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xEES – Analytical indicator for assessing liabilities in pileups

N. Roveri *, S. Milana, A. Culla, G. Pepe, A. Carcaterra

Department of Mechanics and Aeronautics, University of Rome, 'La Sapienza', Via Eudossiana, 18, 00184, Rome, Italy

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ABSTRACT

While pileups involving only two vehicles showcase obvious liability among the drivers, the assessment of liabilities is much more complex in chain collisions. In this work we propose an analytical indicator, named *xEES*, which easily allows to assess the correct liabilities among drivers. The name is mutated by the concept of energy equivalent speed (EES), which is the vehicle speed equivalent to the energy consumed to cause the vehicle deformation: *xEES* is indeed a dimensionless parameter related to the expected EES at the front of the first vehicle requested for a chain reaction car accident and that is energetically coherent with the damages of the hit vehicles. The proposed model needs only the information concerning the damages of the vehicles and does not require any information concerning the accident scene. The model has been tested on real pileups and validated by the software PC-Crash: the analysis has shown how the use of the coefficient xEES leads the engineer to assess the correct liabilities in pileups. Three intervals of variation are defined for *xEES*, which set apart, with due statistical confidence, chain reaction car accidents from collisions involving a column of moving vehicles.

1. Introduction

In line car crashes that involve more than two vehicles are called pile-up. They consist in a sequence of rear-end impacts caused by the front of one vehicle striking the rear end of another vehicle, where the order of impacts may not coincide with the order of vehicles along the queue.

Many studies of this type of accident focus on the study of the risk factor [1,2] and its classification in different contexts; these can be distinguished for example by the type of mobility: urban and suburban [3,4]. One of the most investigated risk factors is visibility due to the presence of fog and smoke [5] and driver behavior induced by the presence of risk factors [6] or by the driver unfair practice [7]. In this context, several studies aim to assess the reduction of the risk of accidents using sensor networks on the infrastructure that are able to provide information to drivers and cooperative driving in the context of smart urban mobility [8–11]. Other studies are more focused on the mechanics of the incident, determining the energy of the crash by estimating the energy equivalent speed (EES) of vehicles, via finite element method [12] and via experimental and analytical method [13].

The novel contributions of the paper to the advancement of knowledge in this field can be summarised in the points ahead:

- It is focused on the use of the EES, and not on an innovative way for estimating it as in Refs. [12,13], to infer liabilities in real life car crashes.
- It introduces a new analytical indicator (xEES) based on the concept of EES, which easily allows to assess the correct liabilities among drivers.
- Three intervals of variation for xEES are found, so that chain reaction car accidents are set apart, with due statistical confidence, from chain collisions involving a column of moving vehicles, even in presence of uncertainties that affects the parameters of the problem, such as masses, restitution coefficients, EESs and deformation energies, whose estimation are generally affected by uncertainties.

These highlights are important since, while collisions involving only two vehicles showcase obvious liability, the correct reconstruction of the sequence of impacts is much more complex in pile-up. However, from a jurisprudential perspective, the main distinction relies indeed on chain collisions involving a column of moving vehicles, from the case in which a vehicle coming from behind collides with a series of vehicles that are already stationary.

The latter case typically happens when the front bumper of the car at the end of a standing queue crashes into the rear bumper of the vehicle ahead, with enough force to push it forward into the car in front, giving rise to a chain reaction, leading to other vehicles in the vicinity colliding with one another as well. In this type of accidents, the driver that

* Corresponding author.
E-mail address: nicola.roveri@uniroma1.it (N. Roveri).

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List of abbreviations

$\widehat{E}_d$	Expected deformation energy
EES	Equivalent Energy speed
ISO	International standard organization
PDOD	Principal Direction of Deformation
PDOF	Principal Direction of Force
xEES	Expected Equivalent Energy speed

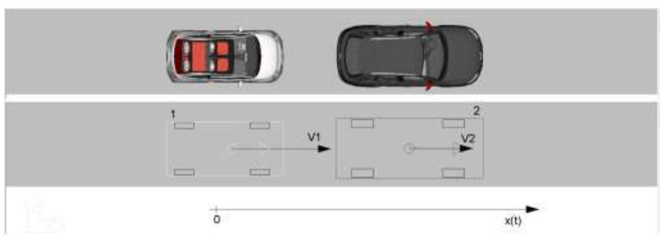

Fig. 1. Reference model for a one-dimensional car crash.

originated the first crash is the at-fault party.

However, this scenario is not always the case. For example, let suppose that the vehicle (3) suddenly stops and the vehicle (2) behind is not able to stop and crashes into the rear bumper of the vehicle (3); soon after the vehicle (1) arrives and crashes into the rear bumper of vehicle (2). In this case, the usual rule of the presumption of fault applies equally, so driver (1) and (2) will have the compensation halved, since each one has not observed the correct safety distance from the vehicle in front. For instance, the Article 2054 of the Italian Civil Code establishes two fundamental principles for the allocation of compensation liability in the event of a road accident: the first imposes the obligation to compensate for damage caused to persons or property during the circulation of a vehicle, if the driver does not prove that has done everything possible to avoid the damage, and the second provides for the presumption of equal responsibility in the case of the accident; it shall be presumed until proven otherwise that each driver contributed equally to the damage suffered by individual vehicles. It is the well-known rule of contributory negligence at 50% for each of vehicle involved: this division of the respective responsibilities operates presumptively, that is, until one of the drivers exceeds it, proving that he has respected the road traffic rules, including the general duties of prudence and attention while driving, while the other did not.

From a technical point of view, the comprehension on how the accident happened is important to identify liabilities in pile-up. It is therefore important to carry out a comprehensive investigation, to reconstruct properly the sequential order of the crashes. However, this task is frequently hampered by lack, or contradictory, information. In fact, often in car accidents with damage to property only and without injuries, the intervention of a judicial authority is not required, and therefore there are no photos of the accident scene, or a planimetric sketch, representing the static positions of the vehicles and the points of impact on the ground. In addition, declarations of the drivers involved are often conflicting with each other and, in any case, cannot be fully trusted as they are parties to the dispute.

In summary, very often the information available in pile-up are only the photos and repair invoices of the damaged vehicles, which hinder the possibility of estimating the energy equivalent speed of vehicles in a very accurate way, as with finite element methods [12] and or via analytical methods [13]. Therefore, the purpose of this work is to provide an analytical technique to reconstruct the correct sequence of the impacts, i.e. which allows an easy, yet rigorous, distinction between chain reaction car accident and chain collisions involving a column of moving vehicles, using only rough information regarding the damages and without any information about the accident scene.

2. Theoretical formulations of the problem

2.1. The governing equations

The theoretical model is formulated considering firstly a two-stage car accident, as depicted in Fig. 1.

In all those cases in which the vector quantities involved have the same direction, or, more generally, to describe coaxial centered collisions, a simplified model of the car with only one degree of freedom is sufficient to described accurately the mechanics of the crash. Therefore, only the longitudinal dynamic is considered, and each car is modelled as a point mass described only by the coordinate position *x(t)* (no yaw rotations are considered); this is the reason why in Fig. 1 the centre of masses of the two vehicles are coaxially aligned.

To get insights on the crash dynamics, the analytical relation between the change of velocity experimented by each vehicle and the crash deformation energy must be written. To this end we must recall that, in each collision, a portion of the kinetic energy initially possessed by the vehicles, E_c, is converted into other forms of energy, partly transferred from one vehicle to another and partly remains as the residual kinetic energy of each vehicle. The kinetic energy of the system can be expressed as the sum of the kinetic energy possessed by the centre of mass, i.e. $\frac{(m_1+m_2)}{2}V_G^2$, plus the kinetic energy of the vehicles computed in the reference frame of the centre of mass, i.e. $\frac{m_c}{2}V_R^2$, namely:

$$E_c = \frac{(m_1 + m_2)}{2}V_G^2 + \frac{m_c}{2}V_R^2 \tag{1}$$

where $m_c = \frac{m_1 m_2}{(m_1+m_2)}$ is the equivalent mass of the systems, m is the mass of the vehicle, $V_G = \frac{(m_1 V_{1i}+m_2 V_{2i})}{m_1+m_2}$ is the velocity of the centre of mass written in terms of initial velocities, and $V_R = V_{1i} - V_{2i}$ is the initial relative velocity between the vehicles, where the index i stands for the initial velocity just before the impact.

To carry out an energy balance and analyse how the energy flows from one vehicle to another, it is useful to split the impact into two phases: the compression and return phases. A part of the initial kinetic energy possessed by the system is absorbed into the deformation of the vehicles; this absorbed energy E_a has two components, one elastic E_r and one anelastic E_d:

$$E_a = E_r + E_d \tag{2}$$

The elastic component is recovered in the next phase of restitution, and, from eq. (1) can be written as:

$$E_a = \frac{m_c}{2}V_R^2 \tag{3}$$

eq. (3) shows the absorbed energy during the impact depends only on the relative velocity between the vehicles. The plastic component is mainly associated with permanent deformations, with the viscous behavior of the materials, and the conversion into other forms of energy, such as sound energy, vibration, heat. The anelastic component is considered as dissipated energy and cannot be recovered in the form of kinetic energy. The elastic energy can be written making use of the restitution coefficient ε, being defined as the ratio between the magnitude of the impulse in the return and that in the compression phase, as follow:

$$E_r = \frac{m_c}{2}V_R^2\varepsilon^2 \tag{4}$$

where $\varepsilon \in [0,1]$, if $\varepsilon = 1$ the impact is elastic, while for $\varepsilon = 0$ the impact is plastic.

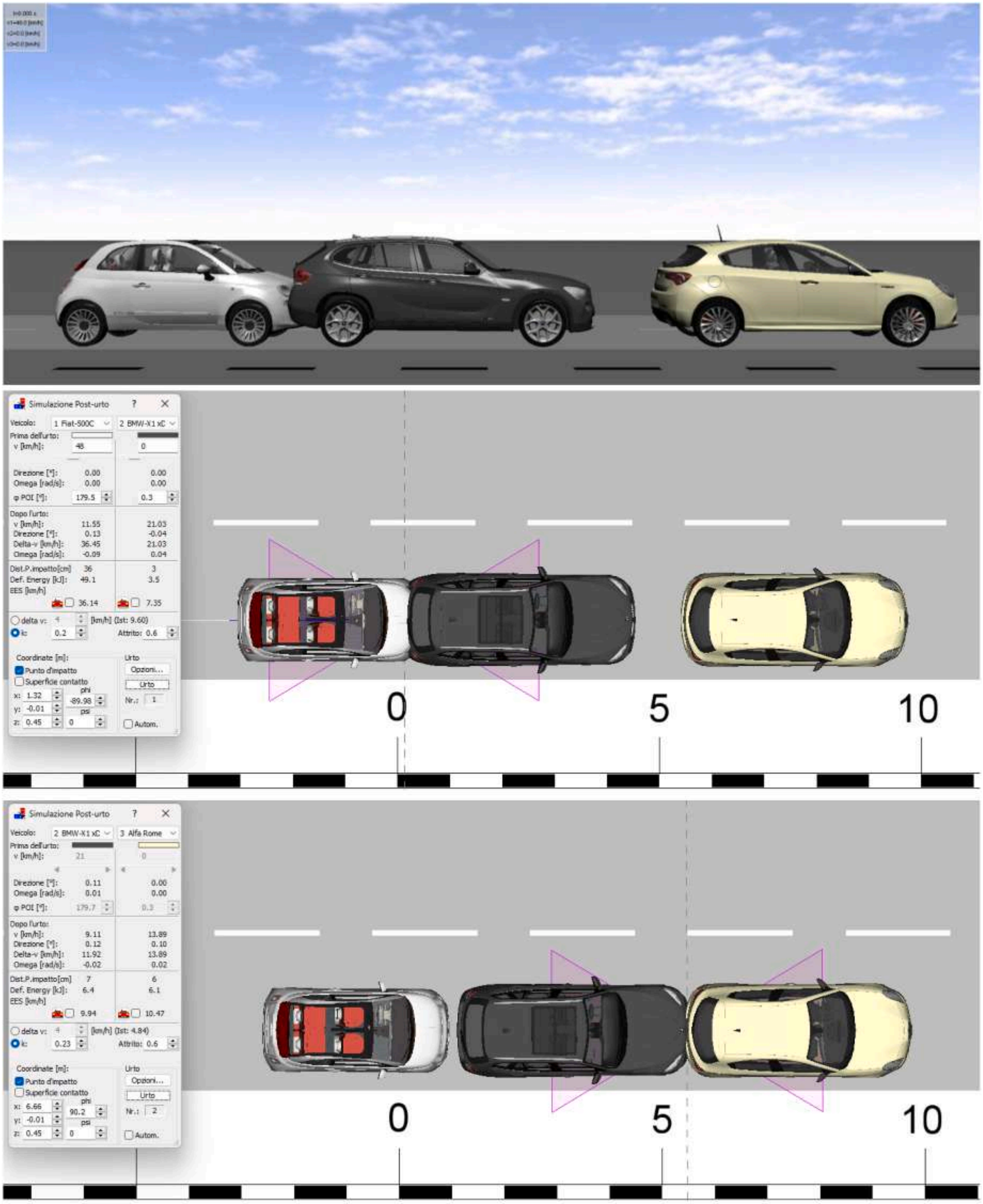

Fig. 2. Simulation with the software PC-Crash of a two-stage chain reaction car accident.

Combining eq.s (2), (3) and (4) produces:

$$E_d = \frac{m_c}{2} V_R^2 \left(1 - \varepsilon^2\right) \tag{5}$$

The previous formula links the dissipated energy, which can be quantified from the deformations on the two vehicles as it will be explained ahead, to the initial relative velocity between them, which is generally unknown. To this end, it is useful to rewrite the previous formula in respect to the unknown relative speed:

$$V_R = \sqrt{\frac{2E_d}{m_c} \frac{1}{(1 - \varepsilon^2)}} \tag{6}$$

recalling the definition of relative velocity, $V_R = V_{1i} - V_{2i}$ in the previous equation only the positive root has been considered, since the pre-crash relative velocity must be positive so that the crash can occur. Making use of the linear momentum conservation and restitution coefficient, the velocity of each vehicle after the impact is related the relative velocity as follow:

$$V_{1f} = V_{1i} - \frac{m_2}{m_1+m_2}(1+\varepsilon)V_R$$
$$V_{2f} = V_{2i} + \frac{m_1}{m_1+m_2}(1+\varepsilon)V_R \tag{7}$$

where the index f stands for the final velocity just after the impact.

Using eq.s (6) and (7) it holds:

$$\Delta V_1 = \frac{-1}{m_1}\sqrt{2E_d\frac{m_1m_2}{m_1+m_2}\frac{1+\varepsilon}{(1-\varepsilon)}}$$
$$\Delta V_2 = \frac{1}{m_2}\sqrt{2E_d\frac{m_1m_2}{m_1+m_2}\frac{1+\varepsilon}{(1-\varepsilon)}} \tag{8}$$

where $\Delta V_1 = V_{1f} - V_{1i}$, $\Delta V_2 = V_{2f} - V_{2i}$, that links the change of velocity of each vehicle to the deformation energy.

The last step required is the estimation of the deformation energy E_d. Among the techniques known for estimating the deformation energy, i.e. Campbel, Principal Direction of Force (PDOF) or Principal Direction of Deformation (PDOD), etc [14,15], it is here employed the one that uses the energy equivalent speed (EES), which was first defined by Burg et al. [16]. EES is a measure for the kinetic energy dissipated by the vehicle during the contact phase, i.e. the energy converted through deformation; it can be thought as the vehicle speed equivalent to the energy consumed to cause the vehicle deformation. International standard organization (ISO) defines EES as equivalent speed, or the speed a vehicle must have during contact with a solid and stiff object, to achieve a deformation energy E_d equivalent to the actual deformation of the vehicle [17], namely:

$$E_d = \frac{mEES^2}{2} \tag{9}$$

The previous formula is valid for a one-dimensional and fully plastic impact where the vehicle comes to a full stop immediately after the collision.

The practical utility of the EES relays on the fact that, even for experienced engineers, the deformation energy is a rather abstract entity, in respect to an equivalent velocity: the reconstructing engineer deals with the velocity – and not with the energy - at which the vehicle should impact against a fixed and non-deformable obstacle, to produce the same deformation in respect to the one suffered from the vehicle in study. However, the concept of EES should not lead to confusion, since the equivalent velocity does not generally correspond to an actual velocity of the vehicle for the crash in study.

Making use of the EES, it is possible to evaluate the strain energy in a rear end collision between two vehicles:

$$E_{d_{12}} = \frac{m_1EES_1^2 + m_2EES_2^2}{2} \tag{10}$$

where the right-hand side is the sum of the deformation energies associated to the actual deformation suffered from each vehicle. Eq. (10) provide the sought deformation energy from the knowledge of EESs.

The EES can be estimated by a visual comparison between the deformation in object with documented crash tests of known EES, which involve vehicles of similar mass, stiffness, and same location of damage. The latter conditions that vehicles compared are of the same type ensures that they have the same structural behavior and, therefore, the same kinetic energy dissipated corresponds to an equal level of deformation. If no documented data of the same vehicle model is available, the comparison can, at the expense of accuracy, be made using data from a vehicle with a similar structural behavior, for example characterised by the same structural stiffness, mass, and wheelbase [14]. The comparison is qualitative, i.e. based on the visual comparison of the damages, which must be substantially similar in width and depth, or through the comparison of the measurements of the extension and depth of the deformations. To obtain reliable data, the material used for the comparison should include comprehensive photographic documentation accompanied by measurements of maximum deformation, the width of the deformed zone, the degree of overlap, whether there are intrusions into the passenger compartment, whether and how much the engine has moved, the mass of the crashed vehicle, etc. Examples of EES estimations will be provided in the next sections.

The present algorithm will be employed to define an analytical index that allows to settle the actual sequence of collisions in a chain reaction car accident.

2.2. The proposed algorithm

Let start the investigation considering a crash involving a queue of three vehicles, where the indexes 1, 2, 3 identify the vehicle at the end, in the middle, and at the beginning of the queue, respectively; the concept is generalizable for an arbitrary number of vehicles. The actual sequence of collisions is unknown: we examine if the accident could have been triggered by the first vehicle at the rear of the queue, with the other two vehicles initially at rest; this hypothetical scenario is represented in Fig. 2. To the scope, it is necessary to assess the initial velocity of vehicle (1) and what damages should have caused, then compare them with the actual damages. The mathematical steps required to assess the order of collisions are the following:

1. Estimate the EES associated to actual damages, by a visual comparison with documented crash tests of known EES. For vehicles (1) and (3) EESs must be estimated for the damage in front and at the rear, respectively; for the vehicle (2) in the middle of the queue, two different estimations of EESs must be caried out, at the front and at the rear.
2. Going backwards with the potential order of crashes, the deformation energy of the second collision between the front bumper of vehicle (2) and the rear bumper of the vehicle (3) is carried out through eq. (10): $E_{d_{23}} = \frac{m_2EES_{fr_2}^2 + m_3EES_{re_3}^2}{2}$.
3. Set a proper value for the restitution coefficient and apply eq. (6) to estimate the relative velocity: $V_{R_{23}} = \sqrt{2E_{d_{23}}\frac{(m_2+m_3)}{m_2m_3}\frac{1}{(1-\varepsilon_{23}^2)}}$.
4. Since the vehicle (3) is at rest before the second impact, the relative velocity of impact coincides with the absolute velocity of impact of the vehicle (2). Also considering the vehicle (2) at rest before the first collision, $V_{R_{23}}$ represents the upper limit of the velocity, V_{2f}, with which the vehicle (2) could exit form the first rear-end collision, in consideration of the engine brake and/or brake action of the driver. Namely: $V_{R_{23}} \approx V_{2f} = \Delta V_2$.
5. Going backwards and analyse the first rear-end collision. From the second eq. (7) it holds: $V_{R_{12}} = \Delta V_2\frac{m_1+m_2}{m_1(1+\varepsilon_{12})}$, where ε_{12} should be generally higher than ε_{23}. Since the vehicle (2) is at rest before the first impact, the relative velocity coincides with the absolute velocity of impact of the vehicle (1): $V_{R_{12}} = V_{1i}$.
6. The deformation energy of the first crash is now indirectly evaluated, i.e. not from actual damages but from kinematic relations, by eq. (6): $\widehat{E}_{d_{12}} = V_{R_{12}}^2\frac{m_1m_2(1-\varepsilon_{12}^2)}{2(m_1+m_2)}$. The estimated deformation energy should be confronted with the one evaluated with eq. (10), $E_{d_{12}} = \frac{m_1EES_{fr_1}^2 + m_2EES_{re_2}^2}{2}$. It is important to stress the conceptual difference between $E_{d_{12}}$, which is the deformation energy emerging from the actual damages suffered by the vehicles, in respect to $\widehat{E}_{d_{12}}$ which is the deformation energy kinematically requested to push forward the vehicle (2), at rest, and to cause a second collision characterised by the actual deformation energy $E_{d_{23}}$. If the two energies are almost equal, $E_{d_{12}} \approx \widehat{E}_{d_{12}}$, then the accident has been triggered by the first vehicle and is characterised by an impact velocity V_{1i}. Otherwise, for instance if $\widehat{E}_{d_{12}} \gg E_{d_{12}}$, the first crash cannot have caused the integral push of the second vehicle against the third and necessarily has

happened after that the vehicle (2) has rear-ended the vehicle (3), or the vehicle (2) would still have collided with vehicle (3) even if it had not been rear-ended by vehicle (1).

7. However, as explained in the previous section, it is more intuitive to work with EESs in respect that with energies. Therefore, from the previous point, the condition for the accident to be triggered by the first vehicle is $E_{d_{12}} = \frac{m_1 EES_{fr_1}^2 + m_2 EES_{re_2}^2}{2} \approx \widehat{E}_{d_{12}}$. Taking as a reference EES_{re_2} associated to the actual damages suffered at the rear from the vehicle (2), from this identity the expected EES at the front of vehicle (1) is: $\widehat{EES}_{fr_1} = \sqrt{\frac{2\widehat{E}_{d_{12}} - m_2 EES_{re_2}^2}{m_1}}$. Again, the superscript $\frown$ stresses the fact that $\widehat{EES}_{fr_1}$ is kinematically needed to push forward the vehicle (2) with enough energy to cause the actual damages emerging in the second collision. If $\widehat{EES}_{fr_1} \approx EES_{fr_1}$, then the accident has been triggered by the first vehicle, otherwise, for instance if $\widehat{EES}_{fr_1} \gg EES_{fr_1}$, this means that the actual damages at the front of vehicle (1) are lower than the one expected to be dynamically coherent with a chain reaction car accident triggered by the first vehicle.

From what shown, it is introduced the expected EES factor, or *xEES* which is the ratio between the expected EES divided by the EES associated to the actual damages at the front of vehicle (1):

$$xEES = \frac{\widehat{EES}_{fr_1}}{EES_{fr_1}} \tag{11}$$

2.3. Statistical validity of xEES

Considering the common estimation errors that affect the parameters of the problem, such as masses, restitution coefficients, EESs and deformation energies, three intervals of variation can be defined, with the statistical analysis outlined ahead.

Let recall the expression $\widehat{EES}_{fr\_1} = \sqrt{\frac{2\widehat{E}_{d_{12}} - m_2 EES_{re_2}^2}{m_1}}$ and remember that $\widehat{E}_{d_{12}} = V_{R_{12}}^2 \frac{m_1 m_2 (1-\varepsilon_{12}^2)}{2(m_1+m_2)}$; for a simplified yet meaningful statistical analysis, the following hypothesis are made:

i. Vehicles with similar masses and rigidities are considered, it holds $m_1 \approx m_2$ and, in first approximation, $EES_{fr_1} \approx EES_{re_2}$;
ii. Thus $\widehat{E}_{d_{12}} \approx V_{R_{12}}^2 \frac{m_1(1-\varepsilon_{12}^2)}{4}$;
iii. Considering Eq. (7), which correlates the velocity variation to the relative velocity, neglecting here the restitution coefficient, which is generally small (<0.35) even for low-speed collisions, with the hypothesis i), it holds $\Delta V_2 \approx \frac{V_{R_{12}}}{2}$.
iv. Recalling the definition of EES, in first approximation it is possible to set $|\Delta V_2| \approx |EES_{re_2}| \approx |EES_{fr_1}|$.
v. Hypotheses iii) and iv) led us to: $|V_{R_{12}}| \approx 2|EES_{fr_1}|$.

Making use of the hypothesis i)-v) it holds:

$$\widehat{EES}_{fr_1} = \sqrt{\frac{2\widehat{E}_{d_{12}} - m_2 EES_{re_2}^2}{m_1}} \approx \sqrt{\frac{8EES_{fr_1}^2 \frac{m_1 m_2 (1-\varepsilon_{12}^2)}{2(m_1+m_2)} - m_2 EES_{fr_1}^2}{m_1}} \approx EES_{fr_1} \sqrt{\frac{8\frac{(1-\varepsilon_{12}^2)}{4} - 1}{1}} \tag{12}$$

which finally reads:

$$\widehat{EES}_{fr_1}(EES_{fr_1}, \varepsilon_{12}) \approx EES_{fr_1} \sqrt{2(1-\varepsilon_{12}^2) - 1} \tag{13}$$

where the expected EES is written in function of the parameters EES_{fr_1}, ε_{12}, whose estimation is affected by random noise and then can be treated as random variables. Eq. (13) is the sought simplified expression that allows to evaluate a meaningful statistical range of variation for *xEES* defined by eq. (11).

To simplify the notation, let $EES_{fr_1} = e_1$ the stochastic variable appearing on the right-hand side in eq. (13), which is related to the deterministic counterpart appearing in the denominator in eq. (11), here indicated with e_{d1}, by the formula:

$$e_1 = e_{d_1}(1+r) \tag{14}$$

where *r* is a random variable that can assume any value within the interval [-0.15, 0.15]; namely the visual estimation of EES is affected by an estimation error of ±15%.

Concerning ε_{12}, in low-speed collision (relative impact velocity smaller than 20 km/h), the restitution coefficient generally is bounded within the interval [0.1, 0.3], being roughly 0.1 or smaller for higher impact speed. Therefore, in analogy to eq. (14), ε_{12} is assumed to be a random variable, defined as follows:

$$\varepsilon_{12} = 0.2(1+q) \tag{15}$$

where *q* is a random variable bounded within the interval [-0.5, 0.5].

Using eq. (13), the stochastic form of eq. (11) is:

$$xEES_{st}(e_1, \varepsilon_{12}) = \frac{e_1\sqrt{(1-2\varepsilon_{12}^2)}}{e_{d1}} \tag{16}$$

where the functional relationship with the stochastic parameters has been highlighted. Substituting in it eq.s (14) and (15), it holds:

$$xEES_{st} = (1+r)\sqrt{1-0.08(1+q)^2} \tag{17}$$

Eq. (17) is the product of two random variables, the first is $(1+r)$, whose expectation and standard deviation are $\mu_1 = 1, \sigma_1 = 0.0863$, respectively, the second is $\sqrt{1-0.08(1+q)^2}$ with $\mu_2 = 0.9592, \sigma_2 = 0.0245$. With the hypothesis that the two random variables are statistically independent, the expectation of their product is the product of their expectations, thus the expectation of eq. (17) is:

$$E[xEES_{st}] = \mu_1 \mu_2 \tag{18}$$

and produces $E[xEES_{st}] = 0.9592$; to simplify it is assumed $E[xEES_{st}] \triangleq 1$.

Similarly, the standard deviation of $xEES_{st}$ is:

$$STD[xEES_{st}] = \sqrt{(\sigma_1^2+\mu_1^2)(\sigma_2^2+\mu_2^2) - \mu_1^2\mu_2^2} \tag{19}$$

and produces $STD[xEES_{st}] = 0.0864$; to simplify it is assumed $STD[xEES_{st}] \triangleq 0.1$.

We are now in the position to define the intervals of variation for *xEES* that set apart, with due statistical confidence, chain reaction car accidents from chain collisions involving a column of moving vehicles. Using eq.s (18) and (19) and recalling the three sigma or empirical rule, the percentage of values that lie within the interval estimate in a normal distribution is 68%, 95%, and 99.7% of the values lie within one, two, and three standard deviations of the mean, respectively. It means that a sample of $xEES_{st}$ that falls within the interval [0.8, 1.2] is, at most, two standard deviations apart from the expected value, thus 95% of the statistical population falls in it and the pileup can be classified as chain reaction car type. In contrast, if $xEES_{st}$ >1.3, the accident does not belong to the former type, since this large value cannot be the fruit of uncertainties that affects the parameters because the whole statistical population falls within 1.3, therefore the car crash has to involve a column of moving vehicles.

In summary, three intervals are defined to statistically classify the expected Equivalent Energy Speed defined by eq. (11):

1. $xEES \in [0.8, 1.2]$. Chain reaction car accident: the accident has been triggered by the first vehicle.

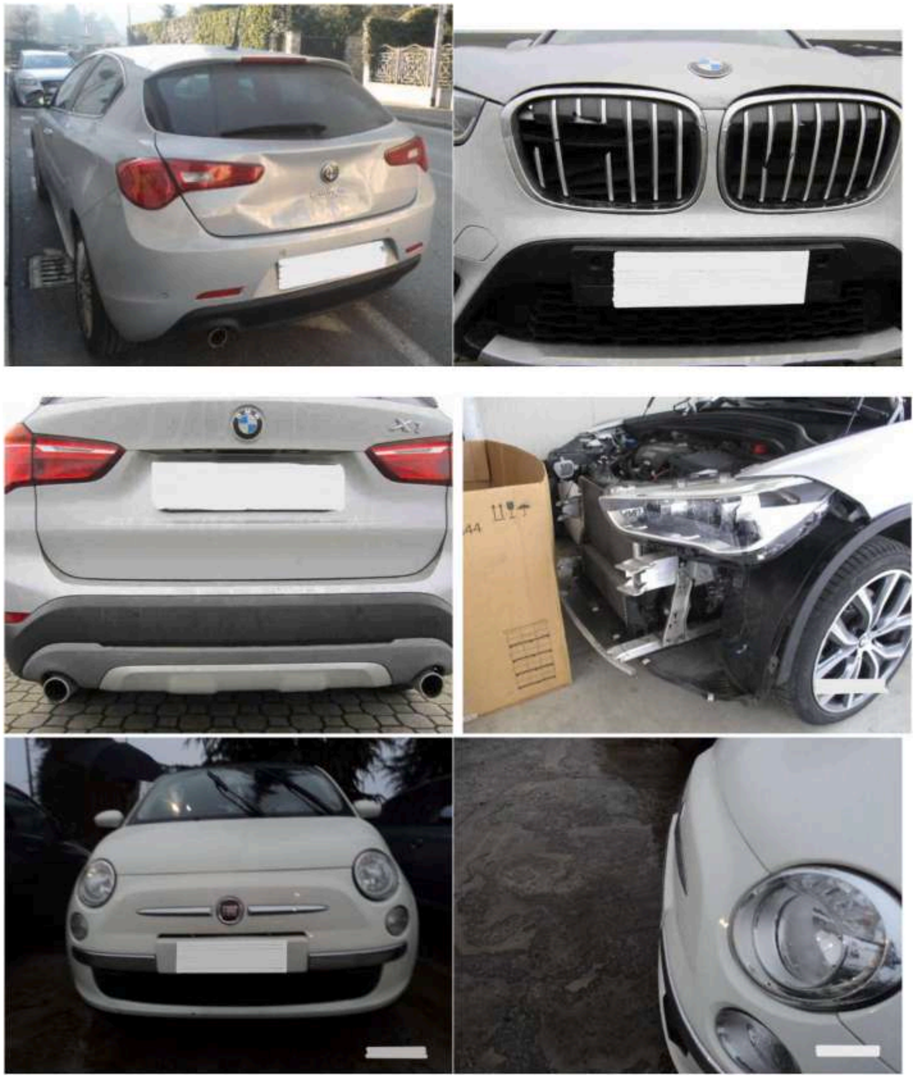

Fig. 3. Damages suffered from the vehicles involved in a multiple rear-end collision.

2. *xEES* $\in [1.2, 1.3]$. Uncertainty, the engineer should use additional information to confidently assess the order of crashes, since less than 5% of the statistical population falls in this interval. The rather large value of *xEES* could not be related to uncertainties affecting the parameters instead could be induced by a sequence of collisions that is different to the one conjectured at the beginning of section 2.2.
3. *xEES* > 1.3. Chain collisions involving a column of moving vehicles: the collision between the first and second vehicle cannot have determined the push of the second against the others and must necessarily have occurred in a second moment.

As a corollary of the analysis, it is presented below how the expected EES is related to the impact force F and to the impact time Δt. Under the i) – v) hypothesis it holds $V_{R_{12}} \approx 2\Delta V_2$ and $\widehat{E}_{d_{12}} \approx 4\Delta V_2{}^2\frac{m_1(1-\varepsilon_{12}^2)}{4}$. Substituting them into $\widehat{EES}_{fr_1} = \sqrt{\frac{2\widehat{E}_{d_{12}} - m_2 EES_{re_2}^2}{m_1}}$, it holds:

$$\widehat{EES}_{fr_1} = \sqrt{\frac{8\Delta V_2{}^2\frac{m_1(1-\varepsilon_{12}^2)}{4} - m_2 EES_{re_2}^2}{m_1}} = \sqrt{\frac{2\Delta V_2{}^2 m_1(1-\varepsilon_{12}^2) - m_2\Delta V_2{}^2}{m_1}} = \Delta V_2\sqrt{2(1-\varepsilon_{12}^2) - 1} \tag{20}$$

that finally reads:

$$\widehat{EES}_{fr_1} = \frac{F}{m_2}\Delta t\sqrt{2(1-\varepsilon_{12}^2) - 1} \tag{21}$$

Eq. (21) states that, for vehicles with similar masses and rigidities, the expected EES is linearly proportional to the impact force and time, is inversely proportional to the mass of the vehicle, and is linearly proportional to the restitution coefficient.

3. Case studies

The use of *xEES* is shown in this section, with the aid of real-life pileups. The section is organised as follows: 3.1 Example of chain collisions involving a column of moving vehicles; 3.2 Numerical validations of the proposed model; 3.3 Further examples of chain reaction car accident and involving a column of moving vehicles.

3.1. Example of chain collisions involving a column of moving vehicles

As a case study, it is considered a civil trial that involved the owners of three cars who were involved in a multiple rear-end collision. The vehicles are a Fiat at the rear of the line, a BMW, in the middle, and an Alfa Romeo, at the head of the line. The driver of the BMW declares that he was rear-ended by the Fiat, while stationary, and pushed against the Alfa Romeo in front, while the driver of the Fiat declares that he had

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Dynamic Test Center AG
Centrum für Dynamische Tests AG
Centre de Tests Dynamiques SA

Heckaufprall 100% 0° gebremst **Übersicht**

HS114-164.jpg

Ein Nissan Qashqai fährt gebremst (7.3 m/s^2) einem Audi A3 (ungebremst, kein Gang eingelegt) auf.

Crash Test Resultate	stossend	gestossen
Fahrzeug	Nissan Qashqai	Audi A3
Masse	1446 kg	1540 kg
Bremse	7.3 m/s^2	-
Kollisionsgeschwindigkeit v_K	19.2 km/h *	0 km/h
Geschwindigkeitsänderung Δv_{Res}	13.5 km/h **	10.1 km/h **
Maximale Beschleunigung $a_{Res\ max}$	6.5 g *	7.6 g *
Durchschnittliche Beschleunigung $a_{Res\ dt}$	3.4 g *	2.5 g *
Stosszeit dt	114 ms *	
Stossfaktor k / Trenngeschwindigkeit $\Delta v'$	0.23 ** / 4.5 km/h **	
Gesamte Deformationsenergie $W_{Def\ Ges}$	8.9 kJ ± 10%	
Energy Equivalent Speed EES	9.9 ± 1 km/h	7.7 ± 1 km/h
Steifigkeit	-	-
Reparaturkalkulation gesamt	7'760.30 CHF	4'120.70 CHF

Fig. 4. Crash test from the database https://crashdb.agu.ch/details.php?crash_id=214.

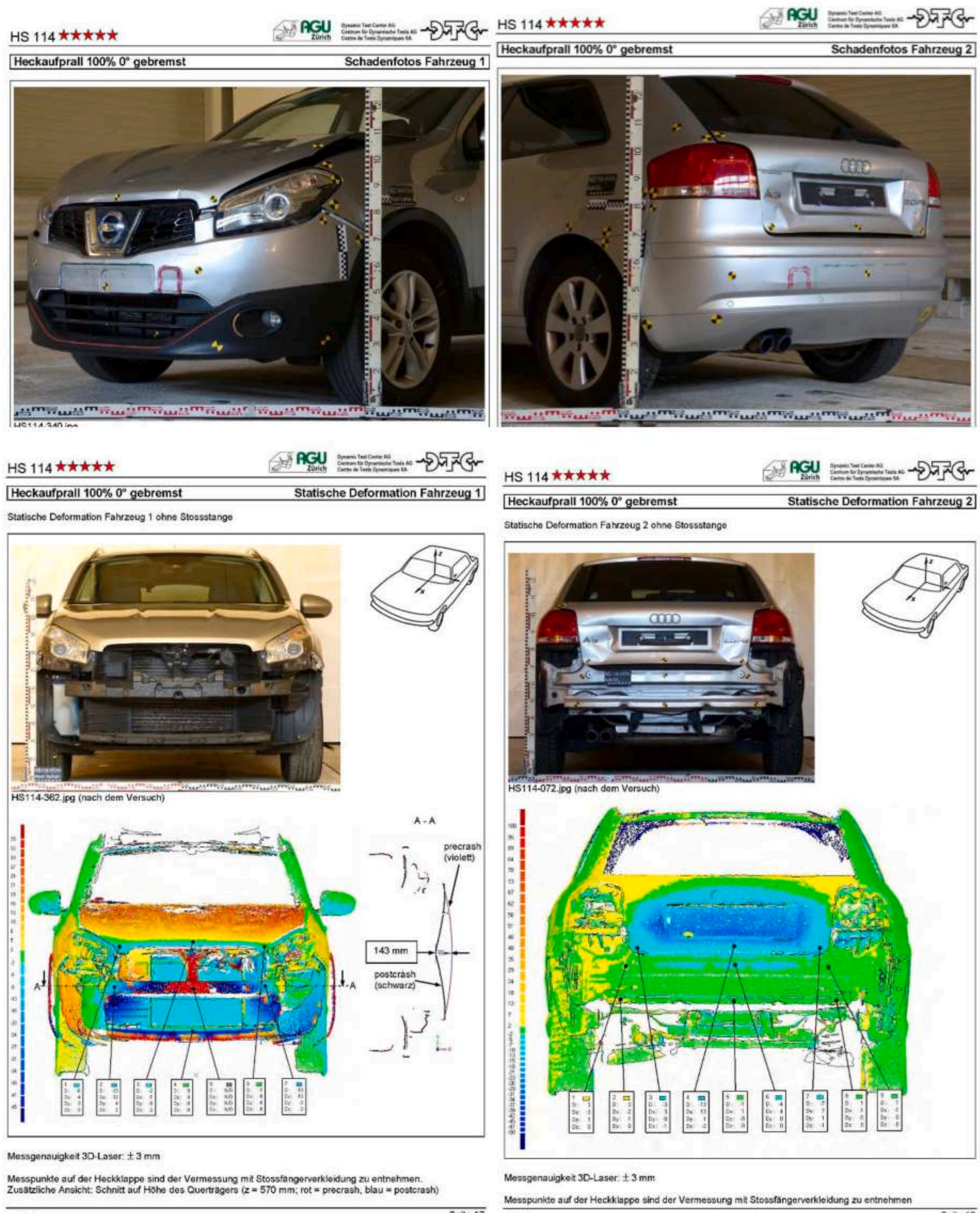

Fig. 5. Details of the damages of the crash test in Fig. 4.

slightly rear-ended the BMW in a second time, after that the BMW had already rear-ended the Alfa Romeo.

The damages found on the three vehicles after the accident are briefly showed in Fig. 3. As far as ascertainable from the photos and from the damage items in the repair invoices attached to the trial documentation, the Fiat, at the front, and the BMW, at the rear, have suffered modest damages mainly involving the bumper and the underlying crossbar. The front of the BMW and the rear of the Alfa have reported mild-damages, with involvement of the bumper, bonnet (BMW), tailgate (Alfa), fenders, of the inner lining, of the crossbar, and of numerous accessories.

In order to analyse the actual sequence of the collisions, i.e. to examine whether the accident was triggered by the first vehicle at the rear of the queue, it is necessary to assess the initial velocity of collision of the Fiat and what damages should have caused. Applying the methodology outlined in section 2.2, it is investigated which of the two versions is compatible with the damages found on the three vehicles.

Starting from the deformation energy associated with the BMW-Alfa impact, the estimate is carried out considering, as a reference, literature data from crash test database https://crashdb.agu.ch/, involving an

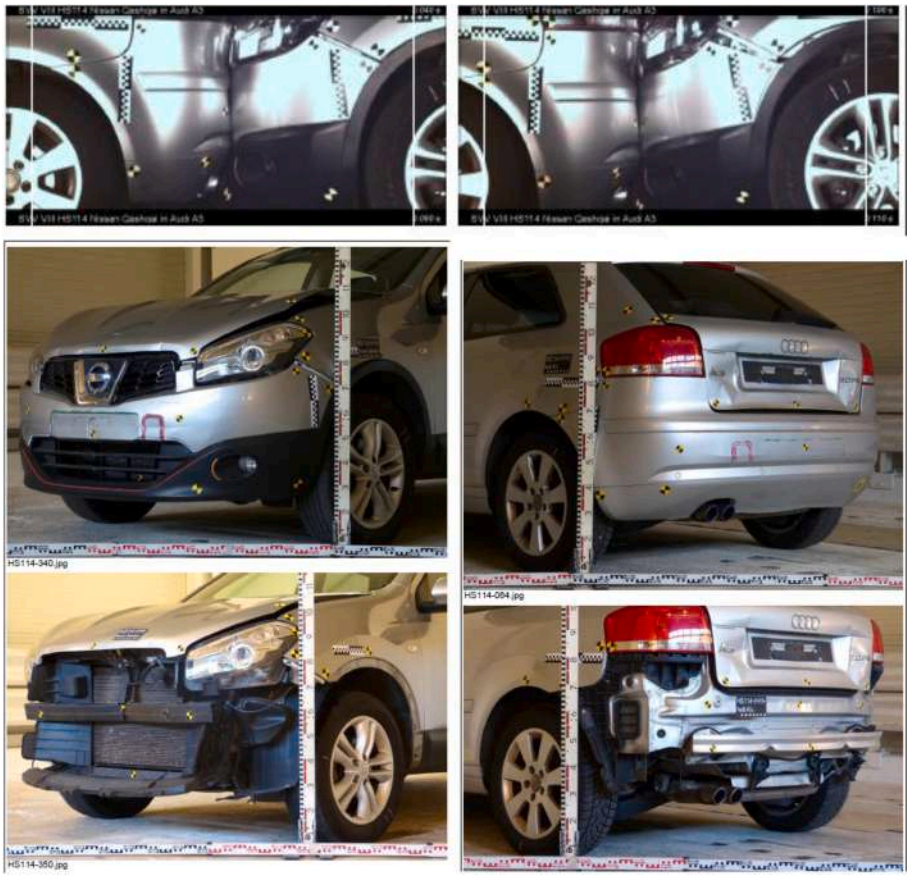

Fig. 6. Details of the damages of the crash test in Fig. 4.

accident between approximately comparable vehicles reported in Figs. 4–6, in which a Nissan Qashqai collides, in a coaxial centered way, against an Audi A3, stationary, with a collision velocity of 19.2 km/h. The full report of the crash test is available here https://crashdb.agu.ch/details.php?crash_id=214. The crash test shows that:

- The dabbing Nissan reported medium severity damages to the central part of the front bumper, the grille, the bonnet and the crossmember. The maximum structural deformation of the crossbar is of 143 mm and the associated EES is 9.9 ± 1 km/h, with a cost of reparation of 7760 CFH (6440 Euro).
- The hit Audi reported medium severity damages to the rear, with involvement of the bumper, the crossmember, the coating, with a residual deformation of about 13 mm and 25 mm at the central part of the hatch. The associated EES 7.7 ± 1 km/h, with a cost of reparation of 4120 CFH (3420 Euro).
- The deformation energy of the crash test in Fig. 4 is 8.9 kJ, the impact time is 0.114 s, the separation velocity is 4.5 km/h, the return factor is 0.23.

By comparison and with the help of Fig. 6, the structural damages to the rear of the Audi in the test of Fig. 4 are lower than those of the Alfa in question, while the damages at the front of the Nissan are of the order of those to the front of the BMW. The resulting EES on the impact points can then be estimated from the results of Fig. 4 by introducing the corrective mass factor [18]: $\frac{EES_M}{EES_{Test}} = \sqrt{\frac{m_{Test}}{m_M}}$, where the Nissan's mass is 1446 kg and 1550 kg is the mass of BMW, the corrective mass factor is about 0.97, so the EES equivalent to the front of the BMW emerging from the previous formula is equal to 9.9·0.97 = 9.6 km/h; the value is here approximated to 10 km/h. In a similar fashion, in consideration of the mass of the Audi of 1540 kg and of 1310 kg for the Alfa, the corrective mass factor is about 1.08, so the EES equivalent to the rear of the Alfa emerging from the previous formula is equal to 7.7·1.8 = 8.3 km/h. Taking into account the greater damage of the Alfa compared to the Audi, the EES associated to the damages at the rear of the Alfa is here assumed at least of 10 km/h.

The deformation energy of the second collision is carried out through eq. (10): $E_{d_{23}} = \frac{m_2 EES_{fr_2}^2 + m_3 EES_{re_3}^2}{2}$.With the estimated EES, the formula provides an overall deformation energy of 11 kJ. Setting $\varepsilon_{23} = 0.23$, eq. (6) is applied to assess the relative velocity, which produces: $V_{R_{23}} = \sqrt{2E_{d_{23}} \frac{(m_2+m_3)}{m_2 m_3} \frac{1}{(1-\varepsilon_{23}^2)}} = 20\ km/h$. The velocity variations provided by eq. (8) are −11 km/h and 13 km/h, respectively, for the BMW and Alfa. Considering the Alfa at rest, the relative velocity of impact coincides with the velocity of impact of the BMW against the Alfa and represents the upper limit for the velocity with which the BMW would have had to come out from the first rear-end collision accordingly to version stated by its driver.

Going backwards and analyse the first rear-end collision. From the second eq. (7) it holds: $V_{R_{12}} = \Delta V_2 \frac{m_1+m_2}{m_1(1+\varepsilon_{12})}$, where $\varepsilon_{12} = 0.20$ to keep into the account the theoretical higher severity of the first crash. The solution produces $V_{R_{12}} \approx 46\ km/h.$ considering the BMW at rest, the relative velocity of impact coincides with the absolute velocity with which the Fiat would have impacted against the BMW. This impact velocity is high and should have caused serious damages both to the front of the Fiat and to the rear of the BMW, which are not evident on the photos in Fig. 3.

In fact, the deformation energy is indirectly evaluated by eq. (6) $\widehat{E}_{d_{12}} = V_{R_{12}}^2 \frac{m_1 m_2 (1-\varepsilon_{12}^2)}{2(m_1+m_2)} \approx 50\ kJ.$

Taking as a reference the rear of the BMW in Fig. 3 and accordingly to the repair invoice, this area has suffered minimum damages, with no

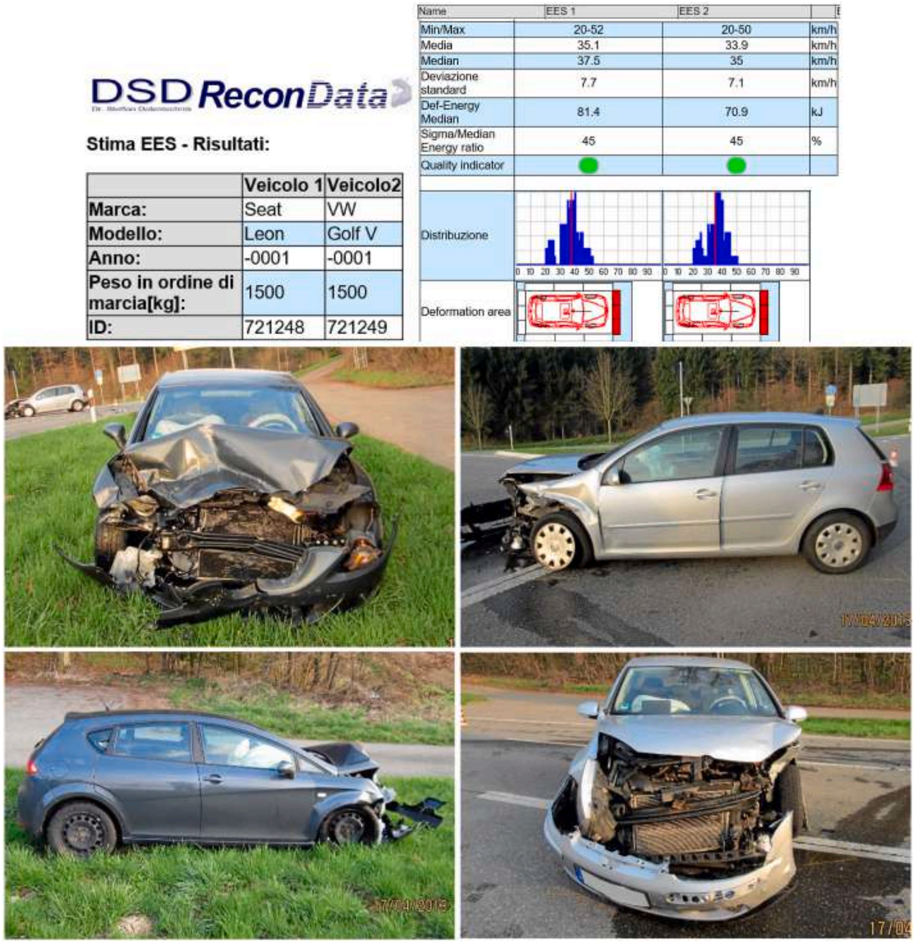

Fig. 7. Crash test from the DSD database, http://www.dsd.at/EESEstimation/Results.php?CaseID=150.

severe involvement of the structural parts below the rear bumper, so the corresponding EES is only of a few km/h. Indeed, it should be considered that modern bumpers must withstand without damage a 4 km/h collision with an immovable object to the body of the car, as required by the ECE R-42 regulation published by the European Economic Commission that regulates the principles for the approval of front and rear protection parts (bumpers, etc.). Damage in which the bumper is visibly damaged, involving the underlying structural parts, indicate that the vehicle has suffered a velocity variation more than that indicated by the legislation. For all that has been explained, the reference EES can be assumed, at the maximum, $EES_{re_2} \approx 6\ km/h$.

With these values the expected EES at the front of vehicle (1) is $\widehat{EES}_{fr_1} \approx 35\ km/h$, this value is high and does not match with the EES actually associated to the minimum damages of the Fiat, which can be considered of a few km/h: $EES_{fr_1} \sim 5/6\ km/h$. In fact, accordingly to eq. (11), $xEES \approx 6$, namely six time higher than the range defined in section 2.3 associated with an accident triggered by the first vehicle.

To get a visual idea of the deformations correlated to an EES of the order of 35 km/h, a crash from the technical literature is considered in Fig. 7, which shows a Seat Leon colliding head-on with a VW Golf. The damage of the Seat involved the bumper, the crossmember, the bonnet and the right fender, to which is associated an EES of about 37.5 km/h, the EES of the VW is 35 km/h, and the crash has a total deformation energy of 81.4 kJ. Neglecting the different mass and stiffness of the Fiat compared to those of the vehicles in Fig. 7, $\widehat{EES}_{fr_1} \approx 35\ km/h$ indicates that the front of the Fiat should exhibit damages of the order of those of the vehicles in Fig. 7, on the contrary, the Fiat exhibits barely perceptible damages, so that: $xEES \approx 6$.

The $xEES$ make clear that the actual damages in front of the Fiat are far lower than those expected in a chain collision triggered by this vehicle. It follows that the Fiat-BMW collision cannot have determined the full push of the BMW against the Alfa and must necessarily have occurred with the Fiat that rear-ended, slightly, the BMW when the latter had already rear-ended the Alfa, as sated by the Fiat's driver.

3.2. Numerical validations of the proposed model

The proposed model is numerically validated using the software PC-Crash version 11.1 that integrates 4 collision calculation models. The algorithms and models used by PC-Crash have been validated with publications and available in SAE's online library, which contains over 80 publications citing the software. The simulate a car accident the software employees both an impulsive approach, which is the extension to 6° of freedom of the Kudlich-Slibar model [19,20], which considers both the conservation of momentum, and the conservation of angular momentum. In the model, the collision calculation is carried out starting from the following parameters: the relative position of the vehicles to the impact, the plane of contact between the vehicles, the position of the point of application of the impact force in the area of contact between the vehicles, the restitution coefficient, the friction in the plane of contact between the vehicles. With regard to vehicle dynamics, PC-Crash allows to reconstruct their movements both before and after the collision. The motion of the vehicles is evaluated by solving the differential equations that describe the forces of contact on the ground: each vehicle is modelled as a rigid body, equipped with suspensions between the vehicle and the tire. The tires are modelled with both the dynamic Seta Tire Model and a linear model, while the vehicle is

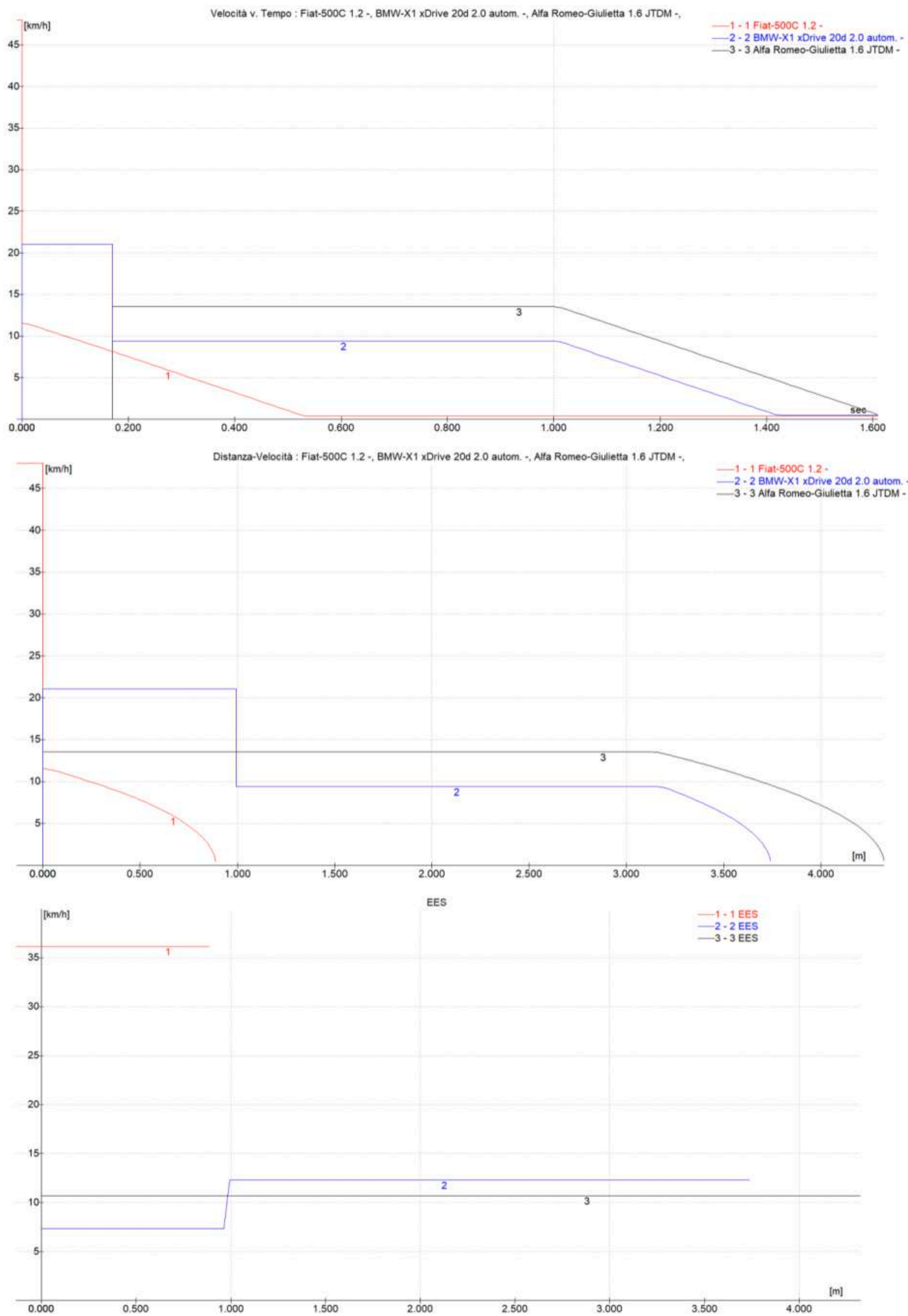

Fig. 8. Starting from the plot on top, vehicles speed in function of the time, of the distance and EES emerging from the numerical simulation.

Table 1
Report of the simulation with PC-Crash. NB: results are in Italian due to the Italian release of the software. Starting from the subplot on top-left, the chart shows the kinematic, dynamic, and energetic parameter of the first collision ("1. Collisione"), of interest: "Vel. Pre Urto" indicates the speed of the vehicle just before the crash; "Vel. Post Urto" indicates the speed just after the crash; on the 9th and 10th rows EES and deformation energy for each vehicle ("Def. Energy") are indicated, on the 20th and 21st rows the total deformation energy ("Energia Totale Deformazione") and the impulsive force ("Impulso") of the crash are indicated. In the subplot on top-right, the chart is analogous to the previous and shows the parameters of the second collision ("2. Collisione"). The subplot on bottom, left and right, shows the geometrical and inertial properties of the three vehicles, such as length ("Lunghezza"), width ("Larghezza"), height ("Altezza"), wheelbase ("Interasse"), dry weihgt ("peso a vuoto"), Roll moment of inertia ("Memento di inerzia Rollè"), Pitch moment of inertia ("Memento di inerzia Beccheggio"), Yaw moment of inertia ("Memento di inerzia Accelerazione"), etc.

1.COLLISIONE **(Simulazione in avanti)**

Veicolo :	1 Fiat-500C	2 BMW-X1 xD
Conducente :		
t [s]:	0.00	0.00
Vel. Pre Urto [km/h]:	**48.00**	**0.00**
Vel. Post Urto [km/h]:	**11.55**	**21.03**
Vel. Ang. [rad/s]:	-0.09	0.04
Cambio di velocità dv [km/h] :	36.45	21.03
Profondità di deformazione [m] :	0.36	0.03
EES [km/h] :	36.14	7.35
Def. Energy [kJ]:	49.13	3.52
Rigidità [kN/m]:	775.3	10825.3
Fattore k (coeff. elasticita) :	0.20	
Velocità dal punto d'impatto [km/h]:	9.6	
Coefficiente d'attrito (MUE) :	0.60	
Coordinata x del punto d'impatto [m] :	1.32	
Coordinata y del punto d'impatto [m] :	-0.01	
Coordinata z del punto d'impatto [m] :	0.45	
Direzione del piano di contatto (phi) [gradi] :	-89.98	
Direzione del piano di contatto (psi) [gradi] :	0.00	
Energia Totale Deformazioni [J] :	52646.49	
Impulso [Ns] :	9871.29	
Direzione della forza dell'impatto [gradi] :	179.96	
Direzione verticale dell'impulso [gradi] :	0.21	
Distanza normale dal punto d'impatto (SHA) [m] :	0.01	0.01
Angolo del punto d'impatto (RHO) [gradi] :	179.96	-0.04
GEV :	1.22	1.22

2.COLLISIONE **(Simulazione in avanti)**

Veicolo :	2 BMW-X1 xD	3 Alfa Rome
Conducente :		
t [s]:	0.17	0.17
Vel. Pre Urto [km/h]:	**21.03**	**0.00**
Vel. Post Urto [km/h]:	**9.40**	**13.56**
Vel. Ang. [rad/s]:	-0.01	0.01
Cambio di velocità dv [km/h] :	11.63	13.56
Profondità di deformazione [m] :	0.06	0.06
EES [km/h] :	9.89	10.68
Def. Energy [kJ]:	6.38	6.38
Rigidità [kN/m]:	3047.8	3047.8
Fattore k (coeff. elasticita) :	0.20	
Velocità dal punto d'impatto [km/h]:	4.2	
Coefficiente d'attrito (MUE) :	0.60	
Coordinata x del punto d'impatto [m] :	6.66	
Coordinata y del punto d'impatto [m] :	-0.01	
Coordinata z del punto d'impatto [m] :	0.45	
Direzione del piano di contatto (phi) [gradi] :	90.20	
Direzione del piano di contatto (psi) [gradi] :	0.00	
Energia Totale Deformazioni [J] :	12753.91	
Impulso [Ns] :	5459.81	
Direzione della forza dell'impatto [gradi] :	-179.91	
Direzione verticale dell'impulso [gradi] :	-0.00	
Distanza normale dal punto d'impatto (SHA) [m] :	0.01	0.00
Angolo del punto d'impatto (RHO) [gradi] :	179.89	0.09
GEV :	1.22	1.22

INSERIMENTO

Veicolo :	Fiat-500C 1.2 -	BMW-X1 xDrive 20d 2.0 autom. -
Database:	DB_DSD	DB_DSD
RecordID:	41306	77049
Lunghezza [m] :	3.54	4.44
Larghezza [m] :	1.63	1.82
Altezza [m] :	1.49	1.60
Num. assi :	2	2
Interasse ant.[m] :	2.30	2.76
Sbalzo anteriore [m] :	0.71	0.89
Larghezza traccia davanti [m] :	1.42	1.50
Larghezza traccia dietro [m] :	1.42	1.50
Peso a vuoto [kg] :	905.00	1550.00
Total weight [kg] :	975.00	1690.00
Peso degli occupanti davanti [kg] :	70.00	140.00
Peso degli occupanti dietro [kg] :	0.00	0.00
Peso nel portabagagli [kg] :	0.00	0.00
Peso sul tetto [kg] :	0.00	0.00
Distanza centro di gravità-asse anteriore [m] :	0.85 (0.85)	1.38 (1.35)
Altezza del centro di gravità [m] :	0.50 (0.50)	0.50 (0.50)
Momento d'inerzia (Rollè) [kgm^2] :	280.92 (301.08)	722.96 (782.84)
Momento d'inerzia (Beccheggio) [kgm^2] :	936.38 (1003.61)	2409.84 (2609.47)
Momento d'inerzia (Accelerazione) [kgm^2] :	936.38 (1003.61)	2409.84 (2609.47)
Elasticità 1. asse sinistro [N/m] :	18643.91	25342.50
Elasticità 1. asse destro [N/m] :	18643.91	25342.50
Elasticità 2. asse sin. [N/m] :	10949.60	25342.50
Elasticità 2. asse des. [N/m] :	10949.60	25342.50
Smorzamento 1. asse sin. [Ns/m] :	2097.44	2851.03
Smorzamento 1. asse des. [Ns/m] :	2097.44	2851.03
Smorzamento 2. asse sin. [Ns/m] :	1231.83	2851.03
Smorzamento 2. asse des. [Ns/m] :	1231.83	2851.03
max. ang. di deriva. 1.asse sin. [gradi] :	10.00	10.00
max. ang. di deriva. 1.asse des. [gradi] :	10.00	10.00
max. angolo di deriva. 2.asse sin.[gradi] :	10.00	10.00
max. angolo di deriva. 2.asse des.[gradi] :	10.00	10.00
ABS :	No	No

INSERIMENTO

Veicolo :	Alfa Romeo-Giulietta 1.6 JTDM -
Database:	DB_DSD
RecordID:	76487
Lunghezza [m] :	4.35
Larghezza [m] :	1.80
Altezza [m] :	1.47
Num. assi :	2
Interasse ant.[m] :	2.63
Sbalzo anteriore [m] :	0.87
Larghezza traccia davanti [m] :	1.55
Larghezza traccia dietro [m] :	1.55
Peso a vuoto [kg] :	1310.00
Total weight [kg] :	1450.00
Peso degli occupanti davanti [kg] :	140.00
Peso degli occupanti dietro [kg] :	0.00
Peso nel portabagagli [kg] :	0.00
Peso sul tetto [kg] :	0.00
Distanza centro di gravità-asse anteriore [m] :	1.31 (1.28)
Altezza del centro di gravità [m] :	0.50 (0.50)
Momento d'inerzia (Rollè) [kgm^2] :	570.56 (625.65)
Momento d'inerzia (Beccheggio) [kgm^2] :	1901.86 (2085.48)
Momento d'inerzia (Accelerazione) [kgm^2] :	1901.86 (2085.48)
Elasticità 1. asse sinistro [N/m] :	21418.50
Elasticità 1. asse destro [N/m] :	21418.50
Elasticità 2. asse sin. [N/m] :	21418.50
Elasticità 2. asse des. [N/m] :	21418.50
Smorzamento 1. asse sin. [Ns/m] :	2409.58
Smorzamento 1. asse des. [Ns/m] :	2409.58
Smorzamento 2. asse sin. [Ns/m] :	2409.58
Smorzamento 2. asse des. [Ns/m] :	2409.58
max. ang. di deriva. 1.asse sin. [gradi] :	10.00
max. ang. di deriva. 1.asse des. [gradi] :	10.00
max. angolo di deriva. 2.asse sin.[gradi] :	10.00
max. angolo di deriva. 2.asse des.[gradi] :	10.00
ABS :	No

modelled as a rigid body with 6° of freedom (translations and rotations on the three axles) and subject to the following forces: tyre-road reaction forces, gravity and aerodynamic drag. In the post-collision phase, the motion of the vehicle is calculated by solving the system of equations of motion and, calculating the forces acting on the vehicle, the displacement that follows. In the Kudlich-Slibar model, the change in translational velocity and angular velocity of vehicles is calculated by solving a system of equations derived from Newton's second law and the second cardinal equation of dynamics, in which the coordinate system is related to the contact plane and originates at the point of impact. From the above system of equations, the conservation of momentum and angular momentum with respect to the point of impact automatically derives. Details of the algorithm can be found in the PC-Crash manual, as well as in the original publications [19].

Said that, the dynamics described by the driver of the BMW, i.e. chain collision triggered by the Fiat with the other vehicles at rest, is simulated with PC-Crash software. The EESs emerging from the simulation must be compatible with the ones associated with the actual damages on the vehicles. PC-Crash performs an iterative forward analysis, once the positions of the vehicles at the time of the collision have been fixed, as shown in the first box in Fig. 2, the initial kinematic parameters of each vehicle are assumed, i.e. the velocity vector at the time

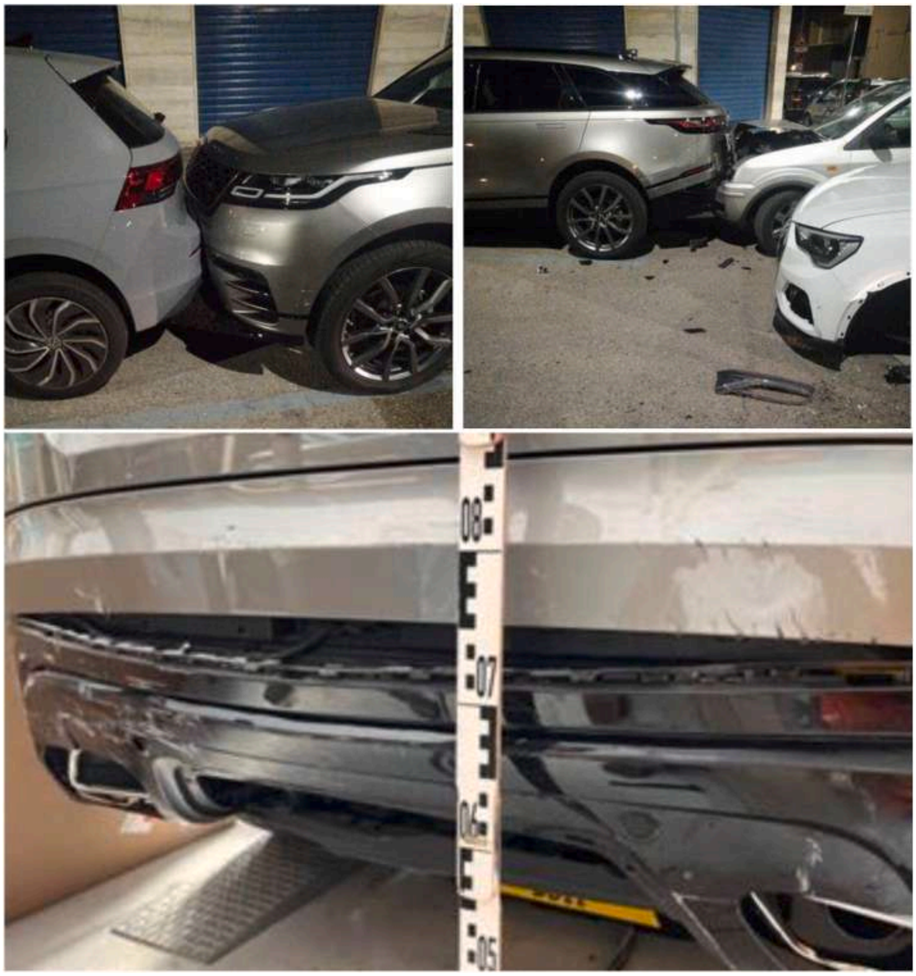

Fig. 9. On top, the static position of the vehicles, the top-left plot shows the front of the Range Rover and the rear of the VW, the top-right plot shows the front of the Ford and the rear of the Range Rover, the plot on bottom shows the damaged rear of the Range Rover.

of the collision, the post-impact speeds and the terminal positions of the vehicles are then calculated, taking care that the EES emerging from the simulation as a result of the second impact, namely at the front and rear of BMW and Alfa, are of the order of 10 km/h each, as estimated in the previous section from the comparison of Fig. 4.

The kinematic parameters of the simulation are summarised in Figs. 2 and 8, and the results are fully shown in Table 1. In the simulation, it was assumed that Fiat was braking with an intensity of 6 m/s^2 at time 0, i.e. the instant of the first impact, while the Alfa and BMW brake with the same intensity, but with a reaction delay of 1 s, as shown from Fig. 8. The restitution coefficients are set at 0.20 and 0.23, according to what was illustrated above. The results of the simulation allow to draw a comparison with the analytical results emerging from the proposed model:

- The vehicles reach the impact as shown in subplot above in Fig. 2, the Alfa and the BMW are supposed to be at rest, while the Fiat has an impact velocity of 48 km/h.

  The impact velocity of the Fiat emerging from the simulation is perfectly in agreement with $V_{R_{12}} \approx 46\ km/h$ previously calculated in section 3.1 making use of the analytical procedure outlined in section 2.2.
- After the first impact, the linear velocity of the Fiat decreases from 48 to about 12 km/h, as in Table 1, with a deltaV of about 36 km/h and an EES of about 36 km/h. The linear velocity of the BMW increases sharply from 0 to about 21 km/h, with an EES at the rear of about 7 km/h.

  The exit velocity from the first impact of the BMW is perfectly in agreement with that analytically calculated by $V_{R_{23}} = 20\ km/h$. Similarly, the EESs emerging from the simulation agree with those of the previous section: $EES_{re_2} \approx 6\ km/h$ and $\widehat{EES}_{fr_1} \approx 35\ km/h$.
- The overall deformation energy of the first impact is about 53 kJ, the impulse exchanged is 9872 Ns, considering qualitatively a shock time of 0.10 s, the impact force is 10063 kg, considering the mass of the vehicles, these experience an average acceleration, in modulus, respectively of 10.3 and 6 g, respectively for the Fiat and BMW.

  The deformation energy emerging from the simulation is in good agreement with that calculated analytically $\widehat{E}_{d_{12}} \approx 50\ kJ$.
- After the first collision, the BMW is pushed against the Alfa. The linear velocity of the BMW decreases from 21 to about 9 km/h, with a velocity variation of about 12 km/h and an EES of about 10 km/h. The Alfa's linear velocity increases from 0 to about 14 km/h, with an EES at the rear of about 11 km/h.

  Theses velocity variations are in line with those analytically provided by eq. (8), which are −11 km/h and 13 km/h, respectively, for the BMW and Alfa.
- The overall deformation energy of the second impact is about 12.7 kJ, the impulse exchanged is 5460 Ns, considering qualitatively and conservatively a shock time of 0.10 s the impact force is 5565 kg, considering the mass of the vehicles, these experience an average acceleration, in module, of 3.3 and 3.8 g, respectively for BMW and Alfa.

Fig. 10. Crash test from the database https://crashdb.agu.ch/details.php?crash_id=152.

The EES and strain energy emerging from the simulation for the second impact agree with the analytical estimate $E_{d_{23}} = 11\ kJ$.

The comparison shows that all kinematic and energy parameters agree with the analytical estimates made previously: in fact, considering that the EES at the front of the Fiat emerging from the simulation is 36 km/h, the *xEES* is equal to 6, namely almost five times larger than the minimum value (1.3) to belong to the third group defined in section 2.3. The *xEES* evaluated from the simulation falls within the third group and clearly indicates a chain collision involving a column of moving vehicles.

The results of the simulation confirm that the damage at the front of the Fiat and at the rear of the BMW are far lower than those expected in a chain collision triggered by Fiat, to justify the push of the BMW against the Alfa and the corresponding damages. Following the results in Table 1, a chain collision triggered by the Fiat would have caused an impulsive force ("Impulso") on the first crash (see subplot on top-left, row 21st) of 9871 Ns, considering a collision time of 0.1 s, which is typical of car crash phenomena [14,15], the impact force is 98710 N, namely 10 tons in the first collision. Considering that the mass of the Fiat is 975 kg (see Table 1, subplot on bottom-left, Total weight), this vehicle would have experienced a mean deceleration of about 10 g (10.32 g exactly). These large values are not related to the minimum damages observed at the front of the Fiat, since they would have caused its destruction, with activation of the airbags, nor they can be related to the modest damages to the rear of the BMW, which should instead present marked deformations of the tailgate sheet, with significant involvement of the bumper and of the underlying structural elements, as observed in

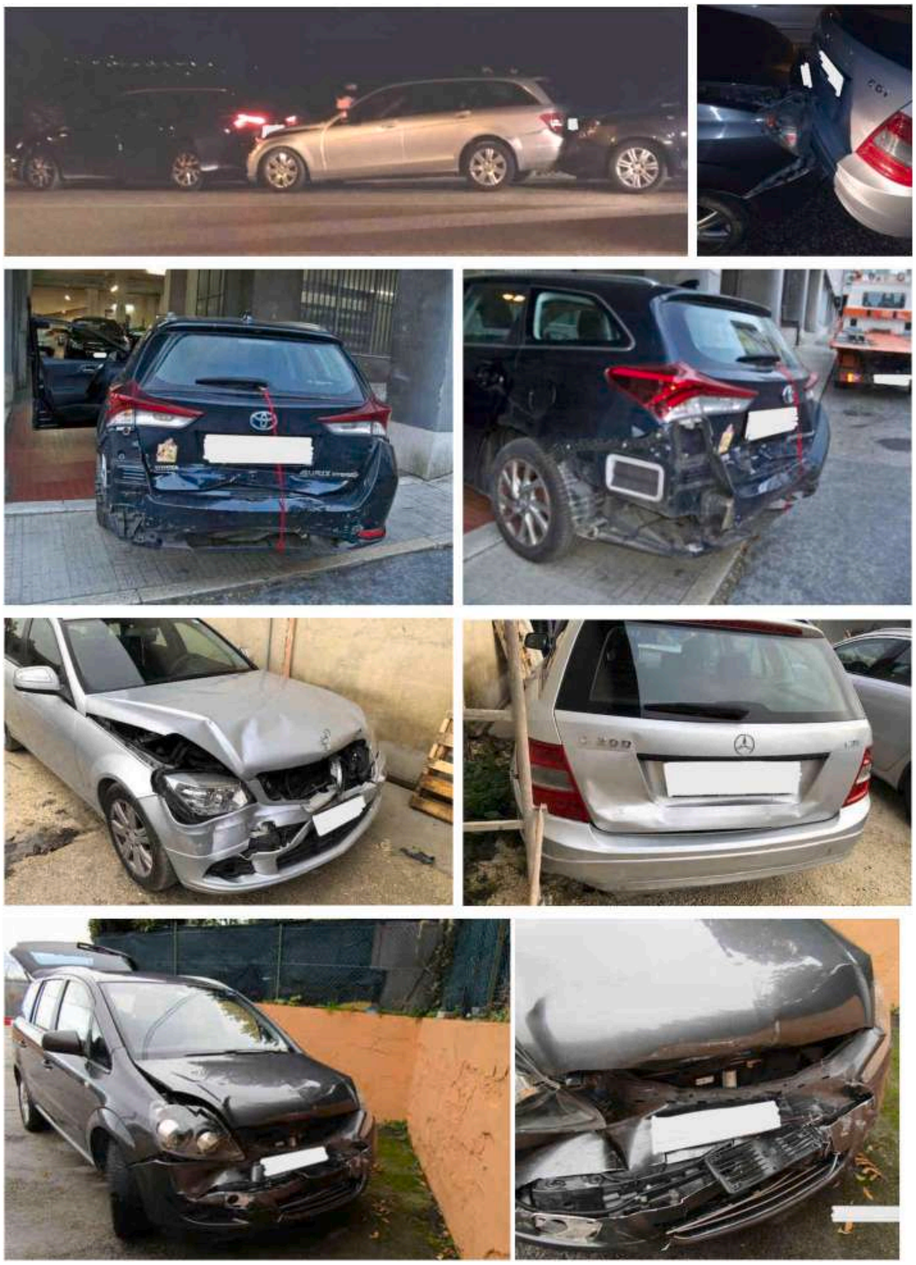

Fig. 11. On top, the static position of the vehicles, on centre and bottom, the damaged of the third, second and first vehicle of the queue.

similar accidents.

3.3. Additional application of xEES to real life pileups

Two further case studies are here considered. The first example involves a chain reaction car accident triggered by a Ford, at the rear of the queue, which collided against the rear end of a parked Range Rover, pushing it against a VW, parked as well; Fig. 9 on top shows the scene of the accident just after the crash. The photos and the damage items in the repair invoices illustrate that the front of the Range Rover and the rear of the VW have reported light-damages, mainly involving the bumper, the underlying crossbar, fenders, and some non-structural accessories. While the rear of the Range Rover has suffered mild-damages, with involvement of the bumper, tailgate, fenders, inner lining, crossbar, and of numerous accessories. Due to the vertical misalignment of the bumpers, the upper half of the front of the Ford has reported a severe compression, with the radiator pushed against the engine.

The methodology outlined in section 2.2 it is here applied. Starting from the Range Rover – VW impact, and with the aid of literature data from crash test database https://crashdb.agu.ch/, the EES associated to the damages at the rear of the VW and at the front of the Range Rover are, respectively, of 8 and 7 km/h.

The deformation energy of the second collision is carried out through eq. (10): $E_{d_{23}} = \frac{m_2 EES_{fr_2}^2 + m_3 EES_{re_3}^2}{2}$, where subscripts 2 and 3 are for the Range Rover and VW, m is their mass of 1850 and 1305 kg respectively, with no occupants on board. With the estimated EES, the above formula provides an overall deformation energy of 6.7 kJ.

Setting $\varepsilon_{23} = 0.25$, eq. (6) is applied to assess the relative velocity, which produces: $V_{R_{23}} = \sqrt{2E_{d_{23}} \frac{(m_2+m_3)}{m_2 m_3} \frac{1}{(1-\varepsilon_{23}^2)}} = 16\ km/h$. The velocity variations provided by eq. (8) are −8 km/h and 11 km/h, respectively, for the Range Rover and VW. Since the VW is parked, the relative velocity coincides with the velocity of impact of the Range Rover and represents the upper limit for the velocity with which the Range Rover

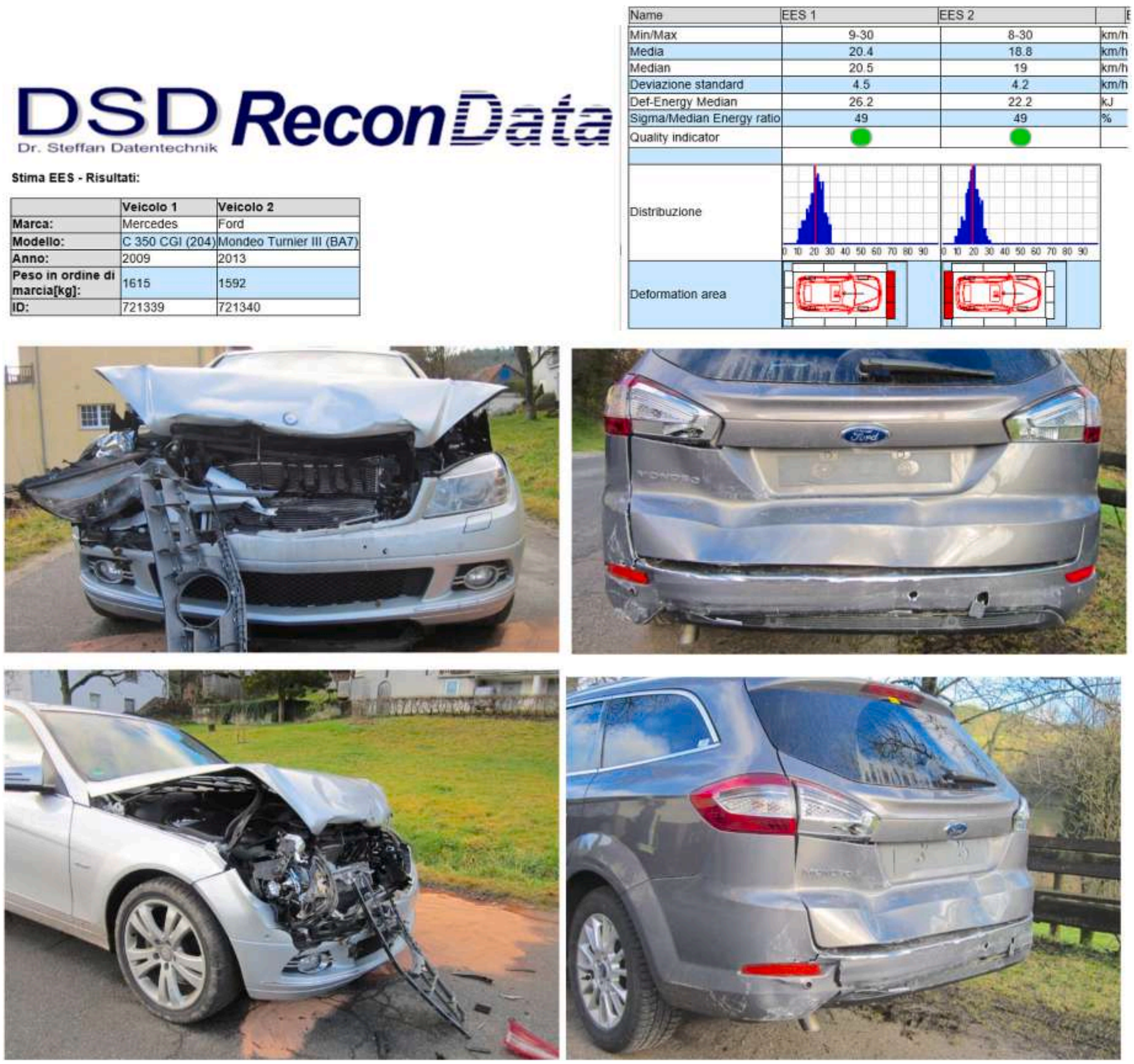

Fig. 12. Crash test from the DSD database, http://www.dsd.at/EESEstimation/Results.php?CaseID=209.

has come out from the first rear-end collision.

Going backwards and analyse the first rear-end collision. At this point it is appropriate to estimate the EES associated to the rear end of the Range Rover, and this will be done considering the accident HS 61 of the CrashDB Database, here summarised in Fig. 10 https://crashdb.agu.ch/details.php?crash_id=152. By comparison, the damage at the rear of the Range Rover is associated with an EES of about 13 km/h, thus the reference EES can be assumed $EES_{re_2} \approx 13\ km/h$. The damaged status at the front of the Ford, of mass 1200 kg, is higher than the one of the Nissan in Fig. 10, thus it is associated an $EES_{fr_1} = 22\ km/\ h$.

From the second eq. (7) it holds: $V_{R_{12}} = \Delta V_2 \frac{m_1+m_2}{m_1(1+\varepsilon_{12})}$, where in this case subscripts 1 and 2 are for Ford and Range Rover; $\varepsilon_{12} = 0.10$ to keep into the account the high plasticity of the first impact, and with $\Delta V_2 = 16\ km/h$, from the previous point. The solution produces $V_{R_{12}} \approx 37\ km/h$, which also represents the impact velocity with which the Ford hit the parked Range Rover.

It is analysed if this value is energetically congruent with the actual damages. Let indirectly evaluate the deformation energy by eq. (6) $\widehat{E}_{d_{12}} = V_{R_{12}}^2 \frac{m_1 m_2 (1-\varepsilon_{12}^2)}{2(m_1+m_2)} \approx 38\ kJ$. For what shown, the reference EES can be assumed $EES_{re_2} \approx 13\ km/h,$ the expected EES at the front of vehicle (1) is: $\widehat{EES}_{fr_1} = \sqrt{\frac{2\widehat{E}_{d_{12}} - m_2 EES_{re_2}^2}{m_1}} \approx 24\ km/h$. With these values, $xEES = 24/22 \approx 1.1$, that falls indeed within the first group defined in section 2.3, associated with an accident triggered by the first vehicle.

One last example is considered, a multiple rear-end collision with conflicting declarations of drivers. The cars involved are shown in Fig. 11, an Opel Zafira at the rear of the line, a Mercedes C200 SW, in the middle, and a Toyota Auris at the head of the line. The driver of the Mercedes declares that he was hit by the Opel, while stationary, and pushed against the Toyota, while the driver of the Opel stated he had rear-ended the Mercedes in a second time, after that the Mercedes had already rear-ended the Toyota. Applying the concept of *xEES,* the following analysis is devoted to understanding which version is true.

Consider a chain reaction car accident, the deformation energy of the collision Mercedes–Toyota is evaluated with the aid of literature data from crash test database in Fig. 12. The damage of the vehicles tested in Fig. 12 are similar of those of the vehicles in question. The resulting EES at the front of the Mercedes and at the rear of the Toyota are therefore 20 km/h each.

The deformation energy of the second collision is carried out through eq. (10): $E_{d_{23}} = \frac{m_2 EES_{fr_2}^2 + m_3 EES_{re_3}^2}{2}$, where subscripts 2 and 3 are for the Mercedes and Toyota, m is their mass of 1600 and 1465 kg respectively, with occupants on board. The above formula provides an overall deformation energy of 47 kJ.

Setting $\varepsilon_{23} = 0.15$ to consider the high plasticity of the impact eq. (6)

is applied to assess the relative velocity: $V_{R_{23}} = \sqrt{2E_{d_{23}}\frac{(m_2+m_3)}{m_2 m_3}\frac{1}{(1-\varepsilon_{23}^2)}} = 41\ km/h$. The velocity variations provided by eq. (8) are -16 km/h and 18 km/h, respectively, for the Mercedes and Toyota. Considering the latter vehicle stationary, the relative velocity coincides with the velocity of impact of the Mercedes and is the upper limit for the velocity with which the Mercedes would have come out from the first rear-end collision.

Going backwards and analyse the first rear-end collision, the damages at the rear end of the Mercedes are of the order than that of the Ford tested in Fig. 12, which has been attributed an EES of 19 km/h, thus the reference EES can be assumed, at maximum, $EES_{re_2} \approx 20\ km/h$. In a similar fashion, at the damages on front of the Opel, of mass 1600 kg, can be associated an $EES_{fr_1} = 21\ km/h$, of the order of the EEs in front of the Mercedes in Fig. 12.

From the second eq. (7) it holds: $V_{R_{12}} = \Delta V_2 \frac{m_1+m_2}{m_1(1+\varepsilon_{12})}$, where the subscripts 1 and 2 are for Opel and Mercedes; $\varepsilon_{12} = 0.20$, and with $\Delta V_2 = 41\,\frac{km}{h}$, from the previous point. The solution produces $V_{R_{12}} \approx 68\,\frac{km}{h}$ which, considering the Mercedes stationary, would be the impact velocity with which the Opel hit the Mercedes. This value is high and should have caused very serious damage both to the front of the Opel and to the rear of the Mercedes, as appreciable by the application of *xEES*. The deformation energy by eq. (6) is $\widehat{E}_{d_{12}} = V_{R_{12}}^2 \frac{m_1 m_2 (1-\varepsilon_{12}^2)}{2(m_1+m_2)} \approx 137\ kJ$. For what shown, the reference EES can be assumed $EES_{re_2} \approx 20\ km/h$, the expected EES at the front of vehicle (1) is: $\widehat{EES}_{fr_1} = \sqrt{\frac{2\widehat{E}_{d_{12}} - m_2 EES_{re_2}^2}{m_1}} \approx 43\ km/h$. With these values $= 43/21 \approx 2$, that falls well within the third interval defined in section 2.3, therefore this *xEES* cannot justify an accident triggered by the first vehicle. In this circumstance, to validate an expected EES of 43 km/h, the Opel should exhibit the marked deformation of the entire front, for over 20 cm, with damages up to the wheels and front suspensions, as happens for VW and Seat vehicles in the crash test of Fig. 7, which have masses and stiffness comparable to those of Opel. On the other hand, even considering the severity of the damages at the front of the Opel, these are energetically lower than those expected. In short, $xEES = 2$ highlights that the Opel hit the back of the Mercedes when this vehicle had already hit the rear of the Toyota.

This last example highlights the utility of the nondimensional coefficient *xEES*, since the marked damages reported by the Opel at the front end could have deceived even an experienced engineer, about the possibility of a chain collision triggered by the first vehicle, if he had not performed an accurate energy analysis of the accident.

4. Concluding remarks

The paper presents an analytical technique that permits to assess the correct sequence of impacts in pile-up using only rough information concerning the damages of the vehicles, in absence of any information about the accident scene, such as the static positions of the vehicles or the identification of the impact points on the ground. More specifically, an indicator called *xEES* is found, which is the ratio between the expected EES at the front of the first vehicle of the queue requested for a chain reaction car accident divided by the EES associated to the actual damages at the front of the first vehicle. If *xEES* is roughly equal to one, then the accident has been triggered by the first vehicle, otherwise and if $xEES > 1.3$ the first crash cannot have pushed the second vehicle against the others and must necessarily have happened in a second moment, or the vehicle in the middle would still have collided with the vehicle in front even if it had not been rear-ended by the first vehicle of the queue. The proposed model has been tested on real car crashes, analytical results have been numerically validated with the software PC-Crash: the comparison have shown a satisfying match between analytical and numerical results, and confirmed the statistical validity of the three intervals of variation defined for xEES.

The analysis has shown how the use of the coefficient *xEES* leads the engineer to assess the correct liabilities in pileups. While collisions involving only two vehicles showcase obvious liability, the correct identification of liabilities is much more complex in pileups. From a jurisprudential perspective, the main distinction relies indeed on the chain collision that occurs in a column of moving vehicles from the case in which a vehicle coming from behind collides with a series of resting vehicles. In this framework, the disposal of *xEES* easily allows to assess the type of collision and, in summary, allows to associate the correct liabilities among the drivers.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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