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DYNAMIC OPTIMAL ASSET ALLOCATION IN A MULTIVARIATE SETTING

Abstract

This article analyzes a portfolio allocation problem to determine how resources should be allocated among several possible investments. Investors aim to maximize the profit of an investment while also considering the risks arising from infrequent events. The global financial crisis, which began with subprime mortgages in the United States, has fundamentally changed the way we invest. As we know, investors want to maximize returns while controlling the risk associated with a particular investment. This behavior must be modeled mathematically using optimal control theory and expected utility maximization. A continuous-time market is considered in a multivariate context in which there exist risky asset classes and a risk-free asset with a constant interest rate. We deviate from the traditional approach by considering co-precision, the inverse of the covariance matrix, as a measure of risk. The optimal weights obtained are proportional (inversely) to the risk measure (volatility). The model is tested on 11 asset classes used by a large company also carrying out a *stress test* on the jump component to analyze the allocation of the investors' portfolio in a real context.

Keywords: asset allocation, jumps, stochastic volatility, Wishart process, dynamic programming.

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1 Introduction

In recent years, the occurrence of some unprecedented events has caused prices to decline across markets. Rarely have investors had to contend with situations such as those arising from a global economy recovering from a devastating pandemic, monetary policy interventions, and the outbreak of wars in Europe.

Market turmoil weighs on the investment outlook, with implications ranging among economic growth and decline, central bank policies for interest rates, credit quality and earnings, changing investor risk-aversion levels, and other key indicators.

The discontinuous movement of asset prices generates jumps in the market causing a deviation of the stock distribution from the Gaussian one and, leading to investment opportunities. As a consequence, the mean-variance approach described by Markowitz (1952) turns out to be no longer efficient, given its static nature. Furthermore, such an approach ignores the presence of price volatility. Therefore, adopting this type of allocation model could result in potential losses. A solution widely adopted in the literature is to consider continuous-time models for price dynamics: by allowing continuous trading, the investor can immediately react to possible changes in price volatility. The randomness and unpredictability of asset prices are some of the reasons why most investors tend to rebalance their portfolios throughout the investment term.

This article analyzes a dynamic allocation problem in a real-world context. We consider a suitable market model, calibrated on real data to evaluate the impact of the jumps. Our findings indicate that jump risk tends to lead investors to adopt a more conservative allocation overall, thereby mitigating the need for dramatic adjustments in dynamic portfolio rebalancing over time.

The wealth of a risk-averse investor is allocated to both risk-free and risky assets by considering an expected constant return. Furthermore, we consider a time-varying diffusion term in the model, which allows for volatility and stochastic correlations, and a jump component driven by a Poisson process.
By analyzing the results in a multivariate context, we specify the dynamics of the inverse of the covariance matrix of risky assets, called co-precision. According to Oliva and Renò (2018), we use a Wishart mean-reverting process, to have a multivariate affine process.

The contribution to the existing literature is to assess the impact of jumps and prove the crucial role of the latter in defining the allocation of a risk-averse investor.

As in Chacko and Viceira (2005), we consider a model where the allocation is proportional to the inverse covariance but not to the risk aversion coefficient. User It seems like there is a correlation between the excess returns, which is the short-sighted part, and the covariance between returns and volatility, which is the hedging aspect. Furthermore, unlike Chacko and Viceira (2005), we model a multivariate context.

In the literature, the pioneering work of Merton (1971) can be considered a starting point for dynamic portfolio management in continuous time. The author derived optimal consumption and investment rules by maximizing the investor's expected utility in an economy including both private activity risk and risky stocks, when the asset price dynamics follow a geometric Brownian motion.

Subsequently, several authors generalized Merton's work within the incomplete market. Framework, to name but a few, we mention Liu et al. (2003), where an investor would hold fewer risky assets when price jumps occur, Liu (2007), where a dynamic portfolio choice is solved by exploiting affine models to face stochastic volatility.

In a complete market framework, the optimal portfolio allocation can be composed of risk-free assets, stocks, and derivatives considering a stochastic volatility environment, with or without the presence of jumps, see e.g. Liu et al. (2003) and Branger (2017).

Wu (2003) demonstrates that the effects of market movements on portfolios are intrinsically linked to the moments of returns. In particular, the investor demand for a specific asset may vary due to market disruptions. Therefore, in the case of a reduction in the demand for the asset, the value of the asymmetry (negative in the case of a negative average jump size) and the positive kurtosis generates greater volatility in the return of the asset, see e.g. Wu (2003). Thus, the variance over time of the returns of an investment opportunity can increase (or decrease).

Differently from the model proposed by Wu (2003), in the present paper described we consider the jump variable constant for each sset and stress its value according to a linera growth, in order to highlight the possible impacts of jumps. However, by observing the descriptive statistics of the returns it is possible to have a first intuition about the portfolio allocation.

Liu (2007), Buraschi (2010) and Liu et al. (2003) use affine stochastic volatility models to manage volatility and the presence of jumps in the allocation. In particular, Liu et al. (2003) studied how investment strategies may change considering changes in prices and volatility. In addition to considering the dynamic aspect in selecting the optimal allocation, other recent works study the role of predictability, see e.g. Brandt (1999), Brennan (1997) and Lynch (1999).

Paper is organized as follows. Section 2 describes the model of an optimal investment allocation problem. Section 3 is dedicated to the empirical application using real data. Section 4 concludes.

2 The optimal investment problem

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbb{P})$ a filtered probability space. We propose a model where the risky asset follows a jump-diffusion process with deterministic jump component.
We assume there exists one riskless asset $B = \{B_t\}_{t\in[0,T]}$ with dynamics:

$$dB_t = rB_t dt, \quad t \in [0,T], \tag{2.1}$$

where $r \in \mathbb{R}$ denotes the risk-free interest rate.
Moreover, we consider N risky assets $S_t = (S_{t,1}, \dots, S_{t,N})' \in \mathbb{R}^{N\times 1}$, with dynamics

$$\begin{cases} dS_t = diag(S_t)\left[\mu dt + \sqrt{Y_t^{-1}}dW_t^{(1)} + JdN_t\right] \\ dY_t = (\Omega\Omega' + KY_t + Y_tK')dt + \sqrt{Y_t}dW_t^{(2)}Q + Q'\left(dW_t^{(2)}\right)'\sqrt{Y_t}, \quad t \in [0,T], \end{cases} \tag{2.2}$$

where $diag(S_t)$ is the square matrix with S_t in the diagonal and 0 on the off-diagonal elements, $\mu \in \mathbb{R}^{N\times 1}$ is the drift and $J \in \mathbb{R}^{N\times 1}$ is the price jump component. We also consider $W_t^1 \in \mathbb{R}^{N\times 1}$ is a Wiener process of the stock and $W_t^{(2)} \in \mathbb{R}^{N\times N}$ is a matrix of Wiener process of inverse of the instantaneous variance-covariance matrix Y_t, called *co-precision*, that follows a Wishart process. The matrices $K, Q \in \mathbb{R}^{N\times N}$, see Kang C. et al (2017).

Discontinuous movements in price are driven by a single Poisson process N_t with intensity $\lambda \in \mathbb{R}$.

Denoting by $GL_N(\mathbb{R})$ the set of real invertible matrices in $\mathbb{R}^{N\times N}$. Also assume $\Omega\Omega' \geq (N+1)QQ'$, implying that equation (2.2) has a unique global strong solution on $S_N^+(\mathbb{R})$ the later being the set of symmetric positive definite matrices of dimension $N \times N$, see e.g. Bru (1991).

In particular, Ω satisfies

$$\Omega\Omega' = \alpha QQ', \tag{2.3}$$

with $\alpha \in \mathbb{R} \geq N-1$. Equation (2.3) is sufficient to guarantee the positive definiteness and the mean-reversion of Y_t for any $tin[0,T]$, see e.g. Bru (1991). The matrix Q is proportional to the volatility of the co-precision matrix.

As highlighted in Oliva and Renò (2018), the matrix K is assumed to be negative semidefinite in such a way as to guarantee the strict positivity and the mean-reverting characteristic of the volatility. Furthermore, the matrix K represents the mean reversion rate of the co-precision at its mean reversion level $\bar{Y}$ is such that:

$$\alpha QQ' + K\bar{Y} + \bar{Y}K'.$$

The Wiener processes determining shocks in prices and variance-covariance matrix $\Sigma_t = Y_t^{-1}$ are correlated according to

$$W_t^{(1)} = W_t^{(2)}\rho + \sqrt{1-\rho'\rho}W_t^{(3)}, \quad t \in [0,T],$$

where $\rho \in \mathbb{R}^{N\times 1}$, $W_t^3 \in \mathbb{R}^{N\times 1}$ is a Wiener process and the elements of W_t^3 and W_t^2 are independent. Equation (2) allows to correlate returns and their variance-covariance matrix, typically in a negative way, see e.g. Branger (2017).

Let $\pi_t \in \mathbb{R}^{N\times 1}$ denote the vector of shares of wealth X_t invested in the risky asset while the proportion of wealth invested in the riskless asset is given by $(\mathbf{1} - \pi_t'\mathbf{1})$ with $\mathbf{1}$ being the $N \times 1$ vector of ones.

The investor's wealth evolves according to

$$dX_t = [\pi_t'(\mu - r\mathbf{1}) + r]X_t dt + \pi_t' X_t \sqrt{Y_t^{-1}} dW_t^1 + \pi_t' J X_t dN_t, \quad t \in [0,T]. \tag{2.4}$$

The investor chooses the portfolio that maximizes her wealth at maturity in terms of a power utility, with risk aversion parameter γ. Considering X_0 the initial wealth, the optimization problem is

$$V(t, Y_t, X_t) := \max_{\pi_t, t\in[0,T]} \mathbb{E}_t \left[\frac{X_T^{1-\gamma}}{1-\gamma}\right], \quad \gamma > 0, \gamma \neq 1 \tag{2.5}$$

subject to the dynamic budget constraint (2.4).

The Hamilton-Jacobi-Bellman (HJB) equation associated with the investment problem, using standard dynamic programming techniques, is given by

$$\begin{aligned} 0 = \max_{\pi_t, t\in[o,T]} \Bigg\{ & \frac{\partial V}{\partial t} + [\pi_t'(\mu - r\mathbf{1}) + r]X_t \frac{\partial V}{\partial X} + Tr\left([\Omega\Omega' + KY_t + Y_t K']\Delta V\right) \\ & + \frac{1}{2}X_t^2 \pi_t' Y_t^{-1}\pi_t \frac{\partial V^2}{\partial X^2} + \left(2\pi_t'\Delta Q'\rho\frac{\partial V}{\partial X}\right)X_t + \frac{1}{2}Tr\left(4Y_t\Delta Q'Q\Delta\right)V \\ & + \lambda\mathbb{E}[V(t, X_t(1+\pi_t'J), Y_t) - V(t, X_t, Y_t)]\Bigg\}, \end{aligned} \tag{2.6}$$

where

$$\nabla := \left(\frac{\partial}{\partial Y_{i,j}}\right)_{1\le i,j\le N}.$$

To solve the optimal investment problem (2.5), the jump component must be linearized through a first-order Taylor expansion, so that

$$(1+\pi_t'J)^{(1-\gamma)} = 1 + (1-\gamma)\pi_t'J + o((\pi_t'J)^2),$$

and ignoring the term $o((\pi_t'J)^2)$.

The approximated solution is provided in the following

Proposition 2.1. *Consider the HJB equation* (2.6)*, the optimal strategy that solves the investment problem* (2.5) *is given by*

$$\pi_t^* = Y_t\left[\frac{(\mu - r\mathbf{1}) + 2A_tQ'\rho + \lambda J}{\gamma}\right] =: Y_t M_t, \tag{2.7}$$

where $M_t \in \mathbb{R}^{N\times 1}$.

The value function is given by

$$V(t, X_t, Y_t) = exp\left\{Tr(A_t Y_t) + G_t\right\} \frac{X_t^{1-\gamma}}{1-\gamma}, \tag{2.8}$$

where the matrix function $A_t \in \mathbb{S}_N$ *and the function* $G_t \in \mathbb{R}$ *satisfy the following system of ODEs*

$$\begin{cases} \dot{A}_t + (1-\gamma)(\mu - r\mathbf{1})B_t' + A_t K + K' A_t - \dfrac{\gamma(1-\gamma)}{2} B_t B_t' + \\ + 2(1-\gamma) A_t Q' \rho B_t' + 2 A_t Q Q' A_t + \lambda(1-\gamma) J B_t' = 0, \quad A_T = 0 \\ \dot{G}_t + (1-\gamma) r + Tr(\Omega\Omega' A_t) = 0, \quad G_T = 0. \end{cases} \tag{2.9}$$

Proof. The proof retraces the steps provided in Oliva and Renò (2018). □

Given Equation (2.7), the optimal weights for portfolio selection are composed of two components. The first component is the of *myopic demand* and is equal to $\frac{(\mu - r\mathbf{1}) + \lambda J}{\gamma}$, the latter highlights the dependence with the performance in excess and risk aversion without considering the multi-period nature of the investment. The second component, $\frac{2A_t Q' \rho}{\gamma}$, is the *intertemporal hedging demand*, characterized by the presence of the correlation coefficient between Brownian motions and co-precision. Its value depends on the sign of both the function A_t and on ρ, which is assumed to be positive. As for the jump component, its amplitude and intensity can positively or negatively influence the portfolio allocation.

3 Empirical application

We apply the methodology described in Section 2 to a challenging financial problem. We consider allocating wealth in a portfolio of 11 indices, mimicking the behavior of large corporate portfolios.

The indices included in our analysis are: ICE BofA 0-1 Year Euro Government Index (EG0A), ICE BofA 1-3 Year Italy Government Index (G1I0), ICE BofA 1-3 Year Euro Government Exchanging Italy Index (N1IT), ICE BofA 1-10 Year Italy Government Index (G5I0), ICE BofA 1-10 Year Euro Government Exchanging Italy Index (N5I0), ICE BofA Global Government Exchanging Euro Governments Index (N0Q1), ICE BofA Euro Large Cap Corporate Index (ERL0), ICE BofA Global Large Cap Corporate Excluding Euro Index (GCXZ), ICE BofA Global High Yield Index (HW00), MSCI Europe Total Return Index (MSDEE15N) and MSCI World A.C. ex Europe Total Return Index (MSDEWXEN).

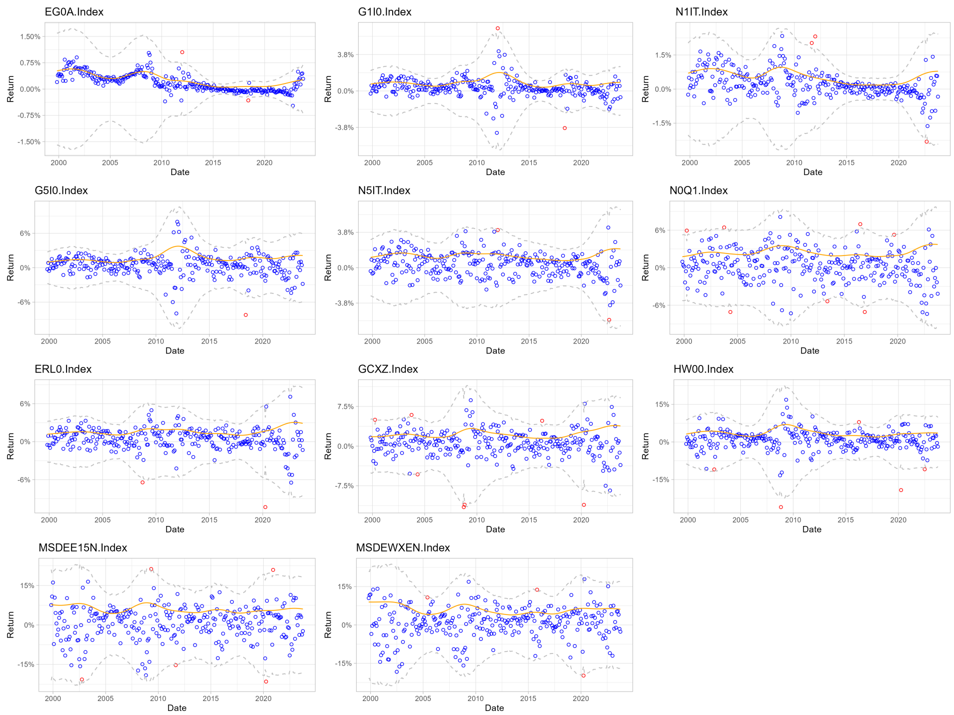
The indices chosen allow for a diversified portfolio allocation, consistent with the investor's needs. Companies can create products that respond to market disruptions and can transfer risk.

We consider the time series of monthly returns from December 1998 to September 2023 for a total of T = 289 months. Table 3.1 shows the descriptive statistics of the monthly returns. The

Table 3.1. Descriptive statistics for the returns of the 11 indices, expressed in percentages and in annualized units.

	EG0A	G1I0	N1IT	G5I0	N5IT	N0Q1	ERL0	GCXZ	HW00	MSDEE15N	MSDEWXEN
Mean	1.54	2.61	1.83	3.52	2.76	2.34	3.42	4.18	6.26	5.28	6.60
Std	0.60	2.24	1.39	4.32	3.02	6.14	4.12	6.10	9.83	15.27	14.68
Skewness	0.63	0.46	0.36	-0.42	-0.33	-0.24	-1.12	-0.87	-1.21	-0.44	-0.48
Kurtosis	3.14	12.10	4.95	7.60	4.54	3.53	9.30	6.99	10.68	4.05	3.62

Figure 3.1. Monthly returns for all indices, along with the calibrated threshold used to generate the jumps and the corresponding estimated standard deviation.



returns show a positive mean and a low standard deviation. We further notice that, except for the first three indices reported in Table 3.1, the indices present a negative asymmetry, demonstrating the typical event risk signals. Despite this, the values are not particularly high.

We calibrate the model according to the method reported in Bandi and Renò (2016), and along the lines of Mancini (2009). Specifically, we implement the methodology for threshold definition on the analyzed dataset. This technique allows to estimate of the amplitude of jumps to evaluate the optimal policies, according to the theoretical findings.

We denote by r_{t_i} the return of the i-th index with $i = 1, \dots, N$ at time t_i with $i = 1, \dots, T$. We define θ_{t_j} the threshold function of the i-th index at time t_i as

$$\theta_{t_j} = 3\sqrt{\frac{\frac{\pi}{2}\sum_{i=1,t_i\neq t_j}^{T-1} \mathcal{K}(\frac{t_i-t_j}{L})|r_{t_i}||r_{t_{i+1}}|}{\sum_{i=1,t_i\neq t_j}^{T-1} \mathcal{K}(\frac{t_i-t_j}{L})}} \quad j = 1, \dots T, \tag{3.1}$$

where $L = 12$ and $\mathcal{K}(x) = \sqrt{\frac{1}{2\pi}}e^{\frac{-x^2}{2}}\mathbf{1}_{\{|x|\leq L\}}$.

The threshold separates the discontinuities in the observed performance of the security from the fluctuations caused by the uncertainty of the security itself.

We identify a jump for each index when the absolute value of the return exceeds the quantity calculated eq. (3.1).

The average covariances and correlations among the indices are reported in Table 3.2. These quantities are calculated by considering only the values of returns less than the threshold calculated in eq. (3.1). Finally, the inverse of the covariance matrix at month t represents the co-precision variable Y_t at month t.

Table 3.2. Average covariances (for returns expressed in percentages and monthly units) and correlation (in bold) among the indices.

Covariances/**Correlations**											
	EG0A	G1I0	N1IT	G5I0	N5IT	N0Q1	ERL0	GCXZ	HW00	MSDEE15N	MSDEWXEN
EG0A	0.04	0.08	0.06	0.11	0.10	0.09	0.08	0.09	0.06	-0.07	-0.12
	-	**0.59**	**0.76**	**0.41**	**0.52**	**0.25**	**0.35**	**0.28**	**0.11**	**-0.09**	**-0.14**
G1I0	-	0.39	0.17	0.70	0.33	0.26	0.36	0.40	0.40	0.09	-0.24
	-	-	**0.67**	**0.91**	**0.61**	**0.25**	**0.51**	**0.41**	**0.26**	**0.03**	**-0.09**
N1IT	-	-	0.16	0.28	0.30	0.30	0.26	0.28	0.10	-0.33	-0.33
	-	-	-	**0.55**	**0.85**	**0.44**	**0.57**	**0.44**	**0.09**	**-0.20**	**-0.20**
G5I0	-	-	-	1.54	0.69	0.58	0.85	0.90	0.78	0.48	-0.07
	-	-	-	-	**0.65**	**0.28**	**0.60**	**0.46**	**0.25**	**0.09**	**-0.01**
N5IT	-	-	-	-	0.75	0.78	0.69	0.76	0.17	-0.43	-0.41
	-	-	-	-	-	**0.54**	**0.70**	**0.55**	**0.08**	**-0.12**	**-0.11**
N0Q1	-	-	-	-	-	2.79	0.87	1.87	0.87	-1.03	-1.42
	-	-	-	-	-	-	**0.46**	**0.70**	**0.21**	**-0.15**	**-0.21**
ERL0	-	-	-	-	-	-	1.29	1.44	1.50	1.31	1.30
	-	-	-	-	-	-	-	**0.80**	**0.52**	**0.28**	**0.27**
GCXZ	-	-	-	-	-	-	-	2.52	2.51	1.49	1.19
	-	-	-	-	-	-	-	-	**0.62**	**0.23**	**0.18**
HW00	-	-	-	-	-	-	-	-	6.40	5.45	4.78
	-	-	-	-	-	-	-	-	-	**0.53**	**0.45**
MSDEE15N	-	-	-	-	-	-	-	-	-	16.75	13.64
	-	-	-	-	-	-	-	-	-	-	**0.80**
MSDEWXEN	-	-	-	-	-	-	-	-	-	-	17.29

In Figure 3.1, the overall graph shows the monthly returns of the 11 asset classes of the companies analyzed. The returns are represented by the blue dots and the gray dashed line represents the calibrated threshold used to separate the highlight jumps (red dots). The solid orange line represents the estimated standard deviation.

In general, negative jumps correspond to the most turbulent periods from an economic point of view such as the 2008 crisis (bankruptcy of Lehman Brothers), the 2020 pandemic, and the outbreak of war between Russia and Ukraine in 2022. These geopolitical events have had a more severe impact on the financial market. Furthermore, for corporate indices, it is possible to notice a jump between 2015 and 2016 when quantitative easing was extended to corporate. In these periods, structural changes and extreme movements in the price dynamics of financial assets have occurred. These extreme changes and movements have consequences for investors.

Table 3.3. Parameter estimates the model of price jumps ($\alpha = 48.3709$) considering the intensity of the jump equal to $\lambda = 0.0123$. The estimates refer to returns expressed in percentages and monthly units.

Only jumps in price											
	EG0A	G1I0	N1IT	G5I0	N5IT	N0Q1	ERL0	GCXZ	HW00	MSDEE15N	MSDEWXEN
μ	0.74	0.60	0.59	0.53	0.60	0.40	0.70	0.57	0.61	0.69	0.61
$diag(Q)$	0.09	0.10	0.03	0.11	0.10	0.14	0.15	0.05	0.22	0.07	0.06
$diag(K)$	-0.12	-0.15	-0.17	-0.13	-0.05	-0.03	-0.16	-0.13	-0.14	-0.16	-0.13
J	0.24	0.88	0.45	-5.49	-0.51	0.48	-5.59	-2.23	-7.89	-2.05	1.05
No-jumps											
	EG0A	G1I0	N1IT	G5I0	N5IT	N0Q1	ERL0	GCXZ	HW00	MSDEE15N	MSDEWXEN
μ	0.54	0.61	0.57	0.56	0.59	0.51	0.6	0.57	0.57	0.54	0.54
$diag(Q)$	0.09	0.10	0.03	0.11	0.10	0.14	0.15	0.05	0.22	0.07	0.06
$diag(K)$	-0.12	-0.15	-0.17	-0.13	-0.05	-0.03	-0.16	-0.13	-0.14	-0.16	-0.13
J	-	-	-	-	-	-	-	-	-	-	-

To the sake of completeness, we report in Table 3.3 the estimates of the model parameters of the monthly percentage returns defined based on the presence of the jump component.

The intensity of the λ jump is equal to 0.0123, meaning that on average the jumps occur every 6.77 years. The risk aversion parameter used is exogenously fixed equal to 3, considering an investment horizon of 1 year. Furthermore, we assume that Y_t equals the average co-precision in our sample.

Table 3.4. Optimal allocation percentage of the portfolio π^* considering 3 cases: No-Jumps (second column), Jumps in Price (third column) and the increasing Jumps component (fourth column).

Indexes	No-Jumps (%)	Jumps in Price Only (%)	Increasing Jumps in Price Only (%)
EG0A	25.8015	17.2892	13.9757
G1I0	1.3939	1.4405	1.2372
N1IT	7.3629	7.1726	6.1764
G5I0	1.1589	1.1554	1.0012
N5IT	3.7897	3.7136	3.1815
N0Q1	-1.5790	-1.8697	-1.6765
ERL0	11.3745	8.5772	6.9509
GCXZ	1.8713	1.8313	1.5773
HW00	1.0694	0.9064	0.7614
MSDEE15N	-0.3678	-0.2689	-0.2215
MSDEWXEN	-0.2082	-0.1867	-0.1599
Total Allocation (%)	51.6673	39.7611	32.8037

The results of the optimal weights for portfolio allocation are reported in table 3.4. We observe that by considering all sources of risk in the model, the overall optimal allocation to risky assets is reduced. Considering only the price jumps, the change in the overall allocation is 0.23%.

From Table 3.4 we further notice that the portfolio allocation is mainly concentrated on the bond part of the portfolio. Compared to past years, the risk premium between the stock and bond markets has reduced due to recent geopolitical and economic events. As a result, investors prefer to risk less, to immediately obtain a more attractive return. Hence, the market is adapting to the geopolitical and economic situation of recent years and in particular to the increase in volatility. We must therefore base our estimates on structural changes and look beyond short-term movements. To reduce interest rates by central banks, holding medium/long duration products can be crucial. This is also confirmed by the negative correlations between bond and stock indices, see Table 3.2.

Within bonds, Government Bonds are preferred over corporate Bonds, it is highlighted the value of the optimal weights in Table 3.4, since thanks to the increase in yields the attractiveness of Government Bonds increases in the short/medium term. Therefore, this asset class is preferred to investment grade (IG) credit, where spreads offer lower rewards for higher risk. This is caused by lower debt levels of companies.

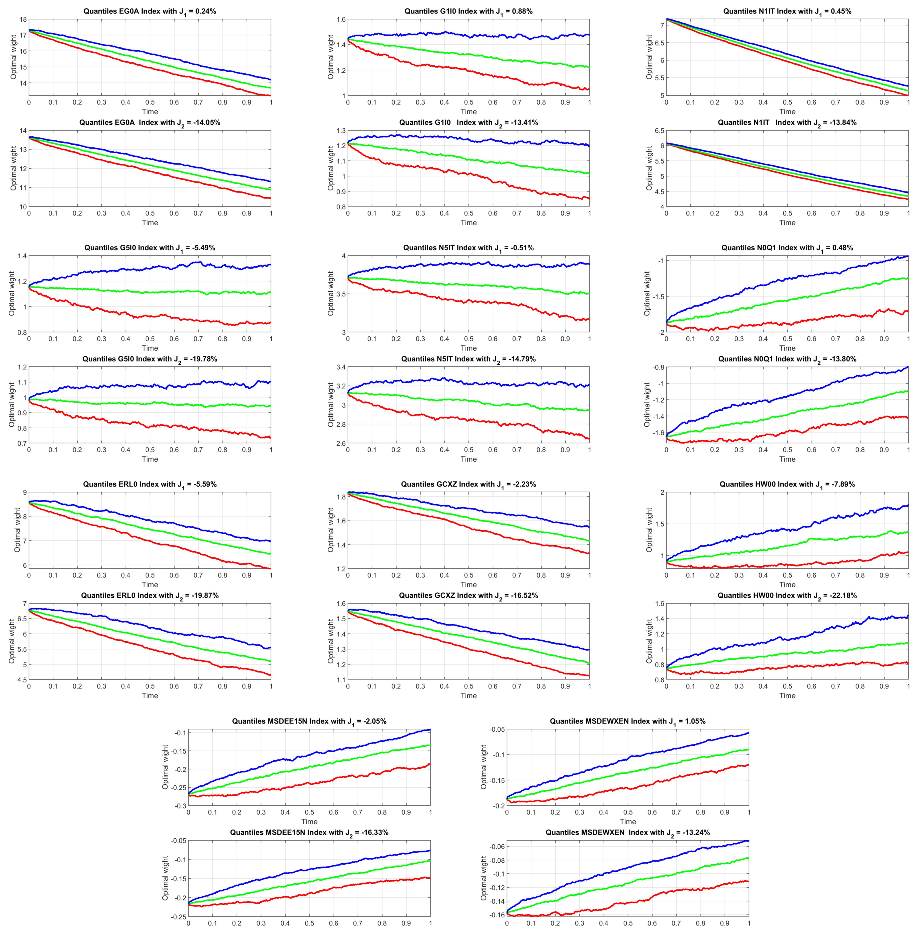
The last column of Table 3.4 shows the optimal weights of a portfolio allocation when the value of the jump component suffers an additive shock equal $2, 5\%$. The overall allocation, in this scenario, is reduced by 0.37% compared to the no-jumps scenario. However, we witness a weak increase in the equity component.

The ex-Europe stock index has a higher weight than the European one. It could most be explained in terms of the corresponding profit growth, especially thanks to the advent of Artificial Intelligence, see e.g. BlackRock (2024). In general, from a medium/long-term strategic perspective, the equity component is preferred over the bond component.

In Figure 3.2 we show that an additive variation of the jump component can influence the performance of activity classes, illustrating three different types of scenarios. In particular, the 95% quantile (red line), the median (green line), and the 5% quantile (blue line) corresponding to the best, median, and worst scenarios respectively are represented.

The presence of jumps on some asset classes leads to having a greater (or lesser) weight based on the correlation, see Table 3.2. , in the case of stocks, the MSDEE15N index and the MSDEWXEN index have a greater weight than other asset classes. As the size of the jump increases, the portfo-

Figure 3.2. Different performance in asset classes following an additive shock of the jump component considering median and extreme scenarios for optimal weights as a function of time (expressed in years).



lio allocation in the risk-free component increases, so that it becomes crucial that companies hold products capable of gradually reaching the riskier component. To increase the risk of the portfolio to have a higher return, we can look at High Yield while remaining prudent given its positive weight in the three scenarios presented in Table 3.4.

In fact, already during and after the Covid pandemic in 2020, many companies have tried to create products that allow this gradual transition, such as. multi-sector insurance policies.

When market crashes occur, especially in the short term, this can lead to a slow adjustment of returns leading to the loss of all market opportunities.

As bond yields have higher levels, the fixed-income asset class has gained relative attractiveness compared to stocks. The benefits of diversification are expected to return when central banks lower interest rates.

4 Conclusion

In this article, we propose an approximate solution to a portfolio problem in a multivariate context considering risky and risk-free assets and considering the presence of price jumps. In our model we consider co-precision, whose dynamics follow a Wishart process. From the theoretical results, we show that the optimal weights depend on two terms: a myopic component and an intertemporal coverage demand. Generalizing the Markowitz intuition, the two components are inversely proportional to the instantaneous volatility. However, we apply the described methodology to a portfolio selection problem in a multivariate context by mimicking the behavior of large Italian companies. Considering a realistic market calibration, we show that investors modify their portfolios in the short term after a market crash by decreasing the allocation to risky assets. The presence of jumps, regardless of their size, confirms a reduction in the allocation so their non-inclusion could lead to considering a riskier allocation and therefore a risky attitude of the investor.

The advantage of this model is the dynamic portfolio allocation where the optimal weights are inversely proportional to the volatility. You might consider a rolling windows-based analysis to capture information on different scenarios (corresponding to the timing of the investment) as best as possible.

This article represents a first step. A possible extension could be considered a smaller period with the non-constant jump component. In particular, it could be interesting to use an analysis based on stress tests to evaluate both the impact of different scenarios on the portfolio allocation and the policies to be implemented to ensure the achievement of the investor's objective by evaluating the trade-off between risk and return. A robust analysis could be done to estimate the jump component to capture the effects of possible crisis periods, along the lines of see Cesarone et al. (2023b). Furthermore, it could also be interesting to carry out an out-of-sample analysis as in Cesarone et al (2019), Cesarone et al (2023a), Cesarone et al (2023c) to test the model and evaluate and calculate the optimal weights. These additions are left for future research.

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