Steering and Formation Control of Unicycles Under Single-Rate Sampling

Mattia Mattioni, *Member, IEEE*, Alessio Moreschini, *Member, IEEE*, Salvatore Monaco *Fellow, IEEE* and Dorothée Normand-Cyrot *Fellow, IEEE*

***Abstract*—In this paper, we propose a new digital feedback law for controlling unicycles in a sampled-data scenario. More in detail, two open problems are solved. First, assuming piecewise constant control and sampled-data measures of the state, we consider the problem of driving one unicycle toward a desired position in the plane with a target fixed orientation. Then, we tackle the problem of forcing a network of unicycles to a desired formation with the same orientation when assuming asynchronous sampled-data communication and piecewise constant controls. The design relies upon discrete-time passivity-based arguments yielding the first feedback law (within the digital literature) that employs single-rate sampling while keeping into account the effects of digital devices. The results are finally illustrated through simulations aimed at showing the benefits of the proposed controller to recover the same performances under the continuous-time control law, whose performances are unavoidably lost when implemented in practice.**

***Index Terms*—Digital Control, Nonlinear Systems, Networks of Autonomous Agents.**

I. Introduction

STEERING a mobile robot to a desired position with an arbitrary orientation represents one of the most common and yet challenging control problems [1]. Because of the failure of the Brockett condition [2], two major classes of approaches have been proposed so far: the one relying on time-varying control [3], [4], dynamic feedback [1], [5] and the one based on time-discontinuous feedback [6], [7]. In this respect, in the former case, the design is lead to a linear time-varying passive model deduced from the original one when considering the angular velocity as a time-varying parameter. Accordingly, stabilization is achieved via proportional feedback on the passive output plus a persistently exciting time-varying component [3]. This approach has proven to be robust, easily implementable, and efficient for solving a variety of control problems including tracking, rendezvous, and formation control in a multi-agent perspective [8]–[13]. Nevertheless, all these methods are only intended for continuous-time control, with natural limitations arising in sampled-data implementation. Roughly speaking, because controllers are typically implemented via digital (sampling and hold) devices, the nominal performance certificates deduced by the continuous-time design are generally lost, even when sampling is *small* [14], [15]. Despite this, most control laws typically used in practice are *emulation-based* with the design uniquely addressed in continuous time while neglecting the effect of the actuation devices, usually employed for the implementation [16], [17]. Therefore, redesigning the feedback law while taking into account, and possibly compensating, the effects of sampling (and maintaining the same performances required by the continuous-time design) is essential. In this context, it is well-known that multi-rate digital methods (yielding controllers that are discontinuous in time) provide a very natural tool for handling control of nonholonomic systems as belonging to the class of dynamics that admit, under preliminary continuous-time feedback, a finitely computable sampled-data model [18]. However, the corresponding closed loop is not robust with respect to both model uncertainties and sample-and-hold implementation due to the preliminary continuous-time loop component aimed at transforming the dynamics into a chained form. Also, finite-time convergence in one step comes with a generally significant control effort that limits its practical implementation. Finally, such control laws do not qualify for a straightforward extension to the multi-agent case as intrinsically require a centralized implementation [19]. A preliminary result partially solving the issues arising in the aforementioned contexts was made in [20] where, however, only position steering was considered with no control at all of the orientation.

This work is contextualized in the aforementioned scenario with the objective of designing a new static sampled-data (single-rate) and time-varying class of controllers for steering unicycles to a desired angular and planar configuration. With a slight abuse of notation, we refer to sampled-data (or digital) systems as systems with piecewise constant control inputs and measures available intermittently over time. In this respect, we model the dynamics at the sampling instants as a time-varying linear system that is shown to be passive with respect to suitably defined outputs. Based on this, a new steering control is proposed via digital damping feedback with a further time-varying component that is required to be persistently exciting in the discrete-time sense. As a result, the new controller requires neither a preliminary continuous-time feedback loop nor multi-rate devices, which are typical of digital imple-

M. Mattioni and S. Monaco are with Dipartimento di Ingegneria Informatica, Automatica e Gestionale *A. Ruberti* (Università degli Studi di Roma *La Sapienza*); Via Ariosto 25, 00185 Rome, Italy {mattia.mattioni, salvatore.monaco}@uniroma1.it.

A. Moreschini is with Department of Electrical and Electronic Engineering, Imperial College London, London, SW7 2AZ, UK a.moreschini@imperial.ac.uk

D. Normand-Cyrot is with Laboratoire de Signaux et Systèmes (L2S, CNRS, CentraleSupélec and *Univ. Paris-Saclay*); 3, Rue Joliot Curie, 91192, Gif-sur-Yvette, France dorothee.normand-cyrot@centralesupelec.fr

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mentations employed in the literature. At the same time, we present a first approach to redesigning time-varying single-rate controllers for digital implementation. As a further contribution, we propose a digital feedback in a multi-agent control perspective under asynchronous sampling. In this latter case, a new decentralized and local control is designed for ensuring the formation of a group of unicycles under both asynchronous sampled-data communication and piecewise constant control inputs. The approach we propose for designing the distributed controller relies upon robustness arguments. Modeling the effect of asynchronicity as a distributed time delay, we deduce upper bounds on the involved gains ensuring that consensus is achieved over the network. Those arguments were exploited in [21] and are strictly reminiscent of the ones in [16]. We underline that this contribution is of interest in the context of sampled-data multi-agent control at large in which the sampled-data design must face several challenges involving the necessity of implementing the control law in a distributed and local fashion (see, for instance, [19]).

To the best of the Authors' knowledge, no sampled-data control laws have been proposed thus far while considering, simultaneously, the following features: (i) the control signals are piecewise constant over the sampling period of fixed and not arbitrary length; (ii) measurements of the states of each robot are available intermittently over time; (iii) the communication over the network is asynchronous and intermittent. Most works on this topic (and related ones) only investigate the case of a total continuous time scenario or intermittent communication; in doing so, it is generally assumed the control input is a continuous-time signal, allowed to arbitrarily change over time. As far as purely continuous-time design, among the immense available literature, we highlight the works in [22]–[24] where the design is carried out via passivity-based control and persistence of excitation. This yields a time-varying static control defined in a scenario that is, in principle, the counterpart of the one we adopt under sampling. In addition, the work in [25] exploits continuous-time passivity-based control in the port-Hamiltonian framework with the output internal model-based regulation principle for formation keeping under matched input disturbances. However, the aforementioned results do not hold in the sampled-data context since passivity is not preserved by sampling [26]. A scenario including including intermittent communication was employed in [27]: after a preliminary dynamical (continuous-time) control action making the closed-loop system linear, the design of the control is based on the assigned linear model using standard tools for formation control of LTI systems under intermittent communication. In [28]–[30], a more general framework was considered. In particular, in [28], [29], the rendez-vous problem on the cartesian positions of the unicycles was solved under ternary control. The resulting angular velocity feedback is fully continuous whereas the linear velocity component is piecewise constant (due to its ternary nature) but with the length of each update interval being a degree of freedom for the designer. In addition, in that case, the average consensus problem does not involve the orientation of the unicycles: orientation consensus is ensured by solving a standard regulation problem, locally, aimed at driving all robots to an arbitrary ϑ_d, a priori fixed and known by all robots. In [30], those results are then extended to solve the formation problem under quantized information and undirected communication topology for passive systems. This work (and most references therein) is settled in a different context than ours: the control input (that in the end results to be piecewise constant) can be arbitrarily changed in time which allows the possibility of using continuous-time arguments for part of the analysis and design. Similar considerations hold for [31]–[33] under intermittent communications and, also, event-triggered control (see, for instance, [34]–[36]).

The paper is organized as follows. The problem is formulated in Section II for the case of single and multi-agent systems under digital control. The main result in the case of a single agent is given in Section III, where computational aspects are also developed. The main result for multi-agent systems is in Section IV, where a new digital control is designed for the asymptotic formation of groups of unicycles under asynchronous sampled-data communication. This opens new perspectives as commented in Section V.

NOTATIONS AND PRELIMINARIES

$\mathbb{R}$ and $\mathbb{N}$ denote the set of real and natural numbers including zero respectively. I denotes the identity matrix whereas 0 is the matrix with all zero entries, both of suitable dimensions depending on the context. Given m column vectors $g_j \in \mathbb{R}^n$ with $j = 1, \ldots, m$ we denote by $\mathrm{diag}\{g_1, \ldots, g_m\} \in \mathbb{R}^{mn \times m}$ the block-diagonal matrix with g_j in the main diagonal whereas $\mathrm{col}\{g_1, \ldots, g_m\} = (g_1^\top \ \ldots g_m^\top)^\top \in \mathbb{R}^{nm}$. Given $B \in \mathbb{R}^{n\times m}$ with $n > m$, $B^\perp$ be denotes the orthogonal complement verifying $B^\perp B = 0$ A function $R(x, \delta) : \mathcal{B} \times \mathbb{R} \to \mathbb{R}^n$ is said in $\mathcal{O}(\delta^p)$, with $p \geq 1$, if it can be written as $R(x, \delta) = \delta^{p-1}\tilde{R}(x, \delta)$ for all $x \in \mathcal{B}$ and there exist $\delta^\star > 0$ and a function $\vartheta \in \mathcal{K}_\infty$ such that $\forall \delta \leq \delta^\star$, $|\tilde{R}(x, \delta)| \leq \vartheta(\delta)$. Given two matrices $A \in \mathbb{R}^{n_1\times n_2}$ and $B \in \mathbb{R}^{m_1\times m_2}$, the Kronecker product is denoted by $A \otimes B \in \mathbb{R}^{n_1m_1\times n_2m_2}$. Consider a digraph (that is an unweighted directed graph) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with $\mathcal{V}$ being the set of vertices with cardinality $|\mathcal{V}| = Q$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ being the set of edges (i.e., the set of ordered pairs of nodes). For all pairs of distinct notes $i, j \in \mathcal{V}$ then $(i, j) \in \mathcal{E}$ if there exists an edge from i to j or, equivalently, i is a neighbour of j for all $i \neq j = 1, \ldots, Q$. $\mathcal{N}_i$ denotes the set of neighborhoods associated with $i \in \mathcal{V}$. The Laplacian of $\mathcal{G}$ is denoted by L. Time dependencies (i.e., $\cdot(t)$ or $\cdot_k$) are omitted when clear from the context.

II. MODELING AND PROBLEM STATEMENT

For the sake of clarity, we first set the problem of steering a single unicycle to a desired position with a given orientation. Then, we formulate the formation control problem under asynchronous sampled-data control.

A. The digital steering problem

Consider the unicycle kinematics modeled as

$$\dot{z} = vr \tag{1a}$$

$$\dot{\vartheta} = \omega \tag{1b}$$

with $z := (x\ y)^\top \in \mathbb{R}^2$ the planar coordinates of the unicycle, $\vartheta \in \mathbb{R}$ the angle described by the chassis respect to the horizontal axis,

$$r = r(\vartheta) = \begin{bmatrix}\cos\vartheta & \sin\vartheta\end{bmatrix}^\top$$

the normalized velocity vector and $v, \omega \in \mathbb{R}$ the linear and angular velocities. Roughly speaking, the problem we address stands in designing a sampled-data controller (that is, with piecewise constant input signals and based on sampled measures of the states) driving the unicycle to a desired Cartesian position $z_d = \mathrm{col}\{x_d, y_d\} \in \mathbb{R}^2$ and a fixed orientation $\vartheta_d \in \mathbb{R}$. For formally stating the problem, the sampled-data equivalent model, describing the trajectories of (1) at all sampling instants, is recalled here below.

Lemma 2.1: Consider the unicycle kinematics (1) and let the inputs be piecewise constant over the sampling period of length $\delta > 0$, that is

$$\omega(t) = \omega_k,\ v(t) = v_k \text{ for all } t \in [k\delta, (k+1)\delta[,\ k \in \mathbb{N}. \quad (2)$$

Then, denoting $z_k = z(k\delta)$, $\vartheta_k = \vartheta(k\delta)$ and

$$s = s(\vartheta) = r^\perp(\vartheta) = \begin{bmatrix}-\sin\vartheta & \cos\vartheta\end{bmatrix}^\top$$

the sampled-data equivalent model of (1) is given by the difference equations below

$$z_{k+1} = z_k - \frac{v_k}{\omega_k}\Big(s(\vartheta_{k+1}) - s(\vartheta_k)\Big) \quad (3a)$$

$$\vartheta_{k+1} = \vartheta_k + \delta\omega_k. \quad (3b)$$

Proof: The proof is carried out by integrating (1) over the sampling period $[k\delta, (k+1)\delta[$ with initial condition $z(k\delta) = z_k$, $\vartheta(k\delta) = \vartheta_k$. This yields

$$z_{k+1} = z_k + v_k \int_{k\delta}^{(k+1)\delta} r(\vartheta(s))\mathrm{d}s \quad (4a)$$

$$\vartheta_{k+1} = \vartheta_k + \delta\omega_k \quad (4b)$$

with $z_{k+1} = z(k\delta + \delta)$ and $\vartheta_{k+1} = \vartheta(k\delta + \delta)$. Since the solution of (1b) for all $s \in [k\delta, (k+1)\delta[$ is given by

$$\vartheta(s) = \vartheta_k + (s - k\delta)\omega_k,$$

the transformation $s \mapsto \ell = \vartheta_k + (s - k\delta)\omega_k$ yields

$$\int_{k\delta}^{(k+1)\delta} r(\vartheta(s))\mathrm{d}s = \frac{1}{\omega_k}\int_{\vartheta_k}^{\vartheta_{k+1}} r(\ell)\mathrm{d}\ell = -\frac{1}{\omega_k}\begin{bmatrix}-\sin(\vartheta_{k+1}) + \sin(\vartheta_k) \\ \cos(\vartheta_{k+1}) - \cos(\vartheta_k)\end{bmatrix}.$$

Substituting the expression above into (4) one gets the result. ∎

In this setting, we address the following problem.

Problem 1 (Single-rate digital steering): Design a suitable sampled-data control law

$$v_k = v^\delta(z_k, z_d, \vartheta_k, \vartheta_d), \quad \omega_k = \omega^\delta(z_k, z_d, \vartheta_k, \vartheta_d),$$

guaranteeing regulation of (1) to a desired Cartesian position $z_d = \mathrm{col}\{x_d, y_d\} \in \mathbb{R}^2$ with orientation $\vartheta_d \in \mathbb{R}$. ∎

In other words, solving the problem above is equivalent to making $\mathrm{col}\{z_d, \vartheta_d\}$ uniformly globally asymptotically stable for the closed-loop sampled-data model (3).

Remark 2.1: As already discussed in the Introduction, a partial solution to Problem 1 has been provided in [20] while considering cartesian steering only. In particular, the control law in [20] is given by

$$v_k = \frac{\kappa}{\delta\omega_k + \frac{\kappa}{\omega_k}(1 - \cos\delta\omega_k)}\big(\Delta_k s\big)^\top(z_k - z_d),\ \kappa > 0 \quad (5a)$$

$$\omega_k = \frac{1}{\delta\omega_0}\Big(\sin\big((k+1)\omega_0\delta\big) - \sin\big(k\omega_0\delta\big)\Big) \quad (5b)$$

provided that $\omega_0 > 0$ is a constant satisfying for a fixed sampling period $\delta > 0$ and $N \in \mathbb{N}$,

$$N\delta = \frac{2\pi}{\omega_0}, \quad N > 2. \quad (6)$$

Such a control law, however, never guarantees convergence to a desired constant orientation as (5b) fixes the angular velocity to a persistently exciting periodic signal.

B. The digital formation control problem

The control of groups of unicycles can be accomplished by suitably adapting the solution to Problem 1 in the framework of multi-agent systems. Accordingly, in a simplified setting (i.e., when allowing collisions among unicycles), we provide the solution to the following problem, under standard assumptions as in continuous time (see e.g., [37]).

Consider a group of $Q > 1$ unicycles each described by the model (1) specified as

$$\dot{z}^i = r^i v^i \quad (7a)$$

$$\dot{\vartheta}^i = \omega^i \quad (7b)$$

with $i = 1, \ldots, Q$ and communicating, at asynchronous and periodic sampling instants, through a given communication digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with each node $i \in \mathcal{V}$ being a unicycle. In particular, each unicycle is a node of the graph with $\delta_i \geq 0$ the corresponding sampling period verifying $\delta_i \in [\delta_{\min}, \delta_{\max}[$ for all $i = 1, \ldots, Q$ with known bounds $\delta_{\max}, \delta_{\min} \in \mathbb{R}_{\geq 0}$; namely, the input signals are piecewise constant over the sampling period $\delta_i > 0$; i.e., $\omega^i(t) = \omega^i(t_k^i)$, $v^i(t) = v^i(t_k^i)$ for $t \in [t_k^i, t_{k+1}^i[$ with $\delta_i = t_{k+1}^i - t_k^i$ and $\Delta^i = \{t_0^i, t_1^i, \ldots\}$ the corresponding sampling sequence. At all sampling instants $t = t_k^i \in \Delta^i$, the unicycle $i \in \mathcal{V}$ measures the relative distance with respect to the corresponding neighbors given by

$$e^{i,j}(t_k^i) = R(\vartheta^i(t_k^i))(z^i(t_k^i) - z^j(t_k^i)), \quad j \in \mathcal{N}_i \quad (8)$$

with the matrix

$$R(\vartheta) = \begin{bmatrix}\cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta\end{bmatrix}. \quad (9)$$

Problem 2 (Formation control under sampling): Given a communication digraph $\mathcal{G}$ containing a spanning tree, design, if any, a distributed sampled-data control making all unicycles converge to a desired formation specified by the vector $\sigma = \mathrm{col}\{\sigma^1, \ldots, \sigma^Q\}$ with $\sigma^i = \mathrm{col}\{\sigma_x^i,\ \sigma_y^i\} \in \mathbb{R}^2$

with a common orientation; i.e., as $t \to \infty$ and for a suitable consensual state $\mathrm{col}\{z_s, \vartheta_s\} \in \mathbb{R}^3$.

$$z^i(t) - \sigma^i \to z_s, \quad \vartheta^i(t) \to \vartheta_s \blacksquare$$

Remark 2.2: We assume that at all sampling instants, the i^{th} unicycle senses the position and orientation of all neighbors. Such a situation occurs in practice, for instance, when each unicycle is equipped with cameras or distance sensors. More realistic situations (e.g., when the control frequency is higher than the acquisition frequency from an exteroceptive sensor) are currently under investigation.

In the following and unless differently specified, all properties are meant to hold globally (with respect to initial conditions) and uniformly (with respect to initial time) even if not indicated explicitly.

III. Steering under single-rate control

For simplicity and for the sake of clarity we first provide the result for $z_d = 0$ and $\vartheta_d = 0$.

A. Steering at the origin

For design purposes, it is worth observing that the sampled-data kinematics (3) is passive [38, Definition 2.2] with respect to suitably defined output mappings as detailed below. In the following, we shall denote

$$\Delta_k s := s(\vartheta_{k+1}) - s(\vartheta_k).$$

Lemma 3.1: Consider the unicycle kinematics (1) and the corresponding sampled-data equivalent model (3). Then the sampled-data kinematics (3) is passive with outputs

$$v \mapsto h_z(z,,\vartheta,v,\omega) := -\frac{1}{\delta\omega}(\Delta_k s)^\top z + \frac{1}{\delta\omega^2}\big(1-\cos(\delta\omega)\big)v \tag{10a}$$

$$\omega \mapsto h_\vartheta(\vartheta,\omega) := \vartheta + \frac{\delta}{2}\omega \tag{10b}$$

and storage function

$$V(z,\vartheta) = \frac{1}{2}\big(z^\top z + \vartheta^2\big); \tag{11}$$

namely, the following equality holds for all $k \in \mathbb{N}$

$$\Delta_k V = \delta v_k h_z(z_k, v_k, \omega_k) + \delta\omega_k h_\vartheta(\vartheta_k, \omega_k) \tag{12}$$

Proof: The proof of passivity follows by computing the one-step increment of the storage function (11) along (3)

$$\Delta_k V = -\frac{v_k}{\omega_k}(\Delta_k s)^\top\Big(z_k - \frac{v_k}{2\omega_k}\Delta_k s\Big) + \delta\omega_k\Big(\vartheta_k + \frac{\delta}{2}\omega_k\Big)$$

so getting, because $(\Delta_k s)^\top \Delta_k s = 2(1-\cos(\delta\omega_k))$, (12) and thus the result. ■

Remark 3.1: The outputs (10) ensuring passivity under sampling of (3) are not the same as in continuous time. It is well-known from [26] that passivity is not preserved under sampling with respect to the same output as in continuous time due to the so-called relative degree zero obstruction. Roughly speaking, if a discrete-time system is passive then it possesses relative degree zero (i.e., a direct input-output throughput). In this case, the sampled-data passivating outputs (10) represent the time-average, over the sampling period, of the outputs making the continuous-time kinematic (1) passive, that is

$$h_z(z,\vartheta,v,\omega) = \frac{1}{\delta}\int_{k\delta}^{(k+1)\delta} r^\top(\vartheta(s))z(s)\mathrm{d}s$$
$$h_\vartheta(\vartheta,\omega) = \frac{1}{\delta}\int_{k\delta}^{(k+1)\delta} \vartheta(s)\mathrm{d}s.$$

Furthermore, as $\delta \to 0$, the sampled-data outputs recover the continuous-time ones; namely, one gets

$$\lim_{\delta\to 0}\frac{1}{\delta}\int_{k\delta}^{(k+1)\delta} r^\top(\vartheta(s))z(s)\mathrm{d}s = r^\top(\vartheta)z$$
$$\lim_{\delta\to 0}\frac{1}{\delta}\int_{k\delta}^{(k+1)\delta} \vartheta(s)\mathrm{d}s = \vartheta.$$

By passivity [26, Corollary 5.2], the feedback law

$$v = \frac{\omega\kappa_v}{\delta\omega^2 + \kappa_v\big(1-\cos(\delta\omega)\big)}(\Delta_k s)^\top z, \quad \kappa_v > 0 \tag{13}$$

$$\omega = -\frac{2\kappa_\omega}{2+\delta\kappa_\omega}\vartheta, \quad \kappa_\omega > 0, \tag{14}$$

that is the solution to the damping equality

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = -\begin{bmatrix} \kappa_v h_z(z,\vartheta,v,\omega) \\ \kappa_\omega h_\vartheta(\vartheta,\omega) \end{bmatrix},$$

makes the closed-loop system

$$z_{k+1} = \big(I - \kappa_v M^\delta(\omega_k)\big)z_k \tag{15a}$$

$$\vartheta_{k+1} = \frac{2-\delta\kappa_\omega}{2+\delta\kappa_\omega}\vartheta_k \tag{15b}$$

with

$$M^\delta(\omega) = \frac{\Delta_k s(\Delta_k s)^\top}{\delta\omega^2 + \kappa_v\big(1-\cos(\delta\omega)\big)} \tag{16}$$

asymptotically stable at the equilibrium

$$z_\star = \mathrm{col}\{1,0\} \text{ and } \vartheta_\star = \vartheta_d = 0.$$

However, the position component of the stabilized equilibrium $z_\star$ is not z_d in general. This issue can be solved by noticing that, for all choices of $\omega \in \mathbb{R}$, the feedback (13) guarantees asymptotic stability of all states such that $((\Delta_k s)^\perp)^\top z = 0$ with

$$(\Delta_k s)^\perp = \Delta_k r, \quad \Delta_k r = r(\vartheta_{k+1}) - r(\vartheta_k).$$

On this basis, asymptotic stabilization at the origin (thus steering to the origin with zero orientation) can be achieved by modifying the feedback law (14) to include a new additional term that is vanishing only if $z \equiv 0$; namely, one sets

$$\omega = -\frac{\kappa_\omega}{2+\delta\kappa_\omega}\vartheta + \big(\alpha_k(\vartheta,\omega)\big)^\top z$$

for a suitably defined $\alpha_k(\vartheta,\omega) \in \mathbb{R}^2$. Rewriting the closed-loop system as

$$\begin{bmatrix} z_{k+1} \\ \vartheta_{k+1} \end{bmatrix} = \begin{bmatrix} z_k \\ \vartheta_k \end{bmatrix} + \begin{bmatrix} \kappa_v M^\delta(\omega_k) & 0 \\ \big(\alpha^\delta(\vartheta_k,\omega_k)\big)^\top & -\frac{2\kappa_\omega}{2+\delta\kappa_\omega} \end{bmatrix}\begin{bmatrix} z_k \\ \vartheta_k \end{bmatrix}$$

the goal is achieved under a bounded signal which makes

$$\begin{bmatrix} M^{\delta}(\omega) & 0 \\ \big(\alpha_k(\vartheta,\omega)\big)^{\top} & -\frac{2}{2+\delta} \end{bmatrix} \tag{17}$$

of rank three as along as $z \neq 0$ and for all $\delta > 0$. From the previous arguments, the following result can be given.

Proposition 3.1: Problem 1 with $z_d = 0$ and $\vartheta_d = 0$ is solved by the controller (13) and $\omega = \omega^{\delta}(z, \vartheta)$ solution to

$$\omega = -\frac{2\kappa_{\omega}}{2+\delta\kappa_{\omega}}\vartheta + a_k \frac{(\Delta_k r)^{\top}}{\delta\omega} z, \quad \kappa_{\omega} > 0. \tag{18}$$

Such control law makes the origin asymptotically stable for the closed loop

$$z_{k+1} = \big(I - \kappa_v M^{\delta}(\omega_k)\big) z_k \tag{19a}$$

$$\vartheta_{k+1} = \frac{2-\kappa_{\omega}\delta}{2+\delta\kappa_{\omega}}\vartheta_k + a_k \frac{(\Delta_k r)^{\top}}{\delta\omega_k} z \tag{19b}$$

if $a_k = a(k\delta)$ is bounded and persistently exciting for all $\delta \geq 0$; i.e., there exist $\bar{k} \in \mathbb{N}$ and $\bar{\mu} > 0$ such that

$$\sum_{j=k}^{k+\bar{k}-1} (a_k)^2 \geq \bar{\mu}, \text{ for all } k \in \mathbb{N}. \tag{20}$$

Proof: First, we note that the matrix (17) is full rank 3 whenever $z \neq 0$ by construction of

$$\alpha_k(\vartheta_k, \omega_k) = a_k \frac{(\Delta_k r)^{\top}}{\delta\omega_k}$$

with $a_k \in \mathbb{R}^n$ and $\Delta_k r = (\Delta_k s)^{\perp}$. Thus the origin is the only equilibrium of the closed-loop system (19a). With this in mind, the proof follows by looking at (19a) as a time-varying linear dynamics (i.e., when setting $M_k^{\delta} = M^{\delta}(\omega_k)$) and considering the overall closed-loop sampled-data kinematics (19a) as a cascade interconnection [39]. Thus, (19) is asymptotically stable if the separate and independent components are such and the corresponding solutions are bounded [40], [41]. By construction of a_k as a persistently exciting signal, (1a) is asymptotically stable at the origin; hence, its solutions are bounded. In addition, the independent part of (19b) (i.e., computed for $z = 0$) is trivially asymptotically stable. Boundedness of the solutions of (19b) is easily verified noticing that the latter is given by the sum of terms that are, by construction, bounded for all $k \in \mathbb{N}$. This concludes the proof. ■

Remark 3.2: The result above holds for all discrete signals that are persistently exciting in the discrete-time sense in (20). In addition, the discrete signal deduced from sampling a continuous-time persistently exciting signal $a(t) : \mathbb{R}_{\geq 0} \to \mathbb{R}$ (i.e., setting $a_k = a(k\delta)$) is not, in general, persistently exciting in the discrete-time sense for all $\delta > 0$ [42].

Remark 3.3: We highlight that the feedback on the linear velocity (13) coincides with (5a), proposed in [20] for Cartesian steering only. As one might expect, the overall controllers differ in the angular velocity component only. However, in the case we are addressing here, this term is the solution of the nonlinear equation (18) so providing a further and not naive difficulty in the design.

B. The general case

At this point we are able to solve the problem of steering the unicycle to a general position $z_d \in \mathbb{R}^2$ with a general orientation $\vartheta_d \in \mathbb{R}$. To this end, we define the steering error

$$e = \begin{bmatrix} e_z \\ e_{\vartheta} \end{bmatrix} := \begin{bmatrix} R(\vartheta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z - z_d \\ \vartheta - \vartheta_d \end{bmatrix} \tag{21}$$

whose sampled-data evolutions are described, at all sampling instants, by the discrete-time model

$$e_{z_{k+1}} = R(\delta\omega_k) e_{z_k} + \delta B(\delta\omega_k) v_k$$
$$e_{\vartheta_{k+1}} = e_{\vartheta_k} + \delta\omega_k$$

with $R(\cdot)$ as in (9) and

$$B(\delta\omega) = \frac{R(\delta\omega) - I}{\delta\omega} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Remark 3.4: The error dynamics above can be shown to be passive with respect to the output

$$h_{e_z}(e_z, \omega, v) = B^{\top}(\delta\omega)\Big(R(\delta\omega) e_z + \frac{\delta}{2} B(\delta\omega) v\Big)$$
$$h_{e_{\vartheta}}(e_{\vartheta}, \omega) = e_{\vartheta} + \frac{\delta}{2}\omega.$$

Theorem 3.1: Problem 1 is solved by the controller

$$v = -\frac{\delta\omega^2\kappa_v}{\delta\omega^2 - \kappa_v\big(1 - \cos(\omega\delta)\big)} B^{\top}(\delta\omega) R(\delta\omega) e_z \tag{22}$$

with $\kappa_v > 0$ and $\omega = \omega^{\delta}(e_z, e_{\vartheta})$ solution to

$$\omega = -\frac{2\kappa_{\omega}}{2+\delta\kappa_{\omega}} e_{\vartheta} + a_k \big(B^{\perp}(\delta\omega)\big)^{\top} e_z, \quad \kappa_{\omega} > 0 \tag{23}$$

$$B^{\perp}(\delta\omega) = \frac{1}{\delta\omega}\big(R(\delta\omega) - I\big) \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \tag{24}$$

namely, the origin is asymptotically stable for the closed-loop error-dynamics

$$e_{z_{k+1}} = \Big(R(\delta\omega_k) - \delta\kappa^{\delta}(\omega_k) B(\delta\omega_k) B^{\top}(\delta\omega_k)\Big) e_{z_k} \tag{25a}$$

$$e_{\vartheta_{k+1}} = \frac{2-\delta\kappa_{\omega}}{2+\delta\kappa_{\omega}} e_{\vartheta_k} + a_k \big(B^{\perp}(\delta\omega_k)\big)^{\top} e_{z_k} \tag{25b}$$

with

$$\kappa^{\delta}(\omega) = \frac{\delta\omega^2\kappa_v}{\delta\omega^2 + \kappa_v\big(1 - \cos(\omega\delta)\big)}$$

provided that $a_k = a(k\delta)$ is bounded and persistently exciting for all $\delta > 0$. Equivalently (22)-(23) make $\mathrm{col}\{z_d, \vartheta_d\}$ asymptotically stable for the sampled-data kinematics (3).

Proof: The proof follows noticing that when setting

$$\varepsilon_z = R^{\top}(\vartheta) e_z, \quad \varepsilon_{\vartheta} = \vartheta - \vartheta_d$$

the closed-loop (25) takes the same structure as (19); namely, it takes the form

$$\varepsilon_{z_{k+1}} = \big(I - \kappa_v M^{\delta}(\omega_k)\big) \varepsilon_{z_k}$$
$$\varepsilon_{\vartheta_{k+1}} = \frac{2-\delta\kappa_{\omega}}{2+\delta\kappa_{\omega}} \varepsilon_{\vartheta_k} + a_k \frac{(\Delta_k r)^{\top}}{\delta\omega_k} \varepsilon_{z_k}$$

with $M^\delta(\omega)$ as in (16). Thus, by Proposition 3.1 the error dynamics (25) is asymptotically stable at the origin and steering at the desired z_d with orientation ϑ_d is guaranteed for (3) in closed loop. ■

Remark 3.5: We fix the linear error component e_z as in (21) to highlight that, for implementation purposes, exact knowledge of z_d is not necessary. In other words, it can be assumed that the unicycle only measures the position of the target point with respect to its own coordinate frame (i.e., $e_z = R(\vartheta)(z - z_d)$) rather than in the global one.

Remark 3.6: We note that in the original dynamics the controller on the linear velocity (35) gets the form

$$v = \frac{\omega\kappa_v}{\delta\omega^2 + \kappa_v\big(1 - \cos(\delta\omega)\big)}(\Delta_k s)^\top (z - z_d) \tag{27}$$

that is similar to (13) evaluated at $z \leftarrow z - z_d$. In addition, the component in the angular velocity is defined by the equation (23) that, in the original coordinates, gets the form

$$\omega = -\frac{\delta\kappa_\omega}{2 + \delta\kappa_\omega}(\vartheta - \vartheta_d) + a_k \frac{(\Delta_k r)^\top}{\delta\omega}(z - z_d)$$

and yields (18) when $z \leftarrow z - z_d$ and $\vartheta \leftarrow \vartheta - \vartheta_d$. By virtue of this, the same computational issues discussed in Section III-A hold true in this situation with the same remarks and constructive aspects.

C. Computational aspects

For simplicity, computational aspects are addressed for the case of stabilization at the origin with a straightforward extension to the general case.

Along the lines of [43], it can be easily shown that a unique solution to (18) in the form of a series expansion in powers of δ exists, as specified in the next result.

Corollary 3.1: There exists $T^\star > 0$ such that for all $\delta \in [0, T^\star[$, the equality (18) admits a unique solution $\omega = \omega^\delta(z, \vartheta)$ of the form

$$\omega^\delta(z, \vartheta) = \omega_0(z, \vartheta) + \sum_{i>0} \frac{\delta}{(i+1)!}\omega_i(z, \vartheta). \tag{28}$$

Proof: The result is proved via the Implicit Function Theorem [44] rewriting (18) as $\mathcal{S}(\delta, z, \vartheta, \omega) = 0$ with

$$\mathcal{S}(\delta, z, \vartheta, \omega) := \omega + \frac{\delta\kappa_\omega}{2 + \delta\kappa_\omega}\vartheta + a_k \frac{(\Delta_k r)^\top}{\delta\omega} z.$$

The result follows because $\lim_{\delta\to 0} \mathcal{S}(\delta, z, \vartheta, \omega) = 1 \neq 0$. ■

Although exact solutions to (18) cannot be easily computed, approximations are easily defined by exploiting Theorem 3.1. As a matter of fact, all terms of the series expansion (28) are exactly computable via a constructive and iterative algorithm solving, at each step, a suitably defined linear equality. To see this, let us replace the terms $\frac{\Delta_k r}{\delta\omega}$ and $\frac{2}{2+\delta\kappa_\omega}$ in (18) with

$$\begin{aligned}\frac{\Delta_k r}{\delta\omega} =& \Big(1 + \sum_{i>0} (-1)^i \frac{(\delta\omega)^{2i}}{(2i+1)!}\Big)s \\ &+ \Big(\sum_{i>0} (-1)^i \frac{(\delta\omega)^{2i-1}}{(2i)!}\Big)r \\ \frac{2}{2+\delta\kappa_\omega} =& 1 + \sum_{i>0} \frac{(-1)^i}{i!}\frac{(\delta\kappa_\omega)^i}{2^i}\end{aligned}$$

so getting

$$\begin{aligned}\omega =& -\kappa_\omega \Big(1 + \sum_{i>0} \frac{(-1)^i}{i!}\frac{(\delta\kappa_\omega)^i}{2^i}\Big)\vartheta \\ &+ a_k \Big(1 + \sum_{i>0} (-1)^i \frac{(\delta\omega)^{2i}}{(2i+1)!}\Big) s^\top z \\ &+ a_k \Big(\sum_{i>0} (-1)^i \frac{(\delta\omega)^{2i-1}}{(2i)!}\Big) r^\top z.\end{aligned}$$

At this point, one substitutes (28) in both sides of the expression above and equates the terms with the same powers of δ. Each term $\omega_i(z, \vartheta)$ is then the solution of the linear equation associated with the corresponding term δ^i in the so-deduced formal series equality. For the first terms, one gets

$$\begin{aligned}\omega_0(z, \vartheta) =& -\kappa_\omega\vartheta + a_k s^\top z \\ \omega_1(z, \vartheta) =& \kappa_\omega^2\vartheta - a_k\omega_0(z, \vartheta) r^\top z.\end{aligned}$$

Accordingly, one can define approximations of the solutions to (18) as the truncation of the series expansion (28) at all finite order $p \in \mathbb{N}$, that is

$$\omega^\delta_{[p]}(z, \vartheta) = \omega_0(z, \vartheta) + \sum_{i=1}^{p} \frac{\delta^i}{(i+1)!}\omega_i(z, \vartheta). \tag{29}$$

The so-defined approximations guarantee, in general, practical asymptotic stability of the closed-loop equilibrium $\mathrm{col}\{z_d, \vartheta_d\}$ [43]; namely, the sampled-data controller is ensured to steer the unicycle to a neighborhood of the desired configuration of radius in $\mathcal{O}(\delta^{p+1})$.

Remark 3.7: The proposed control law extends the emulation-based controller associated with [3], that is the continuous-time solution directly implemented via sample-and-hold devices. This can be readily seen for the angular component by fixing $p = 0$ in (29), so getting

$$\omega^\delta_{[0]}(z, \vartheta) = -\kappa_\omega\vartheta + a_k s^\top z \tag{30}$$

that is exactly the continuous-time angular component of the controller in [3]. Similar considerations hold for the linear velocity that can be computed, starting from (13), as

$$v^\delta_{[0]}(z, \vartheta) = \lim_{\delta\to 0} v^\delta(z, \vartheta) = -\kappa_v r^\top z. \tag{31}$$

With this in mind, the design we propose naturally enriches the sampled-data controller with further *correcting* terms aimed at compensating the effect of sampling and improving performances with respect to mere emulation.

Remark 3.8: Another possible approximation to the solution to (18) can be deduced as follows. It can be easily checked that such equality rewrites as the contribution of two components: one that does not depend on ω and another one that does; i.e., denoting

$$\bar{\omega}^\delta = -\frac{\delta\kappa_\omega}{2+\delta\kappa_\omega}\vartheta, \quad \bar{\Delta}_k r = r(\vartheta + \delta\bar{\omega}^\delta) - r(\vartheta).$$

one gets

$$\omega = -\frac{\delta\kappa_\omega}{2+\delta\kappa_\omega}\vartheta + a_k\frac{(\bar{\Delta}_k r)^\top}{\delta\bar{\omega}^\delta}z + a_k\Big(\frac{(\Delta_k r)^\top}{\delta\omega} - \frac{(\bar{\Delta}_k r)^\top}{\delta\bar{\omega}^\delta}\Big)z \tag{32}$$

Accordingly, neglecting the term

$$\frac{(\Delta_k r)^\top}{\delta\omega} - \frac{(\bar{\Delta}_k r)^\top}{\delta\bar{\omega}^\delta}$$

one computes the approximate solution

$$\omega_a = -\frac{\delta\kappa_\omega}{2+\delta\kappa_\omega}\vartheta + a_k\frac{(\bar{\Delta}_k r)^\top}{\delta\bar{\omega}^\delta}z. \tag{33}$$

IV. Sampled-data formation control

A. The case of synchronous sampling

In this section, we first consider the case of synchronous agents, that is $\delta^i = \delta$ for all $i \in \mathcal{V}$.

In this setting, one can solve Problem 1 by computing a digital control making $\mathrm{col}\{z^i_d, \vartheta^i_d\} = \mathrm{col}\{z_s + \sigma^i, \vartheta_s\} \in \mathbb{R}^3$ asymptotically stable for the sampled-data equivalent model of each unicycle (3) specified, for $i = 1, \dots, Q$, as

$$z^i_{k+1} = z^i_k - \frac{v^i_k}{\omega^i_k}\Delta_k s^i \tag{34a}$$

$$\vartheta^i_{k+1} = \vartheta^i_k + \delta\omega^i_k. \tag{34b}$$

The controller can be designed based on the result in Theorem 3.1 as stated below.

Proposition 4.1: Consider a group of $Q > 1$ unicycles with sampled-data model (34) communicating, at the sampling instants $t = k\delta$, over a synchronous network described by a digraph $\mathcal{G}$ containing a spanning tree. Problem 2 with $\delta_i = \delta$ for all $i \in \mathcal{V}$ is solved by the controller

$$v^i = -\frac{\delta{\omega^i}^2\kappa_v}{\delta{\omega^i}^2 - \kappa_v\big(1-\cos(\omega^i\delta^i)\big)}B^\top(\delta\omega^i)e^i_z \tag{35}$$

and $\omega^i = {\omega^i}^\delta(e^i_z, e^i_\vartheta)$ defined as the solution to

$$\omega^i = -\frac{2\kappa_\omega}{2+\delta\kappa_\omega}e^i_\vartheta + a_k\big(B^\perp(\delta\omega^i)\big)^\top e^i_z \tag{36}$$

for $\kappa_v, \kappa_\omega > 0$, $B^\perp(\cdot)$ in (24), $a_k = a(k\delta)$ a discrete bounded and persistently exciting signal for all $\delta > 0$ and

$$e^i_z = \sum_{j\in\mathcal{N}_i} R(\vartheta^i)(\tilde{z}^i - \tilde{z}^j), \ e^i_\vartheta = \sum_{j\in\mathcal{N}_i}(\vartheta^i - \vartheta^j) \tag{37}$$

for $\tilde{z}^i = z^i - \sigma^i$. Moreover, the consensual state is defined by

$$z_s = (\nu^\top \otimes I_2)\tilde{z}_0, \quad \vartheta_s = \nu^\top\vartheta_0 \tag{38}$$

with $\tilde{\boldsymbol{z}} = \mathrm{col}\{\tilde{z}_1, \dots, \tilde{z}_Q\}$, $\boldsymbol{\vartheta} = \mathrm{col}\{\vartheta_1, \dots, \vartheta_Q\}$ and $\nu \in \mathbb{R}^Q$ verifying $\nu^\top L = 0$ and $\nu^\top \mathbb{1}_Q = 1$.

Proof: The proof is carried out by considering the immersion mapping provided by the error component (37) and the so-called mean field unit [45]

$$z_s = (\nu^\top \otimes I_2)\tilde{\boldsymbol{z}}, \quad \vartheta_s = \nu^\top\boldsymbol{\vartheta}$$

so that, setting $\boldsymbol{v} = \mathrm{col}\{v^1, \dots, v^Q\}$ and $\boldsymbol{\omega} = \mathrm{col}\{\omega^1, \dots, \omega^Q\}$, the overall network dynamics reads

$$\begin{aligned} z_{s_{k+1}} &= z_{s_k} - \kappa_v(\nu^\top \otimes I_2)\boldsymbol{\Delta_k s v}_k \\ \vartheta_{s_{k+1}} &= \vartheta_{s_k} + \delta(\nu^\top \otimes I_2)\boldsymbol{\omega}_k \\ e^i_{z_{k+1}} &= R(\delta\omega^i_k)e^i_{z_k} + \delta B(\delta\omega^i_k)v^i_k \\ e^i_{\vartheta_{k+1}} &= e^i_{\vartheta_{k+1}} + \delta\omega^i_k. \end{aligned}$$

with $\boldsymbol{\Delta_k s} = \mathrm{diag}\{\frac{\Delta_k s^1}{\omega^1_k}, \dots, \frac{\Delta_k s^Q}{\omega^Q_k}\}$. Embedding the local controller into the dynamics above, one gets the error

$$\begin{aligned} e^i_{z_{k+1}} &= \Big(R(\delta\omega^i_k) - \delta\kappa^\delta(\omega^i_k)B(\delta\omega^i_k)B^\top(\delta\omega^i_k)\Big)e^i_{z_k} \\ e^i_{\vartheta_{k+1}} &= \frac{2-\delta\kappa_\omega}{2+\delta\kappa_\omega}e^i_{\vartheta_k} + a_k\big(B^\perp(\delta\omega^i_k)\big)^\top e^i_{z_k} \end{aligned}$$

that coincides with (19) and which is uniformly asymptotically stable at the origin by Theorem 3.1. By the cascade structure and because $v^i \to 0$ and $\omega^i \to 0$ as $e^i_z \to 0$ and $e^i_\vartheta \to 0$, one gets the result as the consensual dynamics is $z_{s_{k+1}} = z_{s_k}$, $\vartheta_{s_{k+1}} = \vartheta_{s_k}$. ■

B. The case of asynchronous sampling

Consider now the case in which the network is asynchronous and periodic. In particular, let $\delta_i = \{t^i_0, t^i_1, \dots\}$ be the sampling sequence associated with unicycle $i \in \mathcal{V}$ with the corresponding bounded sampling period $\delta_i := t^i_{k+1} - t^i_k$ verifying $\delta_i \in [\delta_{\min}, \delta_{\max}[$. Accordingly, each agent dynamics (7) for $t \in [t^i_k, t^i_{k+1}[$ reads

$$\dot{z}^i(t) = r^i(t)v^i(t^i_k) \tag{39a}$$

$$\dot{\vartheta}^i(t) = \omega^i(t^i_k) \tag{39b}$$

and the next result can be given.

Theorem 4.1: Consider a network of Q unicycles kinematics over a sampled-data communication graph $\mathcal{G}$ containing a spanning tree. Let $\lambda_{\max}$ and $\lambda_{\min}$ denote the largest and smallest (non-zero) eigenvalues of the Laplacian. Then, Problem 2 is solved by the sampled-data time-varying control

$$v^i(t^i_k) = -\frac{\delta_i\omega^i(t^i_k)^2\kappa_v B^\top(\delta_i\omega^i(t^i_k))e^i_z(t^i_k)}{\delta_i\omega^i(t^i_k)^2 - \kappa_v\big(1-\cos(\omega^i(t^i_k)\delta_i)\big)} \tag{40}$$

and $\omega^i(t^i_k) = {\omega^i}^{\delta_i}(e^i_z(t^i_k), e^i_\vartheta(t^i_k))$ defined as the solution to

$$\omega^i(t^i_k) = -\frac{2\kappa_\omega}{2+\delta_i\kappa_\omega}e^i_\vartheta(t^i_k) + a_k\big(B^\perp(\delta_i\omega^i(t^i_k))\big)^\top e^i_z(t^i_k) \tag{41}$$

for $B^\perp(\cdot)$ in (24), $a_k = a(t^i_k)$ a discrete bounded and persistently exciting signal for all $\delta_i > 0$ and $e^{i,j}$ as in (37) provided that

$$\kappa_v, \kappa_\omega \in [0, \kappa^\star], \quad \kappa^\star = \frac{\delta_{\min}\lambda_{\min}}{\delta_{\max}\lambda_{\max}(1+\delta_{\max})}. \tag{42}$$

In addition, the consensual state is defined by (38).

Proof: Analogously to [21], the proof relies on modeling the effect of mismatches over the sampling periods as time-delays and then investigating the robustness of the closed-loop dynamics with respect to the nominal one. To this end, let us define the network sampling sequence as

$$\Delta := \cup_{i=1}^{Q} \Delta_i = \{t_0, t_1, \dots\} \tag{43}$$

with, for all $i = 1, \dots, Q$, $\delta_k := t_{k+1} - t_k \leq \delta_i$ and $\delta_k \in [\delta_{\min}, \delta_{\max}[$ denoting the distance from two successive sampling instants in the network. At this point, introduce

$$\tilde{\boldsymbol{z}} = \text{col}\{\tilde{z}^1, \dots, \tilde{z}^Q\}, \quad \boldsymbol{\vartheta} = \text{col}\{\vartheta^1, \dots, \vartheta^Q\}$$

the matrix $\boldsymbol{E}(t_k) = \text{diag}\{E^1(t_k), \dots, E^Q(t_k)\} \in \mathbb{R}^{2Q \times 2Q}$ with

$$E^i(t_k) = \begin{cases} 0, & t_k = t_k^i \\ I_2, & \text{otherwise} \end{cases}$$

and the coordinate transformation

$$\begin{aligned} z_s =& (\nu^\top \otimes I_2)\tilde{\boldsymbol{z}}, \quad \boldsymbol{e}_z = \tilde{\boldsymbol{z}} - (\mathbb{1}_Q \otimes I_2) z_s \\ \vartheta_s =& (\nu^\top \otimes I_2)\boldsymbol{\vartheta}, \quad \boldsymbol{e}_\vartheta = \boldsymbol{\vartheta} - (\mathbb{1}_Q \otimes I_2)\vartheta_s \end{aligned} \tag{44}$$

with $z_s \in \mathbb{R}^2$ and $\vartheta_s \in \mathbb{R}$ the mean-field unit, $\boldsymbol{e}_z$ and $\boldsymbol{e}_\vartheta$ the consensus errors over the position and orientations. Then, at all $t_k \in \Delta$, the agglomerate of the feedback laws (40)-(41) rewrites as

$$\begin{aligned} \boldsymbol{v}(t_k) =& \kappa_v (I - \boldsymbol{E}(t_k))(L \otimes I_2)\boldsymbol{D}(t_k)\boldsymbol{e}_z(t_k) + \boldsymbol{E}(t_k)\boldsymbol{v}(t_{k-1}) \\ \boldsymbol{\omega}(t_k) =& -\kappa_\omega (I - \boldsymbol{E}(t_k))(L \otimes I_2)\boldsymbol{F^\delta}\boldsymbol{e}_\vartheta(t_k) \\ &+ (I - \boldsymbol{E}(t_k))(I_Q \otimes a_k)(\boldsymbol{G}^\perp(t_k))^\top \boldsymbol{e}_z(t_k) \\ &- \kappa_\omega \boldsymbol{E}(t_k)(L \otimes I_2)\boldsymbol{F^\delta}\boldsymbol{e}_\vartheta(t_{k-1}) \\ &+ \boldsymbol{E}(t_k)(I_Q \otimes a_{k-1})(\boldsymbol{G}^\perp(t_{k-1}))^\top \boldsymbol{e}_z(t_{k-1}) \end{aligned}$$

with

$$\begin{aligned} &\boldsymbol{G}(t_k) = \text{diag}\{(I_Q \otimes G^1(t_k)), \dots, (I_Q \otimes G^Q(t_k))\} \\ &G^i(t_k) = \frac{\Delta_k s^i}{\omega^i(t_k)} \\ &\boldsymbol{D}(t_k) = \text{diag}\{I_Q \otimes \boldsymbol{D}^1(t_k), \dots, I_Q \otimes \boldsymbol{D}^Q(t_k)\} \\ &\boldsymbol{F^\delta} = 2\text{diag}\{I_Q \otimes \frac{1}{2 + \delta_1 \kappa_\omega}, \dots, I_Q \otimes \frac{1}{2 + \delta_Q \kappa_\omega}\} \\ &\boldsymbol{D}^i(t_k) = \frac{\Delta_k^\top s^i}{\delta_k \omega^i(t_k) + \frac{\kappa_v}{\omega^i(t_k)}(1 - \cos(\omega^i(t_k)\delta_k)} \\ &\Delta_k s^i = s^i(t_{k+1}) - s^i(t_k) = \begin{bmatrix} -\sin\vartheta^i(t_{k+1}) + \sin\vartheta^i(t_k) \\ \cos\vartheta^i(t_{k+1}) - \cos\vartheta^i(t_k) \end{bmatrix}. \end{aligned}$$

In words, the selecting matrix $\boldsymbol{E}(t_k)$ possesses a zero on the i^{th} diagonal element if the corresponding agent is updating the corresponding feedback and measures at the time instant $t_k \in \Delta$. Accordingly, the product $\boldsymbol{E}(t_k) \dots \boldsymbol{E}(t_{k-M})$ possesses a zero on the i^{th} diagonal element if the corresponding agent has updated the corresponding feedback and measures at least once over the time window $[t_{k-M}, t_k]$ with $t_k, t_{k-M} \in \Delta$ and 1 otherwise. Accordingly, because all $\delta_i \in [\delta_{\min}, \delta_{\max}[$ for a finite upper bound $\delta_{\max} > 0$, there exists $M > 0$ such that all agents have updated the corresponding feedback laws and measures at least once over $[t_{k-M}, t_k]$ with $t_k, t_{k-M} \in \Delta$ so that one gets

$$\boldsymbol{E}(t_k) \dots \boldsymbol{E}(t_{k-M-1}) = 0.$$

Accordingly, fixing

$$\bar{\boldsymbol{E}}(t_k, t_\ell) := \begin{cases} -\boldsymbol{E}(t_k), & \ell = k \\ \boldsymbol{E}(t_k), \dots (I - \boldsymbol{E}(t_{k-M})), & \text{otherwise} \end{cases}$$

the feedback gets the form

$$\begin{aligned} \boldsymbol{v}(t_k) =& \kappa_v (I - \boldsymbol{E}(t_k))(L \otimes I_2)\boldsymbol{D}(t_k)\boldsymbol{e}(t_k) \\ &+ \sum_{\ell=k-M}^{k-1} \bar{\boldsymbol{E}}(t_k, t_\ell)\boldsymbol{v}(t_\ell) \\ =& \kappa_v (L \otimes I_2)\boldsymbol{D}(t_k)\boldsymbol{e}(t_k) \\ &+ \kappa_v \sum_{\ell=k-M}^{k} \bar{\boldsymbol{E}}(t_k, t_\ell)(L \otimes I_2)\boldsymbol{D}(t_\ell)\boldsymbol{e}(t_\ell) \\ \boldsymbol{\omega}(t_k) =& -\kappa_\omega (L \otimes I_2)\boldsymbol{F^\delta}\boldsymbol{e}_\vartheta(t_k) \\ &+ (I_Q \otimes a_k)(\boldsymbol{G}^\perp(t_k))^\top \boldsymbol{e}_z(t_k) \\ &+ \kappa_\omega \sum_{\ell=k-M}^{k-1} \bar{\boldsymbol{E}}(t_k, t_\ell)\Big((L \otimes I_2)\boldsymbol{F^\delta}\boldsymbol{e}_\vartheta(t_\ell) \\ &+ (I_Q \otimes a_\ell)(\boldsymbol{G}^\perp(t_\ell))^\top \boldsymbol{e}_z(t_\ell)\Big) \end{aligned} \tag{45}$$

Thus, the sampled-data agglomerate dynamics of all unicycles in the corresponding coordinates is given by the time-delayed time-varying dynamics below

$$\begin{aligned} z_s(t_{k+1}) =& z_s(t_k) - \kappa_v(\nu^\top \otimes I_2)\boldsymbol{\Delta}_k s\boldsymbol{v}(t_k) \\ \vartheta_s(t_{k+1}) =& \vartheta_s(t_k) + \delta(\nu^\top \otimes I_2)\boldsymbol{\omega}(t_k) \\ \boldsymbol{e}_z(t_{k+1}) =& \boldsymbol{e}_z(t_k) + \boldsymbol{G}(t_k)\boldsymbol{v}(t_k) \\ \boldsymbol{e}_\vartheta(t_{k+1}) =& \boldsymbol{e}_\vartheta(t_k) + \delta\boldsymbol{\omega}(t_k). \end{aligned}$$

As in the previous cases, substituting the expression of the controllers (45) in the agglomerate dynamics above, one gets that the closed-loop system exhibits a cascade structure. Thus, asymptotic stability of the error dynamics at the origin follows if both the position and orientation components (when considered decoupled) are asymptotically stable. Noticing that the angular error dynamics is a set of discrete integrators, the result follows from a straightforward application of [21, Theorem 1]. As far as the position dynamics is considered, it gets the closed-loop form

$$\begin{aligned} \boldsymbol{e}_z(t_{k+1}) =& \Big(I_{2Q} - \boldsymbol{M}(t_k)(L \otimes I_2)\Big)\boldsymbol{e}_z(t_k) \\ &- \kappa_v \sum_{\ell=k-M}^{k} \bar{\boldsymbol{E}}(t_k, t_\ell)\boldsymbol{G}(t_k)\big(L \otimes I_2\big)\boldsymbol{D}(t_\ell)\boldsymbol{e}(t_\ell) \end{aligned}$$

with

$$\begin{aligned} \boldsymbol{M}(t_k) =& \text{diag}\{(I_Q \otimes M^1(t_k)), \dots, (I_Q \otimes M^Q(t_k))\} \\ M^i(t_k) =& \kappa_v G^i(t_k) M^i(t_k) \\ =& \frac{\frac{\kappa}{\omega^i(t_k)}\Delta_k s^i \Delta_k^\top s^i}{\delta_k \omega^i(t_k) + \kappa_v(1 - \cos(\omega^i(t_k)\delta_k))} \end{aligned}$$

and the property that

$$\begin{aligned}\boldsymbol{G}(t_k)\big(L\otimes I_2\big)\boldsymbol{D}(t_\ell)\boldsymbol{e}(t_\ell) =& \big(L\otimes I_2\big)\boldsymbol{G}(t_k)\boldsymbol{D}(t_\ell)\boldsymbol{e}(t_\ell)\\ =& \boldsymbol{G}(t_k)\boldsymbol{D}(t_\ell)\big(L\otimes I_2\big)\boldsymbol{e}(t_\ell).\end{aligned}$$

At this point, introducing in the coordinates

$$\tilde{\boldsymbol{e}} = \mathrm{col}\{\tilde{\boldsymbol{e}}^0, \tilde{\boldsymbol{e}}^1, \dots, \tilde{\boldsymbol{e}}^{Q-1}\} = (I\otimes T)\boldsymbol{e} \tag{46}$$

with T such that

$$TLT^{-1} = \begin{bmatrix}0 & 0\\ 0 & \Lambda_r\end{bmatrix}, \quad \Lambda_r = \mathrm{diag}\{\lambda_1, \dots, \lambda_{Q-1}\} \succ 0$$

and setting, in particular $\tilde{\boldsymbol{e}}_r = \mathrm{col}\{\tilde{\boldsymbol{e}}^1, \dots, \tilde{\boldsymbol{e}}^{Q-1}\}$, uniform asymptotic stability of the origin of the dynamics above is achieved if the origin is uniformly asymptotically stable for the reduced time-delay dynamics

$$\begin{aligned}\tilde{\boldsymbol{e}}_r(t_{k+1}) =& \Big(I_{2Q} - \boldsymbol{M}(t_k)(\Lambda_r\otimes I_2)\Big)\tilde{\boldsymbol{e}}_r(t_k)\\ &- \kappa_v \sum_{\ell=k-M}^{k} \bar{\boldsymbol{E}}(t_k, t_\ell)\boldsymbol{G}(t_k)\boldsymbol{D}(t_\ell)\big(U_r\Lambda_r\otimes I_2\big)\tilde{\boldsymbol{e}}_r(t_\ell)\end{aligned}$$

with $U_r \in \mathbb{R}^{Q\times(Q-1)}$ being the matrix of the right eigenvectors associated with the non-zero eigenvalues of the Laplacian. Setting now the functional

$$\begin{aligned}&\boldsymbol{V}(\tilde{\boldsymbol{e}}_r(t_k), \dots, \tilde{\boldsymbol{e}}_r(t_{k-M})) = V_0(\tilde{\boldsymbol{e}}_r(t_k))\\ &\qquad + \gamma V_1(\tilde{\boldsymbol{e}}_r(t_k), \dots, \tilde{\boldsymbol{e}}_r(t_{k-M}))\\ &V_0(\tilde{\boldsymbol{e}}_r) = \frac{1}{2}\|\tilde{\boldsymbol{e}}_{\boldsymbol{r}}\|^2\\ &V_1(\tilde{\boldsymbol{e}}_r(t_k), \dots, \tilde{\boldsymbol{e}}_r(t_{k-M})) =\\ &\sum_{\ell_1=k-M}^{k}\sum_{\ell_2=\ell_1}^{k}\|\bar{\boldsymbol{E}}(t_k, t_{\ell_2})\boldsymbol{D}^\top(t_k)\boldsymbol{D}(t_{\ell_2})\big(U_r\Lambda_r\otimes I_2\big)\tilde{\boldsymbol{e}}_r(t_{\ell_2})\|^2\end{aligned}$$

with $\gamma > 0$ the proof follows exactly the same lines as the one in [21, Theorem 1] exploiting that for all $i = 1, \dots, Q-1$,

$$1 - \kappa(\omega^i)\Delta_k^\top s^i \Delta_k s^i \leq \frac{1}{\omega^i}.$$

∎

Remark 4.1: From (42), the gain is reduced for coping with asynchronism. Accordingly, the convergence rate to the required formation is proportional to the term $\frac{\delta_{\min}}{\delta_{\max}}$ which can be hence interpreted as the degree of synchronization: as $\delta_{\max} \gg \delta_{\min}$ convergence is slow and faster as $\delta_{\max} \to \delta_{\min}$.

Remark 4.2: In the proposed framework, we embed the possibility for agents to share the same sampling period (i.e., all the δ_i's are not necessarily distinct).

Remark 4.3: The robustness design and analysis we propose rely upon modeling the effect of asynchronism as a distributed state delay over the network and involves Lyapunov-Krasovskii-like arguments. Accordingly, the deduced bound on the gains might be conservative depending on the situation. For instance, when $\frac{\delta_{\min}}{\delta_{\max}} \approx 1$ but $\frac{\lambda_{\min}}{\lambda_{\max}} \ll 1$ (i.e., when the network connectivity degree is large) the resulting estimate might be small even if larger gain might still provide good performances. On the other side, when $\frac{\delta_{\min}}{\delta_{\max}} \ll 1$ (that is when the sampling period of all agents is notably spread and different) the estimate is much closer to the actual maximal allowed gain. The definition of new gains (possibly via adaptive methodologies) bridging this gap is part of perspective works.

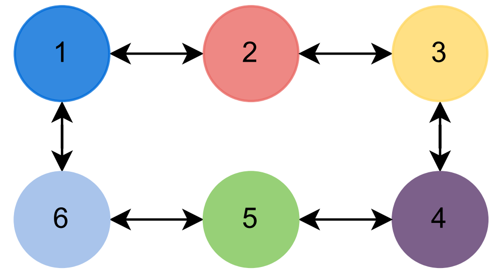

Fig. 1: Communication topology.

C. Simulations

The aim of the reported simulations is to show that the proposed controller is capable of enforcing desired formation despite the effect of both synchronous and asynchronous sampling (Proposition 4.1 and Theorem 4.1), contrarily to the case of emulation-based control (30)-(31) associated with the continuous-time design in [37]. The communication topology is fixed by the graph in Figure 1. The control laws are accordingly defined to guarantee consensus of $Q = 6$ unicycles toward the hexagonal formation specified by the vector

$$\sigma = \begin{bmatrix}1 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -1 & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

with initial conditions

$$\begin{aligned}\boldsymbol{z}(0) &= \begin{bmatrix}2 & 5 & 5 & \frac{11}{2} & 3 & \frac{7}{2} & 3 & 2 & 1 & \frac{7}{2} & 1 & \frac{9}{2}\end{bmatrix}^\top\\ \boldsymbol{\vartheta}(0) &= \begin{bmatrix}\frac{3}{4}\pi & -\frac{\pi}{4} & \frac{\pi}{2} & \frac{\pi}{4} & \frac{\pi}{2} & \frac{\pi}{4}\end{bmatrix}^\top.\end{aligned}$$

The gains are fixed according to 4.1 and Theorem 4.1 while the angular velocity component solution to (36) is approximated as discussed in Remark 3.8. The persistently exciting signal is obtained by sampling the one below

$$a(t) = \begin{cases}0, & \text{if } t \in \left[\frac{2j+1}{\bar{\omega}}\pi, \frac{2(j+1)}{\bar{\omega}}\pi\right], \; j \in \{0, 1, \dots\}\\ \alpha \sin \bar{\omega}t, & \text{otherwise .}\end{cases}$$

with $\alpha = \frac{1}{2}$ and $\bar{\omega} = 2$. For the synchronous case, the results are reported in Figures 2, 3 and 4 when fixing the sampling period as $\delta = 0.5$ seconds and unitary gains for all reported scenario. In this case, the results highlight the capability of the proposed sampled controller (Fig. 4) to guarantee stabilization of the desired formation with performances that are close (and comparable) to the ones of the ideal continuous-time feedback (Fig. 2). In the same situation, the emulation-based controller fails and makes all unicycles diverge (Fig. 3).

As far as the asynchronous case is concerned the results are reported in Figures 5, 6 and 7 the sampling periods are randomly generated within the interval $[\delta_{\min}, \delta_{\max}[$ with $\delta_{\min} = 10^{-2}$ and $\delta_{\max} = 1$ seconds. In that case, similar considerations hold true: the emulation control fails when the proposed one is successful in forcing the desired formation with performances that are similar to the continuous-time one. The transient time significantly increases as the gains are small

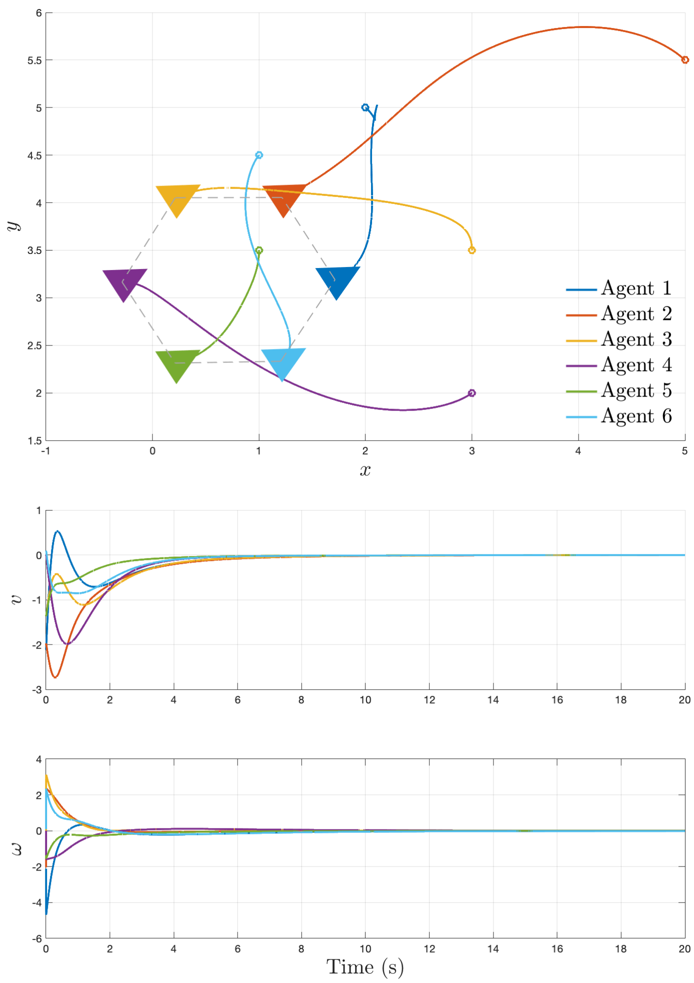

Fig. 2: Continuous-time control [37] with $\kappa_v = \kappa_\omega = 1$.

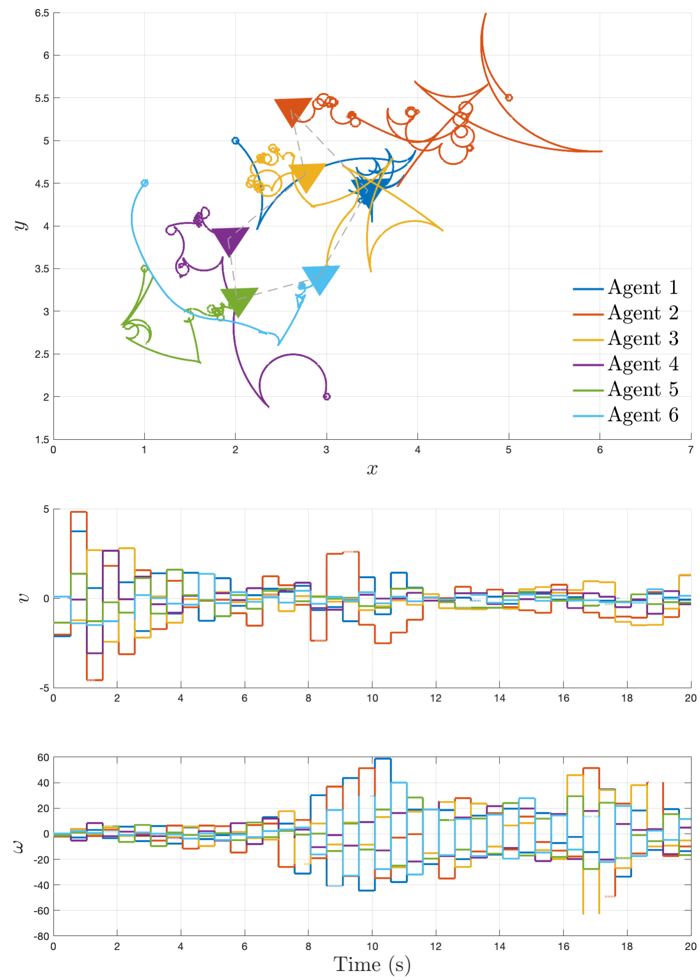

Fig. 3: Emulation-based control under synchronous sampling with $\delta = 0.5$ s and $\kappa_v = \kappa_\omega = 1$.

as we consider an inflated scenario in which the sampling periods can be largely different for each agent.

Remark 4.4: In all cases, the simulated plant for each agent is the continuous-time one (7) fed by the proper piecewise constant signals, computed based on the sampled measures from the network.

Those results testify that the proposed control, even if computed as an approximate solution to the corresponding equalities, preserves the performances of the continuous-time counterpart, despite the effect of sampling. As one might expect, the convergence rate in the asynchronous case is notably affected by the choice of the gain that must take into account the asynchronism of the network. However, despite the large spread among the minimum and the maximum sampling period and even if with a slower convergence, the desired formation is reached with good overall performances.

V. Conclusions and Perspectives

A new sampled-data controller has been proposed for steering unicycles to a desired configuration with a preliminary application to formation consensus control of groups of unicycles under sampled-data asynchronous communication. The sampled-data control is robust with respect to the effect of sampling in both the synchronous and asynchronous cases. Current works concern the extension of those arguments to formation tracking under time-delay noisy communication [46] and time-varying inter-transmission intervals and scheduling constraints [47]. The extension of those arguments to general classes of nonholonomic systems is undergoing too, possibly including actuator dynamics and measurement feedback [48].

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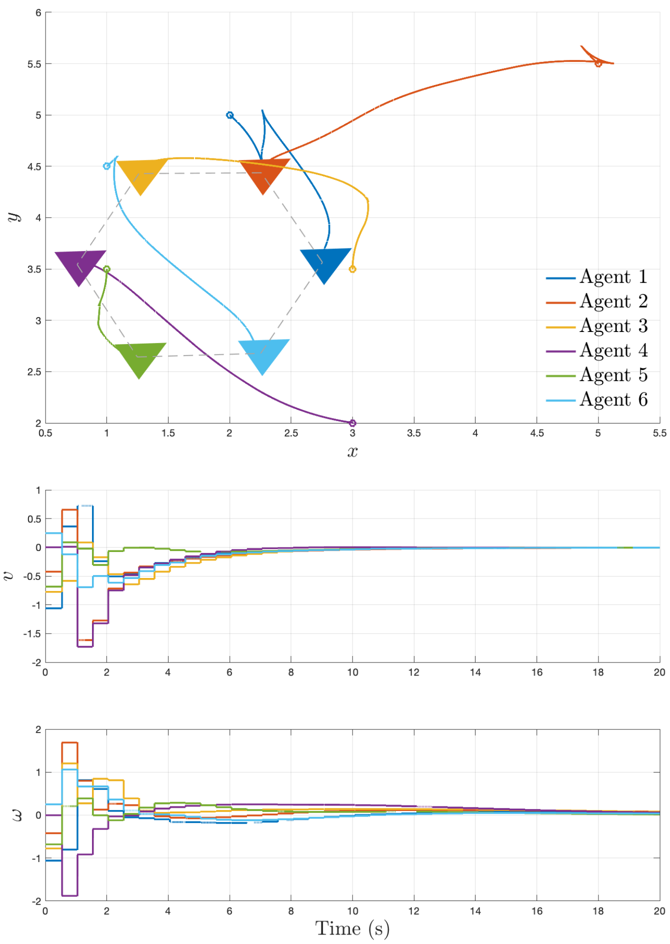

Fig. 4: Sampled-data control (Prop. 4.1) under synchronous sampling with $\delta = 0.5$ s and $\kappa_v = \kappa_\omega = 1$.

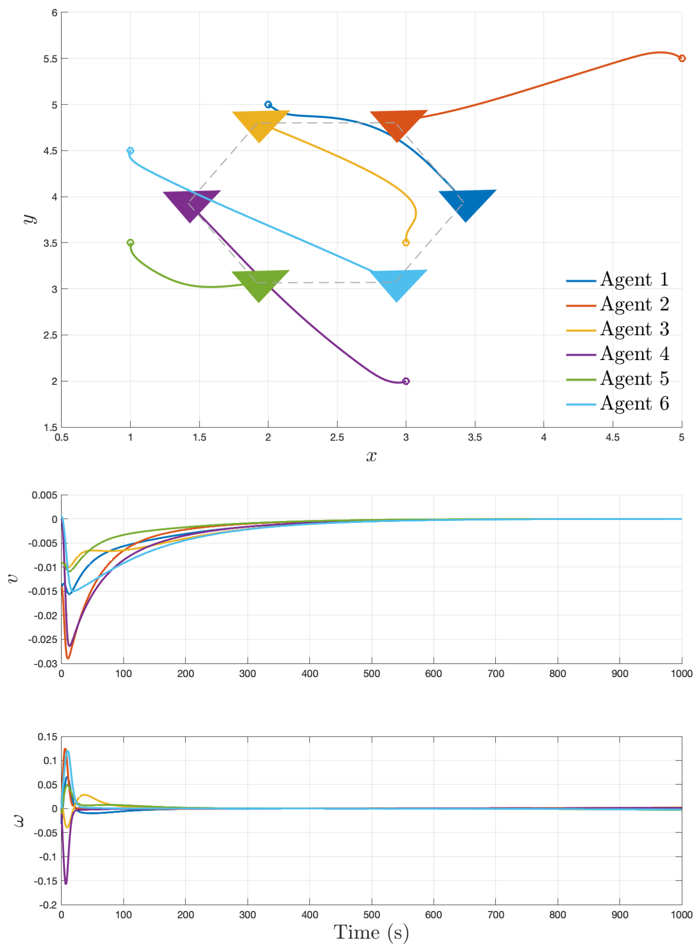

Fig. 5: Continuous-time control [37] with $\kappa_v = \kappa_\omega = 0.0069$.

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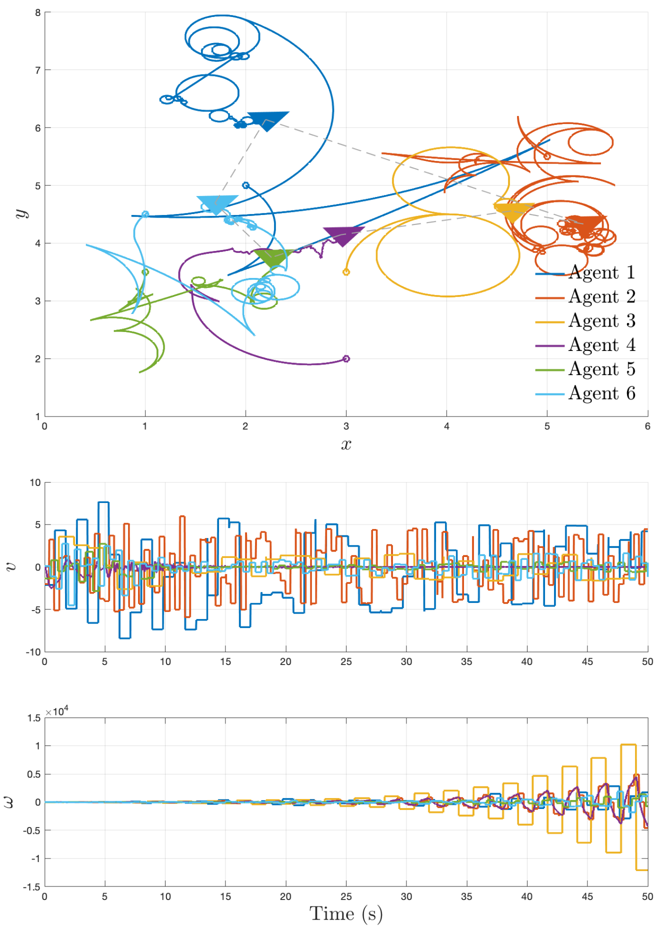

Fig. 6: Emulation-based control under asynchronous sampling with $\delta \in [0.01, 1]$ s and $\kappa_v = \kappa_\omega = 0.0069$.

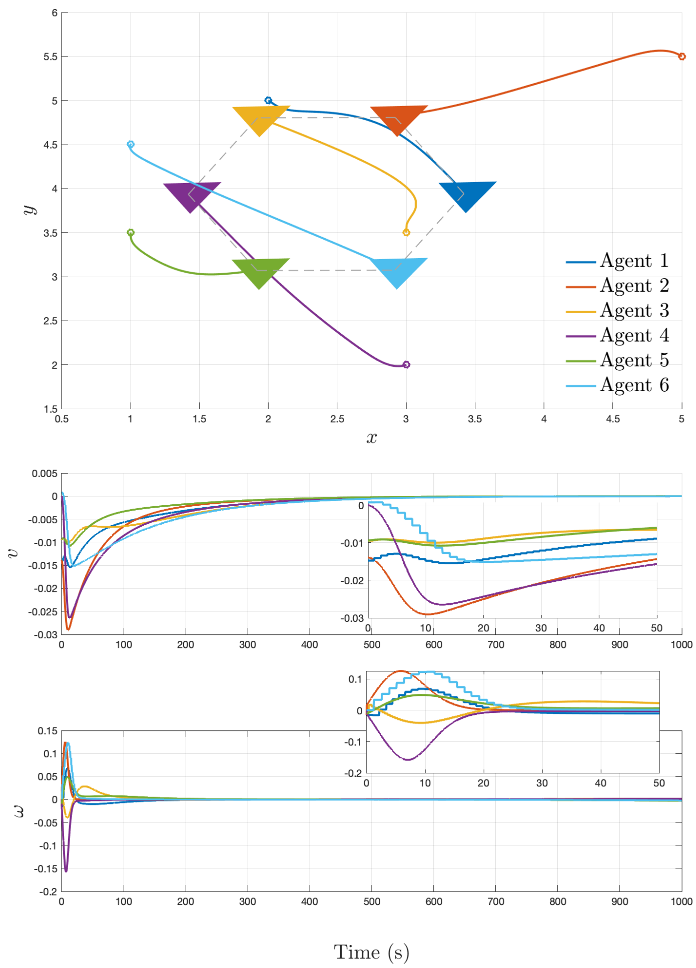

Fig. 7: Sampled-data control (Th. 3.1) under asynchronous sampling with $\delta \in [0.01, 1]$ s and $\kappa_v = \kappa_\omega = 0.0069$.

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