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The gender gap in life expectancy and lifespan disparity as social risk indicators for international countries: A fuzzy clustering approach

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A B S T R A C T

The careful monitoring of longevity risk has a pivotal role in many fields such as the economy, demography, and social sciences. Despite the life expectancy at birth has increased steadily over the last century, it conceals gender-specific differences worldwide. Indeed, longevity risk varies according to gender-specific risk, where, a single-risk-factor effect differs from different lifestyles. This can create problems for life insurers, pension plans and social security schemes that provide benefits linked to people's illness, death or survival. This study considers gender differences in longevity heterogeneity in 32 international countries with the aim of identifying homogeneous groups of countries. To do this, we conduct a longitudinal analysis over the period 1990–2015 using the Dynamic Time Warping-based Fuzzy C-Medoids, and two cross-sectional analyses, one for the first year of the series (1990) and one for the last (2015), using the Fuzzy C-Medoids. We analyze longevity patterns with the double lens of lifespan disparity and life expectancy gender gap. The analyses allow us to highlight the characteristics of the countries considered and their trends according to their longevity dynamics, taking into account gender-related risk differences.

1. Introduction

Social changes are crucially affected by longevity levels and evolution. Let consider, for instance, how the COVID-19 pandemic has influenced longevity levels and has had a considerable effect on health and survival, which threatens to reverse the progress made towards increased longevity over the past 30 years [\[1,](#page-8-0)[2](#page-8-1)]. This led to changes in social and individual habits (for instance, see: [[3–](#page-8-2)[5](#page-8-3)]), also related to the effects of the COVID-19 pandemic on longevity. The careful monitoring of longevity risk plays a central role in many fields of the social sciences (economics, demography, etc.). Since the beginning of the nineteenth century, life expectancy at birth has shown continuous growth in the developed countries, at a steady pace since the end of the Second World War. Researchers, governments, and involved organizations, often recorded a more rapid growth rate than expected according to official forecasts [[6\]](#page-8-4). This has created financial challenges for life insurers, pension plans, and social security schemes involving benefits related to illness, death, or survival of people. Longevity risk is considered a key challenge for these entities. It should also be kept in mind that the overall longevity risk varies according to the gender-specific risk, where the effect of a single risk factor differs from the underlying lifestyle factor in various ways. In fact, gender-related

mortality risks are linked to variations in the population's exposure to certain factors or diseases that are linked to the time of birth or to the development of good or bad habits and lifestyles. The smoking habit is an example, of which women and men show different times in the onset and duration of the habit. Although gender differences appear evident for some aspects and factors, this is not necessarily the case for others. For example, cardiovascular-related diseases are associated with various co-morbidities, which in turn are linked to lifestyle behaviors such as poor diet and exercise as preventable risk factors. Consequently, gender-related mortality risk is the result of a potentially complex combination of single-risk-factors. For these reasons, health indicators such as life expectancy at birth are crucial in measuring the quality of life [[7–](#page-8-5) [9](#page-8-6)] and in studying gender differences in longevity. Although the topic of gender disparities has been the subject of scientific investigation in the literature for a considerable time, its specific adoption in the field of longevity is rather recent $[10-12]$ $[10-12]$ $[10-12]$ $[10-12]$. Its appeal is based on its ability to encapsulate and summarize all factors influencing longevity. For instance, although developed countries have recently experienced stability in mortality and long periods of increased life expectancy, divergent trends and gaps have emerged precisely related to gender differences. It is widely held belief that biological differences suggest

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favoring female life expectancy. Nevertheless, several studies attributed the gap in life expectancy to exogenous factors [\[13](#page-8-9),[14\]](#page-8-10), with higher mortality risk among men due to external mortality and lifestyle-related risks in particular in working ages (see, e.g., [[15\]](#page-8-11)). Thereby, the nature of the gender gap in longevity could be explained by both biological and environmental, social and lifestyle factors, which contribute to its historical and future dynamics. Indeed, females live longer than males in industrialized societies and they also outlive males in most developing countries [\[10](#page-8-7)[,16](#page-8-12)[–18](#page-8-13)]. Nevertheless, since the 1970s, the life expectancies of males and females have converged in most developed countries, where the gender gap in mortality has begun to narrow compared to the previous period. Exogenous factors that affect health and, consequently, gender differences can represent the result of macrolevel changes, attributable to common factors for groups of countries that share similar features, such as socioeconomic factors, and shared improvements in public health. This consideration encourages analyzing gender gaps in a group of countries jointly, identifying groups of countries that have common characteristics as measured by the gender life expectancy ratio.

Notwithstanding the relevance of life expectancy, a sustainable public health policy can benefit from interventions that also reflect individuals' uncertainty about the time of death, i.e. the lifespan disparity [\[19](#page-9-2)[–22](#page-9-3)]. This longevity indicator measures the unevenness of mortality improvements at the population level. It is important for life course decisions [[23\]](#page-9-4) because the greater inequality in life expectancy, the greater vulnerability at the societal level and the consequent ineffectiveness of policies to protect individuals from life course risks. Hence, analyzing both life expectancy and lifespan disparity on a global scale provides a more comprehensive overview of the gender gap evolution. This aspect, still rather neglected in the literature on longevity risk due to the gender gap, could be crucial in the economy and the public system as well as in the determination of sustainability goals. Therefore, we strongly believe that it is urgent to delve into the subject as, so far, there are no scientific contributions that provide monitoring of the gender gap in life span disparity, even more so from a global perspective. Moving from these considerations, we analyze the series' patterns of lifespan disparity and gender gaps in life expectancy, identifying the characteristics and the different stages and transitions that allow countries to be grouped according to their longevity dynamics.

Numerous works in the literature have contributed to the understanding of mortality clustering [\[24](#page-9-5)[–28](#page-9-6)]. Danesi et al. [\[24](#page-9-5)] employed a conventional clustering algorithm utilizing a dissimilarity index derived from life expectancy at birth to group countries before constructing a multi-population model on mortality rates. Conversely, Levantesi et al. [[26,26](#page-9-7)] and Cefalo et al. [\[28](#page-9-6)] utilized Functional Data Clustering (FDC) to study mortality dynamics. In this study, we analyze longevity patterns with the double lens of lifespan disparity and life expectancy gender gap, by performing two different but complementary analyses. A cross sectional analysis considers the first (1990) and the last (2015) available year is conducted by using the Fuzzy C-medoids. This analysis allows us to understand what the initial and final situation is in the period under consideration, taking two 'pictures' that highlight possible differences in the groups of countries, improvements, and/or worsening in the indicators considered. A longitudinal analysis is performed in the period 1990–2015 by employing Dynamic Time Warping-based Fuzzy C-Medoids (for details on the methodological aspects, please see Section [2.3\)](#page-2-0). In this way, it is possible to examine the trends of the different countries over the period considered, highlighting possible groups of them presenting similar trends in the indicators considered. It is, therefore, clear that the information obtained from the two analyses provides a more comprehensive reading of the phenomenon, highlighting the levels (initial and final) and trends over time of the different countries and allowing us to classify and analyze them according to the information obtained through these complementary analyses. To the best of our knowledge, no other work in the literature has performed

such a twofold analysis using a fuzzy clustering approach. Considering the existent literature on this topic, our contribution is manifold. Indeed, exploiting the fuzzy approach, it fills the gap between fuzzy clustering and longevity risk analysis. Secondly, our study provides for the first time a simultaneous multi-population clustering for both, life expectancy and lifespan inequality. In addition, our study presents a cross-sectional and longitudinal analysis, giving different and complementary information on the phenomenon. In this way, we think that this paper can provide useful insights for governments, academics, and policymakers.

Our research builds upon and extends the existing body of knowledge in several significant ways. Our study incorporates novel methodologies, includes a larger view leveraging a multi-population framework, and explores previously unexplored scenarios contributing to the gender gap in longevity. The paper is organized as follows. Section [2](#page-1-0) presents the indicators and the methods used. In Section [3](#page-3-0) the application and the main findings are shown. Section [4](#page-7-0) summarizes and discusses the obtained results and concludes.

2. Data description and methods

2.1. Data

Our analyses are based on historical data provided by the Human Mortality Database (HMD) that contains uniform death rates and life tables for 41 countries, both genders, for different ages and periods. The method adopted to calculate life tables includes the following process:

- Annual counts of live births by sex and by month are collected for each population over the longest time period available and used for estimating the size of individual cohorts.
- Death counts are collected by sex, completed age, year of birth, and year of death if available. Deaths of unknown age may be distributed proportionately across the age range, and aggregated deaths are split into finer age categories $(D_{age, year})$.
- Regarding population size, below age 80, its estimates on January 1st of each year are obtained from official estimates or derived using intercensal survival.
- Estimates of the population exposed to the risk $(E_{age, year})$ of death during some age–time interval are based on annual (January 1st) population estimates, with a small correction that reflects the timing of deaths during the interval and variation in the cohort's birthdays by month.
- For both periods and cohorts, death rates are simply the ratio of deaths to exposure-to-risk in matched intervals of age and time: $(m_{age, year} = \frac{\hat{D}_{age, year}}{F})$ $\frac{E_{age, year}}{E_{age, year}}$).

The choice of countries included in our analysis was determined by the availability of data. It is important to acknowledge that certain populations within the HMD have limited information for certain time periods, particularly notable in the ex-Soviet countries. To ensure consistency and comparability across countries and over time, we carefully selected the period spanning from 1990 to 2015 for our study. This timeframe provided us with a substantial and meaningful duration to explore trends and patterns in relation to our research objectives. By focusing on this specific time range, we aimed to maintain homogeneity in data coverage and facilitate a comprehensive analysis of the gender gap in longevity. Finally, by utilizing the comprehensive and standardized data provided by the HMD, we can ensure the reliability and consistency of our analyses, allowing for a robust examination of the gender gap in longevity across multiple countries and time periods.

2.2. Demographic measures

Period life expectancy at birth is the most widely used indicator of population health and longevity. It refers to the expected average age at death for a synthetic cohort of newborns, that experience the mortality risks of that time throughout their lifespan. We define the life expectancy at age x and time t in a given population as follows:

$$
e_{x,t} = \frac{\int_x^{\infty} S(y,t) dy}{S(x,t)}
$$
(1)

where $S(x,t) = \exp\left(-\int_0^x \mu(a,t)da\right)$ and $\mu(a,t)$ are the survival function and the force of mortality respectively.

Thus, we can introduce the lifespan disparity as an indicator representing the life expectancy lost due to death by an individual aged x at time t .

Formally the lifespan disparity at birth is defined as follows:

$$
e_{0,t}^{\dagger} = -\int_0^{\infty} S(a,t) \cdot \ln S(a,t) da
$$
 (2)

We consider the gender gap in lifespan disparity $(G^{(e_0^{\dagger})} = e_{0,t,Female}^{\dagger}/F_{0,t}^{\dagger})$ $e_{0,t,Male}^{\dagger}$) and in life expectancy ($G^{(e_0)} = e_{0,t,Female}/e_{0,t,Male}$) at birth. The 2 measures used in this work, $G^{(e_0^{\dagger})}$ and $G^{(e_0)}$, are female-to-male ratios, in order to capture gaps between women and men's attainment levels, rather than the levels themselves [[29\]](#page-9-8). This is a choice adopted in a well-established literature (for instance, see: [[30\]](#page-9-9)). We must clarify that, for both the indicators, observed levels equal to 1 indicate an absence of a gender gap. On the contrary, values different (higher or lower) from 1 represent clear evidence of diverging behaviors in longevity between male and female populations. However, the interpretation of the values is opposite for the two indicators. It is worth noting that life expectancy has a growing dynamic; thus, high values correspond to improvements in longevity. On the contrary, the measure of lifespan disparity is characterized by a monotonous decreasing trend, where lower values represent less dispersion and therefore, improvements in longevity levels. Accordingly, for the gender gap in life expectancy $(G^{(e_0)})$ values lower than 1 show a disadvantage for women and values greater than 1 a disadvantage for men; for the gender gap in lifespan disparity, the opposite considerations apply. Moreover, it should be considered that it can be difficult to assess the trend of female-to-male ratios: in fact, it is not immediate to understand (unless the components of the ratio are taken into account) whether an improvement (worsening) in one indicator is related to an improvement (worsening) in the situation of women or a worsening (improvement) in the situation of men.

2.3. Methods

In this work, we deal with a three-way time data array X of the type "units \times variables \times times" [\[31](#page-9-10)[–33](#page-9-11)]. Specifically, we consider 32 countries and the two gender gap variables, $G^{(e_0^{\dagger})}$ and $G^{(e_0)}$. Data are available from 1990 to 2015. Formally: $X = \{x_{ijt} : i = 1, ..., 33; j = 1\}$ $G^{(e_0^{\dagger})}, G^{(e_0)}$; $t = 1990, ..., 2015$.

We perform a cross-sectional and a longitudinal analysis of the two considered gender gap variables, $G^{(e_0^{\dagger})}$ and $G^{(e_0)}$, in the 32 countries in order to identify:

- the presence of different groups of countries in the starting (1990) and the last (2015) year;
- different phases and transitions that allow clustering countries according to their gender gap dynamics in the period 1990–2015.

By using the clustering information, we are able to gain preliminary insights into the gender gap over time and countries.

The gender gap in life expectancy and lifespan disparity is a complex phenomenon, the analysis of which must take into account the various factors and dimensions that define and influence it [[34\]](#page-9-12). In order to

correctly deal with the complexity of the phenomenon investigated, we decided to adopt a fuzzy approach. Fuzzy clustering is an overlapping approach [[35\]](#page-9-13) based on the Fuzzy Set Theory [[36\]](#page-9-14). Approaches of cluster analysis differ in how the different clusters relate to each other. In the so-called crisp methods, the aim is to find partitions mutually exclusive, in which $c_i \cap c_j = 0$ for $i \neq j$. In other words, each unit is exactly assigned to only one cluster obtaining exhaustive partitions characterized by nonempty and pairwise disjoint subsets. However, this procedure can be inadequate, for example, when there is the presence of units that are equally distant from multiple clusters. A crisp partition arbitrarily forces the full assignment of such units to one of the clusters, although they should equally belong to all of them. Overlapping clustering techniques are those that violate the condition of mutual exclusivity; in other words, these techniques allow units to belong to more clusters simultaneously depending on a certain membership degree [\[35](#page-9-13)]. Fuzzy logic is the natural way to address the uncertainty of phenomena. It is based on the evidence that the real world is so complex that it cannot be treated by means of clear rigid propositions. The fuzzy approach has a number of advantages [[37,](#page-9-15)[38\]](#page-9-16) and is particularly suitable for the analysis of socio-economic phenomena, as demonstrated by various works in different fields (for instance, see: [[7](#page-8-5), [38–](#page-9-16)[41\]](#page-9-17)). In many clustering problems one is particularly interested in a characterization of the clusters by means of typical or representative units. These units can be used for further work or research, especially when it is more convenient to use a small set of $k < n$ units. The Partitioning Around Medoids (PAM) clustering technique [[42\]](#page-9-18) makes use of prototypes belonging to the considered dataset (medoid). By adopting the PAM approach, the prototypes of each cluster are units actually observed and not ''virtual'' units. Overall, having non-fictitious representative units available makes interpreting the obtained clusters easier [[38\]](#page-9-16). In fact, ''in many clustering problems one is particularly interested in a characterization of the clusters by means of typical or representative objects. These are objects that represent the various structural aspects of the set of objects being investigated. There can be many reasons for searching for representative objects. Not only can these objects provide a characterization of the clusters, but they can often be used for further work or research'' [\[42](#page-9-18)].

For the longitudinal analysis, we used the Dynamic Time Warpingbased Fuzzy C -Medoids clustering model (DTW-FCMd) (for details, see: [[43\]](#page-9-19)), based on the Dynamic Time Warping distance [[44\]](#page-9-20), formalized as follows:

$$
\begin{cases}\n\min: \quad \sum_{i=1}^{I} \sum_{c=1}^{C} u_{ic}^{m} d_{DTW}^{2}(\mathbf{X}_{i}, \widetilde{\mathbf{X}}_{c}) \\
s.t. \quad \sum_{c=1}^{C} u_{ic} = 1, \ u_{ic} \ge 0\n\end{cases}
$$
\n(3)

where X_i is the *i*th multivariate time series; \widetilde{X}_c is the *c*th medoid (an observed multivariate time series); $d_{DTW}^2(\mathbf{X}_i, \widetilde{\mathbf{X}}_c)$ is the Dynamic Time Warping distance between the *i*th multivariate time series and the medoid of the cth cluster; $m > 1$ is a parameter that controls the fuzziness of the partition (in this case, we set $m = 1.5$).

The three-way time data array X can be represented with a bidimensional matrix by combining 2 of the 3 indices i, j, t on the rows and assigning the remaining index to columns [\[43](#page-9-19)]. In the so-called "time -slices case", we combine i and j indices in rows and assign t to the column, obtaining $X_i = \{x_{ijt} : i = 1, ..., n; j = 1, ..., p\}$. For the cross-sectional analysis, we use the Fuzzy C -medoids (FCMdd); let $\mathbf{X}_t \equiv \{\mathbf{x}_{1t}, ..., \mathbf{x}_{it}, ..., \mathbf{x}_{nt}\}$ a set of objects and $\tilde{\mathbf{X}}_t \{\tilde{\mathbf{x}}_{1t}, ..., \tilde{\mathbf{x}}_{ct}, ..., \tilde{\mathbf{x}}_{kt}\}$ a sub-set of **X** with cardinality k , the FCMdd $[45]$ $[45]$ can be formalized as follows:

$$
\begin{cases}\n\min: & \sum_{i=1}^{n} \sum_{c=1}^{k} u_{ic}^{m} \|\mathbf{x}_{it} - \tilde{\mathbf{x}}_{ct}\|^{2} \\
s.t. & \sum_{c=1}^{k} u_{ic} = 1, u_{ic} \ge 0\n\end{cases}
$$
\n(4)

where u_{ic} is the membership degree of the *i*th unit to the *c*th cluster; $\tilde{\mathbf{x}}_{ct}$ represents the cth medoid; $||\mathbf{x}_{it} - \tilde{\mathbf{x}}_{ct}||^2$ is the squared Euclidean

 ϵ

 \overline{AB}

Fig. 1. Gender gap in life expectancy and in lifespan disparity values in 32 countries: year 1990.

distance between the *i*th unit and the medoid of the *c*th cluster; $m > 1$ is a parameter that controls the fuzziness of the partition (in this case, we set $m = 1.5$). We consider two precise years, $t = 1990$ and $t = 2015$; then, we apply Eq. [\(4\)](#page-2-1) to $X_{t=1990}$ and $X_{t=2015}$.

2.3.1. Choice of the optimal number of clusters

In the literature, different indices for the choice of the optimal partition have been proposed over the years (for a review, please see: [[46](#page-9-22)]). Different works have compared the different indexes under different scenarios (for instance, please see: [[47\]](#page-9-23)), showing that there is no ''best index'' and different indexes often identify different optimal partitions. In this paper, we use the Xie-Beni and the Fuzzy Silhouette, two of the most widely used criteria, as highlighted by many works in the literature. In most cases, only one validation criterion is used. However, since the two indices identify the same partitions, this corroborates the validity of the results obtained.

• The Xie-Beni (XB) criterion [[48\]](#page-9-24); for the longitudinal analysis formalized as follows:

$$
XB = \frac{\sum_{i=1}^{I} \sum_{c=1}^{C} u_{ic}^{m} d_{DTW}^{2}(\mathbf{X}_{i}, \widetilde{\mathbf{X}}_{c})}{n \min_{c \neq c'} \sum_{i=1}^{n} \sum_{c=1}^{k} u_{ic}^{m} d_{DTW}^{2}(\mathbf{X}_{i}, \widetilde{\mathbf{X}}_{c'})}
$$
(5)

and for the cross-sectional analysis as follows:

$$
XB = \frac{\sum_{i=1}^{n} \sum_{c=1}^{k} u_{ic}^{m} d(\mathbf{x}_{it} - \tilde{\mathbf{x}}_{ct})}{n \min_{c \neq c'} \sum_{i=1}^{n} \sum_{c=1}^{k} u_{ic}^{m} d(\tilde{\mathbf{x}}_{ct} - \tilde{\mathbf{x}}_{c't})} \quad \text{for} \quad t = 1990,2015 \quad (6)
$$

The numerator of XB is the total within-cluster distance, i.e., the objective function of the clustering model considered, and represents the overall compactness of the clusters. The minimum distance between centroids at the denominator of XB is called separation. The greater this distance, the more the separation of the partition. The optimal number of clusters c^* is identified in correspondence with the lower value of the index.

• The Fuzzy Silhouette (FS) index [\[49](#page-9-25)], formalized as follows:

$$
FS = \frac{\sum_{i=1}^{I} (u_{pi} - u_{qi})^{\alpha} \cdot \lambda_i}{\sum_{i=1}^{I} (u_{pi} - u_{qi})^{\alpha}}, \qquad \lambda_i = \frac{(b_{pi} - a_{pi})}{\max\{b_{pi}, a_{pi}\}}
$$
(7)

where a_{pi} is the average distance of object *i* to all other objects belonging to the same cluster p ($p = 1, ..., C$) and b_{pi} is the minimum (over clusters) average distance of the *i*th unit to all units belonging to the cluster q with $q \neq p$. $(u_{pi} - u_{qi})^{\alpha}$ is the weight of each λ_i , where u_{pi} and u_{qi} correspond to the first and second largest element of the *i*th column of the fuzzy partition matrix U, respectively; $\alpha \geq 0$ is an optional user-defined weighting coefficient. Setting $\alpha = 0$, it reduces to the crisp Silhouette measure. A higher value of FS means a better assignment of the units to the clusters which implies that, simultaneously, the intracluster distance is minimized while the inter-cluster distance is maximized.

Table 1

Cluster validation: Xie-Beni index and Fuzzy Silhouette index for different value of k; year 1990; year 2015; years 1990–2015.

3. Application and results

We analyze the results of each analysis in individual sub-sections First, we present the data. Subsequently, we analyze the characteristics of the clusters medoids of the optimal partition. Finally, the characteristics of any fuzzy countries are analyzed and compared with those of medoids.

In [Table](#page-3-1) [1](#page-3-1), we report the results of the application of XB and FS, for $2 \le k \le 5$. According to the values of the 2 criteria, for all the analyses we chose the solution with two clusters, $k^* = 2$.

For the evaluation of the fuzziness of the clusters, we need to specify a cut-off point for the membership degree. According to Maharaj and D'Urso [[50\]](#page-9-26), if we have a two-clusters situation and the membership degrees in both clusters are between 0.3 and 0.7, it would be considered that there is a reasonable level of fuzziness in the cluster membership. Consequently, the value 0.7 has been chosen as cut-off. Therefore, those countries that do not have at least that value as membership degree to a cluster are considered fuzzy.^{[2](#page-3-2)}

3.1. Cross-sectional analysis

[Table](#page-4-0) [2](#page-4-0) reports the summary statistics in 1990 of the 2 measures used and [Fig.](#page-3-3) [1](#page-3-3) presents the values of different countries.

Looking at $G^{(e_0)}$, we can observe that in all considered countries, females have a higher life expectancy than males. Indeed, the minimum value is equal to 1.067 (this is the value of GRC, in which the male life expectancy is 74.66, while the female one is equal to 79.63). RUS presents the highest value, 1.165. This data should be interpreted carefully, because, as explained in the previous pages, the ratios tend to capture the gaps between women and men's attainment levels and not the levels themselves. For instance, the Russian value indicates that the gap between the life expectancy of females and males is in favor of females and more marked than in other nations, but ''it tells us nothing'' about the values. In fact, a more detailed analysis shows that RUS has the lowest value of male life expectancy in 1990, 63.76 (the highest is 75.95 for JPN), and the second lowest value, 74.31 for the female one (JPN presents the best value 81.87). The $G^{(e_0^{\dagger})}$ interpretation is

² For more information on the choice of cut-off, see [\[50](#page-9-26)].

Table 2

Table 3

Fig. 2. Gender gap in life expectancy and in lifespan disparity values in 32 countries: year 2015.

opposite to $G^{(e_0)}$: values less than 1 indicate better lifespan disparity values for females than for males. All the countries has values lower than 1, except FIN that highlights an almost gender equality (1.007). SVN has the minimum value, 0.676; this indicates the greater gap in favor of females.

Analyzing the summary statistics shown in [Table](#page-4-1) [3](#page-4-1) and the values of different countries in [Fig.](#page-4-2) [2](#page-4-2), similar considerations can be made for 2015.

All countries have values in $G^{(e_0)}$ ranging from 1.043 (NDL) and 1.172 (RUS); consequently, the gap between the life expectancy of males and females is in favor of the latter. It is interesting to note that RUS again presents the lowest value, 65.26, in male life expectancy and JPN has the highest ones both for males (80.78) and females (87.02). Focusing on $G^{(\bar{e_0^\dagger})}$, we observe that all countries have values lower than 1 (LVA has the lowest, 0.566), except FIN that reports a value of 1.119.

Comparing the 1990 and 2015 data, we observe that the average value of $G^{(e_0)}$ decreases from 1.106 to 1.080, while that of $G^{(e_0^{\dagger})}$ increases from 0.782 to 0.824. This evidence testifies to a trend towards gender equality in both life expectancy and lifespan disparity, that is, an alignment between the values of females and males in the two indicators in different countries. As previously specified, this could be caused by several situations in one country to another. For instance, considering the improvement in $G^{(e_0)}$, this could be related to an improvement in male life expectancy, a worsening in female life expectancy, or both at the same time. Moreover, we might observe a worsening of both life expectancy that is more pronounced in females or a more pronounced improvement in males. Thus, to correctly understand the nature of this alignment, it is necessary to consider the source data from which the ratios are constructed; however, such an analysis is not the purpose of this paper.

3.1.1. Cluster analysis: year 1990

In this Section, we present the results of the analysis on the $G^{(e_0)}$ and $G^{(e_0^{\dagger})}$ indicators for the year 1990 carried out by using the FCMdd described in Section [2.3](#page-2-0). The optimal solution identifies 2 Clusters: Cluster 1 (9 countries) with CHE as medoid and Cluster 2 (18 countries)

Table 4

Gender gap in life expectancy $G^{(e_0)}$ and in lifespan disparity $G^{(e_0^{\dagger})}$: medoids, fuzzy countries and average values; year 1990.

	$G(e_0)$	$G^{(e_0^{\dagger})}$
Medoid cluster 1	1.091	0.855
Medoid cluster 2	1.113	0.740
HUN	1.132	0.800
IRL	1.078	0.789
ITA	1.090	0.791
SWE	1.075	0.796
USA	1.097	0.793
Average value	1.106	0.782

represented by FRA. There are 5 fuzzy countries: HUN, IRL, ITA, SWE, and USA. [Table](#page-8-14) [6](#page-8-14) reports the membership degrees. [Fig.](#page-5-0) [3](#page-5-0) shows the subdivision of the countries according to the cluster to which they belong.

The membership of each country to its respective cluster, except for the fuzzy countries, is clear and unambiguous. Cluster 1 includes those countries having better situation in terms of gender equality, both for life expectancy and lifespan disparity. Countries of Cluster 2 present the worst situation in terms of gender equality; it is interesting to note that it includes the countries of the former Soviet bloc (except LTU). HUN has values in line with the average ones (see [Table](#page-4-3) [4](#page-4-3)). The other fuzzy countries (IRL, ITA, SWE and USA) have values in line with the average for $G^{(e_0^{\dagger})}$ and in line with Cluster 1 with reference to $G^{(e_0)}$.

3.1.2. Cluster analysis: year 2015

This Section reports the results of the analysis on the $G^{(e_0)}$ and $G^{(e_0^{\dagger})}$ indicators for the year 2015 performed by means of the FCMdd described in Section [2.3](#page-2-0). Even in this case, the optimal solution identifies 2 Clusters (see [Table](#page-8-14) [6](#page-8-14) for the membership degrees): Cluster 1 (14 countries) with GRC as medoid and Cluster 2 (13 countries), whose medoid is SVN. There are 5 fuzzy countries: BLR, HUN, NLD, POL, and RUS. [Fig.](#page-5-1) [4](#page-5-1) shows the subdivision of the countries according to the cluster to which they belong.

Fig. 3. Clusters composition: 32 countries; year 1990.

Fig. 4. Clusters composition: 32 countries; year 2015.

Table 5

Gender gap in life expectancy $G^{(e_0)}$ and in lifespan disparity $G^{(e_0^{\dagger})}$: medoids, fuzzy countries and average values; year 2015.

	$G(e_0)$	$G^{(e_0^{\dagger})}$
Medoid cluster 1	1.066	0.777
Medoid cluster 2	1.077	0.886
BLR	1.058	0.810
HUN	1.092	0.833
NLD	1.043	0.831
POL	1.107	0.830
RUS	1.172	0.806
Average value	1.080	0.824

The membership of each country to its respective cluster, except for the fuzzy countries, is clear. The 2 Clusters have practically the same value of $G^{(e_0)}$ almost equal to the average value, while they differ in $G^{(e_0^{\dagger})}$. In detail, Cluster 1 comprises 14 countries having worse situation in terms of gender equality in lifespan disparity; on the contrary, countries of Cluster 2 present a better situation. Regarding the fuzzy countries (see [Table](#page-5-2) [5](#page-5-2)), all present values of $G^{(e_0^{\dagger})}$ intermediate between Cluster 1 and Cluster 2; HUN ($G^{(e_0)} = 1.092$), POL ($G^{(e_0)} = 1.107$), and RUS ($G^{(e_0)} = 1.172$) report values of $G^{(e_0)}$ higher than average, reflecting a gender gap in favor of females that is more pronounced in these nations than in others.

3.2. Longitudinal analysis: years 1990–2015

[Fig.](#page-6-0) [5](#page-6-0) shows the trends of the 2 indicators in the period considered for the 32 countries.

Looking at the gender gap in life expectancy $(G^{(e_0)})$, there is evidence of a downward trend in all countries towards value 1. This should be read positively in terms of gender gap, as it marks a general alignment of life expectancy levels between males and females. The graph highlights the presence of 6 nations that have values greater than all the others: interestingly, these are all countries of the former Soviet bloc (BLR, EST, LTU, LVA, RUS, UKR). The trends of different countries in the gender gap in lifespan disparity $(G^{(e_0^{\dagger})})$ are not as evident as those of $G^{(e_0)}$. It is possible to note that most of the values are concentrated in the range (0*.*70*,* 0*.*85) and the presence of some values above 1.

We performed the cluster analysis on the 32 countries, considering the $G^{(e_0)}$ and $G^{(e_0^{\dagger})}$ time series from 1990 to 2015, by applying the DTW-FCMd (see Section [2.3](#page-2-0)). As shown in [Table](#page-3-1) [1](#page-3-1) the optimal solution identifies 2 Clusters: Cluster 1, composed of 20 countries, with DEU as medoid; Cluster 2, consisting of 9 countries, represented by RUS as medoid; there are 3 fuzzy countries (FIN, SVK, and USA). [Ta](#page-8-14)[ble](#page-8-14) [6](#page-8-14) shows the membership degrees and [Fig.](#page-6-1) [6](#page-6-1) illustrates the clusters' composition.

Clusters are clearly characterized, as we can observe in [Fig.](#page-6-2) [7](#page-6-2). Cluster 1 represents a better situation in terms of the gender gap; in fact, the time series is closer to value 1, which represents perfect equality. Conversely, Cluster 2 includes countries with a more pronounced gender gap (always disadvantaging males).

[Fig.](#page-7-1) [8](#page-7-1) compares the medoids and the fuzzy countries. Looking at gender gap in life expectancy, USA has a trend very similar to the

Fig. 5. Gender gap in life expectancy and in lifespan disparity: 32 countries; time series 1990–2015.

Fig. 7. Comparison between clusters' medoids and average values of life expectancy and lifespan disparity.

Fig. 8. Time series of life expectancy and lifespan disparity 1990–2015: comparison of medoids and fuzzy countries FIN, SVK, and USA.

medoid of Cluster 1, while FIN and SVK show trends between the 2 medoids. Regarding the gender gap in lifespan disparity, USA presents values close to those of Medoid 1 over time, but has a more constant trend. FIN and SVK have very fluctuating time series.

4. Conclusive remarks: longevity risk in a gender gap framework, policy implication

The results provided by the cluster analysis are extremely useful when read and compared with demographic knowledge. Indeed, the role of some countries such as USA and RUS is not surprising. On the contrary, it helps to confirm and prove the longevity dynamics highlighted in recent years. Our results may be linked to the Russian anti-alcohol campaign of 1985–1988 that positively affected longevity narrowing the gender disparities in both life expectancy and lifespan disparity. The conclusion of the aforementioned measures resulted in a sharp decline in life expectancy at the beginning of the 1990s, which matches the first year of our analysis. According to Shkolnikov and Cornia [\[51](#page-9-27)] and Shkolnikov et al. [[52\]](#page-9-28), in 1994, life expectancy in RUS fell to the lowest levels ever recorded in the country, with a relevant disadvantage for males. Another interesting case is provided by USA, where the historical rising in life expectancy stalled after 2010, with different behavior for both genders. Demographic literature focuses on rising drug-related deaths, while others bring evidence of stagnation in cardiovascular disease, which holds back the USA's life expectancy [[53\]](#page-9-29). Finally, as investigated by Seligman et al. [[54\]](#page-9-30), we can underline that improvements in mortality always increase life expectancy, but if these improvements occur at older ages, lifespan disparity may increase too. This is due to the heterogeneity linked to the different causes of death composition for each country, that at this stage is out of the scope of our analysis.

The concept of lifespan or longevity risk is intrinsically crucial to consider in evaluating and planning future sustainability policies. As a result, governments and public systems necessarily go through the study of the number of years that individuals potentially spend contributing to society. Evidence that people are spending more time

in retirement due to improved longevity, has forced Governments to re-frame pension plans to ease the effects of longevity improvements, with peculiar differences between genders. Countries such as Denmark, Finland, and the Netherlands conceived reforms that link retirement age to changes in gender-specific life expectancy. Indeed, longevity is a heterogeneous phenomenon that may change dramatically over time and between countries around the globe.

In this study, we analyzed longevity patterns by leveraging lifespan disparity and life expectancy gender gap. Considering the existent literature on this topic, our contribution is manifold. Indeed, exploiting the fuzzy clustering approach, it fills the gap between fuzzy clustering and longevity risk analysis. Secondly, our study provides for the first time a simultaneous multi-population clustering for both, life expectancy and lifespan inequality, thus providing better guidance for Governments and policymakers.

Thereby, evaluating the gender gap in longevity risk with the double lens of life expectancy and lifespan disparity is essential to developing adequate sustainability policies, especially for public health, social security, and welfare where the knowledge of longevity gender differential provides the equilibrium of the system. However, the international look at the gender differentials in life expectancy and lifespan disparity is not frequently discussed in the longevity risks literature. Our proposal may be considered a prominent practice to exploit a global vision by working directly on gender inequality in longevity, analyze the patterns of both lifespan disparity and life expectancy gender gap series, by identifying the characteristics and the different phases and transitions that allow the clustering of countries according to their longevity dynamics.

A limitation of this study is certainly related to the periodicity of the data, which stops in 2015. While it is true that demographic dynamics tend to change rather slowly over time, undoubtedly having more recent data would add value to the analysis. This paper analyzes longevity patterns with the double lens of lifespan disparity and life expectancy gender gap, by performing a cross sectional and longitudinal analysis adopting a fuzzy clustering approach. This is, to the best of our knowledge, the first work in the literature of this kind. Undoubtedly,

Table 6

it would have been interesting to compare the approach and methods used with others. The absence of such a comparison could be considered a limitation of this work; however, the focus was on the analysis of the phenomenon. Potential future developments of this analysis could, therefore, involve the use of updated data, when they become available, and comparison with other clustering methodologies.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix

See [Table](#page-8-14) [6.](#page-8-14)

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