CURSOS E CONGRESOS

Numerical Methods for Hyperbolic Equations

Theory and Applications

An International Conference to Honour Professor E. F. Toro

Book of Abstracts

Santiago de Compostela
July, 4th-8th, 2011

Edited by
A. Bermúdez
L. Cea
E. Vázquez-Cendón

UNIVERSIDADE DE SANTIAGO DE COMPOSTELA
publicacións

Numerical Methods
for Hyperbolic Equations

CURSOS E CONGRESOS DA
UNIVERSIDADE DE SANTIAGO DE COMPOSTELA
Nº 202

Numerical Methods
for Hyperbolic Equations
Theory and Applications
An International Conference
to Honour Professor E. F. Toro

Santiago de Compostela, July, 4th-8th 2011

BOOK OF ABSTRACTS

Edited by

A. BERMÚDEZ
L. CEA
E. VÁZQUEZ-CENDÓN

2011
UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

Nunerical methods for hyperbolic equations : theory and applications : an international conference to honor professor E. F. Toro, Santiago de Compostela, July 4-8, 2011 / edited by A Bermúdez, L. Cea, E. Vázquez-Cendón. — Santiago de Compostela : Universidade, Servizo de Publicacións e Intercambio Científico, 2011. — p. : cm. – (Cursos e congresos da Universidade de Santiago de Compostela ; 202). – D.L.   . — ISBN 978-84-9887-713-7

1 Análise numérica. 2 Ecuacións hiperbólicas. I. Bermúdez de Castro, A., ed. II. Cea, L. ed. III. Vázquez-Cendón, E., ed. IV. Universidade de Santiago de Compostela. Servizo de Publicacións e Intercambio Científico, ed. V. Serie

519.6
517.956.3

Edita
Servizo de Publicacións e Intercambio Científico
Campus Vida
15782 Santiago de Compostela
www.usc.es/publicacions

ISBN 978-84-9887-713-7 (edición dixital .pdf)

Contents

Numerical Methods for Hyperbolic Equations: Theory an Applications. An international conference to honour Professor E.F. Toro
University of Santiago de Compostela, 4-8 july 2011, Spain

A WENO scheme for the integral form of contravariant shallow water equations

F. Gallerano, **G. Cannata**, **M. Tamburrino**[1]

Keywords: 2D shallow water equations, Upwind WENO scheme, integral form, contravariant formulation, generalized curvilinear coordinates, Exact Riemann Solver

Parallel Session: Shallow water flows

Abstract Many authors solve shallow water equations by using Weighted Essentially Non-Oscillatory (WENO) schemes. Flow simulations over computational domains characterized by a complex boundary can be performed by numerical integrations of the contravariant form of motion equations on a generalized curvilinear boundary conforming grid.

In numerical solutions of motion equations in contravariant formulation, two contradictions appear. The first contradiction is related to the presence of Christoffel symbols in motion equations. It is well known that numerical methods for the solution of the conservation laws in which the convective terms are expressed in non-conservative form do not guarantee the convergence to weak solutions. Consequently in the integrations of the conservation laws in whose solutions shocks are present, convective terms must be expressed in conservative form. In the contravariant formulation of motion equations, covariant derivatives give rise to Christoffel symbols. These terms are extra source terms. They come in with the variability of base vectors and do not permit the definition of convective terms in a conservation form. The second contradiction is related to the difficulty of exactly numerically satisfying the metric identities and preserve the freestream conditions. A well known geometric identity is given by the condition that a cell is closed. In a curvilinear system of reference the summentioned condition becomes the metric identity. If numerical approximations of the metric coefficients do not exactly satisfy the above mentioned identities, the numerical approximations of derivatives of uniform physical quantities do not vanish and freestream conditions are not preserved.

The original contribution of this work is the definition of a new Upwind WENO scheme for the solution of the 2D shallow water equations expressed directly in contravariant formulation. An element of novelty presented in this paper regards the definition of a formal integral expression of the shallow water equations in contravariant formulation. The depth-integrated motion equations (in contravariant form) are integrated on an arbitrary surface and are resolved in the direction identified by a constant parallel field of unit vectors. In this way we present an integral form of the contravariant shallow-water equations in which Christoffel symbols are avoided. In order to correct the effects produced by the spurious source terms related to the difficulties of satisfying numerically the metric identities and in order to guarantee the satisfaction of the freestream preservation condition we propose an original procedure.

We define the water depth as h and the depth averaged velocity vector as $\vec{u}$, whose components are defined in the Cartesian system of reference. Let be $\vec{v} = \vec{u}h$. In order to introduce the notation to be used, we consider a transformation $x^l = x^l(\xi^1, \xi^2)$ from the Cartesian coordinates $\vec{x}$ to the curvilinear coordinates $\vec{\xi}$ (note that superscripts indicate components and not powers in the present notation). Let $\vec{g}_{(l)} = \partial\vec{x}/\partial\xi^l$ be the covariant base vectors and $\vec{g}^{(l)} = \mathrm{grad}(\xi^l)$ the contravariant base vectors. Let r^l be the contravariant components in the curvilinear coordinate system of the vector $\vec{v}$. The integral form of the depth-integrated continuity equation is:

$$\int_{\Delta A} \frac{\partial h}{\partial t} dA + \int_L r^m n_m dL = 0\,, \tag{1}$$

where ΔA is an arbitrary surface element whose contour line is L, and n_m is the covariant outward normal. The integral form of the depth-integrated momentum equation is:

$$\int_{\Delta A} \vec{\tilde{g}}^{(l)} \cdot \vec{g}_{(k)} \frac{\partial r^k}{\partial t} dA + \int_L \vec{\tilde{g}}^{(l)} \cdot \vec{g}_{(k)} \left(\frac{r^k r^m}{h} + g^{mk} \frac{h^2}{2} \right) n_m dL =$$
$$\int_{\Delta A} \vec{\tilde{g}}^{(l)} \cdot \vec{g}_{(k)} \left(G h g^{mk} \frac{\partial H}{\partial \xi^m} + R^k \right) dA\,, \tag{2}$$

where $R^k = G(r^k|\vec{r}|)/(\chi^2 h^2)$ is the bed resistance term, $\chi = (1/M)h^{1/6}$ is the friction coefficient, M is the Manning friction factor, G is the constant of gravity, H is the bed height and $\vec{\tilde{g}}^{(l)}$ indicates the l^{th} contravariant base vector defined at the center of the surface element. Equations (1) and (2) represent the integral expressions of the shallow water equations in contravariant formulation, in which Christoffel symbols are not present.

The numerical integration of Equations (1) and (2) is performed by an Upwind WENO scheme. The Upwind WENO scheme needs a flux calculation at the cell interfaces. These fluxes are calculated by means of the solution of a Riemann problem. An Exact Riemann Solver is used in this framework. All necessary Riemann problems are solved in a locally valid orthonormal basis. This orthonormalization allows one to solve Cartesian Riemann problems that are devoid of geometric terms. The model is validated against several benchmark tests, and the results are compared with theoretical and alternative numerical solutions. The results of these tests show that: the proposed scheme is fifth-order accurate in space and fourth-order accurate in time; the exact C-property for quiescient flow over non-flat bottom profiles is achieved; the original procedure proposed in order to correct the effects produced by the spurious source terms related to the difficulties of satisfying numerically the metric identities, allows to satisfy the freestream preservation property even in highly distorted meshes.

Mathematics Subject Classification 2000: 76D99, 76M12

[1]Department of Civil, Architectural and Ennvironmental Engineering
University of Rome "La Sapienza"
Faculty of Engineering. Via Eudossiana 18, Rome, Italy
francesco.gallerano@uniroma1.it

CURSOS E CONGRESOS
Nº 202

This volume contains the abstracts of the papers presented at the International Conference on *Numerical Methods for Hyperbolic Equations: Theory and Applications* held in the Faculty of Mathematics of the University of Santiago de Compostela, Spain, from 4th to 8th July 2011. The conference was organized to honour Professor Toro in the month of his 65th birthday. We think that all contributions are a valuable state of the art of the most recent research in the topic of numerical methods for hyperbolic equations providing the reader with the latest developments concerning the mathematical aspects and the applications of this active field of mathematics.

USC UNIVERSIDADE DE SANTIAGO DE COMPOSTELA