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Review

Eclipses: A Brief History of Celestial Mechanics, Astrometry and Astrophysics

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Abstract: Solar and lunar eclipses are indeed the first astronomical phenomena which have been recorded since very early antiquity. Their periodicities gave birth to the first luni-solar calendars based on the Methonic cycle since the sixth century before Christ. The Saros cycle of 18.03 years is due to the Chaldean astronomical observations. Their eclipses' observations reported by Ptolemy in the Almagest (Alexandria of Egypt, about 150 a.C.) enabled modern astronomers to recognize the irregular rotation rate of the Earth. The Earth's rotation is some hours in delay after the last three millenia if we use the present rotation to simulate the 721 b.C. total eclipse in Babylon. This is one of the most important issues in modern celestial mechanics, along with the Earth's axis nutation of 18 yr (discovered in 1737), precession of 25.7 Kyr (discovered by Ipparchus around 150 b.C.) and obliquity of 42 Kyr motions (discovered by Arabic astronomers and assessed from the Middle Ages to the modern era, IX to XVIII centuries). Newtonian and Einstenian gravitational theories explain fully these tiny motions, along with the Lense–Thirring gravitodynamic effect, which required great experimental accuracy. The most accurate lunar and solar theories, or their motion in analytical or numerical form, allow us to predict—along with the lunar limb profile recovered by a Japanese lunar orbiter—the appearance of total, annular solar eclipses or lunar occultations for a given place on Earth. The observation of these events, with precise timing, may permit us to verify the sphericity of the solar profile and its variability. The variation of the solar diameter on a global scale was claimed firstly by Angelo Secchi in the 1860s and more recently by Jack Eddy in 1978. In both cases, long and accurate observational campaigns started in Rome (1877–1937) and Greenwich Observatories, as well as at Yale University and the NASA and US Naval Observatory (1979–2011) with eclipses and balloon-borne heliometric observations. The IOTA/ES and US sections as well as the ICRA continued the eclipse campaigns. The global variations of the solar diameter over a decadal timescale, and at the millarcsecond level, may reflect some variation in solar energy output, which may explain some past climatic variations (such as the Allerød and Dryas periods in Pleistocene), involving the outer layers of the Sun. "An eclipse never comes alone"; in the eclipse season, lasting about one month, we can have also lunar eclipses. Including the penumbral lunar eclipses, the probability of occurrence is equi-distributed amongst lunar and solar eclipses, but while the lunar eclipses are visible for a whole hemisphere at once, the solar eclipses are not. The color of the umbral shadow on the Moon was known since antiquity, and Galileo (1632, Dialogo sopra i Massimi Sistemi del Mondo) shows clearly these phenomena from copper color to a totally dark, eclipsed full Moon. Three centuries later, André Danjon was able to correlate that umbral color with the 11-year cycle of solar activity. The forthcoming American total solar eclipse of 8 April 2024 will be probably the eclipse with the largest mediatic impact of the history; we wish that also the scientific impulse toward solar physics and astronomy will be relevant, and the measure of the solar diameter with Baily's beads is indeed one of the topics significantly related to the Sun–Earth connections.

Keywords: eclipses, lunar and solar; occultations; Baily's beads; solar diameter

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1. Introduction

The Sun is the closest star to our planet. The accurate knowledge of its physics is also knowledge of our living environment because of the strong energetic relationship with our climate. Most of the new physics of the XX century is involved in the study of the Sun: General Relativity, Nuclear Physics and Neutrino phase mixing physics. These three domains of modern physics considered the Sun a privileged laboratory. The secular variations of the solar diameter as well as the unexplicable great dimming of solar activity, known as Maunder minimun of 1645–1715 or the great maxima and minima identified in the past millenia [1], do not find a suitable predicting model. In this paper, we briefly review the history of the eclipses' science, both solar and lunar, from antiquity to the present days, focusing on the solar diameter measurements at the lunar shadow's limits through Baily's beads. The variability of the solar diameter is sustained by several accurate measures in the last 130 years as well as from historical eclipses since 1567. In this general context, as celestial mechanics, we consider mainly the geometrical models yielding the predictions of the solar and lunar positions, especially the ones (Methon and Saros "circular motions") which predict eclipses within a given range of decades or centuries.

2. Historical Solar Eclipses and Implications in Celestial Mechanics

2.1. Chaldean Astronomy

For millenia, mankind turned his gaze to the sky to admire and study its wonders. Of all the phenomena, solar and lunar eclipses are those that have influenced the mythology, religion and science of early civilizations the most. Chaldeans identified the periodic recurrence of lunar and solar eclipses, calculating a cycle of 18 years, 10/11 days, 8 h and 42 min, the Saros. Through prolonged and careful observations, the Chaldeans were not only able to predict the following eclipses; they also noted that the same eclipse occurred in the same place after 3 Saros, an Exeligmos cycle of 54 years and 34 days, as also described by Ptolemy in his writings [2–4]. It is safe to assume that the geometrical nature of the eclipses was already clear at that time. In general, the occultation of a body (the Sun, a star or a planet) by the Moon or an asteroid provides information on the shape of the darkening body itself and the relative distance between it and the darkened one (Figure 1). Also, the occultation of the Sun by the Earth produces, on the Moon, a shadow, which was called "defectus", since the Moon is not occulted to our sight.

Figure 1. Venus' occultation of 9 November 2023 at 10:09:48 UT observed in Rome, via Fonteiana 111.

2.2. Occultation'S Astronomy

Figure 1 helps explain how occultations are related to celestial mechanics as it displays Venus' occultation on 9 November 2023. The instant of the disappearance of Venus has been determined within ± 0.01 s of accuracy, which allows for locating the Moon with ± 10 m of space accuracy and 3 millarcsec of angular accuracy [5].

Asteroidal occultations are currently used to assess the asteroidal orbits with great accuracy and, in the case of some stars with large angular diameters like Betelgeuse (60 mas) and Regulus (1.7 mas), also their limb-darkening functions [6]. The eclipses of the Galilean satellite io by Jupiter offered the possibility to measure the speed of light by Roemer on his and Cassini's data (1676) [7].

2.3. Eclipses' Astronomy and Earth's Rotation Rate

In the Sun–Earth–Moon system, two types of eclipses are recognized: lunar and solar. The first occurs when the Moon, whose orbital plane is inclined by 5.9° compared to the ecliptic, passes through one of the intersection nodes of the orbit in opposition to the Sun while being hit by the cone of shadow projected by our planet. These eclipses are visible from any point on the Earth's hemisphere where the Moon is above the horizon. Solar eclipses, on the other hand, occur when the passage through one of the nodes occurs with the Moon in conjunction. The Moon projects its shadow on our planet, and due to the relative size and distances between the Sun, the Earth and itself, the shadow cone can vary in size and duration, giving information on the position of the three bodies in space. The most studied type of eclipse is the total solar eclipse, which is visible when the Earth–Moon distance is such that the angular diameter of our satellite is slightly greater than that of the Sun. This suggestive event has allowed also the first studies on the solar corona. The shadow cone generated during these phenomena covers a narrow band of the Earth's surface, less than few hundreds of kilometers wide, where it is possible to observe a sudden and great variation in brightness of the sky (from 10 magnitudes to 10,000 times in intensity). Ptolemy reported in the Almagest (150 a.C.) the total solar eclipse recorded by the Chaldeans in 721 b.C. [8,9]. The occurrence of this phenomenon in Babylon tells us that the current Earth's rotation rate changed during the last 27 centuries due to the ongoing post-glacial isostatic rebound of the continents [10–12].

2.4. Earth's Axis Millennial Motions

The luni-solar precession (or "of the equinoxes") is responsible for the double-conical movement of the Earth's axis. With a period of approximately 25,700 years, the combined gravitational action of the Sun and Moon on the equatorial bulge tends to align the planet's rotation axis along the direction perpendicular to the ecliptic plane. This gravitational pull is opposed by the rotational movement of the Earth, which keeps the angular momentum fixed. The resulting effect is a shift of the equinoxes by 20 arcminutes westwards [13]. Together with the precession of the equinoxes, and again due to the effect of the tidal forces deriving from the action of the Sun and the Moon, a second motion of our planet is observed: the nutation, which was discovered in 1737 by James Bradley. Nutation is characterized by a subtle wobble in the Earth's rotation axis, which follows a cycle of approximately 18.6 years. The maximum amplitude of nutation in ecliptic latitude is 9 arcseconds. The Earth's obliquity also variates in 42 Ky of $\pm$ 2°. Since the dawn of observative astronomy, the Earth's axis changed from about 24° to the present 23.42°.

2.5. Solar Astrometry: The Eclipse of Clavius

Another example of deducing changes in the properties of a celestial body using an eclipse is represented by the observation of the annular–total solar eclipse in Rome in 1567 AD by the Jesuit mathematician Clavius [14]. He personally observed the 1567 eclipse (9 May) and the eclipse of Coimbra in 1560 (21 August), and he reported, for the one in 1567, the presence of a clear disk of light around the Moon. Clavius deduced that the angular diameter of the Sun was greater than that of our satellite, which went against Ptolemy and medieval Arab astronomers. The nature of the ring of light observed by Christopher Clavius has been investigated by Kepler and by subsequent studies [15,16]. The occurrence of an annular eclipse in 1567 in Rome, instead of total, as predicted by the ephemerides for that day in Rome using current parameters, remains still intriguing. The solar diameter should have been 0.2% larger than expected, and the 1567 eclipse's discussions gave birth to the studies on the secular variations of the diameter of the Sun [17]. It is therefore clear that not only do eclipses give important information on the celestial mechanics of the bodies involved, but they can also shed light on their properties and characteristics and how these influence our planet.

3. Lunar Eclipses from the Metonic Cycle to the Solar Corona Reflection

The celestial mechanics of the Moon are at the basis of civil calendars because, since ancient times, it has been necessary to identify a univocal criterion for organizing religious celebrations and planning social events. The Babylonians were the first to create a calendar based on the lunar motion [18]. However, they were unable to reconcile the discrepancies between the tropical year and the draconic year. The civil year has to correspond to the tropical one, which lasts 365.242 days, or the time between two consecutive Spring equinoxes. The draconic year is the time taken by the Sun to return to the same node of the lunar orbit. Since the line of intersection of the ecliptic plane with that of the lunar orbit moves retrogradely, the position of the nodes moves back annually by $19°33'$, and the Sun does not meet the same node again after a tropical year but rather approximately 19 days earlier, i.e., every 346.62 days. The first to create a synchronized calendar was the Greek astronomer Meton (432 b.C.), who, through the observation of 19 consecutive solar years (corresponding to approximately 235 lunar months and 6940 days) and starting from the Saros cycle, created a calendar so that the motion of the two celestial bodies would return in phase. Numerous calendars from the classical and medieval era are based on it and were later modified according to the uses and needs of each culture [19–21]. The discovery of the Metonic cycle was probably the necessary stimulus to introduce leap years to recover the decimal part of the solar year which is suppressed in the calculation of the civil year, which is made necessarily of an integer number of days. There are testimonies of reform attempts already in the Ptolemaic Egypt [21], but the introduction of a temporally synchronized system is due to Julius Caesar. He, just after having been in Alexandria, introduced the Julian calendar to the Roman Senate in 46 b.C., and it became the basis of the modern Western calendar.

3.1. The Metonic Cycle

The lunar calendar and the Metonic cycle were used by the Catholic Church for the Easter's Computus, the determination of the day of Easter, and it is still used after the Gregorian reformation of the calendar. Although in continental Europe the scientific debate had slowed down in the last centuries of the Roman Empire, the Church made numerous efforts to codify the calculation of the celebration of the resurrection. The principle-rule that fixes the date of Christian Easter was established following the Council of Nicea (325 AD) [22]: Easter falls on the Sunday following the first full moon of spring (at the time of the first calculations, the equinox fell on 21 March, which therefore became the reference date). Consequently, the Easter date is always included in the period from 22 March to 25 April. It is important to point out that among the first controversies in the Catholic Church, there are the ones on the Easter's algorithms based on the local traditions in the geographical area concerned [23–26].

3.2. Lunar Secular Motions

The lunar motion has more than 400 terms in the modern ephemerides, but for eclipses and solar astrometry, we recall the precession of the lunar axis and the variation in the eccentricity of the orbit of our satellite. The improvement of telescopes in the XVII century allowed a more accurate study of such motions. Among these there are librations: apparent movements of the Moon that allow an observer on Earth to see slightly different portions of the lunar surface each time. These variations are caused by the fact that the Moon rotates around its axis at a constant rate but revolves around the Earth at a variable rate, being in an elliptical orbit and moving faster when it is closer to the Earth and slower when it is further away from it. The final effect is that instead of half, only 41% of the lunar surface is always visible, another 41% is always hidden, and a further 18% oscillates between the visible and hidden portions of the surface bringing to 59% the total visible surface of the Moon over the course of an entire libration cycle [27]. The total oscillation effect is given by the contribution of the two previously introduced motions: the inclination of the lunar axis and the libration in latitude discovered by Galileo in 1632 and the variations in the eccentricity of the orbit and the libration in longitude discovered by Hevelius in 1648 [28,29]. The lunar profile during solar total eclipses is of course subjected to all these motions.

3.3. Lunar Colors during the Eclipses from Galileo to Danjon

Galileo Galilei in his writings summarized the discoveries made possible by the telescope, undermining the Ptolemaic certainties on the celestial order: the existence of the seas and the lunar craters and the satellites orbiting around Jupiter were powerful arguments in favor of the Copernican theory [30–33]. In the "Saggiatore" (1623), dedicated to the nature of the comets as real celestial bodies, Galileo also dealt with the colors assumed by the darkened part of the Moon [34,35], as can be seen in Figure 2. It was only in 1921 that the astronomer André-Louis Danjon, through an intensity scale of the eclipsed Moon brightness, was able to connect such phenomenon to the solar activity [36]. The color variations of the shadowed area of the Moon are certainly influenced by atmospheric conditions and by the light reflected from the Earth's surface, but they depend above all on the extension of the solar corona associated with the various phases of our star's activity cycle, as already understood by Angelo Secchi in the second half of the XIX century [37–41].

Figure 2. Partial lunar eclipse of 29 October 2023 at 20:36:37 UT, observed in Ostia.

4. The Sun–Earth Connection

Solar eclipses have been for a long time the only means available for studying the solar corona. This changed with the advent of Bernard Lyot's coronagraph, which is an instrument used to simulate eclipses that can also be employed on satellites dedicated to solar observation, such as the SkyLab and SOHO [42–44]. The study of the Sun from space has also allowed researchers to shed light on other phenomena known since ancient times: the polar auroras. The interaction of high-energy charged particles, components of the solar wind, with the Earth's ionosphere gives life, through radiative de-excitation mechanisms, to the characteristic colored and bright bands in the polar sky, where the shielding effect of the Earth's magnetic field is less effective. The first to relate solar activity to auroras was the astronomer R. C. Carrington, who observed a brief bright light in a group of unusually large sunspots on 31 August 1859 [45–47]. On that day, polar auroras were visible all across the planet, causing the generation of spurious currents in electric circuits, damaging them [48]. This event has been later associated with a powerful solar flare, exactly Earth-facing: it is known as the Carrington event and was also measured by Angelo Secchi's magnetometers in Rome [49]. Since then, studying and monitoring sunspots has played a particularly important role in predicting this type of particles storms, as it would allow us to take the right countermeasures to protect our, now very dense, telecommunications network [50]. Although the nature of sunspots is not yet fully understood, nowadays, we know that they are formed following a decrease in the energy arriving from the star's core to the surface due to a variation in the magnetic field caused by the differential rotation of the celestial body. In these areas of lower temperature (around 4000 K compared to the 6000 K of the surrounding photosphere) a self-sustaining mechanism is established, similar to that of hurricanes, due to the strong magnetic fields. The occurrence of sunspots is linked to the main eleven-year cycle of solar activity; their existence has been known since 800 b.C. [51] and has been recorded since then by various astronomers (Galileo included) [52,53]. They are therefore a good indicator of solar activity throughout the centuries, and their observation will be useful to better understand the internal mechanisms of our star and how, and if, they affect our planet. Cycles longer than the 11-years have also been identified (80–100 years, 800–1200 years, etc.), and they are also linked to the planetary periodicities [54].

5. Recent Eclipses: Preliminary Results on the Solar Radius

In the last few decades, the number of eclipse observers has constantly increased, along with the quality level of their observations. After the first missions to the shadows' limbs in the 1970s [55,56], the solar diameter has been monitored with this method several times; an example is in Figure 3. An Atlas of observed Baily's beads [57] was collected and used to recover the solar limb-darkening function [58] and its inflexion point, which unifies this approach with the solar limb definition used in the oblateness measures since Robert Dicke's in 1967 [59–61].

Figure 3. The long-lasting final Baily's bead at the shadow's limit in Egypt as observed by Zawyet al Mahtallah on 29 March 2006, visible with the full corona [62,63].

On the lunar shadow's limits, the Baily's beads last longer, making this method more accurate, even if it is subjected to the filter's cutoff [63]. For this reason, the International Occultation Timing Association (IOTA) adopted in 2010 a standard filter to fix also the wavelength at which the diameter is measured (520 nm), as in the Solar Disk Sextant SDS [64] balloon-borne mission.

5.1. Solar Diameter's Standard Value

In the last decade, the eclipses, lunar and solar showed a solar radius around 960.0″, which is larger than the 959.63″ IAU standard value [65] at 1 AU. The values of the solar diameter here discussed all refer to 1 AU, so any variation of the Earth–Sun distance along the year or the eccentricity's changes in the orbit does not affect these values; only intrinsic variations do.

5.2. Solar Diameter's Variations

The variation of the solar diameter was confirmed also with photometers' arrays displaced from French missions from 2012 to 2015 [66] in another experiment designed to avoid the cutoff problem. During 2023, two eclipse have occurred: a hybrid on 20 April [67] and an annular on 14 October. In both cases, the preliminary results confirm the solar radius to be in the 960.0″ range [68], with a technique also used in 2022 as can be seen in Figure 4 The statement about the need to change the IAU standard value of 959.63″ dates back to 1891, and it does not give enough importance to the real change of the solar diameter. The measures of 1891 were performed with excellent optics, and careful methods, and they were confirmed by many eclipses, lunar and solar' analyses and by SDS flights. The solar diameter was measured within $\pm 0.02''$ and increased from 959.63″ (1992) to 959.86″ (2011) [64]. The increase in the solar diameter was also detected with the Danjon-modified solar astrolabes (1975–2009) in France, Algery, Turkey, Spain and Brazil [69,70]. Another instrument, the reflecting heliometer, was developed in Rio de Janeiro to monitor the solar diameter especially during the coronal mass ejections, which are the major events in space weather [71]. Other measures in different wavelengths [72] confirmed the diameter's enlargement. The stellar standard model does not explain global solar diameter oscillations on yearly scales but rather only the helioseismic waves of 5 min [73].

Figure 4. The solar partial eclipse of 25 October 2022 at 10:54:20 UT on the Clementine meridian line of 1702 in Rome [74]. This eclipse was a real rare partial one for all the World [75].

6. Conclusions and Perspectives

The solar activity was identified in the XIX century with the 11-year sunspots period and was later corrected to a 22-year cycle after Hale magnetic observations [76]. The corresponding coronal activity was first identified by Secchi [49] and subsequently confirmed by the observations during the eclipses, lunar and solar and with coronographs. Later, at the birth of helioseismology, global solar oscillations of 5 min were detected [77]. A gap of seven decades (1645–1715) in the solar activity was detected by Maunder [78], but the first idea to correlate sunspots activity to the solar diameter variations came out after 1978 with the analysis of Clavius' eclipse of 1567 observed in Rome. Since then, the total eclipses, lunar and solar' accounts have been exploited to measure the solar diameter thanks to the rapid luminosity variation occurring near the solar limb in the last arcseconds. Ancient data were obtained with the naked eye, while photo, video and electronic devices prevailed in the last few decades. General consensus in the present day (2023) establishes the solar radius at 960.0″ not only from eclipse data, and the solar radius increased by about 0.4″ over the last 130 years. Ancient and recent planetary transits have been used to assess this statement also with SOHO and SDO satellites [79,80]; however, there is a lack of consensus here. The instruments devoted to real-time solar diameter's measurement, the reflecting heliometer of Rio de Janeiro and the solar astrolabes in Nice (DORAYSOL) and Rio are currently off duty. There is no news from the other instrument of Nice/Calern Observatory: Picard-Sol [81]. The fascinating eclipses' missions, once possible as a national effort, are now operated also by an increasing number of valent amateur and professional astronomers, who are inspired in their actions by the unforgettable Jay Myron Pasachoff (1943–2022) and Serge Koutchmy (1940–2023) who were able to set new astrophysical experiments for each new eclipse since their first observational missions [82]. The quality of the new incoming data is guaranteeing the validity of the forthcoming research on the secular variability of the solar diameter and on the correlation of the diameter with other observables more closely related with the Earth's climate variability.

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