Machine-learning-aided improvement of mechanics-based code-conforming shear capacity equation for RC elements with stirrups

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Abstract

The development of shear capacity equations for reinforced concrete (RC) beams and columns has been historically pursed starting from the conceptualization of a resisting mechanism. Recently, machine learning techniques are attracting more and more interest in this field. These approaches (i.e., white box and black box modeling, respectively) have been considered independently so far. Conversely, this work aims at exploring a hybrid alternative way (i.e., gray box modeling) for deriving the shear capacity equation for RC beams and columns, in which a mechanics-based code-conforming formulation is improved thanks to a machine-learning-aided approach. Specifically, the capacity equation currently in use within Europe that relies on the variable-angle truss resisting mechanism is enriched by means of Genetic Programming. Easy-to-use novel expressions for the two fundamental coefficients ruling the concrete contribution are defined to better match experimental data. The performance of the newly obtained equation is first discussed within the largest comparative assessment ever presented so far among shear strength formulations reported into existing technical codes around the world. Afterward, it is recast into a code-formatted design capacity equation using a simple, yet reliable, procedure. Overall, the results demonstrate that merging mechanics-based and data-driven methods can be beneficial in the development of capacity equations since it allows preserving the physical meaning of the resisting mechanism while enhancing the accuracy of the final predictions by means of machine learning techniques. Although the methodology is here applied to evaluate the shear strength of RC beams and columns, it is very general and can be readily extended to the development of further capacity equations.

Keywords: Beam, Column, Design code, Genetic programming, Machine learning, Reinforced concrete, Shear capacity, Variable-angle truss model

1. Introduction

It is well known that shear failure of reinforced concrete (RC) elements is typically more brittle and sudden than flexural failure [1]. Consequently, it is of utmost importance to estimate properly the shear capacity of RC members when assessing existing structures or in the design of new constructions. Although the evaluation of the shear capacity in RC members has been extensively addressed in the past decades, the development of safe, reliable and accurate shear capacity equations for RC elements is still a topical research subject because of its implications in daily engineering practice.

Several design methods implemented in current guidelines and codes have a mechanical basis in which the involved parameters are calibrated through statistical regression, based upon experimental evidence. Historical developments of shear design provisions along with the underlying mechanical theories have been comprehensively discussed by the ACI-ASCE Committee 445 [2]. The oldest and most popular shear resisting mechanism for RC elements was developed by Ritter [3] and Mörsch [4] in the early 1900s, and it is referred henceforth to as the Ritter-Mörsch (RM) model. A truss analogy was adopted for the first time in such model, which postulates that a RC element after cracking can be idealized as a truss consisting of two parallel chords with tensile ties representing the transverse reinforcement and compression diagonals simulating the concrete stress fields between adjacent cracks, whose inclination angle is taken equal to 45° with respect to the longitudinal axis. This is the simplest conceptual idealization of the shear resisting mechanism of RC beams and columns with transverse reinforcement. It is worth noting that the original RM model does not directly consider some factors contributing to the shear resisting mechanism, such as residual tensile stress of concrete, aggregate interlock at crack interfaces, dowel action of tensile longitudinal steel, and shear contribution carried across uncracked concrete in the compression zone. Consequently, despite its conceptual simplicity, the original version of the RM model was abandoned because it provided inaccurate predictions when compared to experimental results [5]. However, this pioneering theory inspired the developments of further truss-like capacity models, some of which implemented in recent codes.

From 1980s onward, two families of extended and/or modified formulations have been basically developed as refinement of the original RM model. The first class of formulations exploits an additive-type approach to the shear capacity assessment, in which a truss model (with compression diagonals inclined at a 45° angle) is considered together with an additional corrective contribution (generally calibrated on empirical basis) attributable to the role of concrete. Some examples include the pre-standard version of the Eurocode 2 [6] and the ACI 318 Building Code (from the older version ACI 318-95 up to the recent ACI 318-19 [7]). The second class of formulations for predicting the shear strength is based on a variable-angle truss model with no concrete contribution, wherein the inclination of the compression diagonals is allowed to differ from 45° (up to certain limits based on the plasticity theory [8, 9]). Some examples include the Model Code 90 [10], the final version of the Eurocode 2 [11], as well as related national building codes in European countries such as Germany [12] and Italy [13]. This second approach assumes that, as the load increases until shear failure, the compression struts may rotate and cross two adjacent cracks due to the presence of aggregate interlock and dowel forces, which is also confirmed by experimental evidence [14]. The variable-angle truss model, in turn, has undergone further refinements as well during the years [15–17].

A key issue in the mechanical-based derivation of the shear resisting mechanism for RC beams and columns is how cracked web transfers the shear stress. The class of mechanics-based models mentioned previously relies on simple equilibrium principles (and the theory of plasticity), without any explicit consideration for compatibility conditions. An alternative and well-established class of mechanics-based models (also known as compression field approaches) determines the angle of the compression diagonals by accounting for deformations of transverse reinforcement, longitudinal reinforcement and diagonal stressed concrete, thereby exploiting compatibility conditions and stress-strain relationships in addition to equilibrium equations. Early proposals of such theories accounting for compatibility conditions are presented in the works by Kupfer [18] and Baumann [19], in which the angle of the compression diagonals is determined by assuming a linear elastic behavior for cracked concrete and reinforcement. By removing the assumption of linear elastic behavior, Collins and Mitchell [20] developed the compression field theory (CFT), later on generalized as the modified compression field theory (MCFT) [21, 22], which rationally accounts for the tensile stresses in the diagonally cracked concrete. According to the MCFT, the shear resistance is expressed as the sum of steel and concrete contributions, where the concrete contribution describing the shear stress transferred vertically along the crack is calculated under the assumption that the post-cracking principal stress direction aligned with the compression diagonal is a principal strain direction as well. Simplified versions of the MCFT inspired the development of some guidelines and codes, such as the AASHTO standards [23] and the general method of the Canadian Building Code [24].

A somewhat different method to calculate the tensile stresses in diagonally cracked concrete was then proposed. It is the rotating-angle softened-truss model [25], which predicts that the angle of the principal stress direction decreases (rotates) with increasing shear stress [26], whereas the fixed-angle softened truss model [27] assumes that the concrete struts remain parallel to the initial cracks. Finally, different levels of approximations are presented in the Model Code 2010 [28]. The simplest approach is based on a fixed inclination angle of the compression diagonals (lower than 45°) and ignores the concrete contribution. The most precise level of approximation consists of an additive-type approach where the concrete contribution is calculated through compatibility conditions on the basis of the mid-depth longitudinal strain estimated from cross-sectional analysis [29].

In addition to mechanics-based approaches, a growing number of researchers have recently advanced the use of data-driven methodologies for solving structural engineering problems. Some latest research findings in this field were collected by El-Dakhakhni [30]. Particularly, a pure data-driven polynomial-based regression analysis has been performed by Azadi Kakavand et al. [31] to predict the shear strength of RC columns with rectangular and circular cross-section. A significant part of recent researches about pure data-driven development of capacity equations has basically originated from last progresses in the area of supervised machine learning, which, in its simplest form, is a function (i.e., nonparametric) identification problem: given some outputs collected into a training set, a machine learning technique aims at finding a function that well fits these training data and predicts the results for new ones as best as possible. Within this framework, artificial neural network (ANN) and genetic programming (GP) are the two most commonly adopted nonparametric identification techniques [32], and they have been also widely employed to predict the capacity of RC members. For instance, Mansour et al. [33] have adopted an ANN model to predict the shear strength of RC beams with stirrups. Another application of ANN has been reported by Oreta [34], in such case to predict the shear strength of RC beams without transverse reinforcement. More systematic studies about the role of the net architecture in predicting the shear capacity of RC columns with circular cross-section and RC beams with stirrups have been also presented recently [35, 36]. Applications of GP for estimating the shear capacity of RC elements are fewer than those in which ANN is implemented. As an example, Gandomi et al. [37] made use of GP in order to evaluate the shear strength of RC beams without stirrups. Instead of implementing the classical tree-based solution representation, this study explores a linear-type GP. While the work by Gandomi et al. [37] aggregates accuracy and complexity measures into a single objective function, a fully multi-objective optimization assisted by the non-dominated sorted genetic algorithm is employed by Tahmassebi et al. [38]. Apart from ANN and GP, further data-driven techniques have been considered occasionally in the past years to evaluate the capacity of RC members. For instance, the shear capacity of RC beams without stirrups has been estimated by Fiore et al. [39] through an evolutionary polynomial regression, whereas Zhang et al. [40] have applied the support-vector machine to predict the shear strength of deep RC beams. Random forest, adoptive boosting, gradient boosting regression tree and extreme gradient boosting have been employed by Feng et al. [41] to estimate the shear strength of RC deep beams with or without transverse reinforcement. Recently, Murad [42] exploited a Gene Expression Programming technique for data-driven derivation of the bidirectional shear strength equation for RC columns.

So far, white box (i.e., mechanics-based) and black box (i.e., data-driven) techniques have been considered as the only ways for predicting the capacity of RC members. While white box modeling is attractive because it attempts at explaining the involved resisting mechanism to derive the capacity equation with a broad generality but under simplifying assumptions, higher levels of accuracy are usually achieved by means of a black box modeling optimized for the input parameters range of interest. This dichotomy has produced a lively debate about feasibility and superiority of one approach over the other. Besides the two aforementioned approaches, an alternative hybrid strategy, namely a gray box modeling, might be promising for capacity assessment, but is basically ignored in the relevant literature. This approach consists in the construction of models whose mathematical structure is governed by physical principles but whose parameters are functions that must be identified through a data-driven approach [32]. Using such a gray box modeling, the development of capacity equations is based on a theoretical model derived on a mechanical basis, while their accuracy is improved by employing a data-driven approach in order to find appropriate expressions for some involved parameters, thus alleviating the errors due to the adopted simplifying assumptions.

The present work aims at bridging such gap between white and black box modeling by proposing a machine-learning-aided improvement of a mechanics-based shear capacity equation for RC members with stirrups. Throughout the present study, the focus is on slender RC beams and columns with rectangular cross-section subjected to uniaxial shear loading. Within this framework, the application of GP is here proposed to enhance the accuracy of the shear capacity equation currently in use within the Eurocode 2, which is based on a variable-angle truss model. Since the present contribution deals with a code-conforming capacity model, reliability and accuracy of the proposed approach are discussed within the largest comparative assessment ever presented so far among shear strength formulations reported into existing technical codes. This comparative analysis encompasses shear capacity equations for RC beams and columns with rectangular cross-section proposed by international organisms (European Committee for Standardization and International Federation for Structural Concrete) as well as from national/federal regulatory agencies or standardization bodies in Italy (Ministry of Infrastructure and Transportation), United States and Canada (American Association of State Highway and Transportation Officials, American Concrete Institute), New Zealand (Standards New Zealand), Japan (Japan Society of Civil Engineers), and China (Ministry of Housing and Urban-Rural Development). For practical design purposes, the improved shear capacity equation developed in this work is finally converted into a suitable design format by means of a simple, yet effective, procedure in compliance with current European standards. The present study differs significantly from all previous similar researches in that, here, machine learning is not intended to replace a mechanics-informed model able to explain how structural elements resist to applied loads. Actually, its application is meant at improving the accuracy and alleviate the inherent simplifications of the resulting capacity equations through the data-driven definition of the involved corrective parameters. In this sense, the proposed methodology can also be extended to the development of further capacity equations.

2. Review of code-based shear capacity equations

The effectiveness of the proposed approach will be examined within a large comparative assessment among shear capacity equations reported into some existing national and international technical codes. This is in line with the scopes of the present study, which deals with a machine-learning-aided improvement of a mechanics-based code-conforming capacity model. It is useful to point out that the following comparative assessment will be restricted to formulations already available into final approved official norms or guidelines. However, for the sake of completeness, it is worth remarking that, at the time this paper is being written, some of norms and guidelines are undergoing revision processes [43, 44].

2.1. International standards

2.1.1. Eurocodes

The European Building Codes include two formulations to predict the shear strength of RC beams and columns: while Eurocode 2 [11] (EC2) relies on a mechanics-based formulation, a data-driven formulation due to Biskinis et al. [45] is included within Eurocode 8 [46] (EC8).

According to EC2 [11], the shear strength of RC beams and columns with transverse reinforcement is determined through a mechanics-based formulation derived by resorting to a truss-type resistance mechanism with variable inclination of the diagonal concrete struts. In detail, the shear capacity V is determined as follows:

$$V = \min\{V_{Rs}, V_{Rc}\}, \tag{1}$$

where V_{Rs} and V_{Rc} are the shear capacity due to steel transverse reinforcement yielding and concrete struts crushing, respectively. Equation (1) implies that V can be attained with either the crushing of the concrete diagonal struts or the yielding of the steel reinforcement (i.e., the resisting mechanism resembles a series-type system). Let θ be the angle between the concrete compression strut and the longitudinal axis of the RC member under the limitation $21.81° \leq \theta \leq 45°$ (i.e., $1 \leq \cot\theta \leq 2.5$), if the shear reinforcement is provided by vertical stirrups, then the two resisting contributions read:

$$V_{Rs} = \frac{A_{sw}}{s} z f_{ysw} \cot\theta, \tag{2a}$$

$$V_{Rc} = \alpha_c b z \nu f_c \frac{\cot\theta}{1+\cot^2\theta}, \tag{2b}$$

where A_{sw} is the cross-sectional area of the shear reinforcement, s is the spacing of the stirrups, z is the inner lever arm (with $z = 0.9d$, d being the effective depth of the cross-section), f_{ysw} is the yield strength of the shear reinforcement, b is the (minimum) width of the cross-section, and f_c is the concrete compressive strength. Equation (2b) involves the following two corrective parameters:

$$\nu = 0.6\left[1 - \frac{f_c}{250}\right], \tag{3}$$

$$\alpha_c = \begin{cases} 1 & \text{if } \sigma_c/f_c = 0 \\ 1+\sigma_c/f_c & \text{if } 0 < \sigma_c/f_c \leq 0.25 \\ 1.25 & \text{if } 0.25 < \sigma_c/f_c \leq 0.50 \\ 2.50(1-\sigma_c/f_c) & \text{if } 0.50 < \sigma_c/f_c \leq 1.00 \end{cases}, \tag{4}$$

where f_c is given in [MPa] and σ_c is the (positive) compressive stress of the cross-section. Equations (3)-(4) are the strut efficiency factor and a correction factor for compressed members, respectively.

On the other hand, the capacity equation to predict the shear strength according to EC8 [46] is mostly empirical. It reads:

$$\begin{aligned} V = & \frac{h-x}{2a} \min\{N_E, 0.55 A_g f_c\} + \left(1 - 0.05\min\left\{5, \mu_\Delta^{pl}\right\}\right) \cdot \\ & 0.16\max\{0.5, 100\rho_{tot}\}\left(1 - 0.16\min\{5, a/h\}\right)\sqrt{f_c} A_g + V_{sw}, \end{aligned} \tag{5}$$

where forces and lengths are given in [MN] and [m], respectively. Herein, h is the height of the cross-section, x is the compression zone depth, a is the shear span, N_E is the (positive) compressive axial load, A_g is the area of the cross-section, μ_Δ^{pl} is the plastic displacement ductility demand (i.e., $\mu_\Delta^{pl} = \mu_\Delta - 1$, where μ_Δ is the displacement ductility demand), ρ_{tot} is the total longitudinal reinforcement ratio, $V_{sw} = \rho_{sw} b z f_{ysw}$ is the contribution of the transverse reinforcement to the shear resistance (where ρ_{sw} is the transverse reinforcement ratio). It is remarked that Eq. (5) is the formulation developed by Biskinis et al. [45] to assess the shear strength of slender beams and columns, for which it is assumed that the capacity is attained under diagonal tension failure. Failure of compression diagonals is instead assumed for squat elements (i.e., when $a/h \leq 2$), for which V has to be lower than V_{max} given by the following data-driven capacity equation [45]:

$$V_{max} = \frac{4}{7}\left(1 - 0.02\min\left\{5, \mu_{\Delta}^{pl}\right\}\right)\left(1 + 1.35\frac{\sigma_c}{f_c}\right)(1 + 45\rho_{tot}) \cdot \sqrt{\min\{f_c, 40\}}bz\sin 2\delta, \tag{6}$$

with $\tan\delta = h/(2a)$, δ being the angle between the diagonal and the longitudinal axis of the column.

2.1.2. Model Code

The design philosophy of the Model Code 2010 [28] (MC2010) for the determination of the shear resistance of RC elements is based on the level-of-approximation (LoA) concept [29, 47], with four levels of approximation with increasing complexity. In the general case, the shear resistance V can be expressed as the sum of a steel contribution V_s and of a concrete contribution V_c, and it is limited by an upper bound value $V_{\max}$:

$$V = V_s + V_c \leq V_{\max}. \tag{7}$$

The terms appearing in Eq. (7) take the following expressions:

$$V_s = \frac{A_{sw}}{s} z f_{ysw} \cot\theta, \tag{8a}$$

$$V_c = k_v \sqrt{f_c}\, bz, \tag{8b}$$

$$V_{\max} = k_c f_c bz \sin\theta\cos\theta, \tag{8c}$$

where $k_c = k_\varepsilon \eta_{f_c}$ is a strength reduction factor (whose meaning is similar to that of the parameter ν introduced in Eq. (3) for the EC2 formulation). It incorporates the strain effect and the brittleness effect through the parameters k_ε and η_{f_c}, respectively. Moreover, k_v is a correction factor for the shear resistance attributed to concrete.

Moving from the simplest approach to the most precise method of analysis and design, a variable-angle truss model is implemented in Level I, with a minimum angle of the compressive stress field $\theta_{\min}$ (ranging from 25° to 40°) depending on the axial stress on the concrete element (e.g., $\theta_{\min} = 25°$ for members with significant axial compression or prestress, and $\theta_{\min} = 30°$ for RC members without axial stress), and a constant value of the strength reduction factor $k_\varepsilon = 0.55$. Level II is based on a generalized stress field approach wherein the strut angle $\theta_{\min}$ is expressed as function of the mid-depth longitudinal strain ε_x, which is estimated from cross-sectional analysis as follows:

$$\theta_{\min} = 20° + 10000\,\varepsilon_x, \tag{9a}$$

$$\theta = 29° + 7000\,\varepsilon_x \geq \theta_{\min}, \tag{9b}$$

$$\varepsilon_x = \frac{1}{2E_s A_s}\left(\frac{M_E}{z} + V_E + N_E\left(\frac{1}{2} \mp \frac{e}{z}\right)\right), \tag{9c}$$

where M_E, V_E, N_E are bending moment, shear force and axial force (the latter taken positive for tension and negative for compression) acting on the cross section, respectively, whereas e is the axial load eccentricity. Moreover, E_s and A_s represent the elastic modulus and the area of the steel longitudinal reinforcement bars in the tensile chord, respectively. The strain effects in Level II are estimated based on the principal tensile stress ε_1. This, in turn, depends upon the mid-span longitudinal strain ε_x, so that a relationship of the form $k_\varepsilon = k_\varepsilon(\varepsilon_x)$ exists in such case. In both Level I and Level II, the shear resistance attributed to concrete is neglected, that is, $k_v = 0$. Level III resorts to a simplified version of the MCFT according to the additive approach expressed by Eq. (7), with a nonzero concrete contribution calculated by assuming $k_v = k_v(\varepsilon_x, V_E)$. The most accurate Level IV is commonly confined to research applications and is generally avoided in engineering practice because it involves fulfilling the entire set of equilibrium and compatibility conditions, as well as stress-strain relationships for steel and diagonally cracked concrete, according to the full version of the MCFT. Level III approach only will be considered in the following.

2.2. National standards

2.2.1. Italy

The Italian Building Code consists of a ministerial decree [13] (NTC2018) and a commentary document [48] (Circ2019), both of which are based on the same general concepts as prescribed in the European regulations. In particular, the previous Eqs. (1)-(2) and Eq. (5) are here adopted to predict the shear strength. The only difference with the European standards is concerned with the parameter ν, for which a constant value equal to 0.50 is herein assumed. Additionally, a further empirical equation for RC columns due to Priestley et al. [49] is also reported within the Italian Building Code in the commentary document [48], which reads:

$$V = V_s + V_c + V_n, \tag{10}$$

where V_s, V_c and V_n are the contributions to the shear capacity due to steel transverse reinforcement, concrete, and axial force, respectively. They have the following expressions:

$$V_s = \frac{A_{sw}}{s} f_{ysw} z \cot\left(\frac{\pi}{6}\right), \tag{11a}$$

$$V_c = 0.80 A_g k \sqrt{f_c}, \tag{11b}$$

$$V_n = N_E \frac{h-x}{2a}. \tag{11c}$$

Herein, the parameter k depends on the displacement ductility demand μ_Δ as:

$$k = \begin{cases} 0.29 & \text{if } \mu_\Delta \leq 2 \\ 0.29 - 0.095(\mu_\Delta - 2) & \text{if } 2 < \mu_\Delta < 4 \\ 0.10 & \text{if } \mu_\Delta \geq 4 \end{cases}. \tag{12}$$

2.2.2. United States and Canada

The ACI 318 Building Code, from the older version ACI 318-95 up to the recent ACI 318-19 [7] (ACI318), is based on an additive approach for the determination of the shear strength of RC members V, consisting in a 45° truss model (i.e., RM model) combined with a concrete contribution calibrated on empirical basis:

$$V = V_s + V_c, \tag{13}$$

where the two contributions related to concrete and steel are expressed as follows:

$$V_s = \frac{A_{sw}}{s} f_{ysw} d \leq 0.66\sqrt{f_c}\, bd, \tag{14a}$$

$$V_c = \left(0.17 \min\left\{\sqrt{f_c}, 8.3\right\} + \min\left\{\frac{N_E}{6\,A_g}, 0.05 f_c\right\}\right) bd \leq 0.42\sqrt{f_c}\, bd, \tag{14b}$$

with f_c given in [MPa].

In a similar fashion, according to the American Association of State Highway and Transportation Officials (AASHTO) bridge design specifications [23], the shear resistance is formulated by using an additive approach as follows:

$$V = V_s + V_c \leq V_{\max}, \tag{15}$$

where

$$V_s = \frac{A_{sw}}{s} f_{ysw} z \cot\theta, \tag{16a}$$

$$V_c = 0.083\beta\sqrt{f_c}\, bz, \tag{16b}$$

$$V_{\max} = 0.25 f_c bz, \tag{16c}$$

in which β is a factor expressing the ability of diagonally cracked concrete to transfer tension and f_c is given in [MPa]. The values of θ and β are provided either in tabular format (in older versions of the standard) or through algebraic equations (in more recent versions) as function of the largest calculated longitudinal strain ε_x and, only for sections with less than the minimum transverse reinforcement, of the crack spacing parameter s_{xe}, the latter mainly related to size effects.

It is worth noting that the way the angle θ is computed in the AASHTO standards is identical to that previously explained for the MC2010 in Eq. (9b), and is based on the simplified MCFT [22]. The involved equations are also very similar to those adopted in the Canadian design code [24].

2.2.3. New Zealand

According to the New Zealand Standards for concrete structures [50] (NZS3101), the shear strength V is computed through the sum of two contributions, one (calculated through the RM model) provided by shear reinforcement and one (empirically derived) attributable to concrete:

$$V = V_s + V_c, \tag{17}$$

with

$$V_s = \frac{A_{sw}}{s} f_{ysw} d, \tag{18a}$$

$$V_c = v_c A_{cv}, \tag{18b}$$

where v_c is the shear resisted by concrete and $A_{cv} = bd$ is the effective shear area. The value of v_c is obtained through empirical factors that turn out to be slightly different for beams and columns. For beams, v_c takes the following form:

$$v_c = k_d k_a v_b, \tag{19}$$

where the factor k_d accounts for the influence of member depth and size effects ($k_d = 1.0$ for members with more than the minimum transverse reinforcement or with 200 mm $< d \leq$ 400 mm, otherwise $k_d <$

1.0). The factor k_a incorporates the influence of maximum aggregate size ϕ_a on the shear strength (for $\phi_a \geq 19$ mm, $k_a = 1.0$, otherwise k_a is assumed linearly decreasing up to $k_a = 0.85$ for $\phi_a \leq 10$ mm) and $v_b = \min\{0.07 + 10\rho_{tot}, 0.2\}\sqrt{f_c} \geq 0.08\sqrt{f_c}$ with $f_c \leq 50$ MPa. An expression similar to Eq. (19) can be used for columns, but k_d is replaced by the factor k_n accounting for the influence of axial load on the shear strength, that is:

$$v_c = k_n k_a v_b, \tag{20}$$

where $k_n = 1 + 3\sigma_c/f_c$, under the constraint $\sigma_c/f_c \leq 0.3$.

2.2.4. China

The shear capacity V of RC members with stirrups and rectangular cross-section is estimated in the Chinese Building Code GB50010-2010 [51] (GB50010) by combining the concrete contribution and the shear reinforcement contribution into a single term V_{cs}, to which the additional contribution due to axial load (if any) V_n is superimposed. Therefore:

$$V = V_{cs} + V_n, \tag{21}$$

with

$$V_{cs} = \alpha_{cv} f_t b d + \frac{A_{sw}}{s} f_{ysw} d, \tag{22a}$$

$$V_n = 0.05 \max\{N_E, 0.3 f_c A_g\}, \tag{22b}$$

where α_{cv} depends upon the shear span ratio a/d and may be calculated in case of isolated elements under concentrated loads as $\alpha_{cv} = 1.75/(1 + a/d)$ with a/d equal to 1.5 if it is lower than 1.5 and equal to 3.0 if it is larger than 3.0, while f_t represents the concrete tensile strength (provided in tabular form).

For the sake of completeness, it is highlighted that GB50010 refers to V_n given by Eq. (22b) as "jacking force". A capacity equation identical to that in Eq. (21) with $V_n = 0.07 \max\{N_E, 0.3 f_c A_g\}$ is also reported in GB50010 for members subjected to an eccentric axial force. Since no further expressions seem to be available in GB50010, Eqs. (21)-(22b) are adopted in the following under the assumption that the effects of the "jacking force" on the shear strength are the same as those due to an applied, external centered compressive force.

2.2.5. Japan

According to the JSCE Guidelines for Concrete no. 15 [52] (JSCE15), the shear capacity V of linear (nonprestressed) RC members is obtained as the sum of two contributions, one attributable to the shear reinforcement (calculated through the RM model) and one (empirically derived) due to concrete. Therefore:

$$V = V_s + V_c, \tag{23}$$

where

$$V_s = \frac{A_{sw}}{s} f_{ysw} z, \tag{24a}$$

$$V_c = \beta_d \beta_\rho \beta_n f_{vc} bd, \tag{24b}$$

in which $f_{vc} = 0.20 f_c^{1/3}$ is a parameter related to the tensile strength of concrete (with f_c given in [MPa]), while $\beta_d, \beta_\rho, \beta_n$ are three correction factors. The factor β_d accounts for the influence of the member depth and reads $\beta_d = \min\{(1000/d)^{1/4}, 1.5\}$, with d given in [mm]. The factor β_ρ incorporates the effect of the longitudinal reinforcement and takes the expression $\beta_\rho = \min\{(100\rho_l)^{1/3}, 1.5\}$, where $\rho_l = A_s/(bd)$ is the tensile longitudinal reinforcement ratio. The last factor β_n reflects the contribution of the axial load through the expression (valid for compression axial force) $\beta_n = \min\{1 + 2M_o/M_u, 2.0\}$, where M_o is the decompression bending moment, whereas M_u is the pure flexural strength of the cross-section without axial force.

3. Improved mechanics-based code-conforming shear capacity equation

3.1. Proposed machine-learning-aided approach and its implementation

The previous critical review of code provisions from around the world about the shear capacity assessment of RC beams and columns has shown that they were developed in different ways (i.e., through the analysis of a resisting mechanism or following a pure data-driven approach) and rely on different sets of parameters. Among the reviewed formulations, the capacity equation currently in use within the Eurocode 2 given by Eqs. (1)-(2) seems particularly appealing for two reasons. First, starting from a well-established truss-type resistance mechanism with variable inclination of the diagonal concrete struts, the capacity equation can be obtained in a fully analytical fashion exactly as it appears in Eqs. (1)-(2), with the only exception of the involved corrective parameters. Second, it is rather compact and easy to use for practitioners. The main criticism thus deals with the corrective parameters ν and α_c in Eqs. (3)-(4), which were likely derived empirically but whose definition is not very clear. Hence, it seems appropriate to explore whether a machine-learning-based approach can serve at defining new corrective parameters in place of ν and α_c in such a way to improve the accuracy of the final predictions provided by Eqs. (1)-(2), thus alleviating the impacts of the inherent simplifications in the underlying truss-type resisting mechanism while retaining the advantage of using a mechanics-based and code-conforming formulation. To this end, the capacity model given by Eqs. (1)-(2) is initially rewritten as follows:

$$V = \min\{V_{Rs}, V_{Rc}\}, \tag{25}$$

$$V_{Rs} = \frac{A_{sw}}{s} z f_{ysw} \cot\theta, \tag{26a}$$

$$V_{Rc} = \eta_c b z \eta f_c \frac{\cot\theta}{1+\cot^2\theta}, \tag{26b}$$

where η and η_c are the new corrective parameters that have to be defined in place of ν and α_c through a machine-learning-based approach. While the new factor η still accounts for the strut efficiency in the variable-angle truss model as ν, the new factor η_c differs from α_c in that it mostly takes into account the effects of cyclic loading and applied compressive stress. The following methodology is herein proposed and implemented to define new expressions for η and η_c.

Initially, a set of relevant experimental data about the shear capacity of RC beams under monotonic loading condition (and null compressive stress) is collected. Therefore, the optimal value of the parameter η in Eq. (26b) is calculated for each sample by solving the following problem:

$$\begin{aligned} \min_{\eta} \quad & \frac{|V_{\text{num}}(\eta|\eta_c = 1) - V_{\text{exp}}|}{V_{\text{exp}}} \\ \text{s.t.} \quad & \eta_{\min} \le \eta \le \eta_{\max} \\ & \theta_{\min} \le \theta \le \theta_{\max} \end{aligned}, \tag{27}$$

where $V_{\text{num}}(\eta|\eta_c = 1)$ is the numerical prediction of the shear capacity according to Eq. (25)-(26) as function of the variable η and assuming $\eta_c = 1$ in Eq. (26b) (i.e., it is assumed that the parameter η_c plays no role under monotonic loading condition). Hence, the optimal values of η are determined for each sample in such a way that $V_{\text{num}}(\eta|\eta_c = 1)$ is as close as possible to the corresponding experimental data V_{exp} while satisfying predefined lower and upper bounds given by $\eta_{\min}$ and $\eta_{\max}$, respectively. Furthermore, the angle θ between the concrete compression strut and the longitudinal axis of the RC member is also constrained to vary between a minimum and a maximum value given by $\theta_{\min}$ and $\theta_{\max}$, respectively. Once the problem in Eq. (27) has been solved for each sample, the optimal values of η, namely η_{opt}, are available. A suitable machine learning technique is thus employed to search for a new expression for η, namely $\eta = f(\boldsymbol{\vartheta})$, in such a way that its predictions fit the retrieved optimal values η_{opt} in the best possible manner. Herein,

$\boldsymbol{\vartheta}$ is the set of all model variables upon which η can depend, and f is the optimal functional relationship to be searched within the class of candidate mathematical models given by all possible combinations of the operators available into a user-assigned functions set.

Next, a set of relevant experimental data about the shear capacity of RC columns under cyclic loading condition is collected. Therefore, the optimal value of the parameter η_c in Eq. (26b) is calculated for each sample by solving the following problem:

$$\begin{aligned} \min_{\eta_c} \quad & \frac{|V_{\text{num}}(\eta_c|\eta = f(\boldsymbol{\vartheta})) - V_{\text{exp}}|}{V_{\text{exp}}} \\ \text{s.t.} \quad & \eta_{c,\min} \leq \eta_c \leq \eta_{c,\max} \\ & \theta_{\min} \leq \theta \leq \theta_{\max} \end{aligned} , \tag{28}$$

where $V_{\text{num}}(\eta_c|\eta = f(\boldsymbol{\vartheta}))$ is the numerical prediction of the shear capacity according to Eq. (25)-(26) as function of the variable η_c and assuming $\eta = f(\boldsymbol{\vartheta})$ as obtained previously. Similarly to Eq. (27), the optimal values of η_c in Eq. (28) are determined for each sample in such a way that $V_{\text{num}}(\eta_c|\eta = f(\boldsymbol{\vartheta}))$ is as close as possible to the corresponding experimental data V_{exp} while satisfying predefined lower and upper bounds given by $\eta_{c,\min}$ and $\eta_{c,\max}$, respectively. So doing, a minimum and a maximum value given by $\theta_{\min}$ and $\theta_{\max}$, respectively, are again imposed to the angle θ between the concrete compression strut and the longitudinal axis of the RC member. Once the problem in Eq. (28) has been solved for each sample, the optimal values of η_c, namely $\eta_{c,\text{opt}}$, are obtained. In a similar fashion, a suitable machine learning technique is thus employed to search for a new expression for η_c, namely $\eta_c = g(\boldsymbol{\varphi})$, in such a way that its predictions fit the retrieved optimal values $\eta_{c,\text{opt}}$ in the best possible manner. Herein, $\boldsymbol{\varphi}$ is the set of all model variables upon which η_c can depend, whereas g is the optimal functional relationship to be searched within the class of candidate mathematical models given by all possible combinations of the operators available into an user-assigned functions set. It is important to remark that the final expression for η_c is primarily expected to reflect the differences between the shear failure of RC columns under cyclic loading condition as compared to that of RC beams under monotonic loading conditions. Secondarily, it inherently introduces the corrections that allow using a unique expression of η for both typologies of RC members, although range and distribution of the model variables in columns might be slightly different from that in beams (indeed, mechanical and geometrical data into the databases of beams and columns can span over different ranges with different frequencies).

Lower and upper bounds for η and η_c in Eq. (27) and Eq. (28), respectively, are herein assumed equal to $\eta_{\min} = 1.001\omega_{sw}$, $\eta_{\max} = 1.0$, $\eta_{c,\min} = 0.3$ and $\eta_{c,\max} = 3.0$. On the other hand, the upper limit for θ in Eqs. (27)-(28) is taken as $\theta_{\max} = 45°$ in agreement with EC2 since this threshold can be motivated on mechanical basis and is also supported by data [45]. Actually, with regard to the lower limit for θ, Biskinis et al. [45] pointed out that it can result much lower than the EC2 lower limit of 21.81°. It is worth noting that lower values of θ might require larger elongation of the shear reinforcement (i.e., higher amount of redistribution of internal forces) that, in turn, would widen the inclined cracks and would reduce the resistance of the inclined compressive struts [53]. Considering the range of variation of θ recommended in the EC2 (i.e., $1 \leq \cot\theta \leq 2.5$), the effective concrete compressive strength is assumed around $0.5f_c$ owing to the efficiency factor ν in Eq. (3). In the present work, the corrective term η defined through the machine learning approach is allowed to take on values lower than those assumed by ν in the EC2 formulation. Similar remarks can be made when comparing η_c with α_c in the EC2 formulation. Based on these considerations, the data reported by Biskinis et al. [45] as well as the suggestions from other truss-type mechanical models from the literature [16, 54], a lower limit equal to $\theta_{\min} = 11.31°$ (i.e., $1 \leq \cot\theta \leq 5$) is assumed in this work.

The selection of homogeneous and reliable databases is essential for the proposed machine-learning-aided approach. Experimental data for RC beams and columns needed in this study have been collected from the pertinent literature. Three popular databases for RC beams failing in shear are considered, namely the database prepared by Mansour et al. [33] including 133 samples, the database prepared by Zhang et al. [55] including 194 samples, and the ACI-DAfStb database prepared by Reineck et al. [56, 57] including 170 samples. Specimens not failing in shear were excluded a priori by the authors presenting these databases. Moreover, for RC columns failing in shear, the NEES ACI 369 rectangular column database prepared by Ghannoum et al. [58] is considered, with the recent extensions provided by Azadi Kakavand et al. [59] in the PRJ-2526 database. The latter database was also recently adopted by the same authors to develop a pure data-driven model for the shear strength of RC rectangular and circular columns [31]. Among the 325 samples in the PRJ-2526 database, some specimens exhibited a flexural failure and were excluded from the database, thus keeping only shear and flexure-shear failures.

Although the experimental samples belonging to these databases were selected by the original authors on the basis of reasonable and clear control criteria, some additional filters have been applied so as to select data consistent with the underlying truss-type resistance mechanism considered in the present work. More specifically, only slender RC members with a shear span-to-effective depth ratio $a/d \geq 2.2$ are considered, in order to exclude the possibility that a significant direct transfer of the applied load towards the supports (also known as "arch effect") can occur together with, or in place of, the relevant resisting mechanism based on the variable-angle truss model. Furthermore, samples characterized by a transverse reinforcement mechanical ratio $\omega_{sw} \leq 0.25$ are taken into account. Finally, the selected samples for RC columns are characterized by a ratio between applied compressive stress and concrete compressive strength $\sigma_c/f_c \leq 0.50$. Apart from that, it is worth remarking that the variable-angle truss model that underlies Eq. (25)-(26) does not pose special restrictions. Particularly, it does not require special limitations about compressive concrete strength or yielding stress of the steel reinforcement, thus samples with high-strength concrete and/or steel are also considered. The distribution of the main variables within the considered databases for RC beams (373 samples) and columns (119 samples) accounting for this further filtering on the collected literature data is provided in Fig. 1 and Fig. 2, respectively.

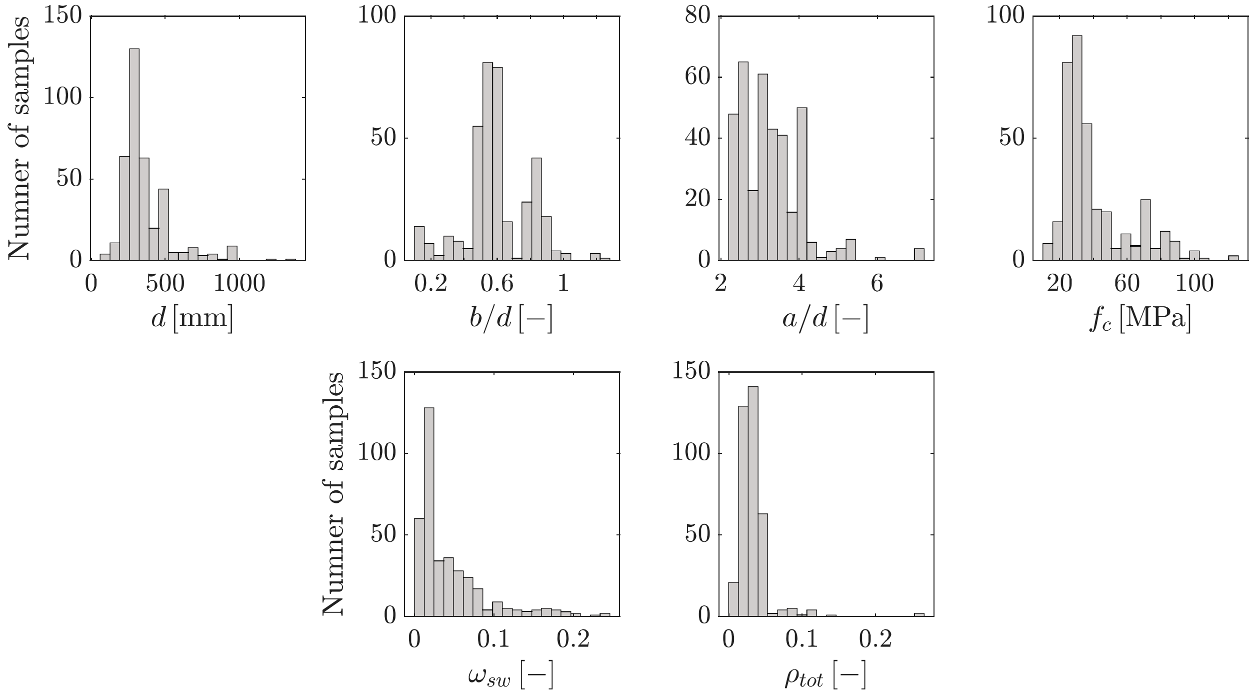

Figure 1: Distribution of some variables within the considered experimental database of shear capacity values for RC beams.

For the sake of completeness, it is mentioned that a few variables are sometimes missing within the collected experimental data. In order to avoid reducing the size of the databases for this analysis and the next comparative assessment, the following assumptions have been made: i) if only one parameter between h or d is explicitly available, then it is assumed $d = 0.90h$; ii) if ρ_{tot} only is explicitly available, then it is assumed that the compressive longitudinal reinforcement in beams is one half of the tensile longitudinal reinforcement, while a symmetric longitudinal reinforcement is presumed for columns; iii) the maximum aggregate size ϕ_a is taken equal to 16 mm if not available. Without more detailed information, it is expected that such assumptions are reasonable, on average, and have no large effects on the final results

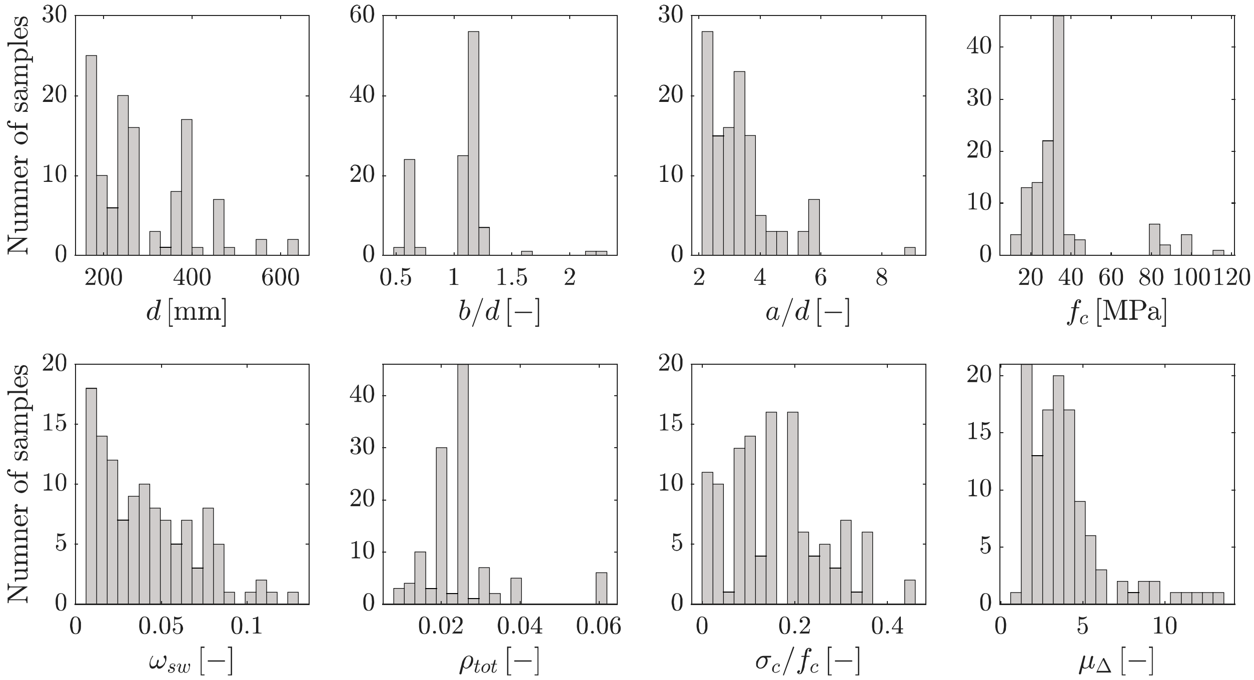

Figure 2: Distribution of some variables within the considered experimental database of shear capacity values for RC columns.

and the relevant statistics and conclusions.

The search for suitable mathematical models for η and η_c can be done by means of several machine learning techniques. Herein, a standard GP with a tree-based representation of the candidate solutions is employed (the interested reader can refer, for instance, to the state-ot-the-art review by Quaranta et al. [32] and references cited therein for details). The use of GP was found appropriate in the context of this study since it allows to define inline expressions for η and η_c that can be readily used in practical applications. Although alternative machine learning techniques can be possibly adopted, a comparative-based selection of the best one is not deemed a relevant task for the scopes of the present work. The optimal models are searched with the aim of maximizing the accuracy of the numerical predictions given by either $\eta = f(\boldsymbol{\vartheta})$ or $\eta_c = g(\boldsymbol{\varphi})$ as compared to the corresponding optimal values (either η_{opt} or $\eta_{c,\text{opt}}$, respectively). The adopted accuracy measure is the Pearson correlation coefficient R^2, which has to be maximized throughout the GP-based search by combining optimally the candidate model variables using the operators available into the functions set (this set consists of standard arithmetic operators only, in such a way to derive easy-to-use formulations for practitioners). Common dimensional and dimensionless explanatory model variables are taken into account (i.e., $\boldsymbol{\vartheta} = \{f_c, b/d, a/d, \omega_{sw}\}$ and $\boldsymbol{\varphi} = \{f_c, b/d, a/d, \omega_{sw}, \sigma_c/f_c, \mu_\Delta\}$). The population of candidate solutions is initially generated according to the ramped half-and-half method and then manipulated iteratively by applying classical genetic operators. After a preliminary evaluation based on different combinations of most common genetic operators, the GP is launched for the final analyses using a population size equal to 1,000, tournament selection (with a tournament size equal to 10), subtree crossover (with crossover rate equal to 0.90, functions and leaves being selected as crossover point 90% and 10% of the times, respectively), point mutation (with mutation rate equal to 0.15) and reproduction (by duplicating 5 candidate solutions within the current population to the next without changes). The iterative procedure is stopped when there is no improvement after 100 generations.

From an operative standpoint, the data-driven search for $\eta = f(\boldsymbol{\vartheta})$ and $\eta_c = g(\boldsymbol{\varphi})$ via GP starts with the preparation of training and testing datasets, which are obtained by sampling the complete databases randomly as follows: 80% of the samples for the training dataset (70% as learning dataset, 10% as validation dataset), and 20% for the testing dataset. An intermediate rough final solution is first defined by analyzing the results carried out for tree depths up to 16 and comparing the final accuracy values for different arrangements of the learning and validation datasets in order to minimize the degradation of the predictive ability for unseen data due to overfitting. This task also accounted for the function complexity and a possible mechanics-based explanation of the final formulations. When an intermediate rough final solution that also well-behaves over the testing dataset is identified, then a sensitivity analysis is performed in order to get a refined final solution after a further model simplification. This step concludes the search for $\eta = f(\boldsymbol{\vartheta})$ or $\eta_c = g(\boldsymbol{\varphi})$.

It is important to remark that a machine learning technique is here implemented to identify suitable expressions of the involved corrective parameters whereas the capacity estimation is ultimately governed by a mechanics-based resisting mechanism. This strongly differs from the usual approach followed in previous studies in the area of data-driven strength prediction, in which a machine learning technique is applied to predict directly the final capacity. Although a pure data-driven approach is likely to produce more accurate predictions, building a capacity model on a mechanical ground is deemed an equally important objective. Moreover, while strength prediction based on pure data-driven approaches can be prone to overfitting, the fact that the final capacity estimation is still ruled by a mechanical model is helpful to enhance the robustness of the predictive capability.

3.2. New shear capacity equation

Based on the proposed machine-learning-aided approach, the shear capacity V for RC beams and columns is still determined in compliance with the EC2 formulation through the mechanics-based unified formulation given by Eqs. (25)-(26) wherein the new corrective factors $\eta = f(\boldsymbol{\vartheta})$ and $\eta_c = g(\boldsymbol{\varphi})$ are defined as follows:

$$\eta = 0.12 + \frac{3.86 + 3.94(b/d)}{8.21 - 0.08(b/d) - 0.08f_c + f_c(b/d)} \text{ (with } f_c \text{ in [MPa])}, \tag{29}$$

$$\eta_c = \begin{cases} 1 & \text{for beams, monotonic loading} \\ 0.37 + \dfrac{0.30 + 0.75(\sigma_c/f_c) - 1.39(a/d)\omega_{sw}}{a/d}\left(3.79 + 57.52\dfrac{\sigma_c/f_c}{\mu_\Delta^2}\right) & \text{for columns, cyclic loading} \end{cases}. \tag{30}$$

Equations (29)-(30) are valid within the range of values of the explanatory model parameters defined by the considered databases (it is also recalled that σ_c is the compressive stress of the cross-section, which is assumed as positive value), subjected to the constraints $0.1 \leq \eta \leq 1.0$, $1/3 \leq \eta_c \leq 2.6$, and $11.31° \leq \theta \leq 45°$ (i.e., $1 \leq \cot\theta \leq 5$). The θ value used in the shear capacity equations is determined, for each data point depending on the mechanical and geometrical input data, as the angle leading to simultaneous crushing of compression struts and yielding of transverse reinforcement. This condition, also termed "web crushing criterion" by Hoang and Nielsen [60], is obtained by equating Eq. (26a) with Eq. (26b):

$$\cot\theta^* = \sqrt{\frac{\eta \cdot \eta_c}{\omega_{sw}} - 1}. \tag{31}$$

It is worth noting that this choice is a rational design procedure adopted in other literature studies [16, 17, 61] as it is consistent with the lower-bound theorem of plasticity: it inherently leads to the largest limit load among all possible statically and plastically admissible solutions. This approach is adopted for selecting the θ angle not only in the proposed model, but also in assessing the original EC2 formulation, by suitably replacing the η and η_c coefficients in Eq. (31) with ν and α_c. For completeness, it is noted that the designer would be, in principle, free to subjectively select any value of θ within the acceptable range, which may lead to more conservative estimates (i.e., lower shear strength values) than those reported in the accuracy assessment discussed in this paper, which have been carried out avoiding any non-objective choice that would lead to inconsistent comparisons. The R^2 values resulting from the calibration of η were found rather low, equal to 22.47% and 19.76% for the training and the complete dataset, respectively. Those resulting from the calibration of η_c were found much larger, equal to 75.64% and 73.52% for the training and the complete dataset, respectively.

Equation (29) discloses an inverse relationship between η and f_c, in agreement with the existing relationship between ν and f_c already present in the original EC2 formulation given by Eq. (3), see also Fig. 3. Moreover, Fig. 3 highlights that η decreases as b/d increases, which means that the proposed efficiency factor η reduces as the flexural inertia of the concrete diagonals cross-section decreases. Such outcome is a significant novelty with respect to the existing relationship between ν and f_c in the original EC2 formulation of Eq. (3). It might be explained by accepting that such formulation for η aims at taking into account the effects due to the bending moment in the concrete diagonals, which is not considered in the formulation of the original truss-based mechanism.

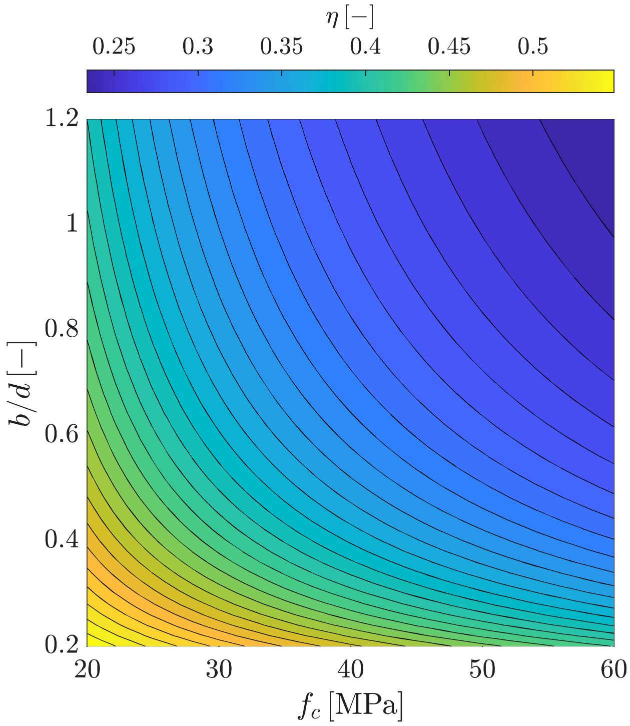

Figure 3: Evaluation of the proposed formulation for η.

The formulation for η_c in Eq. (30), as expected, involves the contribution of the applied compression in terms of σ_c/f_c. In agreement with the original EC2 formulation for α_c given by Eq. (4), the larger σ_c/f_c, the greater η_c, see also Fig. 4. On the other hand, the larger μ_Δ, the lower η_c. This outcome is a significant novelty with respect to the existing formulation for α_c in the original EC2 formulation of Eq. (4). It is in agreement with available experimental findings from the literature and indicates that the shear strength of RC columns under cycling loading condition degrades as the displacement ductility demand μ_Δ increases. It is noted in Fig. 4 that such reduction of the shear strength approaches a constant value when the displacement ductility demand μ_Δ is large enough, in agreement with the conclusions by Biskinis et al. [45]. However, while Biskinis et al. [45] states that the shear strength degradation due to displacement ductility demand does no longer increase beyond a fixed threshold, Fig. 4 suggests that such a limit value can vary. Finally, Eq. (30) and Fig. 4 also highlight an inverse relationship between η_c and ω_{sw} as well as between η_c

and a/d, whereas the term b/d does not appear in Eq. (30) essentially because it is almost constant in the database for RC columns.

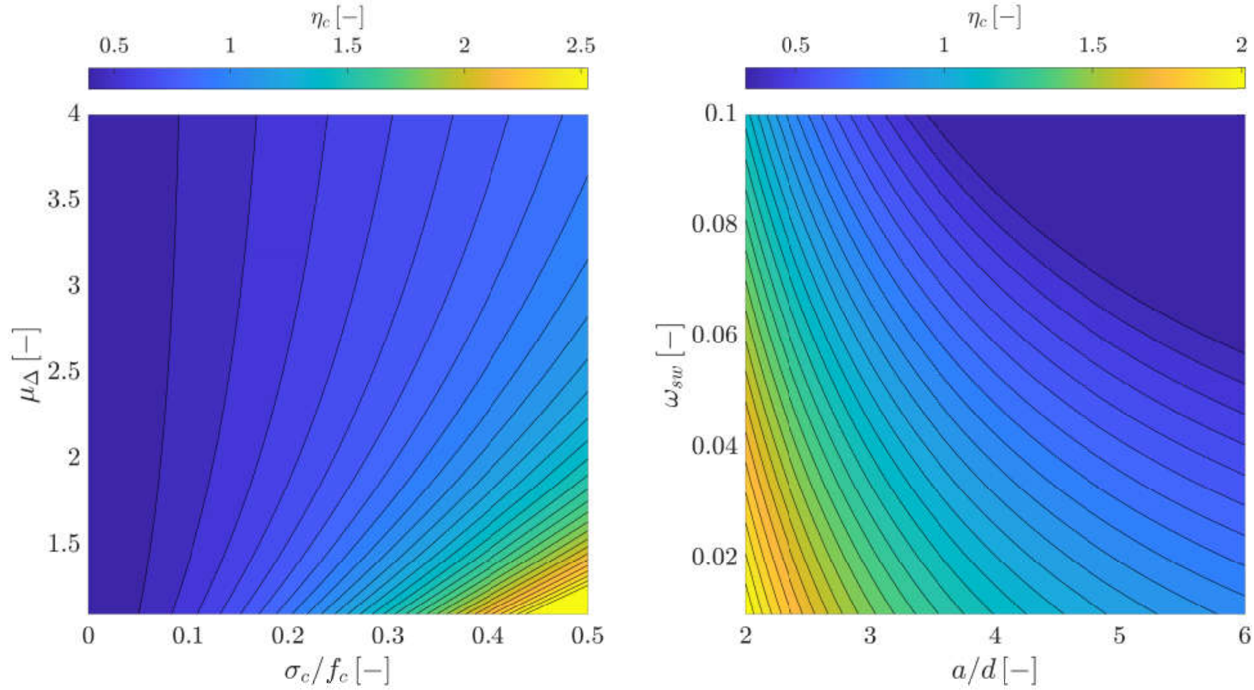

Figure 4: Evaluation of the proposed formulation for η_c ($a/d = 4$ and $\omega_{sw} = 0.05$ in the left-side picture, whereas $\sigma_c/f_c = 0.25$ and $\mu_\Delta = 2$ in the right-side picture).

The main outputs of the two-step machine-learning-aided procedure as applied to beams are provided in Figs. 5-6 whereas the results for columns are given in Figs. 7-8. Specifically, Fig. 5 and Fig. 7 provide the values of $V_{\rm opt} = V_{\rm num}(\eta_{\rm opt}|\eta_c = 1)$ and $V_{\rm opt} = V_{\rm num}(\eta_{c,\rm opt}|\eta = f(\boldsymbol{\vartheta}))$ for beams and columns, respectively. Since the values of $V_{\rm opt}$ are obtained from a sample-by-sample calibration of η and η_c according to Eq. (27) and Eq. (28), respectively, they are very close to the corresponding test values. On the other hand, the values of the shear strength $V_{\rm proposed}$ obtained according to Eqs. (25)-(26) and Eqs. (29)-(30) for beams and columns are given in Fig. 6 and Fig. 8, respectively. As expected, the use of the general expressions for η and η_c given by Eq. (29) and Eq. (30) reduces the accuracy and increases the dispersion of the shear capacity predictions as compared to a sample-by-sample calibration. Nonetheless, Fig. 6 and Fig. 8 basically highlight a general good agreement between numerical estimates and experimental values, thus demonstrating the satisfactory accuracy of the proposed procedure.

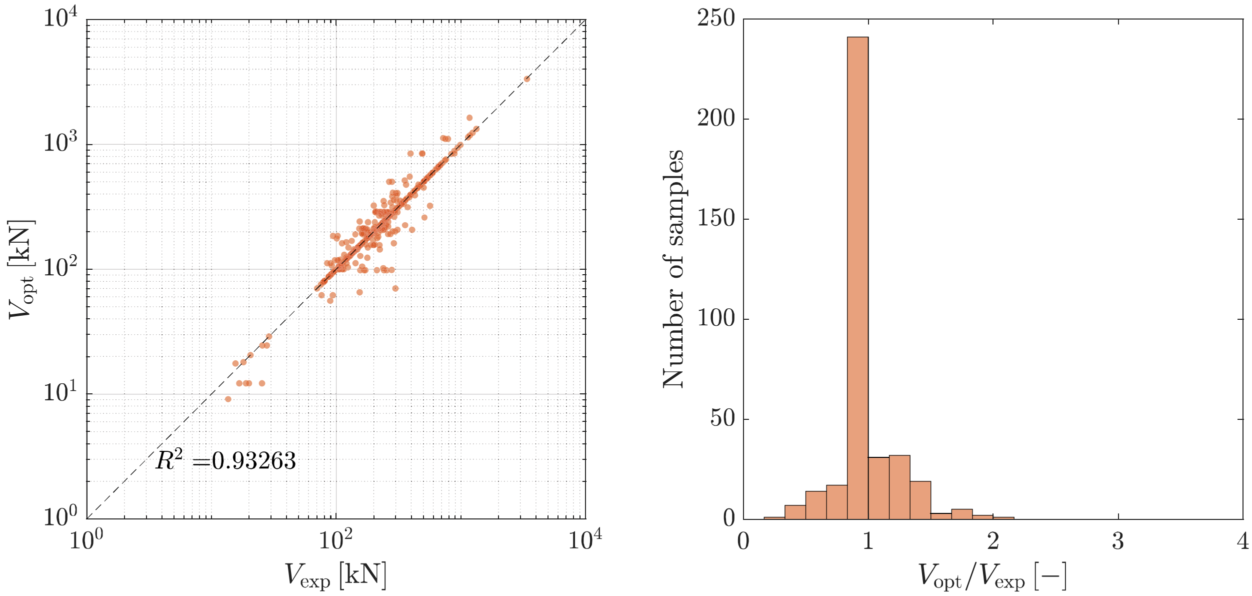

Figure 5: Comparison between optimized predictions of the shear capacity V_{opt} and corresponding experimental values V_{exp} for RC beams.

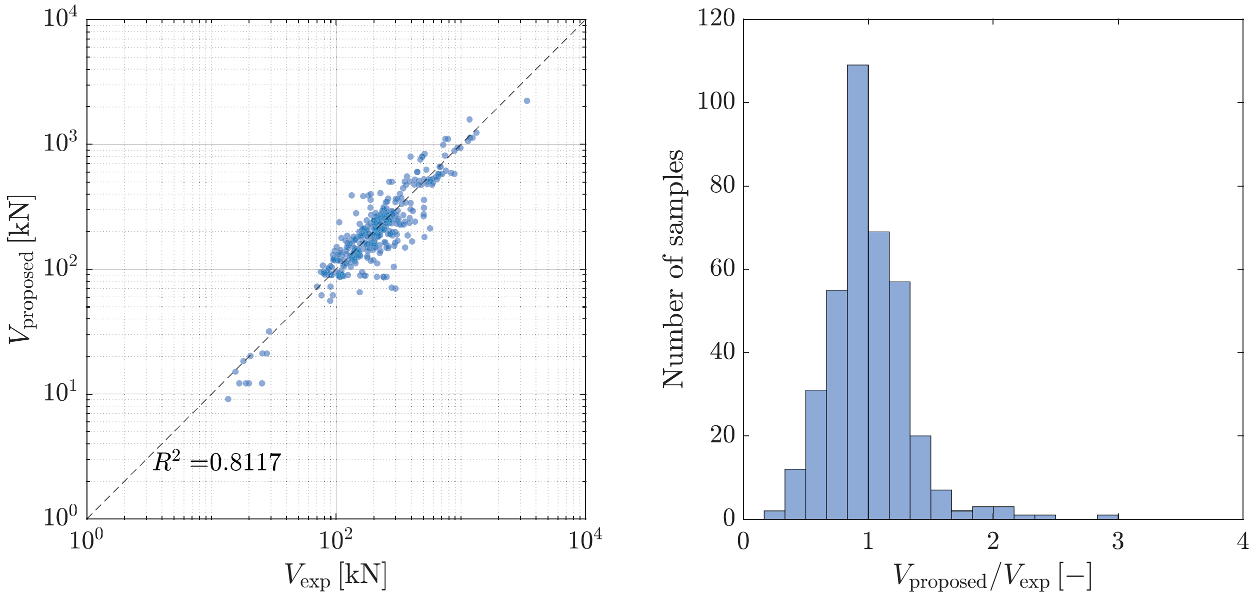

Figure 6: Comparison between shear capacity predictions obtained by means of the proposed approach V_{proposed} and corresponding experimental values V_{exp} for RC beams.

3.3. Comparative assessment of code-conforming shear capacity equations

3.3.1. Comparison of shear capacity equations for beams

A comparative assessment of shear capacity equations for RC beams in terms of mean, coefficient of variation, mean squared error (i.e., sum of variance and squared bias), median, and interquatile range (i.e., difference between the 75th and the 25th percentiles) of the ratio between code-conforming predictions of

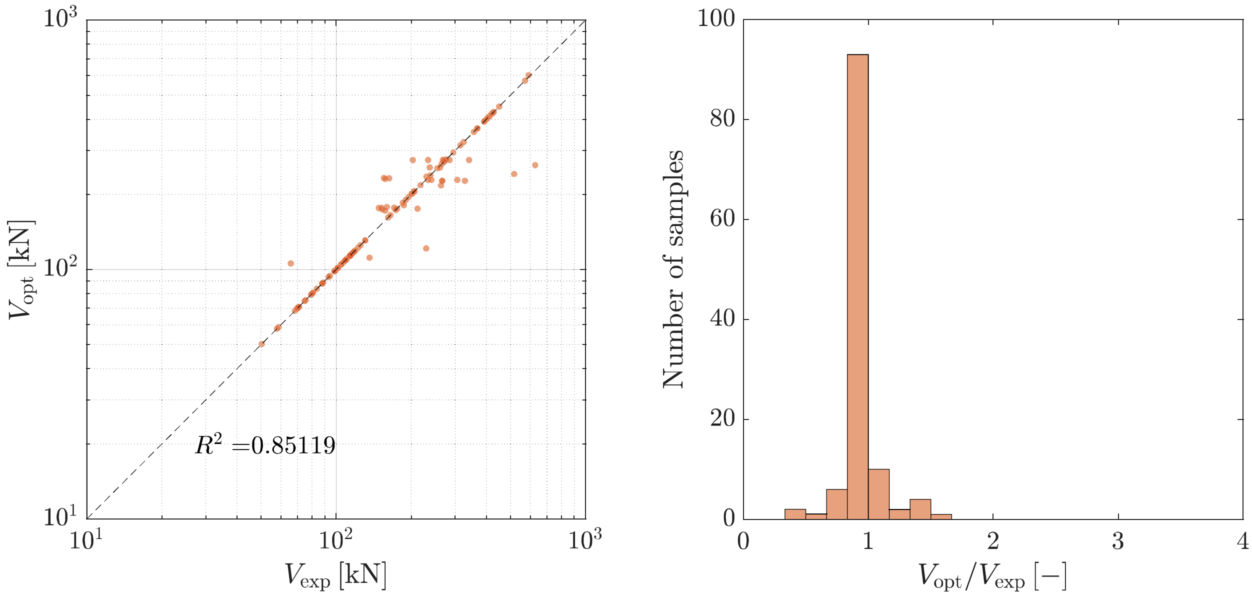

Figure 7: Comparison between optimized predictions of the shear capacity V_{opt} and corresponding experimental values V_{exp} for RC columns.

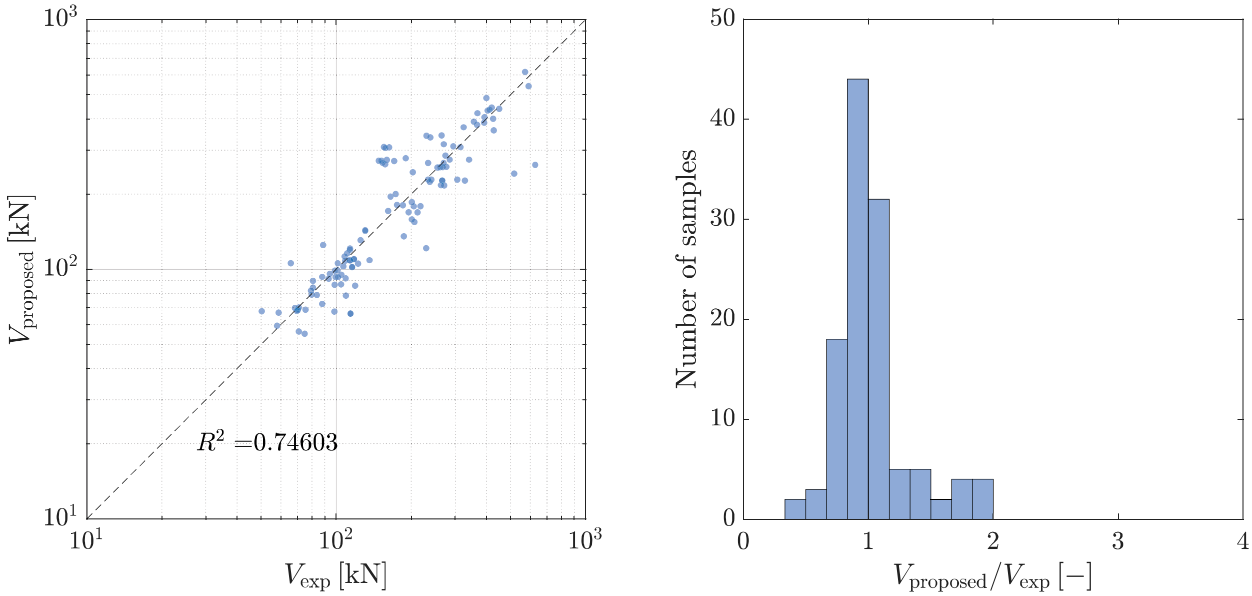

Figure 8: Comparison between shear capacity predictions obtained by means of the proposed approach V_{proposed} and corresponding experimental values V_{exp} for RC columns.

the shear capacity for RC beams and corresponding experimental values $V_{\mathrm{num}}/V_{\mathrm{exp}}$ is provided in Tab. 1 and Fig. 9. It is highlighted that the statistical metrics in Tab. 1 refer to the full database of RC beams. For the sake of completeness, it is also pointed out that mean and coefficient of variation of the $V_{\mathrm{num}}/V_{\mathrm{exp}}$ ratio corresponding to the proposed capacity equation are equal to 1.0011 and 0.3354 for the training dataset, respectively; while they are equal to 1.0128 and 0.2803 for the testing dataset, respectively. This comparative assessment is deemed useful for a further insight about the obtained results rather than for a competitive evaluation, since other formulations might not strictly valid for the intervals in the considered dataset, or because imply the use of design (rather than mean) material strength parameters.

It is evident from Tab. 1 and Fig. 9 that almost all the existing code-conforming formulations underestimate, on average, the shear capacity of RC beams to a rather larger extent as compared to the proposed equation, which exhibits a almost null bias. It is useful to remark that these overly conservative estimates of the shear capacity obtained through most of the existing code-conforming formulations are not due to the use of design values because safety factors are not taken into account. A good performance on average is also obtained by means of the formulation reported by AASHTO, which however exhibits one of the worst performance in terms of coefficient of variation and mean squared error. It is also noted that mean and median value of the ratio $V_{\mathrm{num}}/V_{\mathrm{exp}}$ are very close each other for the proposed capacity equation, while a significant difference between them is observed for some existing code-conforming formulations. Coefficient of variation and mean square error corresponding to the proposed approach are also rather low compared to the others existing code-conforming formulations. Notably, while the capacity equation in use within EC2 turns out to be one of the worst among the reviewed codes, Tab. 1 and Fig. 9 demonstrate that its machine-learning-based refinement as proposed in the present study was able to improve drastically its performance, thereby making the original formulation one of the best predictive models after fine-tuning of its corrective parameters. This, in turn, implies that the low accuracy of the original formulation into EC2 is only partially attributable to the simplifying hypotheses related to the variable-angle truss-based shear resistance mechanism, whereas it basically depends on the way its corrective parameters were defined. Repeating the calculation for the EC2 model with a hypothetical larger range of variation of the inclination angle $1 \leq \cot\theta \leq 5$ as adopted in the proposed model while keeping the same expressions for the corrective parameters ν and α_c would lead to unconservative results (mean value of the ratio $V_{\mathrm{num}}/V_{\mathrm{exp}}$ equal to 1.16), although the dispersion slightly decreases (coefficient of variation 0.36).

Figure 10 shows the variability of the ratio between numerical predictions and corresponding experimental values as obtained according to the proposed approach and the best models among the examined code-conforming formulations in terms of mean squared error. Figure 10 basically demonstrates that shear capacity predictions obtained according to the proposed procedure are not biased, and they are almost uniformly distributed in the proximity of the corresponding test values.

Table 1: Mean, coefficient of variation, mean squared error, median and interquatile range of the ratio between code-conforming predictions of the shear capacity for RC beams and corresponding experimental values $V_{\mathrm{num}}/V_{\mathrm{exp}}$.

Capacity equation (V_{num})	$V_{\mathrm{num}}/V_{\mathrm{exp}}$				
	Mean value	Coefficient of variation	Mean squared error	Median	Interquantile range
Proposed (V_{proposed})	1.0034	0.32458	0.10608	0.96805	0.35127
EC2 (V_{EC2})	0.79264	0.54165	0.22733	0.6959	0.40128
EC8 (V_{EC8})	0.79243	0.47412	0.18424	0.7358	0.34827
MC2010 (V_{MC2010})	0.76615	0.39167	0.14473	0.72435	0.26293
NTC2018 (V_{NTC2018})	0.78251	0.52456	0.21579	0.6959	0.39194
ACI318 (V_{ACI318})	0.71504	0.3592	0.14717	0.69249	0.24465
AASHTO (V_{AASHTO})	0.97896	0.52864	0.26827	0.85623	0.40014
NZS3101 (V_{NZS3101})	0.76423	0.43164	0.16441	0.72155	0.25344
GB50010 (V_{GB50010})	0.81861	0.4218	0.15213	0.79192	0.28994
JSCE15 (V_{JSCE15})	0.73144	0.39903	0.15731	0.69659	0.22936

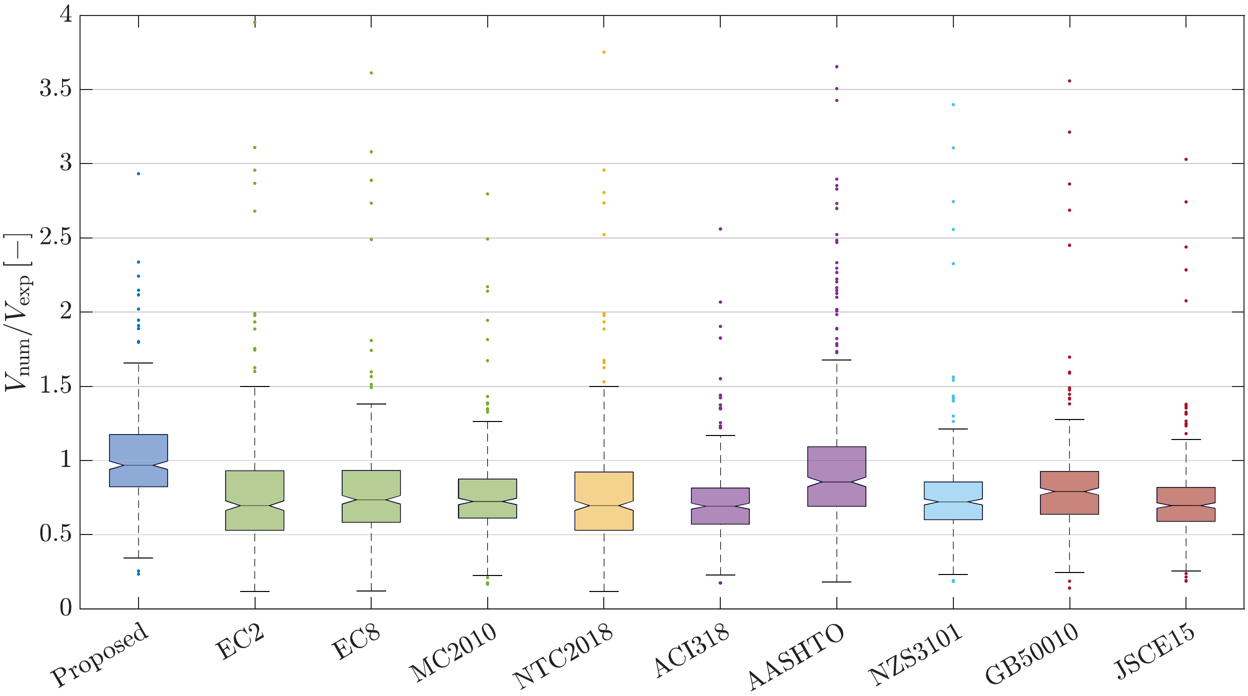

Figure 9: Comparison among code-conforming predictions of the shear capacity for RC beams in terms of box plot of the ratio between numerical predictions and corresponding experimental values $V_{\mathrm{num}}/V_{\mathrm{exp}}$.

3.3.2. Comparison of shear capacity equations for columns

In a similar fashion, Tab. 2 and Fig. 11 present a comparative assessment of the obtained results for RC columns considering the complete dataset. As regards the proposed capacity equation, mean and coefficient of variation of the $V_{\mathrm{num}}/V_{\mathrm{exp}}$ ratio are equal to 1.0301 and 0.2871 for the training dataset, respectively; while they are equal to 1.0416 and 0.2601 for the testing dataset, respectively. Once again, it is pointed out that

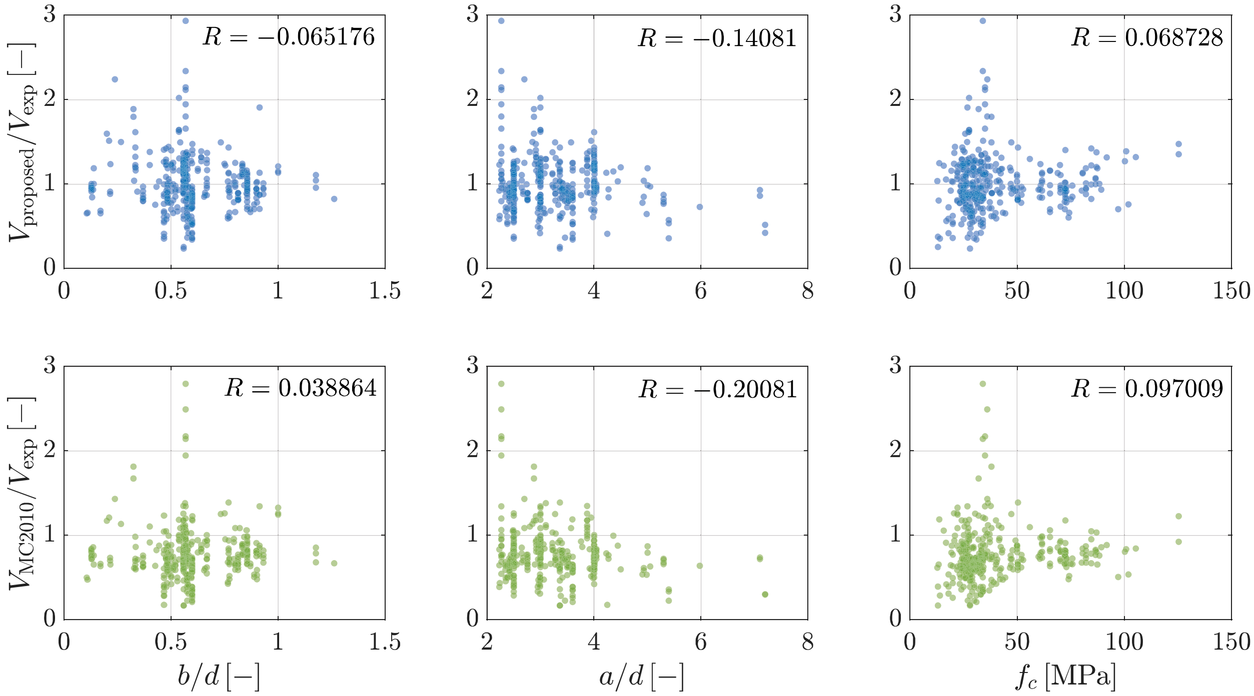

Figure 10: Variability of the ratio between numerical predictions of the shear capacity obtained by means of the proposed approach and corresponding experimental values $V_{\text{proposed}}/V_{\text{exp}}$ for RC beams (first row). Variability of the ratio between numerical predictions of the shear capacity according to MC2010 and corresponding experimental values $V_{\text{MC2010}}/V_{\text{exp}}$ for RC beams (second row).

it is not appropriate to consider such comparison as a competitive evaluation because other formulations might not strictly valid for the intervals in the considered dataset, or because imply the use of design (rather than mean) material strength parameters. Conversely, this analysis is mostly intended to draw further conclusions about the obtained results.

While Tab. 2 and Fig. 11 confirm the satisfactory accuracy of the shear capacity predictions for columns as obtained following the proposed procedure, they also show that some of the examined code-conforming formulations overestimate in this case the actual value. On average, the proposed capacity equation is the most accurate one together with the formulation reported by AASHTO. The performances of these two capacity equations are also very similar in terms of coefficient of variation, median and mean square error, while the proposed formulation shows a significantly smaller interquatile range. Table 2 and Fig. 11 show that the proposed machine-learning based approach leads to an impressive improvement of the original formulation in use within EC2 – which, once again, classifies among the worst capacity equation – without rejecting the underlying mechanics-based capacity model, but properly revisiting the definitions of the involved corrective parameters. Overall, the proposed code-conforming capacity equation represents the best one in predicting the shear strength for both typologies of RC members (i.e., RC beams under monotonic loading and RC columns under cyclic loading). Repeating the calculation for the EC2 model with a hypothetical larger range of variation of the inclination angle $1 \leq \cot\theta \leq 5$ as adopted in the proposed model while keeping the same expressions for the corrective parameter ν and α_c would lead to extremely unconservative results (mean value of the ratio $V_{\text{num}}/V_{\text{exp}}$ equal to 1.71), with a slightly reduced dispersion (coefficient of variation 0.38). This comparison demonstrates that the refinement achieved by the calibration of the η and η_c corrective parameters cannot be obtained by simply changing the assumptions for the range of $\cot\theta$ in the EC2 formulation.

Table 2: Mean, coefficient of variation, mean squared error, median and interquatile range of the ratio between code-conforming predictions of the shear capacity for RC columns and corresponding experimental values $V_{\text{num}}/V_{\text{exp}}$.

Capacity equation (V_{num})	$V_{\text{num}}/V_{\text{exp}}$				
	Mean value	Coefficient of variation	Mean squared error	Median	Interquantile range
Proposed (V_{proposed})	1.0324	0.28077	0.085067	0.98217	0.20249
EC2 (V_{EC2})	1.1816	0.46123	0.33001	1.1804	0.86508
EC8 (V_{EC8})	1.2383	0.20691	0.12244	1.1966	0.29024
MC2010 (V_{MC2010})	0.89265	0.23813	0.056708	0.85767	0.24017
NTC2018 (V_{NTC2018})	1.1722	0.45498	0.31408	1.1804	0.86508
Circ2019 (V_{Circ2019})	1.5216	0.30026	0.39596	1.4985	0.47416
ACI318 (V_{ACI318})	1.3196	0.28843	0.24699	1.2983	0.49279
AASHTO (V_{AASHTO})	1.0132	0.23768	0.05816	0.98678	0.34259
NZS3101 (V_{NZS3101})	1.1795	0.27094	0.13434	1.1502	0.44239
GB50010 (V_{GB50010})	1.0813	0.33866	0.14069	1.0087	0.39971
JSCE15 (V_{JSCE15})	1.0965	0.26203	0.09187	1.0674	0.3611

Figure 12 illustrates the variability of the ratio between numerical predictions and corresponding experimental values as obtained according to the proposed approach and the best models among the examined code-conforming formulations in terms of mean squared error. Figure 12 highlights the fact that shear capacity predictions obtained according to the proposed procedure are not significantly biased as compared to existing code-conforming formulations.

4. Design format of the proposed shear capacity equation

The calibration of the shear capacity equation based on the proposed formulations for η and η_c given by Eqs. (29)-(30) is finally addressed in view of potential design applications. In general terms, it means searching for a suitable value for the model uncertainty factor γ_{Rd} in a code-formatted design capacity equation, as follows [62]:

$$V_d = \frac{1}{\gamma_{Rd}} \kappa V(\mathbf{X}_d), \tag{32}$$

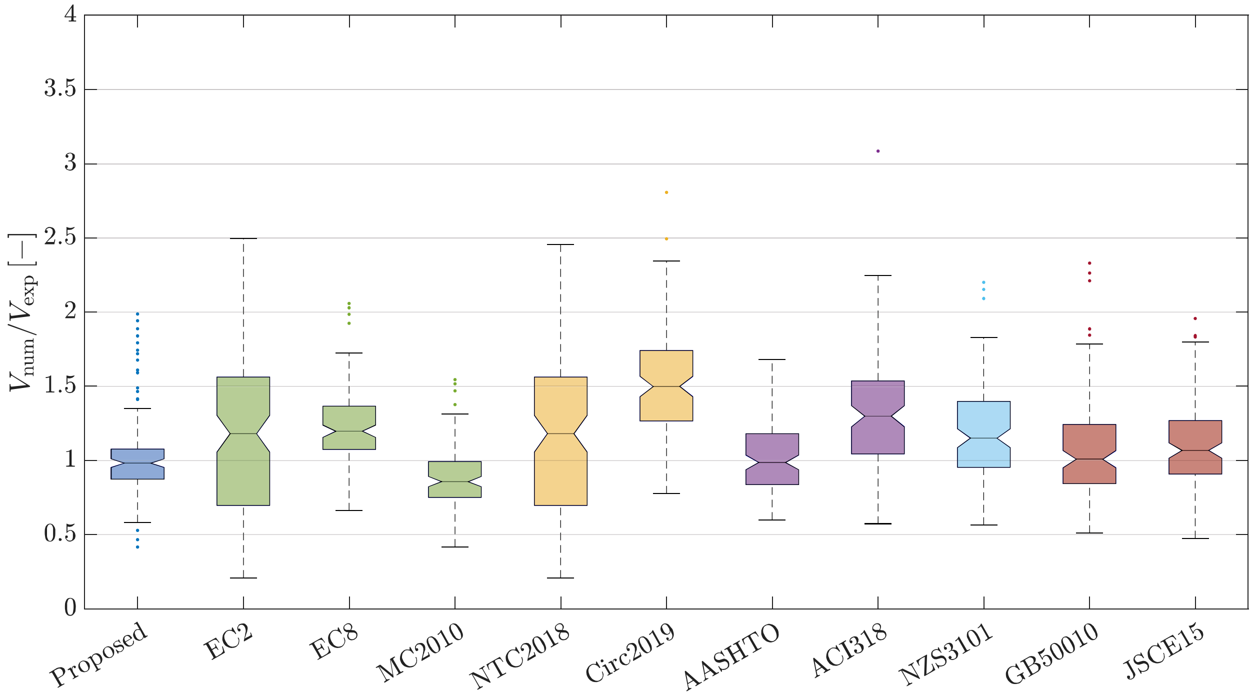

Figure 11: Comparison among code-conforming predictions of the shear capacity for RC columns in terms of box plot of the ratio between numerical predictions and corresponding experimental values V_{num}/V_{exp}.

where V_d is the design value of the shear capacity and κ is a conversion factor. Moreover, $V(\mathbf{X}_d)$ is the shear capacity evaluated for $\mathbf{X}_d$, which is the vector collecting the design values of the basic variables. The ith design basic variable X_{di}, in turn, is defined as follows:

$$X_{di} = \frac{X_{ki}}{\gamma_{X_i}} = \frac{1}{\gamma_{X_i}} \left(\mu_{X_i} + k_{p_i}\sigma_{X_i}\right) \tag{33}$$

where X_{ki} is the characteristic value of the basic variable and γ_{X_i} is its partial safety factor in use within the reference code, whereas μ_{X_i} and σ_{X_i} are mean and standard deviation of the basic variable, respectively. Finally, k_{p_i} is defined by the relevant code to attain the characteristic value of interest X_{ki} (e.g., $k_{p_i} = -1.645$ for a lower 5% fractile according to the Normal probability density function). With these premises, γ_{Rd} is estimated in such a way that the design capacity value falls below the corresponding test value with an assigned probability, in compliance with the general approach in use within the European Building Codes [62]. Formally, it reads [63]:

$$\text{Find } \gamma_{Rd} : P\left[V_d \leq V_{\text{exp}}(\mathbf{X})\right] = \Phi\left(\alpha_R \beta_{LS}\right), \tag{34}$$

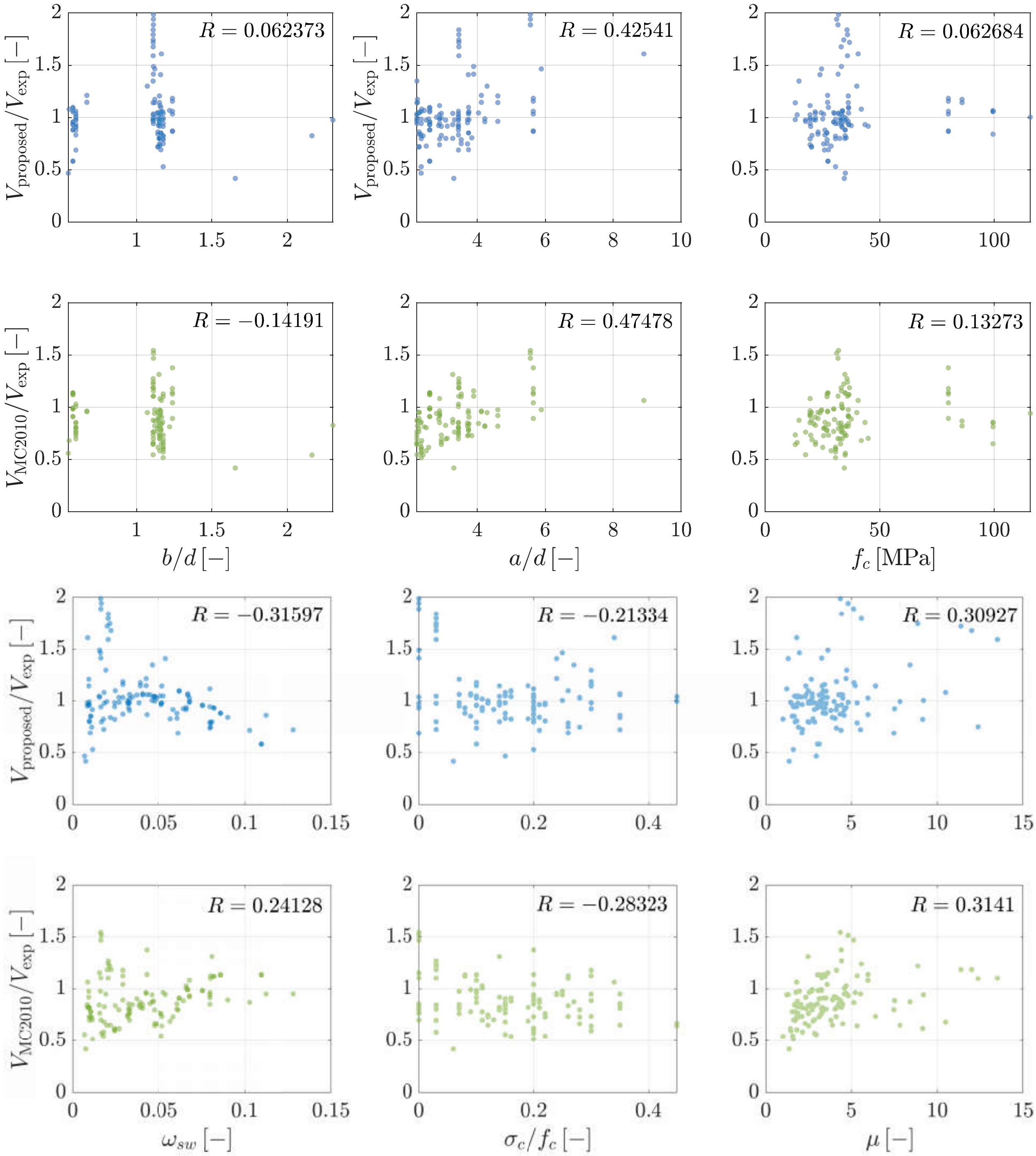

Figure 12: Variability of the ratio between numerical predictions of the shear capacity obtained by means of the proposed approach and corresponding experimental values $V_{\mathrm{proposed}}/V_{\mathrm{exp}}$ for RC columns (first and third row). Variability of the ratio between numerical predictions of the shear capacity according to MC2010 and corresponding experimental values $V_{\mathrm{MC2010}}/V_{\mathrm{exp}}$ for RC columns (second and forth row).

where $P[\cdot]$ is the probability operator and $V_{\exp}(\mathbf{X})$ is the experimental shear capacity corresponding to test value of the basic variables $\mathbf{X}$. According to the European Building Codes [62], the constant α_R is the sensitivity factor for the capacity, usually taken equal to 0.8, whereas β_{LS} is the safety index relevant to the considered limit state, usually assumed equal to 3.8 for ultimate limit states. Both α_R and β_{LS} are here taken for a 50-years reference period. Moreover, $\Phi(\cdot)$ is the standard Normal cumulative distribution function. After straightforward manipulations, the problem in Eq. (34) can be rewritten as follows:

$$\text{Find } \gamma_{Rd} : P\left[\varrho \leq \gamma_{Rd}\right] = \Phi\left(\alpha_R \beta_{LS}\right), \tag{35}$$

where it is introduced $\varrho = \eta V(\mathbf{X}_d)/V_{\exp}(\mathbf{X})$. If ϱ follows a Normal probability density function with $\varrho \sim \mathcal{N}(\mu_\varrho, \sigma_\varrho)$, then:

$$\gamma_{Rd} = \mu_\varrho + \alpha_R \beta_{LS} \sigma_\varrho. \tag{36}$$

In this study, the estimation of γ_{Rd} based on Eq. (36) is performed for beams and columns independently, using the corresponding complete experimental databases. It is assumed $\kappa = 1$, whereas geometrical variables are taken with the corresponding deterministic nominal values in agreement with the European Building Codes and as usual in most standards. Conversely, compressive concrete strength and yielding stress of reinforcement are assumed as random Normal variables, the lower 5% fractile being their characteristic value. The mean value is assumed equal to that reported in the collected experimental databases. The definition of the coefficient of variation is a more complicated task, as it depends on many factors (e.g., age, country, quality controls). For the sake of simplicity, a constant value is adopted. Specifically, based on the extensive investigation by Shimizu et al. [64], the adopted coefficient of variation for the compressive concrete strength is equal to 0.20, whereas 0.13 is considered for the yield stress of reinforcement taking into account the studies by Jaskulski et al. [65] and Croce et al. [66]. The design values for the compressive concrete strength and the yield stress of reinforcement are obtained in agreement with European Building Codes [11] by assuming partial safety factors equal to 1.50 and 1.15, respectively. Long-term effects on the concrete compressive strength are considered as per European Building Codes [11] through the use of a reductive coefficient equal to 0.85.

Following this simple procedure, a good agreement is found between the probability that the design value of the shear capacity $V_{\text{proposed},d}$ estimated according to the proposed formulation is lower than the corresponding test value $V_{\exp}$ and the target value, being the absolute value of the maximum relative difference equal to 1.76%, see Tab. 3. This certifies the correctness of the presented procedure. Herein, the target value of $P\left[V_{\text{proposed},d} \leq V_{\text{exp}}\right]$ is simply given by $\Phi\left(\alpha_R \beta_{LS}\right) = \Phi\left(0.8 \cdot 3.8\right) = 0.9988$. The actual value of $P\left[V_{\text{proposed},d} \leq V_{\text{exp}}\right]$ is numerically estimated as the ratio of the number of samples for which $V_{\text{proposed},d} \leq V_{\text{exp}}$ and the total number of samples.

The numerical value of γ_{Rd} for beams under monotonic loading condition is lower than the one obtained for columns under cyclic loading condition, see Tab. 3. A closer inspection to Figs. 13-14 also reveals that design estimates $V_{\text{proposed},d}$ are not too lower than the corresponding test values V_{exp}, i.e. the code-formatted capacity equation provides safe-side evaluations of the shear capacity in good agreement with the desired threshold, but it does not lead to over-conservative estimates on average.

Table 3: Model uncertainty factor γ_{Rd} for the proposed code-formatted shear capacity equation.

Loading/Member	γ_{Rd}	$P\left[V_{\text{proposed},d} \leq V_{\text{exp}}\right]$	
		Target value	Actual value
Monotonic/Beam	1.2183	0.9988	0.9812
Cyclic/Column	1.4055	0.9988	1.0000

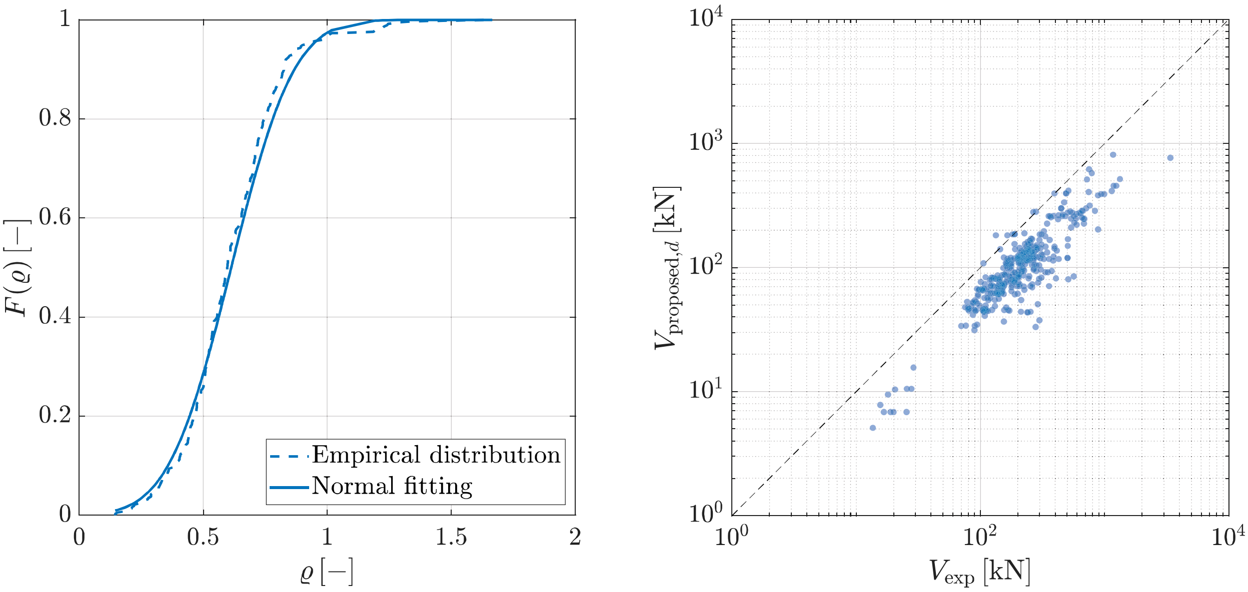

Figure 13: Empirical cumulative distribution function of the variable ϱ for RC beams and corresponding Normal fitting (left). Design values of the shear capacity obtained by means of the proposed approach $V_{\text{proposed},d}$ and corresponding experimental values V_{exp} for RC beams (right).

Conclusions

Starting from the first intuitions by Ritter and Mörsch, the knowledge of the mechanisms underlying the shear resistance of RC elements has steadily advanced, thanks to a continuous research effort spanning over more than a century. Some design equations nowadays used with confidence in most advanced codes are still

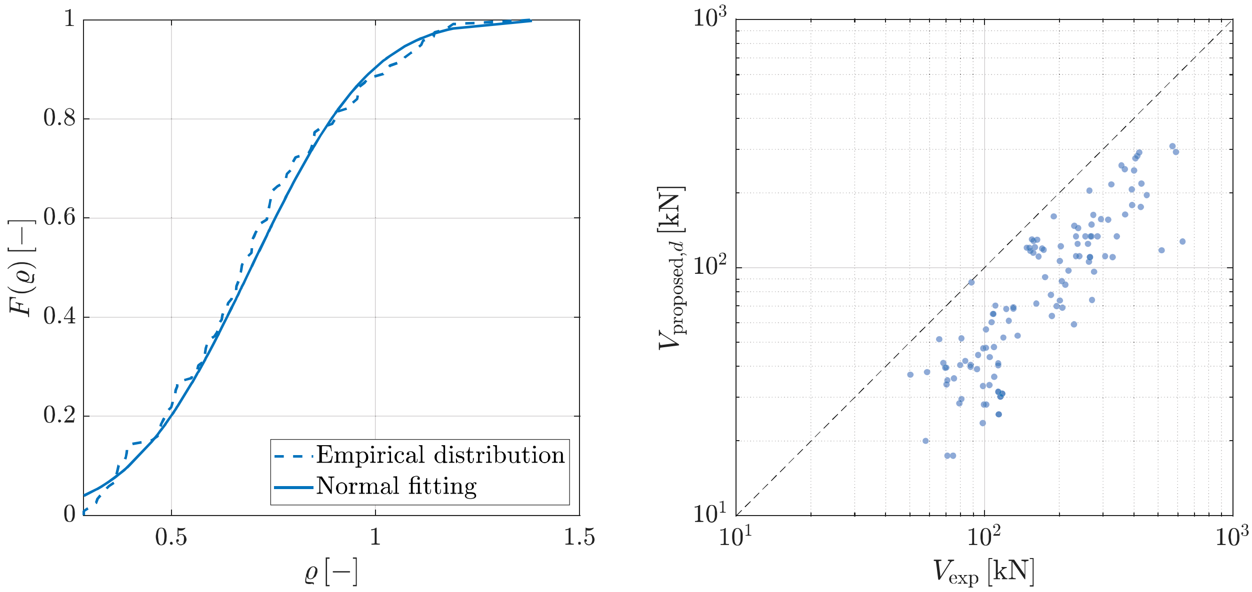

Figure 14: Empirical cumulative distribution function of the variable ϱ for RC columns and corresponding Normal fitting (left). Design values of the shear capacity obtained by means of the proposed approach $V_{\mathrm{proposed},d}$ and corresponding experimental values V_{exp} for RC columns (right).

essentially based on those intuitions, with some progressively added adjustments. Alternative formulations have also gradually emerged, either based on different theoretical interpretations, or those of empirical nature that follow a purely data-driven approach. The underlying idea of this study was to explore a new hybrid approach for the development of the shear capacity equation, whereby a mechanics-based code-conforming formulation is enhanced via machine learning technique. In fact, the shear capacity equation based on the variable-angle truss model currently in use in Europe has been improved by means of Genetic Programming, aiming at obtaining novel and more accurate expressions of the fundamental coefficients governing the concrete contribution. Such refined expressions of the model parameters have been sought by restricting the functions set to standard arithmetic operators only, thus preserving the overall simplicity of the final formulation. It has been demonstrated that the EC2 formulation can be considerably improved by calibrating two corrective parameters governing the concrete contribution through Genetic Programming. Based on the performed calibration, it has been found that the efficiency factor η is linked to the concrete strength through an inverse relationship, similar with the existing relationship between ν and f_c already present in the original EC2 formulation. Moreover, η depends on the b/d ratio, somehow suggesting that the effective concrete compressive strength in the truss model decreases as the flexural inertia of the concrete diagonals cross-section decreases. This may be explained by accepting that such formulation for η aims at taking into account the effects due to the bending moment in the concrete diagonals, which is not considered in the formulation of the original truss-based mechanism. Additionally, the η_c factor increases with the applied compression, similar with the existing relationship between α_c and σ_c/f_c already present in the original EC2 formulation. An important role on the determination of η_c is also played by the ductility factor μ_Δ (which is, instead, not present in the EC2 formulation), suggesting that shear strength of RC columns under cycling loading condition decreases as the displacement ductility demand μ_Δ increases, in line with available experimental findings. The enhanced predictive performance achieved by the proposed hybrid approach has been scrutinized and demonstrated within a large comparative assessment involving several shear capacity equations reported in major technical codes from all over the world. A code calibration procedure has been finally carried out to obtain an equation meeting a predefined reliability level, which can be used in the design of beams and columns.

The methodology has been successfully applied herein to assess the shear strength of RC members, but it can be extended to elaborate further capacity equations.

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