

Proceedings

of the 45th conference
of the international group
for the psychology
of mathematics education

July 18-23, 2022

EDITORS

Ceneida Fernández / Salvador Llinares
Ángel Gutiérrez / Núria Planas

VOLUME 2

Research Reports (A - H)



Universitat d'Alacant
Universidad de Alicante

**Proceedings of the 45th Conference of the International Group
for the Psychology of Mathematics Education**

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Editors:

Ceneida Fernández
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**Psychology in Mathematics
Education
(PME) Conference
18-23 July 2022
Alicante, Spain**

Website:
<https://web.ua.es/pme45>

Cite as:

Fernández, C., Llinares, S., Gutiérrez, A., & Planas, N. (Eds.) (2022). *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2). Alicante, Spain: PME.

Website: <https://web.ua.es/pme45>

Proceedings are also available on the IGPME website:
<http://www.igpme.org>

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ISBN: 978-84-1302-176-8

Legal Deposit: A 289-2022

ISSN 0771-100X

Printed by Universidad de Alicante.

Logo designed by the Gabinete de Imagen y Comunicación Gráfica
(Universidad de Alicante)

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