

Correction

Correction: Cerqueti, R.; Lupi, C. Some New Tests of Conformity with Benford's Law. *Stats* 2021, 4, 745–761

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Text Correction

There were several errors in the original publication [1].

1—Convergence in Equation (10) is approximate.

A correction has been made to Proposition 2, in the first text line after Equations (9) and (10):

where $e_n^* := (e_{n1}, \dots, e_{n,k-1})'$ and Σ^* is made of the first $k - 1$ rows and columns of Σ . Furthermore, using the symbol $\overset{d}{\rightsquigarrow}$ to denote approximate convergence in distribution as $n \rightarrow \infty$,

$$MAD^* := \frac{\sqrt{n}}{k} \sum_{j=1}^k \frac{|f_{nj} - p_j|}{\sqrt{p_j(1-p_j)}} \overset{d}{\rightsquigarrow} N\left(\sqrt{\frac{2}{\pi}}, \frac{1}{k^2} \sum_{i=1}^k \sum_{j=1}^k r_{ij}\right) \quad (10)$$

2—Explicit reference to quantities e and e_n^+ was missing.

A correction has been made to the text line before Equation (14):

To prove (10), first note that, since $\sqrt{n}e \xrightarrow{d} N(0, \Sigma)$, $e_n^+ := (e_{n1}^+, \dots, e_{nk}^+)'$ is such that the e_{nj}^+ 's are individually $N(0, 1)$ but are correlated. We also exploit the fact that if $Y \sim N(0, 1)$, then see [36].

3—The distribution in Equation (16) was wrong.

A correction has been made to Equation (16) and the following three text lines:

$$\sqrt{ne_{nj}^*} := \frac{\sqrt{n}|f_{nj} - p_j|}{\sqrt{p_j(1-p_j)}} \xrightarrow{d} \text{HN}\left(\sqrt{\frac{2}{\pi}}, 1 - \frac{2}{\pi}\right) \quad (16)$$

where $\text{HN}(\cdot, \cdot)$ denotes the half-normal random variable.

Furthermore, $\sqrt{n}E(e_n^*) := \sqrt{n}E(e_{n1}^*, \dots, e_{nk}^*)' = \mathbf{1}\sqrt{\frac{2}{\pi}}$, with $\mathbf{1}$ a k -vector of ones, and $\text{Cov}(\sqrt{ne_n^*}) = R$, with R to be defined below.

4—A comma was missing in the LHS of Equation (20).

A comma was added in the LHS of Equation (20):

$$\begin{aligned} \text{cov}\left(|e_{ni}^+|, |e_{nj}^+|\right) &= E\left(|e_{ni}^+| |e_{nj}^+|\right) - E\left(|e_{ni}^+|\right) E\left(|e_{nj}^+|\right) \\ &= \frac{2}{\pi} \left(\rho_{ij} \arcsin(\rho_{ij}) + \sqrt{1 - \rho_{ij}^2} \right) - \frac{2}{\pi}. \end{aligned} \quad (20)$$

5—The reference to k large was missing.

A correction has been made to the text line before Equation (23), and two text lines were added following Equation (23):



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Finally, as $n \rightarrow \infty$ and for k large enough

$$\frac{\sqrt{n}}{k} \sum_{j=1}^k e_{nj}^* = \frac{1}{k} \sum_{j=1}^k \frac{\sqrt{n}|f_{nj} - p_j|}{\sqrt{p_j(1-p_j)}} \xrightarrow{d} N\left(\sqrt{\frac{2}{\pi}} \frac{1}{k^2} \mathbf{t}' \mathbf{R} \mathbf{t}\right). \quad (23)$$

Of course, k is such that it cannot tend to ∞ . The usual values of k are $k \in \{9, 10, 90\}$. This implies that convergence to the normal can only be approximate, leading to (10).

6—Remark 6 was missing.

Remark 6 was added:

Remark 6. Simulations (available from the authors upon request) show that, as $n \rightarrow \infty$, the convergence of MAD^* to the normal distribution is very fast as k increases. Even for $k = 9$ (first-digit case) or $k = 10$ (second-digit case), the approximation is highly accurate. Since the distribution of $\sqrt{n} e_{nj}^*$ is asymmetric, a very slight skewness is still observed for small values of k ; however, this vanishes as k increases. In practice, agreement with the asymptotic distribution is essentially perfect in the first two-digit case.

The authors state that the scientific conclusions are unaffected. This correction was approved by the Academic Editor. The original publication has also been updated.

Reference

1. Cerqueti, R.; Lupi, C. Some New Tests of Conformity with Benford's Law. *Stats* **2021**, *4*, 745–761. [[CrossRef](#)]

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