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MULTISCALE FINITE ELEMENT MODELING LINKING SHELL ELEMENTS TO 3D CONTINUUM

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Abstract. The present paper investigates the response of masonry structural elements with periodic texture adopting an advanced multiscale finite element model, coupling different formulations at the two selected scales of analysis. At the macroscopic structural level, a homogeneous thick shell is considered and its constitutive response is derived by the detailed analysis of the masonry repetitive Unit Cell (UC), analyzed at the microlevel in the framework of the three-dimensional (3D) Cauchy continuum. The UC is formed by the assembly of elastic bricks and nonlinear mortar joints, modeled as zero-thickness interfaces. The Transformation Field Analysis procedure is invoked to address the nonlinear homogenization problem of the regular masonry. The performance of the model in reproducing various masonry textures is explored by referring to an experimentally tested pointed vault under different profiles of prescribed differential settlements. The structural behavior of the vault is studied in terms of global load-displacement curves and damaging patterns and the numerical results are compared with those recovered by detailed micromechanical analyses and experimental evidences.

1 INTRODUCTION

In the last decades, many numerical and analytical procedures have been developed to understand and predict the response of masonry structures, in view of their safety assessment. The proposed strategies usually involve the limit analysis concepts, sometimes used in conjunction with advanced computer-based approaches [1], the discrete element, and the finite element (FE) methods [2]. A detailed review of classical and modern methodologies specifically developed for masonry is given in [3].

Focusing on the FE models, a common classification is based on the scale of the analysis, thus distinguishing macromechanical [4], micromechanical [5] and multiscale [6] approaches. These latter are a powerful tool to study the response of heterogeneous material, like masonry, as they allow for detailed geometric/mechanical modeling and, when properly formulated, can limit the computational efforts [7]. Within the computational homogenization framework, the multiscale models here referred to split the structural problem in two scales: the macroscale where an equivalent homogenized medium is considered and a microscale where a representative volume element is analyzed. In such a way, the composite nature of the material, usually made of the regular or irregular arrangement of bricks/blocks and mortar joints, is properly accounted for.

In the case of curved masonry structural elements, like arches, domes and vaults, evaluation of the mechanical response is even more difficult due to the complexity of the curved structures mechanics. Hence, specific computational methods have been developed [8], many of them also finalized to predict the influence of masonry texture on the overall behavior [9].

Based on the above considerations, this work explores the response of masonry structural elements with periodic texture by using the multiscale model recently presented by the authors in [10]. At the macroscopic level, the real heterogeneous masonry material is modeled as an equivalent homogenized shell, whose constitutive response is derived by the detailed analysis of a 3D representative unit cell (UC), made of elastic bricks and nonlinear interfaces. A Transformation Field Analysis (TFA) based procedure is used to link the two scales, thus resorting to a reduced order model with indubitable computational advantages. FE numerical applications are performed to investigate the effects of texture and loading conditions on the ultimate strength and failure mode of an experimentally tested masonry pointed vault [11]. Results obtained with the proposed model are compared with both micromechanical and experimental outcomes.

2 SHELL-3D MULTISCALE MODEL

The multiscale model proposed by the authors in [10] is here briefly described. This connects two different formulations at the two scales of the analysis, exploiting the combined advantage of each approach. At the macroscopic structural level, a homogeneous thick shell is assumed, based on the Mindlin-Reissner theory for plates. The constitutive response of the shell is evaluated by linking each material point to the repetitive masonry UC, modeled at the microlevel by a 3D Cauchy continuum. The UC accounts for the actual masonry texture and is formed by elastic bricks and nonlinear zero-thickness interfaces representing the mortar joints.

The microscopic displacement field, $\mathbf{u}$, is computed as the sum of two contributions:

$$\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}^{\star} \tag{1}$$

being the term $\bar{\mathbf{u}}$ the known contribution depending on the macroscopic shell strains, i.e. the membrane strains, the flexural curvatures and plate shear strains, collected in vector $\mathbf{E}$, and the term $\mathbf{u}^{\star}$ representing the perturbation due to the material heterogeneity and subjected to the periodic conditions imposed at the UC boundaries.

Accordingly, the strains in the bricks, listed in vector $\boldsymbol{\varepsilon}^b$, are derived by compatibility with the displacements and result as:

$$\boldsymbol{\varepsilon}^b = \mathbf{B}\,\mathbf{E} + \boldsymbol{\varepsilon}^{b\star} \tag{2}$$

being $\mathbf{B}$ the matrix ruling the macro-micro information transition [10]. The corresponding brick stresses, $\boldsymbol{\sigma}^b$, are evaluated by assuming the classical linear elastic relation $\boldsymbol{\sigma}^b = \mathbf{C}^b\boldsymbol{\varepsilon}^b$.

The kinematic descriptors of the interfaces, i.e. the displacement jumps $\mathbf{s}$, are defined only by the displacement perturbations and are related to the work-conjugate tractions, $\mathbf{t}$, by means of a damage-friction nonlinear relationship [7]. This is expressed in the local system (x_{T1}, x_{T2}, x_N) of each interface (Figure 1(a)) as:

$$\mathbf{t} = \mathbf{C}(\mathbf{s} - \boldsymbol{\pi}) \qquad \text{with} \quad \boldsymbol{\pi} = D(\mathbf{c} + \mathbf{p}) \tag{3}$$

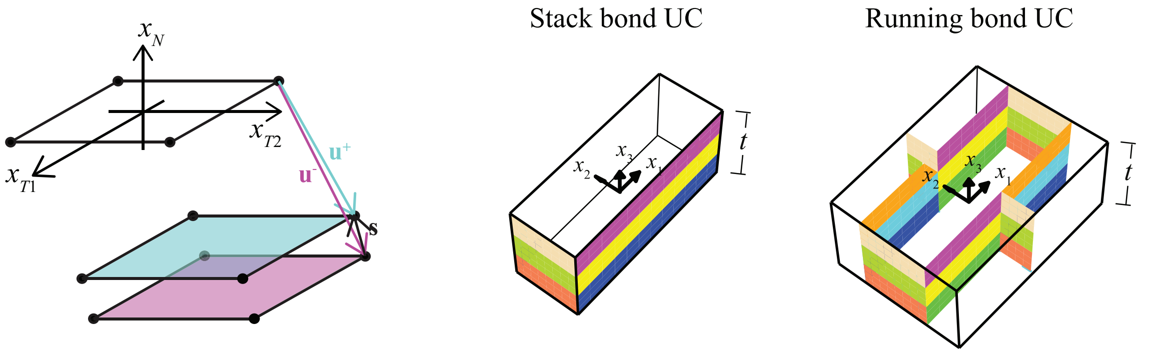
where $\mathbf{C}$ denotes the matrix collecting the stiffness values in the tangential, C_{T1} and C_{T2}, and normal, C_N, directions to the interface plane. In this work it is assumed $C_{T1} = C_{T2} = C_T$. The inelastic vector $\boldsymbol{\pi}$ considers the unilateral contact, $\mathbf{c}$, and sliding friction, $\mathbf{p}$, displacement jumps. The activation of these latter is ruled by the classical Mohr-Coulomb yield function, dependent on the friction coefficient μ. The effect of the degrading mechanisms due to the crack evolution is introduced by means of the damage variable, D, whose evolution law accounts for the fracture energies, G_{cN} and G_{cT}, and the peak stresses, t_N^0 and t_T^0, associated to fracture mode I and II, respectively.

Finally, the up-scaling process is performed by invoking the generalized Hill-Mandel principle. Hence, the shell stress vector, $\boldsymbol{\Sigma}$, collecting the membrane, bending and shear stress resultants, is determined as follows:

$$\boldsymbol{\Sigma} = \frac{1}{A} \int_A \int_{-t/2}^{t/2} \mathbf{B}^T \boldsymbol{\sigma}^b \, dx_3 \, dA \tag{4}$$

being A the area of the UC-mid plane and x_3 the local axis running along the thickness, t, of the UC (Figure 1(b)). To be noted is that interfaces give null contributions to the integral in Eq. (4)

The model has been implemented in the finite element program FEAP [12] adopting a non-local integral formulation for the shell at the macroscopic level, to avoid the mesh-dependency issues typical of strain softening constitutive behavior.



(a) joint interface

(b) possible subset layouts

Figure 1: Local reference system for the interfaces and examples of subset arrangements for the UCs

2.1 TFA-based homogenization

The homogenization process that links the two scales is based on the Transformation Field Analysis procedure, which assumes piece-wise uniform distributions of damaging and frictional mechanisms over the mortar joints. Accordingly, the nonlinear interfaces are divided in regions, called *subsets*, where the inelastic displacement jump vector $\boldsymbol{\pi}$ is assumed as uniform. Figure

1(b) shows two possible subset layouts for UCs having different textures. Here, regions belonging to the same subset are depicted with the same color. For the running bond UC, the example considers 2 and 1 subdivisions for bed and head joints, respectively, in the direction parallel to the UC plane, i.e $n_b = 2$ and $n_h = 1$. Each of them is further discretized into 3 partitions along the thickness, i.e. $n_t = 3$. By contrast, the case of $n_b = 1$, $n_h = 1$ and $n_t = 3$ is shown for the stack bond UC. Detailed discussion on the optimal subset arrangement to improve the homogenization technique is addressed in [13].

The procedure requires to perform a set of preliminary elastic analyses to evaluate the effects on the UC response of the macroscopic strains $\mathbf{E}$ and those of the inelastic jumps $\boldsymbol{\pi}^i$ $(i = 1, .., (n_b + n_h) \times n_t)$ applied one-by-one in each subset. Hence, proper localization operators are computed and, then, used to solve only at the macroscale the evolutive nonlinear problem of the UC linked to each point of the structural shell model. This allows to avoid the time-consuming nonlinear micromechanical analysis of the UC at each step and iteration of the global analysis, as it is typical of the classical FE^2 approach.

3 MASONRY VAULTS UNDER GROUND SETTLEMENTS

The structural response of the pointed masonry vault schematically depicted in Figure 2 is analyzed. The specimen, characterized by running bond texture, was experimentally tested [11] by imposing a linear vertical differential settlement pattern. Here, in addition to the experimental settlement profile, uniform vertical displacements of the sagging impost are considered (see Figure 3(a)). Moreover, the effect of the masonry texture on the vault behavior is investigated, by comparing the results with the structural responses obtained in [13] for a stack bond arrangement. Figure 2 shows the UCs selected for both arrangements, considering: mortar thickness $l = 10\,\text{mm}$, brick width $b_b = 60\,\text{mm}$, brick height $h_b = 14.5\,\text{mm}$ and thickness $t = 30\,\text{mm}$.

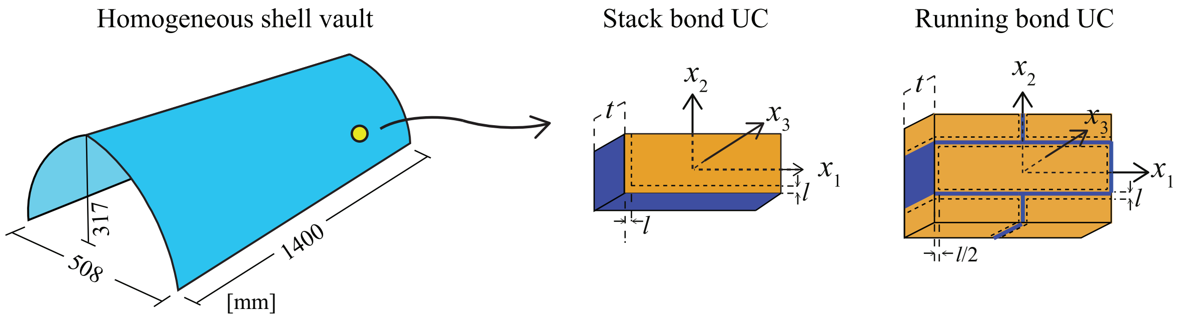
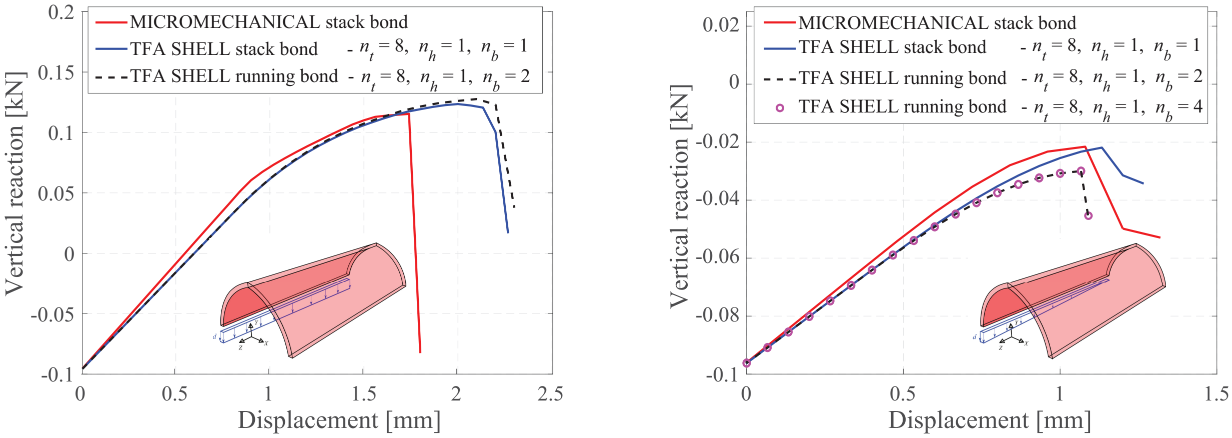


Figure 2: Multiscale model of the analyzed masonry vaults: overall geometry and representative UCs

The mechanical parameters assumed for bricks ('b') and mortar ('m') are contained in Table 1 and are set according to the data reported in [14]. The corresponding interface properties are derived on the basis of the mortar thickness, that is $C_N = E_m/l$, $C_T = G_m/l$, $G_{cN} = g_{cN} \times l$ and $G_{cT} = g_{cT} \times l$.

Figure 3(a) shows the vault response under the uniform settlement profile in terms of total vertical reaction of the fixed abutment versus the imposed displacement (downward direction is assumed as positive). Red and blue solid curves refer to the stack bond solutions obtained with



(a) uniform settlement

(b) linear settlement

Figure 3: Global responses of the vault under differential vertical settlements considering stack and running bond textures

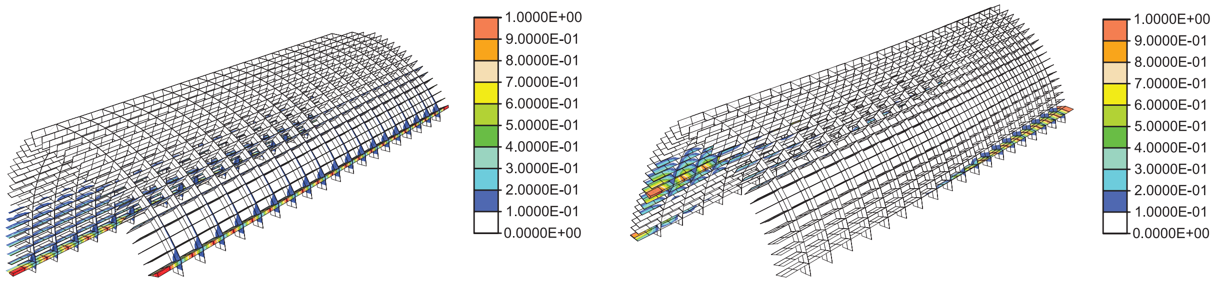
a detailed micromechanical model and with the proposed multiscale approach, respectively. For the latter, 16 subsets are considered for the UC, uniformly spaced across the masonry thickness. The black dashed curve refers to the multiscale solution for the running bond texture derived with 2 and 1 subdivisions for bed and head joints and 8 partitions along the thickness.

Table 1: Material parameters for bricks and mortar

E_b [MPa]	G_b [MPa]	E_m [MPa]	G_m [MPa]	t_N^0 [MPa]	t_T^0 [MPa]	g_{cN} [MPa]	g_{cT} [MPa]	μ [-]
9000	3600	750	75	0.15	0.10	2.48×10^{-4}	7.98×10^{-4}	0.5

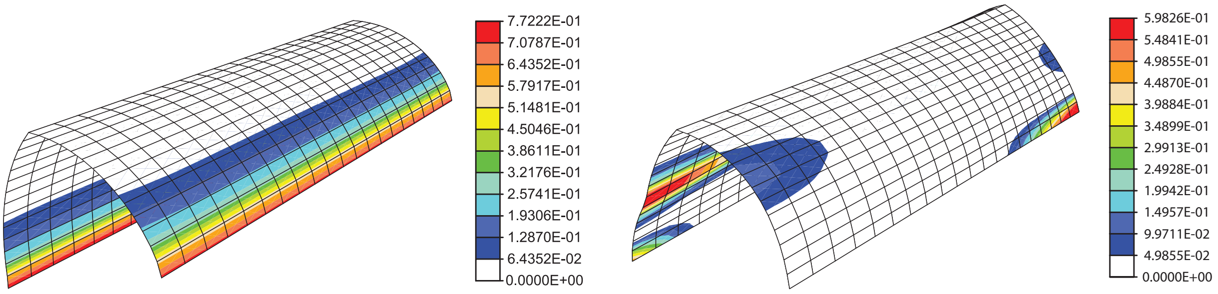
As expected, under the imposed uniform displacements, the overall behavior is governed by the opening of the mortar bed joints (i.e. the mortar layers directed as the vault generatrix) and, consequently, the curve responses obtained for the two masonry textures practically overlap. However, slight discrepancies emerge between the multiscale and micromechanical solutions in terms of ultimate displacement. This is due to the stronger localization of the damage obtained for the micromechanical model that affects the two bed mortar joints right above the vault abutments (Figure 4(a)). For the shell models, smoother distributions of the subset average damage at bed joints result, as shown by the contour plots in Figures 5(a) and 6(a).

As concerns the case of linear settlement pattern, more differences are expected for the two analyzed masonry arrangements, due to eventually severe damage of the head joint. However, Figure 3(b) shows only slight discrepancies between the two solutions obtained with the TFA shell models (blue and black curves), that are in good agreement with the micromechanical result referring to the stack bond case. Indeed, for this loading condition, although the crack patterns are more complex than those obtained for the uniform settlement case (Figures 4(b), 5(b) and 6(b)), low head joint damage occurs, which slightly affects the global vault performance. Damage starts and evolves at the haunch of the vault where higher displacements are prescribed and, then, it develops in the bed joints above the abutments, causing severe sliding of the joints on the fixed side. Similar trend of the cracking evolution was observed in the experimental test, which also resulted in a diagonal shear crack at a later stage of the loading process, not considered in this study.



(a) uniform settlement at ultimate displacement (b) linear settlement at the peak response

Figure 4: Distribution of the damage variable D in the interface elements representing the mortar joints in the micromechanical model of the vault with stack bond texture



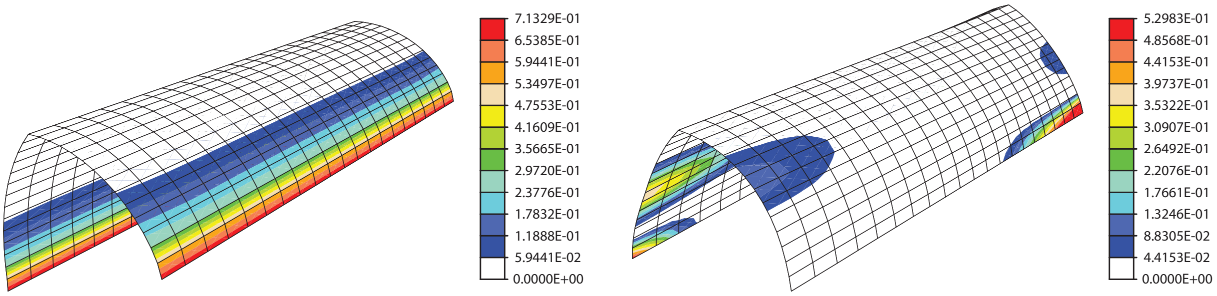
(a) uniform settlement at ultimate displacement (b) linear settlement at the peak response

Figure 5: Distribution of the average damage in the mortar joint subsets for the TFA shell model of the vault with stack bond texture

Finally, the influence of a denser subset partition of the bed joints has been considered for the running bond case. The resulting response curve (violet circles in Figure 3(b)) perfectly matches that corresponding to the coarser discretization (black dashed curve), testifying that the adopted procedure well describes the vault structural behavior, even just assuming a limited number of subsets.

4 CONCLUSIONS

- This paper investigated the structural response of periodic masonry elements by using an advanced multiscale procedure that links a macroscale thick shell to a microscale Cauchy continuum. The Transformation Field Analysis technique was adopted to address the nonlinear homogenization problem, thus resorting to a reduced order model with indubitable



(a) uniform settlement at ultimate displacement (b) linear settlement at the peak response

Figure 6: Distribution of the average damage in the mortar joint subsets for the TFA shell model of the vault with running bond texture

computational advantages, when compared with the classical multiscale FE^2 approach.

- The behavior of an experimentally tested masonry pointed vault was numerically analyzed considering uniform and linear differential settlement patterns and two different masonry textures. The numerical results proved that, for the considered loading and boundary conditions, texture geometry does not significantly affect the overall behavior. In fact, the global failure mechanisms were characterized, for both masonry bonds, by opening and sliding of the longitudinal bed joints and, consequently, similar response curves were obtained.

- Comparison between numerical outcomes derived from the multiscale model and those recovered by detailed micromechanical analyses proved that the proposed formulation is a very promising tool and encourage to extend the study to other types of geometry and loading. Although some slight discrepancies emerged due to more diffused and smooth damage distributions, the proposed model was able to reproduce the overall degrading phenomena evolving in the structure up to collapse.

- Enrichments of the model will be devoted to overcome the limit related to the assumption of piece-wise uniform distributions of the nonlinear quantities over mortar joints. To this end, higher order variations of the nonlinearity will be considered. Finally, further improvements will be finalized to capture global failure mechanisms where strong damage localizations are expected.

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