Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Space Charge Forces analytical model for emittance compensation

To cite this article: M. Carillo *et al* 2024 *J. Phys.: Conf. Ser.* **2687** 062009

Space Charge Forces analytical model for emittance compensation

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Abstract. Space charge forces represent main induced effects in an RF-injector that degrade the beam quality. In this scenario the laser distribution sent on the photocathode acquires an important role in the emittance compensation process, as the slice analysis shows. Starting from the preliminary studies performed on [1], a novel semi-analytical model of space charge forces is proposed in detail for bunch with arbitrary charge distribution to derive expressions of self-induced forces. The performance of the fields at low energy regime (as the field has not expired RF forces) is under present analysis, we can investigate use of this model in low charge regime. Further, the model has been bench-marked with the behavior of the distributions present in the literature and studied for new ones. It has also been applied for the study of the optimization of a C-band hybrid photoinjector now being commissioned, thus explaining the factor two reduction of the emittance observed at the exit of the gun by changing the initial distribution at the cathode.

1. Introduction

In modern accelerator applications such as free-electron lasers [2, 3], inverse Compton scattering sources[4, 5] and ultrafast electron diffraction[7, 8], producing high brightness beams with large peak currents and low transverse emittances is essential [9, 10, 11, 12]. However, the presence of space charge can lead to unwanted emittance growth [13], which negatively impacts accelerator performance. To address this issue, researchers have been working to better understand the mechanisms behind space charge growth [14] and develop effective strategies to prevent it.
One promising approach to mitigating emittance growth involves linearizing the spatial charge forces [15, 16]. Recently, a new analytical model based on the Green function method has been proposed, which can be used to study the behavior of space charge fields in the presence of any longitudinal and transverse laser distribution. This model has the potential to significantly improve our understanding of space charge effects in particle accelerators and could lead to the development of more effective mitigation strategies.
In this study, we applied this analytical model to investigate the impact of the transverse distribution of a laser on the emittance of a C-band hybrid photoinjector developed by Luigi Faillace et all [17]. The C-band hybrid photoinjector[18, 19]is a type of particle accelerator that is

designed to produce high-brightness electron beams for use in a variety of scientific applications, including free-electron lasers, synchrotron radiation sources, and ultrafast electron diffraction. The design of the C-band hybrid photoinjector is based on a combination of radiofrequency (RF) acceleration and photoinjection, which allows for precise control over the energy and emittance of the electron beam, thanks also to velocity bunching approach [20].
Through simulation, we found that modifying the transverse distribution of the laser resulted in a significant reduction of the emittance, by a factor of 2. By applying slice analysis[21] and an analytical model[1], an explanation for the observed phenomenon has been then obtained.

2. Space charge analytical model

A beam with a charge density of $\rho(\mathbf{r},t)$ and traveling at a speed of $\mathbf{v}=\hat{z}\beta c$ within a circular pipe with a radius of b generates a scalar potential ϕ, which satisfies the inhomogeneous wave equation [22]:

$$(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2})\phi=-\frac{\rho}{\epsilon_0}. \tag{1}$$

Assuming cylindrical symmetry of the beam pipe, where the position vector is defined as $\mathbf{r}=(r,\theta,z)$, the Green's function method can be used to describe an arbitrary beam distribution [23, 24]. The scalar potential $\phi(r,z)$ can be expressed as the integral of the Green's function $G(r,z;r',z')$ with respect to the charge density $\rho(r',z')$, where the variables in superscript refer to the beam sizes. The Green's function $G(r,z;r',z')$ for an asymmetric charge distribution $\rho(r',z')$ with no θ dependence can be computed by solving the inhomogeneous wave equation for a unit ring of charge (in r,z space) located at the source point (r',z'). The Green's function method can then be used to analyze the beam distribution and the resulting solution can be expressed as the sum of two terms: a direct charge term and an image charge contribution.

$$\tilde{G}(r,r',k)=\frac{1}{2\pi\epsilon_0}\Bigg[I_0\Big(\Big|\frac{k}{\gamma}r_<\Big|\Big)K_0\Big(\Big|\frac{k}{\gamma}r_>\Big|\Big) \tag{2}$$

$$-\frac{K_0\Big(\Big|\frac{k}{\gamma}b\Big|\Big)}{I_0\Big(\Big|\frac{k}{\gamma}b\Big|\Big)}I_0\Big(\Big|\frac{k}{\gamma}r\Big|\Big)I_0\Big(\Big|\frac{k}{\gamma}r'\Big|\Big)\Bigg] \tag{3}$$

where k is the wave number in the Fourer space, $r_<=\min\{r,r'\}$, $r_>=\max\{r,r'\}$ and $I_0(x)$ and $K_0(x)$ are the modified Bessel function.
The image charge is due to the presence of the circular beam pipe and can be ignored if the transverse dimensions of the beam are much smaller than the pipe radius.
The scalar potential for an arbitrary charge distribution density using standard properties of Fourier transform is:

$$\begin{aligned}\phi(r,z)&=\int d\tau' G(r,r',z-z')\rho(r',z')\\&=\int dS'\int\frac{dk}{2\pi}e^{ikz}\tilde{G}(r,r',k)\tilde{\rho}(r',k)\end{aligned} \tag{4}$$

where $\tilde{\rho}(r',k)$ is the volumetric charge distribution on the Fourier space.

3. Hybrid photoinjector Space charge Analysis

The hybrid photoinjector use a new way to create bright beams. This device combines a photocathode with a 2.5 gun cell SW section and a TW section through an input coupling cell[17]. It has advantages over a standard split SW-TW system, including eliminating rf reflections and avoiding the bunch lengthening effect. The rf coupling between the SW and TW sections creates a 90° phase shift, which produces a strong velocity bunching effect resulting in very short bunch lengths [20]. In the framework of beam dynamics optimization the process of shaping beams during photocathode injection has been studied using two different transverse laser distributions. The first distribution is a uniform flattop distribution in r, while the second is a truncated Gaussian with a hard radial edge achieved through collimation at a radius of 0.5 mm. This study showed that using a truncated Gaussian transverse laser distribution provides two significant operational advantages. The use of the truncated Gaussian transverse laser distribution provides a significantly emittance lowering compared to previous designs. In initial studies, a uniform transverse laser illumination yielded an optimized rms normalized emittance of approximately 0.75 $mm\ mrad$. However, when using the truncated Gaussian laser profile on the photocathode, the normalized emittance decreased dramatically to 0.46 $mm\ mrad$. The simulation of this case (performed by using the General Particle Tracer (GPT) code [25]), using 1 million macro-particles to ensure conclusive results, is depicted in Figure 1.

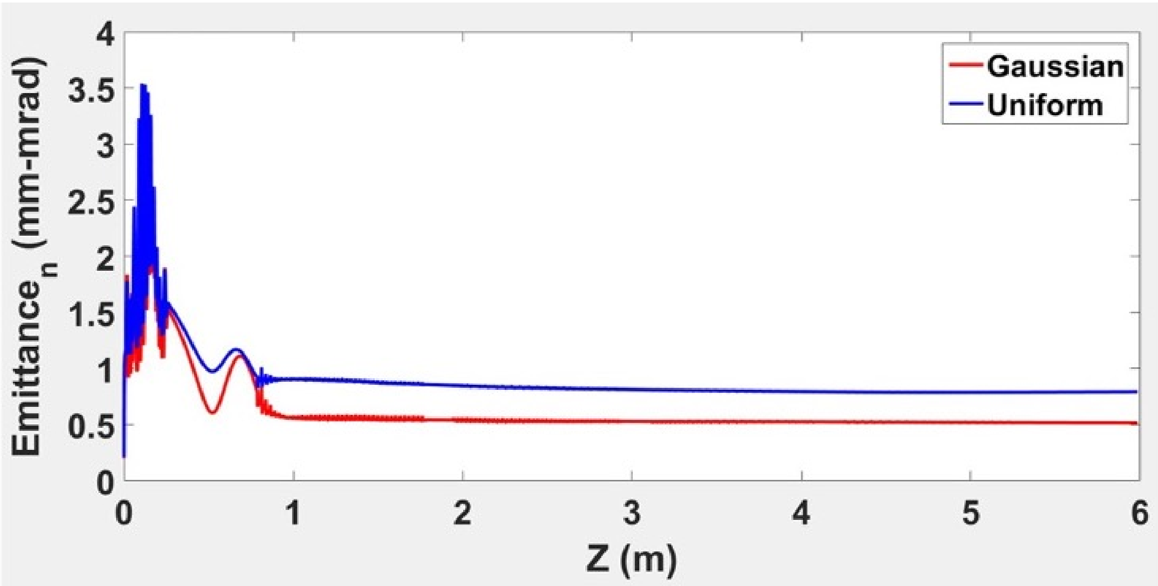

Figure 1. Comparison of Cut-Gaussian and Uniform transverse distributions in emittance performance of the hybrid photoinjector and two subsequent accelerating sections. Red curve: Cut-Gaussian distribution emittance. Blue curve: Uniform distribution emittance.

To explain the observed behavior, we used slice analysis to divide the particle distribution into longitudinal slices and track the beam's evolution over time. Then, the analysis of the transverse phase space of the beam showed significant non-linearities for the uniform distribution, even at low energies, in contrast with the truncated Gaussian distribution. These results are displayed in Figure 2. Where the respective longitudinal slices were overlapped for the two different distributions. To reproduce and justify the observed behavior of the transverse phase space, the described model was applied to the beam generated by the hybrid photocathode. Then, the longitudinal distributions used was:

$$\lambda(z) = \frac{e^{-z^2/2\sigma_z^2}}{\sqrt{2\pi}\sigma_z} \tag{5}$$

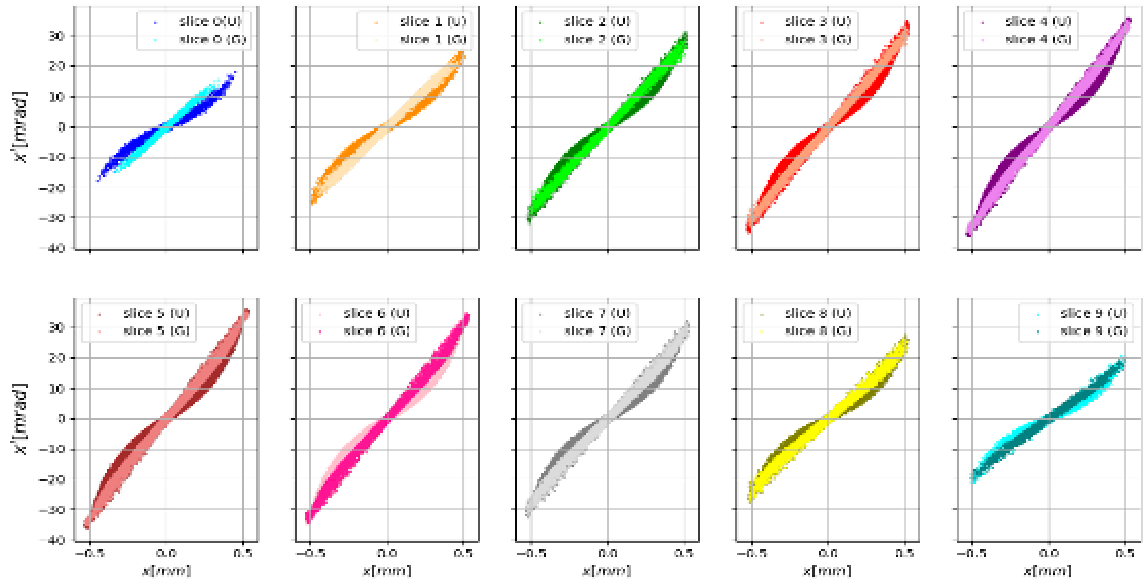

Figure 2. Comparison of Uniform and Cut-Gaussian Transverse Distributions: Beam Slice Analysis. The figure displays the transverse phase space of a newly generated beam, using uniform and cut-gaussian distributions. The plot consists of 10 small images, each representing a slice of the beam. Nonlinearities are observed in the transverse phase space of the uniform distribution, even at low energies, while the cut-gaussian distribution shows a smoother and more uniform behavior, leading to enhanced beam quality and efficiency.

while the uniform transverse distribution is given by:

$$R(r) = \frac{1}{\pi a} \begin{cases} 1, & \text{if } 0 < r \leq a \\ 0, & otherwise \end{cases} \tag{6}$$

and the truncated Gaussian one is given by:

$$R(r) = \begin{cases} R_0 + (R_1 - R_0)\Big(1 - \frac{r^2}{a^2}\Big), & \text{if } 0 < r \leq a \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where a and σ_z are respectively C-Band hybrid photo injector beam radius and the beam length at the cathode (beam just emitted) and the constants R_0 and R_1 are defined taking into account the 1σ cut and normalization conditions.
Therefore, by applying equations 5 and 6, 7, the radial spatial charge distribution shown in figure 3 was obtained using the parameters described by the hybrid model.

The figure3 shows the variation of the space charge field as a function of the beam radius r. As evident from the plot, there are non-linear components for both distributions, i.e., Cut-Gaussian (continuous red curve) and Uniform (dashed black curve). However, in the case of the Cut-Gaussian distribution, the linear components dominate in the core of the beam. The image was analyzed for different values of fixed z, but the shape of the field does not significantly change with the longitudinal position within the beam. This is also confirmed by the observation of the slice analysis reported in Figure 2, where it is evident that the distortion effect of the phase space due to space charge forces is more or less the same for all the slices.
The qualitative agreement between the transverse space charge electric field (Fig. 3) and the

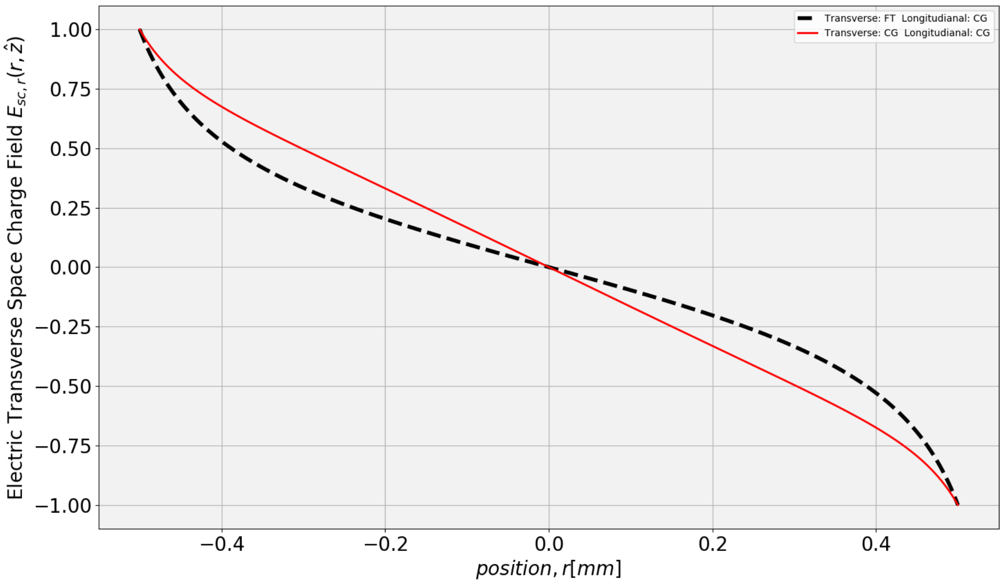

Figure 3. The plot depicts the variations of the normalized radial space charge fields within the beam as a function of the normalized transverse position. Specifically, the results are presented for two distinct transverse distributions: the Cut-Gaussian distribution, which is illustrated by the continuous red curve, and the Flat-Top distribution, which is represented by the black dashed curve.

prediction derived from the slice analysis model (Fig. 2) provides compelling evidence of the accuracy of the model in describing the underlying dynamics of the beam. Moreover, the agreement between the two suggests that the model is capturing the essential features of the beam's transverse behavior, as well as the influence of the space charge forces that are known to impact the beam's emittance.

4. CONCLUSION

In conclusion, we have applied a new analytical model based on the Green function method to study the impact of the transverse distribution of a laser on the emittance of a C-band hybrid photoinjector. Through simulation, we found that modifying the transverse distribution of the laser resulted in a significant reduction of the emittance, by a factor of 2. We also used slice analysis and an analytical model to explain the observed phenomenon. Our findings suggest that linearizing the spatial charge forces can be an effective approach to mitigating emittance growth in particle accelerators. The analytical model presented in this study can be used to further investigate the behavior of space charge fields in the presence of arbitrary longitudinal and transverse laser distributions. This could lead to the development of more effective mitigation strategies and ultimately improve the performance of particle accelerators used in a wide range of scientific applications.

Acknowledgements
This work is supported by DARPA under Contract N.HR001120C0072, by DOE Contract DE-SC0009914 and DE-SC0020409, by the National Science Foundation Grant N.PHY-1549132 and by INFN through the project ARYA.

References

[1] M Carillo1, M Behtouei, F Bosco, L Faillace, L Ficcadenti, A Fukasaway, L Giuliano, A Mostacci, M Migliorati, B Spataro, J Rosenzweig and L Palumbo *Space charge analysis for low energy photoinjector*, 2023 J. Phys.: Conf. Ser. 2420 012058.

[2] C. Pellegrini, A. Marinelli, and S. Reiche, "The physics of X-ray free-electron lasers", *Rev. Mod. Phys. 88, 015006 (2016).*

[3] P. Emma, R. Akre, J. Arthur, R. Bionta, C. Bostedt, J. Bozek, et al., First lasing and operation of an angstrom-wavelength free-electron laser, Nature Photonics 4, 641 (2010).

[4] C. Curatolo, I. Drebot, V. Petrillo, and L. Serafini, Analytical description of photon beam phase spaces in inverse Compton scattering sources, Phys. Rev. Accel. Beams 20, 080701 (2017).

[5] L. Faillace, R. Agostino, A. Bacci, R. Barberi, A. Bosotti, F. Broggi, P. Cardarelli, S. Cialdi, I. Drebot, V. Formoso, et al., Status of compact inverse compton sources in Italy: BriXS and STAR, in Advances in Laboratory-based X-Ray Sources, Optics, and Applications VII, Vol. 11110 (International Society for Optics and Photonics, 2019) p.11110-05.

[6] M. Ferrario, D. Alesini, M. Anania, M. Artioli, A. Bacci, S. Bartocci, et al., EuPRAXIA@SPARC_LAB Design study towards a compact FEL facility at LNF, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 909, 134 (2018), 3rd European Advanced Accelerator Concepts workshop (EAAC2017)

[7] A. H. Zewail, 4D ultrafast electron diffraction, crystallography, and microscopy, Annu. Rev. Phys. Chem. 57, 65 (2006).

[8] S. P. Weathersby et al., Mega-electron-volt ultrafast electron diffraction at SLAC National Accelerator Laboratory, Rev. Sci. Instrum. 86, 073702 (2015).

[9] B. Carlsten, New photoelectric injector design for the Losalamos National Laboratory XUV FEL Accelerator, Nucl. Instrum. Methods Phys. Res., Sect. A 285, 313 (1989).

[10] F. Zhou, C. Adolphsen, A. Benwell, G. Brown, D.H. Dowell, M. Dunning, S. Gilevich, K. Grouev, G. Huang, B. Jacobson, X.H. Liu, A. Miahnahri, F. Sannibale, J. Schmerge, and T. Vecchione, Commissioning of the SLAC linac coherent light source II electron source, Phys. Rev. Accel. Beams 24, 073401 (2021).

[11] R. R. Robles, O. Camacho, A. Fukasawa, N. Majernik, and J.B. Rosenzweig, Versatile, high brightness, cryogenic photoinjector electron source, Phys. Rev. Accel. Beams 24, 063401 (2021).

[12] J. B. Rosenzweig, A. Cahill, V. Dolgashev, C. Emma, A. Fukasawa, R. Li, C. Limborg, J. Maxson, P. Musumeci, A. Nause, and R. Pakter, Next generation high brightness electron beams from ultrahigh field cryogenic rf photo- cathode sources, Phys. Rev. Accel. Beams 22, 023403 (2019).

[13] K.Floettmann, *Emittance compensation in split photoinjector*, Phys. Rev. Accel. Beams 20, 013401 (2017) https://doi.org/10.1103/PhysRevAccelBeams.20.013401

[14] M. Ferrario, M. Migliorati, and L. Palumbo, *Space charge effects*, arXiv preprint arXiv:1601.05214 (2016). https://doi.org/10.48550/arXiv.1601.05214

[15] L. Serafini and J.B. Rosenzweig, Envelope analysis of intense relativistic quasilaminar beams in rf photoinjectors: A theory of emittance compensation, Phys. Rev. E 55, 7565 (1997).

[16] L. Serafini, The short bunch blow-out regime in rf photo- injectors, AIP Conf. Proc. 413, 321 (1997).

[17] Faillace, L. e al., High field hybrid photoinjector electron source for advanced light source applications, PhysRevAccelBeams.25.063401,(2022) American Physical Society
doi = 10.1103/PhysRevAccelBeams.25.063401

[18] L. Faillace, R. Agusstson, M. Behtouei, F. Bosco, M. Carillo, A. Fukasawa, L. Giuliano, A. Giribono, S. Kutsaev, N. Majernik, M. Migliorati, A. Mostacci, A. Murokh, L. Palumbo, B. Spataro, S. Tantawi, C. Vaccarezza, O. Williams, J.B. Rosenzweig. *Beam Dynamics for a High Field C-band Hybrid Photoinjector*, in 12th Int. Particle Accelerator Conf.(IPAC'21), Campinas, Brazil (2021).
http://dx.doi.org/10.18429/JACoW-IPAC2021-WEPAB051

[19] Faillace L., Behtouei M., Giribono A., Spataro B., Vaccarezza C., Palumbo L., Bosco F., Carillo M., Giuliano L., Migliorati M., Mostacci A., Rosenzweig, J.B., Camacho O., Fukasawa A., Majernik N., Williams O., Agustsson R., Bruhwiler D., Gadjev I., Murokh A.,*Start-To-End Beam-Dynamics Simulations of a Compact C-Band Electron Beam Source for High Spectral Brilliance Applications*, (2022) in 13th Int. Particle Accelerator Conf.(IPAC'22), Bangkok, Thailand
doi:10.18429/JACoW-IPAC2022-MOPOMS023

[20] L. Serafini and M. Ferrario, AIP Conf. Proc. 581, 87 (2001).

[21] M. Carillo, M. Behtouei, F. Bosco, L. Faillace, L. Ficcadenti, A. Giribono, L. Giuliano, A. Mostacci, M. Migliorati, B. Spataro, J. Rosenzweig, C. Vaccarezza, L. Palumbo, *Three-Dimensional Space Charge Oscillations in a Hybrid Photoinjector*, in 12th Int. Particle Accelerator Conf.(IPAC'21), Campinas, Brazil

(2021).
http://dx.doi.org/10.18429/jacow-ipac2021-wepab256

[22] John David Jackson, *Classical Electrodynamics* John Wiley and Sons, INC. 1998.

[23] L. Palumbo, V. G. Vaccaro, and M. Zobov, Wake fields and impedance, 10.15161/oar.it/1448441728.46 (1994).

[24] Shobuda, Y., Chin, Y.H. Rigorous formulation of space-charge wake function and impedance by solving the three-dimensional Poisson equation. Sci Rep 8, 12805 (2018). https://doi.org/10.1038/s41598-018-30960

[25] http://www.pulsar.nl/gpt/index.html