elements. As an example, if we had included the monopole in the input maps, it would have been modulated to higher frequencies.

Constant terms (i.e., independent of ωt) are supposed to be filtered out in the map-making procedure.

The amended expressions for Eqs. (A3)–(A4) in the appendix are as follows. Considering a polarimeter along the x-axis ($\xi = 0$):

$$\begin{aligned}
M^{TT}_x &\simeq \frac{1}{2}\Big(1 + h_1 + h_2 + (h_1 - h_2)\cos(2\omega t) + (\zeta_1\cos\chi_1\cos\beta - \zeta_2\cos\chi_2)\sin(2\omega t)\Big),\\
M^{TQ}_x &\simeq \frac{1}{4}(1 + h_1 + h_2)(1 - \cos\beta)\cos(2\psi) + \frac{1}{2}(h_1 - h_2)\cos(2\omega t + 2\psi) + \frac{1}{4}(1 + h_1 + h_2)(1 + \cos\beta)\cos(4\omega t + 2\psi) -\\
&\quad - \frac{1}{2}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2\cos\beta)\sin(2\omega t + 2\psi) - \frac{1}{4}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2)(1 - \cos\beta)\sin(2\psi)\\
&\quad - \frac{1}{4}(\zeta_1\cos\chi_1 + \zeta_2\cos\chi_2)(1 + \cos\beta)\sin(4\omega t + 2\psi),\\
M^{TU}_x &\simeq \frac{1}{4}(1 + h_1 + h_2)(1 - \cos\beta)\sin(2\psi) + \frac{1}{2}(h_1 - h_2)\sin(2\omega t + 2\psi) + \frac{1}{4}(1 + h_1 + h_2)(1 + \cos\beta)\sin(4\omega t + 2\psi)\\
&\quad + \frac{1}{2}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2\cos\beta)\cos(2\omega t + 2\psi) + \frac{1}{4}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2)(1 - \cos\beta)\cos(2\psi)\\
&\quad + \frac{1}{4}(\zeta_1\cos\chi_1 + \zeta_2\cos\chi_2)(1 + \cos\beta)\cos(4\omega t + 2\psi),\\
M^{TV}_x &\simeq -\frac{1}{2}\sin\beta\sin(2\omega t).
\end{aligned} \tag{6}$$

The corresponding elements when the polarizer is along the y-axis ($\xi = \pi/2$) direction are:

$$\begin{aligned}
M^{TT}_y &\simeq \frac{1}{2}\Big(1 + h_1 + h_2 - (h_1 - h_2)\cos(2\omega t) - (\zeta_1\cos\chi_1\cos\beta - \zeta_2\cos\chi_2)\sin(2\omega t)\Big),\\
M^{TQ}_y &\simeq -\frac{1}{4}(1 + h_1 + h_2)(1 - \cos\beta)\cos(2\psi) + \frac{1}{2}(h_1 - h_2)\cos(2\omega t + 2\psi) - \frac{1}{4}(1 + h_1 + h_2)(1 + \cos\beta)\cos(4\omega t + 2\psi) -\\
&\quad - \frac{1}{2}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2\cos\beta)\sin(2\omega t + 2\psi) + \frac{1}{4}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2)(1 - \cos\beta)\sin(2\psi)\\
&\quad + \frac{1}{4}(\zeta_1\cos\chi_1 + \zeta_2\cos\chi_2)(1 + \cos\beta)\sin(4\omega t + 2\psi),\\
M^{TU}_y &\simeq -\frac{1}{4}(1 + h_1 + h_2)(1 - \cos\beta)\sin(2\psi) + \frac{1}{2}(h_1 - h_2)\sin(2\omega t + 2\psi) - \frac{1}{4}(1 + h_1 + h_2)(1 + \cos\beta)\sin(4\omega t + 2\psi)\\
&\quad + \frac{1}{2}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2\cos\beta)\cos(2\omega t + 2\psi) - \frac{1}{4}(\zeta_1\cos\chi_1 - \zeta_2\cos\chi_2)(1 - \cos\beta)\cos(2\psi)\\
&\quad - \frac{1}{4}(\zeta_1\cos\chi_1 + \zeta_2\cos\chi_2)(1 + \cos\beta)\cos(4\omega t + 2\psi),\\
M^{TV}_y &\simeq +\frac{1}{2}\sin\beta\sin(2\omega t).
\end{aligned} \tag{7}$$

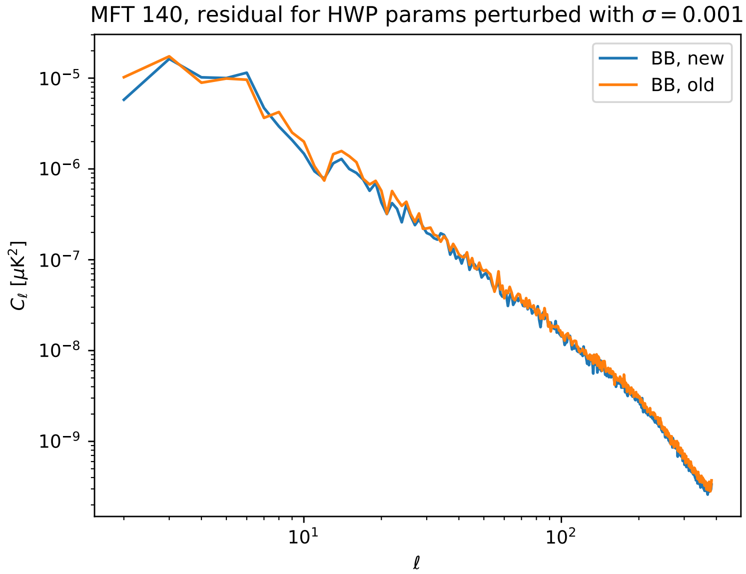

Fig. 1. Residual C_ℓ^{BB} computed with the old, wrong expressions for the Mueller matrices of the optical chain (in orange) and the corrected ones (in blue).

We computed the residual power spectrum caused by perturbing all the HWP parameters in the TOD with a Gaussian distributed error with $\sigma = 0.001$, both with the wrong $M^{TT/TQ/TU}$ expressions and the corrected ones. We followed the same procedure outlined in the paper, just considering one realization of errors, which is the same for the two output maps. The residuals are computed with respect to a template map where the HWP parameters are unperturbed in both TOD and map-making (again, we have computed two template maps with the wrong and corrected expressions). The two residual power spectra are very similar, confirming the claim that the error in the Mueller matrix expressions does not significantly affect the overall result of the work.