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Preface

This book includes the contributions presented at the 51st Scientific Meeting of the Italian Statistical Society (SIS) held in Caserta at the Università della Campania “Luigi Vanvitelli”, from the 22nd to 24th of June, 2022.

The conference has registered more than 300 presentations, including 4 keynotes in plenary invited sessions and 9 presentations in 3 guest sessions, 48 presentations collected in 16 specialized sessions and 68 presentations in 17 solicited sessions, all dealing with specific themes in methodological and/or applied statistics and demography. Furthermore, more than 200 contributions, with one or more authors, have been spontaneously submitted to the Program Committee and arranged in 43 contributed sessions.

The high number of contributions and the large participation at the conference show that researchers have met the challenge of pursuing working even in the face of the pandemic period from which we are only now emerging. The research activity in our field therefore has never stopped, and the desire to participate in scientific events, as a place for exchange and discussion on new developments in our field, remains a living characteristic of our scientific community.

With the publication of this book, we wish to offer to all members of the Italian Statistical Society, all international academics, researchers, Ph.D. students, and all interested practitioners, a good snapshot of the on-going research in the statistical and demographic fields. We deeply thank all contributors for having submitted their works to the conference and all the researchers for their remarkable job in acting as referees accurately and timely. We also would like to thank the International Biometric Society (IBS) – Italian region, the European Network for Business and Industrial Statistics (ENBIS) and the Italian Society of Statistical Physics (SIFS) we had the pleasure of hosting. A special thanks is addressed to the Scientific and Organizational Committees for their great efforts devoted to all the organizational aspects, to the Università della Campania “Luigi Vanvitelli” and to the Department of Mathematics and Physics who made this event possible, as well as to the Municipality of the Town of Caserta who has patronized the event and to all the funders for their supports.

Finally, we wish to express our gratitude to the publisher Pearson Italia for all the support received.

Asymmetric Spectral Clustering: a comparison between symmetrizations

Spectral clustering asimmetrico: un confronto tra simmetrizzazioni

Cinzia Di Nuzzo and Donatella Vicari

Abstract In this work, spectral clustering of asymmetric data is addressed. In particular, two different methods to perform clustering have been compared: the application of spectral clustering on directed graphs represented by an asymmetric matrix; and the application of the classical spectral clustering algorithm once transformed the directed graph into an undirected one. To this end, some symmetrizations are described to convert the directed graph to an undirected one.

Abstract *In questo lavoro viene trattato lo spectral clustering di dati asimmetrici. In particolare, sono state confrontate due diverse metodologie per effettuare il clustering: l'applicazione dello spectral clustering su grafi diretti rappresentati da una matrice asimmetrica; e l'applicazione del classico algoritmo di spectral clustering dopo la trasformazione del grafo diretto in uno non diretto. A tal fine, sono stati descritte alcune simmetrizzazioni per convertire il grafo diretto in uno non diretto.*

Key words: spectral clustering, directed graph, symmetrizations

1 Introduction

Clustering on directed graphs has several useful applications, just think of network and social network analysis. So far, few spectral methods have been proposed for clustering this type of data, see [8], [3] and [2]. Many clustering methods on directed graphs rely on the symmetrization of the weighted matrix of the data by taking into account the out-degrees between the vertices of the graph. Once such a symmetrization has been computed, an undirected graph can be associated with the resulting symmetric matrix, and the standard clustering methods on undirected graphs can be applied. Here, we propose to apply the spectral clustering method proposed by [5]. In particular, three different ways to symmetrize an asymmetric matrix are presented: the first method is based on the symmetrization of the asymmetric weighted matrix associated with the directed graph; the second one is based on the symmetrization of the weighted matrix considering the out-degrees of the graph, while the last symmetrization takes into account both the in-degrees and out-degrees of

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the directed graph. The spectral clustering, proposed in [5] and applied to these three symmetrizations, has been compared with the asymmetric spectral clustering proposed by [8]. The rest of the paper is structured as follows: in Section 2, some definitions on directed graphs are given. In Section 3, the symmetrizations of the asymmetric matrix associated with the directed graph, and the undirected spectral clustering method are summarized. In Section 4 the asymmetric spectral clustering proposed by [8] is summarized. Section 5 provides a numerical example where a comparison is carried out.

2 Background theory

Let $\mathcal{G} = (V, E)$ be a directed graph, where $V = \{v_1, \dots, v_n\}$ is the set of n vertices or nodes, $E \subseteq V \times V$ is the set of the edges or arcs. In a directed graph, an edge is an ordered pair (v_i, v_j) , where $v_i, v_j \in V$ for $i, j \in \{1, \dots, n\}$. A tuple of vertices (v_1, v_2, \dots, v_p) with $(v_h, v_{h+1}) \in E$ for $1 \leq h \leq p-1$, is a *path*. If for each pair of vertices v_i and v_j there is a path where $v_1 = v_i$ and $v_p = v_j$, then the directed graph is *strongly connected*. When \mathcal{G} is a strongly connected graph, there is an integer $l \geq 1$ and a unique partition $V = V_0 \cup V_1 \cup \dots \cup V_{l-1}$ such that for all $0 \leq r \leq l-1$ each edge $(v_i, v_j) \in E$ with $v_i \in V_r$ and $v_j \in V_{r+1}$, and where $V_l = V_0$. The graph \mathcal{G} is *weighted* when there is a weight function $w : E \rightarrow \mathbb{R}^+$ which associates a positive value $w((v_i, v_j))$ to each edge $(v_i, v_j) \in E$. Let $\mathcal{G} = (V, E, W)$ be a weighted directed graph, where W is the *asymmetric weighted matrix* associated to \mathcal{G} . The out-degree and the in-degree of a vertex v_i are defined by $d_i^+ = \sum_{j=1}^n w_{ij}$ and $d_i^- = \sum_{j=1}^n w_{ji}$, respectively. Let $D_o = \text{diag}(d^+)$ be the diagonal matrix of the vertex-out degrees. Given a weighted directed graph \mathcal{G} , there is a natural random walk on the graph with *transition probability matrix of the Markov chain* equal to matrix $P = D_o^{-1}W$ with entries $p_{ij} = w_{ij}/d_i^+$. When \mathcal{G} is assumed to be a strongly connected graph, the Markov chain P is *irreducible* and has a unique stationary distribution π , i.e. a unique probability distribution π (with n non-negative entries summing to one) that satisfies the following balance equations

$$P' \pi = \pi, \quad (1)$$

where $\pi = (\pi_1, \dots, \pi_n)'$. Let us denote Π the diagonal matrix having π as main diagonal.

3 Symmetrizations of the asymmetric matrix W and spectral clustering algorithm

Several attempts have been made to generalize the spectral graph theory of undirected graphs to directed graphs and they have been mainly addressed towards the symmetrization of the weight matrix associated to the graph. The graph symmetrizations proposed in literature are summarized below (see [6] for further details).

First symmetrization: $W + W'$. The simplest symmetrization is obtained as

$$U = W + W'. \quad (2)$$

Note that this symmetrization does not take into account the in- or out- links between the vertices of the graph.

Second symmetrization: Random walk. The random walk symmetrization proposed by [1] is defined as

$$U_{rw} = \frac{\Pi P + P' \Pi}{2}. \quad (3)$$

This symmetrization is based on the stationary distribution Π of the graph and does not take into account the in-degrees in each vertex.

Third symmetrization: Degree-discounted. In order to symmetrize the graph, both in- and out- degrees are taken into account in [6]. The idea behind this is to look for a similarity measure that takes into account the in- and out-degrees between the vertices of the directed graph. In fact, nodes with high degrees will share many common edges (in- or out-) with other nodes simply by virtue of their higher degrees. The idea is to weigh the edges so that vertices with high (in- or out-) degree are weighted less in the symmetrization process. The reason is very simple: vertices with high (in- or out-) degrees are actually less informative in describing the links between the vertices of the graph than nodes with lower degrees. The symmetrization proposed in [6] is given by the following expression

$$U_d = D_o^{-1/2} W D_i^{-1/2} W' D_o^{-1/2} + D_i^{-1/2} W' D_o^{-1/2} W D_i^{-1/2} \quad (4)$$

where $D_i = \text{diag}(d^-)$ is the diagonal matrix of the in-degrees.

Once a directed graph has been transformed into an undirected graph, the spectral clustering algorithm proposed in [5] can be run as summarized in Algorithm 1.

Algorithm 1 Spectral Clustering algorithm (see [5])

Let $\mathcal{G}_u = (V_u, E_u, W_u)$ be a undirected weighted graph, where $V_u = \{v_1^u, \dots, v_n^u\}$ is the set of the vertices, E_u is the set of the edges, and $W_u = (w_{ij}^u)$ is the symmetric weighted matrix associated to \mathcal{G}_u .

Input: Symmetric weighted matrix W_u ; number of clusters K .

1. Compute the degree matrix $D_u = \text{diag}(d_i^u)$ associated to the undirected graph \mathcal{G}_u , where d_i^u is the degree of the vertex v_i^u associated to the undirected graph \mathcal{G}_u and it is computed as $d_i^u = \sum_{i \neq j} w_{ij}^u$.
 2. Calculate the normalized symmetric Laplacian matrix $L_{\text{sym}} = D_u^{-1/2} W_u D_u^{-1/2}$.
 3. Consider matrix Y having the eigenvectors associated to the K largest eigenvalues of L_{sym} as columns
 4. Re-normalize the rows of Y to have unit length yielding $X \in \mathbb{R}^{n \times K}$.
 5. Perform k -means algorithm on matrix X .
-

4 Asymmetric Spectral Clustering

Let us suppose to divide the graph into K subgraphs. In order to find the best partition of the graph \mathcal{G} , in the standard undirected spectral clustering method a normalized cut is introduced and the clustering is obtained thanks to a minimization of this cut (see [7] and [4]). An extension to directed graphs has been introduced by [8], where the directed $Ncut$ of a cluster C is defined as

$$Ncut_{dir}(C) = \frac{\sum_{i \in C, j \in \bar{C}} \pi_i p_{ij}}{\sum_{i \in C} \pi_i} + \frac{\sum_{i \in C, j \in \bar{C}} \pi_j p_{ji}}{\sum_{j \in \bar{C}} \pi_j}. \quad (5)$$

In [8], the $Ncut_{dir}$ is minimized by the eigenvectors of the following matrix

$$\Theta = \frac{\Pi^{1/2} P \Pi^{-1/2} + \Pi^{-1/2} P' \Pi^{1/2}}{2}. \quad (6)$$

The Asymmetric Spectral Clustering is summarized in Algorithm 2.

Algorithm 2 Asymmetric Spectral Clustering algorithm (see [8])

Input: Asymmetric weighted matrix W associated to directed graph \mathcal{G} ; number of clusters K .

1. Compute the transition matrix P and the diagonal matrix Π from its unique stationary distribution P .
 2. Compute the (symmetric) Laplacian matrix Θ (6).
 3. Consider the eigenvectors of Θ and introduce matrix Z with columns equal to the eigenvectors $\gamma_1, \dots, \gamma_K$ associated with the K largest eigenvalues of Θ .
 4. Perform k -means algorithm on matrix Z .
-

5 Numerical experiments

In order to compare the methods described in Sections 3 and 4, we present an example with artificial data.

The artificial data are described in Figure 1, the graph $\mathcal{G} = (V, E, W)$ is composed by three groups: $C_1 = \{v_1, v_2, v_3\}$, $C_2 = \{v_4, v_5\}$ and $C_3 = \{v_6, v_7, v_8\}$. Specifically, in Figure 1-a) the edges of the graph \mathcal{G} are displayed with different thickness according to their weights (i.e., solid, dashed and dotted lines represent weights in descending order, respectively).

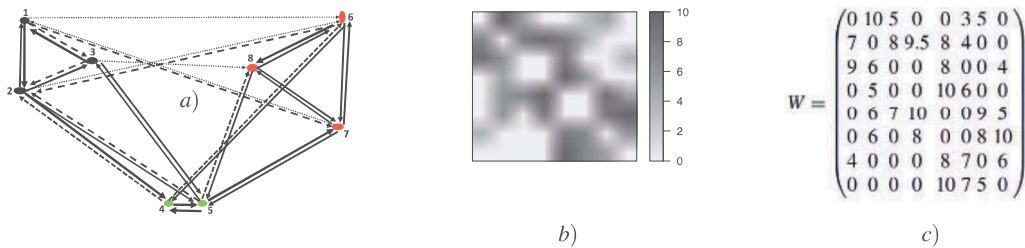


Fig. 1: *Artificial data.* a) Representation of the graph \mathcal{G} . b) Heat map of the asymmetric matrix W . c) Asymmetric weighted matrix W associated to graph \mathcal{G} .

In Figure 2, the geometric features of the spectral clustering algorithm (Algorithm 1) applied on the three symmetrizations described in Section 3 have been represented. The correct clustering results are attained from the U and U_{rw} symmetrizations; conversely, the spectral clustering applied on the U_d symmetrization does not provide the correct classification, because the accuracy is equal to 0.5 and the confusion matrix is

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

The asymmetric spectral clustering described in Algorithm 2 has been also carried out, the geometric features of the algorithm are shown in Figure 3, and the correct clustering is obtained.

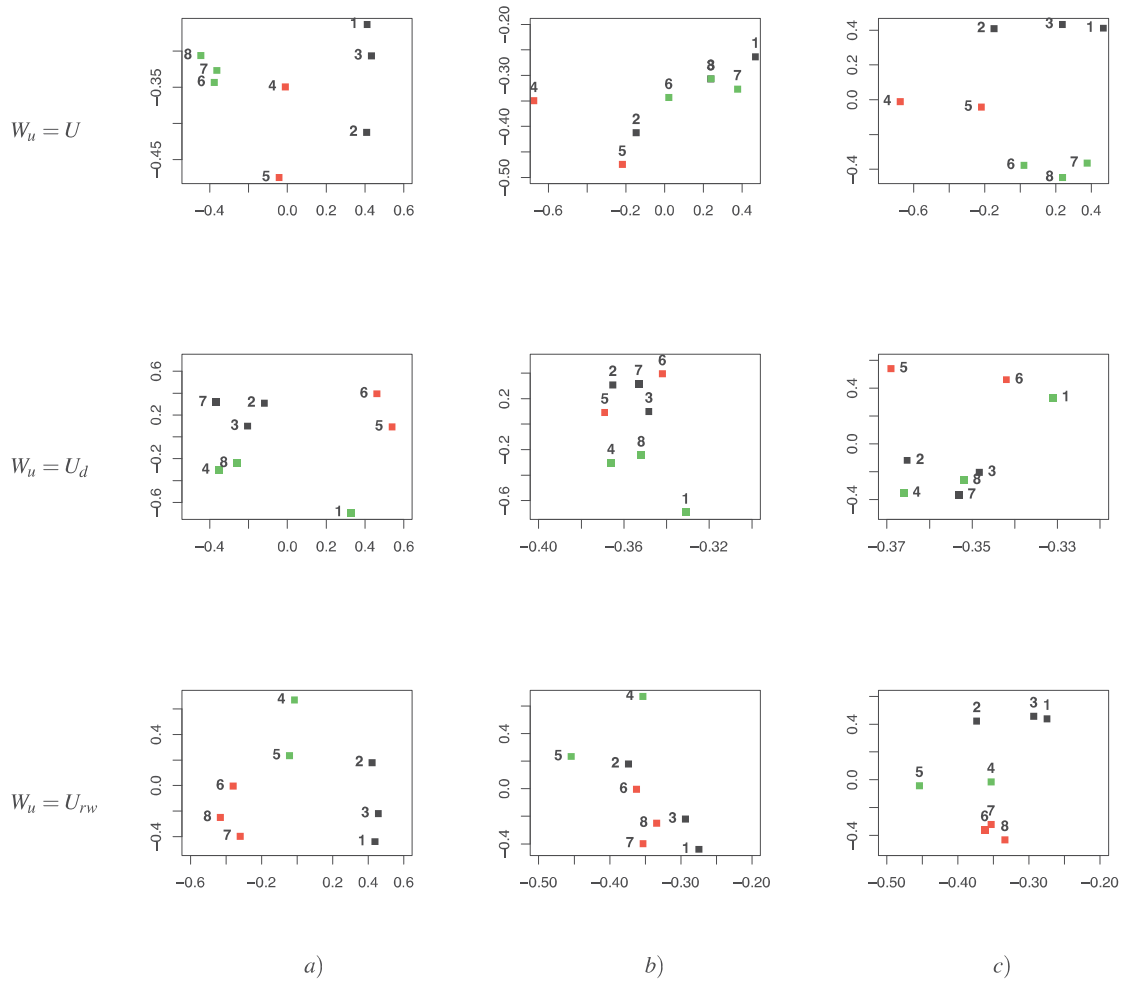


Fig. 2: Spectral clustering after symmetrizations. Embedded data associated to L_{sym} according to: a) the first two eigenvectors; c) the first and third eigenvector; d) the last two eigenvectors (different colors denote different clusters).

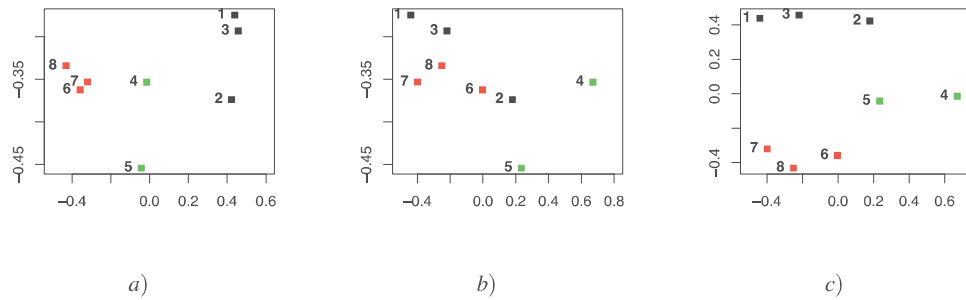


Fig. 3: *Asymmetric spectral clustering (Algorithm 2)*. Embedded data associated to Θ according to: a) the first two eigenvectors; c) the first and third eigenvector; d) the last two eigenvectors (different colors denote different clusters).

To conclude, the asymmetric spectral clustering described in Algorithm 2 provides the correct classification, while the spectral clustering applied after the symmetrizations provides the correct results only when the U_{rw} and U symmetrizations are used. Finally, the Laplacian embedding structure provided by the U_d symmetrization, shown in Figure 2, is different from the others, because U_d also takes into account and incorporates the information from the in-degrees of the nodes of the directed graph \mathcal{G} . Therefore, this lays the groundwork for further studies on the information extracted from the different symmetrizations by carrying out studies on both artificial and real data.

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