Digital Reconstruction of the Paradox—Escher's Relativity

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1 Escher's Relativity: From Perception to Paradox

To talk about Escher and the paradox that the latter pursued throughout his artistic career, it is necessary to focus on the meaning of simulation of reality. The latter is defined by G. Bettetini as an imitation, a representation, a reproduction, but also a fiction, a deception: "The art of simulation involves the executive ability of the portrait, the statue, the scenographic setting, of the representation of the idea; but also, that of cheating, of stratagem. Furthermore, to complicate things, the temporal component of the root simul is added, which opens the spaces of meanings in play towards perspectives of contemporaneity and, metaphorically, of quantitative equivalence: the simulacrum, the fictitious reconstruction of reality, seem to be valid as much and perhaps more than reality itself […]" [1]. The *trait d 'union* between the real and its representation, therefore, materializes in the dematerialization of the

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tangible, simulacrum of things capable of leading us towards a world neither true nor false, however illusory.

The art of simulation as just defined, sees the natural implication of the works of the engraver and graphic designer Maurits Cornelis Escher, a Dutch artist known in the last century for his representations with logical irregularities [2], capable of involving the perceptive illusion, aesthetic, and perspective, making use of visual and mathematical expedients [3]. As argued by D'Isa: "Escher prefers to unmask the mechanisms of language without offering a substitute, to force the viewer to admire the naked inconsistency of the rules" [4]. If his first works can be understood as the masterful use of traditional drawing techniques, it is with time that aesthetic gratification gives way to a deeper language, extremely rooted in the laws that govern the world. The artistic characterization that unites all his works created from 1946 to 1955 can be defined as the period of perspective paintings, in which Escher manifests a profound interest in simple geometric figures -regular polyhedrons, spatial spirals, Moebius rings- and the fusion of the latter towards infinite interpretations. In general, the ultimate objective of all the works associated with this period is to generate in the viewer a sense of vertigo and insecurity, alternating points of view with multiple vanishing points, useful to restore an unreal unity.

The well-known lithograph *Relativity* (Fig. 1), produced in 1953, is defined as a particularly fitting example, belonging to this type of representation. The environment represented is characterized by seven differently oriented scales, in which the laws of physics are overturned, causing different gravitational sources to act in space [5]. The composition is characterized, in the top left corner, by a landing oriented according to the observer's vertical, delimited by a balustrade, and framed by an arch, from which it is possible to glimpse an external space defined by sparse vegetation. Here, a small tree and two plants are struggling to grow, while a vase is placed on the wall. In the same frame, an animated mannequin leans against the wall on the left and a couple walks outside, defining the first as an impossible world.

In contrast to the gravitational force associated with the upper part of the work, in the lower part of the drawing, we notice a mannequin climbing horizontal stairs from the observer's point of view. Having reached the top of this first ramp, turning left, and placing a garden in front of it, the mannequin placed in the center of the work can therefore continue its journey along two new stairs, one to its left and the

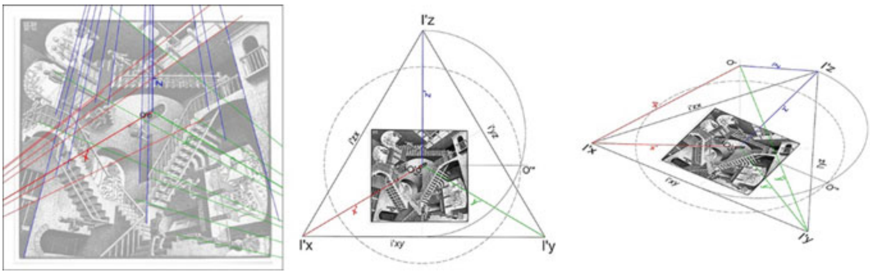

Fig. 1 Phases of perspective decoding. *Image* The authors

other to its right. Each of these scales is used by other creatures, placed in a world where gravitational forces change, forces opposed to the first creature. On the right staircase, in the world opposite to the central creature, we find another mannequin intent on carrying a tray with a glass and a bottle. In this same world, we find several other mannequins: one sitting next to a staircase, intent on reading a book; a second one is climbing a ladder; a third one and a fourth one are having lunch on the table located outside [6].

In Relativity, Escher, therefore, uses three different vanishing points -lying outside the depicted space and therefore capable of generating a unified vision in the viewer-configured as belonging to the different impossible worlds, to give life to a unitary representation capable to represent, simultaneously, three distinct worlds. Each group of mannequins inhabits one of the three worlds, perfectly coherent with themselves, in which the spaces and beings are characterized by an unreal everyday life, in contrast to the paradox of the work. The doors for some mannequins become trap doors for others, the walls are transformed into floors, the ceilings into walls, stairs that can be walked on both sides, that is, solutions capable of creating disorientation and confusion in the viewer, as well as a technique extremely connected to perception.

The paradox of the image comes into play in the ability to understand and define the simulation of reality, i.e. cognitive illusions in which the geometry of the image is erroneously perceived. Perception therefore becomes the process of being deceived, through which the acquisition of the image is deformed. But before analyzing the paradox, it is necessary to delve deeper into the process of image acquisition and how this can be modified according to our perception.

It is Gestalt that underlines how we cannot perceive everything about an image, as the human eye is created to organize and process information as quickly as possible [7]. Through a series of rapid involuntary movements, the eye manages to capture a first summary vision of the real or pictorial space, only subsequently entering the details. Knowledge cannot be broken down into simple elements, therefore the totality of what is perceived is defined not by the sum of the individual parts but rather by the sensory activations that the individual parts arouse side by side, in a complex totality [8].

The visual paradox comes into play in this process, distinguishable into optical paradox, caused by purely optical phenomena independent of human physiology; perceptive paradoxes, generated by the physiology of the eye; and cognitive paradoxes, the result of the brain's interpretation of images, in this case, impossible figures and perspective paradoxes. It is precisely the impossible figures that characterize Relativity and Escher's entire work, through which he confirms how the perceptions received, clearly in contrast with the laws of physics and three-dimensional construction, cannot be corrected in any way by the intellect in how much the brain modules act independently of each other, linking the drawing and the reality we see indissoluble. The representation of impossible worlds therefore manages to deceive the brain, a receptor of the eye's distorted stimuli, making the representation of the impossible possible, placing preconceptions in the foreground instead of reality.

In this sense, as supported by Maria Teresa Tuccio, professor of physics for the biological sciences: "Perception is a reconstructive simulation generated by the brain, under the control of a genetic determinant, of the interactions between us and the material environment that surrounds us and based on our knowledge and our previous experiences: what is perceived is different from the external object it represents. With a beautiful expression of neurolinguistic programming, we can say: the map is not the territory, and each of us builds different maps of the same territory and also different maps from moment to moment, based on our level of attention, our needs, our motivations" [9].

2 Extended Reality (XR): Between Real and Virtual

As just discussed, Escher's illusion is based on the different perceptions of users concerning the paradoxical space created [10], a concept with enormous potential if transposed into a three-dimensional space based on immersive reality. More specifically, the term Extended Reality (XR) is an all-encompassing term, capable of encompassing and combining augmented reality, virtual reality, and mixed reality experiences.

Extended reality allows the perception of reality to be changed concretely, moving the user towards a new level of perception. Using XR, it is possible to find oneself in a virtual world or to interact in a virtually augmented world, with content perceived in a way that is closely associated with reality. It is, therefore, a question of the fusion of real and virtual worlds, through which to produce new environments and visualizations, in which physical and digital objects coexist and interact in real-time.

In Escher's duality between reality and perception, the XR is configured as the optimal tool, through which to project the observer into a virtual reality, superimposing the three-dimensional model of the paradox in a real space, generating several different impossible worlds. As claimed by Escher: "two inhabitants of two different worlds" [11], that is, belonging to the same representation but to different spatial configurations, cannot be considered lying on the same floor as they do not have the same concept of what is vertical and what is horizontal [12]. Therefore, the observer, depending on the point of view from which he will make the surrounding space his own, will see an above and a below always different, and never reconcilable with the rest of the users of the surrounding environment, making the two realities differ, although coinciding in time and space. In the environment used in AR, this phase shift will therefore be highlighted by the transposition of the lithography into real space, calling into question the concept of gravitational force in the case of uses using viewers.

The process of mutation of reality therefore begins, from lithography to real space, which can be summarized in three specific steps, detailed in the next paragraph: perspective analysis of the work and creation of the digital model; transposition of the digital model into a real scale perfectly coinciding with real space; testing and visualization of the model using smartphones (AR) or viewers (XR). In this process, XR is configured as the tool through which to project the user into a world in which the laws of physics change, at the same time making the work of art more comprehensible and increasing its interest.

3 Geometric Reconstruction

The purpose of the proposed experimentation arises from the desire to leverage the semi-immersive experience of extended reality to immerse the user in Escher's paradoxical space. To achieve this goal, the first step involves formulating a three-dimensional model that corresponds to the illusory image created by the artist.

To do this, it is necessary to carry out a perspective decoding of the image, recognizing three-dimensional architectural elements in the drawing with constructive logic; in this way, it is also possible to verify the architectural space and its spatial coherence. As Vincenzo Fasolo [13] asserts in his critical reading of architectural perspectives, the two-dimensional images under analysis are, in all respects, architectures and can be studied as such.

When addressing the theme of the interpretation of Architectural Perspectives [14], two classes of objectives for the restitution of a perspective image are identified: the first, of a disseminative nature, aims to highlight the theoretical reasons and scientific knowledge surrounding perspective and its history, such as the identification of geometric construction, the peculiarities of a particular work, and the solutions employed by the author to achieve a convincing degree of illusory spatiality. The second class of objectives, which is informative in nature, uses three-dimensional modeling to reconstruct space, architecture, and the various elements represented in the two-dimensional image, also identifying the best position from which, when observed, the work assumes its maximum illusory power.

During the process of perspective decoding and three-dimensional modeling, it is essential to explore the interpretative choices that have shaped the representation of Escher's space. The interaction between theory and practice clearly emerges in how the conceptions of Escher's paradoxical spaces have guided the decisions made in creating the model. A critical analysis of these choices can provide a further understanding of the challenges and opportunities encountered in translating a two-dimensional work into a three-dimensional environment, emphasizing the synergy between artistic vision and the technical concreteness of spatial interpretation.

Although the creation of the perspective image and the reverse process of restitution are widely recognized and have already been illustrated and discussed in numerous studies, it is deemed necessary to declare the principles and methods that will be employed in the subsequent phases.

Perspective is a method of representation, "capable of constructing an image that simulates the visual perception that humans have of the space around them (…) perspective restores space" [15], allowing the representation of a three-dimensional space on a surface called the picture plane. The relationship between the model and its represented image is bijective, as it is possible to transition from space to its representation and vice-versa.

The theoretical process supporting the transition from image to representation is known as the inverse case of perspective and enables perspective restitution. This process comprises the identification of a few essential elements: identification of classes of parallel lines and subsequent identification of vanishing points and vanishing lines, allowing the identification of a perspective system; identification of the projection center in space as a consequence of identifying all geometric entities related to it, such as the principal point and the distance circle. These three elements, belonging to the picture plane, are sufficient to identify the projection center in space. Starting by considering what is depicted in perspectives as an isotropic architectural space, a configuration has been hypothesized in which the three classes of lines, converging respectively at three distinct vanishing points, form a tri-orthogonal spatial reference system.

The work under examination features an inclined perspective; having identified the vanishing points of the three main directions, x, y, and z, it is immediately possible to formulate the position of the principal point O'0, the orthocenter of the triangle of vanishing planes. Despite the lithograph not being of large dimensions, surprisingly, even at this stage, the coherence of all examined perspective directions emerges, providing a certain data point for perspective decoding (Fig. 1).

To recreate the depicted space in illusory perspectives, it is necessary to understand the issue of model indeterminacy [16]. In other words, in a perspective image, there are infinite spatial configurations that satisfy it; therefore, it is essential to identify one of the possible models through a hypothesis and base the restoration on it.

In the case under consideration, dealing with a perspective with a tilted frame where the position of the projection center relative to the perspective reference system is unambiguously identifiable, the only variable to consider is the position of the picture plane. In this case, the arbitrary choice of the picture plane's position does not determine the morphology of the final model but only its scale of representation [15]. Thus, an intermediate position, spanning the model, was chosen to use the lines' traces in the image as reference points.

The next phase involves identifying the projection center in space and the resulting three directions through the projection lines of the x, y, and z axes. This reconstructed perspective system allows to formulate spatial hypothesis for the represented space using the three-dimensional model. For constructing the represented space, one can envision a simplified model where each element is akin to a parallelepiped volume consisting of edges. Decomposing the image's complexity into simpler geometric elements enables working part by part to recompose the initial complexity.

Once the edges to be restored are identified, the reconstructive investigation takes place directly in space by constructing projection planes generated from the projection center, passing through the investigated edges, and extending to intersect the corresponding volumes in space.

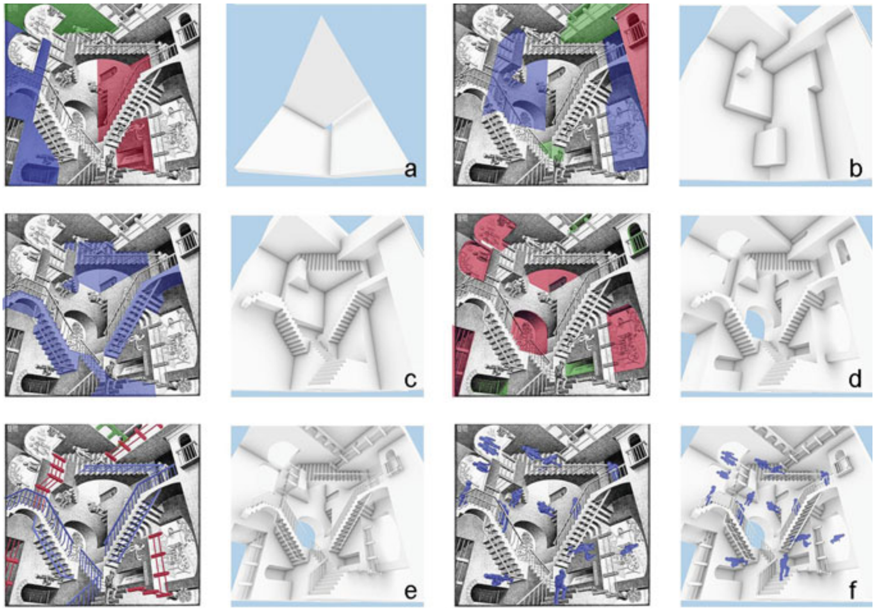

Fig. 2 Decomposition of typological elements for the composition of the three-dimensional model. *Image* The authors

The model construction began by identifying three main volumes determining the spatial configuration (Fig. 2a). From these main volumes, the space was further articulated with secondary structures (Fig. 2b). Once the spatial configuration was established, continually determining spatial limits through the investigation of perspective restitution, typological elements were identified and investigated as characterizing subgroups of space. Escher's presented environment consists of typologically recurring elements such as stairs, handrails, doors, and large openings.

If the articulation of spatial volumes alone couldn't answer whether the artist created a realistic space, certainly the spatial transposition of stairs into the digital model begins to configure a space that can be traversed and explored (Fig. 2c). The analysis of these elements, in fact, revealed no significant spatial incongruities, except those arising from graphic approximation. Nevertheless, a perfect overlap between image and the model was achieved. Even the restoration of doors and openings doesn't show strong inconsistencies, except for the lack of correspondence of rounded arches: the curves describing curved elements in the image do not correspond to conical curves [17], so their projection in space does not match a circumference (Fig. 2d). Finally, handrails were investigated. These elements, in complexity, articulation, and graphic approximation, deviate the most from the original image but, given their role, do not influence the spatial articulation and perceptual component of the created environment (Fig. 2e).

Mannequins animating the space were not included because, in recreating a life-sized immersive environment, their presence in the model might risk creating a sense of dissociation from the sought illusory effect in the semi-immersive experience (Fig. 2f).

The result of the restoration generates a model showing an excellent adherence to the perspective image (Fig. 3), whose constructive logic has been extended even beyond what is visible in the representation (Fig. 4a). However, to achieve this correspondence, the model exhibits large spatial extensions, especially to describe areas glimpsed beyond the large openings (Fig. 4b).

These extensive dimensions are not easily manageable when considering the purpose for which the model was created, i.e., visualization within a real environment with dimensions that, although ample, are finite. Therefore, a version of the model has been formulated for this purpose, limiting spaces in a more compact configuration.

Fig. 3 Overlay between perspective image and three-dimensional model. *Image* The authors

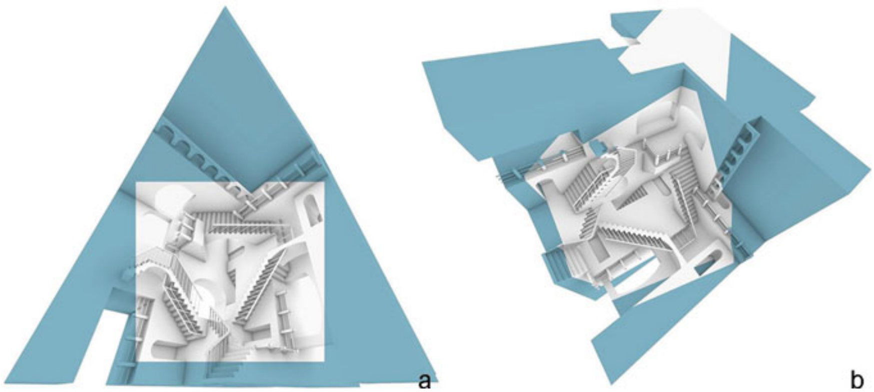

Fig. 4 Completion of the model beyond the limits of the image (left) and extension of the perspective space observed from an external viewpoint (right). *Image* The authors

The outcome of the perspective restoration operation reveals a striking spatial coherence in the resulting space, without significant perspective contractions or dilations. The formulated model provides an architecturally coherent space that can be virtually explored without the need for further transformations. The architectural coherence of the model is not the only noteworthy aspect of its realization.

The spatial transposition adds a layer of complexity and a fascinating paradox. The model not only restores a three-dimensional image coherently from an architectural standpoint but also preserves the paradoxical nature typical of Escher's works. Its versatility is evident in the fact that it can be viewed and explored coherently with different orientations. This phenomenon adds a dynamic dimension to the virtual experience, as users can discover new perspectives and spatial perceptions, emphasizing the continuous resonance with Escher's poetics. The model's ability to maintain its visual integrity from different angles further enriches the immersive experience, allowing observers to delve into an architecturally metamorphic world. Its adaptability underscores the mastery in interpretation and fidelity to Escher's original vision, making the model not only an expression of technical expertise but also of a profound understanding of illusionistic art (Fig. 5).

4 Experimentation: Augmented Reality

Once the NURBS model was completed, a mesh transformation was carried out with a low number of polygons but with particular attention to the correct direction of the normals, and its possible representation with materials was evaluated. Unfortunately, it was found that a model that was too similar to the framed environment would have been less faithful to the original lithograph, while on the contrary, a material similar to the lithograph would have been unrealistic and in any case an imperfect adaptation.

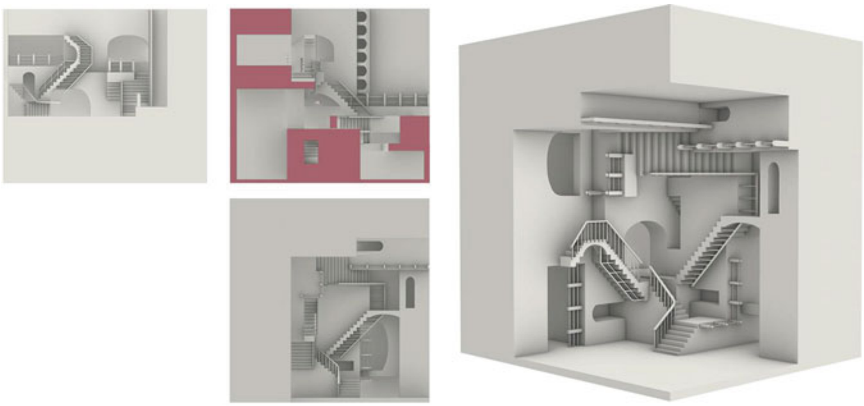

Fig. 5 Model with optimized space for visualization in AR. *Image* The authors

For this reason, a clay version was chosen as the first case study (Fig. 6). The app used to trigger the model in AR is Augment, which allows the room plans' tracking without the use of markers or targets, and a useful model-sharing function for any tests.

Normally, the content of immersive reality is placed as a specific object smaller than the user, which appears when a target is triggered. Thanks to the tracking of the walls of the Augment app, it was, therefore, possible to create digital objects to appear even above or far from the target, and in this way reproduce an entire environmental envelope that entirely covers the room. In this way the content differs

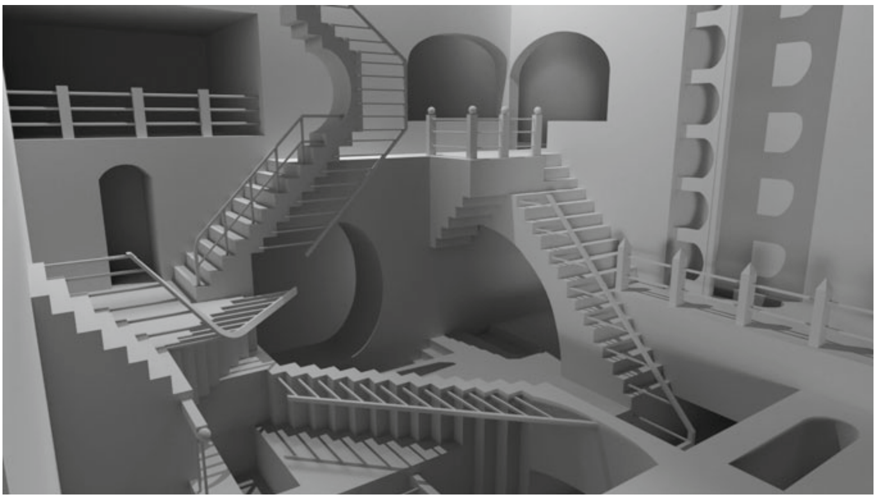

Fig. 6 Rendered model, clay version. *Image* The authors

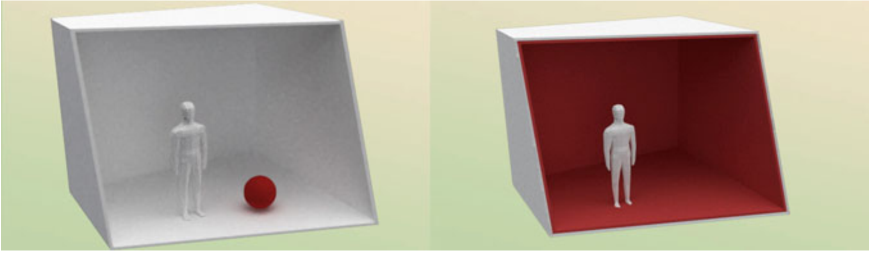

Fig. 7 AR use dynamics compared: in red the virtual content displayed, which in common AR is a small object, and in this case a shell of the environment itself. *Image* The authors

from common AR in that it is indoor, and in showing the user large-scale objects that he can walk through himself (Fig. 7).

The resulting experience is therefore achieved in the use of architectural environments in a way like that of VR, which completely immerses the user in an artificial environment, stimulating new sensations and interest. Figure 8 shows the comparison between the user and his point of view. Of particular interest is the interaction with the stairs, and with the voids on which it is possible to lean freely, but above all with the paradoxical experience of walking upside down streets or walking on balustrades sideways. Likewise, it is possible to fly over the models to observe them in their entirety.

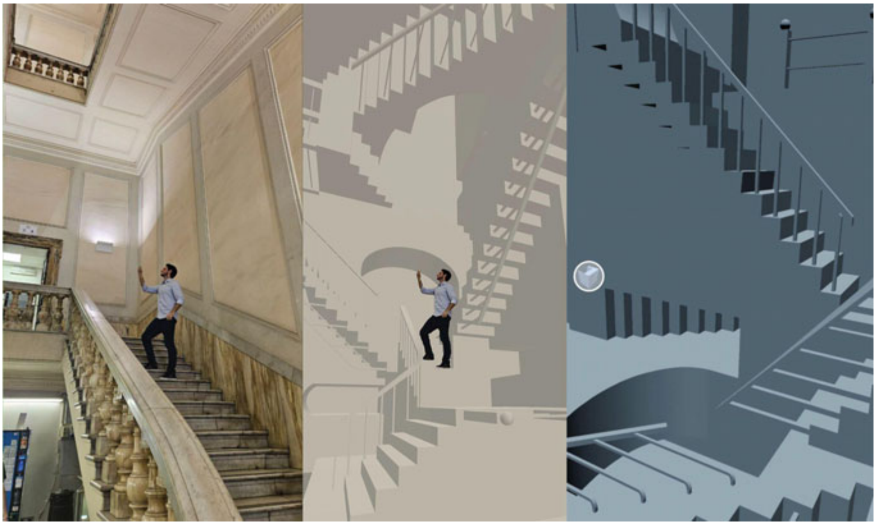

Fig. 8 Comparison between the observer in the real environment, in the virtual one, and his point of view. *Image* The authors

5 Test and Results

The experience produced was then tested by a group of students in two different locations of the Faculty of Architecture in Rome, and finally, they were subjected to a survey using questionnaires. The questions mainly revealed satisfaction, the sensations encountered, the likelihood of the original, and technical feedback on the experience.

The model seemed fairly similar to the original and produced good involvement according to the students themselves (Fig. 9).

As regards the sensations aroused (Fig. 10), a clear prevalence is found on disorientation and surprise, perhaps also linked to technological innovation, while secondarily curiosity and interest were found. These were actually the most coveted parameters in the creation of the prototype, demonstrating the success of the model, while negative feelings such as fear or boredom were not expressed by anyone.

Less success and great margins for improvement were highlighted instead in the adaptation of the model to the surrounding environment (Fig. 11): this problem can be attributed to the complexity of the work and its distance from reality, but greater experimental work and level of detail can certainly fill this void.

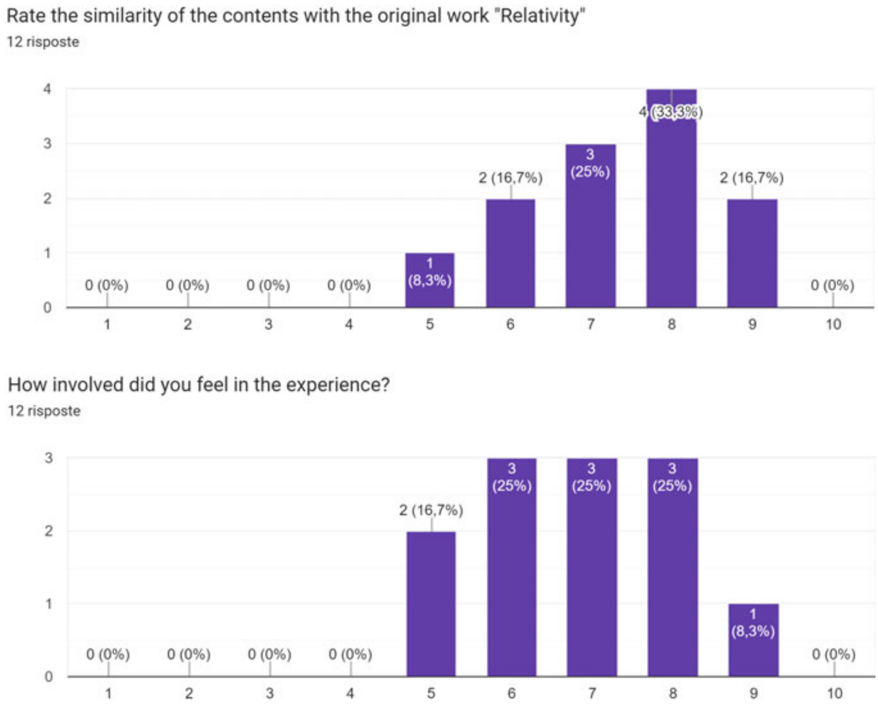

Fig. 9 Answers on verisimilitude and involvement. *Image* The authors

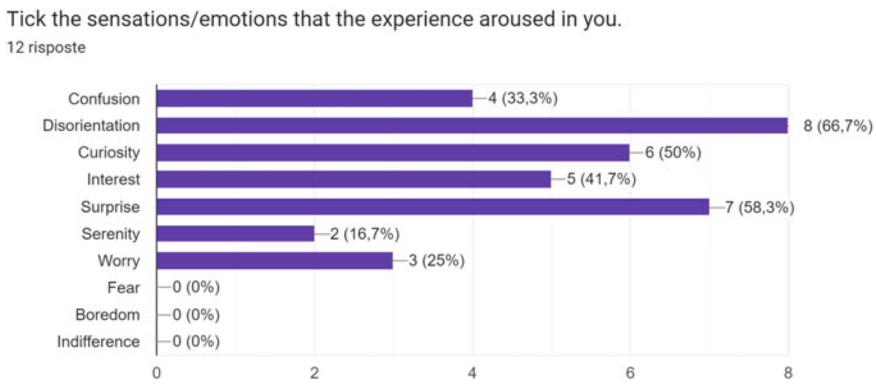

Fig. 10 Answers on sensations and emotions felt during the experience. *Image* The authors

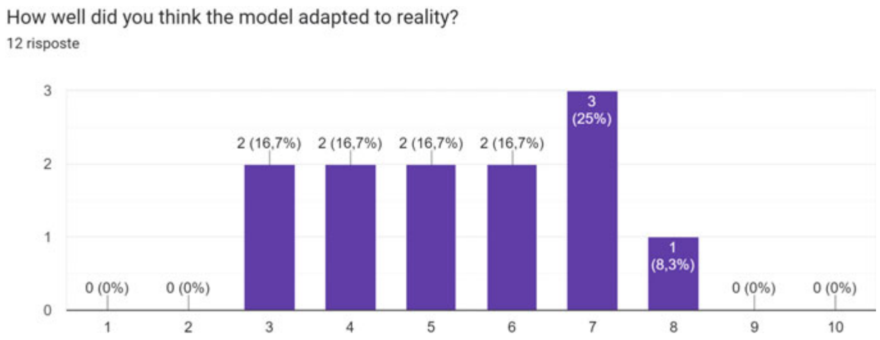

Fig. 11 Answers on the model adaptation. *Image* The authors

Finally, as regards the perception of the media as actual augmented reality (Fig. 12), a difference was actually perceived compared to its more canonical use, and an approach to virtual reality. This observation technique can offer various ideas and possibilities for the reconstruction and visualization of indoor architectural environments.

6 Conclusions

In conclusion, the geometric method for building a rigorous model worked perfectly, and the shapes were respected according to perspective theory. The 3D model imported into AR made it possible to demonstrate the potential of this innovative method of use, as a shell of pre-existing architectural environments, which could be helpful in many other cases of reconstruction. The theme of paradox in this experience

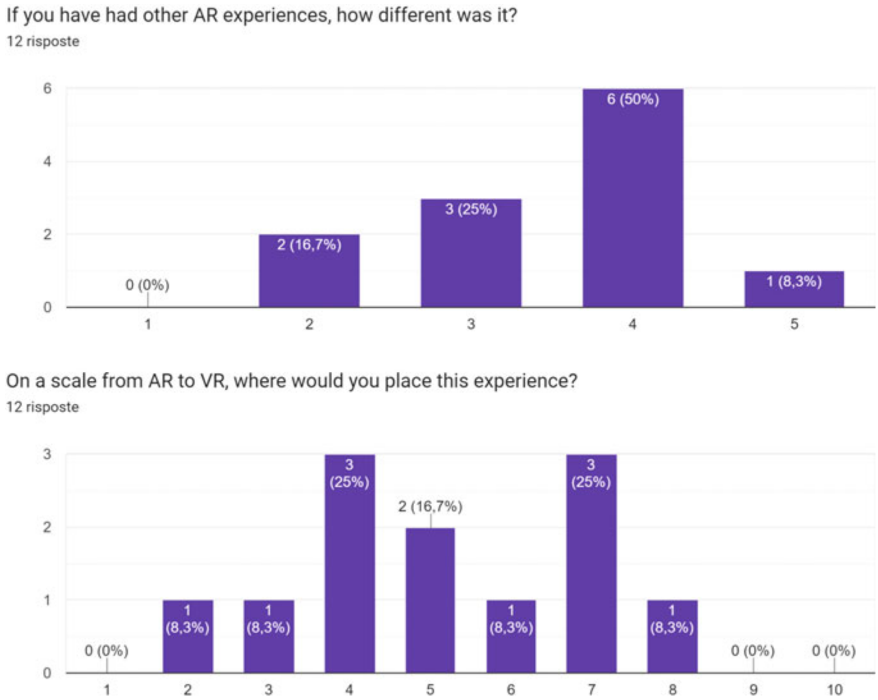

Fig. 12 Answers on the virtual method perceived. The applied technique, despite being AR, is very reminiscent of the sense of immersion experienced in VR. *Image* The authors

allowed us to generate involvement and interest in the students, as well as disorientation and amazement. As for the future potential of research, it will be useful to implement real paradoxical portals in an immersive environment that connect various sections of the model, in order to complete the experience.

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References

1. Bettetini, G.: La simulazione visiva. Inganno, finzione, poesia, computer graphics. Bompiani, Milano (1991)
2. Taschen (ed.): MC Escher: the graphic work. Taschen, Colonia (2021)
3. Schattschneider, D.: The mathematical side of MC Escher. Notices Am. Math. Soc. **57**(6), 706–718 (2010)
4. D'Isa, F.: Paradossi della visione: L'arte di M. C. Escher. https://www.indiscreto.org/paradossi-escher/. Last accessed 21 Aug 2023

5. Hofstadter, D. R., Trautteur, G.: Gödel, Escher, Bach. Un'eterna ghirlanda brillante. Una fuga metaforica su menti e macchine nello spirito di Lewis Carroll. Adelphi, Milano (1990)
6. Falletta, N.: Il libro dei paradossi. Longanesi, Milano (2002)
7. Kanizsa, G.: Grammatica del vedere. Saggi su percezione e Gestalt. Il Mulino, Bologna (1997)
8. Zerbetto, R.: La Gestalt. Terapia della consapevolezza. Xenia, Pavia (1998)
9. Cella, S.: Creatore di spazi, 1st edn. Youcanprint, Italia (2015)
10. Arnheim, R.: Arte e percezione visiva. Feltrinelli, Milano (2008)
11. Escher, M. C.: Grafica e disegni. Benedikt Taschen, Colonia (1990)
12. Ernst, B.: Lo specchio magico di M. C. Escher. Taschen, Colonia (2007)
13. Fasolo, O.: Illusioni prospettiche unitarie d'architettura (Architettura prospettica nella Wandmalerei e Deckenmalerei). XY, dimensioni del disegno **16**, 83–85 (1992)
14. Baglioni, L., Migliari, R., Fasolo, M.: Sulla Interpretazione delle prospettive architettoniche. In: Bertocci S., Bini, M. (eds.) Le ragioni del disegno-The reasons of drawing. Atti del 38° Convegno internazionale dei docenti della rappresentazione, pp. 1027–1032. Gangemi Editore, Roma (2016)
15. Migliari, R.: La prospettiva: una conversazione su questioni solo apparentemente banali. In: Carlevaris, L., De Carlo, L., Migliari, R. (eds.) Attualita della geometria descrittiva, pp. 99–142. Gangemi Editore, Roma (2012)
16. Casale, A.: Forme della percezione dal pensiero all'immagine. Franco Angeli, Milano (2018)
17. Di Paola, F.: Le curve di Apollonio. Tradizione e innovazione nei processi risolutivi. Aracne, Roma (2010)