Coexistence of collapsed-snaking-related dark and bright Kerr localized states

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Dissipative solitons are localized structures (LSs) found in a plethora of different fields of sciences, ranging from plant population ecology to nonlinear optics [1]. These states form due to a double balance between antagonistic phenomena: on one hand dispersion counteract nonlinearity, while on the other hand energy gain balances losses. In optical resonators, LSs appear in the form of dark and bright pulses of light, and underline important applications such as the generation of frequency combs [2]. In the mean-field approximation, the modified Lugiato-Lefever equation including second- and fourth-order chromatic dispersion

$$\partial_t A = -(1+i\Delta)A - i\beta_2\partial_x^2 A + i\beta_4\partial_x^4 A + i|A|^2 A + S,$$

describes the dynamics of such states [3]. Here, A is the complex electric field amplitude, t represents the slow time coordinate, and x corresponds to the fast time in fiber cavities or angular variable in microresonators. The second- and fourth-order dispersion coefficients are respectively β_2 and β_4, the non-linearity is of Kerr-type, the gain is modeled by S, the losses are linear and Δ describes the detuning.

When $\beta_4 = 0$, and $\beta_2 = 1$, this system supports dark LSs like those depicted in Figs. (i)-(ii). These states undergo a collapsed homoclinic snaking bifurcation structure which is depicted in Figs. (a),(c), where stable (unstable) states are represented using thick (thin) lines [4]. The formation of these states is related to the locking of fronts connecting two homogeneous states [5]. In this work we study the modification of this scenario when $\beta_4 \neq 0$. Diagrams shown in Figs. (b),(d) illustrate such modification when $\beta_4 = 10$. We have shown that at this value dark LSs survive, and bright LSs arise, leading to the collapsed snaking depicted in Fig. (d). Two examples of such states are shown in Fig. (iii)-(iv).

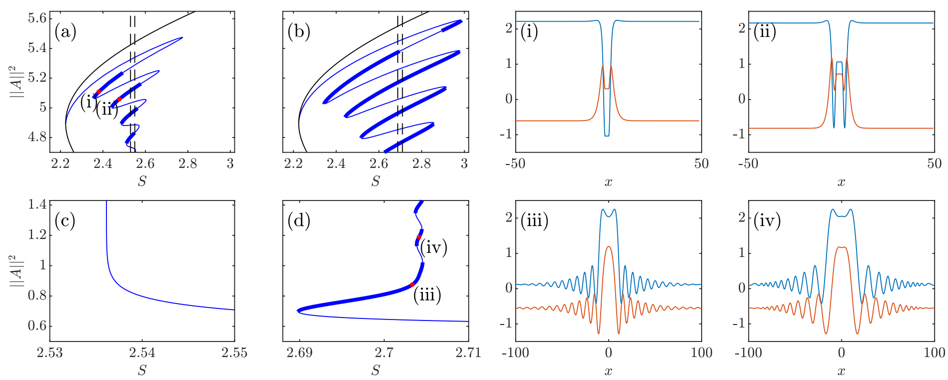

Fig. 1 Panels (a) and (c) show the collapsed snaking undergone by dark LSs [see (i) and (ii)] when $\beta_4 = 0$. Panels (b) and (d) show the modification of such structure. When $\beta_4 = 10$, and the stabilization of bright LSs [see (iii), (iv)]. Here $\Delta = 5$.

We have performed a detailed bifurcation analysis of these states, computed their stability and established their existence extension in the parameter space. Funding: Marie Sklodowska-Curie Actions (101023717).

References

[1] N. Akhmediev and A. Ankiewicz, eds., *Dissipative Solitons*. Lecture Notes in Physics, Berlin Heidelberg: Springer-Verlag, 2005.

[2] T. J. Kippenberg, R. Holzwarth, and S. A. Diddams, "Microresonator-Based Optical Frequency Combs," *Science*, vol. 332, pp. 555–559, Apr. 2011.

[3] M. Haelterman, S. Trillo, and S. Wabnitz, "Dissipative modulation instability in a nonlinear dispersive ring cavity," *Optics Communications*, vol. 91, pp. 401–407, Aug. 1992.

[4] P. Parra-Rivas, E. Knobloch, D. Gomila, and L. Gelens, "Dark solitons in the Lugiato-Lefever equation with normal dispersion," *Physical Review A*, vol. 93, p. 063839, June 2016.

[5] P. Coullet, "Localized patterns and fronts in nonequilibrium systems," *International Journal of Bifurcation and Chaos*, vol. 12, pp. 2445–2457, Nov. 2002.