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From difficulty to competency: cognitive abilities beyond mathematics and their impact on mathematical performance

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ABSTRACT

Mathematics represents a discipline in which there is increasing interest in recent years. Indeed, achieving good mathematical competence seems to positively affect employment success, socioeconomic status, financial choices and even health status (Ritchie & Bates, 2013).

Precisely because of the importance of this discipline, the present work aims to investigate mathematical skills in children and young adults.

Both the studies will be introduced with a systematic review aimed at identifying the domain-general abilities more involved in mathematical difficulties (children) or performance (young adults).

In light of some critical issues that emerged from the reviews, such as the lack of consensus in defining the groups with mathematical difficulties, it was considered appropriate to alter the course of the experimental studies of this work focusing on mathematical competence, and not on the mathematical disorder.

However, to investigate mathematical competence, it is necessary to consider some characteristics of this construct.

First, mathematics is a composite discipline, and therefore its measurement is heterogeneous.

That implies that to define overall mathematical competence it is necessary to identify "sub-competencies" which may be influenced by different cognitive abilities.

All that considerations, combined with the purpose of identifying the cognitive functions that most influence these skills, will be the cross-cutting aim of the present work.

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Introduction

The research activity presented in this thesis work has been carried out in a context in which "numeracy" is recognized as a fundamental skill in everyday life that seems to positively affect employment success, socioeconomic status, financial choices, and even health status (Ritchie & Bates, 2013).

However, at the same time, it is estimated that between 3 and 6 percent of the population has a Specific Learning Disorder in Mathematics (Shalev et al., 2005; Swanson et al., 2009). But this prevalence seems to vary depending on the criteria adopted to define the disorder (Murphy et al., 2007; Jovanović et al., 2013; Devine et al., 2013), on which the international literature seems to have not yet reached an agreement.

Within the Italian context, the latest surveys conducted by the Ministry of Education, University and Research (MIUR, 2019) in schools show that only 1.6% of students with SLD (which corresponds to 4.9 percent of total scholars) have a diagnosis of Specific Learning Disorder in mathematics (or Dyscalculia).

Of the total number of students attending secondary school, 1.9 percent have a certification of Dyscalculia, compared to 24 percent of students of the same age who, on the other hand, do not reach the basic level of proficiency in mathematics (Programme for International Student Assessment, 2018).

Such evidence should make us reflect on two main aspects.

First, it is necessary to distinguish between disorder and difficulty in learning mathematics. In fact, the percentage of students with a Specific Learning Disorder in mathematics is smaller than that of students who struggle in this area.

Moreover, the increasing demand for evaluations for suspected Specific Learning Disorders during middle school (Genovese et al., 2013), combined with a lower prevalence than estimated, suggests the existence of a large percentage of children and youth with undiagnosed and unrecognized Specific Learning Disorders (Fenzi & Cornoldi, 2015), for whom identification is necessary.

In response to the growing necessity to correctly identify an SLD in mathematics and to accurately distinguish it from difficulty, some points have been defined in the clinical practice (AID-AIRIPA Agreement, 2012).

Among these is essential the assessment of domain-specific skills, such as numerical competence (including the processes of subitizing, quantification, seriation, and comparison, as well as computational strategies) and arithmetic competence (involving both the processes of transcoding symbolic, verbal, and Arabic codes and the procedures for performing written calculations and retrieving arithmetic facts and algorithms).

However, in order to best describe the functional profile, it is crucial to assess also some domain-general skills that may influence mathematical performance.

Specifically, the literature considers fundamental abilities such as processing speed (Kulp et al., 2004; Haist et al., 2015); mnestic skills (Geary, 2004; Swanson & Jerman, 2006; Szűcs, 2016; Peng et al., 2018); executive functions (Toll et al., 2011; Cragg et al., 2017); and attention (Askenazi & Henik, 2010; Anobile et al., 2013).

However, due to the heterogeneity with which mathematical disorders profiles manifest themselves (Träff et al., 2017), the cognitive processes that can most support the diagnosis o in clinical practice have not been defined. Finally, some emotional-motivational variables, such as Math Anxiety and Math Attitude, may influence mathematics performance (Rubinstein & Tannock, 2010; Zhang et al., 2019; Kapetanas & Zachariades, 2007).

In light of these considerations, this work will be divided into two macro-areas: assessment and analysis of mathematical performance in scholar populations (primary and secondary school) and analysis of math competence in adulthood.

The study on school-age children will be introduced with a systematic review aimed at identifying the domain-general abilities more involved in mathematical deficits. Next, the results of experimental research on this population will be presented to deepen and exceed what emerged from the literature review.

On the other hand, the work focusing on the adult population aims to evaluate the cognitive abilities that most predict mathematical proficiency in young adults with typical development. This aim will be reached through a systematic review of this phenomenon and the presentation of experimental research aimed to exceed observed limits.

I. Domain-general cognitive skills and mathematical difficulties: a systematic review of the literature

Introduction

Many studies have highlighted the important role played by arithmetic and mathematical skills in everyday life (McCloskey, 2007; Reyna & Brainerd, 2007; Ojose, 2011) also in terms of job opportunities and professional success (Parsons & Bynner, 1997; Geary, 2011).

However, many school-age children have difficulties learning mathematics, a problem with an incidence ranging between 5% and 7% (Kosc, 1974; Shalev, 2005; Swanson et al., 2009). Given the clinical relevance of this phenomenon, it is important to understand which factors cause or contribute to mathematical difficulties (MD) in order to intervene more effectively. Mathematics is a composite discipline, including various domains such as arithmetic, algebra, geometry, and statistics. Individuals' performance in each of these domains implies developing different skills such as the sense of number, understanding mathematical concepts and procedures (Dehaene, 2001, Aunola, 2004). Thus, mathematical performance depends on a series of specific domain skills that, however, also require the simultaneous development of general cognitive- domain abilities. The impairment of any of these domains could determine a cascade effect on the learning of mathematics.

Several studies support the existence of a domain-specific deficit in children with MD (Krajewski, & Schneider, 2009 De Smedt, 2013) or dyscalculia (Piazza, 22010; Mazzocco et al., 2019)). Butterworth (1999, 2008) proposed that mathematical difficulties in children with Developmental Dyscalculia (DD) are due to a deficit in understanding the basic numerical concept, such as counting or magnitude comparison (i.e., number processing system). By contrast, Geary (2004) highlighted that competencies in each mathematic domain are based on different conceptual and procedural processes supported by various cognitive abilities. However, the role of such cognitive abilities is still unclear. Indeed, while some studies have shown that domain-general cognitive skills strongly predict mathematical ability (Hassinger-Das, 2014; Purpura et al., 2017), others reported that mathematical skills depend on both domain-specific and general cognitive processes (Passolunghi & Lanfranchi, 2012; Vanbinst, 2014).

Many cognitive functions may be involved in learning mathematics. Processing speed may facilitate carrying out simple tasks, such as decoding numbers and counting quickly, which are useful for speeding up mathematical operations. Furthermore, processing speed is directly proportional to short-term memory store capacity (Case, 1982). Consequently, a higher processing speed will allow keeping more information in memory, allowing an association between operations and results (Geary, 1993; Peng et al., 2018). The frequent repetition of this process enables the information consolidation in long-term memory, becoming an easily and quickly recoverable arithmetic fact (Geary, 2011; Bull & Johnston, 1997; Fuchs et al., 2008), which can increase the automatization of the calculation process. However, the role of phonological awareness may also be important in this process. To solve any calculation, it is first necessary to convert the terms of the operation into a verbal code (i.e., transcoding processes) (Peng et al., 2018). Then, the attentional and inhibitory processes of the central executive system support the procedural and conceptual knowledge underlying each mathematical domain (Geary, 2004; Andersson, 2008), as well as the ability to pass quickly from a rule to a procedure or strategy (e.g., shifting, or cognitive flexibility). Furthermore, keeping in memory and manipulating visual and verbal information (e.g., working memory) contributes to mathematical performance (Geary, 2004; Geary, 011; Bull & Scerif, 2001; Clark et al., 2010). As already highlighted, mathematical domains are numerous, and each of them requires different numerical, conceptual, and procedural knowledge. Different cognitive functions may influence the development of each mathematical skill in different ways. Accordingly, the problems related to the definition and identification of MD are also open. At a diagnostic level, we refer to math learning disorder (or developmental dyscalculia), when there are persistent difficulties in numerical information processing, memorization of arithmetic formulas, accurate and fluent calculation, with onset in the school years (APA,2013). The diagnosis of dyscalculia is made after a complete and accurate evaluation of specific abilities of numerical cognition (such as subitizing, quantification, seriation, comparison, and calculation strategies) and the procedural level of arithmetic (such as reading and writing, the ability to perform written operations in the column, to retrieve arithmetic information and algorithms). However, the instruments currently used to evaluate math skills and identify children with MD do not always evaluate both the formal and "innate" aspects of such skills. This difficulty may lead to incorrect estimates of children's skill levels (Geary, 2004; Murphy et al., 2007).

A further element of complexity in highlighting MDs and their relationship with cognitive functioning is using different classification cut-offs to indicate different severity levels (Murphy et al., 2007). Moreover, the cognitive domains are usually assessed with "impure" tasks that requires a contribution of several processes or abilities (Sörqvist, 2014, Neath, 2019). On the one hand, that implies that the same task could be interpreted as a measure of different domains. For instance, the Stroop Task has generally considered an inhibition task (Diamond, 2013), but sometimes it has been considered an attentional one (Cai et al., 2013; MacLeod, 1992). On the other hand, task as the Rapid Automatized Naming (RAN), might require various abilities like processing speed, phonological processing, visual, temporal, lexical or attentional processes (Wolf & Bowers, 1999; Donker et al., 2016). To deal with this "task impurity" question, we choose to define the cognitive demands assess by each task according to the authors' interpretation and theoretical model, except for the RAN. Considering the several abilities involved in this task, we considered the RAN as a measure of general processing speed, in line with the idea that naming speed requires rapidly encode visual stimuli (Wolf et al., 2000; Shanahan et al., 2006).

In light of these considerations, the present systematic review aims to identify the most impaired cognitive functions in school-age children with MD. Understanding these relationships would allow clinicians to better evaluate the domain-general cognitive skills that influence mathematical abilities, obtaining a more comprehensive profile of their children's expertise. This knowledge can help clinicians to propose more effective and tuned interventions.

Method

The review was conducted according to the PRISMA statement (Liberati et al., 2009;

Moher et al., 2009). The protocol was registered on PROSPERO (CRD4202019707).

Research Strategies

The systematic search of the international literature was conducted until February 20, 2020, on the following electronic databases: *PsycArticles; PsycInfo; Scopus, and Web of Science*. The results were limited to articles in English and academic publications. The search was conducted using the following script on each database: ((math* disability OR math* difficulty OR dyscalculia) AND (Cognitive Function*)) and produced a total of 2977 records. After eliminating duplicates (N = 273) through the Mendeley software, 2704 records were screened based on title and abstract. Then, 2448 records were excluded, while the remaining 256 were assessed for eligibility based on reading the full texts.

To update the results, on November 8th, 2021, the search was re-run on each database limiting by publication data range (e.g., 2020-2021). A total of 261 new records was found, and 219 records was screened based on title and abstract after eliminating duplicates (N= 42). Finally, thirteen records were assessed based on full texts.

Eligibility criteria

To be included in this systematic review, the studies must meet the following eligibility criteria: (a) school-age participants, i.e., they must be aged between 6 and 12; (b) the study had to evaluate at least one of the following cognitive domains: Processing speed, Phonological processing; Memory (long or short term, verbal and visual, spatial or visuospatial); Executive functions such as working memory, inhibition and cognitive flexibility (switching or set-shifting), and attention; (c) the study had to assess the participants' mathematical abilities and intelligence (e.g., fluid intelligence or verbal or non-verbal IQ).

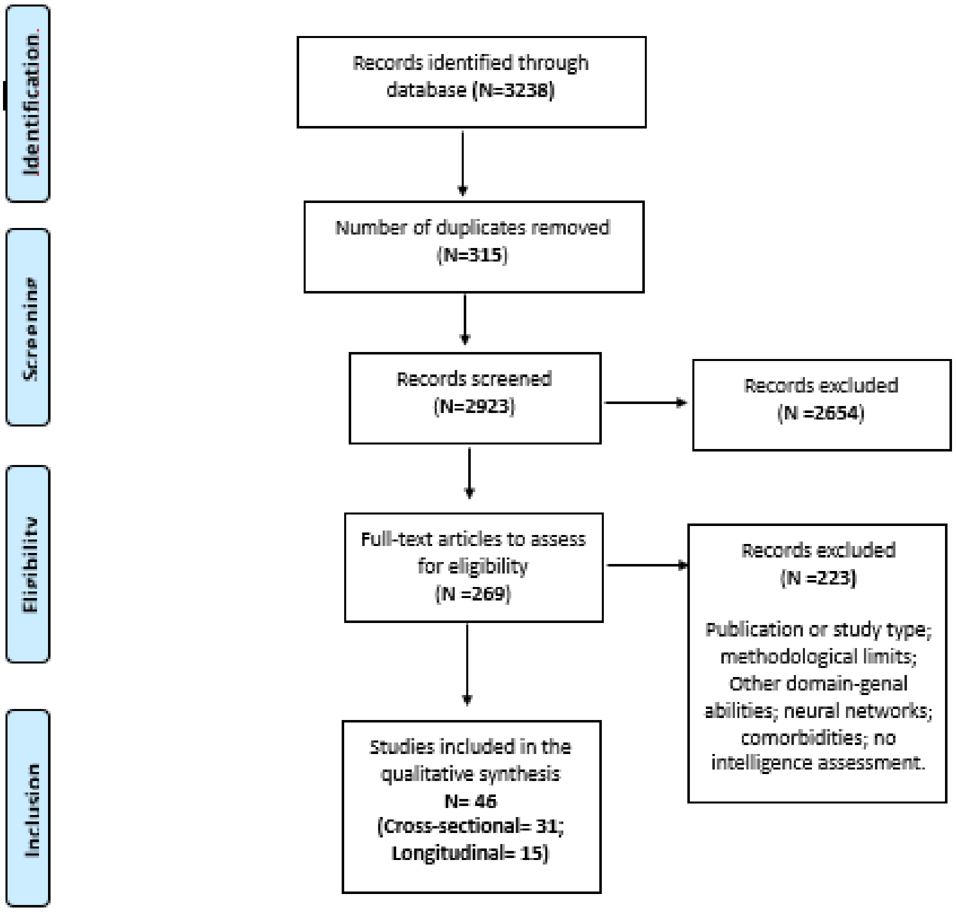
Cross-sectional and longitudinal studies were included. The cross-sectional studies had to report the measures used for mathematical skills assessment in the screening phase and the criteria adopted to define the group with MD. Furthermore, they had to include a control group. In longitudinal studies, children assessed in the preschool-age must have at least one follow-up during primary school, i.e., during the period of formal math learning.

Longitudinal studies also had to include children with persistent MD. Sometimes, MD can be temporarily spontaneously resolved; therefore, longitudinal studies that did not take this feature into account would not allow us to grasp any cognitive difficulties characterizing the population of interest.

Out of the 269 articles assessed for eligibility, three were excluded because they were in a language other than English, 44 studies were excluded because they were not experimental studies (e.g., book chapters; reviews, metanalyses, theoretical issues, commentaries, or editorials), 42 were excluded because they did not evaluate the functions of our interest. Furthermore, 10 studies were excluded because they were correlational and, evaluating mathematical skills along a continuum, did not distinguish between children with and without MD. Other ten studies were excluded because they evaluated the neural networks involved in mathematical skills, while five were excluded as they assessed the effectiveness of rehabilitation or enhancement of mathematical skills. Other 65 studies were excluded for methodological reasons (N = 32; absence of validated measures for the screening of mathematical skills, classification of the experimental group based on executive and non-mathematical skills), or because they did not involve primary school participants (N = 35; preschoolers, adolescents, and adults). Finally, 44 studies were excluded as they assessed the comorbidity between MD and other disorders (N = 10), because they did not have a control group without MD (N = 6), or because they did not carry out an intelligence assessment of the participants (N = 27).

Forty-six articles were included in the systematic review, 31 cross-sectional and 15 longitudinal studies. Figure 1 reports the flowchart showing the number of studies identified from the databases and the number of studies examined, assessed for eligibility, and included in the review. The reasons for possible exclusions are also reported.

Figure 1. PRISMA flow chart of the selected studies on mathematical difficulties and cognitive functioning



Data Collection and Quality Assessment

The selection was independently conducted by two researchers; a supervisor resolved any doubt. The data of the 46 articles included in this systematic review were extracted according to the PICOS approach (50). The following information was extrapolated: author(s) and year of publication; study design, characteristics of participants (gender, mean age), tests used to assess intelligence, instruments, and criteria adopted to define the group with MD, cognitive domains assessed and results. Moreover, for the longitudinal studies, the number of measurements carried out over time, and the cognitive domains (with related instruments) evaluated in the various follow-ups, were considered. The extracted data were reported in Appendix A1. The results are summarized reporting the performance differences between the MD and control group.

The quality of the studies was assessed using the Cochrane Handbook for Systematic Reviews criteria (Higgnis et al., 2011), adapted ad hoc according to the objective of this review. For each study, the evaluated domains were: (a) selection of sample and control of any variables that could play a role in mathematical difficulties (e.g., IQ, socioeconomic status, motivation or performance in reading tests; selection bias); (b) the use of standardized instruments to assess mathematical skills and a clear definition of MD group (selection bias); (c) the use of appropriate tasks or tests for assessment of the cognitive domains considered (detection bias); (d) incomplete outcome data about cognitive function (attrition bias), (e) selective outcome reporting in discussion (reporting bias) and (f) other risks of bias.

The quality of the studies was categorized as with unclear/low/high risk of bias for each item ("0" for a low risk of bias, "1" for a high risk of bias, "Unclear" otherwise). For each study, a mean score was calculated and multiplied by 100. Then, studies were categorized into a low risk of bias (lower than 75%) or a high risk of bias (higher than 75%). Finally, if at least two items were unclear, the studies were classified as with an unclear risk of bias.

Results

Studies Selection

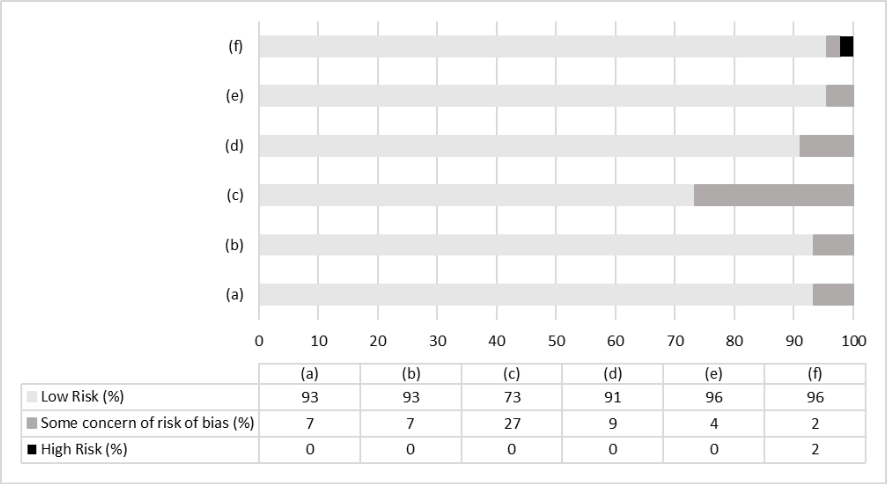
The systematic search produced a total of 3196 records. After eliminating duplicates (N = 315) and the screening based on title and abstract, 269 articles were evaluated for eligibility, then 46 were included in the qualitative analysis, i.e., 31 cross-sectional and 15 longitudinal studies (see Figure 1).

The studies meeting the inclusion criteria were conducted from 1980 to 2020 and involved 8398 children. Participants were aged between 7 (Rousselle & Noël, 2007; Cirino et al., 2015) and 11 years (Webster, 1980; Keeler & Swanson, 2001; Peng et al., 2012; Cai et al., 2013; Willcutt et al., 2013). The percentage of females in the studies ranged from 31.8% (Murphy et al., 2007) to 83.3% (Lafay & St-Pierre, 2017). In five studies, information on the participants' gender was not reported (Webster, 1980; Chan & Ho, 2010; Slot et al., 2016; Chu et al., 2019; Zhang et al., 2020).

Quality Assessment

Figure 2 shows the percentage of articles fulfilling each quality criterion assessed. All the studies had a generally good quality, with an average risk of bias lower than 75%. The high percentage of studies with low (37.8%) or no risk (55.5%) of bias highlights the validity of this systematic review. No study reports a high risk of bias, while three studies (6.7%) showed an unclear risk of bias. All studies clearly defined the criteria for the MD group and used appropriate statistical analyses. The highest risk of bias was on the detection bias domain (27%) and was due to the assessment of cognitive functions with non-standardized tools, that produced some concern of risk of bias.

Figure 2. Percentage of risk of bias for each domain of tool assessment for the selected studies



	(a)	(b)	(c)	(d)	(e)	(f)
▫ Low Risk (%)	93	93	73	91	96	96
▨ Some concern of risk of bias (%)	7	7	27	9	4	2
■ High Risk (%)	0	0	0	0	0	2

Characteristics of selected studies

Characteristics of selected studies are organized into two subsections: characteristics of cross-sectional and longitudinal studies.

Characteristics of cross-sectional studies (N = 31)

The mean age of children with MD ranged from 7 to 11 years. The most represented age group was 9 years, considered in 12 out of 31 studies (38.7%) (Lafay & Macoir, 2017, Cirino et al., 2007; Raghubar et al., 2009; Passolunghi, 2011; Passolunghi & Mammarella, 2012; De Weerdt et al., 2013a; De Weerdt et al., 2013b; Moura et al., 2013; Szucs et al., 2013; Lafay & St-Pierre, 2017; Lambert & Spinath, 2018; Mammarella et al., 2018; McDonald & Berg, 2018). Two studies did not report the mean age of the sample, but the participants were recruited in primary school classes; therefore, they fall within the age range of our interest (Geary et al., 2004; Fuchs et al., 2008).

Studies included in this review assessed intelligence using different tests, such as Raven's Standard Progressive Matrices (RSPM; Keeler & Swanson, 2001-Peng et al., 2012, Chan & Ho, 2010; Peng et al., 2012; Cai et al., 2013), Raven's Colored Progressive Matrices (CPM; Costa et al., 2011; Moura et al., 2013; Szucs et al., 2013; Attout & Majerus, 2015; Lafay & St-Pierre, 2017; McDonald & Berg, 2018), Cultural Fair Intelligence Test (CFT; Lambert & Spinath, 2018), reduced versions of different editions of the Wechsler Intelligence Scale for Children (WISC-R- (Willcutt et al., 2013); WISC III- (Geary et al., 1999; Censabella & Noël, 2007; Rousselle & Noël, 2007 ; De Weerdt et al., 2013a; De Weerdt et al., 2013b; Kroesbergen & Van Dijk, 2015; Donker et al., 2016; Slot et al., 2016); WISC IV (Kuhn et al., 2016; Mammarella et al., 2016; al., 2018), the Wechsler Abbreviated Scale of Intelligence (WASI) (Cirino et al., 2007; Fuchs et al., 2008; Raghubar et al., 2009, Cirino et al., 2015), the Stanford Binet Intelligence Scales (Geary et al., 2004), the Primary Mental Abilities (PMA; Passolunghi, 2011; Passolunghi & Mammarella, 2012);); the Intelligence and Developmental Scale (IDS, Reimman et al., 2013), and the Peabody Picture Vocabulary Test (PPVT, Dunn & Dunn, 2009).

The instruments used for the initial assessment of mathematical skills, i.e., for the definition of groups with MD, are reported in the Table 1 in appendix A2. Table 1, also includes the mathematical domains assessed and the cut-offs applied to define children with MD.

Characteristics of longitudinal studies (N = 15)

Each longitudinal study included at least one assessment in the first-grade primary school and the definition of MD according to at least two assessments of math achievement.

The studies assessed intelligence using either the Raven’s Colored Progressive Matrices (CPM; Cowan & Powell, 2014; Chan & Wong, 2019; Zhang et al., 2020), Raven's Standard Progressive Matrices (RSPM; Wong & Chan, 2019), Vocabulary and Matrix Reasoning subtests of the Weschler Intelligence Scale for Children (WISC-III; Geary et al., 2000), the Weschler Abbreviated Scale for Intelligence (WASI; Mazzocco & Kover, 2007; Murphy et al., 2007), the Receptive Vocabulary subtests and the Drawing with cubes of the WPPSI-III (Chu et al., 2019). Other studies used two intelligence measurements, i.e., CPM and some subtests of WISC-III (Geary et al., 2008) or WASI (Geary et al., 2007; Geary et al., 2012a; Geary et al., 2012b) at two different points in the study. Finally, a study evaluated verbal IQ through the

PMA battery (Passolunghi & Siegel, 2004), while in the Mazzocco and Grimm' study (2013), the test used is not specified, but an IQ higher than 80 is reported in the participants. The instruments and criteria used for assessing mathematical skills and their persistence, i.e., for the definition of groups with MD, are reported in appendix A2 (Table 1).

Results on cognitive functioning (N = 46)

The studies included in this systematic review refer to developmental dyscalculia (N = 5), mathematic learning disabilities (N = 13), MD (N = 19), or mathematical disability (N = 9) to consider conditions that appear similar. Notably, the use of these terms was not clearly influenced by the cut-off criteria used to define the severity of mathematical deficit. To report the results of the studies, we chose to refer more generally to Mathematical Difficulties. In such a way, we included both children who performed well below average (e.g., 10° percentile) and those performing at or below the 35th percentile (e.g., less restrictive criteria).

The studies that evaluated the difference between groups with and without MD considered the following cognitive domains: processing speed (N = 22), short-term (N = 13) and long-term memory (N = 2), attention (N = 10); executive functions such working memory (N = 32), cognitive flexibility (N = 7) and inhibition (N = 8), and phonological awareness (N = 4). The results will be separately presented for each cognitive domain, and a summary of results was reported for each cognitive domain.

Processing Speed (N = 22)

Twenty-two articles evaluated the processing speed of children with MD comparing them with a control group. Among these, 15 were cross-sectional studies, while 7 were longitudinal studies.

Thirteen studies evaluated the ability to process visual stimuli, and most of them adopted barrage tasks, i.e., visual search tasks (Fuchs et al., 2008; Chan & Ho, 2010; Passolunghi et al., 2011; Cirino et al., 2015). Lafay and St-Pierre (2017) used a coding task, while composite scores derived from barrage and coding tests were used in two other studies (Cowan & Powell, 2014; Willcutt et al., 2013). A lower accuracy in perform these pencil and paper task within a time limit was observed in all studies (Fuchs et al., 2008; Passolunghi et al., 2011; Cirino et al., 2015; Lafay & St-Pierre, 2017) except in the Chan and Ho's study (2010). Worse performance of children with MD was also observed in a task demanding identifying the total number of dots on certain cards (Lambert & Spinath, 2018).

By contrast, no difference emerged in studies using simple reaction times tasks (Costa et al., 2011; Cai et al., 2013) or choice reaction times tasks (Kuhn et al., 2016).

In this task, participants had to name alphanumeric, such as digit and letters (Chan & Ho, 2010; Cirino et al., 2015; Donker et al., 2016; Slot et al., 2016; Geary et al., 2007; Murphy et al., 2007; Geary et al., 2008; Geary et al., 2012a; Geary et al., 2012b; Mazzocco & Grimm, 2013; Cowan & Powell, 2014), or non-alphanumeric stimuli, such as colors or pictures (Donker et al., 2016; Slot et al., 2016; Murphy et al., 2007; Mazzocco & Grimm, 201; Zhang et al., 2020). Other authors used a composite score obtained from the speed in naming letters, digits, and colors (Willcutt et al., 2013). In studies requiring participants to quickly name colors or pictures, a worse performance was found in children with MD (Donker et al., 2016; Slot et al., 2016; Mazzocco & Grimm, 2013; Murphy et al., 2016; al., 2007; Zhang et al., 2020).

Longitudinal studies using a RAN task with color naming identified an MD group's persistent slowness even at follow-up (Mazzocco & Grimm, 2013; Murphy et al., 2007). Specifically, this difficulty persisted only in children classified according to the 10th percentile (Mazzocco & Grimm, 2013; Murphy et al., 2007), while children classified with the 25th percentile were slower than the control group only until 6 years of age. One study (Träff et al., 2020) did not found a worse performance in the MD group compared to the typical achievement group.

Three cross-sectional studies (Chan & Ho, 2010; Cirino et al., 2015; Slot et al., 2016) and six longitudinal studies (Geary et al., 2007; Geary et al., 2008; Geary et al., 2012a; Geary et al., 2012b; Mazzocco & Grimm, 2013; Cowan & Powell) used alphanumeric stimuli and found worse performance in children with MD than in the control group. Only Donker and colleagues (2016) did not observe any difference between groups in the speed of naming alphanumeric stimuli.

Furthermore, two studies (Chan and Ho, 2010; Mazzocco and Grimm, 2013) found worse performance in naming digits only in the group of younger children with MD (mean age = 8.3), but not in older ones (10 years; Chan & Ho, 2010) and at the follow-up (14 years;

Mazzocco & Grimm, 2013). Murphy and colleagues (2007) reported a persistent slowness in naming digits at all follow-ups (up to the third grade, 8 years) only in children with MD classified considering the 10th percentile; conversely, the children classified according to the 25th percentile at the last follow-up (third grade) presented a performance equivalent to that of the control group. The slowness of naming of letters and colors found in preschoolers persisted even at the follow-up when the children were ranked at the 10th percentile (in 8th grade; Mazzocco & Grimm, 2013).

Summary of results on Processing speed

Processing speed could be assessed with several task that involves different variables of the ability to automatically achieve relatively easy cognitive task (Salthouse, 2000).

This systematic review highlights low processing speed for visual and verbal stimuli in children with MD. Specifically, in visual processing, these difficulties were manifest in the execution of visual search tasks that required the participant to identify the target stimulus among other distractors as quickly as possible (Fuchs et al., 2008; Passolunghi et al., 2011; Cirino et al., 2015) or when the task consisted in reproducing symbols associated with single numbers or letters (Cowan & Powell, 2014). The only study that used a composite score (barrage and coding tasks) identified a worse performance in children with MD than in the control group (Willcutt et al., 2013). The same difficulty occurred in the numerosity processing task (Lambert & Spinath, 2018), demanding to process the numerosity of the elements represented on a map and quickly perform the calculation to give the solution. It seems interesting to note how these pencil and paper tasks were more sensitive to identifying any difficulties in children with MD than the control group. Indeed, studies using simple reaction times as an indicator of processing speed did not identify differences between children with and without MD (Cai et al., 2013; Costa et al., 2011; Kuhn et al., 2016), regardless of the cut-off scores and the screening measures used to define the experimental group. In the Rapid Automatized Naming tasks, children with MD have greater difficulty, mainly linked to a slow execution compared to the control group, regardless of the type of stimulus presented (Murphy et al., 2007; Geary et al., 2008; Geary et al., 2012a; Geary et al., 2012b;

Mazzocco & Grimm, 2013; Cowan & Powell, 2014; Cirino et al., 2015; Donker et al., 2016; Slot et al., 2016). However, in Chan and Ho's study (2010), only younger children with MD were slower in naming a series of pictures, while this difference disappeared when the older group was considered.

In the longitudinal studies, in which the group of participants with MD was classified according to the persistence of the difficulties, a slower performance in processing speed tasks persisted only in children classified using a cut-off at the 10th percentile (Mazzocco & Grimm, 2013; Murphy et al., 2007).

Short- and long-term memory (N = 13)

Among the 12 studies that evaluated verbal short-term memory, most of them did not report worse performance in the MD group compared to the control group (Geary et al., 1999; Costa et al., 2011; Passolunghi et al., 2011; Peng et al., 2012; De Weerdt et al., 2013; Szucs et al., 2013; Reimman et al., 2013; Attout & Mejerus, 2015; Lafay & St-Pierre, 2017; Geary et al., 2000; Passolunghi et al., 2004). These studies used as stimuli words (Passolunghi & Siegel, 2004; Peng et al., 2012; De Weerdt et al., 2013a; Szucs et al., 2013), non-words (Attout & Mejerus, 2015), or numbers (Geary et al., 1999; Costa et al., 2011; Reimman et al., 2013; Lafay & St-Pierre, 2017; Passolunghi et al., 2011; Peng et al., 2012; De Weerdt et al., 2013a; Szucs et al., 2013; Geary et al., 2000; Passolunghi & Siegel, 2004). In order to verify whether the type of stimulus can influence the performance of children with MD, some studies compared their performance in digit and letter (Reimman et al., 2013) or word span tasks (Passolunghi & Siegel, 2004; Passolunghi et al., 2011; Peng et al., 2012; De Weerdt et al., 2013a; Szucs et al., 2013). The results did not highlight differences based on the type of stimulus proposed (Passolunghi & Siegel, 2004; Passolunghi et al., 2011; Szucs et al., 2013; Reimman et al., 2013). Other authors (Peng et al., 2012; De Weerdt et al., 2013) found a worse performance in children with MD exclusively in digit span tasks and not in word span tasks. Webster (1980) confirmed this finding regardless of the nature (visual or verbal) of the stimulus and the type of response (written or verbal).

Visuospatial short-term memory was evaluated in four studies. Three studies found no difference between the groups with and without MD using the Corsi Block test Forward

(Costa et al., 2011; De Weerdt et al., 2013; Lafay & St-Pierre, 2017), while Szucs and colleagues (2013) observed worse performance in children with MD using a Dot Matrix task. Long-term memory was analyzed in its verbal component only in two studies. Reimman and colleagues (2013) found no difference in the ability to recall a story among children with and without MD, while children with MD performed worse on a semantic fluency task (Fuchs et al., 2008).

Summary of results on short- and long- term memory

Nine out of twelve studies evaluating short-term verbal memory found worse performance in children with MD than the control group in tasks that used numbers, letters, words, or non-words as stimuli. Only three studies using the Digit span forward did not identify differences between the two groups.

All the studies that analyzed verbal memory have presented the stimuli a required the response verbally. Webster's study (1980) assessed whether the performance of children with MD could depend on the modality of presentation and recall of the stimuli and found that children with MD recalled more elements when they had to reproduce them verbally than in a graphic-symbolic way; the opposite trend occurred in children with adequate mathematical skills (Webster, 1980).

Concerning the short-term visuospatial memory, the Corsi Block task did not highlight differences between children with and without MD (Costa et al., 2011; De Weerdt et al., 2013; Lafay & St-Pierre, 2017).

Long-term memory has been evaluated only in its verbal component. Using a semantic fluency task, the performance was worse in children with MD aged 8 years; this finding did not occur in children of 10 years or older (Fuchs et al., 2008). However, Reimman and colleagues (2013) did not find differences between groups requiring children to recall a story after a latency period.

Attention (N = 9)

Attention was assessed in nine studies. One of them observe the worst performance in children with MD than the control group in divided attention tasks (e.g., dual task) that required to read a sentence or an operation on the computer screen and remember the last word or the result (Peng et al., 2012).

Children with MD also seem to show greater difficulty in selective attention tasks, in which they were asked to identify elements with a given characteristic, ignoring irrelevant information (Reimman et al., 2013); this task also implies processing speed. Willcutt and colleagues (2013) observed a greater number of omissions in children with MD than in the control group in an 18-minute task in which they were required to press a button when the number "9" appears immediately after the number "1". With a similar task using images rather than numbers, Kuhn and colleagues (2016) observed only a greater number of false alarms in children with MD than in the control group. Worse performance in attentional tasks also emerged in the Cai and colleagues' study (2013), who assessed this ability through Expressive attention (e.g., Stroop Task), Number detection (e.g., visual search), and Receptive attention (e.g., determining whether letters presented were physically the same or if they have the same name). Finally, four studies evaluated attention through the Strengths and Weaknesses of ADHD and Normal Behavior (SWAN) administration and reported higher scores in the inattention subscale (Cirino et al., 2017; Fuchs et al., 2008; Raghubar et al., 2009; Geary et al., 2012a) and hyperactivity/impulsivity scale (Cirino et al., 2007) in children with MD, compared to the control group.

Summary of results on attention

Most of the included studies observed a worse performance in attentional tasks: children with MD presented difficulties in both vigilance/sustained attention tasks Willcutt et al., 2013; Kuhn et al., 2016) and those evaluating selective attention in a limited time (Reimman et al., 2013). Moreover, higher inattention ratings were detected through SWAN in the MD group.

Executive functions: Working memory (N = 32)

Twenty studies assessed verbal working memory and seventeen visuospatial working memory. Five studies evaluated working memory according to the Baddeley model

(Baddeley, 1992).

Concerning the verbal working memory, fifteen studies found significantly lower scores in children with MD, compared to the control group, both in Digit Span Backward tasks (Geary et al., 1999; Fuchs et al., 2008; Chan & Ho, 2010; Cai et al., 2013; De Weerdt et al., 2013a; Moura et al., 2013; Willcutt et al., 2013; Attout & Majerus, 2015; Cirino et al., 2015; Lafay & st-Pierre, 2017; Geary et al., 2000; Passolunghi & Siegel, 2004; Chan & Wong, 2019), and in Word Span Backward tasks (Passolunghi & Siegel, 2004; De Weerdt et al., 2013a; Kuhn et al., 2016). A significant difference also emerged in the Listening Span task (Passolunghi et al., 2011) and Sentence Digit Task (Keeler & Swanson, 2001) in which the participant was required, respectively, to recall the last word of a sentence pronounced by the experimenter after having given a judgment of its truthfulness, and to recall the number of the street/address pronounced by the investigator. However, controlling for the number of intrusion, any difference in performance disappeared in the Listening Span task (Passolunghi & Siegel, 2004), but the difference persists in the Listening Span completion Task, that required the participant to recall the words he/she used to complete some incomplete sentences.

Five studies did not observe different performance between the two groups, using the Listening Span Test (Szucs et al., 2013), the Digit Span backward (Costa et al., 2011; Wong & Chan, 2019), the Word Span Backward (Passolunghi & Siegel, 2004) or a composite score derived from the number of correct responses in an Auditory digit sequence and a Semantic categorization test. In this task, the participant had to recall an address and place a series of words in the correct semantic category (McDonald & Berg, 2018).

Out of eight longitudinal studies evaluating working memory, five referred to the Baddeley model (Geary et al., 2007; Geary et al., 2008; Geary et al., 2012a; Geary et al., 2012b; Cowan & Powell, 2014) and identified a worse performance of children with MD compared to the control groups both in the tasks evaluating the phonological loop and in those assessing the central executive. Even in Cai and colleagues' study (2013) in which the central executive was assessed through the Stop-signal and the Flanker tasks, children with MD performed worse than the control group.

Only one study (Träff et al., 2020) did not find a difference between groups using a task that requires to recall in the correct serial order a sequence of words while managing an interferential task (e.g,. decide whether each presented word was an animal or not).

Concerning the results in tasks measuring the visuospatial sketchpad, two studies reported a worse performance in children with MD (Geary et al., 2007; Geary et al., 2012a), while three did not detect any difference (Geary et al., 2008; Geary et al., 2012b; Cowan & Powell, 2014), although they used the same tool (WMBT-C).

Out of the 14 cross-sectional studies evaluating visuospatial working memory, nine identified a significant difference between groups with and without MD in visuospatial working memory (Keeler & Swanson, 2001; Chan & Ho, 2010; Cai et al., 2013; De Weerdt et al., 2013a; Szucs et al., 2013; Kroesbergen & Van Dijk, 2015; Kuhn et al., 2016; Mammarella et al., 2018; McDonald & Berg, 2018). The visuospatial memory tasks proposed to the participants were different. In fact, some used the Mapping and direction task (Keeler & Swanson, 2001; McDonald & Berg, 2018) that requires the participant to memorize the symbols found on a path and then recall them. In other studies, different versions of the Visual Matrix task (Kuhn et al., 2016; McDonald & Berg, 2018) or the Spatial Span (De Weerdt et al., 2013; Kroesbergen & Van Dijk, 2015) were used in which the participants were required to recognize in which spaces of a grid (Visual Matrix span, Dot Matrix, Nine-grid task) or a figure (Spatial Span and Odd One Out) the dots were previously shown. In other cases (Chan & Ho, 2010; Mammarella et al., 2018), the child had to recognize the figures previously shown among distractors, while in the Cai and colleagues' study (2013), the 2-back task was proposed in which the child has to press a button if the figure appearing on the screen was the same as that showed one or two times before.

On the other hand, five studies, using the *Corsi Block recall Backward* test (Costa et al., 2011; De Weerdt et al., 2013a; Moura et al., 2013; Lafay & St-Pierre, 2017; Chan & Wong, 2019) and one study (Reimman et al., 2013), using a subtest of IDS (recognition of tridimensional figures), did not confirm any difference between children with and without MD.

Executive Functions: Inhibition and interference control (N = 8)

Four studies used the Color-Word Stroop Task (Censabella & Noël, 2007; Peng et al., 2012; Willcutt et al., 2013; McDonald & Berg, 2018), and only one of them (Willcutt et al., 2013) observed a worse performance in children with MD than in the control group.

Worse performance in children with MD emerged using the Number Inhibition task (Peng et al., 2012; McDonald & Berg, 2018), while no differences were observed employing the Numerical Stroop task (Rousselle & Noël, 2007).

Poorer performance in children with MD than in the control group was reported with the Stop-Signal Task (Willcutt et al., 2013; Szucs et al., 2013), which requires to inhibit an automatic response (press a button when targets appeared) in the presence of a given stimulus (alert sound). Children with MD also had a higher number of false alarms in the Visual Continuous Performance (CPT) task in which they were required to press a button when a "9" appeared immediately after a "1" (Willcutt et al., 2013). A difficulty of children with MD in cognitive control tasks was also confirmed by Cai and colleagues (2013).

The only study using the Go/No-Go task (De Weerdt et al., 2013b) did not find impaired inhibition in children with MD compared to the control group. Equally, Censabella and Noël (2007) did not find a different performance in children with MD and in the control group in a task requiring suppressing irrelevant information from working memory or perform a Flanker Task.

Executive Function: Cognitive flexibility (N = 7)

Within the four cross-sectional studies evaluating the cognitive flexibility, two (Szucs et al., 2013; McDonald & Berg, 2018) observed a worse performance in children with MD compared to the control group in a *Trail Making Test* (TMT).

Willcutt and colleagues (2013) found that children with MD made more perseverative errors in the *Wisconsin Card Sorting Test* (WCST). However, Kuhn and colleagues (2016) did not observe any difference between children with and without MD by using a PC base flexibility task.

Three longitudinal studies (Murphy et al., 2007; Mazzocco & Kover, 2007; Chu et al., 2019) adopted composite tasks involving different executive functions (e.g., working memory, inhibition, cognitive flexibility). Specifically, two studies (Murphy et al., 2007; Mazzocco & Kover, 2007) used the *Contingency Naming Test* (CNT) that required children to name the stimulus according to one attribute (e.g., color or form, based on the stimulus congruence) or two-attribute (color or form based on the stimulus congruence and the presence/absence of an arrow) rules. In the first assessment (e.g., 1st grade) the MD group defined by a 10° cut-off showed less efficient performance than the control group on the one-attribute subtest (Murphy et al., 2007; Mazzocco & Kover, 2007), while the MD group defined by a 25° cut-off did not (Murphy et al., 2007). Regarding the two attribute subtest, Murphy and colleagues (2007) assessed the MD performance only in 4th grade showing a worst performance in both MD groups (defined on 10° or 25° percentile), while Mazzocco and Kover (2007) did not analyze this subtest because only one child with MD completed the subtest on the first assessment.

The worst performance in children with MD was also observed in Chu and colleagues' study (2019), adopting the *Conflict Executive Function Scale* (Beck et al., 2011) that required to place cards inside two boxes based on different rules (congruence or incongruence of the stimuli; color or shape; color or shape based on the presence/absence of the border on the card).

Summary of results on Executive Function

In visuospatial working memory tasks, generally, children with MD presented critical performance (Keeler & Swanson, 2001; Chan & Ho, 2010; Cai et al., 2013; De Weerdt et al., 2013a; Szucs et al., 2013; Kroesbergen & Van Dijk, 2015; Kuhn et al., 2016; Mammarella et al., 2018; McDonald & Berg, 2018). Nevertheless, it is interesting to note that such difficulties emerged in many types of tasks, but not when the Corsi Block task was used (Costa et al., 2011; De Weerdt et al., 2013a; Moura et al., 2013; Lafay & St-Pierre, 2017). This finding suggests that this task may not be sensitive in identifying specific difficulties in visuospatial working memory in children with MD. Concerning other executive functions, children with MD were generally impaired in tasks evaluating cognitive flexibility (Szucs et al., 2013;

Willcutt et al., 2013; McDonald & Berg, 2018), inhibition of automatic responses, interference control (i.e., the Stroop Task or its "numerical" variants), and attentional control (i.e., the dual tasks independently from the numerical nature of the stimuli; (Peng et al., 2012). Only the Kuhn and colleagues' study (2016) did not identify any difference using a choice reaction times task.

Phonological processing and phonological awareness (N= 4)

Three cross-sectional studies (Willcutt et al., 2013; Cirino et al., 2015; Slot et al., 2016) and one longitudinal study (Zhang et al., 2020) assessed phonological processing or phonological awareness.

One study observed worse performance in phonological awareness assessed through a composite score, including both deletion (phoneme deletion from a word or a non-word) and manipulation of phonemes (move the first phoneme of a word to the end, and then add a sound; Willcutt et al., 2013). The study of Slot and colleagues (2016) found a worse performance of MD comparing to control group when the task requires to delete the onset, middle or last sound from a word (e.g., phonemic deletion task). No differences between the groups with and without MD were found in tasks that require switching the first sound of two given words (Slot et al., 2016), removing a sound (syllables or phonemes) varying in position (Cirino et al., 2015), or identifying the initial phoneme (Zhang et al., 2020).

Summary of results on Phonological processing and phonological awareness

The few studies including the assessment of phonological processing and awareness indicate mixed results. The lack of homogeneity between the tasks proposed does not allow inferences.

Discussion

The purpose of this review was to identify the cognitive skills involved in MD. Finding general skill deficits in children with MD would be advantageous in clinical assessment because it could help recognizing children with specific mathematic learning disabilities.

During the diagnostic process, an assessment of cognitive skills is already recommended (Kaufamm & von Aster, 2012; Kucian & von Aster, 2015), but there is no agreement on which skills should be of greatest interest.

This review highlights that children with MD have greater difficulties in processing speed, working memory, inhibition, and cognitive flexibility.

In particular, the difficulties related to the processing of visual stimuli would seem to be more manifest in tasks requiring visual and perceptual discrimination, such as in visual search tasks (Fuchs et al., 2008; Passolunghi et al., 2011; Cai et al., 2013; Cirino et al., 2015; Lambert & Spinath, 2018) or coding tasks (Cai et al., 2013; Lafay & St-Pierre, 2017). By contrast, children with MD are no slower than children without MD in responding to visual stimuli, as shown by studies using simple reaction time tasks to assess processing speed (Costa et al., 2011; Cai et al., 2013; Kuhn et al., 2016). Therefore, their impairment would not depend on the ability to respond promptly to a stimulus but rather from the request to process this stimulus and recognizing its relevant characteristics quickly. This cognitive aspect would also imply the ability to discriminate stimuli correctly. From this point of view, a deficit in visual processing would entail difficulty discriminating between numbers and arithmetic signs (Kulp et al., 2004; Sortor & Kulp, 2003). These difficulties would affect formal mathematic learning (Pieters et al., 2012; Haist et al., 2015).

Conversely, children with MD do not appear to have difficulties in verbal and visuospatial short-term memory. However, the results concerning verbal short-term memory seem to contrast with the findings of some recent reviews, in which worse performance of children with MD has been identified by using these tasks (Peng et al., 2018; Swanson & Jerman, 2006). Moreover, according to some authors, children with MD would have especially difficulty in memorizing numerical information. In the studies included in this review, only two out of the eleven studies evaluating performance in verbal and numerical span tasks identify this trend in children with MD (Peng et al., 2012; De Weerdt et al., 2013).

The results on verbal working memory are in line with previous reviews (Peng et al., 2018), showing worse performance in children with MD (Geary et al., 1999; Keeler & Swanson, 2001; Passolunghi & Siegel, 2004; Fuchs et al., 2008; Chan & Ho, 2010; Passolunghi et al., 2011; Cai et al., 2013; De Weerdt et al., 2013a; Moura et al., 2013; Willcutt et al., 2013a; Attout & Majerus, 2015; Cirino et al., 2015; Kuhn et al., 2016; Lafay et al., 2017). Similarly, visuospatial working memory is impaired in children with MD (Keeler & Swanson, 2001; Chan & Ho, 2010; Cai et al., 2013; De Weerdt et al., 2013a; Szucs et al., 2013; Kroesbergen & Van Dijk, 2015; Kuhn et al., 2016; Mammarella et al., 2018; McDonald & Berg, 2018), in line with previous systematic reviews on this topic (Szűcs, 2016)

The limited working memory capacity of children with MD, linked to normal short-term memory, could indicate a specific difficulty in retaining information and simultaneously performing manipulations or operations (Geary et al., 1999). This difficulty would not emerge in tasks in which the cognitive load is lower, as in the direct memory span task, in which passive repetition of elements is required (Swason & Jergman, 2006).

Another finding of this review is the impairment in attentional control (Peng et al., 2012) and sustained attention over time (Willcutt et al., 2013; Kuhn et al., 2016) in children with MD. The difficulties in tasks requiring manipulating information, both verbal and visual, and involving attentional processes could explain the high comorbidity between attention deficit hyperactivity disorder (ADHD) and MD (Zentall et al., 1994; Lucangeli & Cabrele, 2006; Platt, 2017).

Deficits in working memory and attentional control could affect mathematical performance, especially in those tasks that require multistep planning and processing of information, such as occur in the case of MD (Raghubar et al., 2010; Peng et al. al., 2018).

Children with MD present normal inhibition abilities when assessed through the classic Stroop tasks (Censabella & Noël, 2007; Peng et al., 2012; Szucs et al., 2013 McDonald & Berg, 2018) or the Stroop task using numbers (Rousselle & Noël, 2007). However, they show greater difficulties in solving the Number Inhibition tasks (Peng et al., 2012; McDonald & Berg, 2018); in fact, they ignore the presented number, indicating only the quantity of digits (e.g., in the presence of the stimulus "444", they may say: three, referring to the number of digits rather to the quantity indicated by the number). An inhibition difficulty also emerged in the Stop-signal task that requires to inhibit a response (press a button) previously made automatic (Willcutt et al., 2013; Szucs et al., 2013; Cai et al., al., 2013). This impairment appears evident in some typical errors that children with MD commit in retrieving arithmetic tables (Barrouillet et al., 1997; Gilmore et al., 2017). However, the inhibition could also be linked to the ability to suppress ineffective strategies in favor of new, more efficient strategies, revealing high cognitive flexibility (Lemaire & Lecacheur, 2011). In fact, children with MD have more difficulty changing their response based on the demands of the context (Szucs et al., 2013; Willcutt et al., 2013; McDonald & Berg, 2018; Murphy et al., 2007; Chu et al., 2019), and this difficulty could affect children's ability to perform complex mathematical calculations in which it is necessary to go from one procedure (e.g., subtraction) to another (e.g., multiplication).

Furthermore, slowness in rapid naming tasks appears to be a common feature in children with MD (Chan & ho, 2010; Cirino et al., 2015; Donker et al., 2016; Willcutt et al., 2013), as it shares with arithmetic some basic processes, such as the rapid retrieval of phonological representations from long-term memory (Koponen et al., 2017). However, when compared to the alphanumeric RAN, the results appear to be mixed. According to Donker and colleagues (2016), the alphanumeric RAN would mainly involve phonological processing ability, but this skill was not examined in depth in their study. The only study that also evaluated phonological processing in addition to rapid naming (Willcutt et al., 2016) observed worse performance of children with MD in both tasks, supporting the Donker's hypothesis.

In the non-alphanumeric RAN, children with MD showed worse performance presumably because it involves elements related to the conceptual and perceptual processing of objects but also the ability to use the verbal and visual code interactively (Donker et al., 2016; Roelofs, 2006). Furthermore, it involves the retrieval of semantic information (Bruffaerts et al., 2019). Therefore, the non-alphanumeric RAN requires additional processes compared to the alphanumeric RAN, in which children with MD could be specifically and uniquely compromised (Donker et al., 2016). On the other hand, other authors (e.g., Kruk & Ruban, 2018) found that visual processes, such as visual discrimination, visual problem solving (e.g., reasoning tasks with matrices), and attention are central in non-alphanumeric RAN tasks.

Words have both a semantic and a phonological representation in the mental lexicon. Letters have phonological representations, but they do not have a meaning (Poulsen & Elbro, 2013). So, the naming of letters activates mainly phonological access; conversely, naming an image requires semantic access. Phonological and semantic accesses are two separate mechanisms that could independently contribute to mathematical skills. Specifically, the difficulty in non-alphanumeric RAN tasks could reflect difficulty integrating the visual-perceptive information (the image of the stimulus) with its semantic representation. It could also be interesting to verify whether children with MD have a specific difficulty in the RAN of numbers, reflecting the same difficulty of symbolization present in the RAN of pictures.

Consequently, the core deficit in children with MD could be linked to visual-perceptual discrimination and rapid scanning of visual information; this impairment would undoubtedly explain the difficulties in visuospatial working memory, and perhaps also those in verbal working memory.

This result, combined with those concerning visuospatial skills, would support the hypothesis of a "visual system" (Geary, 2004) necessary to organize and manipulate the visual information that should be involved in the procedural knowledge that allows achieving good math performance.

This finding, combined with those concerning visuospatial skills, would support the hypothesis of a "visual system" (Geary, 2004) necessary to organize and manipulate the visual information that should be involved in the procedural knowledge that allows achieving good math performance.

Limitations

Some cautions must be considered when interpreting the results of the present review.

First, some limitations concern the definition of the mathematical deficit itself. On the one hand, there is no agreement on which term better describes the condition (Butterworth, 2005; Szűcs & Goswami, 2013). On the other hand, the standardized measures to assess math achievement do not evaluate all the numerical, mathematical, and arithmetic domains that could be compromised in specific learning disabilities in mathematics. Moreover, there is no full consensus about the cut-off criteria. Consequently, identifying different deficits in mathematical learning disabilities (MLD) could be due to the variability of MLD definition and classification across studies.

Another limitation is the high variability of the tests used to evaluate cognitive domains that prevented from performing a meta-analysis of the results. Consequently, a limitation of this review is the lack of quantitative analysis that would have given greater force to the inferences through the effects-size analysis.

One additional limit could be publication bias. Some methodological choices allow defining rigorous inclusion criteria of the studies that did not lead to analyze some grey literature. The choice to include only academic articles published in peer-review journals may have limited the selection only of those studies that have obtained results in line with the literature.

Conclusions

Mathematical performance implies a series of numerical and mathematical skills that are strictly linked to some general cognitive abilities that, if impaired, may have a cascading effect on math learning. This systematic review was aimed to identify the most impaired cognitive functions in school-age children with MD. Despite some variability in the tests used to evaluate the various cognitive domains, the main findings revealed poor executive function performance, such as inhibition, flexibility, working memory, and processing speed.

Furthermore, this review highlights the need to develop a standardized protocol to assess the specific mathematical learning disability. This protocol should consider many mathematical skills representing this complex domain, including formal and informal competencies and general domain abilities like working memory, processing speed, and executive functions.

Future studies should better investigate the role in mathematical learning and disability of factors that have to date have received little attention, such as long-term memory and phonological awareness.

II. Unraveling Mathematical Competencies in School-Age Children: The Influence of Domain-General Abilities

Introduction

The systematic review regarding the cognitive functions involved in mathematical difficulties revealed some critical issues related to the construct investigated. Primarily it highlighted the great heterogeneity concerning the terms used to define impairment in mathematics (e.g., Developmental Dyscalculia, Mathematics learning disabilities, mathematical difficulties, mathematical disability). The same heterogeneity emerged in the cut-off criteria used to define deficits in this area (from below the 10th to below the 35th percentile).

Moreover, the studies included in the systematic review assessed a variety of different mathematical skills to define the group with difficulties.

The different degrees of severity of mathematical difficulties, and the different domain-specific abilities assessed raise doubts about the possibility to generalize the results achieved.

These critical issues in the field of research collide with the clinical practice that requires the adoption of unambiguous cut-off criteria and suggests assessing the domain-general skills that could impact the expression of the disorder.

Although significant progress has been made in clinical practice through the consensus documents produced with the support of the scientific community (Consensus Conference, 2011; AID-AIRIPA Agreement, 2012; New Guidelines for Specific Learning Disorders, 2022), the discrepancy between clinical practices and scientific research contributes to the difficulty in identifying the possible core deficits of dyscalculia.

Moreover, it was observed that children with dyscalculia show heterogeneous profiles in both domain-specific and domain-general deficits. That various profile of functioning was scarcely explored systematically, preventing to recognize the potential presence of subtypes of dyscalculia.

Considering these issues, there is an evident necessity to untie some knots related to mathematical ability.

We believe that to enhance our knowledge about deficits in mathematics, we need to take a step back, reasoning deeply about mathematical ability.

This represents a competence that encompasses several skills and that -globally- moves along a continuum of mastering the fundamental aspects of mathematical ability. Over the past decades, some scholars, including the research group of Niss (2016, 2019), have often questioned how to define mathematical competence.

Niss and collaborators (2016), in fact, initially differentiated between mathematical knowledge (such as formulas, theorems, methods and definitions) and mathematical "actions," that is, the enactment or execution of mathematical processes to respond to challenges involving mathematics in different situations and contexts. The latter aspect would represent the so-called mathematical competence, which had to be discerned from mathematical competency, which refers to the ability to solve problems involving different mathematical domains by choosing the appropriate strategy and activating the specific mathematical categories necessary to reach the solution (Niss & Højgaard, 2019).

Currently, clinical and research contexts, and even the educational environment, focus on factual knowledge of mathematics.

Despite knowing and understanding mathematical definitions, concepts, and procedures helping to build different competencies, there is the need to comprehend how individual formal learnings can combine itself to assemble mathematical domains that supposedly will underlie the development of competencies as described by the group of Niss.

Aim of the study

Considering the critical issues and reflections related to the construct of mathematics mentioned, this research aims to answer two questions:

1) *Which is the domain-specific abilities, defined as a set of individual factual knowledge, assessed in the clinical practice through the standardized instruments available?*

Answering this question would allow us to move toward an "alignment" of practices used in clinical and research. In the short term, this could have the positive effect of helping clinicians to interpret the impairments of children with specific learning disorder in mathematics more accurately. However, in the long term, it could encourage reflections in the scientific scene by leading the study of math difficulties to a different level.

2) Which cognitive functions predict performance in these mathematical domains?

Identifying the cognitive abilities that mainly contribute to mathematical performance had two main advantages. On the one hand, it could guide the diagnostic procedure of clinicians by orienting it in the choice of instruments useful to define the functioning profile of children with dyscalculia. On the other hand, it could provide a starting point for identifying possible subtypes of dyscalculia, in which domain-general impairments may explain or support domain-specific impairments.

Materials and Method

Participants

Seventy-one children (31 female; 40 male) aged 9-13 years (mean age = 10.91 years; SD = 1.15) attending a primary (4th and 5th grades) and secondary (6th and 7th grades) school in central Italy (city of Gaeta, Lazio region) participated in the study.

Three participants were excluded due to certified neurodevelopmental disorders such as ADHD (N=2) and Specific Learning Disorder-Mixed Type (N=1).

Therefore, the final sample is composed of 67 children (30 female; 37 male) with a mean age of 10.87 years (SD= 1.17). All children included in the final sample have neither a diagnosis of Specific Learning Disorder nor the presence of suspected difficulties in mathematics, as referred by parents.

Instruments

To assess mathematical skills, the tests from the BDE-2 (Biancardi et al., 2016) and ACMT-3 (Cornoldi et al., 2020) batteries were administered. Both the batteries are standardized Italian instrument used in clinical practice to screen for and diagnose a developmental dyscalculia assessing the different numerical and arithmetical abilities in children between 8 and 13 years old.

From the BDE-2 battery the following tests were proposed: Counting (counting forward and then backwards in an interval of time, the score is the number of numbers counted backwards); Number reading (reading numbers aloud); Multiplication by the mind (retrieving arithmetic facts, such as multiplication tables); Mental calculation (addition and subtraction to be solved by mind); Written operations (addition, subtraction, and multiplication to be solved in writing); Inserts (placing a target number respecting the ordinality of numbers pre-filled); Approximative calculation (indicating among four alternatives the correct solution operations; it requires to estimate the result and not to solve it by mental computation).

In addition, some subtest from the AC-MT3 battery were administered. These include the Computational Fluency task (solving simple written operations quickly and accurately); Mathematical Inferences (solving inferential mathematical reasoning tasks); Numerical Matrix (solving reasoning tasks about numerical properties).

The tests from the BDE-2 are the same for each grade, while the ACMT-3 tests consist of items differentiated by grade with increasing difficulty.

A series of test or subtests from different batteries were used to assess different cognitive abilities.

To assess **processing speed** was proposed the *Parrots* subtest of the Intelligence and Development Scales- 2nd edition (IDS-2; Grob & Hagmann-von Arx, 2018; Italian version by Ferri et al., 2022).

Regarding the **memory domains**, the *Shape Memory* subtest of the IDS-2 battery was administered to assess the short-term visuospatial memory; while the recognition of the Rey-Osterrieth Complex Figure (Rey, 1941; Italian validation by Caffarra et al., 2002) was used as a measure of long-term visuospatial memory. The *Digit Span forward and backward* from the WISC-IV were used to assessing short-term verbal memory and working memory. While the *recall of a Word List* previously presented (Nepsy-II battery) was used to assess long-term verbal memory. The C*opy of the Rey-Osterrieth Complex Figure* was adopted to assess the **visual-perceptive skills**.

Finally, some computerized tests were used from Inquisit Millisecond Library to assess executive function: t**he cognitive inhibition** ability was assessed through a Color-Word Stroop Task (Stroop, 1935), while the **shifting ability** was evaluated with the Color Shape Task (Miyake et al., 2004). Finally, the **planning and problem-solving** skills were evaluated using the computerized version of the Tower of London task (Shallice, 1982). All the tasks were in the library in English, so the instructions were translated into Italian so as to be understandable to the participants.

Procedure

The research was conducted according to the Helsinki Declaration and was approved by the Local Ethics Committee (Department of Dynamic, Clinical Psychology and Health Studies, University of Rome "Sapienza"; Prot. N. 0002103; 13/12/2022) and by the authorities of the school involved.

The research was proposed to the parents by teachers and comprised two 4th and 5th grade classes, and three middle school classes (one 6th grade, and two 7th grade).

Only children whose parents provided written consent participated in the study. In addition, parents filled out a brief anamnestic form to collect demographic information (date of birth; language spoken at home), the potential presence of diagnosed neurodevelopmental disorders, and suspected difficulties in learning,

The overall assessment, lasting about 2 hours, included three sessions of 40 minutes each. One session was collective, while the last two were one-to-one.

All sessions were carried out during school time in a quiet, well-lit room placed at the disposal by the school.

The first session was the collective one and involved a presentation of the step of the project to participants and the administration of some timed tests from the BDE-2 battery (Approximate Calculation, Inserts; Rapid Calculation).

The second session involved individual administration of the mathematical and cognitive paper-and-pencil tests, while in the last one, the computerized tasks (Classic Stroop; Switching Task; Tower of London) were proposed.

As a symbolic reward for participation, children received stickers to attach to their "Researcher's Badge."

Data Analysis

Data analyses were conducted through Jamovi software. The scores performed by the participants in each cognitive and mathematical test were transformed into z-scores.

First, correlation analysis was conducted considering age and mathematical variables.

Next, an exploratory factor analysis (EFA) was conducted to identify a small number of mathematical domains (i.e., latent factors) that could explain the variability in performance on each mathematical test. The score of participants on the mathematical test was converted into factorial scores based on the factors loading of each variable on each latent factor that emerged through EFA.

Then, a correlational analysis was conducted considering age, factorial scores of mathematical dimensions (Appendix B1), and performance on cognitive tasks (Appendix B2).

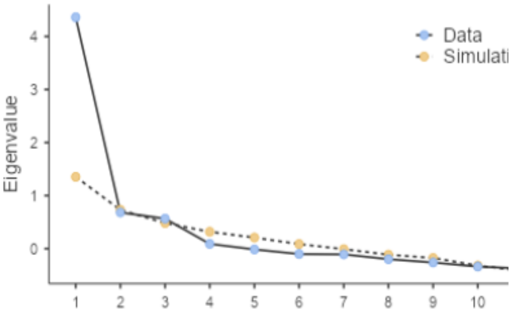
Finally, a series of hierarchical multiple regressions were conducted considering as dependent variables the mathematical factors identified by the EFA and as predictor variables 1) the age and 2) the cognitive tasks.

Results

The correlational matrix considering age and the score on mathematical measurements (Appendix B1) show that age is positively and significantly correlated with most of the mathematical performance ($p < 0.02$). There is not a significant correlation exclusively between age and Written Operation, Counting; and Mathematical Inferences ($p > 0.05$). Through the scree test, there were identified three latent factors (Figure 1).

That factorial structure was confirmed through the Maximum likelihood extraction method

Figure 1. Scree Test



used in combination with an OBLIMIN rotation.

Based on the saturation of each variable on the factor, it was possible to identify three different mathematical domains that explain about 56.4% of the total variance (Table 1).

Factor 1 contains all those tests that require speed and accuracy of computation (Mental Calculation; Rapid Calculation; Approximate Calculation; Computational Fluency); Factor 2 includes all those tests that require some the acquisition of basic mathematical skills (Counting; Multiplication Tables; Written Calculation; Number Reading); while the third and final factor encloses the tests that measure mathematical reasoning ability (Mathematical Matrix and Inferences).

For each domain, a factorial score was computed: the raw scores were summed based on their factor loading on each factor (weighted sum). However, we intended to consider only the secondary saturation identified in the factorial structure therefore, only the variables with a secondary saturation greater than .30 were summed to define the factorial scores.

Table 1.. Saturations and communalities after oblique rotation

	OBLIMIN Rotation			
Variables	F1	F2	F3	*h*
Counting	-.06	**.72**	.01	.52
Multiplication tables	.45	**.50**	-.18	.39
Mental Calculation	**.51**	.02	.47	.32
Written Calculation	-.14	**.41**	-.12	.88
Number reading	.03	**.80**	.10	.27
Rapid Calculation	**.87**	.11	.03	.12
Inserts	.24	.14	.15	.83
Approximate Calculation	**.63**	.02	.44	.19
Computational Fluency	**.91**	-.03	-.11	.27
Mathematical Matrix	.03	.28	**.49**	.58
Mathematical Inferences	-.05	.06	**.75**	.43
% of Variance	25.5	17.3	13.3	
Cumulative %	25.5	42.8	56.4	

The factorial scores so calculated were used as dependent variables in a series of regressions in which the different cognitive functions were considered as predictor variables.

Since the correlation matrix showed that each mathematical domain significantly and positively correlated with age (Table 2), also this variable has been included as a predictor.

Table 2. Correlational matrix considering age and each mathematical domain.

		1	**2**	**3**	**4**
1. Age	r	—			
	p	—			
2. Computational Ability	r	.81	—		
	p	< .001	—		
3. Basic Mathematicl Skills	r	.42	.58	—	
	p	< .001	< .001	—	
4. Mathematical Reasoning	r	.46	.72	.44	—
	p	< .001	< .001	< .001	—

Since this study aimed to identify the relationship between the cognitive functions (i.e., predictors) and mathematical domains, it was considered appropriate to control for variability due to age using a hierarchical regression model.

Thus, in the analyses considering the specific mathematical dimensions as the dependent variable, two blocks of predictive variables were included: age was included in the first block (Model 1), and all the cognitive variables were entered into the second block (Model 2).

Results showed that the first model was significant in all mathematical domain (Table 3). Age is significantly associated with Computational Ability (ß= .81; t= 10.5; p < .001), Basic mathematical skills (ß= .40; t= 3.30; p = .002), and Mathematical reasoning (ß= .44; t= 3.75; p < .001).

Table 3. Hierarchical Multiple regressions considering age as predictor (model 1)

Block 1	**Computational Ability**			**Basic mathematical skills**			**Mathematical Reasoning**		
Predictor	**ß**	**SE**	**p**	**ß**	**SE**	**p**	**ß**	**SE**	**p**
Age	**.81**	**.19**	**<.001**	**.40**	**.16**	**.002**	**.44**	**.16**	**<.001**
R^2	.65			.16			.19		
R^2 Adjusted	.65			.14			.18		
F $_{(1,58)}$	109.4			10.88			14.08		
p	<.001			.002			<.001		

Including the second block of variables (Table 4), which comprises the different cognitive domains, is observed a significant improvement over Model 1 only for the dimensions of

Basic mathematical skills ($\Delta F_{12,46}$=3.06; ΔR^2 = .37; p = .003). Therefore, the inclusion of cognitive variables in the model (model 2) explains about 37% more variance than in model 1, with a total of explained variance corresponding to 41% (R^2 Adjusted = .41; $F_{12,47}$ = 4.39; p < .001).

In particular, may be observed (Table 5) that in Model 2 some cognitive variables are significantly and positively associated with Basic mathematical skills: these are verbal short-term memory (ß= .26; t= 2.29; p = .03) and working memory (ß= .32; t= 2.74; p = .009); while the execution time at the Tower of London is negatively associated (ß= -.54; t=-3.61; p < .001), Compared to the other two mathematical domains, the additional contribution of cognitive variables does not significantly improve the first model (Comparison model 1 vs model 2 = p > 0.5).

However, some results of the model including cognitive variables appear of interest.

In particular, the second model considering Computational ability as the dependent variable maintains a significant and strong effect of age (ß= .76; t= 5.12; p < .001) indicating that as Computational Ability increases by one unit, age increases by 1.82 units, corresponding to a standardized effect of 0.76.

Finally, concerning Model 2 which considers Mathematical Reasoning as the dependent variable, by including the cognitive variables (Block 2), age is no longer significant (ß= .76; t= 5.12; p < .001). However, a significant effect of working memory occurs (ß= .33; t=2.39; p=.02).

Table 4 Model fit and model comparisons of hierarchical multiple regressions

Model Fit*		**R^2**	**R^2 Adj**	**ΔR^2**	**F $_{(df)}$**	**p**
Computational ability	Model 1	.65	.65	-	$F_{1,58}$ =109.4	<.001
	Model 2	.75	.69	-	$F_{12,47}$ =11.8	<.001
	Comparison	-	-	.10	$F_{11,47}$ =1.68	.11
Basic mathematical skills	Model 1	.16	.14	-	$F_{1,58}$ =10.88	.002
	Model 2	.53	.41	-	$F_{12,47}$ =4.39	<.001
	Comparison	-	-	.37	$F_{11,47}$ =3.36	.002
Mathemetical Reasoning	Model 1	.19	.18	-	$F_{1,58}$ =14.08	<.001
	Model 2	.35	.18	-	$F_{12,47}$ =2.08	.04
	Comparison	-	-	.15	$F_{11,47}$ =0.99	.47

**Model 1 include only age as predictor; Model 2 includes age and cognitive variables as predictors*

Table 5. Hierarchical Multiple regressions considering age and cognitive variables as predictors (model 2)

Domain	**Predictors**	**Computational Ability**			**Basic mathematical skills**			**Mathematical Reasoning**		
		ß	**SE**	**p**	**ß**	**SE**	**p**	**ß**	**SE**	**p**
Control variable	Age	**.76**	**.36**	**< .001**	-.28	.28	.17	.34	.33	.16
Processing Speed	Parrots	-.07	.45	.63	.34	.35	.07	-.02	.42	.93
Verbal Memory	Digit Span Forward	.13	.23	.13	**.26**	**.18**	**.03**	.03	.21	.83
	List Memory – Delayed	-.05	.24	.54	-.23	.19	.051	.07	.22	.59
Visuo-Spatial Memory	Shape Memory	.06	.27	.49	.10	.21	.37	-.01	.25	.95
	Rey's Figure- Recall	.12	.34	.31	.29	.26	.08	-.13	.31	.49
Visuo-perceptual integration	Rey's Figure - Copy	-.02	.38	.86	-.18	.30	.32	.19	.36	.35
Working Memory	Digit Span Backward	.17	.25	.052	**.32**	**.20**	**.01**	**.33**	**.23**	**.02**
Inhibition	Stroop Effect (TR)	.16	.23	.06	.11	.18	.34	.18	.21	.19
Switching	Switch Cost (TR)	.03	.21	.68	.04	.16	.69	.13	.19	.32
Planning and problem solving	Tower of London – Correct.	-.12	.23	.14	-.16	.18	.15	-.11	.21	.42
	Tower of London – Execution time	-.08	.32	.49	**-.54**	**.25**	**< .001**	-.01	.30	.94

Discussion

Today is widely recognized that developing good competencies in mathematics plays a central role in academic and professional success, as well as everyday functioning. The main intention of this study is to reflect on the construct of mathematics, keeping in mind some characteristics of this discipline that often make its operationalization difficult.

One aspect that we think is essential to consider is the multitude of specific skills that represent the broader construct of mathematics. In clinical contexts, numerous tests were administered to assess domain-specific skills. Considering these various tests, clinicians can calculate quotients to define the level of impairment in each mathematical area. However, the mathematical areas identified through these standardized instruments only conceptually estimate the same mathematical component.

This observation is partially confirmed in the guidelines provided to clinicians to accurately identify children with dyscalculia (AID-AIRIPA Agreement, 2012). This document states that the hypothesis of dyscalculia should only be considered in the presence of critical scores in at least 50% of the tests in a sufficiently representative battery.

Leaving aside the definition of a "sufficiently representative battery," let us focus instead on the need to have critical scores in half of the proposed tests. In our opinion, the main problem with this indicator is that "the proposed tests" in most batteries represent only a series of formal skills and knowledge. This would not be a problem in itself if the relationships between the ability assessed by each test and the mathematical "area" (or competency) estimated through them are interpreted.

Without a clear definition of the mathematical area (and not of the specific ability or test), we could only identify a significant impairment in isolated mathematical skill and not a specific deficit in a mathematical domain or competency.

This means that to define a dyscalculia profile, it may potentially be sufficient to identify a poor performance in many tests that evaluate similar skills, which can be more or less directly influenced by other domain-specific or even domain-general skills.

Therefore, while acknowledging the significant work and contribution that agreement and consensus documents have provided to clinical practice in the short term and daily life, it is believed that to increase our understanding of a phenomenon such as dyscalculia, on which there are still many areas of uncertainty, is necessary to switch from a quantitative approach (e.g., focused on the number of critical scores) to a qualitative approach.

In this sense, a qualitative approach should focus more on understanding the relationships between each mathematical skill assessed to identify areas of competency that, if compromised, interfere with their mastery.

This study fits into this complex framework.

The first step to changing to a more qualitative approach is to identify the latent dimensions of standardized tests used in clinical practice. The factor analysis conducted for this purpose revealed the presence of three latent factors that we defined as Computational Ability, Basic mathematical skills, and Mathematical Reasoning.

Because we think that it is crucial to deeply understand what is exactly assessed when mathematical tasks are administered, it seems appropriate to delve into each domain and explain the latent factors behind them.

Regarding Computational Ability, this dimension explains the highest percentage of variance compared to all other domains (25.5%). In this domain, primary saturations of Computational Fluency, Rapid Calculation, Approximate Calculation, and Mental Calculation tests are observed. All of these tests share the fact that they measure the ability to solve a series of operations quickly and accurately using knowledge related to the properties of operations (i.e., commutative and distributive properties) and the retrieval of arithmetic facts. These abilities are crucial for developing competence in the calculation area as they simplify the performance of operations and support subsequent learning (Russel, 2000; Ding et al., 2021).

The second mathematical domain identified shows primary saturations in the Number Reading, Counting, Multiplication Tables, and Written Operations tests and has been defined as a factor that measures Basic mathematical skills.

This domain is defined by specific-domain abilities that relate to the acquisition and understanding of basic skills such as transcoding Arabic numbers to verbal numerals (Reading numbers), counting rapidly and accurately (counting backwards), automatization of arithmetic facts retrieval thanks to consolidation of the association between operators and results (Rapid Calculation), and the ability to solve simple written operations, due to knowledge related to both the alignment in columns method and the algorithms needed to reach the solution (e.g., carrying and borrowing; Written operations).

These competencies are critical to build overall mathematical competence as they provide the basement and the instruments to understand and apply more advanced skills (Clements & Sarama, 2011; Nguyen et al., 2016; Moura et al., 2022; Geary, 2011; Jordan et al., 2009).

The third mathematical domain identified is the easiest to interpret; in fact, it comprises performance in the Numerical Matrix and Mathematical Inferences tests. Both tests require one to perform mathematical reasoning starting from number series (*What number completes the series?* 60; 57; 50; __) or starting from knowledge related to mathematical symbols (*What operation that considers 6 and 5 as terms gives 30 as a result?* 6 _ 5 = 30) and the principles of arithmetic (*Knowing that* 3x4 = 42. *How much will do* 14 + 14 + 14 = ___?). The results relative to this domain seem particularly interesting to us because the literature when talking about Mathematical Reasoning mainly refers to arithmetic problem-solving (Lithner, 2000; Herman, 2018). However, in our study, this domain does not involve all aspects related to arithmetic problem solving (e.g., text comprehension, data extraction, generalization of strategies, etc.). On the contrary, the competency of Mathematical Reasoning identified in this study represents the ability to enact a logical-mathematical process starting exclusively from numerical information.

Finally, it seems worth mentioning that among the proposed tests, the Inserts task had no saturation values (<.30) in either of the factors. This result may have depended on the fact that the proposed test did not cover all the existent mathematical domains. In fact, among the proposed mathematical task those considered to assess skills such as number sense and transcoding ability were limited (Inserts and Number Reading, respectively).

The second aim of this study is to identify the domain-general skills that can explain performance in mathematical competency defined.

Since our sample consisted of children ranging from 9 to 13 years old, it seemed appropriate to consider the impact of age on mathematical performance. We observed a significant and positive correlation between the age of participants and most of the mathematical tasks. Similarly, the validation and standardization manual of the BDE-2 battery also reported a significant increase in performance with increasing age in most of the tests used in this study (except for the Written Calculation task).

Therefore, to investigate the cognitive functions involved in mathematical performance, hierarchical multiple regressions were conducted, which showed that age is an important predictor variable of competence in each mathematical domain, explaining about 65% of the variance in the Computational Ability domain and 18% of the variance in the Mathematical Reasoning domain. When cognitive variables were also included in the regression, they did not significantly increase the percentage of explained variance.

However, while age is the only variable that continues to contribute significantly to performance in the Computational Ability competency, in the Mathematical Reasoning domain, an additional contribution of working memory emerges when cognitive variables are considered.

These results show that in typical development the increase in age and school experience improves some domain-specific mathematical skills.

In particular, regarding Computational Ability, this competency especially involves arithmetic knowledge, which gradually improves from the 4th grade onwards (Shalev et al., 1993). The evidence that age contributes to explaining a large portion of the variance in this domain is due to the acquisition and consolidation of arithmetic principles and operation properties (e.g., factual knowledge of arithmetic), which tends to increase with practice and schooling. Similarly, children who frequently solve the same simple operations (i.e., 7 + 5) tend to memorize a series of basic number facts (Baroody, 2006), contributing to achieving good computational fluency.

The factual knowledge and the acquisition of fluency in performing simple calculations allow children with typical development to choose the most suitable strategy among the various acquired to solve a precise calculation task (Peters & De Smedt, 2018; Geary, 2011).

Also in the Mathematical Reasoning competency, age and schooling contribute to explaining the majority of the variance. However, the model that additionally considers cognitive variables shows a significant contribution of working memory. The role of working memory in Mathematical Reasoning may lie in the fact that the tasks defining this competency are less familiar and more complex, involving the central executive to a greater extent (Geary, 2011).

Finally, for the domain defined as Basic arithmetic skills, the cognitive variables explain about 37% additional variance than the age variable alone, significantly increasing the predictive ability of the model. Specifically, the tests that significantly predict Basic mathematical skills are short-term memory, working memory and execution time at the Tower of London (the latter with a negative direction).

Both short-term memory and working memory play a central role in the development of Basic mathematical skills because they are involved in the processing, maintenance, and active manipulation of numerical information needed to perform intermediate steps and respond to task demands (Swanson & Kim, 2007). In this sense, intermediate steps can be both the steps required to solve a 'written operation (e.g., carry and borrow), but also the identification of the positional value of a digit to be able to apply the lexical and syntactic rules of the numerical code (reading numbers). Time of execution at the Tower of London enters into this process since it represents how quickly an individual enacts a series of steps (e.g., moves) previously generated and kept in mind during the execution (Krikorian et al., 1994).

Conclusion

In conclusion, this study shows a different perspective to approach the not-simple construct of mathematics.

The results obtained regarding the different mathematical domains help us in understanding how domain-specific abilities are interrelated to form a set of competencies that go beyond the simple knowledge of facts, formulas, and procedures (often emphasized in diagnostic and educational contexts).

Therefore, it is important to realize that domain-specific skills may not be assessed (and interpreted) separately but must be considered within their reciprocal relationships. This awareness could positively affect clinical settings, enabling specialists to gain a comprehensive understanding of the functioning profile of children with dyscalculia.

However, it should be remembered that these skills represent only a part of the more comprehensive mathematical competence. For this reason, it is essential to further examine this complex construct and understand all of its facets and connections. That will enable us to develop a more comprehensive view of mathematical competence and provide more effective support for children with deficits in this area.

Limitation and future perspectives

The present study addresses important issues concerning mathematical competencies and cognitive variables; however, it has some significant limitations and offers some consideration for likely future perspectives.

One of the main limitations of this study concerns the small sample size. This restriction did not allow multiple regression analyses to be performed on the different age or class groups to assess the association between the various mathematical domains and cognitive variables. For this reason, in the future, it would be hopeful to extend the sample, so that the role of cognitive variables on individual mathematical domains can be explored in children at different developmental stages.

Another aspect to consider is the selection of the mathematical tests included in the study. Even if the tests proposed were chosen among those commonly used in clinical practice, some mathematical domains may not have been identified because of their inadequate representation in the study. These include, for example, the area related to number sense and transcoding abilities.

For a complete understanding of mathematical skills, it would be interesting to include standardized tests covering additional domains in the procedure. In this way, it would be possible to investigate how specific latent factors are expressed and adjusted by the tests considered.

Finally, it should be noted that the sample used consisted exclusively of children with typical development (with only one participant meeting the diagnostic criteria for dyscalculia). Therefore, to deepen knowledge about the specific learning disorder in mathematics, it would be appropriate to replicate the study on a clinical population with a diagnosis of dyscalculia. This future research could offer worthwhile insights into the mathematical skills particularly impaired in children with dyscalculia.

In conclusion, despite the limitations presented, future perspectives could offer an opportunity to broaden our understanding of the interactions between cognitive variables and mathematical competencies in different age groups through larger samples and the inclusion of various standardized tests. In addition, the inclusion of a clinical population diagnosed with dyscalculia would allow the identification of the most impaired mathematical skills in these individuals, providing useful information for clinical practice and research.

III. Evaluating Cognitive Predictors of Mathematical Proficiency in Young Healthy Adults: A Systematic Literature Review

Introduction

Mathematical competence is critical to the academic and vocational success of young adults. The ability to deal confidently with complex mathematical tasks influences academic choices and is crucial in several aspects of daily life and career choices (Parsons & Bynner, 2005; Vigna et al., 2022). In this context, the scientific literature sought better to understand the link between cognitive abilities and math achievement and to identify the cognitive skills that contribute most to performance. Indeed, studying the cognitive abilities that underlie math performance is a significant step in understanding which processes may support (or impede) math proficiency.

Our previous systematic review on school-age children (Agostini et al., 2022; Chapter 1) highlighted how children with math difficulties perform worse on tasks that assess cognitive skills, such as processing speed, working memory, inhibition, and cognitive flexibility. Impairment in these areas could result in a cascading effect on domain-specific abilities, leading to difficulties in learning mathematics.

Since some impairments identified in children with developmental dyscalculia seem to persist into adulthood (Attout et al., 2015), the present systematic review aims to analyze the numerous studies conducted in the international arena to synthesize current knowledge on the relationships between cognitive abilities and mathematical problem-solving skills in young adults to identify cognitive abilities that contribute to predicting mathematical competence.

Method

The systematic review was conducted in accordance with the PRISMA method (Preferred Reporting Items for Systematic Reviews and Meta-Analyses; Page et al., 2020)

Research Strategies

The systematic search of the international literature was conducted until October 19, 2022 on the following electronic databases: PsycArticles; PsycINFO; Scopus; and Web of Science. Results were limited to English-language articles and academic publications. The search was conducted using the following script on each database: (("Math* performance" OR "Math* skills" OR "Math* ability*") AND (Cognitive Function*)) and produced a total of 29,850 records. Duplicates were eliminated using Mendeley software (N=14,796), after which 15,081 records were examined according to title and abstract.

Elegibility Criteria

The articles were selected by two independent researchers, with any doubt solved by a third supervisor (M.C.).

An initial selection of studies was made based on reading the title and abstract of the records. At this stage, records had to meet the following inclusion criteria to be eligible: (a) involve healthy participants (i.e., without medical or psychiatric conditions) with an age of at least 18 years; (d) experimental studies with behavioral measures; (b) assessment of at least one of the following cognitive domains: processing speed, inhibition, flexibility, problem-solving, working memory, long-term memory, and short-term memory; (c) evaluation of the mathematical abilities of participants.

The initial search produced a total of 29,850 items. After excluding 14,796 duplicates, 15,081 articles were analyzed based on title and abstract. During this phase, all studies that did not meet the inclusion criteria were excluded. So, records that involved a sample with an age of less than 18 years, i.e., preschool or school-age children and adolescents (n=5403); articles that addressed topics not relevant to the subject of this study, such as animal model studies, genetic studies, or studies of pedagogy and teaching (n=4,982) were excluded. In addition, studies that did not investigate any cognitive domains or that investigated cognitive domains other than those of interest to us (i.e., attention, language, praxis components; mental rotation or visual-perceptual abilities) were excluded (n = 1,940), as well studies that did not investigate any mathematical skills or those that investigated more complex and specific mathematical skills, such as algebra, statistics, probability, or principles of physics or geometry (n = 1,102). In addition, 139 records were excluded because they investigated mathematical skills and cognitive functions by analyzing the neural networks involved in these processes, using invasive protocols and tools (e.g., neuroimaging; ERP, TMVS). Records that focused on clinical populations such as ADHD, Parkinson's, Alzheimer's, Broca's Aphasia, or Fragile X were also excluded (N =41). Finally, literature reviews (n=528), meta-analyses (874), and other types of nonexperimental studies (such as book chapters, theoretical issues, or commentaries; n=20) were excluded.

Following the exclusion of 15,028 records based on title and abstract, the remaining 52 records were evaluated for eligibility based on full-text.

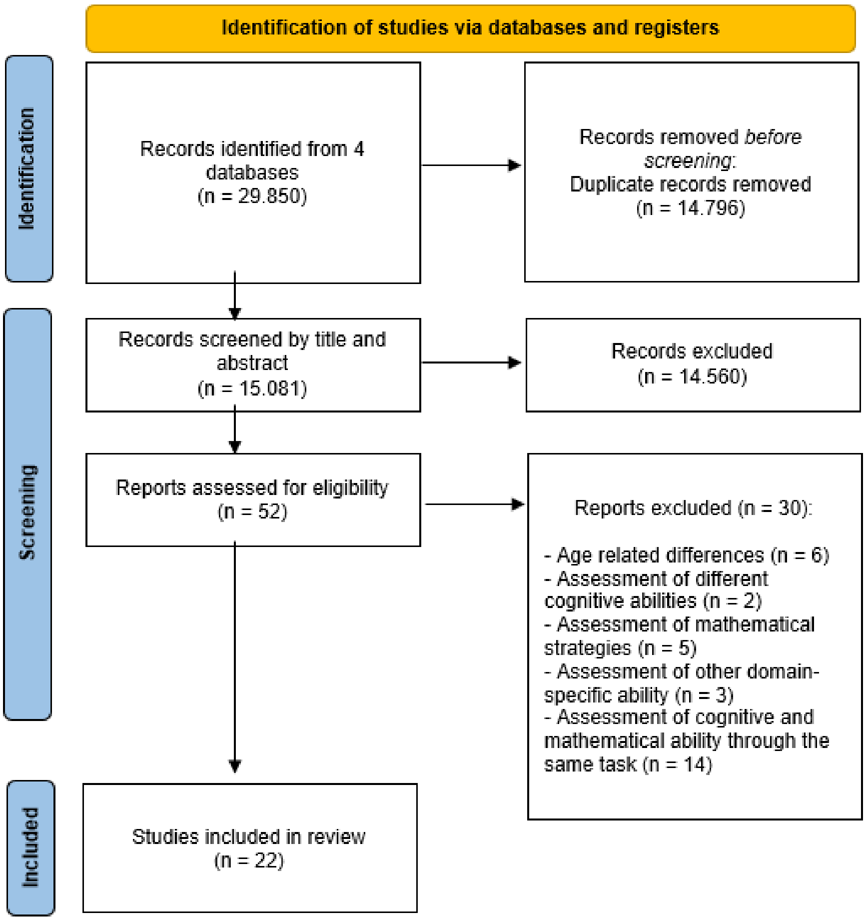
At this point, to be included in this systematic review, the studies had to meet the following eligibility criteria in addition to the abovementioned inclusion criteria: 1) clearly report the instruments used to assess both the mathematical and cognitive domains; 2) each cognitive or mathematical domain had to be assessed through an isolated task; 3) report findings regarding the relationship between cognitive and mathematical abilities.

In light of the following eligibility criteria, of the 52 full texts evaluated, an additional six articles were excluded because they focused on age-related differences regarding math achievement, while five articles were excluded because they focused on strategies used to solve math tasks. Two more studies were excluded because they assessed different cognitive skills than those included in this review (e.g., encoding ability and spatial coding). Finally, 14 studies were excluded because they assessed cognitive skills through the use of numerical tasks that, therefore, would not have allowed us to split the two processes (cognitive and mathematical) and understand their relationship. Finally, three studies were excluded because they focused on the assessment of domain-specific abilities related to basic numerical processing (e.g., Approximate Number System; SNARC effect)

Thus, a total of 22 articles were included in the qualitative analysis.

Figure 1 shows the study selection process.

Figure 3. PRISMA flow-diagram of the selection process



Data Collection and Quality Assessment

The study selection was conducted independently by two researchers (G.P. and F.A.); any doubt was resolved by a supervisor (M.C).

Data for the 22 articles included in this systematic review were extracted following the PICOS approach (Page et al., 2020). Appendix C1 shows the extracted data for the included studies. Specifically, the following information is reported for each article: author and year of publication, study design, characteristics of participants (such as gender, mean age, and standard deviation), criteria used to define groups, mathematical skills investigated, cognitive functions assessed, tasks used, and results.

The quality of the studies was assessed using the criteria of the Cochrane Handbook for Systematic Reviews (Higgins et al., 2011), adapted to the specifics of this review. The domains considered for the assessment of quality and risk of bias are as follows: (a) sample bias (clear definition of the sample, with reported information about age and gender, and criteria for defining the group with mathematical difficulties, if any); (b) clear definition of the instruments used to measure mathematical performance; (c) clear definition of tasks or tests used to assess the cognitive domains ; (d) confounding factors potentially handled; (e) selective reporting of outcomes in the discussion; and (f) other potential risks of bias. Study quality was categorized as uncertain/low/high risk of bias for each domain (with "0" indicating low risk of bias, "2" high risk of bias, and "1" uncertain risk of bias). An average score was calculated for each study, multiplied by 100. Next, studies were categorized as either low risk of bias (score below 75%) or high risk of bias (score above 75%). Finally, if at least two domains were uncertain, the study was categorized as uncertain risk of bias.

Results

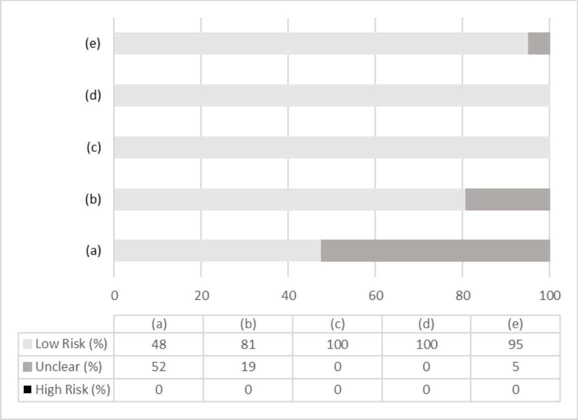
Studies Selection

The systematic review was conducted on four electronic databases (PsycArticles; PsycInfo; Scopus; and Web of Science) and produced 29,850 records. After the elimination of duplicates (N=14,796) and screening based on title and abstract, 53 articles were evaluated for eligibility, of which 22 studies were included in the qualitative analysis

Quality Assessment

Figure 2 shows the percentage of articles fulfilling each quality criterion assessed. All the studies had a generally good quality, with an average risk of bias lower than 75%. The high percentage of studies with low (45.5%) or no risk (36.4%) of bias highlights the validity of this systematic review. No study reports a high risk of bias, while three studies (13.6%) showed an unclear risk of bias. The Sample Characteristics domain (a) is the one that produces some concern of risk of bias, and it was due to the about half of the studies (52%) that do not report detailed gender or age information.

Figure 4. Percentage of risk of bias for each domain of tool assessment for the selected studies.



	(a)	(b)	(c)	(d)	(e)
▢ Low Risk (%)	48	81	100	100	95
■ Unclear (%)	52	19	0	0	5
■ High Risk (%)	0	0	0	0	0

Characteristics of selected studies

Regarding the 22 studies included in this systematic review, the average age of the samples varies from a minimum of about 18 years (Wilson & Swans, 2001) to a maximum of about 27 years (Osmon et al., 2006); however, some studies report only the age range of the participants.

The range most represented in the included articles is 18-29 years (He et al., 2021; Zeleznik et al., 2022; Dowker & Sherida, 2022). Only the study by Miller and Bischel (2004) reports a wider range (e.g., 18-66 years), while the study by He and colleagues (2021) also includes participants younger than 18 years (e.g., 16 years). Only one study (Laski et al., 2015) does report neither mean age nor age range but includes undergraduate students.

Regarding gender distribution in the sample, five studies report any information (Wilson & Swanson, 2001, Orrantia et al., 2019; Wang & Carr, 2020; Reynvoet et al., 2021; Silver et al., 2022), while the remaining studies report a prevalence of female gender ranging from 40 to 60% (Osmon et al., 2006, Laski et al., 2015; Cragg et al., 2017; Skagerlund et al., 2019; Chemerisova, & Martynova, 2019; He et al., 2021; Dowker, & Sherida, 2022) or above 60% (Miller & Bischel, 2004; Vallée-Tourangeau, 2013; Gilmore et al., 2015; Norris & Castronovo, 2016; Goffin & Ansari, 2016; Van den Bussche, et al., 2020; Coulanges, et al., 2021; Zaleznik, et al., 2022). In the study by Hoffmann and coworkers (2014), the percentage of females in the control group and the group of mathematics experts was around 50%, while in the group with mathematics difficulties, the percentage of females was above 85%. Finally, in the study by Berkowitz and coworkers (2022), the percentage of females in the sample is 14%.

Concerning the mathematical skills investigated, most studies (15) studies assess performance in computational tasks (Wilson & Swanson, 2011; Miller & Bichsel, 2004; Osmon et al., 2004; Vallée-Tourangeau, 2013; Hoffmann et al., 2014; Norris & Castronovo, 2016; Goffin & Ansari, 2016; Orrantia et al., 2019; Skagerlund et al., 2019; Chemerisova & Martynova, 2019; He et al., 2021; Reynvoet et al., 2021; Coulanges et al. 2021; Zaleznik et al., 2022; Silver et al., 2022). Seven studies assess the ability to apply mathematical knowledge to solve simple arithmetic problems (Miller & Bichsel, 2004; Osmon et al., 2006; Cragg et al., 2017; Van den Bussche et al., 2020; Reynvoet et al., 2021; Silver et al., 2022; Dowker & Sherida, 2022), while TOT studies assess different aspects related to the area of number sense, such as the Number Line Estimation (Laski et al., 2015;) or the Approximate Number System (ANS; Norris & Castronovo, 2016; Silver et al., 2022) or number processing (Orrantia et al., 2019; Skagerlund et al., 2019; He et al., 2021; Reynvoet et al., 2021; Coulanges et al., 2021; Zaleznik et al., 2022).

Finally, regarding cognitive abilities, most studies (63.6%) assessed working memory in its verbal component (Vallée-Tourangeau, 2013; Orrantia et al., 2019; Skagerlund et al., 2019; Chemerisova & Martynova, 2019; Dowker & Sherida, 2022), visuospatial (Hoffmann et al.,2014; Goffin & Ansari, 2016; Coulanges et al. 2021) or both (Wilson & Swanson, 2001; Miller & Bichsel, 2004; Cragg et al., 2017; Wang, & Carr, 2020; Zaleznik et al., 2022; Berkowitz et al., 2022). In contrast, 45.45% of studies focused on assessing inhibitory control (Gilmore et al., 2015; Laski et al., 2015; Norris & Castronovo, 2016; Goffin & Ansari, 2016; Cragg et al., 2017; Orrantia et al., 2019; Van den Bussche et al., 2020; Reynvoet et al., 2021; Coulanges et al. 2021; Silver et al., 2022). Processing speed was assessed by 18% of studies (Osmon et al., 2006; Vallée-Tourangeau, 2013; Hoffmann et al.,2014; He et al., 2021), and cognitive flexibility by 13.6% (Vallée-Tourangeau, 2013; Cragg et al., 2017; Coulanges et al. 2021).

The tasks used in each study to assess cognitive abilities are shown in Appendix C1, along with information about the sample, potential criteria for classifying the group with mathematical difficulties, and outcomes.

Synthesis of Results

To identify which cognitive abilities most predict mathematical performance, the results analyzed in the present review are regression analyses or structural equation models, whether available.

To better understand the phenomenon, it will be also analyzed the studies that exclusively conduct correlations and analysis of variance.

Moreover, the studies that differentially analyze the performance of young adults and school-age children (Gilmore et al., 2015; Cragg et al., 2017) will be presented and discussed only referring to the group of our interest (e.g., young adults) since our aim is not to identify differences across the lifespan.

Synthesis of results on cognitive predictors of math performance

Half of the 22 studies included in this systematic review analyzed the predictive role of cognitive variables on mathematical ability (Wilson & Swanson, 2001; Miller & Bichsel, 2004; Vallée-Tourangeau, 2013; Gilmore et al., 2015; Laski et al., 2015; Goffin & Ansari, 2016; Cragg et al., 2017; Orrantia et al., 2019; He et al., 2021; Coulanges et al. 2021; Dowker & Sherida, 2022).

Specifically, verbal WM and visuospatial WM explain from 25% (Wilson & Swanson, 2001) to 34% (Cragg et al., 2017) of variance in Math Achievement. Miller & Bichsel's (2004) study shows how the regression model that includes - in addition to WM components- mathematics anxiety can explain 28% variance in calculation tasks and 17% in applied problem tasks.

Only the study of Dowker and Sherida (2022) did not identify any predictive role of verbal WM on the ability to resolve arithmetic reasoning tasks.

Furthermore, the study by Cragg and coworkers (2017) shows how different mathematical abilities are predicted in varying degrees by different cognitive abilities: on the one hand, 5% of the variance in conceptual knowledge is explained exclusively by verbal WM. On the other hand, the verbal and visual WM and the inhibitory control explain the 12% and 15% of factual mathematics knowledge and procedural skills, respectively.

Concerning the inhibitory control, the included studies show that this domain predicts about 8% variance in composite scores of Math Achievement (Gilmore et al., 2015). However, when considering individual math skills the percentage of variance explained by inhibitory control explains 25% of the variance in Number Line Estimation tasks with non-standard endpoints (Laski et al., 2015), 26% in Calculation tasks (Coulanges, et al. 2021) and 29% in Conceptual knowledge tasks (Gilmore et al., 2015).

Goffin and Ansari's (2016) study showed that the predictive role of cognitive skills such as visual WM and inhibition do not contribute to explaining math achievement variance when other domain-specific skills (e.g., distance effects) are included in the model. The study by Orrantia and collaborators (2019), on the other hand, shows that domain-specific abilities (e.g., numeral order and magnitude processing) help explain 14% more variance than cognitive variables alone (e.g., Intellectual ability, verbal WM, Inhibition). When efficiency (accuracy/time ratio) in overlearning mental operations is considered, it can be seen that 45% of the variance is explained by the model that includes arithmetic skills, but WM ability and attention-switching skills (Vallée-Tourangeau, 2013).

Some studies analyzing the predictive role of other cognitive skills in addition to those included in this review (He et al., 2021), show that skills such as Mental Rotation and Visual Perception help explain -along with age and processing speed- between 16 and 21% of the variance in tasks requiring simple and complex subtraction, respectively.

Three studies have analyzed the effect of cognitive ability on math performance through structural equation models or path analyses (Wang & Carr, 2020; Reynvoet, et al., 2021; Silver et al., 2022). The results show that cognitive inhibition (e.g., Animal Stroop Task) was directly related to Arithmetic Computation Fluency (Reynvoet, et al., 2021), while motor inhibition (e.g., Go/No Go Task) does not affect mathematical performance (Silver et al., 2022). Finally, Visual WM would appear to indirectly affect math performance via the mental rotation ability (Wang & Carr, 2020).

Synthesis of results on math performance: relations and differences with cognitive abilities

Two studies investigated differences in cognitive ability in the groups with and without math difficulties (Osmon et al., 2016; Hoffmann et al., 2014). No differences emerged between the groups with and without math difficulties in performing tasks involving the visual-spatial component of memory (Osmon et al., 2016; Hoffmann et al., 2014). No difference also emerges for tasks assessing general processing speed (Osmon et al., 2016; Hoffmann et al., 2014), while significantly worse performance is observed in the group with mathematical difficulties compared to the control group in a numerical processing speed test (Hoffmann et al., 2014).

Finally, some studies have observed a significant correlation between performance in verbal WM tasks and performance in numeracy, arithmetic calculation, and logical reasoning tasks (Skagerlund et al., 2019; Chemerisova & Martynova, 2019; Zaleznik et al., 2022). A positive correlation can also be observed between Visual WM and arithmetic calculations (Zaleznik et al., 2022).

A positive and significant correlation also emerges between inhibitory control and Math Achievement (Goffin & Ansari, 2016; Van den Bussche et al., 2020;); between ANS acuity and inhibitory control measured through numerical tasks (e.g., Number Stroop Task; Norris & Castronovo, 2016).

Discussion

The present review aimed to identify the cognitive abilities that most contribute to predicting performance in mathematical tasks. For this reason, 22 studies focusing on a population of young adults were included in this qualitative analysis.

The results show that a single cognitive skill can rarely explain performance in any mathematical task; this is only the case when the relationship between verbal WM and conceptual knowledge is considered (Cragg et al., 2017). As the authors themselves report, this result could be because conceptual knowledge is stored through the verbal code and, as a result, verbal working memory is necessary to activate and retrieve information from long-term memory.

In general, both working memory and cognitive inhibitory processes help explain mathematical competence, albeit to varying degrees depending on the other cognitive abilities included in the model and the specific mathematical skill being investigated. In fact, concerning inhibitory control, we can see that if we consider a composite math achievement score, inhibition explains about 8 %of the variance (Gilmore et al., 2015), while if we consider individual math domains (but different from each other) its contribution seems quite homogeneous, varying in the 25-29% range (Laski et al., 2015; Gilmore et al., 2015; Coulanges, et al. 2021).

Even for working memory, it can be seen that it only contributes to explaining between 5 and 15 percent of the variance when it comes to different math domains (Cragg et al., 2017), while the percentage of variance explained is higher (25-34%) when considering a composite Math Achievement score (Wilson & Swanson, 2001; Cragg et al., 2017).

Only one study has shown that performance in the Digit Span task of the WAIS-IV battery does not predict performance in the Arithmetic Reasoning task of the same battery (Dowker & Sherida, 2022). In this regard, we feel it is worth pointing out that in the WAIS-IV scale, these two subtests contribute to the Working Memory Index as they are both believed to assess the restricted ability of Working Memory capacity according to the Cattel-Horn-Carrol model of intelligence (Lang et al., 2015).

However, it seems worth noting that in most cases the results show that the "synergistic" work of several cognitive abilities influences mathematical performance, regardless of the type of ability investigated.

Moreover, some studies show that mathematical performance in adulthood is influenced by both domain-general and domain-specific skills (Orrantia et al., 2019; Vallée-Tourangeau, 2013; Goffin & Ansari, 2016). These findings support the idea that the simultaneous development of cognitive skills and basal number-processing processes influence mathematical learning and competence (Dehaene, 2001; Aunola et al., 2004; Geary, 2001; Geary 2011).

Finally, although it was not the aim of this review to evaluate the role of emotional factors, some studies have considered mathematics anxiety in their models, showing how it affects performance either uniquely (Silver et al., 2022) or indirectly via verbal WM (Skagerlund et al., 2019; Wilson & Swanson, 2001).

In any case, it seems appropriate to emphasize that the choice of predictors to investigate and their contribution to mathematical performance always depends on how mathematical ability is measured and conceptualized.

Limitations and Future Perspectives

The present work allowed us to identify how some cognitive abilities, such as working memory and inhibitory control, help to explain mathematical performance. However, it seems appropriate to point out that we still need to understand which specific component or mechanism most influences the relationship between cognitive and mathematical processes.

Moreover, although most studies have evaluated the role of multiple cognitive functions, it is hard to reach an unambiguous conclusion because the cognitive skills assessed were "combined" differently in the various studies. For this reason, it would be interesting to evaluate in a unique sample the different cognitive abilities that could explain the differences in mathematical performance in young adults. It would also be interesting to assess how different cognitive affect performance in different mathematical domains.

IV. Exploring the Relationship between Cognitive Functioning, Emotional Aspects, and Numeracy Skills: Implications for Young Adults

Introduction

Considering the evidence of the study of mathematical competence in school-age children, and the results of the systematic review about the predictive role of cognitive abilities in math performance it was decided to conduct a parallel study of young adults.

This choice is motivated by several reasons: first of all, young adults with typical development turn out to be a particularly interesting population since it can help us understand how a competence such as mathematics is expressed net of learning (and life) experiences. This competence seems to be particularly affected by formal learning by changing in its manifestation (Geary, 2000), and being able to identify mathematical competencies common to all young adults independent of background could stimulate important insights.

On the other hand, the study of Hartshorne and Germine (2015) showed that some cognitive skills reach their peak development in high school graduation, while others tend to stabilize in early adulthood. This makes college freshmen the ideal population to study the relationship between cognitive and mathematical skills.

However, in approaching the study of mathematical skills in young adults, it is necessary to consider some important elements that may lead individuals to reach different levels of proficiency.

First, mathematics is a cumulative discipline, and then elementary knowledge directly impacts subsequently "higher" knowledge.

Consequently, the assessment of math competence in adulthood should consider these elementary skills that, if not adequately automatizated and acquired, will impede "higher" learning.

Moreover, unlike in other disciplines (i.e., reading and writing), learning mathematics turns out to be influenced by both cognitive and more purely emotional aspects (i.e., math anxiety, self-perceived competence) from the preschooler age through college (see Zhang et al., 2019 for a meta-analysis). Thus, the role of emotional aspects must also be considered in this relation.

Aim of the study

The present study simultaneously assesses different cognitive abilities to identify which one appears to be most associated with the development and learning of mathematics.

Compared to previous studies, this study has the advantage of assessing this in a sample of young adults with typical development. By focusing on this type of population, it is feasible to limit age-related effects and the impact of undiagnosed mathematical disorders.

Moreover, we aim to consider how math anxiety and the self-perception of experiencing difficulties in this area impact math performance by assessing these emotional aspects.

Materials and Method

Participants

A total of 109 (84 female; 25 male) Italian university students (mean age = 20.40 years; SD = 1.64; range = 18,7 – 26,3 years) were recruited from the University of Rome "La Sapienza" and voluntarily participated in the study. Six participants were excluded from the analysis: one of them was excluded because he was not a native Italian speaker, while additional five students were excluded due to compromised performance in the tests used to control the presence of difficulties in reading and spelling (assessed with standardized test from the battery LSC-SUA) Specifically, two students showed an overall clinical performance in the reading tests (e.g., reading word and pseudoword); two students showed a deficit in the spelling tests (writing words with and without articulation suppression), and one more student obtained critical scores in both reading and spelling performance.

Thus, the final sample comprised103 (81 female; 22 male) college students (mean age = 20.40 years; SD = 1.62; range = 18,7 – 26,3 years), of whom 95,15% were attending a degree program in Psychology.

Concerning the type of high school attended, which could have an impact on mathematic competencies, about half (55.3%) of the sample pursued scientific studies; the remaining half is divided between those who pursued classical studies (15.5%), other high schools (24.3%), and technical studies (4,9%).

Instruments

The tests used are briefly described, divided into the following domains: mathematical competencies, general cognitive functions, and self-report measures.

Mathematical Competencies

To assess mathematical skills, the tests from the LSC-SUA battery (Montesano et al., 2020) were administered. The LSC-SUA is a standardized Italian instrument used in clinical practice to assess the essential aspect of reading, writing, and computation in college students and adults.

The mathematical tests used to assess math competence are 1) dictation of numbers, 2) number reading, 3) mental calculation (+; -; x; :), 4) Arithmetic facts retrieval (e.g., multiplication tables; basic addition and subtraction, basic properties of numbers), 5) approximate calculation (e.g., /630 x 6/ = a)30.810; b)520; c)3780), and 6) transcription of word numbers in digit (e.g., /*ventitremiladuecento*/{twenty-three thousand two hundred} = 23.200).

General Cognitive Functions

According to the aim of the study and the literature, the general cognitive functions assessed in the study are 1) processing speed, 2) memory, in its verbal and visuospatial components, 3) visual-perceptive skills, and 4) planning and problem solving.

To assess **processing speed** are proposed both the *Parrots* and *Boxes* subtests of the Intelligence and Development Scales- 2nd edition (IDS-2; Grob & Hagmann-von Arx, 2018; Italian version by Ferri et al., 2022). Both the tasks assess the visual processing speed, but the subtest Parrots requires rows processing, similar to that one required by the reading and writing task (e.g., from left to right), while the subtest *Boxes* requires a column processing (e.g., from top to down), correspondingly to the processing necessary to solve written operations.

Regarding the **memory domains**, the *Shape Memory* subtest of the IDS-2 battery was administered to assess the short-term visuospatial memory, while the recognition of the

Rey-Osterrieth Complex Figure (Rey, 1941; Italian validation by Caffarra et al., 2002) was used as a measure of long-term visuospatial memory. The Digit Span forward (Monaco et al., 2013) and the Rey Auditory Verbal Learning Test (RAVLT; Rey, 1941) were used to assess short-term verbal memory. Moreover, the Digit Span backward (Monaco et al., 2013) was used to evaluate verbal working memory, while the recognition of the Rey Auditory Verbal Learning Test was used to measure long-term memory capacity.

The planning and problem-solving skills were evaluated using the Tower of London test (Shallice, 1982; Italian validation Sannio Fancello et al., 2021), while the copy of the Rey-Osterrieth Complex Figure was adopted to assess the visual-perceptive skills.

Self-report questionnaires

Two self-report questionnaires were administered to assess the emotional aspects that may affect mathematics performance.

For the assessment of math anxiety, the Abbreviated Math Anxiety Scale (AMAS, Primi et al., 2014) questionnaire standardized for Italian college students was proposed.

For the assessment of perceived difficulty in performing math-related tasks, the Vinegrad Plus questionnaire was used; the Italian adaptation and validation of this questionnaire can be found in the LSC-SUA Battery (Montesano et al., 2020)

Procedure

The Local Ethics Committee (Department of Dynamic, Clinical Psychology and Health Studies, University of Rome "Sapienza") approved the research (Prot. N. 0000800; 11/06/2021) that was conducted according to the Helsinki Declaration.

Before starting the experimental session, the researcher carefully explained the procedure to each participant, and written informed consent was obtained.

After a brief anamnestic interview aimed at gathering information about age, educational background, potential diagnosis, or familiarity with neurodevelopmental disorders, participants completed some self-report questionnaire to assess the presence of anxiety and the self-perceived difficulties in performing activities involving learning in the domain of reading, writing, and math. These are administered at the beginning of the experimental session to prevent responses from being influenced by participants' perceptions of their performance on the standardized tasks.

Afterward, the experimental session began with the randomized administration of tests to assess cognitive functions, mathematical skills, and reading and writing skills.

Participants were tested individually in a single session lasting about 2 hours, with a break about halfway through the evaluation. The whole experimental procedure took place in a quiet and well-lit room.

Data Analysis

The scores obtained by the participants in each cognitive and mathematical test were converted into z-scores based on the normative data.

Regarding the mathematical tests, to obtain an overall measure of mathematical competence (calculated as the average of z-scores), it was necessary to standardize the direction of z-scores, regardless of the type of measure that was considered (e.g., accuracy, number of errors, response time). Therefore, the polarity of tests was changed when it has the number of errors or response time as units (e.g., more errors and longer response time correspond to worse performance).

Data analyses were conducted mainly through Jamovi software; however, because that software does not support the application of some statistical models, R software was used in some cases. When not otherwise specified, the software Jamovi has been used.

First, to verify the age-related impact on test performance, a correlational analysis (Pearson's r) was conducted considering the age, the scores in cognitive domains, and the various mathematical skills examined as variables. Then an exploratory factor analysis (EFA) was conducted to combine and synthesize the different mathematical variables into a minor number of latent variables. A correlation analysis among the various mathematical tests was conducted to determine the rotation to adopt following the EFA. Based on the results of the factor analysis, aggregate scores were calculated to define the score of participants for each factor identified.

Next, multiple regression analyses were conducted to assess the predictive role of cognitive variables (Independent Variables) on the different mathematical domains (Dependent Variables). Moreover, given the large number of predictors included in the multiple regression model, these results were compared with a stepwise regression model in which only the predictors most correlated with the dependent variable were considered.

Concerning the evaluation of emotional aspects, a correlation analysis was first conducted by including the scores of the self-report questionnaires and the aggregate scores of the previously assessed mathematical domains. Then, to understand the impact of the emotional variables on the individual math domains, multiple regressions were conducted considering mathematical dimensions as the dependent variable and the scores on the self-report questionnaires as the independent one.

Results

The correlational analysis conducted considering age and both cognitive and mathematical variables proved that age did not correlate with any of the variables considered (see Appendix D1 e D2).

Therefore, to identify the mathematical domains assessed through the proposed tests was conducted an exploratory factor analysis to group the variables observed into distinct domains (or factors).

A Maximum likelihood extraction method was used. Since all variables considered in the factor analysis assess certain mathematical abilities, the correlational matrix showed significant correlations among all these variables (Appendix D1). For this reason, an oblique rotation was applied. or factors).

Table 1. Saturations and communalities before and after oblique rotation

	Matrix			Matrix rotation OBLIMIN		
Variabili	F1	F2	*h*	F1	F2	*h*
Mental Calculation - Time	.79	.39	.22	**.90**	-.02	.21
Mental Calculationn - Correct.	.82	.17	.30	**.66**	.24	.30

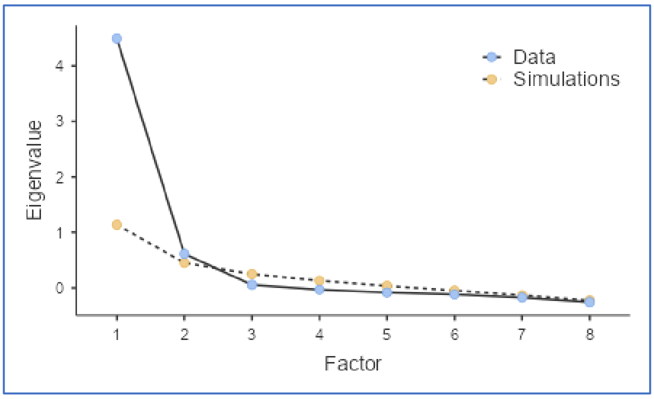
Approximate Calculation	.54	.42	.53	**.77**	-.18	.55
Arithmetic Facts	.86	.17	.23	**.68**	.27	.23
Transcoding digits	.69	-.45	.32	-.11	**.88**	.32
Numbers writing	.72	-‘.31	.38	.07	**.74**	.38
Numbers reading – Time	.79	-.08	.36	.36	**.52**	.37
Numbers reading – Errors	.80	-.28	.28	.14	**.76**	.27

As can be seen from Table 1, after rotation the scores of mental calculation, approximate calculation, and arithmetic facts saturate primarily in the first factor, while scores in the tests of transcoding digits, writing, and reading numbers saturate primarily in the second factor. Also the scree test (Figure 1) confirmed the two-factor structure.

The two latent variables revealed by factorial structure could be termed Computational System (factor 1) and Numerical Transcoding Ability (factor 2).

For each factor was calculated an aggregate score (e.g., mean of the scores of the variables defining each domain), which will be considered in the next analysis along with overall mathematical proficiency (e.g., mean of the z-scores obtained in all mathematical tests).

Figure 1. Scree Plot for EFA of Mathematical Factors



In light of the domains identified through the factor analysis (e.g., Computational system and Numerical transcoding ability), a series of multiple regressions were conducted considering the global mathematical competence and each mathematical domain as the dependent variable and the performance in the different cognitive tests as predictor variables.

The results of the multiple regressions (Table 2) showed that the model considering the complessive mathematical performance and the different cognitive functions as predictor variables explain about 42% of the variance (R^2 Adjusted = 0.425; $F_{12,90}$=7,28; p= <0.001) with significant effects of the of processing speed as measured by the Tetris test (ß= 0.31; t= 3.31; p = 0.001), the verbal short term memory as measured by the Digit Span Forward (ß= 0.13; t= 2.31; p = 0.02), and the execution time at the Tower of London test (ß= 0.20; t= 2.70; p = 0.008).

When the transcoding ability is considered as the dependent variable the model explains the 35% of the variance (R^2 Adjusted = 0.351) and is statistically significant ($F_{12.90}$= 5.59; p = < 0.001). In particular, the effects of processing speed (ß= 0.33; t= 2.78; p = 0.007) and execution time at the Tower of London test (ß= 0.16; t= 1.958; p = 0.054) were significant.

Finally, considering the Computational system as the dependent variable, the model explains about 39% of the variance (R^2 Adjusted = 0.386) and a significant effect of verbal short-term memory is observed (ß= 0. 19; t= 2.773; p = 0.007), as well as of processing speed as measured by the Tetris test (ß= 0.33; t= 3.00; p = 0.003) and execution time at the Tower of London task (ß= 0.24; t=2.78; p = 0.007).

Table 2. Multiple regressions considering the mathematical domains as dependent variables.

		Mathematical Competence			**Numerical Transcoding ability**			**Computational System**		
Cognitive Function	**Predictors**	ß	SE	p	ß	SE	p	ß	SE	p
Processing Speed	Parrots	-.006	.08	.95	-.04	.09	.70	.03	.09	.79
	Tetris	**.40**	**.09**	**.001**	**.36**	**.11**	**.007**	**.38**	**.11**	**.003**
Short-Term Memory (Verbal)	Digit Span Forward	**.19**	**.06**	**.02**	.10	.06	.22	**.23**	**.07**	**.007**
	Rey's Words - Immediate	.11	.07	.24	.10	.08	.28	.09	.08	.34
Long-Term Memory (Verbal)	Rey's Word – Recall	-.10	.07	.29	-.04	.08	.71	-.15	.09	.15
Working Memory (Verbal)	Digit Span Backward	.13	.07	.15	.11	.08	.25	.12	.08	.17

Visuo-Spatial Memory	Shape Memory	-.02	.06	.79	.03	.07	.77	-.07	.08	.45
Long-Term Memory (Visuo-Spatial)	Rey's Figure- Recall	-.01	.07	.29	-.08	.08	.40	.06	.08	.54
Visuo-perceptual integration	Rey's Figure - Copy	.05	.05	.53	.15	.06	.09	-.05	.06	.55
Planning and problem solving	Tower of London – Correct.	.02	.07	.85	.13	.08	.18	-.09	.08	.34
	Tower of London – Decision time	.007	.08	.93	-.02	.09	.85	.03	.09	.74
	Tower of London – Execution time	**.27**	**.07**	**.008**	.21	.08	.054	.29	.09	**.007**
Model Fit	**R²**	.492			0.427			0.458		
	R² Adjusted	.425			0.351			0.386		
	Test *F*	7.28			5.59			6.33		
	p	<.001			<.001			<.001		

Given the large number of predictors included in the previous model, it was considered appropriate to use a stepwise regression method to identify the model that best fits the data. This method enables us to suggest the variables that represent the best predictors of the Numerical Transcoding abilities and Computational System (i.e., dependent variables), identifying them among the various cognitive functions assessed (i.e., independent variables). These analyses were conducted using R software.

From the results of these analyses (Table 3), the model including the tests of Tetris, Digit Span Forward, Copy of Rey's Figure, and the Tower of London measures of correctness and execution time seem to jointly explain about 40% of the performance in the mathematical domain of transcoding, although only the score obtained at Tetris (ß= 0. 28; t= 3.851; $p < .001$) and execution times at the Tower of London (ß= 0.17; t= 2.219; $p < .002$) appear to have a significant effect.

As for the Computational System domain, however, it is observed that performance at Tetris, Digit Span Forward, and execution times at the Tower of London alone explain about

42% of the variance.

Table 3. Stepwise regressions considering the two mathematical domains as dependent variables.

	Mathematical Competence			**Numerical Transcoding ability**			**Computational System**		
Predittore	ß	SE	p	ß	SE	p	ß	SE	p
Tetris	**.33**	**.06**	**<.001**	**.28**	**.23**	**<.001**	**.35**	**.08**	**<.001**
Digit Span Forward	.15	.05	.005	.11	.06	.08	**.20**	**.06**	**< .001**
Rey's Figure- Copy	/	/	/	.09	.05	.09	/	/	/
Tower of London – Correct.	/	/	/	.11	.07	.14	/	/	/
Tower of London – Execution time	**.22**	**.06**	**<.001**	**.17**	**.08**	**.03**	**.21**	**.07**	**<.001**
R^2	.469			0.404			0.421		
R^2 Adjusted	.452			0.373			0.404		
Test *F*	29.14			13.17			24.04		
p	<.001			<.001			<.001		

The second aim of the present study intends to understand the role of emotional factors on mathematical competence.

For this purpose, the scores obtained on both the questionnaires to assess mathematical anxiety and the self-perception of encountering difficulties (Vinegrad+) were analyzed.

Regarding the scores on the Vinegrad+ questionnaire, the descriptive analysis shows that 63,7% of participants do not refer difficulties in performing activities and tasks involving mathematical skills, the remaining 36,3 % of participants experience some (20.6%) or many (15.7%) efforts in this area.

Even concerning Math Anxiety, most of the participants obtain scores in the normal range on the AMAS questionnaire (81.8%), and only 18,2% of the participants report a moderate or high level of math anxiety.

A correlation analysis was conducted by including both the emotive variables (e.g., scores on the Vinegrad+ and AMAS) and the performance obtained in each math domain. The results showed that the two measures of emotional aspects were significantly correlated (r = 0.57; p <.001) and the correlations between the emotive variables and the mathematical domains were significant (Table 5).

Table 4. Correlation between emotive and mathematical variables

Variables		1	2	3	4
1. Self-perceived difficulties	r	-			
	p	-			
2. Math Anxiety	r	.57	-		
	p	<.001	-		
3. Numerical Transcoding ability	r	-.49	-.30	-	
	p	<.001	.002	-	
4. Computational System	r	-.56	-.24	.68	-
	p	<001	.02	<001	-

To understand the predictive role of the abovementioned emotive variables on mathematical performance, a series of multiple regression analyses were conducted considering the scores on the Vinegrad+ and AMAS questionnaires as independent variables.

Regarding the analyses considering Computational System as the dependent variable, the regression model explains about 32% of the variance (R^2 Adjusted= 0. 318) and is significant ($F_{2,95}$=2.6; p < 0.001). But only the effect of self-perceived difficulties in mathematics was significant (ß= - 0.63; p < 0.001). The effect of Mathematics Anxiety is not substantial when measured at the net of perceived difficulties (ß= - 0.12; p =0.26).

A similar trend can be observed in the regression that considers Numerical Transcoding Ability as the dependent variable. The model explains about 24% of the variance, but only the effect of perceived difficulty is significant (ß= - 0.49; p < 0.001), while the effect of Math Anxiety is non-significant (ß= - 0.03; p = 0.78).

In both cases, the predictor variable of perceived difficulty is significant with a negative direction. Thus, as the perceived struggle in math tasks increases, performance in both the

Computational System and Numerical Transcoding Ability decreases.

Tabella 5. Regressione multipla con i singoli domini matematici come variabile dipendente e gli aspetti emotivi come predittori

	Mathematical Competence			**Numerical Transcoding ability**			**Computational System**		
Predictors	ß	SE	p	ß	SE	p	ß	SE	p
Self-perceived Difficulties	**-.62**	**.05**	**<.001**	**-.49**	**.05**	**<.001**	**-.63**	**.06**	**<.001**
Math Anxiety	.05	.01	.63	-.03	.01	.78	.12	.01	.26
R^2	.349			0.260			0.332		
R^2 Adjusted	.335			0.245			0.318		
Test *F*	25.4			16.7			23.6		
p	<.001			<.001			<.001		

Discussion

This study aimed to explore the influence of cognitive and affective aspects on mathematical competence in young adult.

The sample consists of college students with typical development, and their mathematical performance was assessed through the administration of an Italian standardized instrument for the college population (LSC-SUA).

The factor analysis conducted to identify the mathematical domains revealed two factors, which are named Computational System and Numerical Transcoding Ability.

These domains represent two main areas of mathematical competence that, if impaired, may suggest the presence of a specific learning disorder in mathematics (Moura et al., 2013).

Specifically, numerical transcoding ability involves the transformation of the verbal code (numbers) into Arabic code (digits) and vice versa (Deloche & Seron, 1997; Dehaene, 1992). The transcoding ability represents one basic mathematical ability: it is acquired during the earliest years of schooling and could predict mathematical performance in afterwards grades (Moeller et al., 2011; Gobel et al., 2013).

Regarding computational skills, much emphasis is placed on this ability in educational and clinical contexts.

Mastery of the arithmetical procedures required to perform basic computational tasks is crucial to acquiring the following and more complex skills (Calhoon et al., 2007; Geary et al., 2017).

However, defining how this ability develops and which specific skills help computational fluency is not always simple to establish.

The exploratory factor analysis conducted in this study showed how this domain includes the performance in tasks such as arithmetic fact retrieval, approximative calculation (or computational estimation), and mental calculation. These tasks share the purpose of correctly detecting the results of an operation within a limited time, although the process to get it is different for each assignment.

If we analyze the process to perform the cited task, we can note that the demand for retrieving the exact result of some arithmetic facts requires that an operation-result association was consolidated and automatized in previous phases (Siegler, 1996).

Request to choose among several options the correct result without doing the precise calculation invites the individual to estimate the right solution (approximative calculation task). Finally, when an operation has to be solved by the mind, we need to regain the procedures and strategies learned in the years of formal schooling to reach the correct answer.

All the processes and skills required to perform these different tasks are part of the Computational System because these allow adults to solve mathematical calculations efficiently.

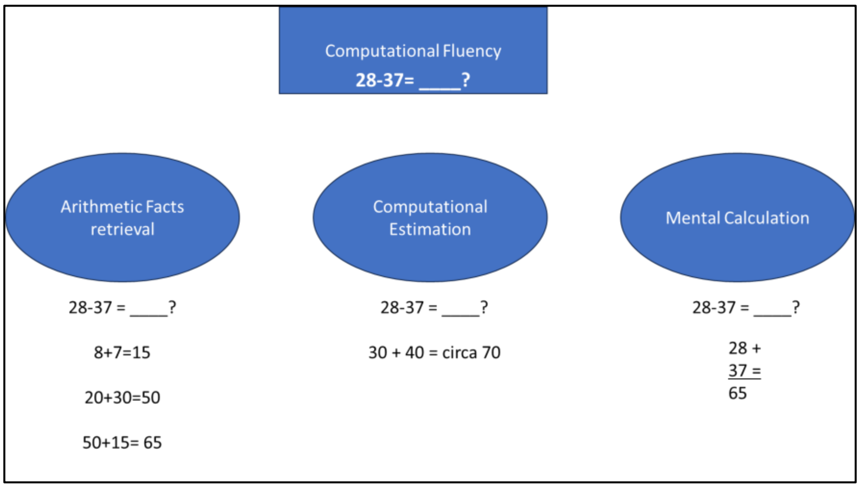
An interesting point of view to interpret this evidence is that these abilities are acquired sequentially and trained during school attendance, but in adulthood, these skills interact and are employed in a versatile way to solve problems that require both exact and approximate computation.

As already discussed by Ashcraft (1992), performance in progressively more complex mathematical tasks will still require the involvement of already acquired and established knowledge and processes. For example, solving 28+37 will be involved both automatic retrieval of arithmetic facts (8+7, but also 20+30, i.e., 2+5) and the procedure of carrying.

Figure 2 reports how the same operation can be solved using different strategies. The strategy choice will depend partly on the request of the environment (approximate vs exact calculation) and partly on the characteristics of both the problem (e.g., type of operation, problem size) and the individual (e.g., cognitive resource available; Threlfall, 2009; Imbo & Vandierendonck, 2008).

Such evidence makes us suppose that computational fluency is the outcome of a complex computational system that requires the development of different mathematical processes.

Figure 2. Example of resolution processes of an arithmetic operation in adult



Regarding the cognitive skills most involved in mathematical performance, the literature has focused primarily on studying the predictors of math performance and the domain-general skills most impaired in students with math difficulties.

However, this study focuses on mathematical competence intended as a continuum from mathematical disorders to mathematical proficiency.

For this reason, this study aims to recognize the cognitive skills that mainly contribute to explaining the level of competence acquired in young adults with typical development.

Results of stepwise regression analyses show that about 45% of the variance in math competence is explained by tasks measuring processing speed (e.g., Tetris), executive-procedural aspects (e.g., Execution Time at ToL) and verbal short-term memory (e.g., Digit Span Forward). Specifically, while numerical transcoding ability is impacted most by processing speed and executive-procedural aspects, for the Computational System, the highest percentage of variance (42%) is explained by the model that also includes the verbal short-term memory.

Specifically, in the present study, the strongest predictor of both global mathematical competence and performance in each domain is processing speed. This result is in line with the idea that the speed of processing information represents a "mental capacity" that can explain individual differences in performance on basic and complex cognitive tasks (Kail & Salthouse, 1994).

Similarly, the execution time at the Tower of London task helps to explain performance in the different domains assessed. This parameter implies the involvement of multiple skills, including planning, information updating and organization of motor sequencing. For these reasons, it can be considered a measure of executive-procedural skills.

These skills, according to the ADAPT model (A Developmental, Asemantic, And Procedural Transcoding; Barrouillet et al., 2004), influence the ability to convert more complex numbers for which is necessary to rely on syntactic-lexical knowledge and, therefore, procedural rules.

On the other hand, tasks requiring multi-digit operations are proposed in adulthood to assess basic computational skills. The process leading to solving these uncomplicated operations requires choosing the strategy to be applied (e.g., decomposition, rounding, etc.) and then transforming and updating the operators following a multi-step path and respecting rule-based procedures.

Finally, the fact that there is also a crucial contribution of verbal short-term memory in the computational process supports the fact that in performing simple calculations, the task is decomposed into several steps, the results of which need to be preserved in the short-term memory storage.

We feel it is also appropriate to discuss here the lack of a significant contribution of working memory in predicting performance in the two domains assessed. This result appears to be in line with some studies (Vanbinst & De Smedt, 2016; Peng et al., 2015) that have observed age-related effects on the association between mathematical competence and working memory. Indeed, the working memory role in math performance seems to be more robust in the early stages of mathematics learning (e.g., first grades of schooling), and it would decrease with increasing age and formal education.

This pattern may depend on the fact that working memory is highly involved in new and complex tasks that requires a cognitive load and more effort (Raghubar et al., 2010).

Because our sample performed basic mathematical tasks, which do not involve new learning but rather the enactment of past practices and knowledge, would explain the low involvement of working memory in performing the proposed tasks.

Finally, the second aim of this study was to explore the role of emotional aspects on mathematical performance.

Results show how experiencing math difficulties contributes exclusively and significantly to explaining math competence. These results are not in line with evidence that shows how math anxiety significantly affects math achievement even in adulthood (Barroso et al., 2021; Foley et al., 2017). However, our results may be the result of bias sampling: only 3 participants in our sample report high levels of math anxiety, compared with 37 who report, instead, experiencing difficulties.

Nonetheless, in our view, it should be appropriate to focus on and discuss how math self-efficacy negatively affects math performance in transcoding and computational tasks.

This result is consistent with studies showing that confidence in successfully solving math-related problems and tasks is a significant predictor of math performance (Pajares & Miller, 1995; Peters, 2012; Gatabu et al., 2014). These findings are in line with the self-efficacy theory proposed by Bandura (1997) which recognizes that all cognitive performance (including mathematics performance) is influenced by personal judgment of capabilities to solve a given task.

Conclusion

Studies on mathematics in adulthood have mainly focused on its role in daily life. Mathematical proficiency seems to positively affect employment success, socioeconomic status, financial choices and even health status (Ritchie & Bates, 2013). Given the impact that math competence has in life chances, in the last decades additional attention has been placed to the assessment of this ability in adulthood (Duchhardt et al., 2017; *Programme for the*

Assessment of Adult Competencies, 2013).

Precisely the ability to "generalize" to ecological contexts a set of mathematical skills learned in a formal context (e.g., school) and in an abstract way is what has been referred to in recent years as numeracy. Achieving a good level of numeracy is crucial in adulthood as it allows one to use the knowledge acquired during formal learning and growth to manage and cope with everyday situations and, therefore, to earn functional autonomy (Gal et al., 2020; Dennis & Barnes, 2010; Bynner & Parsons, 1997).

In this regard, numeracy refers to the ability requested to adulthood to manage money and time, calculate lengths, areas, and volumes, maintain numerical and graphical records (Bynner & Parsons, 1997; Ginsburg et al., 2007).

Even though our study has not investigated all the mathematical domains that can more fully define mathematical competence and numeracy (e.g., measures related to number sense are missing), the presented results can be a starting point to examine what influences mathematical proficiency. As we have already anticipated, we believe that the best way to consider mathematical competence is along an impairment-exceptional continuum. That implies that between those with significant impairments in this area and those who have developed above-average logical-mathematical intelligence (for example, physicists and mathematicians), there are all the typical adults who achieved the level of numeracy necessary to solve math-related tasks in everyday life.

This study shows us that mathematical proficiency is predicted by general domain skills such as processing speed, executive-procedural skills and verbal short-term memory, but also the perceived self-efficacy to solve mathematical tasks impacts this competence.

These findings on typical development may provide some insights to better understand atypical development and, therefore, the potential presence of specific disorders in the area of mathematical competence that may impair daily functioning and limit life chances.

In fact, in future, it could be interesting to evaluate whether adults with overt learning disorders in the mathematical area (and specifically in transcoding and the computational system) show significant impairments in the cognitive domain that we identified as predictive of mathematical performance or whether they show additional (or exclusively) deficits in other cognitive skills (i.e., working memory).

Results in line with this second option would, on the one hand, allow the detection of domain-general "symptoms/sign" to be evaluated in the clinical setting to support the identification of unrecognized math disorders. On the other hand, the potential identification of domain-general deficits that interfere with numeracy would help in proposing and setting up ad hoc interventions or instruments to support adults with difficulties in performing math-related tasks in everyday life.

General Discussion

The present thesis focuses on investigating the complex relationship between cognitive abilities and mathematical competence.

To this end, two systematic reviews were conducted to understand which cognitive skills are most impaired in children with Mathematical Difficulties (MD) and which cognitive skills predict mathematical performance in young adults with typical development.

On the one hand, results from these reviews have shown that school-age children with varying degrees of severity of math difficulties perform worse on several tests assessing executive functioning. In particular, skills such as verbal working memory (in line with Peng et al., 2012), visuospatial working memory (in line with Szűcs, 2016) and cognitive inhibition with number stimuli (e.g., Number Inhibition Task) are found to be impaired in children with MD. On the other hand, we observed that both visual and verbal components of working memory, and inhibition can predict the mathematical proficiency of young adults with typical development.

Starting from this evidence, the two experimental studies presented in this thesis work focused separately on school-age children and young adults. Italian standardized test batteries were administered to assess mathematical skills, differentiated by age and schooling to avoid floor or ceiling effects. In addition, the cognitive variables assessed in the two studies are mostly the same (e.g., Processing Speed; Verbal Short Term Memory; Verbal Working Memory; Verbal Long Term Memory; Visuo-spatial Memory; Long-term Term Visual Memory; Visuo-perceptual integration; Planning and Problem Solving). The study of children additionally included assessment of inhibition and switching through computerized tasks.

To overcome some of the limits found in the literature, mainly related to the heterogeneity of the mathematical skills investigated, Exploratory Factor Analyses (EFA) were conducted in both studies to identify the mathematical domains assessed through the proposed tasks. The mathematical domains identified through the EFA were used as dependent variables for a series of multiple regressions to identify the cognitive variables that most predict performance in each mathematical domain.

In the study conducted on school-age children (age range: 9-12 years old), the EFA showed three latent variables, defined as the domains of Computational Ability, Basic Mathematical Skills and Mathematical Reasoning. Regarding the predictor variables, the results show that age is the only predictor of performance in the Computational ability domain, which encloses tasks that require solving both written and mental operations rapidly and accurately. As for the Mathematical Reasoning domain, which involves solving tasks with numerical information using logical-mathematical reasoning, verbal working memory represents the only significant predictor.

Finally, the verbal component of working memory and short-term memory contribute, along with execution time at the Tower of London, to explain performance in the Basic Mathematical Skills domain, understood as those skills that are the basis for building future learning (counting, transcoding, rapid facts retrieval, procedural skills).

The EFA conducted in the study with young adults showed the presence of two latent variables defined as the Numerical Transcoding Ability and the Computational System domains representing, respectively, the mathematical abilities related to numerical transcoding and the computational fluency, seen as the ability to solve calculations efficiently (rapidly and accurately) recurring to different specific skills (e.g., number facts retrieval, estimation or procedures and strategies acquired in the schooling years).

Performance in the Numerical Transcoding Ability domain is predicted exclusively by processing speed, while the performance in the Computational System domain is explained by execution time at the Tower of London, short-term memory and processing speed.

To integrate the results obtained in the two experimental studies, we can see that verbal WM does not predict any mathematical performance in the young adult group, while it explains performance in the domain of Basic Mathematical Skills and Mathematical Reasoning in children.

We already discussed the role of WM in Mathematical Reasoning, and it does not seem appropriate to us to examine any group differences in this regard since the assessment in adults did not include the administration of tests assessing logical-mathematical reasoning. In contrast, it seems interesting to discuss the result related to Basic Arithmetic Skills. In fact, some overlap can be found between the tasks that define Basic Arithmetic Skills in children and those that define Computational System in young adults.

Both domains hold the performance in tasks that require retrieving arithmetic facts from long-term memory and applying procedures to solve mental or written calculations. The fact that verbal WM helps explain performance in these tasks in children but not in adults supports the idea that working memory is involved more in the early stages of learning, concurrently with the presentation of new assignments that require greater cognitive and executive effort (Vanbinst & De Smedt, 2016; Peng et al., 2015). Accordingly, these tasks become automatic in adulthood, favouring the ability to perform them effectively and effortlessly (Hasher & Zacks, 1979; Bargh et al; 1992). At the same time, however, we can observe how executive-procedural skills and short-term memory contribute to explaining performance in both of these domains (e.g., Computational System and Basic Mathematical Skills). This result does not sound odd if we consider that the tasks included in these domains require both the maintenance of information in the memory storage and the enactment of a series of steps and procedures to solve multi-step tasks.

Finally, in young adults, we can identify, in both mathematical domains, a predictive role for processing speed, a variable that does not help to explain any mathematical performance in children. This result would support the theory of processing speed (Salthouse, 1996), according to which processing speed increases gradually during childhood and adolescence and then peaks in young adulthood. In this view, different performances in cognitive tasks in the adult population would depend on different cognitive speeds, meant as a basic component of higher cognitive functioning, which, therefore, would influence performance in many cognitive tasks (Kail & Salthouse, 1994).

General Conclusion

The present work has allowed us to investigate the relationship between cognitive abilities and mathematical competence by trying to shed light on some aspects of the phenomenon still overshadowed.

In particular, we have already discussed the heterogeneity of mathematical domains assessed in the international literature, which often makes it difficult for us to generalize the results regarding the cognitive skills most involved. This study wanted to start precisely from this point: from an attempt to reflect on each mathematical ability, aware of the strong association that often exists between apparently distinct abilities.

Although the mathematical domains identified in this study have some limitations related to the current possibilities of assessing mathematical skills through standardized instruments, this study allowed for a deeper understanding of the latent constructs that characterize these tasks.

That, in our opinion, represents a more detailed description of what might be the mathematical domains to be considered in clinical and educational settings.

These considerations, however, point to the urgent need for the development and validation of new instruments for the assessment of mathematical competence, although this is not an easy step.

References

Agostini, F., Zoccolotti, P., & Casagrande, M. (2022). Domain-general cognitive skills in children with mathematical difficulties and dyscalculia: A systematic review of the literature. *Brain Sciences*, *12*(2), 239.

American Psychiatric Association. (2013). *Diagnostic and statistical manual of mental disorders* (5th ed.).

Andersson, U. (2008). Mathematical competencies in children with different types of learning difficulties. *Journal of Educational Psychology*, *100*(1), 48.

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, *44*(1–2), 75–106.

Attout, L., & Majerus, S. (2015). Working memory deficits in developmental dyscalculia: The importance of serial order. *Child Neuropsychology*, *21*(4), 432-450.*

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of educational psychology*, *96*(4), 699.

Baddeley, A. (1992). Working memory. *Science*, *255*(5044), 556-559.

Bargh, J. A. (1992). The ecology of automaticity: Toward establishing the conditions needed to produce automatic processing effects. *The American journal of psychology*, 181-199.

Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2021). A meta-analysis of the relation between math anxiety and math achievement. *Psychological Bulletin*, *147*(2), 134–168.

Barrouillet, P., Camos, V., Perruchet, P., & Seron, X. (2004). ADAPT: A Developmental, Asemantic, and Procedural Model for Transcoding From Verbal to Arabic Numerals. *Psychological Review*, *111*(2), 368–394.

Barrouillet, P., Fayol, M., & Lathulière, E. (1997). Selecting between competitors in multiplication tasks: An explanation of the errors produced by adolescents with learning difficulties. *International Journal of Behavioral Development*, *21*(2), 253-275.

Berkowitz, M., Edelsbrunner, P., & Stern, E. (2022). The relation between working memory and mathematics performance among students in math-intensive STEM programs. *Intelligence*, *92*, 101649.**

Biancardi, A., Bachmann, C., Nicoletti, C., (2016). BDE-2. Batteria Discalculia Evolutiva. Test per la diagnosi dei disturbi dell'elaborazione numerica e del calcolo in età evolutiva – 8-13 anni. Trento. Erickson.

*Articles included in the qualitative analyses of "Domain-general cognitive skills and mathematical difficulties: a systematic review of the literature" (Chapter I)

**Articles included in the qualitative analyses of "Evaluating Cognitive Predictors of Mathematical Proficiency in Young Healthy Adults: A Systematic Literature Review" (Chapter III)

Bruffaerts, R., Tyler, L.K., Shafto, M. *et al.* Perceptual and conceptual processing of visual objects across the adult lifespan. *Sci Rep* **9,** 13771 (2019). https://doi.org/10.1038/s41598-019-50254-5

Bull, R., & Johnston, R. S. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of experimental child psychology*, *65*(1), 1-24.

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental neuropsychology*, *19*(3), 273-293.

Butterworth, B. (1999). A head for figures. *Science*, *284*(5416), 928-929.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of child psychology and psychiatry*, *46*(1), 3-18.

Butterworth, B. (2008). Developmental dyscalculia. *Child neuropsychology: Concepts, theory, and practice*, 357-374.

Bynner, J. M., & Parsons, S. (1997). *Does numeracy matter?: Evidence from the National Child Development Study on the impact of poor numeracy on adult life*. Basic Skills Agency.

Caffarra, P., Vezzadini, G., Dieci, F., Zonato, F., & Venneri, A. (2002). Rey-Osterrieth complex figure: Normative values in an Italian population sample. *Neurological Sciences*, *22*(6), 443–447.

Cai, D., Li, Q. W., & Deng, C. P. (2013). Cognitive processing characteristics of 6th to 8th grade Chinese students with mathematics learning disability: Relationships among working memory, PASS processes, and processing speed. *Learning and Individual Differences*, *27*, 120-127.*

Case, R., Kurland, D. M., & Goldberg, J. (1982). Operational efficiency and the growth of short-term memory span. *Journal of experimental child psychology*, *33*(3), 386-404.

Caviola, S., Primi, C., Chiesi, F., & Mammarella, I. C. (2017). Psychometric properties of the Abbreviated Math Anxiety Scale (AMAS) in Italian primary school children. *Learning and Individual Differences*, *55*, 174–182.

Censabella, S., & Noël, M. P. (2007). The inhibition capacities of children with mathematical disabilities. *Child neuropsychology*, *14*(1), 1-20.*

Chan, B. M. Y., & Ho, C. S. H. (2010). The cognitive profile of Chinese children with mathematics difficulties. *Journal of Experimental Child Psychology*, *107*(3), 260-279.*

Chan, W. W. L., & Wong, T. T. Y. (2020). Subtypes of mathematical difficulties and their stability. *Journal of Educational Psychology*, *112*(3), 649*.

Chemerisova, E. V., & Martynova, O. V. (2019). Effects of the Phonological Loop of Working Memory on the Productivity of solving Mathematical and Verbal Tasks in specialists in Mathematics and the Humanities. *Neuroscience and Behavioral Physiology*, *49*, 857-862.**

Chu, F. W., Hoard, M. K., Nugent, L., Scofield, J. E., & Geary, D. C. (2019). Preschool deficits in cardinal knowledge and executive function contribute to longer-term mathematical learning disability. *Journal of experimental child psychology*, *188*, 104668.*

Cirino, P. T., Fletcher, J. M., Ewing-Cobbs, L., Barnes, M. A., & Fuchs, L. S. (2007). Cognitive arithmetic differences in learning difficulty groups and the role of behavioral inattention. *Learning Disabilities Research & Practice*, *22*(1), 25-35.*

Cirino, P. T., Fuchs, L. S., Elias, J. T., Powell, S. R., & Schumacher, R. F. (2015). Cognitive and mathematical profiles for different forms of learning difficulties. *Journal of learning disabilities*, *48*(2), 156-175.*

Cirino, P. T., Morris, M. K., & Morris, R. D. (2002). Neuropsychological Concomitants of Calculation Skills in College Students Referred for Learning Difficulties. *Developmental Neuropsychology*, *21*(2), 201–218.

Clark, C. A., Pritchard, V. E., & Woodward, L. J. (2010). Preschool executive functioning abilities predict early mathematics achievement. *Developmental psychology*, *46*(5), 1176.

Costa, A. J., Silva, J. B. L., Pinheiro-Chagas, P., Krinzinger, H., Lonnemann, J., Willmes, K., ... & Haase, V. G. (2011). A hand full of numbers: a role for offloading in arithmetics learning?. *Frontiers in psychology*, *2*, 368.*

Coulanges, L., Abreu-Mendoza, R. A., Varma, S., Uncapher, M. R., Gazzaley, A., Anguera, J., & Rosenberg-Lee, M. (2021). Linking inhibitory control to math achievement via comparison of conflicting decimal numbers. *Cognition*, *214*, 104767.**

Cowan, R., & Powell, D. (2014). The contributions of domain-general and numerical factors to third-grade arithmetic skills and mathematical learning disability. *Journal of educational psychology*, *106*(1), 214.*

Cragg, L., Keeble, S., Richardson, S., Roome, H. E., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition*, *162*, 12-26.**

De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). The relationship between symbolic and non-symbolic numerical magnitude processing skills and the typical and atypical development of mathematics: a review of evidence from brain and behavior. *Trends in Neuroscience & Education*, *2*(2), 48-55.

De Weerdt, F., Desoete, A., & Roeyers, H. (2013a). Behavioral inhibition in children with learning disabilities. *Research in developmental disabilities*, *34*(6), 1998-2007.*

De Weerdt, F., Desoete, A., & Roeyers, H. (2013b). Working memory in children with reading disabilities and/or mathematical disabilities. *Journal of learning disabilities*, *46*(5), 461-472.*

Dehaene, S. (2001). Précis of the number sense. *Mind & language*, *16*(1), 16-36.

Dennis, M., & Barnes, M. (2002). Math and Numeracy in Young Adults With Spina Bifida and Hydrocephalus. *Developmental Neuropsychology*, *21*(2), 141–155.

Diamond, A. (2013). Executive functions. *Annual review of psychology*, *64*, 135-168.

Donker, M., Kroesbergen, E., Slot, E., Van Viersen, S., & De Bree, E. (2016). Alphanumeric and non-alphanumeric Rapid Automatized Naming in children with reading and/or spelling difficulties and mathematical difficulties. *Learning and Individual Differences*, *47*, 80-87.*

Dowker, A., & Sheridan, H. (2022). Relationships between mathematics performance and attitude to mathematics: Influences of gender, test anxiety, and working memory. *Frontiers in Psychology*, *13*, 814992.**

Duchhardt, C., Jordan, A.-K., & Ehmke, T. (2017). Adults' Use of Mathematics and Its Influence on Mathematical Competence. *International Journal of Science and Mathematics Education*, *15*(1), 155–174.

Foley, A. E., Herts, J. B., Borgonovi, F., Guerriero, S., Levine, S. C., & Beilock, S. L. (2017). The Math Anxiety-Performance Link: A Global Phenomenon. *Current Directions in Psychological Science*, *26*(1), 52–58.

Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition?. *Journal of educational psychology*, *100*(1), 30.*

Gal, I., Grotlüschen, A., Tout, D., & Kaiser, G. (2020). Numeracy, adult education, and vulnerable adults: A critical view of a neglected field. *ZDM*, *52*(3), 377–394

Gatobu, S., Arocha, J. F., & Hoffman-Goetz, L. (2014). Numeracy and Health Numeracy Among Chinese and Kenyan Immigrants to Canada: Role of Math Self-Efficacy. *SAGE Open*, *4*(1), 215824401452143.

Geary, D. C. (1993). Mathematical disabilities: cognitive, neuropsychological, and genetic components. *Psychological bulletin*, *114*(2), 345.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of learning disabilities*, *37*(1), 4-15.

Geary, D. C. (2011). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of developmental and behavioral pediatrics: JDBP*, *32*(3), 250.

Geary, D. C., & Brown, S. C. (1991). *Cognitive Addition: Strategy Choice and Speed-of-Processing Differences in Gifted, Normal, and Mathematically Disabled Children*.

Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of experimental child psychology*, *77*(3), 236-263.*

Geary, D. C., Hoard, M. K., & Bailey, D. H. (2012b). Fact retrieval deficits in low achieving children and children with mathematical learning disability. *Journal of learning disabilities*, *45*(4), 291-307.*

Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of experimental child psychology*, *74*(3), 213-239.*

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of experimental child psychology*, *88*(2), 121-151.*

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child development*, *78*(4), 1343-1359.*

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012a). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: a five-year prospective study. *Journal of educational psychology*, *104*(1), 206.*

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental neuropsychology*, *33*(3), 277-299.*

Gilmore, C., Keeble, S., Richardson, S., & Cragg, L. (2015). The role of cognitive inhibition in different components of arithmetic. *Zdm*, *47*(5), 771-782.

Gilmore, C., Keeble, S., Richardson, S., & Cragg, L. (2015). The role of cognitive inhibition in different components of arithmetic. *Zdm*, *47*, 771-782.**

Göbel, S. M., Watson, S. E., Lervåg, A., & Hulme, C. (2014). Children's Arithmetic Development: It Is Number Knowledge, Not the Approximate Number Sense, That Counts. *Psychological Science*, *25*(3), 789–798.

Goffin, C., & Ansari, D. (2016). Beyond magnitude: Judging ordinality of symbolic number is unrelated to magnitude comparison and independently relates to individual differences in arithmetic. *Cognition*, *150*, 68-76.**

Grob, A., Hagmann-von Arx, P., IDS-2 (2022). Intelligence and Development Scales-2nd Edition. Edizione italiana Ferri, R., Casagrande, M., Rea, M.,. (a cura di). Hogrefe Editore.

Haist, F., Wazny, J. H., Toomarian, E., & Adamo, M. (2015). Development of brain systems for nonsymbolic numerosity and the relationship to formal math academic achievement. *Human brain mapping*, *36*(2), 804-826.

Hasher, L., & Zacks, R. T. (1979). Automatic and effortful processes in memory. *Journal of experimental psychology: General*, *108*(3), 356.

Hassinger-Das, B., Jordan, N. C., Glutting, J., Irwin, C., & Dyson, N. (2014). Domain-general mediators of the relation between kindergarten number sense and first-grade mathematics achievement. *Journal of experimental child psychology*, *118*, 78-92.

He, X., Zhou, X., Zhao, J., & Zhang, Y. (2021). Visual perception supports adults in numerosity processing and arithmetical performance. *Frontiers in Psychology*, *12*, 722261.**

Hemmings, B., Grootenboer, P., & Kay, R. (2011). Predicting mathematics achievement: the influence of prior achievement and attitudes. *International Journal of Science and Mathematics Education*, *9*(3), 691–705.

Higgins, J. P., Altman, D. G., Gøtzsche, P. C., Jüni, P., Moher, D., Oxman, A. D., ... & Sterne, J. A. (2011). The Cochrane Collaboration's tool for assessing risk of bias in randomised trials. *Bmj*, *343*.

Hoffmann, D., Mussolin, C., Martin, R., & Schiltz, C. (2014). The impact of mathematical proficiency on the number-space association. *PLoS one*, *9*(1), e85048.**

Imbo, I., & Vandierendonck, A. (2008). Effects of problem size, operation, and working-memory span on simple-arithmetic strategies: Differences between children and adults? *Psychological Research*, *72*(3), 331–346.

Kail, R., & Salthouse, T. A. (1994). Processing speed as a mental capacity. *Acta psychologica*, *86*(2-3), 199-225.

Kail, R., & Salthouse, T. A. (1994). Processing speed as a mental capacity. *Acta Psychologica*, *86*(2–3), 199–225.

Kalaman, D. A., & Lefevre, J. A. (2007). Working memory demands of exact and approximate addition. *European Journal of Cognitive Psychology*, *19*(2), 187-212.

Kaufmann, L., & von Aster, M. (2012). The diagnosis and management of dyscalculia. *Deutsches Ärzteblatt International*, *109*(45), 767.

Keeler, M. L., & Swanson, H. L. (2001). Does strategy knowledge influence working memory in children with mathematical disabilities?. *Journal of learning disabilities*, *34*(5), 418-434.*

Koponen, T., Salmi, P., Eklund, K., & Aro, T. (2013). Counting and RAN: Predictors of arithmetic calculation and reading fluency. *Journal of educational psychology*, *105*(1), 162.

Kosc, L. (1974). Developmental dyscalculia. *Journal of learning disabilities*, *7*(3), 164-177.

Krajewski, K., & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. Learning and Instruction, 19, 513-526

Kroesbergen, E. H., & van Dijk, M. (2015). Working Memory and Number Sense as Predictors of Mathematical (Dis-) Ability. *Zeitschrift für Psychologie*, *223*(2), 102-109.*

Kruk, R. S., & Luther Ruban, C. (2018). Beyond phonology: visual processes predict alphanumeric and nonalphanumeric rapid naming in poor early readers. *Journal of learning disabilities*, *51*(1), 18-31.

Kucian, K., & von Aster, M. (2015). Developmental dyscalculia. *European journal of pediatrics*, *174*(1), 1-13.

Kuhn, J. T., Ise, E., Raddatz, J., Schwenk, C., & Dobel, C. (2016). Basic Numerical Processing, Calculation, and Working Memory in Children With Dyscalculia and/or ADHD Symptoms. *Zeitschrift fur Kinder-und Jugendpsychiatrie und Psychotherapie*, *44*(5), 365-375.*

Kulp, M. T., Earley, M. J., Mitchell, G. L., Timmerman, L. M., Frasco, C. S., & Geiger, M. E. (2004). Are visual perceptual skills related to mathematics ability in second through sixth grade children?. *Focus on Learning Problems in Mathematics*, *26*(4), 44.

Lafay, A., St-Pierre, M. C., & Macoir, J. (2017). The Mental Number Line in Dyscalculia: Impaired Number Sense or Access From Symbolic Numbers?. *Journal of learning disabilities*, *50*(6), 672-683.*

Lambert, K., & Spinath, B. (2018). Conservation abilities, visuospatial skills, and numerosity processing speed: association with math achievement and math difficulties in elementary school children. *Journal of learning disabilities*, *51*(3), 223-235.*

Lang, M., Michelotti, C., Bardelli, E., (2015). Wais-IV. Wechsler adult intelligence-Scale-IV: lettura dei risultati e interpretazione clinica. *Raffaello Cortina Editore*

Laski, E. V., & Dulaney, A. (2015). When prior knowledge interferes, inhibitory control matters for learning: The case of numerical magnitude representations. *Journal of Educational Psychology*, *107*(4), 1035.**

Lemaire, P., & Lecacheur, M. (2011). Age-related changes in children's executive functions and strategy selection: A study in computational estimation. *Cognitive Development*, *26*(3), 282-294.

Liberati, A., Altman, D. G., Tetzlaff, J., Mulrow, C., Gøtzsche, P. C., Ioannidis, J. P., ... & Moher, D. (2009). The PRISMA statement for reporting systematic reviews and meta-analyses of studies that evaluate health care interventions: explanation and elaboration. *Journal of clinical epidemiology*, *62*(10), e1-e34.

Lucangeli, D., & Cabrele, S. (2006). Mathematical difficulties and ADHD. *Exceptionality*, *14*(1), 53-62.

MacLeod, C. M. (1992). The Stroop task: The "gold standard" of attentional measures. *Journal of Experimental Psychology: General*, *121*(1), 12.

Mammarella, I. C., Caviola, S., Giofrè, D., & Szűcs, D. (2018). The underlying structure of visuospatial working memory in children with mathematical learning disability. *British Journal of Developmental Psychology*, *36*(2), 220-235.*

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (Dyscalculia): Impaired numerical acuity contributes to MLD. Child Development, 82, 1224-1237.

Mazzocco, M. M., & Grimm, K. J. (2013). Growth in rapid automatized naming from grades K to 8 in children with math or reading disabilities. *Journal of learning disabilities*, *46*(6), 517-533.*

Mazzocco, M. M., & Kover, S. T. (2007). A longitudinal assessment of executive function skills and their association with math performance. *Child neuropsychology*, *13*(1), 18-45.*

McCloskey, M. (2007). Quantitative literacy and developmental dyscalculias. In D. B. Berch & M. M. M. Mazzocco (Eds.), Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities (p. 415–429). Paul H. Brookes Publishing.

McDonald, P. A., & Berg, D. H. (2018). Identifying the nature of impairments in executive functioning and working memory of children with severe difficulties in arithmetic. *Child Neuropsychology*, *24*(8), 1047-1062.*

Miller, H., & Bichsel, J. (2004). Anxiety, working memory, gender, and math performance. *Personality and Individual Differences*, *37*(3), 591-606.**

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H.-C. (2011). Early place-value understanding as a precursor for later arithmetic performance—A longitudinal study on numerical development. *Research in Developmental Disabilities*, *32*(5), 1837–1851.

Moher, D., Liberati, A., Tetzlaff, J., Altman, D. G., & Prisma Group. (2009). Reprint—preferred reporting items for systematic reviews and meta-analyses: the PRISMA statement. *Physical therapy*, *89*(9), 873-880.

Monaco, M., Costa, A., Caltagirone, C., & Carlesimo, G. A. (2013). Forward and backward span for verbal and visuo-spatial data: Standardization and normative data from an Italian adult population. *Neurological Sciences*, *34*(5), 749–754.

Montesano, L., Valenti, A., Cornoldi, C., (2021). LSC-SUA: Prove di lettura, comprensione del testo, scrittura e calcolo. Batteria per la valutazione dei DSA e altri disturbi in studenti universitari e adulti. Trento. Erickson

Moura, R., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: The role of working memory and procedural and lexical competencies. *Journal of Experimental Child Psychology*, *116*(3), 707-727.*

Moura, R., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: The role of working memory and procedural and lexical competencies. *Journal of Experimental Child Psychology*, *116*(3), 707–727.

Murphy, M. M., Mazzocco, M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of learning disabilities*, *40*(5), 458-478.

Neath, I., & Surprenant, A. M. (2019). Set size and long-term memory/lexical effects in immediate serial recall: Testing the impurity principle. *Memory & cognition*, *47*(3), 455-472.

Norris, J. E., & Castronovo, J. (2016). Dot display affects approximate number system acuity and relationships with mathematical achievement and inhibitory control. *PloS one*, *11*(5), e0155543.**

Ojose, B. (2011). Mathematics literacy: Are we able to put the mathematics we learn into everyday use. *Journal of Mathematics Education*, *4*(1), 89-100.

Orrantia, J., Muñez, D., Matilla, L., Sanchez, R., San Romualdo, S., & Verschaffel, L. (2019). Disentangling the mechanisms of symbolic number processing in adults' mathematics and arithmetic achievement. *Cognitive Science*, *43*(1).**

Osmon, D. C., Smerz, J. M., Braun, M. M., & Plambeck, E. (2006). Processing abilities associated with math skills in adult learning disability. *Journal of Clinical and Experimental neuropsychology*, *28*(1), 84-

95.**

Page MJ, McKenzie JE, Bossuyt PM, Boutron I, Hoffmann TC, Mulrow CD, et al. The PRISMA 2020 statement: an updated guideline for reporting systematic reviews. BMJ 2021;372:n71. doi: 10.1136/bmj.n71

Parsons, S., & Bynner, J. (1997). Numeracy and Employment. *Education+ Training*, *39*(2), 43-51.

Passolunghi, M. C. (2011). Cognitive and emotional factors in children with mathematical learning disabilities. *International Journal of Disability, Development and Education*, *58*(1), 61-73.*

Passolunghi, M. C., & Lanfranchi, S. (2012). Domain‐specific and domain‐general precursors of mathematical achievement: A longitudinal study from kindergarten to first grade. *British Journal of Educational Psychology*, *82*(1), 42-63.

Passolunghi, M. C., & Mammarella, I. C. (2012). Selective spatial working memory impairment in a group of children with mathematics learning disabilities and poor problem-solving skills. *Journal of learning disabilities*, *45*(4), 341-350.*

Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of experimental child psychology*, *88*(4), 348-367.*

Peng, P., Congying, S., Beilei, L., & Sha, T. (2012). Phonological storage and executive function deficits in children with mathematics difficulties. *Journal of Experimental Child Psychology*, *112*(4), 452-466.*

Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics. *Journal of Educational Psychology*, *108*(4), 455–473

Peng, P., Wang, C., & Namkung, J. (2018). Understanding the cognition related to mathematics difficulties: A meta-analysis on the cognitive deficit profiles and the bottleneck theory. *Review of Educational Research*, *88*(3), 434-476.

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. Trends in Cognitive Sciences, 14, 542-551.

Pieters, S., Desoete, A., Roeyers, H., Vanderswalmen, R., & Van Waelvelde, H. (2012). Behind mathematical learning disabilities: What about visual perception and motor skills?. *Learning and Individual Differences*, *22*(4), 498-504.

Platt, A. (2017). ADHD and math disabilities: Cognitive similarities and instructional interventions.

Poulsen, M., & Elbro, C. (2013). What's in a name depends on the type of name: The relationships between semantic and phonological access, reading fluency, and reading comprehension. *Scientific Studies of Reading*, *17*(4), 303-314.

Purpura, D. J., Daya, E., Napolia, A. R., & Hartbc, S. A. (2017). Identifying Domain-General and Domain-Specific Predictors of Low Mathematics Performance: A Classification and Regression Tree Analysis. *Journal of Numerical Cognition*, *3*(2), 365-399.

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and individual differences*, *20*(2), 110-122.

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, *20*(2), 110–122.

Raghubar, K., Cirino, P., Barnes, M., Ewing-Cobbs, L., Fletcher, J., & Fuchs, L. (2009). Errors in multi-digit arithmetic and behavioral inattention in children with math difficulties. *Journal of learning disabilities*, *42*(4), 356-371.*

Reimann, G., Gut, J., Frischknecht, M. C., & Grob, A. (2013). Memory abilities in children with mathematical difficulties: Comorbid language difficulties matter. *Learning and individual differences*, *23*, 108-113.*

Rey, A. (1941). L'examen psychologique dans les cas d'encéphalopathie traumatique.(Les problems.). *Archives de psychologie.*

Reyna, V. F., & Brainerd, C. J. (2007). The importance of mathematics in health and human judgment: Numeracy, risk communication, and medical decision making. *Learning and Individual Differences*, *17*(2), 147-159.

Reynvoet, B., Ribner, A. D., Elliott, L., Van Steenkiste, M., Sasanguie, D., & Libertus, M. E. (2021). Making sense of the relation between number sense and math. *Journal of Numerical Cognition*, *7*(3), 308-327.**

Roelofs, A. (2006). Context effects of pictures and words in naming objects, reading words, and generating simple phrases. *Quarterly Journal of Experimental Psychology*, *59*(10), 1764-1784.

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, *102*(3), 361-395.*

Salthouse, T. A. (1996). The processing-speed theory of adult age differences in cognition. *Psychological review*, *103*(3), 403.

Salthouse, T. A. (2000). Aging and measures of processing speed. *Biological psychology*, *54*(1-3), 35-54.

Shalev, R. S., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: A prospective six-year follow-up. *Developmental Medicine & Child Neurology*, *47*(2), 121-125.

Shallice, T. (1982). Specific impairments of planning. *Philosophical Transactions of the Royal Society of London. B, Biological Sciences,* 298(1089), 199-209.

Shanahan, M. A., Pennington, B. F., Yerys, B. E., Scott, A., Boada, R., Willcutt, E. G., ... & DeFries, J. C. (2006). Processing speed deficits in attention deficit/hyperactivity disorder and reading disability. *Journal of abnormal child psychology*, *34*(5), 584-601.

Silver, A. M., Elliott, L., Reynvoet, B., Sasanguie, D., & Libertus, M. E. (2022). Teasing apart the unique contributions of cognitive and affective predictors of math performance. *Annals of the New York Academy of Sciences*, *1511*(1), 173-190.**

Skagerlund, K., Östergren, R., Västfjäll, D., & Träff, U. (2019). How does mathematics anxiety impair mathematical abilities? Investigating the link between math anxiety, working memory, and number processing. *PloS one*, *14*(1), e0211283.**

Slot, E. M., van Viersen, S., de Bree, E. H., & Kroesbergen, E. H. (2016). Shared and unique risk factors underlying mathematical disability and reading and spelling disability. *Frontiers in psychology*, *7*, 803.*

Sörqvist, P. (2014). On interpretation and task selection in studies on the effects of noise on cognitive performance. *Frontiers in psychology*, *5*, 1249.

Sortor, J. M., & KULP, M. T. (2003). Are the results of the Beery-Buktenica Developmental Test of Visual-Motor Integration and its subtests related to achievement test scores?. *Optometry and vision science*, *80*(11), 758-763.

Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of educational Research*, *76*(2), 249-274.

Swanson, H. L., Jerman, O., & Zheng, X. (2009). Math disabilities and reading disabilities: Can they be separated?. *Journal of Psychoeducational Assessment*, *27*(3), 175-196.

Szűcs, D. (2016). Subtypes and comorbidity in mathematical learning disabilities: Multidimensional study of verbal and visual memory processes is key to understanding. *Progress in brain research*, *227*, 277-304.

Szűcs, D., & Goswami, U. (2013). Developmental dyscalculia: Fresh perspectives. *Trends in*

Neuroscience and Education, *2*(2), 33-37.

Szucs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyscalculia is related to visuo-spatial memory and inhibition impairment. *cortex*, *49*(10), 2674-2688.*

Taub, G. E., Keith, T. Z., Floyd, R. G., & Mcgrew, K. S. (2008). Effects of general and broad cognitive abilities on mathematics achievement. *School Psychology Quarterly*, *23*(2), 187–198.

Threlfall, J. (2002). Flexible mental calculation. *Educational studies in Mathematics*, *50*, 29-47.

Threlfall, J. (2009). Strategies and flexibility in mental calculation. *ZDM*, *41*(5), 541–555.

Unterrainer, J. M., Rahm, B., Leonhart, R., Ruff, C. C., & Halsband, U. (2003). The Tower of London: The impact of instructions, cueing, and learning on planning abilities. *Cognitive Brain Research*, *17*(3), 675–683.

Valleé-Tourangeau, F. (2013). Interactivity, efficiency, and individual differences in mental arithmetic. *Experimental psychology*.**

Van den Bussche, E., Vanmeert, K., Aben, B., & Sasanguie, D. (2020). Too anxious to control: the relation between math anxiety and inhibitory control processes. *Scientific Reports*, *10*(1), 19922.**

Vanbinst, K., Ghesquiere, P., & De Smedt, B. (2014). Arithmetic strategy development and its domain-specific and domain-general cognitive correlates: A longitudinal study in children with persistent mathematical learning difficulties. *Research in developmental disabilities*, *35*(11), 3001-3013.

Vanbinst, K., Ghesquière, P., & De Smedt, B. (2015). Does numerical processing uniquely predict first graders' future development of single-digit arithmetic?. *Learning and Individual Differences*, 37, 153-160.

Vigna, G., Ghidoni, E., Burgio, F., Danesin, L., Angelini, D., Benavides-Varela, S., & Semenza, C. (2022). Dyscalculia in early adulthood: implications for numerical activities of daily living. *Brain Sciences*, *12*(3), 373.

Wang, L., & Carr, M. (2020). Understanding the Relationship between Spatial Ability and Mathematics Achievement through the Lens of Information Processing Theory. *North American Journal of Psychology*, *22*(2).**

Webster, R. E. (1980). Short-Term Memory in Mathematics-Proficient and Math Ematics-Disabled Students as a Function of Input-Modality/Output-Modality Pairings. *The Journal of Special Education*, *14*(1), 67-78.*

Willcutt, E. G., Petrill, S. A., Wu, S., Boada, R., DeFries, J. C., Olson, R. K., & Pennington, B. F. (2013). Comorbidity between reading disability and math disability: Concurrent psychopathology, functional impairment, and neuropsychological functioning. *Journal of learning disabilities*, *46*(6), 500-516.*

Wilson, K. M., & Swanson, H. L. (2001). Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit?. *Journal of Learning disabilities*, *34*(3), 237-248.**

Wolf, M., & Bowers, P. G. (1999). The double-deficit hypothesis for the developmental dyslexias. *Journal of educational psychology*, *91*(3), 415.

Wolf, M., Bowers, P. G., & Biddle, K. (2000). Naming-speed processes, timing, and reading: A conceptual review. *Journal of learning disabilities*, *33*(4), 387-407.

Wong, T. T. Y., & Chan, W. W. L. (2019). Identifying children with persistent low math achievement: The role of number-magnitude mapping and symbolic numerical processing. *Learning and Instruction*, *60*, 29-40.*

Zaleznik, E., Comeau, O., & Park, J. (2023). Arithmetic operations without symbols are unimpaired in adults with math anxiety. *Quarterly Journal of Experimental Psychology*, *76*(6), 1264-1274.**

Zentall, S. S., Smith, Y. N., Lee, Y. B. B., & Wieczorek, C. (1994). Mathematical outcomes of attention-deficit hyperactivity disorder. *Journal of Learning Disabilities*, *27*(8), 510-519.

Zhang, J., Zhao, N., & Kong, Q. P. (2019). The Relationship Between Math Anxiety and Math Performance: A Meta-Analytic Investigation. *Frontiers in Psychology*, *10*, 1613.

Zhang, X., Räsänen, P., Koponen, T., Aunola, K., Lerkkanen, M. K., & Nurmi, J. E. (2020). Early cognitive precursors of children's mathematics learning disability and persistent low achievement: A 5-year longitudinal study. *Child development*, *91*(1), 7-27.*

APPENDIX

I. Domain-general cognitive skills and mathematical difficulties: a systematic review of the literature

- Appendix A1. Characteristics of the studies included in the qualitative analysis
- Appendix A2. Tools, domains, and criteria used to define the group with MD in included studies

II. Unraveling Mathematical Competencies in School-Age Children: The Influence of Domain-General Abilities

- Appendix B1. Correlational Matrix considering Age and performance in Mathematical tasks in school-age children
- Appendix B2. Correlational Matrix considering Age and performance in cognitive variables in school-age children

III. Evaluating Cognitive Predictors of Mathematical Proficiency in Young Healthy Adults: A Systematic Literature Review

- Appendix C1. Characteristics of the study included in the qualitative analysis

IV. Exploring the Relationship between Cognitive Functioning, Emotional Aspects, and Numeracy Skills: Implications for Young Adults

- Appendix D1. Correlational Matrix considering Age and performance in Mathematical tasks in young adults.
- Appendix D2. Correlational Matrix considering Age and performance in cognitive variables in young adults