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Discussion of Frank Kleibergen Invited Lecture

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Comment on: Identification Robust Testing of Risk Premia in Finite Samples

Paolo Zaffaroni
Imperial College Business School, London, UK

Address correspondence to Paolo Zaffaroni, Imperial College Business School, London SW7 2AZ, UK, or e-mail: p.zaffaroni@imperial.ac.uk

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This paper of Kleibergen, Kong, and Zhan (2021), hereafter KKZ, represents an excellent contribution to the vast literature that concerns testing the validity of asset pricing models. I will first outline the problem and then provide some thoughts on when are the proposed methodologies relevant or not.

Factor asset pricing models can be represented, for the one-factor example, and when the model is correctly specified and exact pricing holds, as

$$R_{it} = \lambda_0 + \beta_i \lambda_F + \beta_i F_t + u_{it}, \tag{1}$$

where we adopt the notation of KKZ, and thus where F_t is the de-meaned risk factor, and u_{it} is the asset-specific component. KKZ goal is to study inference on the risk premium λ_F, that is to evaluate the null hypothesis $H_0 : \lambda_F = \lambda_{F,0}$ for some prespecified value $\lambda_{F,0}$, an important example being $\lambda_{F,0} = 0$. Simplifying the notation, assume that the risk-free asset is tradable and denote $R^e_{it} = R_{it} - R_f$ the excess return, that is we set $\lambda_0 = R_f$. The most popular inferential procedure is based on the two-pass estimator of Fama and MacBeth (1973) (FM) for λ_F:

$$\hat{\lambda}_F = (\hat{\beta}\prime\hat{\beta})^{-1}\hat{\beta}'\overline{R}^e, \tag{2}$$

where $\overline{R}^e = T^{-1}\sum_{t=1}^{T} R^e_t$, with $R^e_t = R_t - R_f 1_N$, and $\hat{\beta}$ equals the OLS estimator (3) from KKZ.

Building on the important work of Kleibergen (2009), KKZ are concerned with testing H_0 when the covariance between the test assets R^e_{it} and the factor F_t is small, that is when the population values of β_i are equal or close to zero, the so-called case of spurious factor, which dramatically affects the FM procedure (see Kan and Zhang 1999). KKZ concern is that many of the existing procedures that mitigate the poor performance of the FM procedure are valid asymptotically when T, the time-series length of the data, diverges but N, the number of assets under consideration, remains fixed. However, the same procedures become less reliable, in terms of size and power, when one considers the case of a large N, as the discrepancy between the finite-sample and asymptotic distribution becomes sizeable.

Therefore, the focus on KKZ is to derive the finite-sample distribution of the existing asset pricing tests robust to spurious factors. To accomplish this goal, KKZ make distributional assumptions, in particular, Gaussianity of asset returns, following previous exact finite-sample results on testing asset pricing models (see, among others, Gibbons, Ross, and Shanken 1989) although these studies ruled out the possibility of spuriousness of the examined factors.

KKZ examine a variety of tests, for which they establish both the asymptotic (when T is large) and finite-sample distribution. The reason for this showcase is that these various tests sometimes exhibit exact-sample distributions that are independent of N, the drawback being that they could be based on a non-standard distribution that needs to be simulated, for practical use of the test. In particular, the factor Anderson–Rubin (FAR) test, proposed by Kleibergen (2009), has a χ^2_{N-1} limiting distribution, but KKZ show that it has a $F_{N-1,T-K-N+1}$ distribution when T is fixed. Note that, at minimum, one requires $T > N + K + 1$. Next, KKZ examine the split of the FAR test into two orthogonal components, the so-called *GLS-LM* and *JGLS*, each of which can also be used as a testing procedure. The advantage is that the *GLS-LM* test has a χ^2_K distribution, whereas the *JGLS* test has a χ^2_{N-K-1} distribution, and are mutually independent, thus mitigating the power and size problems arising when N is assumed even moderately large. However, their finite-sample distributions are, respectively, non-standard (difference of quadratic forms in the Wishart distribution) and an $F_{N-K-1,T-N+1}$. KKZ provide a further decomposition of the FAR, leading to the FM-LM and JFM statistics, with the further advantage that the exact-sample distribution of the former is standard (an F distribution) and does not depend on N. When considering joint hypotheses, KKZ derive upper bounds of the exact-distribution, a well-known difficulty in deriving finite-sample results in a multi-parameter setting. This implies that these results are only conclusive in case of rejection, but not when the p-values are large, that is, small test statistics. KKZ provide a set of extensive Monte Carlo experiments that demonstrates the better size and power properties of the proposed tests when N and T are comparable, that is, $T = 55$ and $N = 31$. The experiments also show that no difference arises between the asymptotic and finite results when T is much larger than N, that is, $T = 500$ and same N as in the previous cases. KKZ revisit the C-CAPM empirical case and conclude that the evidence in favor of the significance of consumption growth and cay (see, among others, Lettau and Ludvigson 2001) is not warranted.

This line of research is extremely important, as the issue of spuriousness of the candidate risk factors in beta-pricing models is often overlooked by financial economists. A limitation of the proposed approaches is that N needs to be smaller than T. This sometimes can limit the applicability of these approaches, for example when considering large panels of individual stocks where N is often much larger than T. In this circumstance, the standard FM procedure exhibits first-order biases and these biases are likely to carry through the present testing procedures. However, Raponi, Robotti, and Zaffaroni (2020), building on Shanken (1992), show how to extend the FM procedure to draw inference on the asset pricing model (1) through a modified Cross Sectional Regression (CSR) OLS estimator. The only relevant aspect is that when T is fixed, then one cannot identify λ_F but rather $\lambda_F^* = \lambda_F + \overline{F} - EF_t$, where $\overline{F} = 1/T \sum_{t=1}^{T} F_t$, denominated as the *ex post* risk premium by Shanken (1992). A very important result is that although λ_F^* differs from λ_F, one can still construct a valid test of the pricing ability of λ_F based on the empirical *ex post* pricing errors that correspond to the estimator for λ_F^*. Moreover, the limiting distributions involved are always standard, although the convergence rate is now $\sqrt{N}$, as opposed to $\sqrt{T}$. I conjecture that a large-N analysis of the FAR-type tests proposed by KKZ can be successfully developed, providing a further tool for inference on linear asset pricing models. Another fruitful generalization would be to extend the analysis of KKZ to conditional factor models, whereby loadings and risk premia are driven by a set of predetermined state variables, that is, instruments (see, among others, Gagliardini Ossola, and Scaillet 2016). For instance, one can envisage a scenario where the risk premia are on average very small over a given interval of time, but taking large negative and positive values within such interval. An inferential procedure, robust to spuriousness of the candidate risk factors, able to assess the significance of time-varying risk premia would certainly be useful to empiricists working in empirical asset pricing.

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