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DEVELOPMENT OF A SIMPLIFIED METHOD FOR THE ESTIMATION OF SEISMIC-INDUCED EXCESS PORE WATER PRESSURES UNDER UNDRAINED CONDITIONS

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Abstract

The reduction of effective stress caused by the seismic-induced pore water pressure build-up occurring in saturated soils can be a remarkable issue. To avoid catastrophic consequences, the development of a reliable but still simplified method is needed to fairly estimate the liquefaction hazard of the site at hand. This work presents an update of the *decoupled* approach firstly proposed by Seed *et al*. (1975), where the term representing the rate of excess pore pressures under fully-undrained conditions is added to the well-known 1D consolidation equation. The assumption of a uniform distribution of the equivalent number of cycles over the earthquake duration has been hereby replaced by a more realistic assumption. This hypothesis has been then tested through a series of fully-coupled, nonlinear dynamic 2D Finite Element analyses.

1. Introduction

The development of excess pore water pressures in saturated sandy soils occurring during a seismic event entails a reduction of effective stresses, with consequent degradation of shear strength and stiffness, which may lead to catastrophic consequences. Although the topic has been addressed by several researchers over the last fifty years, reliable but easy-to-use procedures to evaluate the pore water pressure build-up in liquefiable sandy soils are still lacking.

In the literature, build-up of pore water pressure is typically evaluated via two different methods, namely *coupled* and *decoupled* approaches (Chiaradonna *et al*. 2018). While the former relies on rigorous nonlinear dynamic analyses performed in terms of effective stress, the latter allows the user to evaluate the excess pore water pressures by adopting semi-empirical relationships and results of seismic response analyses carried out in terms of total stress (*i.e.*, neglecting the bi-phasic nature of saturated soils).

In this framework, Seed *et al*. (1975) proposed a decoupled approach to evaluate the earthquake-induced pore pressures in liquefiable soils. The Authors modified the 1D consolidation equation (Terzaghi, 1923) by adding a source term, which represents the rate of excess pore water pressure occurring under fully-undrained conditions. This term depends on the shear stress time history induced by the earthquake, which in turn has to be replaced by an equivalent cyclic loading with constant amplitude, characterised by cycles uniformly distributed over the loading duration.

This paper focuses on the evaluation of the source term. In particular, a new assumption on the distribution of equivalent cycles along the earthquake duration is first made. Then, the hypothesis is validated against the results of 2D fully-coupled dynamic Finite Element (FE) analyses.

In the FE analyses, a Representative Elementary Volume (REV) of saturated sandy soil was subjected to an irregular shear stress time history, under fully-undrained conditions, and the soil cyclic behaviour was described by the advanced constitutive model SANISAND (Dafalias and Manzari 2004). The FE outcomes were compared with those resulting from a homemade Matlab script (Mathworks, 2021), where the source term by Seed *et al.* (1975) was implemented together with the new hypothesis.

2. Decoupled approach and definition of the source term

Seed *et al.* (1975) proposed a simplified method where a source term was added to the well-known 1D consolidation equation (Terzaghi, 1923), to evaluate the generation and redistribution of seismic-induced excess pore water pressure, u, in a horizontally-stratified liquefiable soil layer. The equation governing the phenomenon is the following:

$$\frac{\partial u}{\partial t} = c_\mathrm{v} \cdot \frac{\partial^2 u}{\partial z^2} + \frac{du_\mathrm{g}}{dt} \tag{1}$$

Equilibrium between the well-known dissipative term (*i.e.*, first term on the right side, proportional to the coefficient of consolidation, c_v) and the source term (i.e. second term on the right side, representing the rate of development of the excess pore water pressure under fully-undrained conditions, u_g) provides an immediate picture of the pore pressure generation phenomenon under partially-drained conditions (Boccieri *et al.*, 2022).

Based on the *decoupled* approach, the source term is related to the shear stresses caused by the seismic event, evaluated through a site response analysis. Seed and co-authors replaced the shear stress time history with a cyclic loading, defined by a constant amplitude τ_eq, equal to the 65% of the maximum shear stress τ_max, and by a number of cycles N_eq, uniformly distributed over the cyclic loading duration, T_d (this typically being equal to the strong motion duration of the input signal, $D_{5\text{-}95}$). Following these assumptions, the source term can be rewritten as

$$\frac{du_\mathrm{g}}{dt} = \sigma'_\mathrm{v0} \cdot \frac{dr_\mathrm{u}}{dr_\mathrm{N}} \cdot \frac{dr_\mathrm{N}}{dN} \cdot \frac{dN}{dt} = \frac{\sigma'_\mathrm{v0}}{N_\mathrm{L}} \cdot \frac{dr_\mathrm{u}}{dr_\mathrm{N}} \cdot \frac{N_\mathrm{eq}}{T_\mathrm{d}} \tag{2}$$

where $r_\mathrm{u} = u_\mathrm{g}/\sigma'_\mathrm{v0}$ is the pore pressure ratio; $r_\mathrm{N} = N/N_\mathrm{L}$ is the cyclic ratio; N is the n-th cycle of loading; and N_L is the number of cycles needed to trigger liquefaction (i.e. $u_\mathrm{g} = \sigma'_\mathrm{v0}$ and therefore $r_\mathrm{u} = 1$). Fitting the results of undrained cyclic laboratory tests, Seed and Booker (1977) proposed a relationship between r_u and r_N:

$$r_\mathrm{u} = \frac{2}{\pi} \cdot \sin^{-1}\left(r_\mathrm{N}^{1/2\alpha} \right) \tag{3}$$

where α is a function of soil current state and test conditions, typically assumed equal to 0.7.

The hypotheses assumed to evaluate the $CSR – N_\mathrm{L}$ curve (with $CSR = \tau_\mathrm{eq}/\sigma'_\mathrm{v0}$), which are necessary to obtain the number of cycles to trigger liquefaction N_L, the number of cycles N_eq and the loading duration T_d, are reported in Section 4. Equation (3) was used to define the $r_\mathrm{u} = \mathrm{f}(r_\mathrm{N})$ relationship ($\alpha = 0.7$).

In a liquefiable sandy soil column, the generation and distribution of excess pore water pressures u is strongly influenced by drainage conditions and signal propagation. Hence, to evaluate the accuracy in the definition of the sole source term du_g/dt, a Representative Elementary Volume (REV) of saturated soil was considered, subjected to an irregular shear stress time history under fully-undrained conditions (Fig 1 (a)). Following this procedure, the dissipative term in Equation (1) can be neglected.

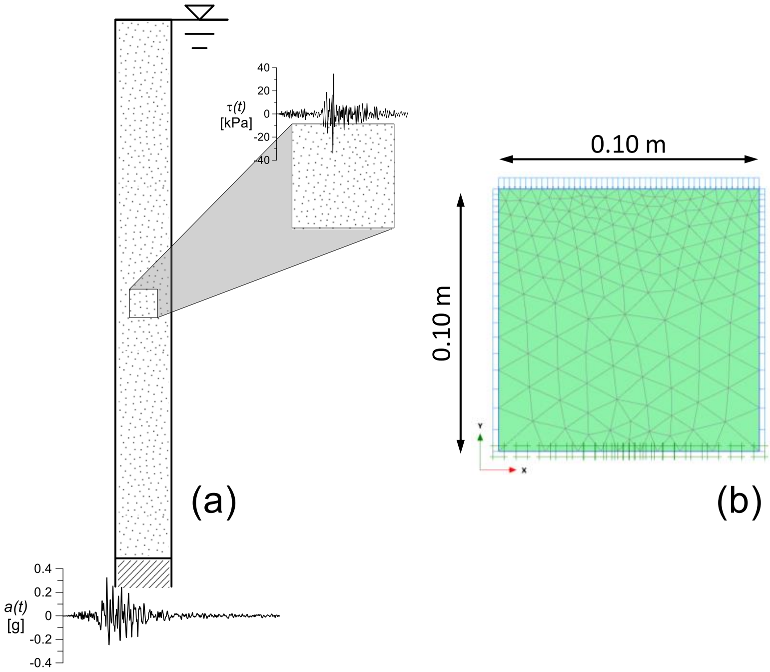

Fig 1. (a) Representative Elementary Volume (REV) subjected to an irregular shear stress time history and (b) REV implemented as in Plaxis 2D

3. Numerical analyses

A series of fully-coupled dynamic 2D analyses were carried out through the software Plaxis 2D CE v20 (Bentley, 2020) to study the undrained behaviour of a Representative Elementary Volume (REV). These analyses aimed at simulating an undrained cyclic shear test where an irregular shear stress time history was applied, rather than a uniform cyclic load as typically assumed in a decoupled approach.

Figure 1(b) shows the plane strain model as implemented in the code, characterised by 304 6-node triangular finite elements (second-order approximation for displacements) and 665 nodes. The REV saturated soil had dimensions $X = 0.10$ m and $Y = 0.10$ m. The advanced constitutive model SANISAND was adopted to describe the mechanical soil behaviour: the relevant parameters were obtained assuming a uniform Toyoura sand sample with a relative density D_r = 50% (Dafalias and Manzari, 2004). A normal effective stresses σ'_{v0} and $\sigma'_{h0} = K_0 \cdot \sigma'_{v0}$ was applied atop the REV and on the lateral sides, respectively, with $K_0 = 0.5$. The base was fully fixed and, on the top of the REV, an irregular time history of shear stress was applied. The numerical steps were as follows: (i) geostatic phase (K_0 procedure), in which an elastic soil behaviour is assumed, with $\nu = 1/3$ and $K_0 = 0.5$. To compute the initial horizontal effective stresses (σ'_{x0} and σ'_{z0}), $\sigma'_{y0} = \sigma'_{v0}$ was applied in plane strain and under drained conditions, while restraining the horizontal displacements along the vertical sides of the REV; (ii) horizontal stress phase, where the static fixities in the REV lateral sides were replaced by the σ'_{h0}; (iii) soil model phase, in which the elastic soil behaviour was replaced with the SANISAND constitutive model; (iv) undrained cyclic stress phase, in which the shear stress irregular time history was applied under undrained conditions and periodic boundaries (*i.e.*, tied degrees of freedom) were applied along the vertical sides of the REV.

To validate the numerical model of the REV, a comparison with the Plaxis *Soil Test* tool was made. Here it is worth mentioning that the *Soil Test* tool allows the user to apply regular cyclic time histories only. A static normal effective stress σ'_{v0} = 150 kPa and a regular shear stress cyclic time history were applied atop the REV, the latter characterized by 40 cycles/s and an amplitude of 7.5 kPa (*i.e.*, $CSR = 0.05$). The results obtained through the REV and the Plaxis *Soil Test*, in terms of excess pore water pressure u and stress invariants q-p', are plotted in Figure 2, showing a very good agreement between the two. This proved the reliability of the REV, adopted in the following as a benchmark.

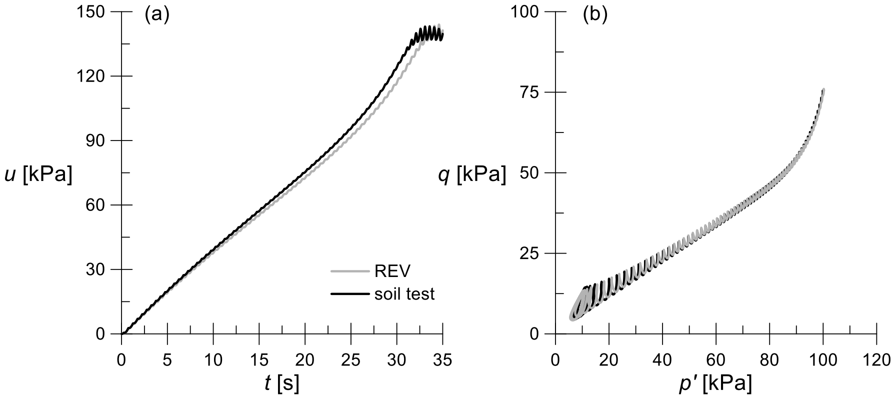

Fig 2. Comparison of (a) excess pore water pressure and (b) stress state obtained through the REV and Plaxis Soil Test tool

4. Implementation of the source term

A homemade Matlab script was implemented to compute the source term introduced by Seed *et al.* (1975) for the fully-undrained conditions. In the following, the assumptions made in this work to evaluate the number of cycles needed to trigger liquefaction, N_L, the number of equivalent cycles N_{eq}, and the distribution of the cycles on the loading duration T_d, are provided and discussed. In particular, the hypothesis of a uniform distribution of cycles over the loading duration is too simplified, since it does not represent the actual release of energy by the input signal. To this end, a new hypothesis for the cycles distribution was proposed.

The $CSR – N_L$ function was considered as follows:

$$CSR = CSR_t + a \cdot N_L^{-b} \tag{4}$$

where CSR_t, a, and b represent a threshold below which no excess pore pressure generation occurs, the intercept and the slope of the curve in a semi-logarithmic plane, respectively. These three parameters were determined by best-fitting a set of results from the Plaxis *Soil Test* (Fig. 3(a)), thus obtaining $CSR_t = 0.0109$, $a = 0.475$, and $b = 0.73$.

The number of equivalent cyclic loading N_{eq} was evaluated to generate the same increase of excess pore water pressure of the relevant irregular shear stress time history, under fully-undrained conditions. Hence, considering the $CSR – N_L$ curve as the *locus* of same damage level (*i.e.*, initial liquefaction of soil sample) and the peak-counting method (Hancock and Bommer (2005)), N_{eq} can be calculated as:

$$N_{eq} = \sum_i 0.5 \cdot \left(\frac{CSR_{0.65} - CSR_t}{CSR_i - CSR_t} \right)^{-1/b} \tag{5}$$

where $CSR_{0.65} = \tau_{eq}/\sigma'_{v0}$, and $CSR_i = \tau_i/\sigma'_{v0}$. It is worth noting that no threshold proportional to $CSR_{max} = \tau_{max}/\sigma'_{v0}$ has been considered (typical hypothesis as considered in the literature, see Hancock and Bommer, 2005; Biondi *et al.*, 2012) since the CSR_t already excludes peaks which would not generate any excess pore pressure.

The hypothesis adopted in this paper about the time distribution of the number of cycles about the calculation of N_{eq} over time. To represent more realistically the energy content of the shear stress loading, the rate of the number of cycles, dN/dt, was taken equal to the time derivative of the cumulative N(t), obtained using Equation (5) up to time instant t. Figure 3(b) shows the comparison between the N(t) time history, normalized by its final value N_{eq}, and the normalized Arias Intensity, both computed for an irregular shear stress time history representative of the Northridge (1994) earthquake. Specifically, it was assumed $\tau(t) = a(t)/PGA \cdot \tau_{max} = a(t)/PGA \cdot \sigma'_{v0} \cdot CSR/0.65$, so as to obtain $CSR = 0.65 \cdot \tau_{max}/\sigma'_{v0} = 0.1$ (with $\sigma'_{v0} = 150$ kPa). The two time traces result to be in a very good agreement, which denotes a successful representation of the irregular loading energy content. For the

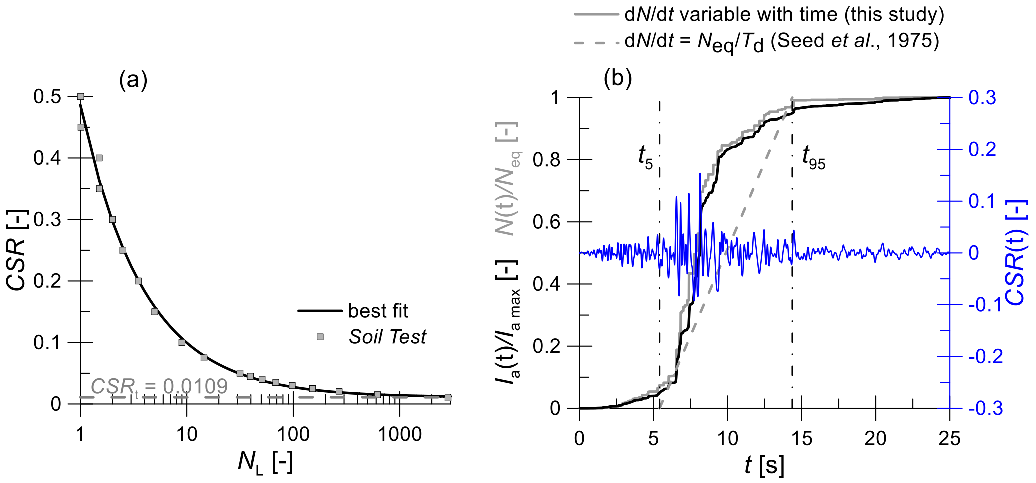

Fig 3. (a) CSR-N_L curve obtained through the Plaxis Soil test best-fitting and (b) comparison between the cumulative N(t), the Arias Intensity, and the uniform distribution of cycles

sake of completeness, Figure 3(b) also shows the uniform distribution of cycles proposed by Seed *et al.* (1975), with the loading duration T_d defined as the time range between 5% (t_5) and 95% (t_{95}) of the Arias Intensity of shear stress time history (Trifunac and Brady, 1975).

5. Comparison with fully-coupled 2D analyses

A comparison with the results obtained through the REV 2D numerical model was made to verify the new hypothesis. To this aim, the irregular shear stress time histories to be applied both atop the REV and in the Matlab script were obtained by scaling three accelerograms recorded during real earthquakes (Friuli 1976, Northridge 1994, and Trinidad 1983, low-pass filtered at 10 Hz, *e.g.*, Tab. 1) to achieve a Cyclic Stress Ratio $CSR = 0.65 \cdot \tau_{max}/\sigma'_{v0} = 0.1$ ($\sigma'_{v0} = 150$ kPa). The excess pore water pressures obtained in the numerical analyses and in the simplified method are plotted in Figure 4. Taking into account the grade of complexity, the results achieved through the simplified method are deemed satisfactory, when considering the new hypothesis for the rate of the number of cycles *dN/dt*. The good estimation of the energy content carried out by the irregular shear loading is evidenced by the good prediction of the excess pore water pressure rate, both in magnitude and trend.

The computed final excess pore water pressures are still in good agreement with the numerical ones, even when considering a uniform distribution of cycles during the strong motion duration (as done by Seed *et al.*, 1975). Nonetheless, the development of *u* over time does not fit the results obtained through the Plaxis model for all seismic inputs in this case: this further demonstrates the need of introducing the new hypothesis discussed in this paper.

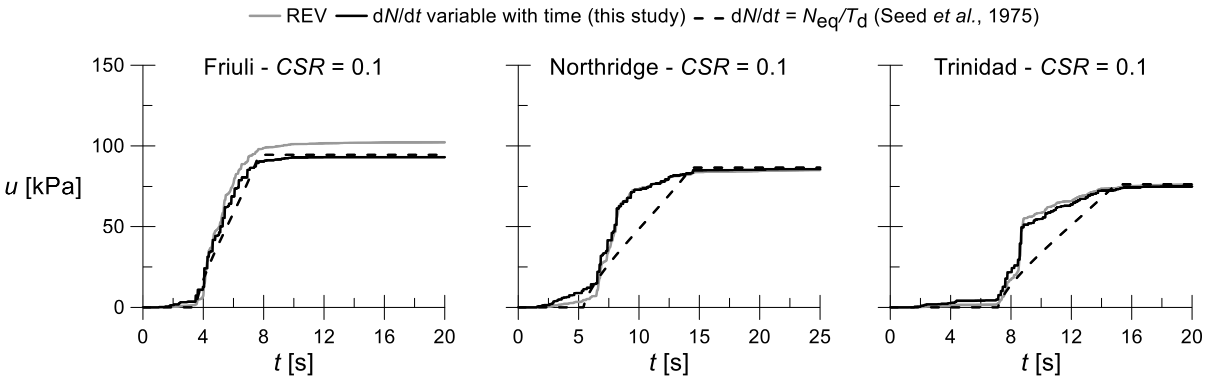

Fig 4. Comparison between the excess pore water pressure obtained through the fully-coupled 2D analyses and the decoupled approach considering a uniform distribution of N_{eq} (dashed lines) or the new hypothesis proposed in this paper (solid lines), for three irregular shear stress time histories

Tab 1. Seismic events properties (PGA = Peak Ground Acceleration, $I_{A\,max}$ *= Arias intensity,* $D_{5\text{-}95}$ *= strong motion duration, Trifunac and Brady, 1975) and properties of the shear stress event* (τ_{max} = maximum shear stress to obtain CSR=0.65$\cdot\tau_{max}/\sigma'_{v0}$ = 0.1, N_L = number of cycles needed to trigger liquefaction, N_{eq} = Number of equivalent cycles)

earthquake	*PGA* (*g*)	$I_{A\,max}$ (m/s)	t_5 (s)	t_{95} (s)	$D_{5\text{-}95}$ (s)	τ_{max} (kPa)	N_L (s)	N_{eq} (-)
Trinidad (1983)	0.169	0.158	7.14	14.95	7.81	23.08	9.90	6.23
Friuli (1976)	0.324	0.757	3.50	7.69	4.19	23.08	9.90	7.75
Northridge (1994)	0.582	2.698	5.40	14.36	8.96	23.08	9.90	7.11

6. Conclusions

A reliable although simplified method to estimate the liquefaction hazard is necessary to assess the risk of liquefaction accurately. To this end, in this paper the Authors tried to improve the *decoupled* approach proposed by Seed *et al*. (1975) by introducing a novel hypothesis on the rate of number of cycles, *dN/dt*. Indeed, the strong hypothesis of considering a uniform distribution of cycles over the loading duration is replaced by a more realistic assumption, based on assuming the time distribution of cycles equal to the cumulative N(t) obtained through the N_{eq} calculation over time. To test the validity of the new hypothesis, a comparison was made between the results obtained through a series of numerical analyses on a Representative Elementary Volume (REV) under fully-undrained conditions and a homemade Matlab script, where the source term with the new hypothesis was implemented. The results obtained are in a really good agreement with the numerical model, which justifies the hypothesis. In future developments, the Authors will try to implement these considerations in a *decoupled* 1D column analysis, considering both the source and the well-known dissipative term.

References

Bentley (2020). Plaxis 2D CE v20 – Reference Manual. Delft University of Technology. Delft, The Netherlands.

Biondi G., Cascone E., Di Filippo G. (2012). “Affidabilità di alcune correlazioni empiriche per la stima del numero di cicli di carico equivalente”, *Rivista Italiana di Geotecnica*, 46(2), 9-39. [in italian]

Boccieri G., Gaudio D., Conti R. (2022). A simplified method for the estimation of earthquake-induced pore pressure. Geotechnical Engineering for the Preservation of Monuments and Historic Sites III. CRC Press, 812-822. DOI: 10.1201/9781003308867-62

Chiaradonna A., Tropeano G., d’Onofrio A., Silvestri F. (2018). “Development of a simplified model for pore water pressure build-up induced by cyclic loading”, *Bulletin of Earthquake Engineering*, 16(9), 3627-3652.

Dafalias Y.F., Manzari M.T. (2004). “Simple plasticity sand model accounting for fabric change effects”, *Journal of Engineering mechanics*, 130(6), 622-634.

Hancock J., Bommer J.J. (2005). “The effective number of cycles of earthquake ground motion”, *Earthquake Engineering & Structural Dynamics*, 34(6), 637-664.

Mathworks Inc. (2021). Matlab version 9.10.0 (R2021a). Natick, Massachusetts.

Seed H.B., Booker J.R. (1977). “Stabilization of potentially liquefiable sand deposits using gravel drains”, *Journal of Geotechnical Engineering Division*, ASCE, 103(7), 757-768.

Seed H.B., Martin P.P., Lysmer J. (1975). The generation and dissipation of pore water pressures during soil liquefaction. College of Engineering, University of California.

Terzaghi K. (1923). Die Berechnung der Durchassigkeitsziffer des Tones aus dem Verlauf der hydrodynamischen Spannungs. erscheinungen. Sitzungsber. Akad. Wiss. Math. Naturwiss. Kl. Abt. 2A 132: 105-124.

Trifunac M.D., Brady A.G. (1975). “A study on the duration of strong earthquake ground motion”, *Bulletin of the Seismological Society of America*, 65(3), 581-626.