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Nonlinear Time-Delay Systems

A Geometric Approach

Springer

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Preface

This book is devoted to nonlinear time-delay control systems. Although they include the class of linear time-delay systems, the specific mathematical tools valid (only) for this subclass of systems will not be developed or used herein.

Thus, in this introductory chapter, a sketch is given of what can be found elsewhere (Richard 2003) and which will not be considered herein.

Linear Time-Delay Systems

For the subclass of linear time-delay systems in continuous time, the use of the Laplace transform yields quasi-polynomials in the Laplace variable s and in e^{-s} Gu et al. (2003), Michiels et al. (2007), Niculescu (2001). Among those systems, one may distinguish betweem the so-called retarded systems (Fridman 2014) described by differential equations where the highest differentiation order of the output, or the state, is not delayed, and the so-called neutral systems (Fridman 2001) whose equations involve delayed values of the highest differentiation order of the output, or the state.

Thus, the Laplace transform still enables an input–output analysis of linear time-delay systems (Olgac and Sipahi 2002, Sipahi et al. 2011). This approach can hardly be extended to more general nonlinear time-delay systems which are the main focus of this book. Finite dimension approximations may be appealing, but have a limited interest due to stability issues (Insperger 2015).

Discrete-time linear systems with unknown delays are under interest in Shi et al. (1999). Discrete-time linear systems with varying delays are under interest with an *ad hoc* predictor design in Mazenc (2008).

Stability analysis and stabilization of linear time-delay systems require general tools derived from the Lyapunov theory as in Fridman (2001), Kharitonov and Zhabko (2003).

Stability of linear systems with switching delays is tackled in Mazenc (2021) using trajectory-based methods and the so-called sup-delay inequalities.

Stability and Stabilization of Nonlinear Time-Delay Systems

Some of the most significant historic results obtained for the general class of nonlinear time-delay systems are about the analysis of their stability, thanks to a generalization of the Lyapunov theory, the so-called Krasovskii-type approach (Gu et al. 2003). This approach can hardly be circumvented even in the case of linear time-delay systems (Fridman 2001).

Early design problems for nonlinear time-delay control systems focus on the stabilization. Considering a positive definite functional, the stabilizing control has to render its time derivative negative definite. Advanced control methods as backstepping make use of Lyapunov–Razumikhin–Krasovskii-type theorems as in Battilotti (2020), Krstic and Bekiaris-Liberis (2012), Mazenc and Bliman (2006), Pepe and Jiang (2006). Also, delay-free nonlinear systems may be subject to a delayed state feedback whose stability is tackled in Mazenc et al. (2008). Nonlinear observer designs for systems subject to delayed output measurements are found in Battilotti (2020), Van Assche (2011).

Discrete-time nonlinear systems including delays on the input are considered in Ushio (1996). A reduction process is introduced to define a delay-free system with equivalent stability properties in Mattioni et al. (2018), Mattioni et al. (2021).

Content

This book is essentially devoted to the so-called structure of nonlinear time-delay systems. Thanks to adapted (non-commutative) algebraic mathematical tools, one is able to follow the main features valid for delay-free systems displayed in Conte (2007) which are extended to the time-delay case under interest.

The structure of a system includes notions as controllability, observability, and the related decompositions. The so-called structural control problems are based on inversion techniques like feedback linearization Oguchi et al. (2002), disturbance decoupling (Moog et al. 2000, Velasco et al. 1997) or noninteracting control.

More precisely, geometric tools such as the Lie Bracket are adapted for the class of nonlinear time-delay systems involving a finite number of commensurable delays. Thanks to these tools, the very first criterion for accessibility is provided for this class of systems. Specialized to linear time-delay systems, it corresponds to weak controllability (Fliess 1998).

The realization problem was partially solved for delay-free nonlinear systems either in the continuous-time case in Crouch (1995) or in the discrete-time case (Kotta, et al. 2001, Monaco and Normand-Cyrot 1984). The realization problem is tackled in this book in relation to strongly or weakly observable retarded-type or neutral-type state equations or input–output equations. The notions of order of a system and whether it is of neutral or retarded type are questioned since a higher order retarded-type realization may admit a lower order neutral-type realization as well, both being either weakly observable or strongly observable.

As for control design problems, the exact linearization using feedback is one mainstream to control delay-free nonlinear systems. The same path is followed in this book, although causality of the feedback adds a new constraint and new conditions for the solvability of the problem.

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