A Novel Method for Transverse beam-coupling Impedance Measurements in Particle Accelerator Devices Using the Bead-pull Method

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***Abstract*—In high-energy particle accelerators, circulating particle beams with high intensity interact electromagnetically with their surroundings (i.e. metallic vacuum chambers of various geometries). These electromagnetic interactions, which are typically described through the concept of beam-coupling impedance, can represent a serious issue in terms of stability of the particle beam and heating on the accelerator structures. It is therefore crucial to account for beam-coupling impedance through simulations and measurements during the design phase of an accelerator component. However, standard methods used in the accelerator field have known limitations due to the perturbations that they induce in the device geometry. Thus, the bead-pull method, which is a possible alternative approach, is presented in this study. The longitudinal beam-coupling impedance measurement technique is introduced, and it is further developed for novel use in transverse impedance measurements.**

***Index Terms*—Beam-Coupling Impedance, Bead-Pull Method, Transverse Impedance, Slater Theorem, RF Cavity.**

I. INTRODUCTION

RESEARCH in energy physics uses particle accelerator machines which are ultra-high vacuum metallic chambers hosting an ultra-relativistic beam of charged particles. The trend for future high-energy particle accelerator physics is moving toward increasing the energy and the intensity (i.e. the number of circulating particles in the machine) of the beams [1], [2]. Going toward this goal, some physical phenomena start becoming more relevant and potentially detrimental [3]; one of the main issues is given by the interaction between the electromagnetic field irradiated by the travelling particles and the metallic pipes containing the beam. This complex interaction is synthetically described through the concept of beam-coupling impedance [4]. More specifically, the beam-coupling impedance contribution can be separated into two components, one associated with the longitudinal direction (the beam propagation direction) and the other associated with the transverse plane (orthogonal to the beam propagation direction). These components can be responsible for beam-induced heating deposition [5] on the accelerator devices and beam stability and quality degradation [6]. Both aspects are undeniably critical for ensuring the proper operation and enhancing the overall performance of the accelerators and, this explains the need for i) accurately monitoring the device temperature during operation [7], ii) quantifying the impedance contribution of an accelerator device both during its design phase and in post-mortem scenarios.

In the context of measuring the beam-coupling impedance, the ideal approach involves the excitation of the device using the beam itself [6]. However, this may not always be feasible. In such cases, alternative methods such as simulations or bench measurements [8] become necessary.

The most commonly employed bench methods, i.e. the coaxial-wire and the coaxial probe methods, have notable limitations. In the coaxial-wire method [9], [10], a wire is stretched through the structure and it is excited with a current pulse (which mimics the beam passage). The stretched wire perturbs the electromagnetic boundary conditions of the original design, potentially resulting in inaccurate results, as demonstrated and documented in [11], [12]. On the other hand, wireless methods, such as the coaxial probe technique, introduce less perturbations in the boundary conditions of the structure but they do not supply all the information needed to quantify the beam-coupling impedance.

An alternative method, typically employed in the Radio-Frequency (RF) community for fine tuning of modes in an electromagnetic resonant cavity [13], [14], is the bead-pull method. In the context of accelerator physics, the bead-pull method has been so far used only for measurements of longitudinal beam-coupling impedance. In addition to the consolidation of the bead-pull method as a longitudinal impedance bench measurement technique, the novelty of this work is the extension to transverse impedance measurements which will be showcased and bench-marked with simulations. To move toward this goal a portable and general-purpose bead-pull setup will be presented as well. The paper starts with a background section explaining the concepts and definitions of the beam-coupling impedance. It follows a review of the standard bench measurement techniques underlying their limitations with respect to the specific device we are using for the benchmark (a pill-box cavity). There is a section dedicated

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This article has been accepted for publication in IEEE Transactions on Instrumentation and Measurement. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIM.2024.3458042

to the theory of the standard bead-pull method, in which the setup implemented will be showcased. Finally, the idea of the extension of the method to transverse measurements will be explained and the resulting benchmark will be shown and discussed.

II. Wakefields and Beam-coupling impedance

When a beam of charged particles travels in the vacuum chamber of a particle accelerator it is subject to external electromagnetic fields (typically produced by magnets for bending or focusing and RF cavities for acceleration). However, the particles are themselves source of an electromagnetic field that can act directly on other particles or indirectly through scattering on the walls of the vacuum chamber. In high-energy physics accelerators, the particles are travelling at ultra-relativistic speed. Then, the field resulting from the interaction with the vacuum chamber is completely behind the particle that generated it. For this reason, these fields are commonly called wakefields [15].

As the beam intensity increases, the wakefields become stronger in amplitude and they can interact with the trailing bunches of particles disturbing their motion. More specifically, due to the Lorentz force, the fields can induce kicks on the particles with the undesired effects of deflecting the beam, causing energy loss, increasing the beam spot-size and, ultimately, triggering instabilities [6], [15]. All these effects can cause a severe degradation of the performance of the accelerator. Moreover, the wakefields can be the cause for electromagnetic power deposition on the accelerator component leading to a phenomenon called beam-induced heating [5], [16], [17]. For describing these phenomena, it is usually more convenient to work with a simplified description of the wakefields which considers integrated effects.

We consider an idealized scenario where particles move all at the same speed, that is the speed of light (c). A particle (with charge q_s) enters an accelerator device (which, for these analyses, can be schematically considered either as a cavity-like structure or as a pipe with a certain cross-section) moving in the $\hat{z}$ direction. This source particle generates a time-varying electromagnetic field at the point in space $\mathbf{r} = (x, y, z)$ ($\mathbf{E}(\mathbf{r}, t)$, $\mathbf{B}(\mathbf{r}, t)$) in the structure located all behind it. The momentum change of a "test" particle (with charge q_t) entering the cavity at a certain distance (s) behind the source particle would be [4]:

$$\Delta \mathbf{p}_t = q_t \int_{-\infty}^{+\infty} dt \left[\mathbf{E}(\mathbf{r}_t(t), t) + c\hat{z} \times \mathbf{B}(\mathbf{r}_t(t), t) \right]. \quad (1)$$

The considered scenario is schematically depicted in Fig. 1. The wake function is defined as:

$$\boldsymbol{w}(\mathbf{r}_{s,\perp}, \mathbf{r}_{t,\perp}, s) = \frac{c}{q_s q_t} \Delta \mathbf{p}_t. \quad (2)$$

The wake function is usually split into a longitudinal (w_l) and a transverse ($\boldsymbol{w}_t$) components:

$$w_l = -\frac{c}{q_s q_t} \Delta p_{t,z} = -\frac{c}{q_s} \int_{-\infty}^{\infty} dt E_z(\mathbf{r}_t(t), t), \quad (3)$$

$$\boldsymbol{w}_t = \frac{c}{q_s q_t} \Delta \mathbf{p}_{t,\perp}, \quad (4)$$

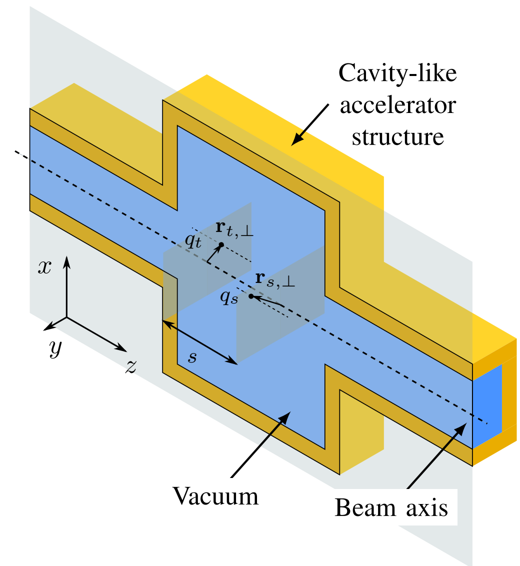

Fig. 1. Cavity-like accelerator structure cut-view. The source and test charges (q_s and q_t respectively) are travelling at the speed of light spaced at a constant distance s. Their positions are given by the vectors $\mathbf{r}_s = (x_s, y_s, ct) = (\mathbf{r}_{s,\perp}, \mathbf{r}_{s,\parallel})$ and $\mathbf{r}_t = (x_t, y_t, ct - s) = (\mathbf{r}_{t,\perp}, \mathbf{r}_{t,\parallel})$.

where the longitudinal wake function is usually taken with a minus sign so that a positive value is associated with energy loss of the test particle. Moreover, since the longitudinal beam-coupling impedance only depends on the longitudinal component of the electric field, it is evident from Eq. 3 that only TM modes in the structure will contribute to it.

The longitudinal and transverse wake functions are linked through their transverse space derivatives thanks to the Panofsky-Wenzel theorem [18]:

$$-\frac{\partial}{\partial z} \boldsymbol{w_t} = \nabla_t w_l. \quad (5)$$

Wake functions represent a description of the wakefields in the time domain, however, in a number of practical applications, it is more convenient to have a description in the frequency domain. Thus, the Fourier transform of the wake function is the quantity called beam-coupling impedance.

$$Z_l = \frac{1}{c} \int_{-\infty}^{+\infty} ds \; w_l e^{j\omega s/c}, \quad (6)$$

$$\mathbf{Z}_t = -\frac{j}{c} \int_{-\infty}^{+\infty} ds \; \boldsymbol{w}_t e^{j\omega s/c}, \quad (7)$$

where j is the imaginary unit and $\omega = 2\pi f$, with f the frequency. The longitudinal and transverse beam-coupling impedances are related through the Panofsky-Wenzel theorem [4]:

$$\omega \mathbf{Z}_t = c \nabla_\perp Z_l. \quad (8)$$

In the context of impedance bench measurements, it is more practical to measure the longitudinal beam coupling impedance and through Panofski-Wenzel theorem it is possible to evaluate the transverse impedance.

When the major impedance contribution is due to modes resonating in the structure (and this is the case of the pill-box cavity considered in this paper) it can be helpful to write the beam-coupling impedance (both longitudinal and transverse) with an analytical resonator model [19]:

$$Z(\omega) = \frac{R_s}{1 + jQ(\frac{\omega}{\omega_R} - \frac{\omega_R}{\omega})}, \tag{9}$$

where R_s is the shunt impedance [20], Q is the quality factor and ω_R is the angular resonant frequency. It is therefore possible to characterize a cavity-like structure in terms of beam coupling impedance considering these three parameters for each mode excited in the cavity.

The beam-coupling impedance concept is typically used in particle accelerator physics to describe both some longitudinal and transverse beam instabilities [21]–[23] and for accounting for heating of devices due to power dissipated by the beam [24]. This explains the need for an accurate estimation of the beam-coupling impedance and the demand for improved bench measurement techniques like the one that will be presented in this paper.

III. Standard measurement techniques

Although analytical computations or numerical simulations are widely employed, in most cases, they are not enough to properly characterize the Device Under Test (DUT) particularly when dealing with complex geometries or unknown material properties within the relevant frequency range. Moreover, measurements are crucial both prior to device installation (to verify conformity) and after installation, especially in cases where operational issues arise during accelerator operation [25], [26].

The standard bench measurement techniques to measure the beam-coupling impedance of accelerator devices are: i) the wire method [9], and ii) the probe method. However, both of them present limitations in some scenarios. A detailed description is provided in the following paragraphs, where these methods are tested on a very simple DUT which still underlines their criticalities. Moreover, the use of the bead-pull method for the evaluation of the solely longitudinal beam impedance (already present in the literature [8], [27]), will be further treated in Sec. IV.

A. The wire method

The stretched wire method is widely employed for the longitudinal and transverse beam-coupling impedance measurements [9]. The method has been developed, improved and tested in several decades [10], [28]–[30] proving to be extremely effective in some scenarios with complex devices such as kickers [31]–[34]. Typically, the DUT is closed by two metallic hollow flanges and a copper wire is stretched along the axis of the DUT. Both ends of the wire are then connected to a Vector Network Analyzer (VNA), as illustrated in Fig. 2.

The field excitation generated by the relativistic particle beam passing through the DUT, especially its longitudinal pulsed current distribution, can be approximated by a current pulse (with identical temporal behaviour) flowing through the wire aligned with the beam axis.

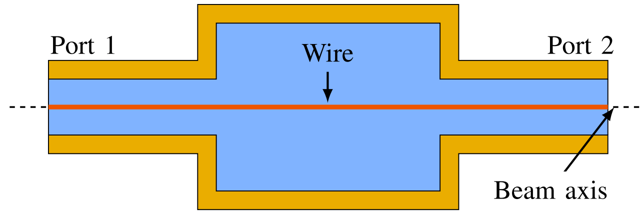

Fig. 2. Wire method setup: cross section of a cavity-like structure with the stretched metallic wire (typically copper) along the beam axis.

The longitudinal beam coupling impedance can be related to the measured scattering parameters through the following general formula [9]:

$$Z_l = -Z_c \ln \frac{S_{21}^{\mathrm{DUT}}}{S_{21}^{\mathrm{REF}}} \left[1 + \frac{\ln S_{21}^{\mathrm{DUT}}}{\ln S_{21}^{\mathrm{REF}}} \right], \tag{10}$$

where the S_{21}^{DUT} is the transmission scattering parameter of the DUT measured in the configuration shown in Figure 2. The S_{21}^{REF} is the reference transmission scattering parameter. It refers to the case in which the scattering parameters are measured or assessed on the DUT when its impedance contribution is removed (i.e. a wire and beam-pipe ensemble, resembling a TEM coaxial line). The Z_c is the characteristic impedance of this TEM coaxial line.

This method presents some critical aspects: i) it is not always possible to perform a reference measurement, and ii) the introduction of a metallic conductor along the device axis alters the electromagnetic boundary conditions of the original DUT. More specifically, the insertion of a second conductor (the wire) other than the cavity walls creates a non-simply connected geometry which allows TEM wave propagation. This TEM propagation should not have been present in the cavity when excited by the particle beam alone. This has the implication of introducing a detuning/shift of the frequency of the impedance resonances with respect to the expectations. Moreover, it could also introduce some additional losses leading to incorrect magnitude of impedance resonances and incorrect evaluation of the quality factors. This is particularly detrimental below the cut-off frequency of the beam pipe of the device, as previously demonstrated in [11]. For these reasons, the wire method provides inaccurate results for resonant structures.

Once the longitudinal impedance has been measured, the wire method can be employed to determine the transverse impedance by applying the Panowsky-Wenzel theorem [29]. However, this requires offsetting the wire with respect to the beam chamber axis. From a mechanical point of view, for each specific device, there is the need to develop a moving support system to implement the offset of the wire. This can be challenging and expensive.

B. The probe method

The probe method relies on a transmission measurement conducted by inserting two electric probes (ending with straight pins) or magnetic probes (ending with loops) in the

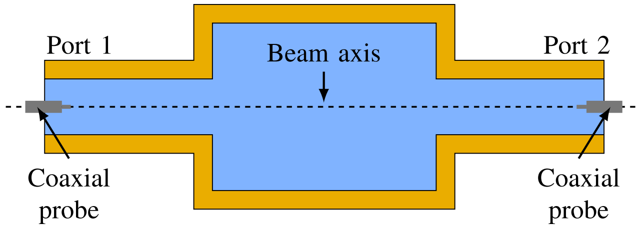

Fig. 3. Probe method setup: cross-section of a cavity-like structure with the electric probes inserted along the beam axis.

DUT closed with hollow flanges (see Fig. 3). The probes are connected to the two ports of a VNA and oriented in such a way as to excite TM modes.

The intrinsic resonances of the device typically remain unchanged in frequency when the penetration of the probes in the device varies. This is true only if the probes' position does not allow excessive coupling with the signal and there is no relevant geometric modification. Therefore, it is crucial to conduct measurements in an under-coupled configuration with the excitation probes, monitoring not only the transmission parameters but also the reflection scattering parameters [19].

This technique, through the excitation of TM modes in the structure, enables the identification of the intrinsic resonances contributing to the longitudinal impedance. This is a major limitation for the comprehensive characterization of the beam coupling impedance unless these measurements are coupled with simulations. Efforts are being made to develop a wireless method using the probe excitation for the beam coupling impedance measurements [35].

C. The characterization of the DUT

As already said, the previously shown methods can present some limitations when measuring resonant structures. To showcase these limitations, in this section, we present the DUT of our interest and the information extracted with the wire and probe methods compared to simulations. The DUT considered is a circular pillbox cavity, shown schematically in Fig. 4.

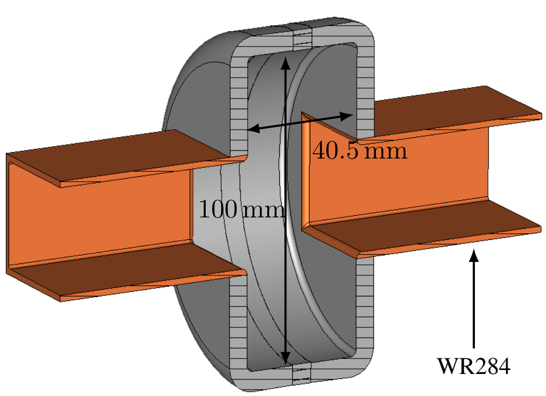

Fig. 4. Cross section of the 3D model of the DUT. The pillbox cavity has an inner radius of $50\,\mathrm{mm}$ and a length of $40.5\,\mathrm{mm}$. It is terminated with two copper WR284 waveguides.

The results of measurements and simulations are shown in Fig. 5 and summarized in Table I. The simulations are performed using the Wake Field (WF) solver of CST Studio Suite [36] which provides the beam coupling impedance simulating a particle beam travelling along the DUT and therefore it provides the appropriate excitation.

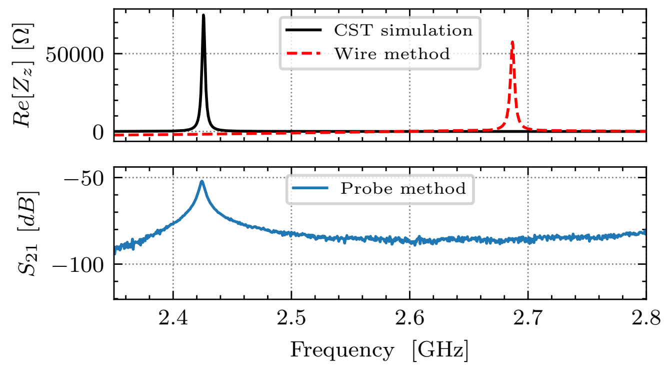

Fig. 5. Characterization of the DUT. Top, the real part of the longitudinal beam-coupling impedance as a function of frequency with a comparison between the wire method and a CST WF simulation. Bottom, the transmission scattering parameter obtained through a probe measurement. All measurements were performed using a standard Short-Open-Load-Through (SOLT) 2-Port calibration. The reference planes were at the end of the cables.

TABLE I. Comparison between the methods and simulation.

	f_R [GHz]	R_S [kΩ]	Q
Wire	2.686	57.4	650
Probes	2.424	-	770
Simulation	2.425	75	777

When comparing expected values from WF simulations and the two standard bench measurement techniques, their limitations are evident. In the case of the wire method, as already discussed at the end of Sec. III-A, the resonant frequency is shifted by almost $11\,\%$ with respect to the expectation. Moreover, also the R_S and Q show discrepancies ($23\,\%$ and $16\,\%$ respectively). In the case of probe method, the transmission scattering parameter shows accurate values of resonant frequency and Q (less than $0.1\,\%$ and less than $1\,\%$ respectively). However, the biggest limitation is given by the incomplete information about the R_S that cannot be estimated with this method.

IV. The Bead Pull Method

A. Theory and concept: the Slater theorem

The bead-pull method relies on Slater's small perturbation theory [37], [38], which states that, when a resonant cavity is perturbed by a small bead, whether metallic or dielectric, the resonant frequency varies from its unperturbed value. The relative frequency shift is proportional to the combination of the squared amplitudes of the electrical and magnetic fields at the location of the bead. The frequency shift due to the material medium perturbation can be written as follows [19]:

$$\frac{\Delta f}{f_R} \approx \frac{\int_{\Delta V} \left[\mu_r\mu_0|\mathbf{H_0}|^2 - \epsilon_r\epsilon_0|\mathbf{E_0}|^2\right] dV}{\int_{V_0}(\mu_r\mu_0|\mathbf{H_0}|^2 + \epsilon_r\epsilon_0|\mathbf{E_0}|^2)dV}, \tag{11}$$

where the first integration is done on the bead volume ΔV, and the second one on the unperturbed cavity volume V_0. $\mathbf{E_0}$ and $\mathbf{H_0}$ are the fields of the unperturbed cavity. The constants μ_r and ϵ_r are the relative magnetic permeability and electric permittivity of the bead. Hence, pulling a dielectric bead with $\epsilon_r > 1$ and $\mu_r = 1$ means measuring the electric field only. This can be exploited during transverse movements of the bead to couple mostly with the electric field. This is the main idea behind the use of a dielectric bead for the transverse impedance measurement that will be presented in the paper.

If the dielectric bead is assumed to be small enough so that the fields can be regarded as uniform for an appreciable distance behind and beyond it, the relative frequency shift can be written as:

$$\frac{\Delta f}{f_R} \approx -k_{SL}\frac{|\mathbf{E_0}|^2}{U}, \tag{12}$$

where k_{SL} is a constant parameter named calibration constant of the bead and it is a function only of the bead's geometry and material. U is the total stored energy in the cavity (i.e. the denominator of Eq. (11)). However, in practical measurements, the phase shift of the transmission scattering parameter can be obtained more accurately than the frequency shift. The relation between this phase shift and the relative frequency shift is:

$$\frac{\Delta f(z)}{f_R} \approx \frac{\Delta\phi(z)\cdot(\pi/180)}{2\cdot Q_L}, \tag{13}$$

where z is the location of the small perturbation and Q_L is the loaded quality factor of the cavity.

From the frequency shift, one can compute the shunt impedance to reconstruct the beam coupling impedance, through the following equation [27]:

$$R_S = \frac{Q_0}{2\omega_R k_{SL}}\left(\int_0^D \sqrt{\frac{|\Delta f(z)|}{f_R}}dz\right)^2, \tag{14}$$

where D is the length of the cavity and Q_0 is the unloaded quality factor of the cavity.

B. Bead-pull setup and technique

In this paragraph, the developed portable setup and the steps required to perform the bead-pull measurement of the longitudinal beam-coupling impedance are described.

1) Description of the developed setup: A general-purpose and portable bead-pull system has been developed and is depicted in Fig. 6. A nylon wire is stretched through the structure and held in place by two aluminium profiles with a pulley system attached. This is controlled by an Arduino-driven motor which allows the movement of the bead (glued to the nylon wire) through the cavity. A graduated scale is mounted on the two aluminium profile holders, enabling controlled transverse movements of the wire and bead. These movements are necessary for conducting transverse impedance measurements, as will be described in Sec. V.

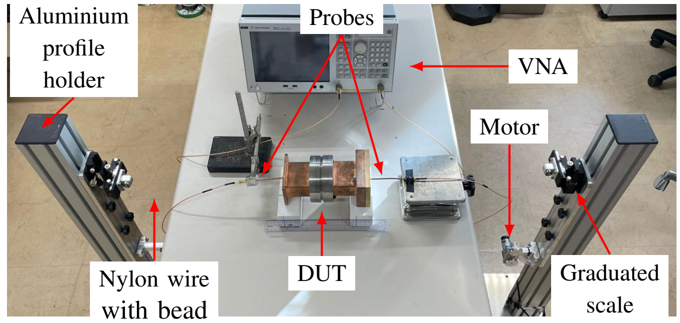

Fig. 6. Bead-pull method setup with the portable and general purpose pulley system.

2) Bead-pull method for longitudinal impedance measurements:

Obtaining the longitudinal beam-coupling impedance with different offsets is the starting point for the retrieval of the transverse impedance. In this section, the bead-pull method is explained in depth while showing the measurement process of the longitudinal beam-coupling impedance in the case of our DUT. The general bead-pull technique involves the following steps:

1) Calibrate the bead through simulations. Modelling the geometry of the bead in a known reference cavity, it is possible, through several simulations, to compute the frequency shift $\Delta f(z)$ by changing the longitudinal position of the bead. Furthermore, the f_R, Q_0 and R_S are also output of the simulations. Finally, using the inverse of Eq. (14) it is possible to obtain the bead calibration constants k_{SL}. The dielectric bead that was used in the simulations and measurement process of our DUT is shown in Fig. 7.

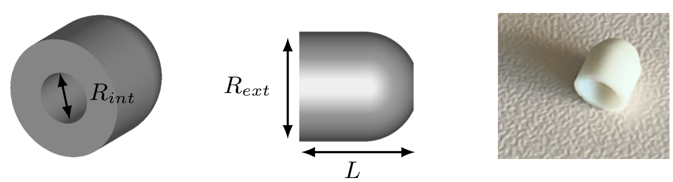

Fig. 7. From left to right: the 3D model of the bullet shaped dielectric bead, a side view of it and the real bead that was used. L=3.75 mm, R_{ext}=1.97 mm, R_{int}=0.85 mm, $\epsilon_r = 5.5$.

The calibration of the bead is a time-consuming process that involves multiple eigenmode simulations (all performed in CST). Primarily because the simulations must be highly accurate to resolve properly the frequency shift. Additionally, the geometric modelling of the bead requires meticulous study to ensure an accurate representation.

2) Excite the resonances of the cavity and characterize them in terms of resonant frequency and quality factor with the probe method (as described in section III-B). The results of this step are shown in Fig. 5 in the bottom plot.

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3) With the probes inserted, measure the phase $\phi(z)$ of the transmission parameter S_{21} at the resonant frequency while the bead is travelling inside the cavity (along the beam axis). In post-processing, the normalized Δf is computed through Eq. 13. For the DUT this measurement is reported in Fig. 8. It is worth mentioning that, to insert the bead in the structure and to allow its transverse movements, it is required to leave the terminating waveguides open (without any measurement flange). This will introduce only a minimal perturbation on the measurements since the waveguides that are terminating the structure are chosen to have a cut-off frequency well below the fundamental cavity mode that is being characterized.

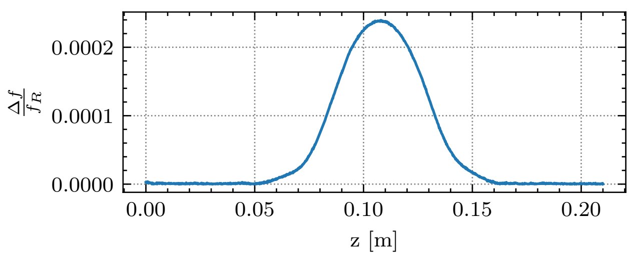

Fig. 8. The normalized Δf as a function of the longitudinal position z of the bead in the DUT.

4) Compute the shunt impedance through Eq. 14.
5) Compute the longitudinal beam-coupling impedance using the resonator formula of Eq. 9.

In Fig. 9 it is depicted the obtained longitudinal impedance of the DUT.

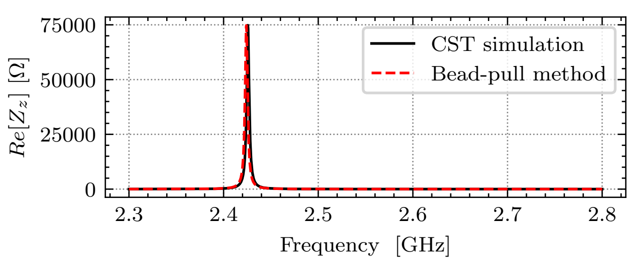

Fig. 9. Longitudinal beam-coupling impedance obtained with the bead-pull method (red curve). The expected value from CST simulations is reported as well (black curve).

V. Transverse beam-coupling impedance measurements: a new approach

The idea of measuring the transverse impedance by applying the bead-pull technique is inspired by the approach used in the wire method [8]. In the simple case of a device with top/bottom and left/right symmetry (which is the case for our DUT), the measured longitudinal impedance can be expanded as a power series [3], [4], [39] as follows:

$$Z_l(f) = Z_{l,0}(f) + Z_{l,1x}(f)x_0^2 + Z_{l,1y}(f)y_0^2, \tag{15}$$

where x_0 and y_0 are the transverse offsets of the source of excitation with respect to the centre of the chamber (i.e the metallic wire in the case of the wire method and the bead trajectory in the case of the bead-pull method). The first term $Z_{l,0}(f)$ is the classical longitudinal impedance (when the excitation is centred in the symmetry axis). The frequency-dependent coefficients $Z_{l,1x}(f)$ and $Z_{l,1y}(f)$ are linked to the transverse impedance Z_x and Z_y through Eq. (8):

$$Z_x(f) = \frac{c}{2\pi f} Z_{l,1x}(f), \tag{16}$$
$$Z_y(f) = \frac{c}{2\pi f} Z_{l,1y}(f). \tag{17}$$

The new approach, in the case of the bead-pull method, consists of measuring the longitudinal impedance Z_l as a function of frequency, as described in IV-B2, for different transverse offsets of the bead. The offsets are given separately on the two axes by shifting the nylon wire location. This can be viewed in Fig. 10. Considering now the measured resonant frequency f_R, the related total shunt impedance R_S is a function of the bead offset with a parabolic trend (as in Eq. (15)) which can be expressed in the form:

$$R_{S,x} = A \cdot x_0^2 + C \tag{18}$$
$$R_{S,y} = B \cdot y_0^2 + C \tag{19}$$

where, C is the shunt impedance R_S when the bead is at the center, A and B are the shunt impedances related to the impedance terms $Z_{l,1x}$, $Z_{l,1y}$ respectively.
From the parabola equations (18) and (19), using Eq. (8), the shunt impedance related to the transverse beam-coupling impedances (on x and y) can be then obtained as follows:

$$R_{T,x} = \frac{A \cdot c}{2\pi f_R} \tag{20}$$
$$R_{T,y} = \frac{B \cdot c}{2\pi f_R} \tag{21}$$

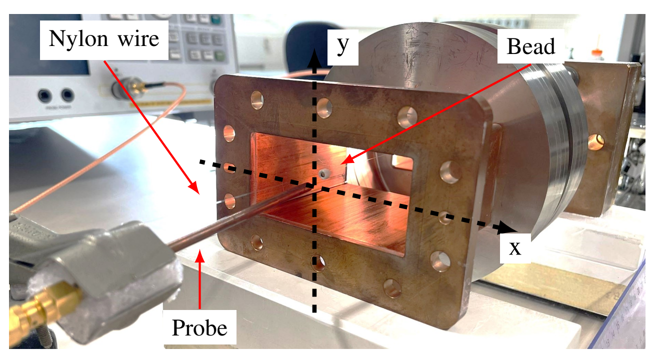

Fig. 10. Reference axis system in the structure. Offsets are given with respect to the zero, which is the centre of the cavity.

In the case of our DUT, measurements of $R_{S,x}$ and $R_{S,y}$ for different transverse offsets are performed. In Fig. 11 it is reported the results of measurements in the case of offsets of the dielectric bead on the x axis. The expected parabola is obtained with parameters $A \approx -66.45\,\mathrm{M\Omega/m^2}$ and $C \approx 74.84\,\mathrm{k\Omega}$.

This article has been accepted for publication in IEEE Transactions on Instrumentation and Measurement. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIM.2024.3458042

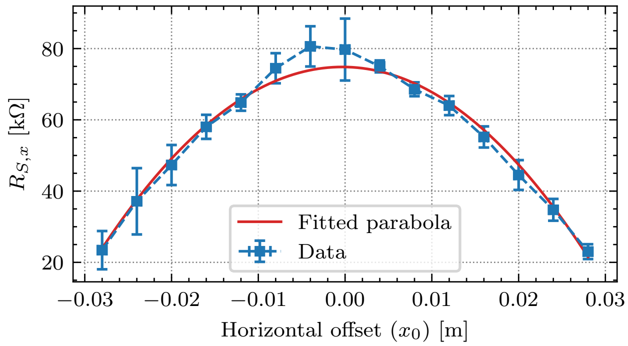

Fig. 11. $R_{S,x}$ vs x_0 offsets. The error bar was calculated from repeated measurement data. The goodness of the fit is justified by $P(\chi^2 > 5.7) = 0.93$.

The shunt impedance related to Z_x is:

$$R_{T,x} = \frac{A \cdot c}{2\pi f_R} = -1.308 \ \frac{\text{M}\Omega}{\text{m}}$$

Finally, in Fig. 12, both transverse impedances are reported and compared to the expectations coming from simulations.

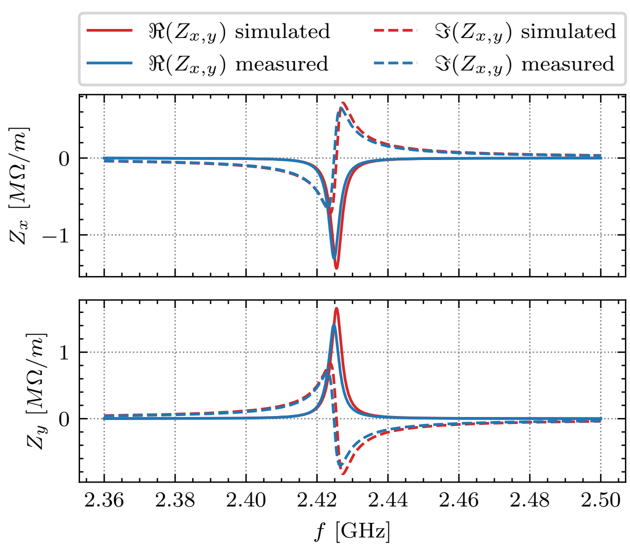

Fig. 12. Transverse beam-coupling impedances (both on x and y) obtained with the bead-pull method (blue curves). Expected values from CST simulations are reported as well (red curves) .

It is possible to appreciate that not only it is possible to measure accurately the longitudinal beam-coupling impedance, but that, with the novel approach, there is also a good agreement between measurements and simulations for the transverse impedances in the general case study reported.

VI. Conclusion

In this paper, a novel approach for transverse beam-coupling impedance measurements has been proposed. The new method, based on the bead-pull technique, has been described and its experimental proof-of-concept was showcased for a pillbox cavity. The method was implemented and tested developing a general-purpose, low cost and portable setup with the potential to be easily applied to other accelerator devices. In this paper, other standard bench measurement techniques (wire method and probe method) were introduced to underline their limitations when applied to resonant structures. The setup employed allowed for accurate measurements of the longitudinal beam-coupling impedance, overcoming the limitations of the existing methods. Moreover, the idea of using a dielectric bead, which couples with the electric field when giving a transverse offset to the bead, allowed to measure the longitudinal impedance in several transverse locations. This, combined with the Panofsky-Wenzel theorem, proved the possibility of measuring the transverse beam-coupling impedance by adopting the bead-pull technique. The results on the pillbox cavity showed an excellent agreement with the expected values from simulations. Minimal disagreement can be explained by the absence of the flanges closing the structure.

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