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Threefold layered optical wireless network: illumination, communication, and positioning

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Abstract: Visible light communication (VLC) technology has traditionally emerged as a complementary solution to radio frequency, due to its huge available bandwidth and high data rates. Relying on visible spectrum, VLC allows both illumination and communication, thus representing a green technology with reduced energy impact. However, VLC can also be exploited for localization, and thanks to its huge bandwidth it can reach very high accuracy (i.e., <0.1 m). In this paper, we deal with a VLC network intended as a fully integrated indoor system providing at the same time both illumination, communication, and positioning tasks. Three different optimization problems are presented, aiming to seek for the minimum number of white LEDs that can achieve different constraints of illumination, data rate, and localization accuracy. Different types of LEDs are considered, according to which tasks they are intended to pursue. We consider traditional white LEDs aiming to provide illumination, communication, and positioning; otherwise, we distinguish between devices designed for localization-only and communication-only. Such distinction results in different optimization problems, and related solutions, as confirmed through extensive simulation results.

1. Introduction

Visible light communication (VLC) is expected as a prominent technology for future Beyond 5G (B5G) wireless networks due to its spectral, energy, and cost efficiencies [1,2]. Also known as a *green technology*, it is largely adopted in indoor environments, working as a complementary solution to radio frequency (RF) networks, especially at places where RF signals fail to support end-users requirements.

The main benefits of VLC rely on its achievable high data rates, while guaranteeing illumination and security requirements. The remarkable research advances in VLC motivated researchers to employ light-based access points (APs) also for localization services [3], thus resulting in visible light positioning (VLP) technology. This is considered as a key technology for B5G wireless networks [4] and is particularly important as the widely-used RF-based positioning technology, i.e., the global positioning system (GPS), fails at indoor environments. However, the system requirements for data transmission and positioning applications are quite different. This is simply as the former applications aim at maximizing the spectral efficiency, while the latter ones target on minimizing the localization resolution, which measures the ability to separate two users in the spatial domain. In this context, from radionavigation theory, at least three sources at known positions (i.e., anchor nodes) are required to estimate the user position with high accuracy.

Leveraging on the previous considerations, in practical realizations, the constraint of having at least three APs in each position of a given environment is not always feasible. As for instance, in [5] it was demonstrated that in an indoor area of 9×10 m^2, only two APs can be sufficient to satisfy certain illumination and data rate requirements in a multi-user scenario, but they do not meet localization requirements. It follows that adding more access points is needed for

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localization purpose, but at the same time it is necessary to find a proper number of APs for allowing the achievement of desired localization accuracy and also avoid increasing installation costs, and inter-LED interference [5].

In this paper, we focus on the three pillars of VLC networks, aiming to provide (*i*) illumination, (*ii*) positioning, and (*iii*) communication in a unified solution. Furthermore, it must be specified that for what concerns illumination we focus on a twofold optimization since we pay attention both to minimum illumination to grant and also its distribution by balancing maximum and minimum levels. For each feature, we consider different Quality of Service (QoS) requirements and aim to optimize the LED deployment that can meet such requirements. Specifically, we distinguish three different optimization problems, each of them considers different kinds of white LEDs based on the tasks they are designed to provide, i.e., communication or localization. Problem 1 seeks for the minimum number of white LEDs that can provide at the same time both illumination, communication, and localization constraints, in each position of a given environment. Since in this case the LEDs can act both for communication and localization, they are defined as "peer" LEDs. On the other side, Problem 2 seeks for the minimum number of white LEDs aiming to provide only localization and illumination constraints, neglecting data communication purpose. This case suits in all those environments where localization service is required and communication has been already setup (e.g., industry, hospitals, etc.). In this second approach, LEDs are defined as localization-only devices. Finally, Problem 3 considers two different types of white LEDs, i.e., those designed for communication and those for localization purpose. In this case, the optimization problem aims to distinguish the minimum number of communication-LEDs and localization-LEDs, meeting communication and localization accuracy requirements, respectively. This problem reflects a VLC system enriched with the ability to support positioning service, by means of some (extra) non-data, i.e., localization-only APs, to be installed at different positions in order to improve the localization accuracy within the whole environment. By the term "localization-only", we mean that such APs do not transmit data but only send some predefined signals with certain power to facilitate the positioning process. Therefore, these APs do not cause interference to the existing data-APs.

In this paper, we propose the network architecture design and, more, three LEDs deployment optimization approaches. Solutions are shown for different environments (i.e., indoor room sizes) and network requirements. Specifically, for each LED deployment approach, we focus on a minimization process that aims to guarantee specific QoS requirements, such as the illumination and localization accuracy below a given threshold, and the achievable data rate higher than a given threshold, while providing a minimum number of APs. Extended simulation results highlight a minimum of 4 (8) white LEDs are needed to provide at least 30 Mbps, with at least 250 lux, in each position in a $4 \times 5 \times 3$ m^3 ($9 \times 10 \times 3$ m^3) room, respectively. However, adding also localization requirements, the minimum number of LEDs increases to at least 7 and 41 in a $4 \times 5 \times 3$ m^3 and $9 \times 10 \times 3$ m^3 room, respectively.

This paper is organized as follows. Section 2. provides a state-of-the-art of the most relevant studies about indoor VLP. Section 3. deals with the proposed indoor VLC/VLP system and describes the main elements of network architecture. Different design constraints related to the threefold pillar of illumination, communication and positioning are introduced in Section 4, that are fully integrated in our proposed approach. Then, the APs deployment optimization problem is introduced in Section 5, where we distinguish three different approaches in case of (*i*) "peer", (*ii*) localization-only, and (*iii*) joint communication and localization LEDs. Extended simulation results are described in Section 6, in case of variable geometric features and room setup. Results have been achieved also in case of blockage and variable tilting angle, in order to consider a most realistic scenario. Finally, conclusions are drawn at the end of this paper.

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2. Related works

Traditionally, VLC has been investigated from the point of view of data communication. At the same time, the topic of VLP has been largely investigated and still represents an open issue, especially in the context of B5G networks that are expected to require high accuracy localization. The use of Optical Wireless Communications (OWC), and in particular Line of Sight (LoS) VLC connectivity links, is envisioned to provide under-centimeter accuracies, mainly due to its high bandwidth and link directivity.

VLC and VLP look as separated issues that should be investigated each in a dedicated manner. Commonly, research on VLP is split by VLC component, while both VLP and VLC should be fully integrated according to the vision of next generation services. For instance, in [6], Tran and Ha propose an innovative and easily implemented solution for indoor positioning, based on dual-function machine learning (ML) algorithms. As a benefit, increased positioning accuracy is observed under the negative effect of multipath reflections. An interesting approach is presented by Chaudhary *et al.* [7], where the received signal strengthand the information of tilt angles of transmitters and receivers are exploited, as well as multipath reflections. The tilting effect has been shown to be beneficial in VLP systems considering both LoS and Non-Line of Sight (NLoS) transmission modes.

Contributions dealing with both VLP and VLC as a joint system are limited. For instance, Li *et al.* [8] investigate the optimal configuration of LEDs in indoor environments from the sole point of view of communications, expressed through the constraints quality of experience (QoE), and also illumination. Singh *et al.* [9] present an optimal LED power allocation framework that maximizes the average data rate across the room, while satisfying the bit error rate and illumination constraint. This solution is mainly focused on LED energy-saving, and does not consider a fully integrated VLC/VLP indoor system, but still splits the communication pillar from the localization one. Also, Dang *et al.* [10] propose a new metric to evaluate the positioning accuracy based on information theory. The optimum LED deployment for communication and positioning has been investigated, and it was found that LEDs concentrating in the middle of the room are efficient for communication-optimal scenario, while for positioning-optimal scenario, LEDs should be placed at the corners of the room. However, the proposed approach still separates the localization and communication goals, since no optimal solution for joint VLC and VLP has been identified. Yazar and Gezic [11] address the problem of localization performance maximization for pulse design, with consideration of practical limitations related to power consumption, illumination levels, and effective bandwidths. In [12], Xu *et al.* propose a system that realizes simultaneously both positioning and communications in the same band by using orthogonal frequency division multiple access (OFDMA). Position estimation is carried out through received signal strength measured in each subcarrier block, and therefore, VLP does not deteriorate the performance of VLC. However, in this work, the authors do not consider communication requirements, but only focus on localization accuracy.

In all previous works, VLP has been addressed from the point of view of achievable accuracy, ignoring the network design and the LED deployment related issues, specially in a joint VLC/VLP approach. Indeed, VLP should not neglect neither communication nor illumination constraints, as they are the essential pillars of VLC systems and VLP should be built upon them. Leveraging on such research gap, in this paper we aim to build a fully integrated VLC and VLP system, aiming to satisfy both communication, localization, and illumination requirements. This paper represents an evolution of a previous work [5], dealing with the optimal deployment of an indoor VLC, according to different data rate requirements. Moreover, in [5] we aimed at considering the outage in a probabilistic sense, while here localization is also taken into account as well as illumination distribution and the constraints are punctually considered. Moreover, differently from [5], we present a framework to communicate, that is, the whole system architecture of the threefold layered optical wireless network. Hence, that work presented the optimal number of

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white LEDs achieving different data rates in a given environment, while neglecting localization and illumination constraints. Now, we investigate an evolved approach dealing with the three main pillars of VLC technology, resulting in an indoor illumination-plus-communication-plus-localization system.

This paper presents an optimization technique related to three different criteria, such as the illumination, the achievable data rate, and the localization estimation error, applied to indoor VLC systems. The optimal deployment solutions will be presented as expressed in terms of minimum number of LEDs achieving at the same time different localization, illumination, and communication requirements. The objective is then unique since this contribution presents a threefold optimization approach. From an overview of the state-of-the-art, the network optimization of VLC systems has been focused on communication-only or positioning-only aspects. A joint optimization of communication and positioning, in addition to illumination requirements to fulfill, has not been investigated yet in visible light systems. In RF systems as well, the issue of cell coverage optimization is related to data rate only, and localization is not investigated as a joint metric to optimize. For instance, in [13] the authors investigate the cell coverage optimization problem for a massive multiple-input multiple-output (MIMO) uplink, and their goal is the maximization of cell throughput, which is shown to be mainly determined by the user distribution within several key geometrical regions. Similarly, in [14] Wei and Hwang present a multiple-objective optimization model in a multi-user 5G networks where users are grouped based on their propagation conditions. However, also in this case the optimal cell size is estimated to maximize the total network capacity and minimize the deployment cost, and do not consider localization service.

3. Proposed system and network architecture

Let us consider an indoor VLC network where N APs are assumed able to provide both illumination, localization, and communication services. In such a network, the APs might be insufficient to support all the services with the required quality levels. For example, the value of N may be sufficient to provide illumination and/or communication requests and insufficient to grant the desired level of localization accuracy. This implicitly suggests that we aim at determining a sufficient number of APs so as to guarantee all the systems requests/constraints.

3.1. System signaling

The APs are simple LED devices each one with an identification code that is represented by a direct-sequence optical code division multiple access (DS-OCDMA), [15]. Since the communication is bi-directional we assume the nodes to be equipped with a photodiode (PD) for receiving signals from APs and also IR-LEDs to transmit towards the ceiling of the room. For this reason we consider that the uplink requires the ceiling to have IR-PDs according to the framework in [16].

Herein, we consider a two-phase localization process, such that each phase corresponds to a different localization level, as follows:

3.1.1. Phase I

In this phase, the user detects the received signals. Due to orthogonality of the codes, it is able to *measure* the signal strength of each AP since under the realistic approach of orthogonality (it is reasonable in downlink) by considering all the possible sequences of a given spreading factor, the user collects the power levels from each AP.

As anticipated, due to the orthogonality of optical codes and to clarify the decoding process at the user side, in this paper we assume that the N APs do not cause interference to each other. This does not mean that we neglect signal overlapping, however we consider the opportunity, by the receiver, to resolve each code. Specifically, the APs transmit orthogonal sequences by employing Walsh-Hadamard codes so as to grant orthogonality. In this context, we should regard the special requirements for VLC transmission, particularly the condition that the transmitted symbols should be real and positive. Such requirements should be carefully considered before applying normal CDMA scheme, i.e., that used for the RF systems. Herein, we consider the simple solution of employing the VLC-adapted version of the DS-CDMA, in which a fixed DC offset is added to convert the bipolar signal into a unipolar signal before modulating the light source, [17,18].

Hence, *Phase I* identifies the APs covering a certain location, while *Phase II* calculates the distances to those APs. Analytically speaking, let us describe the channel LoS propagation as a single coefficient. In fact, dealing with VLC channel model, this is typically composed of both LoS and diffuse (multi-path) components. However, it was observed in [19] that, diffuse components can be ignored in typical indoor scenarios as the majority of the collected energy at the photodetector (more than 95%) comes from the LoS component. Therefore, in this paper we mainly assume that the VLC link has a dominant LoS component [20]. More in detail, we assume that the LED-based AP follows the Lambertian radiation pattern, and that the VLC AP is directed downwards and the user PD is directed upwards. It follows that, the channel gain at a given distance $d_{j,k}$ [m] between the j-th LED and the k-th PD is expressed as [21]:

$$h_{j,k} = \frac{(m+1)A_k}{2\pi d_{j,k}^2} T_s(\psi_k) G(\psi_k) \cos^n(\phi_j) \cos(\psi_k), \tag{1}$$

where m is the Lambertian emission order, given as $-\log_2[\cos(\phi_{\frac{1}{2}})]$, with $\phi_{\frac{1}{2}}$ as the LED's semi-angle at half-power. In Eq. (1), $T_s(\psi_k)$ and $G(\psi_k)$ are the gains of the optical filter and of the optical non-imaging concentrator of the k-th PD, respectively. We assume $T_s(\psi_k) = 1$ and $G(\psi_C) = n^2/\sin^2(\psi_C)$, with n as the refractive index, ψ_k and ψ_C as the angle of incidence to the k-th PD and the Field of View (FOV) of the k-th PD, respectively. Furthermore, A_k [cm^2] is the physical area of the k-th PD and ϕ_j is the angle of irradiance of the j-th LED.

Formally, let us first express the received signal of the k-th PD at the l-th chip as follows:

$$r_{l,k} = \sum_{j=1}^{N} (c_{l,j} + P_{bias}) h_{j,k} + w_{l,k} \quad \text{for} \quad l = 1, 2, \ldots, SF, \tag{2}$$

where $c_{l,j}$ is the l-th chip of the j-th AP sequence, while $w_{l,k}$ is the additive white Gaussian noise sampled at chip level, and P_{bias} is the light bias. It is important to note at this stage that using a light bias P_{bias} allows the transmitter to obtain non-negative chip values. Moreover, it can be considered as a dummy spreading sequence where all the chips are set to 1 so excluding the sequence of all ones from those available by the APs. It it important to highlight here that there is a relationship between the spreading factor (SF) to be used and the number of APs. Since one sequence is used for the light bias, the SF should be properly set according to $SF = 2^{\lceil \log_2(N+1) \rceil}$. Moving to a sequence-base description, we have for the k-th PD the following vector/matrix description:

$$\mathbf{r}_k = \mathbf{h}_k^T \mathbf{C} + \mathbf{w}_k = [h_{1,k}\, h_{2,k}\, \ldots\, h_{j,k} \ldots h_{N,k}] \begin{bmatrix} c_{1,1}\, c_{2,1}\, \ldots\, c_{SF,1} \\ c_{1,2}\, c_{2,2}\, \ldots\, c_{SF,2} \\ \ldots \\ c_{1,j}\, c_{2,j}\, \ldots\, c_{SF,j} \\ \ldots \\ c_{1,N}\, c_{2,N}\, \ldots\, c_{SF,N} \end{bmatrix} + [w_{1,k}\, w_{2,k}\, \ldots\, w_{l,k} \ldots w_{SF,k}], \tag{3}$$

where $\mathbf{h}_k$ is the N-elements column vector reporting the DC gains, while the $(N \times SF)$ matrix $\mathbf{C}$ reports the spreading sequences of the APs in the N rows. Moreover the $(1 \times SF)$ vector $\mathbf{w}_k$ gathers the noise samples on a sequence basis, while $\mathbf{r}_k$ is the sum of all the sequences weighted by the channel coefficients. From Eq. (3) it is possible to infer that all the chips related to the same transmitting node (e.g., $c_{1,1}\, c_{2,1} \ldots c_{SF,1}$) are multiplied by the same channel gain (e.g., $h_{1,k}$). However each received chip is the *linear combination* of all chips in the same position within the spreading sequences, weighted by the channel coefficients.

In order to proceed to detect the $h_{j,k}$ value, the point is to remove the light bias as for Eq. (2). The light bias is a constant signal. This can be considered as a dummy user whose spreading sequence is characterized by constant chips (that is a unit vector $\mathbf{1}$ with SF elements). Since it is assigned to a –dummy– user, the spreading sequence is orthogonal with respect to the other $(SF - 1)$ sequences. Hence, by operating the despreading operation related to the j-th sequence, we have that *all* the spreading sequences are orthogonal to the j-th one, including the dummy sequence characterized by all ones. It follows that the estimated $\tilde{h}_{j,k}$ can be inferred as

$$\tilde{h}_{j,k} = \frac{1}{SF}\mathbf{r}_k\mathbf{C}[k,:]^T = \frac{1}{SF}\left(\mathbf{h}_k^T\mathbf{C}\mathbf{C}[k,:]^T + [w_{1,k}w_{2,k}\ldots w_{l,k}\ldots w_{SF,k}]\mathbf{C}[k,:]^T\right), \quad (4)$$

where $\mathbf{C}[k,:]$ is the k-th row of the matrix, that is, the signature (spreading sequence) related to the k-th AP. It is important to highlight that the product of $\mathbf{C}$ by $\mathbf{C}[k,:]^T$ leads to obtain a column vector since the dimensions of $\mathbf{C}$ are $(N \times SF)$, while those of $\mathbf{C}[k,:]^T$ are $(SF \times 1)$. Moreover, since the product of a row of $\mathbf{C}$ by the vector $\mathbf{C}[k,:]^T$ is a scalar product, we have that due to the orthogonality of the sequences only one element of the vector given by $\mathbf{C}\mathbf{C}[k,:]^T$ will be different from 0 and it equates SF, exactly in the k-th position of the vector. In fact, due to the orthogonality property of the spreading sequences we have that $\mathbf{C}[l,:]\mathbf{C}[k,:]^T = 0$ when $k \neq l$. As a consequence, the product of the vector collecting the channel gains $\mathbf{h}_k$ by the above mentioned quasi-null vector given by $(\mathbf{C}\mathbf{C}[k,:]^T)$ is characterized only by the term $h_{j,k}$ with the exception of an additional filtered noise $[w_{1,k}w_{2,k}\ldots w_{l,k}\ldots w_{SF,k}]\mathbf{C}[k,:]^T$.

3.1.2. Phase II

Having the power levels detected, each node can send the measures by using an uplink connection (that is mandatory to have, also for the communication process and it can be infra-red [16]) hence the network is responsible for process the information and apply a simple triangulation algorithm by selecting the three highest signal levels so as to provide accurate localization. It should be remarked here, that the exact user location might not be found without phase I and by depending only on this phase. This is because most of the algorithms applied in this phase mainly depend on calculating the distances between the user and the APs forming the overlapping area. Thus, these algorithms identify the user location relative to the positions of the associated APs, and not to a reference (global) point within the entire environment, unless the positions of these APs are known to that reference point. This latter information is provided by *Phase I*.

The system design proposed in this paper is, principally, independent of the algorithm used in this phase. However, the applied algorithm will have an indirect impact on the system design through the estimation accuracy, which is a key performance criteria to be regarded by system designers.

4. Design constraints

As previously anticipated, we focus on optimizing the proposed network architecture, according to three constraints. The first one is the localization that can be useful especially for identifying the LEDs that should provide the communication service so as to allow the transmission by multiple LEDs. In this regard the communication rate with a target error probability constitutes the second constraint. Finally, the third one twofold since it is related to the minimum illumination level and the ratio between minimum and maximum.

4.1. Localization error

Let us consider the task of estimating the position of the k-th user, which is covered by a number of APs that are assumed to be greater or equal to 3, otherwise it is not possible to grant the absence of ambiguity. Although this is a very well known result from the literature, it must be underlined that using less than 3 APs leads to have huge estimation error and/or location ambiguity.

More in detail, we consider the k-th user is in a 3D position, where the z-component is assumed as constant. So, the localization algorithm provides the estimation error considering all the three components along x, y, and z, then representing a 3D localization. We assume the localization estimation relies on the received signal strength indicator (RSSI) metric [22], and we refer to the Cramer-Rao bound (CRB), that is the estimation error variance of an efficient estimator [23]. According to [24], we exploit the best three signals the k-th user receives so as to obtain the following expression for the CRB, mathematically indicated as Θ [m^2], i.e.,

$$\Theta = \sum_{i \in \mathcal{I}_3} \frac{c^2 d_{j,k}^2}{2h_{j,k}^2 \frac{P_L}{N_0 B_L}\left(4c^2 + \frac{d_{j,k}^2 (2\pi B_L)^3}{3} + 2cd_{j,k}\right)}, \tag{5}$$

where c [m/s] is the speed of light in free space, P_L [W] is the electrical transmission power used for localization purposes that already accounts for optical-to-electrical conversion, thus including the PD responsivity and the load resistance. Furthermore, N_0 [W/Hz] is the noise spectral density, and B_L [Hz] is the bandwidth of the signal to be utilized for localization purposes. It is important to highlight that the set $\mathcal{I}_3$ collects the labels of the three best signal components at the k-th PD. As it is possible to notice, when the ratio between the received power (or transmitted since they are direct proportional) and the noise increases, i.e., P_L/N_0, we have that the CRB quickly decreases. From this point of view, we expect to have the mean square error σ_ϵ^2 that perfectly fits the value of CRB. This can be performed with RSS methods, [22].

4.2. Communication/access power constraint

Although each LED utilizes its own DS-OCDMA sequence, the access within a cell is assumed occurring according to a TDMA fashion. In order to grant a minimum rate we focus on some geometrical, transmission as well as, access features. In this regard, the achievable rate per single user for the VLC link is expressed according to the following well-known formula [25]:

$$R = B \log_2\left(1 + \frac{\zeta}{\Gamma}\right), \quad \text{bps} \tag{6}$$

where B [Hz] is the communication bandwidth, that is, the bandwidth related to the signal utilized for the communication in downlink between LED and PD, ζ [dB] describes the signal-to-noise-ratio (SNR) and Γ is a factor that considers the SNR increase allowing to achieve required performance in terms of symbol error rate (SER). As an example we can consider a unity SNR value that allows to achieve a spectral efficiency of 1 bit/s/Hz. However at 0 dB of SNR the performance in terms of error rate are really poor. The Γ term allows to consider the error rate, and it is defined as

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{SER}{4}\right)\right]^2, \tag{7}$$

where Q function is expressed as

$$Q(x) = \int_x^\infty \frac{e^{-u^3/2}}{\sqrt{2\pi}} du. \tag{8}$$

This allows to grant not only a rate but also an error rate.

In case of a multi-user scenario, we introduce, as an innovative contribution, a correction factor expressed as the product of the lighting cell coverage and the user density per area, in order to measure the average rate per user. In fact, usually when TDMA is considered as enabling access procedure, the correct way of scaling the available bandwidth is to divide the single user rate expression in Eq. (6) by the number of users, since the channel is shared. This is correct when we consider only one LED in the network. When we consider multiple LEDs in the network it is not true that we need to scale the rate by all the users but simply by a *fraction* of them, corresponding to the (average) number of users covered by a single LED. In this regard, we consider the average number of users covered one –out of N– LED by referring the the user density in the room. Following the above considerations, Eq. (6) becomes

$$R = \frac{B}{\lceil \mathcal{A}\Delta \rceil} \log_2\left(1 + \frac{\zeta}{\Gamma}\right), \quad \text{bps} \tag{9}$$

where the term $\mathcal{A}$ [m^2] is the area of the footprint of the lighting cell at the considered height, while the term Δ [user/m^2] takes into account for the user density in the room expressed as the number of users per squared meter. This approach is highly realistic since it takes into account how many users can populate the network, and in a spatial sense, the room. This allows to avoid to allocate resources (i.e., time slots) for all the user by each LED. Hence, we focus on this approach that considers how many users are *covered* by a single LED. More regarding the parameter Δ, this number usually is smaller than 1 since considering more than 1 user per squared meter is a bit unrealistic. As a simple example, in a cell with $\mathcal{A} = 2$ [m^2] and $\Delta = 1$ [m^{-2}], it is expected that the minimum rate per user will be $B/2 \log_2(1 + \zeta/\Gamma)$. Besides, since it is largely possible that the footprint overlaps, in those positions we assume the user to be able to combine signals since the LEDs, in that case, transmit the same information, and hence the rate becomes

$$R = \frac{B}{\lceil \mathcal{A}\Delta \rceil} \log_2\left(1 + \sum_{i \in \mathcal{I}_3} \frac{\zeta_i}{\Gamma}\right), \quad \text{bps} \tag{10}$$

where we consider the sum of the three best RSS signals belonging to $\mathcal{I}_3$, measured at a given PD position. It is important to underline that the above modeling for what concerns rate is able to jointly account for both number of users and BER. In fact the SNR gap is able to grant the target BER while the denominator, that is, $\lceil \mathcal{A}\Delta \rceil$ takes into account the number of users since we multiply the spatial user density Δ by the area $\mathcal{A}$, and by considering the ceiling function we resort to the worst case for the rate since we take the upper integer number.

Till now, it appears that the number of LEDs in the network that influences the chip-rate, is unlinked to the communication bandwidth. However, the communication bandwidth is essentially $B = B_t/SF$, where B_t [Hz] is the bandwidth of the signal in air (after spreading). At a first glance this suggests that if we consider a high number of LEDs for a fixed B we have a large increase of B_t, or converse, for a fixed B_t, a large number of LED reduces B. However, since not all the light footprints overlap (for example, light footprint of the LEDs posed in corners of room) we can think to apply the policy of spreading sequence re-use as for carrier frequencies in cellular networks, hence utilizing the same spreading sequence by different LEDs that are far one from each other, thus meaning not interfering in any position of the floor.

4.3. Illumination constraint

Since the main role of a LED is to provide the illumination service, we must take care of providing a minimum level of illumination everywhere in the room, thus meaning that a target light intensity must be granted. From this point of view, the constraint on illumination does not go in contrast with the above two considerations about localization accuracy and achieved rate. Higher is the number of LEDs, higher is the illumination, even though, for sake of energy saving as well as eye safety, uncomfortable situations should be avoided. Indeed, in principle also a maximum illumination level should be set as a constraint since we must pay attention to eye safety issues/regulations.

There is also another important aspect in the illumination service that deals with uniformity. Generally speaking, obtaining a flat illumination level all over the room/environment is often an unfeasible target. Hence, the key point to solve the minimum requirements, a certain degree of uniformity and preserve eye-safety is to set two constraints. One is on the minimum illumination level achievable in the room, while the other one is a fairness ratio in the illumination that can be defined as

$$\mathcal{F} = I_{\text{min}}/I_{\text{max}}, \tag{11}$$

which tends to zero when the minimum illumination measured in the room I_{min} approaches very low value with respect to the maximum one I_{max}, while it approaches 1 when the minimum is very close to the maximum, i.e., $I_{\text{min}} \approx I_{\text{max}}$. Besides requiring a certain degree of fairness is equivalent, due to the request of a minimum illumination level, also to requiring the maximum illumination level usually set according to eye-safety regulations and/or to use of room. It is known that different room use leads to different illumination requirements.

In principle, the illumination level at a given position depends mainly on the transmission profile of the light source, in addition to the distance to that light source. Further, notice that the illumination is an additive feature, i.e., if a certain location is covered by multiple light sources then the total illumination at that location is the sum of the illumination levels due to the individual light sources. Analytically speaking, let the k-th user be covered by multiple APs, whose indexes define the set $\mathcal{I}$. Then, the total illumination perceived by the k-th user is given by

$$I_k = \sum_{i \in \mathcal{I}} P_i h_{i,k}, \tag{12}$$

where P_i [W] is the optical transmission power of the i-th AP.

4.4. Regarding blockage

Since the propagation is for the most part in LoS, the event of blockage, that occurs when an obstacle impedes the signal to propagate from LED to PD, is more than relevant and affects system performance. About this aspect two elements must be properly highlighted. First, blockage cannot be inserted directly in the optimization since it can occur in an unpredictable way and this deals both with time (when) and position (where), and also in which way (how). If fact, let us suppose that a user hand is posed so as to cover the PD. This can happen at different distances from the PD itself, and this is not in the possibility of being predicted or assumed. Furthermore, even though blockage can be linked to movements of the user in the network, our optimization is performed on the whole room, thus meaning that also in case of mobility all the constraints are met.

The second element is that we show in the numerical performance the effect of blocking, by posing an obstruction at different distances from the PD. The blockage can affect the results in terms of constraints met or not met. As last comment, about the network planning, we consider two typologies of room the dimensions of which are different. However, no furniture (layout) has been considered, differently from [5] where user spatial distribution has been assumed. This is

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because we require in each point of the room to meet the constraints. If we consider the spatial distribution of objects in the room (i.e., tables, closet, and so forth) a change in the distribution may lead to fail to meet the constraints if the optimization is performed for a specific room configuration.

5. LEDs-constrained optimal placement

In this section, we aim at solving one key problem, and two additional ones that represent two particular cases. The main problem consists in building a network from the scratch by minimizing the number of access points needed to guarantee (*i*) a given location estimation accuracy, (*ii*) a minimum data rate level, (*iii*) a minimum illumination level, all measured in a given position, and also (*iv*) a minimum illumination fairness, measured all over the room. Analytically speaking, we have the following constrained problem to be met for each position of coordinates represented by the couple (x, y), i.e.,

$$\textbf{Problem 1 :} \min N \tag{13a}$$

$$\text{s.t. } \Theta(x, y) \le \Theta^* \tag{13b}$$

$$R(x, y) > R^* \tag{13c}$$

$$I(x, y) \ge I^* \tag{13d}$$

$$\mathcal{F} \ge \mathcal{F}^* \tag{13e}$$

where $\Theta(x, y)$, $R(x, y)$, and $I(x, y)$ are the CRB value, the achieved rate, and the illumination level, at each location, respectively, while $\mathcal{F}$ is the illumination fairness all over the room. Since $I(x, y) \ge I^*$ and $\mathcal{F} \ge \mathcal{F}^*$, it follows that $I^* \ge I_{\text{min}}$ and then the limitation on the maximum illumination level is provided by the following relationship, i.e.,

$$I_{\text{max}} \le \frac{I^*}{\mathcal{F}^*}. \tag{14}$$

Hence, posing a limitation on minimum fairness and minimum illumination is an implicit requirement of bounding also the maximum illumination. This is worth especially for eye safety issues/regulation.

Let us consider a room of area $\mathcal{L}_x \times \mathcal{L}_y$ [m^2], where $\mathcal{L}_x$ and $\mathcal{L}_y$ are the lengths in the horizontal and vertical direction, respectively. In this scenario, we assume a grid structure where different tiles on the floor are along the horizontal and vertical direction of the room, whose dimensions are defined as τ_x [m] and τ_y [m], respectively. Then, a given position (x, y) is identified by a given tile of size $\tau_x \times \tau_y$ [m^2]. Additionally, we consider a similar structure on the ceiling, with tiles of size $q_x \times q_y$ [m^2] that determine the possible LEDs positions on the horizontal and vertical directions, respectively. It is important to notice that in general we can have $\tau_x \ne q_x$ and $\tau_y \ne q_y$. Figure 1 depicts the grid structure of the room environment.

In Problem 1, it is interesting to note that the constraints do not present a trade-off behavior, thus meaning that by increasing N we can improve both the estimation accuracy, the transmission rate, as well as the illumination level. Hence apparently, the optimization does not present constraints that are in contrast each other. Besides, even though the output of the problem, that is, the solution, seems to be a number (i.e., N) it must be highlighted that the position of the APs is able to considerably change the performance and/or constraints met/not met. It is an off-line problem since it is not an optimization that should be carried out during localization, communication and illumination, and takes place during system planning. The way to solve this off-line problem is the exhaustive search. We set two step sizes, not necessary the same, one used as a measure tool to explore all the possible positions of LEDs in a given area as possible solution of Problem 1 (e.g., the ceiling tile), and one to measure the constraints $\Theta(x, y)$, $R(x, y)$, and $I(x, y)$ assuming a floor/height tile step size.

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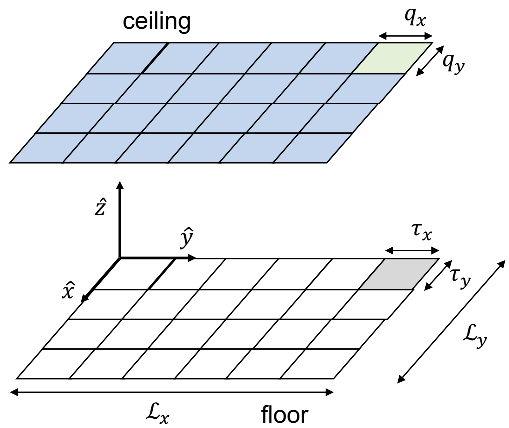

Fig. 1. Room grid-structure model, with $\tau_x = q_x$ [m] and $\tau_y = q_y$ [m].

The second problem that can be of interest is referred to the following situation. A network is already setup, but localization accuracy is not guarantee all over the environment (e.g., in all the positions of the room). This may suggest the opportunity of inserting localization-only APs (i.e., N_ℓ), that is, LED AP aiming to provide localization estimation only. In this latter case the problem becomes the following:

$$\textbf{Problem 2} : \min N_\ell \tag{15a}$$

$$\text{s.t. } \textit{set } N \tag{15b}$$

$$\Theta(x,y) \leq \Theta^* \tag{15c}$$

$$I(x,y) \geq I^* \tag{15d}$$

$$\mathcal{F} \geq \mathcal{F}^* \tag{15e}$$

that is really similar to Problem 1 in Eq. (13), but it presents as a constraint the number and positions of the N access points able to communicate and localize. Hence, *set N* indicates that the number of these APs and their location are the constraint. Furthermore, notice that in Problem 2 the constraint on data rate in Eq. (13c) disappears since we assume the rate constrained is achieved since a network is already present. Also in this case, Eq. (14) applies.

The third problem we consider is when we separate the functions of communication and localization. In this sense we want to minimize the number of communication APs (i.e., N_c) and localization ones (i.e., N_ℓ) by assuming that each LED can only communicate or localize. In this case we have the following mathematical expressions:

$$\textbf{Problem 3} : \min\,(N_c, N_\ell) \tag{16a}$$

$$\text{s.t. } \Theta(x,y) \leq \Theta^* \tag{16b}$$

$$R(x,y) > R^* \tag{16c}$$

$$I(x,y) \geq I^* \tag{16d}$$

$$\mathcal{F} \geq \mathcal{F}^* \tag{16e}$$

with the limitation on the maximum illumination provided by Eq. (14).

In this third problem, the goal is still to minimize the whole number of APs by considering that they belong to two different categories, i.e., communication-only, and localization-only APs.

It follows that the N_ℓ APs contribute both to illumination and localization constraints, while the N_c APs contribute to rate and illumination constraints.

Both two problems worth to be presented and investigated since they represent two realistic scenarios (i.e., *i*) the case where LEDs are deployed for localization-only service (i.e., Problem 2), and (*ii*) the case where two different types of LEDs are deployed, one working for communication, and one for localization (i.e., Problem 3). As an instance, Problem 2 occurs when we need to integrate a positioning service in an existing VLC indoor network. On the other side, Problem 3 is investigated for providing two different services (i.e., communication and localization) in a separated way. Leveraging on the previous motivations, we solve both Problem 2 and 3 according to the mechanism proposed for Problem 1, that is, the exhaustive search. Once more we want to emphasize that despite of the computational cost we detail later, these problems are solved before setting up the network and no calculations are required during the network operation, hence the cost is outside the network functionalities (i.e., offline setup).

5.1. About computational cost

The effort required for obtaining the solution depends on several parameters. First of all the room dimension. Second the ceiling tile and also floor tile. In fact, smaller the tile is higher is the number of positions considered. In general, when an exhaustive search is considered for a number N of access points considered as solution, we have that the maximum number of *for* cycles needed to find the solution is given by the following expression

$$\mathfrak{C} = \mathcal{P}_x \mathcal{P}_y \sum_{j=1}^{L_x L_y-(N-1)} \sum_{i=1}^{j} i, \tag{17}$$

where $\mathcal{P}_x$ and $\mathcal{P}_y$ are the number tiles in the horizontal and vertical direction of the room given by $\mathcal{P}_x = \mathcal{L}_x/\tau_x$ and $\mathcal{P}_y = \mathcal{L}_y/\tau_y$, respectively. Additionally, L_x and L_y are the number of tiles on the ceiling for determining the possible LEDs positions on the horizontal and vertical directions, respectively. The number of tiles L_x is given by $L_x = \mathcal{L}_x/q_x$ and L_y is given by $L_y = \mathcal{L}_y/q_y$ for the horizontal and vertical directions, respectively. A detailed pseudocode for the computational cost in Eq. (17) is shown in Algorithm 1. The output is a Boolean variable, i.e., O, expressing the outage condition. Specifically, for a given number N of LEDs, if all the QoS requirements are satisfied (i.e., $O = 1$), then the system is not in outage and N is a solution of the optimization problem. Algorithm 1 will finally compute the computational cost $\mathfrak{C}$ according to Eq. (17).

It follows that not only the room dimension and step size (i.e., tile) influences the cost but also the *solution* since the term N states the summation upper limit. This is *a priori* unknown when we run the optimization. So, the computational cost expressed in terms of *for* cycles related to the exhaustive search procedure is given by Eq. (17) for what concerns Problem 1 and Problem 3, while Problem 2 presents a slightly lower cost due to the fact that we assume that N is already established. In this case the cost in terms of nested *for* cycles is given by

$$\mathfrak{C} = \mathcal{P}_x \mathcal{P}_y \sum_{j=1}^{L_x L_y-(N_\ell-1)} \sum_{i=1}^{j} i - N. \tag{18}$$

Problem 2 considers a similar approach as Algorithm 1, where N_ℓ LEDs are considered (with $N_\ell < N$). At this stage it is fundamental to stress that this optimization is performed before the network starts to work, since this is the network planning/setup operation. In this sense, the cost of optimization is a secondary issue. The solution for a number of access points in the range $[15, 30]$ leads to have a processing time around $[4, 5]$ minutes with a Processor with 3.4 GHz CPU running on Matlab 2020a. Although there is not the necessity of reducing this phase, since it is largely lower with respect to LEDs installation on the ceiling that requires several minutes/hours

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Algorithm 1. Pseudocode for Problem 1 and 3

```
Input  : N                                        ▹ Number of LEDs
         L_{x,y}  ▹ Number of tiles on the ceiling in the horizontal/vertical direction
         Θ*                                       ▹ Localization requirement
         R*                                       ▹ Data rate requirement
         I*                                       ▹ Illumination requirement
         F*                                       ▹ Fairness requirement
Output : O                                        ▹ Boolean variable for total outage
         ℭ                                        ▹ Computational cost
foreach n = [1, N] do
  foreach Configuration of LEDs positions with the constraint that LED n always
   follows LED (n − 1) do
    foreach l = [1, L_x] do
      foreach m = [1, L_y] do
        compute CRB, Θ                            ▹ Eq. 5
        compute data rate, R                      ▹ Eq. 10
        compute illumination, I                   ▹ Eq. 12
        compute fairness, F                       ▹ Eq. 11
        if Θ ≤ Θ* then
          out_Θ = 1                               ▹ Localization is in outage
          exit
        else
          out_Θ = 0                               ▹ Localization requirement is guaranteed
          if R ≤ R* then
            out_R = 1                             ▹ Data rate is in outage
            exit
          else
            out_R = 0                             ▹ Data rate requirement is guaranteed
            if I ≤ I* then
              out_I = 1                           ▹ Illumination is in outage
              exit
            else
              out_I = 0                           ▹ Illumination requirement is guaranteed
              if F ≤ F* then
                out_F = 1                         ▹ Fairness is in outage
                exit
              else
                out_F = 0                         ▹ Fairness requirement is guaranteed
        if O == [out_I AND out_R AND out_Θ AND out_F]
         then
          compute ℭ                               ▹ Eq. 17
          exit
                                                  ▹ Update the LEDs configuration
```

by considering also the cabling, we want to look at more efficient solutions. Hence, in order to lower the cost, we can simply proceed not to scan all the opportunities. This can be overcome by randomly allocating LEDs and checking if all the constraints are met. In fact, despite of the analysis of each possible LED location, we can recognize easily that some configurations are far from being a solution. As an example, let us figure the performance when we pose all the LEDs on one corner of the room. Even in our exhaustive search we consider that case, while it is *a priori* justified that we do not really need to check for that situation. We experienced a reduction in terms of processing time ranging from 70% to 95% to find a solution for an increasing number of LEDs, so lowering the [4, 5] minutes of the exhaustive search up to 20 seconds.

6. Simulation results

In this section, we provide the simulation results to evaluate the performance of the proposed system design, and to visualize the impacts of different system parameters on the overall performance. If not otherwise specified, the vertical distance d_v m between transmitting and receiving planes is 2 m. In addition, we assume all the APs are transmitting with an average power of 10 Watt. Other main parameters are shown in Table 1. Notice that we have considered two room sizes, i.e., one small and one large room. The room size essentially impacts on the number of LEDs required to fulfill the user requirements, since they are expected to be satisfied in the whole room. If we consider a more realistic scenario, like an office or a furnished room where a user cannot lay in all the positions, the optimization technique is still effective and the impact of different room layout is irrelevant.

Table 1. Parameters used in the simulation results.

LED angle of irradiance, ϕ_j	70°
PD field of view (FOV), ψ_C	60°
PD physical area, A	1 cm^2
Communication bandwidth, B	10 MHz
Refractive index, n	1.5
Optical filter gain, $T_s(\psi)$	1
Noise power spectral density, N_0	10^{-21} A^2/ Hz
Localization Bandwidth, B_L	50 KHz
Room size (small)	$4 \times 5 \times 3$ m^3
Room size (large)	$9 \times 10 \times 3$ m^3
Vertical distance, d_v	2 m
1st use-case	$\Theta^* = 0.01$ m^2, $R^* = 30$ Mbps, $I^* = 375$ lux, $\mathcal{F}^* = 0.25$
2nd use-case	$\Theta^* = 0.1$ m^2, $R^* = 50$ Mbps, $I^* = 250$ lux, $\mathcal{F}^* = 0.4$

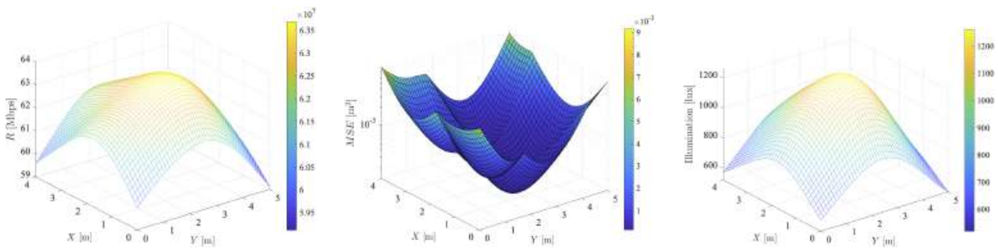
We initially consider Problem 1 and aim to describe two use-case scenarios for which possible solutions exist.

6.1. Performance for Problem 1

The first use-case is defined by the following constraints, i.e., $\Theta^* = 0.01$ m^2, $R^* = 30$ Mbps, $I^* = 375$ lux, and $\mathcal{F}^* = 0.25$, and applies to both small and large rooms, whose sizes are defined in Table 1. It is important to notice that we translate the illumination requirements (that is measured in lumen or watts) into lux by considering the illuminated area. Due to high localization accuracy, this use-case well depicts a localization-oriented scenario, characterized by lower data rate and illumination constraints. On the other hand, the second use-case is more oriented to data rate and illumination requirements, while it presents lower localization accuracy, i.e., $\Theta^* = 0.1$ m^2, $R^* = 50$ Mbps, $I^* = 250$ lux, and $\mathcal{F}^* = 0.4$, and again applies to both small and large rooms.

Figure 2 presents the data rate, the mean square error (MSE), and the illumination spatial distributions in a small room when the constraints are $\Theta^* = 0.01$ m^2, $R^* = 30$ Mbps, $I^* = 375$ lux, and $\mathcal{F}^* = 0.25$. The optimal solution is $N = 6$ LEDs and we report one possible allocation that meets the constraints of Problem 1. Indeed, $N = 6$ is the minimum number of LEDs that can satisfy the requirements of the first use-case. In Fig. 2(a), we can notice the data rate shows small fluctuations within the room, ranging in $\approx [59, 62]$ Mbps, largely overcoming the minimum requirements of $R^* = 30$ Mbps. Similar considerations also apply to the illumination spatial distribution depicted in Fig. 2(c), which guarantees the constraint of $I > I^* = 375$ lux. Moreover, it must be noted that here the illumination fluctuation is limited by the constraint about fairness and this, as previously disclosed, limits also the maximum allowed illumination. Furthermore, dealing with the localization error, Fig. 2(b) depicts MSE values lower than $\Theta^* = 0.01$ m^2, with peaks corresponding to each LED position.

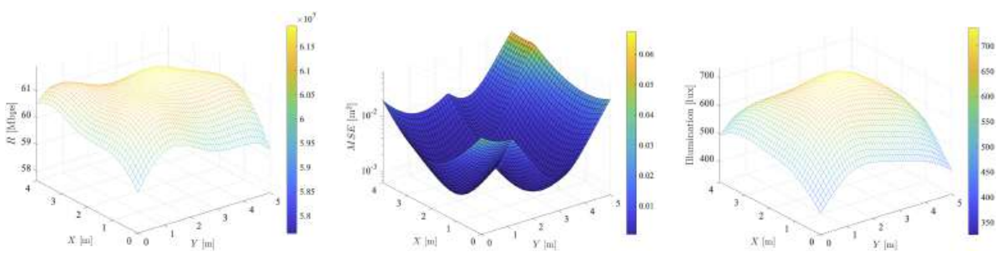
Results about the second use-case are depicted in Fig. 3. This scenario aims to guarantee high data rate all over the room, as well as higher illumination and fairness requirements. In this case, due to increasing data rate requirements, i.e., $R^* = 50$ Mbps, but lower accuracy, i.e., $\Theta^* = 0.1$



(*a*) (*b*) (*c*)

Fig. 2. Problem 1. First use-case in a small room. 6 LEDs can guarantee $\Theta^* = 0.01$ m^2, $R^* = 30$ Mbps, $I^* = 375$ lux, and $\mathcal{F}^* = 0.25$, thus resulting in (a) data rate, (b) localization error, and (c) illumination spatial distributions, respectively.

m^2, as well as $I^* = 250$ lux, and $\mathcal{F}^* = 0.4$, the solution of Problem 1 is $N = 4$ LEDs, which is lower than previous 6-LEDs case, needed to satisfy the first use-case scenario. This can be justified by considering that having relaxed the constraint on localization leads to the need of lower number of LEDs even though we have increased the constraint about rate. As compared to Fig. 2(a), notice that the data rate is almost the same by confirming that the constraint about rate, in this particular case, is not an issue and it is still around ≈ 60 Mbps. Similarly, also the other performance are the same as in the first use-case.



(*a*) (*b*) (*c*)

Fig. 3. Problem 1. Second use-case in a small room. 4 LEDs can guarantee $\Theta^* = 0.1$ m^2, $R^* = 50$ Mbps, $I^* = 250$ lux, and $\mathcal{F}^* = 0.4$, thus resulting in (a) data rate, (b) localization error, and (c) illumination spatial distributions, respectively.

Moving to the second typology of room, in Fig. 4 we report the data rate, the localization error, and the illumination spatial distributions, achieved with the first use-case in a large room for $\Theta^* = 0.01$ m^2 5, $R^* = 30$ Mbps, $I^* = 375$ lux, and $\mathcal{F}^* = 0.25$ constraints. As expected, the number of LEDs increases with respect to the previous situations since we have a wider area to cover with the three services (i.e., illumination, communication, and positioning) hence the minimum value is $N = 41$ LEDs. Again, we notice that the data rate spatial distribution is in the interval ranging from 60 Mbps to 65 Mbps as depicted in Fig. 4(a). It follows that the increase of the number of LEDs as a solution of Problem 1 is not necessary to reach performance constraints, as they are overcome indeed, but it is needed to guarantee the constraints all over the room that is larger than the small one. Moreover, since the constraint in terms of rate is largely satisfied, such a high number of LEDs (with respect to the previous case) strictly depends on the other constraints. This aspect will be deeply tackled later. The illumination level achieved is higher with respect to the constraints as shown in Fig. 4(c) hence this directly leads to conclude that the major issue is

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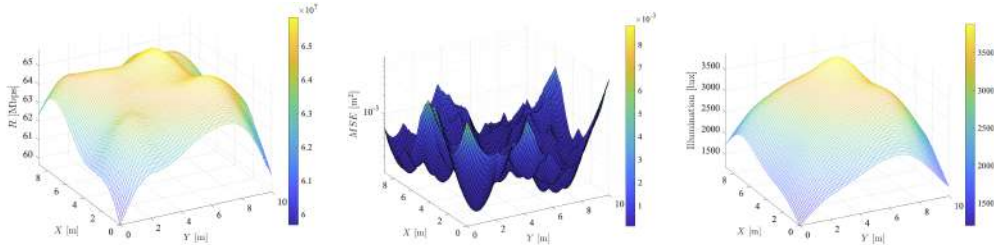
represented by the localization constraint. A deep analysis of the surface reporting the MSE in Fig. 4(b) shows that on one corner we have a value of MSE approaching the constraint value.



(*a*) (*b*) (*c*)

Fig. 4. Problem 1. First use-case in a large room. 41 LEDs can guarantee $\Theta^* = 0.01$ m^2, $R^* = 30$ Mbps, $I^* = 375$ lux, and $\mathcal{F}^* = 0.25$, thus resulting in (a) data rate, (b) localization error, and (c) illumination spatial distributions, respectively.

To conclude the use-cases, in Fig. 5, we report the data rate, the localization error, and the illumination spatial distributions achieved when the second use-case in a large room is considered for constraints $\Theta^* = 0.1$ m^2, $R^* = 50$ Mbps, $I^* = 250$ lux, and $\mathcal{F}^* = 0.4$. For these requirements, the number of LEDs needed is $N = 8$. Also in this case, we notice that the data rate and illumination spatial distribution is similar to the previous results, as reported in Fig. 5(a) and (c), respectively. In detail, with respect to the previous case, here the constraints about rate and fairness are stricter while the ones about minimum illumination and mean square error are a bit relaxed. It is possible to appreciate how the rate in Fig. 5(a) is lower with respect to the previous case since the number of LEDs is considerably lower and this can be easily justified by observing that the constraint in terms of MSE has been relaxed. In fact, in Fig. 5(b), we can see how the MSE values are higher with respect to the previous case and this is due to constraint relaxation. Moreover, in terms of illumination, we can note that the whole illumination levels decreased with respect to the previous case still meeting the required constraint.

Even we obtain in Fig. 5 similar behavior with respect to Fig. 4, it is worth to underline that the shape is different. This is mainly due to the choice of showing how different LEDs placement can lead to different surfaces even though they are able to meet all the constraints. It follows that our approach seeks for a solution to a minimization problem, under different QoS requirements that need to be satisfied in each position of a given room. However, different LED deployment can change the spatial distribution of the QoS requirements, but still guaranteeing the QoS constraints. Just to provide an example, in Fig. 6 and Fig. 7 we show the performance achieved by two different sets of LEDs that are solutions of Problem 1 and guarantee the second use-case requirements in all the positions of a small room. Specifically, each LED position is indicated in the figures. We observe that, while guaranteeing the QoS requirements, different shapes of the QoS spatial distributions are achieved. Similarly, in Fig. 8(a) and (b) we depict the normalized illumination spatial distribution achieved for the first and second solution, respectively. Also in this case, we observe that the ratio $I/I_{\max}$ is always lower than 1, but different shapes are obtained in case of the two different LED deployments.

Passing now from use cases to more general situations, we want to emphasize the role played by different values of the constraints. In Fig. 9 we report the number of APs needed to grant the constraints when a small room scenario is considered. In particular in Fig. 9(a) we detail the number of access points needed as a function of illumination level ranging from 250 lux to 562.5 lux when different values of rates per user are considered (namely, $R^* = 30$ Mbps, 37 Mbps, 45 Mbps). The main comment is that increasing rate leads to increasing the number of APs. In other words, as from the case of a rate of 30 Mbps requiring 4 LEDs, when the rate increases

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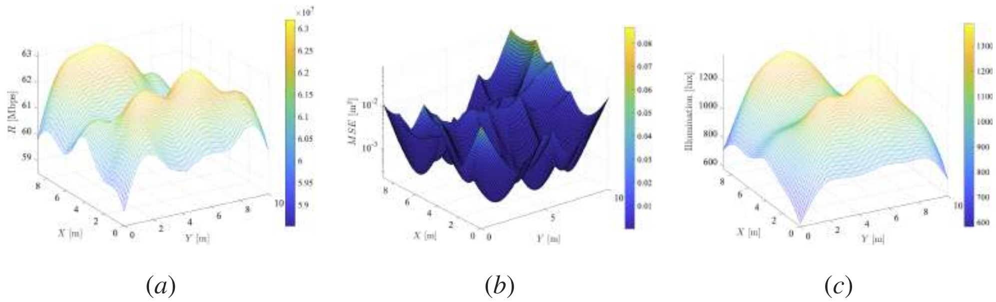

Fig. 5. Problem 1. Second use-case in a large room. 16 LEDs can guarantee $\Theta^* = 0.1$ m^2, $R^* = 50$ Mbps, $I^* = 250$ lux, and $\mathcal{F}^* = 0.4$, thus resulting in (a) data rate, (b) localization error, and (c) illumination spatial distributions, respectively.

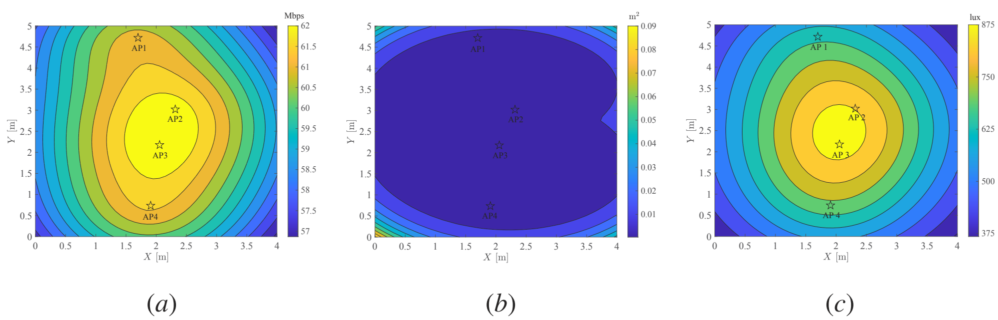

Fig. 6. Deployment of 4 LED APs, as a first solution of Problem 1 with second use-case requirements in a small room, achieving (a) data rate [Mbps], (b) localization error [m^2], and (c) illumination [lux].

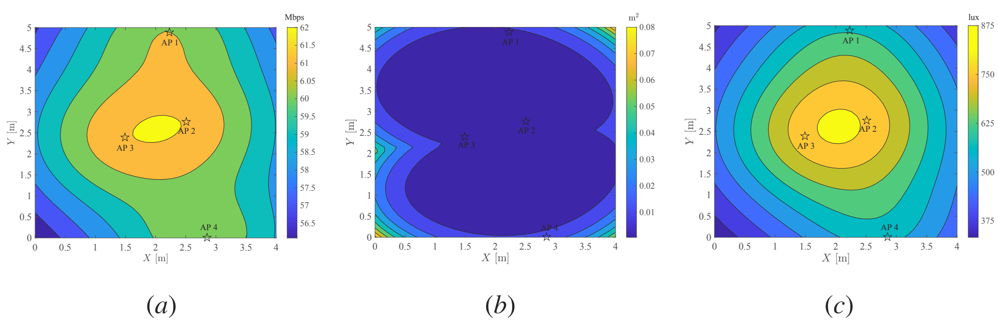

Fig. 7. Deployment of 4 LED APs, as a second solution of Problem 1 with second use-case requirements in a small room, achieving (a) data rate [Mbps], (b) localization error [m^2], and (c) illumination [lux].

the illumination constraint is always satisfied. We obtain these results by asking for fairness of 30% and $\Theta^* = 0.01$ m^2. Moreover, from the same figure it is possible to note while asking more illumination level slightly increases the number of LEDs. Besides, for the particular case of rate equating 45 Mbps, increasing the required illumination does not affect too much the number of LEDs, since $N = 9$ LEDs can guarantee illumination constraint up to 562.5 lux.

Dealing the impact of fairness and illumination level, we report in Fig. 9(b) the performance in terms of APs needed to achieve $R^* = 30$ Mbps and $\Theta^* = 0.01$ m^2, by considering different

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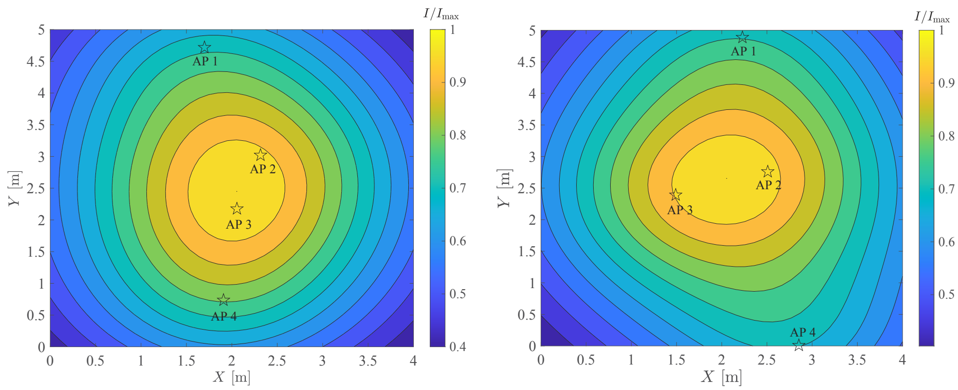
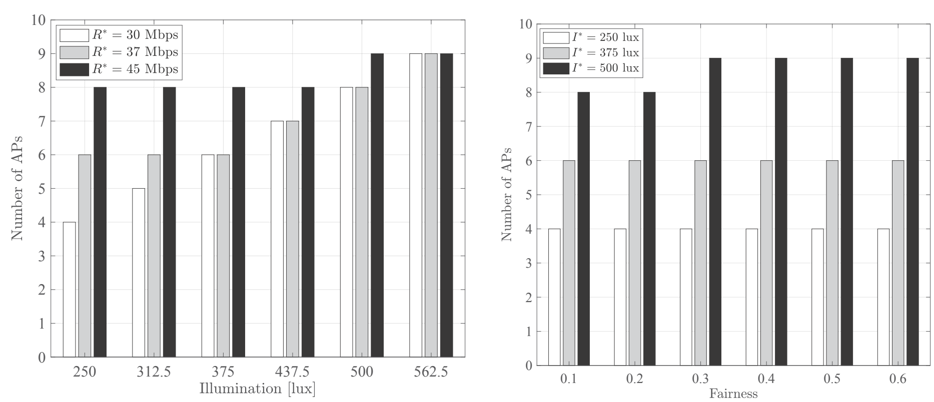


(a) (b)

Fig. 8. Normalized illumination distribution achieved through 4 LED APs, as a (a) first and (b) second solution of Problem 1 with second use-case requirements in a small room.



(a) (b)

Fig. 9. Problem 1. Minimum number of LED APs in a small room vs. (a) illumination [lux], and (b) fairness.

illumination levels (namely 250 lux, 375 lux and 500 lux) and ranging the fairness level from 10% to 60%. The most important aspect is that for illumination levels of 250 lux and 375 lux we are already able to guarantee the fairness value of 60% with a fixed value of APs that are 4 LEDs for 250 lux and 6 APs for 375 lux. In the case of 500 lux, increasing the required fairness increases the number of APs needed from 8 to 9. Hence in this particular performance description it seems that the number of LEDs is not so sensible (it is still sensible but less than other constraints) to the illumination constraints.

Analogously, we report in Fig. 10 exactly the same performance already presented in Fig. 9 but computed in the case of large room. A quick comparison of Fig. 10(a) with Fig. 9(a) gives evidence of the impact of the dimension of the room since all the constraints and parameter settings are exactly the same. We observe a strong increase of the number of APs needed to guarantee all the requirements, exhibiting values in the range $\approx [40, 50]$ APs. Moreover, by comparing Fig. 10(b) with Fig. 9(b) we can highlight a flat variation on the number of APs for different illumination constraints, given a fixed fairness requirement. This means that the minimum number of APs can largely satisfy the illumination constraints. Also, differently from the small room scenario, we can observe a short dynamicity on the number of APs, which are still in the range $\approx [40, 50]$. Indeed, for a small room scenario, we can appreciate different achievement for different values of APs, while in case of large room a high number of APs such as ≈ 40 can largely guarantee all the constraints.

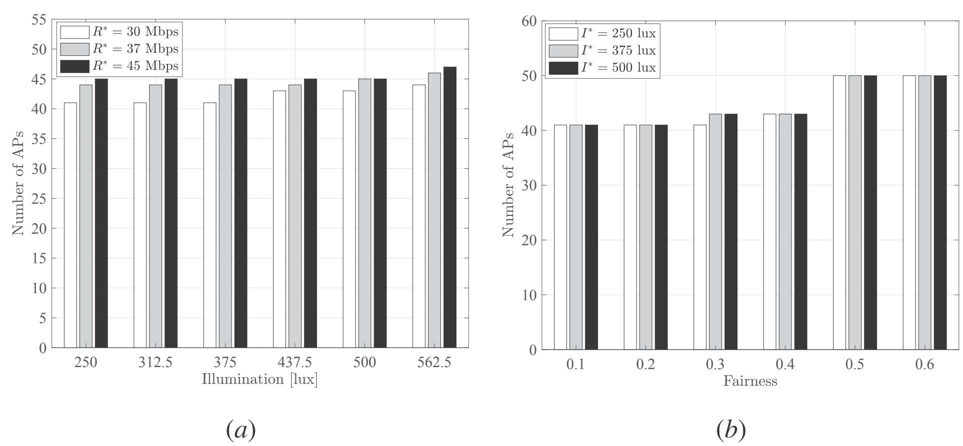


Fig. 10. Problem 1. Minimum number of LED APs in a large room vs. (a) illumination [lux], and (b) fairness.

In order to study the joint impact of localization and data rate constraints, in Fig. 11 we report the solutions of Problem 1 when the two use-cases are considered for the small room scenario. In particular, by observing Fig. 11(a) for the first use-case we can appreciate how increasing rate does not lead to a particular sensible increase of the number of LEDs since for the same value of MSE we have ranges from $7 \div 8$ for $\Theta = 0.005$ m^2 to $4 \div 6$ for $\Theta = 0.01$ m^2. However, spanning Fig. 11(a) for the same required rate allows to give evidence of the fact that stricter estimation leads to higher number of APs required. Furthermore, notice that as expected for increasing localization requirements, the number of APs increases.

The second use-case is analyzed still for MSE and rate constraints in Fig. 11(b). The behavior is very similar to the first use-case since also in this situation higher estimation accuracy leads to higher number of LEDs for the same required rate and, more, flat or increasing number of LEDs for increasing rate at the same MSE. Hence, also in this case we have that the MSE ranges from $8 \div 9$ for $\Theta = 0.005$ m^2 to $4 \div 6$ for $\Theta = 0.01$ m^2.

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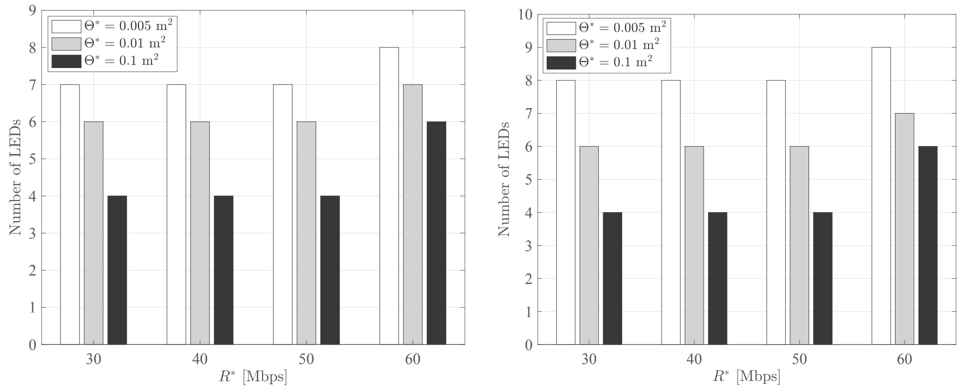
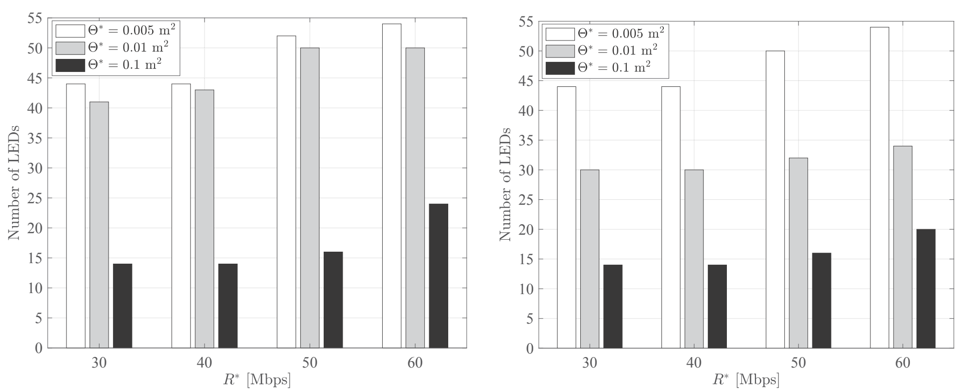


(*a*) (*b*)

Fig. 11. Problem 1. Minimum number of LEDs in a small room vs. data rate [Mbps], in case of the (a) first and (b) second use-case, respectively.

The first and second use-case are also presented for the large room context in Fig. 12, still considering the impact of MSE and required data rate per user. If we compare the results for the large room with those of the first use-case but in small room, as expected we have a considerably higher value for what concerns the solution in terms of number of LEDs. The behavior in terms of inter-relationships between data rates and MSE is similar in Fig. 12 and in Fig. 11. Specifically, in Fig. 12(a) we can note that the maximum achieved number of LEDs is now $N = 41$ and no more, as in the small room, limited to $N = 6$. Once more, this huge increasing is due to the higher area to be covered since it is now 90 m^2 and no more 20 m^2.



(*a*) (*b*)

Fig. 12. Problem 1. Minimum number of LEDs in a large room vs. data rate [Mbps], in case of the (a) first and (b) second use-case, respectively.

Similarly to the first use-case in small room, we have that increasing the data rate per user requested increases the number of APs to be employed. However, here, the behavior is sufficiently flat thus meaning that probably the required rate is not so responsible for the number of APs. The problem seems to be more influenced by the request in terms of MSE since for a rate of 30 Mbps we have a number of APs ranging from $N = 14$ (for $\Theta^* = 0.1$ m^2) to $N = 44$ (for $\Theta^* = 0.005$ m^2), while for a data rate of 60 Mbps the number of APs ranges from $N = 24$ (for $\Theta^* = 0.1$ m^2) to

$N = 54$ (for $\Theta^* = 0.005$ m^2). Similar considerations apply to the second use-case for large room, as depicted in Fig. 12(b).

6.2. *Space of solutions for Problem* 1

Passing now to discuss the role played by the number of LEDs, in Fig. 13 we report the position in the space of constraints (i.e., data rate, illumination and estimation error variance) for the values granted by the *best* LED configurations. In order to specify, best is intended in terms of Euclidean distance with respect to constraints.

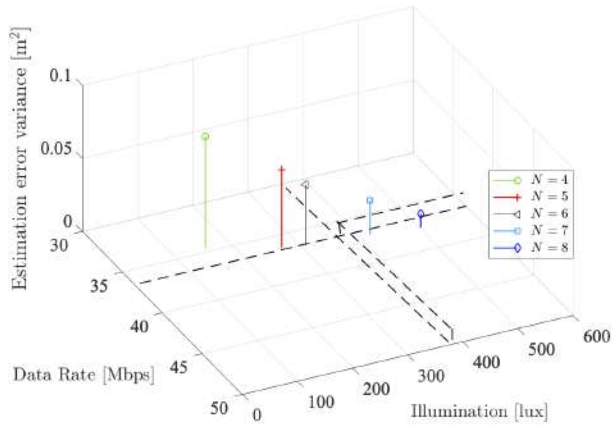


Fig. 13. Problem 1. Position in the constraints space of different configurations for the number of LEDs.

Let us detail the structure of Fig. 13. The dashed lines represent the constraints, hence the open parallelepiped that starts where the constraints lines cross each other, individuates the locus of points that are solution of the problem. This implicitly means that for the following constraints, $R^* = 37$ Mbps, $I^* = 375$ lux and $\Theta^* = 0.01$ m^2, the cases of $N = [4, 5, 6, 7]$ do not lead to any solution since the point is outside the parallelepiped. In particular, the cases with $N = [4, 5, 6]$ do not allow to achieve the minimum rate, the minimum illumination and maximum mean square error. On the other hand, $N = 7$ meets both illumination and data rate constraints. However the estimation error variance is not met. The case of $N = 8$ is able to meet the constraints and the point is within the parallelepiped.

Still considering a geometrical interpretation, in Fig. 14 we plot the sets of different solutions (points) achieving the requirements of use-case 1, for different numbers of LEDs and room sizes. Specifically, in Fig. 14(a) we considered the small room and variable numbers of LEDs that can meet the use-case 1 requirements. We observe that for increasing number of LEDs, i.e., $N = [7, 9, 10, 12]$, the achieved average data rate increases, as well as the average illumination all over the room. It is worth noticing that, for increasing LED sets the average MSE is lower (*right side*). Also, the number of solutions increases for higher number of LEDs. Hence each point is the values we achieve with an allocation of LEDs (position) related to a given number of LEDs as reported in the legend. Similar considerations apply to the large room as depicted in Fig. 14(b). Here the number of LEDs we consider is higher since we are referring to large room. We take into account the following values, that are $N = [41, 45, 50, 55]$. However, we still experience that increasing the number of LEDs move the scatter plot toward a specific direction that is characterized by higher illumination, higher rate per user and lower localization error since, as previously discussed, the constraints are not in trade-off since increasing the number of LEDs help the system to meet the constraints. However, basing on the room and constraints there are some of the characteristics (i.e., illumination, fairness, rate and localization) that can have

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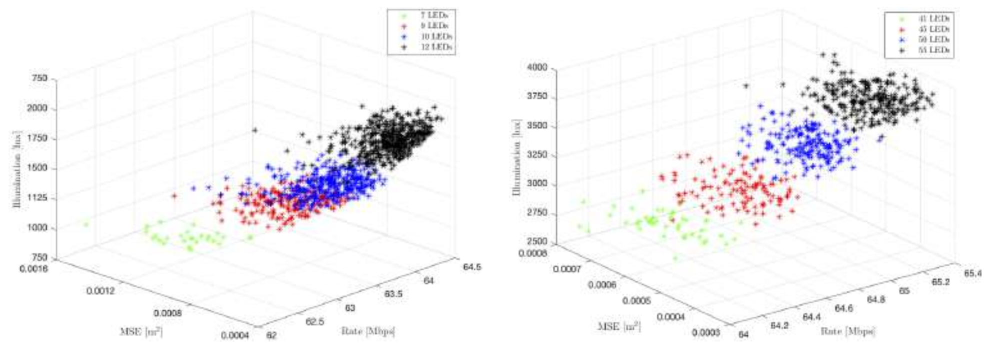
a higher impact since the relative constraint is hard to meet with a low number of LEDs. As a results, sometimes, it may happen that by satisfying a strict constraint leads to have the ones always satisfied.



(*a*) (*b*)

Fig. 14. Problem 1. Distribution of different numbers of LEDs achieving data rate, MSE, and illumination requirements, in case of (a) small and (b) large room, respectively.

Finally, about the number of solutions, we already mentioned that depending on tile dimensions and with exhaustive search we may find more that one solution. This is well represented by the number reported in Table 2. In particular, we detail the number of solutions when $\Theta^* = 0.05$ m^2, $I^* = 150$ lux and $\mathcal{F}^* = 0.3$ for different rates requested that is, 30 Mbps and 37 Mbps. It is worth to highlight that increasing the number of LEDs means to have more opportunities to meet the constraints since it is possible that some configurations that are not solution with N LEDs may be solution with an additional LED (i.e., $N + 1$). This justifies while by increasing the number of LEDs the number of solutions increases. Besides, increasing the rate per user reduces the number of solutions since some configurations that, for example, provide 30 Mbps per user, may not be feasible for 37 Mbps.

Table 2. Number of solutions for different number of LEDs and target when $I^* = 150$ lux is considered for a tile of 0.25 m ×0.25 m, $\Theta^* = 0.05$ m^2, $\mathcal{F}^* = 0.3$ and $4 \times 5 \times 3$ m^3 room.

N	Solutions @$R^* = 30$ Mbps	Solutions @$R^* = 37$ Mbps
4	1920	1520
5	2156	1890
6	2400	2240

6.3. Robustness with respect to tilting and blocking

The above optimization results rely on the assumption that the PDs are always pointing the ceiling and no block occurs. However, it is important to test the reliability of the solution in terms of robustness. To do so, we consider the receiving devices with different tilting angles and then test if the optimization constraints are still met. In fact, proceeding the optimization under the hypothesis of a specific tilting angle does not represent a realistic approach. In Table 3 we report different tilting angles ranging from 0° to 50° and in case of 7 APs. Moreover, we indicate in the first row the constraints so as to show how much far we are. 3pt

For what concerns the achieved rate (see the third column), we observe that the constraint of 30 Mbps is always satisfied since the minimum data rate obtained for a tilting angle of 50° is higher

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Table 3. Achieved values of positioning error, rate and illumination with different values of tiling angles, for 7 APs.

Tilting angle	$\Theta^* = 0.01\text{m}^2$	$R^* = 30$ Mbps	$I^* = 250$ lux	$\mathcal{F}^* = 0.4$
0°	0.006	38.2	372	0.5
10°	0.0076	36.5	361	0.47
30°	0.0092	32.7	336	0.42
50°	0.013	30.2	307	0.37

than the constraint. The same is for the illumination level (see the fourth column). However, with a tilting angle of 50° it is no more possible to grant the system to meeting all the constraints since both localization error and fairness ones fail to be verified. At the same time, running the simulations with a different number of AP (i.e., 8) leads to meet all the constraints. Hence, we can conclude that the price to be paid to assure the achievement of all the goals (constraints) is to increase, in this particular case, the number of APs by 1, thus meaning increasing it from 7 to 8.

Moving now to discuss the effect of blocking, we consider a worst case scenario when the center of mass of an object of 18×10 cm^2 dimensions is posed in correspondence to the (imaginary) vertical line connecting a LED and a PD. The above mentioned object dimensions are comparable a human hand covering signal. Furthermore, we want to point out that considering LED and PD vertically aligned leads to consider a worst case with respect to a situation in which the LED light is not perpendicular since the effect of blocking is largely mitigated in this latter case.

From Table 4, we evince that for a distance from the blocking object to the PD from 170 cm to 70 cm the performance worsens, still guaranteeing all the constraints. Hence the localization error, rate and illumination constraints are met even though by decreasing the performance. When the distance achieves 20 cm, the performance degrade and the constraints are no more met. However, it is really interesting to note that the value achieved in this last case (i.e., 20 cm) is not very far from other results; this is due to the other signals, coming with different angles to the PD, since the blocking object is not able to block also the other light paths propagating from the other LEDs deployed in the room.

Table 4. Achieved values of positioning error, rate and illumination with different values of distance with a blocking object.

Block-obj distance	$\Theta^* = 0.01$ m^2	$R^* = 30$ Mbps	$I^* = 250$ lux	$\mathcal{F}^* = 0.4$
170 cm	0.0078	35.4	308	0.48
120 cm	0.0081	34.7	297	0.46
70 cm	0.0096	32.2	266	0.41
20 cm	0.012	12.6	192	0.23

6.4. Achieved performance for Problem 2 and Problem 3

We finally show the solutions of Problem 2 and Problem 3. We remind that these two problems consider different types of LEDs and distinguish communication and localization LEDs. Specifically, Problem 2 focuses on localization and illumination constraints only, thus separating localization issue from the communication one, which is not investigated in this problem. On the other side, in Problem 3 communication and localization aspects are still separated (i.e., it distinguishes localization and communication LEDs), but jointly investigated so that it seeks for both the minimum number of communication LEDs that satisfy data rate constraint (i.e., $\min N_c$), and the minimum number of localization LEDs that satisfy localization constraint (i.e., $\min N_\ell$). Notice that the illumination constraint should be satisfied in all the position of the room and it is achieved by both communication and localization LEDs.

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Leveraging on such considerations, we expect that the minimum number of LEDs that are solution of Problem 2 will be lower than those as solution of Problem 1, since they are limited to N_ℓ LEDs, i.e., $N_\ell < N$. Also, for Problem 3 we can observe that $N = N_c + N_\ell$, as expected. In Fig. 15 we present the minimum number of LEDs that are solution of Problem 2, so focusing only on N_ℓ LEDs. We notice that for small room, we need from $N_\ell = 4$ to $N_\ell = 8$ LEDs to achieve localization accuracy of $\Theta^* = 0.1$ m^2 and $\Theta^* = 0.005$ m^2, respectively. As expected, N_ℓ increases for higher accuracy, as well as in large room where N_ℓ ranges from 14 to 44 for $\Theta^* = 0.1$ m^2 and $\Theta^* = 0.005$ m^2, respectively.

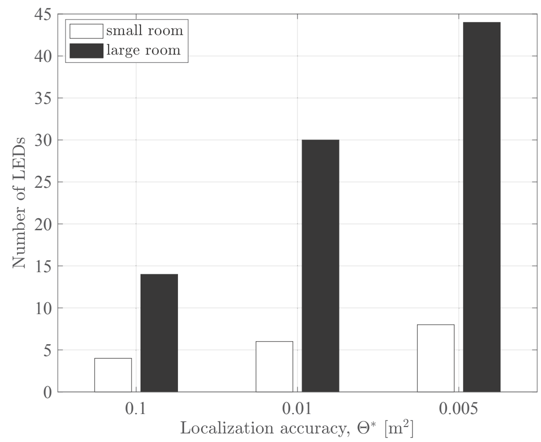
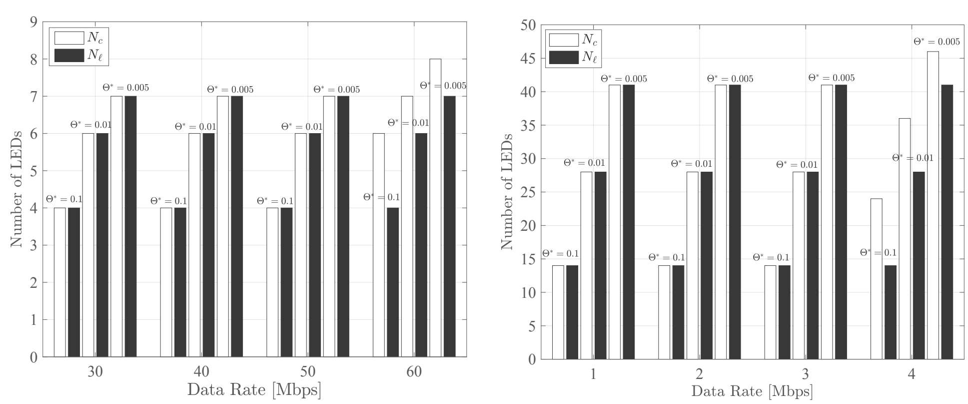


Fig. 15. Minimum number of localization-only LEDs as a solution of Problem 2, vs. localization accuracy Θ^* [m^2] and in case of small and large room.

The solutions of Problem 3 are reported in Fig. 16, in case of small and large room. We show the number of LEDs N_c and N_ℓ for different data rate constraints and MSE values (i.e., 0.1, 0.01 and 0.005 m^2). Specifically, in Fig. 16(a), for increasing data rate, N_c is always constant till $R^* = 60$ Mbps where the communication LEDs increase till 8. Notice that for different values of Θ^* m^2, we observe an increase of N_ℓ for higher accuracy, as well as an increase of N_c due to



(a) (b)

Fig. 16. Problem 3. Minimum number of LEDs vs. data rate [Mbps], achieving different localization accuracy requirements Θ^* [m^2], in case of (a) small and (b) large room, respectively.

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illumination constraints. Similarly, in Fig. 16(b), the number N_c is constant for different data rate requirements, while increases for $R^* = 60$ Mbps. Also in this case, N_ℓ varies for different MSE constraints. In general, we observe an increase on both N_c and N_ℓ in case of large room.

7. Conclusions

An optimization problem related to the deployment of white LEDs in indoor scenarios has been investigated in this paper. Considering a full integrated VLC/VLP indoor system, we rely on the threefold pillar of illumination, communication, and positioning of visible spectrum technology. The aim is how to guarantee in any position of a given room both illumination, communication, and positioning requirements through a minimum number of resources (i.e., LED devices).

Since communication and positioning requirements may need of different number of LEDs, we distinguish three different optimization problems, assuming (*i*) "peer" LEDs, i.e., each LED is aimed for both communication and localization, (*ii*) "localization-only" LEDs, and (*iii*) localization/communication LEDs, i.e., communication LEDs distinguish from localization-only ones. Different problems are adopted for different goals, and so the first problem occurs when no LED distinction is considered, while the second problem occurs when a VLC network is already set and we aim to build localization services upon that. Finally, the third problem aims to introduce different types of LEDs, and communication and localization are optimized separately.

Following the main simulation results, in Problem 1, according to specific geometric and channel features, high data rate (i.e., $R^* = 60$ Mbps) and accuracy requirements (i.e., $\Theta^* = 0.005$ m^2) are achieved with around $N = 8$ and $N = 40$ LEDs, in a small and large room, respectively. On the other side, focusing on localization-only LEDs as in Problem 2, and so neglecting the data communication task, for high accuracy (i.e., $\Theta^* = 0.005$ m^2), we need 7 and 41 LEDs in small and large room, respectively. Furthermore, the solutions of Problem 3 correspond to those achieved with previous Problem 1. Last, we tested the validity of the proposed approach by comparing the results obtained though our simulations.

Notice that our algorithm provides an exhaustive search that checks for all the possible positions of a given number of LEDs that are expected to be the solution of the minimization problems. As soon as a LED deployment configuration is a solution of a given problem, the algorithm stops and returns the LEDs' positions. If the need is to reduce costs, while guaranteeing the minimum QoS constraints, our approach provides the minimum number of LEDs that can satisfy all the requirements. Of course, this algorithm has been tested in case of an empty room; in case of a furnished indoor scenario, the LED deployment will be forced to avoid those areas of the room where occlusions can happen (e.g., in proximity of a wardrobe or a bookcase). However, the algorithm will be still effective since the impact of different room layout is irrelevant. On the other hand, if the main goal is to maximize the QoS requirements, and no matter the costs, then we can abuse and exceed the number of LEDs. Of course, we should respect the maximum illumination bound.

As a conclusive remark, we can state that increasing the number of LEDs always leads to satisfy the first three constraints (i.e., rate, localization, and illumination). Fairness is different since it is a constraint on light distribution (in terms of maximum and minimum) so, a low fairness requirement means that increasing LEDs without paying too much attention to light distribution works fine, while a high value of fairness means that only few configurations –positions– of LEDs can be solution since guarantee a good level of light uniformity.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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