

Precise orbit determination for the orbital phase of BepiColombo

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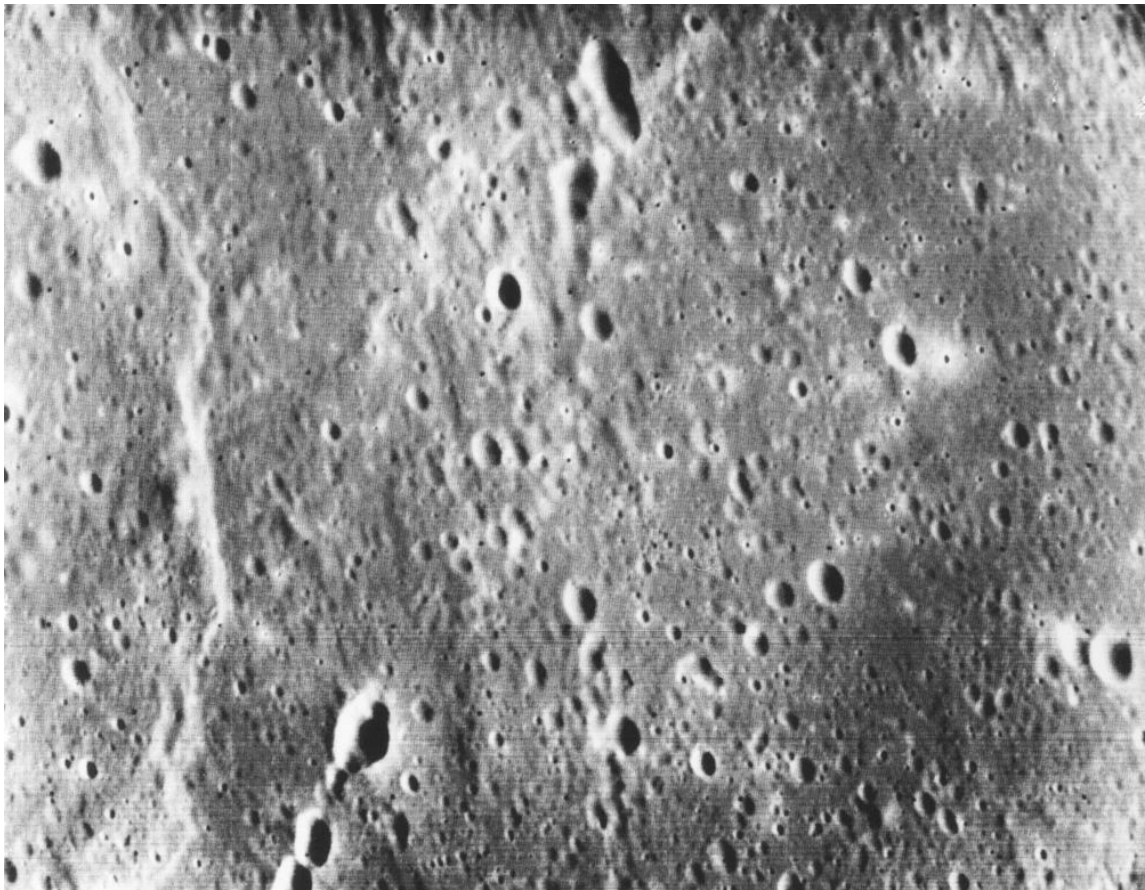
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Mercury at first encounter closest approach from Mariner 10 first flyby.

Credit: NASA/JPL.

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Abstract

The BepiColombo spacecraft, designed by ESA and JAXA, is expected to begin its scientific phase in orbit around Mercury in early 2027. Among the scientific investigations is the Mercury Orbiter Radio-science Experiment (MORE), which will exploit a multi-frequency microwave tracking system with an advanced Ka-band transponder to achieve its objectives related to Mercury's geodesy and fundamental physics. Building on the legacy of the MESSENGER mission, MORE aims to expand and refine our knowledge of the planet's geophysical properties.

This thesis supports the preparation of the MORE geodesy investigation during BepiColombo's orbital phase through three complementary tasks. First, an independent reanalysis of the complete MESSENGER two-way Doppler radiometric dataset is carried out to provide a reliable basis for a future joint analysis with MORE data. Second, numerical simulations of MORE are conducted to evaluate its performance under realistic dynamical modelling errors and in combination with MESSENGER data. Finally, a ray tracing software package for advanced modelling of non-gravitational accelerations is validated, demonstrating its potential for the orbit determination of future missions such as BepiColombo.

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List of Acronyms

AU	Astronomical Unit
CMD	Commanded Momentum Desaturation
DPU	Data Processing Unit
DSA	Deep Space Antenna
DSN	Deep Space Network
DST	Deep Space Transponder
ESA	European Space Agency
FBA	FanBeam Antenna
GR	General Relativity
G/S	Ground Station
HGA	High-Gain Antenna
IAU	International Astronomical Union
ICRF	International Celestial Reference Frame
IERS	International Earth Rotation Service
IR	Infrared Radiation
ISA	Italian Spring Accelerometer
JAXA	Japan Aerospace Exploration Agency
KaT	Ka-band Transponder
LGA	Low-Gain Antenna
LRO	Lunar Reconnaissance Orbiter
MCB	Mantle-Crust Boundary
MCS	Mercury Composite Spacecraft
MDIS	Mercury Dual Imaging System
MGA	Medium-Gain Antenna

MLA	Mercury Laser Altimeter
MLI	Multi-Layer Insulation
MMO	Mercury Magnetospheric Orbiter
MORE	Mercury Orbiter Radio-science Experiment
MOSIF	Mio Sunshield and Interface Structure
MPO	Mercury Planetary Orbiter
MTM	Mercury Transfer Module
NASA	National Aeronautics and Space Administration
NGA	Non-Gravitational Acceleration
OCM	Orbit Correction Maneuver
OD	Orbit Determination
PAA	Phased-Array Antenna
PDS	Planetary Data System
PLL	Phase-Locked Loop
PN	Pseudo Noise
RMS	Root Mean Square
RS	Radio Science
RTLT	Round-Trip Light Time
SCE	Solar Conjunction Experiment
SEPS	Solar Electric Propulsion System
SNR	Signal to Noise Ratio
SPE	Sun-Probe-Earth angle
SRP	Solar Radiation Pressure
SSA	Solid State Amplifier
SSC	Superior Solar Conjunction
TRP	Thermal Recoil Pressure
TSI	Total Solar Irradiance
TTC	Telemetry, Tracking and Commands
TWTA	Travelling Wave Tube Amplifier
USN	Universal Space Network
USO	Ultra-Stable Oscillator
WOL	Wheel Off-Loading

Introduction

Mercury exhibits many unique characteristics that distinguish it from all terrestrial planets and the Moon. Its high bulk density, anomalously large iron-rich core, the unexpected presence of a magnetic field, and the peculiar 3:2 spin-orbit resonance are fascinating features for investigations into planetary formation and evolution. Moreover, Mercury is the innermost and fastest planet of the Solar System, thus the most affected by relativistic corrections in its orbital dynamics. It is therefore an excellent laboratory to carry out tests of Einstein's general relativity. However, despite the exceptional scientific interest, the planet remained relatively unexplored due to the significant technical challenges associated with approaching and orbiting a body so close to the Sun. The large energy required for the orbital transfer and the harsh thermal and radiation environment impose severe constraints on spacecraft design and mission operations.

Until recently, only two missions had visited Mercury. Mariner 10 performed three flybys in 1974–1975, providing the first images of the planet from space and discovering its intrinsic magnetic field. More than three decades later, NASA's MESSENGER mission entered orbit around Mercury in 2011, operating until 2015. MESSENGER returned a wealth of data on Mercury's surface composition, exosphere, magnetosphere, and gravity field, leading to major advances in our understanding of the planet. However, many fundamental questions remain open, particularly concerning Mercury's internal structure, rotational state, thermal evolution, and the origin of its magnetic field.

Some of these questions can be addressed through precise gravity measurements which, in accordance with the equivalence principle, require determining the relative acceleration between two bodies. In planetary geodesy, the spacecraft acts as a test mass in free fall within the combined gravity field of the planet and other Solar System bodies, while the ground antenna on Earth is the fiducial reference point. Radio signals exchanged between the two provide radiometric observables, which allow to determine the spacecraft orbit and derive parameters relevant to Solar System dynamics, tests of relativistic gravity, planetary geodesy, and the study of planetary atmospheres and ionospheres. This type of investigation is sometimes considered a radio science (RS) experiment due to its use of radio tracking systems.

The primary radiometric observables used to determine the orbit of the spacecraft are range and Doppler. These are generated by establishing microwave radio links between the spacecraft and the ground stations and are related to their distance and relative radial velocity. More precisely, these observable quantities are the time of flight of radio signals and its time derivative. Over the years, improvements in onboard and on ground radio systems have substantially enhanced the measurement accuracy, thus leading to a much increased scientific return from planetary missions. ESA's BepiColombo mission adopts a very advanced tracking system, needed to bring the geodesy and geophysics investigations of Mercury to a far more advanced level. The mission comprises two spacecraft: the Mercury Planetary Orbiter (MPO), developed by ESA, and the Mercury Magnetospheric Orbiter (MMO), developed by JAXA. Following an eight-year interplanetary cruise involving multiple planetary flybys and long periods of thrust with an electric propulsion system, BepiColombo is expected to enter orbit around Mercury in late 2026, with nominal science operations beginning in April 2027.

The MPO hosts a suite of eleven scientific instruments, including the Mercury Orbiter Radio-science Experiment (MORE), devoted to geodesy, and geophysics. In addition, MORE has carried out tests of general relativity and alternative theories during six superior solar conjunctions (SSC) in the cruise phase, as will further extend such tests in the orbital phase. The experiment relies on an advanced multi-frequency tracking system, including a dedicated, fully digital Ka-band transponder (KaT). By combining X band (7.2

GHz - 8.4 GHz) and Ka-band (34.4 GHz - 32.2 GHz) radio links, MORE data will be free from the noise due to solar plasma at almost all solar elongation angles, enabling range and Doppler measurements with unprecedented accuracy. Another critical aspect to accomplish the RS investigation is the accurate modelling of the spacecraft dynamics, which includes not only gravitational forces but also non-gravitational perturbations due to solar and planetary radiation pressure, as well as the recoil from thermal imbalances of the spacecraft. The presence of the Italian Spring Accelerometer (ISA) onboard MPO allows for direct measurement of these non-gravitational accelerations. However, in the event of degraded or unavailable accelerometer data, the modelling of these forces must rely on precise numerical models, which must be carefully implemented to ensure robust scientific results.

This thesis contributes to the ongoing preparation for the orbital phase of the BepiColombo mission with three complementary objectives. Firstly, it presents a complete reanalysis of the entire MESSENGER Doppler dataset. This effort yields independent estimates of Mercury's gravity field, spin axis orientation and tidal Love number (controlling the response of the body to tidal fields, mostly induced by the Sun), providing an essential step for a future joint analysis with MORE radiometric data. Then, two numerical simulations of the MORE experiment have been carried out for the orbital phase of the mission. The first one is intended to assess the reliability of the gravity estimates obtained by using numerical models for non-gravitational accelerations in the presence of modelling errors. The second simulation evaluates the benefits of combining MORE and MESSENGER datasets in a joint inversion of the gravity field. As a third objective, this work validates a software tool for improving the computation of non-gravitational accelerations due to photons and neutral particles through ray tracing methods. This step may be beneficial not only for BepiColombo, but also for future applications to Venus, Earth, the Moon, and Mars, where strong non-gravitational effects significantly influence the spacecraft dynamics, making accurate numerical modelling essential.

By combining a reanalysis of MESSENGER data with high-fidelity simulations of the BepiColombo mission and the development of advanced numerical models for non-

gravitational effects, this thesis aims to support the processing of future observations, improve the robustness of dynamical models, and hopefully contribute to a more accurate and complete investigation of Mercury's properties and its role in Solar System evolution.

This thesis is organized into seven chapters. Chapter 1 reviews the general characteristics of Mercury and the remaining open questions. Chapter 2 introduces the fundamentals of orbit determination procedures used in this work. Chapter 3 presents the dynamical and geodetic models relevant to Mercury. Chapter 4 provides an overview of the MESSENGER mission and reports the results of the reanalysis of its radiometric dataset. Chapter 5 describes the mission BepiColombo and presents updated simulations of the MORE experiment. Chapter 6 focuses on the validation of a software tool for the precise computation of non-gravitational accelerations, with an application to the Lunar Reconnaissance Orbiter. The main results are summarised in Chapter 7.

Chapter I

Mercury

Mercury is the innermost and smallest planet in the Solar System. Its unique characteristics distinguish it from all other terrestrial bodies and have always captured the interest of the scientific community. Studying Mercury is crucial for advancing our understanding of the early evolution of the inner Solar System and the formation processes of terrestrial planets. However, due to the high energy required for orbital transfer and the planet's extreme thermal and radiation environment, Mercury has remained relatively unexplored. To date, MESSENGER has been the only spacecraft to orbit the planet. Despite the major advances achieved through its observations, many fundamental aspects of Mercury remain poorly understood. To address these open questions, the dual-spacecraft BepiColombo is expected to begin its scientific operations in April 2027 to provide an exhaustive investigation of the planet.

This chapter provides an overview of Mercury's distinctive features and explains why it is a compelling target for space exploration in Section 1.1. Past missions are briefly reviewed in Section 1.2, along with a look toward the future of Mercury's exploration: BepiColombo.

I.1 Scientific background

Mercury exhibits a set of fascinating characteristics that distinguish it from other bodies in the Solar System. Its proximity to the Sun, unusual internal structure, and long-term geological and magnetic activity make it a unique object of investigation among the terrestrial planets. Understanding Mercury's interior and external environment provides crucial insights into early Solar System processes, planetary differentiation, and the evolution of rocky bodies.

I.1.1 Interior structure and rotation

The uncompressed density of Mercury, approximately 5300 kg/m^3 , is significantly higher with respect to other terrestrial planets and the Moon, suggesting a unique internal structure. As shown in Figure 1.1, Mercury is expected to have a solid silicate crust and mantle overlying a large metallic core, made of an external liquid layer that surrounds a solid inner core (Genova et al., 2019). The thickness and density of each layer for a plausible interior model are listed in Table 1.1. The crust shows estimated average thickness ranging between 26 ± 11 and 35 ± 18 km (Padovan et al., 2015; Sori, 2018) and is composed primarily of silicates enriched in sulfur, sodium, and magnesium. Beneath it lies a comparatively thin mantle, extending to depths of roughly 300–400 km, dominated by olivine (forsterite) and orthopyroxene (enstatite), and notably depleted in iron (Namur, Collinet, et al., 2016).

The core occupies nearly 80% of the planet's radius and contributes to about 55–60% of its total mass, an unusually large fraction compared to other terrestrial bodies. This has supported hypotheses involving early giant impacts or high-temperature formation near the Sun, which could have stripped away much of Mercury's original silicate mantle (Grard et al., 2000). The composition of the core remains an open question. Early models assumed that sulfur was the dominant light element alloyed with iron, as it lowers the melting temperature and could explain the persistence of a molten outer core (Hauck et

al., 2007; Rivoldini & Van Hoolst, 2013; Stevenson et al., 1983). However, geochemical evidence from MESSENGER indicates that Mercury formed under highly reducing conditions characterized by low oxygen levels (McCubbin et al., 2012; Nittler et al., 2011), favouring the incorporation of silicon into the metal phase and reducing the solubility of sulfur. Alternative models therefore favour Fe–Si or Fe–S–Si alloys with relatively low sulfur content and are consistent with geodesy data (Cartier et al., 2020; Chabot et al., 2014; J.-L. Margot et al., 2018; Namur, Charlier, et al., 2016).

Layer	Thickness (km)	Density (kg/m ³)
Solid inner core	~ 875	~ 7000
Liquid core	~ 1092	
Mantle	~ 437 km	~ 3150
Crust	~ 35 km	~ 2700

Table 1.1. Thickness and density values for a plausible interior model of Mercury (Genova et al., 2019).

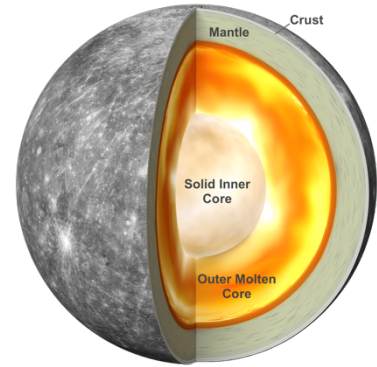


Figure 1.1. Representation of Mercury's interior (Genova et al., 2019).

The existence, size, and composition of a solid inner layer depend critically on the composition of the core. While sulfur barely partitions into solid iron, silicon does so almost equally between solid and liquid phases (Kuwayama & Hirose, 2004; J. Li et al., 2001). As a result, a silicon-rich core would yield a less dense and larger solid inner core than a sulfur-dominated one (Knibbe & van Westrenen, 2018; Rivoldini et al., 2018).

The joint analysis of the gravity field and rotation of Mercury is one of the approaches to probe its internal structure (Peale, 2005; Peale et al., 2002). Gravity measurements reveal how mass is distributed within the planet and provide a powerful tool for probing the size and state of the core when combined with the precise knowledge of its rotation (see Section 3.4.1). Complementary constraints are given by the tides and magnetic field, all of which will be measured with unprecedented accuracy by the BepiColombo mission (see Chapter 5). Although the absence of an inner core cannot be excluded (Rivoldini &

Van Hoolst, 2013), one of the latest gravity and rotation solutions favours the presence of a small solid inner core surrounded by a molten outer shell (Genova et al., 2019).

The rotation of Mercury consists of a stable 3:2 resonance revealed by radar observations in the 1960s. Under this configuration, the planet rotates three times on its axis for every two revolutions around the Sun (Pettengill & Dyce, 1965). This condition generates librations in longitude, whose amplitudes carry information about the size and state of the core. Observations of Mercury's heliocentric motion led Urbain Le Verrier to discover an anomalous precession of its perihelion in the mid-nineteenth century. This discrepancy was later explained by Einstein's theory of general relativity (GR) in 1915. Owing to its proximity to the Sun, Mercury is one of the most favourable target for testing relativistic gravity, a goal that the BepiColombo mission will pursue through its dedicated high-precision experiment.

1.1.2 Magnetic field, surface and exosphere

One of the most distinctive feature of Mercury is its global magnetic field (Johnson et al., 2012). Although relatively weak, about 1.1% the strength of Earth's, it is strong enough to sustain a well-defined magnetosphere (Grard et al., 2000). The origin of this field remains a matter of debate, but the prevailing interpretation attributes it to an active dynamo operating within the planet's molten outer core. The presence of such a magnetic field provides indirect evidence for a large, iron-rich core that must be at least partially liquid and convective to sustain dynamo action. Understanding the generation and long-term evolution of this magnetic field thus offers crucial insight into the planet's deep interior, thermal history, and present state of differentiation.

The surface of Mercury is shaped by a combination of intense impact cratering, volcanic activity, and tectonics. Multispectral imaging, spectroscopy, and topographic observations from the MESSENGER mission revealed that nearly the entire surface is a secondary crust formed by extensive partial melting of the mantle, ascent of buoyant magma, and emplacement through extrusive and intrusive volcanism between about 4.2

and 3.5 Gyr ago (Byrne et al., 2018). For this reason, the surface composition provides valuable clues about the composition and evolution of the deeper mantle source regions. Mercury's surface features both ancient intercrater plains and younger smooth plains likely formed by lava flows and pyroclastic eruptions. It is dominated by enstatite and plagioclase, with smaller amounts of sulfides like oldhamite, the presence of which is a strong clue of the extremely reducing conditions which have led to Mercury's accretion and differentiation (Weider et al., 2012). Darker regions are likely enriched in carbon, possibly graphite. Large impact basins such as Caloris, along with widespread lobate scarps and fault systems, point to a complex geological history marked by cooling and global contraction. As Mercury's interior cooled over time, the progressive solidification of its metallic core and thermal contraction of the silicate mantle led to a global decrease in radius. Early Mariner 10 images provided evidence of a total contraction of less than 1 km (Watters et al., 2004), but MESSENGER data later suggested values up to 7 km (Byrne et al., 2014).

Although long considered geologically inactive, MESSENGER revealed features that indicate Mercury may still experience surface modification today. The observations of bright and shallow depressions, or hollows, associated with the loss of volatile-rich materials, indicates relatively recent (and possibly ongoing) geological processes (Blewett et al., 2011). In addition, deposits of water ice and other volatiles have been detected in permanently shadowed polar craters, where temperatures are sufficiently low to trap them despite the planet's extreme thermal variations (Lawrence et al., 2013). Whether these volatiles represent ancient remnants or the result of more recent processes remains an open question.

Finally, Mercury possesses a thin surface-bound exosphere composed of hydrogen, helium, sodium, potassium, calcium, and other species (Domingue et al., 2007). This exosphere is highly dynamic and tightly coupled to both surface and magnetospheric processes. Its composition varies with local time, solar distance, and solar activity, implying multiple source mechanisms such as diffusion, photon-stimulated desorption, ion sputtering, and micrometeoroid vaporization. These interactions make Mercury an

important case study for understanding volatile evolution and the coupling of surface, exosphere and magnetosphere.

I.2 Missions to Mercury

The first spacecraft to explore Mercury was Mariner 10 (Figure 1.2), launched in 1973 within NASA's Mariner program (Shirley, 2003). Direct transfer to Mercury was energetically prohibitive, but the mission pioneered the use of gravity assist by exploiting Venus's gravity to reach Mercury. Under Giuseppe Colombo's guidance, the spacecraft was placed on a trajectory that enabled three flybys of Mercury between 1974 and 1975. Mariner 10 mapped about 45% of Mercury's surface, capturing over 2800 images. It revealed its heavily cratered terrain, detected a thin exosphere dominated by helium, and made the first measurements of Mercury's magnetic field and iron-rich core. It also provided the first constraints on the planet's extreme surface temperatures, magnetosphere, and surface composition, laying the foundation for future exploration.

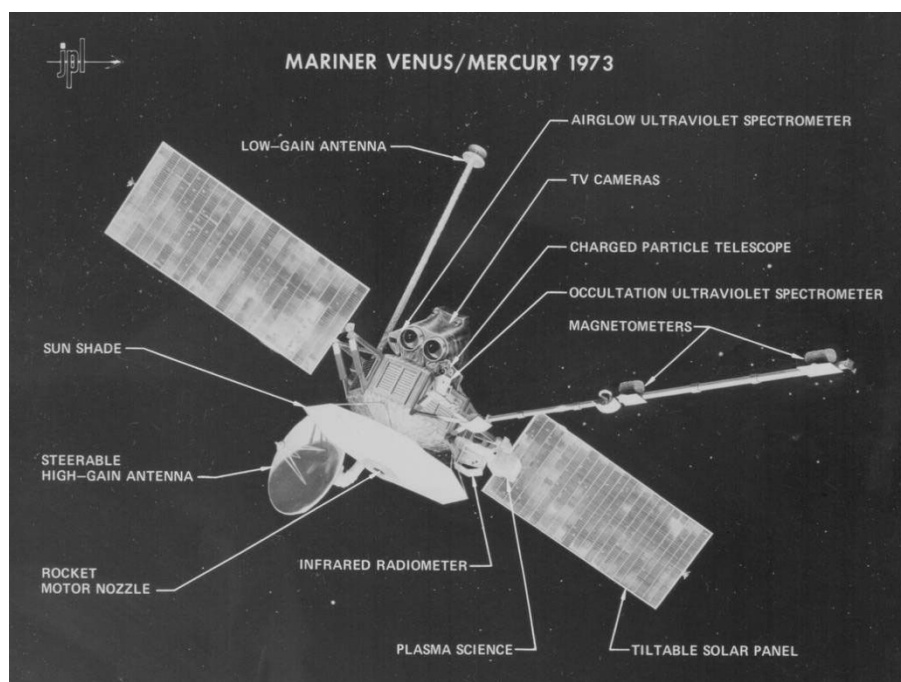


Figure 1.2. Mariner 10 spacecraft and onboard instrumentation. Credit: NASA.

More than three decades later, NASA's MESSENGER mission (Figure 1.3) was launched in 2004 under the Discovery program (Solomon et al., 2007). After a seven-year cruise it entered orbit around Mercury in March 2011. The spacecraft operated until April 2015, when it impacted the planet's surface. MESSENGER orbited in a highly elliptical, near-polar trajectory with pericenter near 200 km altitude at high northern latitudes and apocenter at $\sim 15,000$ km. Its design was driven by the harsh thermal environment, with innovative solar panels partly covered by mirrors for thermal control, passive cooling techniques, and electronically steered phased-array antennas, the first used in deep space. Equipped with eight instruments, MESSENGER revolutionized our knowledge of Mercury by probing its gravity field and rotation, providing geochemical and topographic data, as well as mapping its magnetic field and exosphere. A more detailed discussion of the mission design, payload and scientific results is provided in Chapter 4.

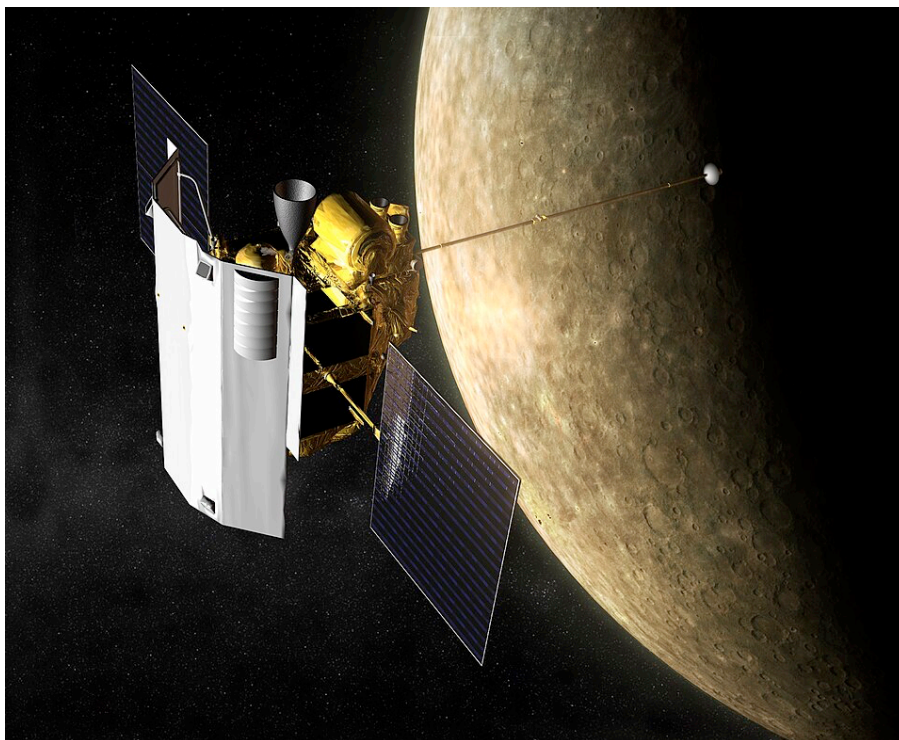


Figure 1.3. Artist's rendering of MESSENGER orbiting Mercury. Credit: NASA.

To solve the unveiled mysteries of Mercury, the BepiColombo spacecraft, jointly designed by ESA and JAXA, was launched in October 2018 on an Ariane 5 rocket (Benkhoff et al., 2021). It consists of two orbiters: ESA's Mercury Planetary Orbiter (MPO), dedicated

primarily to study Mercury's surface and interior, and JAXA's Mercury Magnetospheric Orbiter (MMO, also known as Mio), focused on the planet's magnetic environment and its interaction with the solar wind. The two spacecraft are transported by the Mercury Transfer Module (MTM), which provides propulsion and power during the cruise phase (Figure 1.4).

After a complex eight-year interplanetary transfer involving nine planetary flybys (one of Earth, two of Venus, and six of Mercury), the two probes will separate and enter their respective orbits around Mercury in late 2026. They carry a comprehensive suite of instruments aimed at exploring the planet's interior structure, surface composition, exosphere, and magnetosphere. The combination of high-resolution imaging, altimetry, spectroscopy, and radio science is expected to provide new insights on Mercury's properties. The mission's coordinated dual-orbiter configuration will, for the first time, permit simultaneous observations of Mercury's surface and magnetosphere, providing detailed studies of the coupling between the planet and its surrounding environment. In addition, the MPO will improve upon the spatial coverage and data accuracy achieved by MESSENGER, particularly in the southern hemisphere. A more detailed description of the mission and its scientific objectives is presented in Chapter 5.

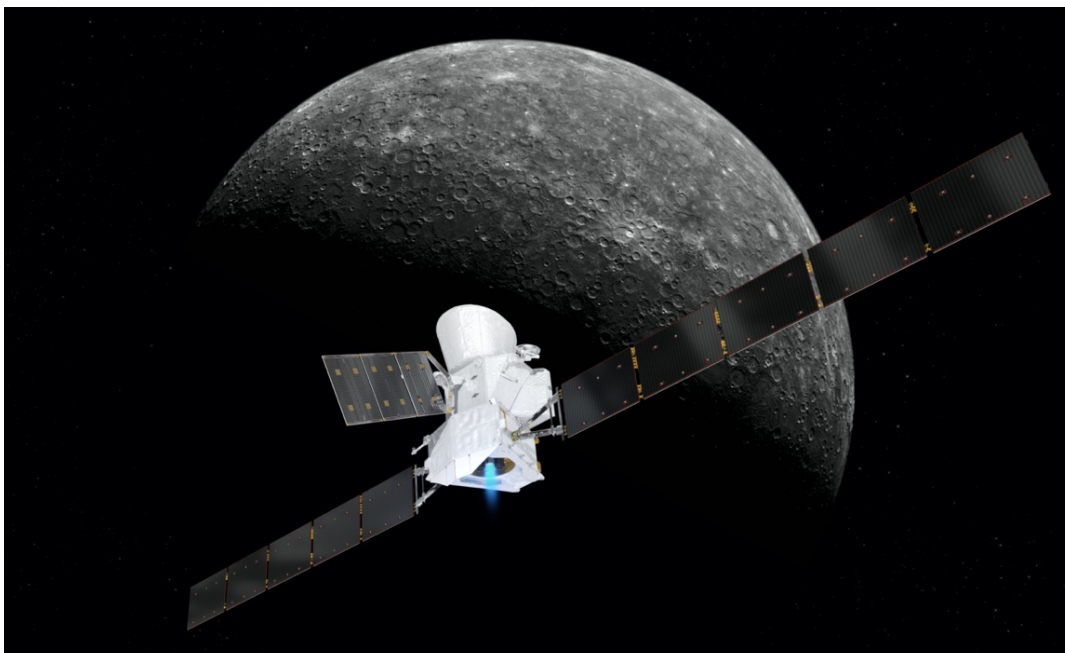


Figure 1.4. Artist's rendering of BepiColombo approaching Mercury. Credit: ESA.

Chapter 2

Orbit determination

Most of the current knowledge of planetary gravity fields and interiors has been gained through radio science experiments, in which radio observables are fitted within an orbit determination process. Orbit determination broadly refers to the methods used to infer the trajectory of an orbiting object from measured observable quantities, such as range and Doppler. In the case of Mercury, for example, these observables are used to reconstruct the spacecraft orbit around the planet. The orbit is estimated by adjusting, in a least-squares sense, quantities collected in a state vector, which enables to predict the evolution of the system in time by integrating the equations of motion. The minimal state vector contains the object's position and velocity at a reference epoch, but it can be extended to include additional parameters that influence either the dynamics or the observations, such as the gravity field coefficients, planetary ephemerides, or relativistic parameters for tests of fundamental physics.

This chapter provides a brief overview of the orbit determination problem and the main solution techniques in Section 2.1, followed by a description of the specific methods applied in this thesis, presented in Sections 2.2 and 2.3. Section 2.4 reviews the primary radiometric observables used in orbit determination. Finally, Section 2.5 concludes the chapter by discussing the differences between real data analyses and numerical simulations.

2.1 Fundamental concepts

The main idea behind orbit determination (OD) is that observables measured at the ground stations can be compared with predictions (Tapley et al., 2004). The latter are computed by propagating the orbital motion of the spacecraft, starting from an a priori initial guess of the state vector at a reference epoch. The accuracy of the propagated trajectory depends heavily on the dynamical model, which represents the dynamical laws governing the evolution of the system in time. Since the equations are nonlinear, an iterative procedure to correct the reference state is required. By comparing the observed observables with the computed observables, a vector of observation residuals $\mathbf{y}$ (actual observables minus computed observables) is retrieved. A cost function, defined upon this vector, is minimised by iteratively correcting the spacecraft state and the other solved-for parameters in a weighted least-squares filter. The process continues until the improvement in the parameters of interest becomes smaller than a defined criterion. When convergence is reached, the estimated values are retrieved from the last iteration together with a covariance matrix that contains the associated formal uncertainties. The weighted least-squares differential correction $\hat{\mathbf{x}}_k$ to the state vector at iteration k can be expressed as:

$$\hat{\mathbf{x}}_k = (\mathbf{H}_k^T \mathbf{W} \mathbf{H}_k + \overline{\mathbf{W}}_k)^{-1} (\mathbf{H}_k^T \mathbf{W} \mathbf{y}_k + \overline{\mathbf{W}}_k \bar{\mathbf{x}}_k) \quad (2.1)$$

where the matrix $\mathbf{H}_k$ contains the partial derivatives of the observable quantities with respect to the solve-for parameters and $\mathbf{W}$ is the matrix which contains the weights associated to the observables. The quantity $\mathbf{P}_k = (\mathbf{H}_k^T \mathbf{W} \mathbf{H}_k + \overline{\mathbf{W}}_k)^{-1}$ is the covariance matrix of the estimated state vector. The square root of the main diagonal elements provides the formal uncertainties of the estimated parameters. The quantities $\bar{\mathbf{x}}_k$ and $\overline{\mathbf{W}}_k = \bar{\mathbf{P}}_0^{-1}$ represent, the a priori state deviation vector and the inverse of the a priori covariance matrix, respectively. The a priori information acts as a constraint in the estimation process, guiding the solution toward previous knowledge. Smaller a priori

uncertainties give more weight to the prior knowledge, reducing the influence of new data in the correction.

2.2 Multi-arc approach

The MESSENGER and BepiColombo analysis presented in this thesis relies on a multi-arc strategy. The idea is to reduce the effects of unmodelled accelerations by introducing an over-parameterization of the problem (Milani & Gronchi, 2010). This approach divides the spacecraft trajectory into distinct, non-overlapping arcs, each treated independently with its own estimated set of initial conditions. By handling each arc separately, errors that accumulate over time due to dynamical mismodelling can be minimised. However, this comes at the expense of discontinuities in the estimated trajectory at arc boundaries.

In a multi-arc approach, the solve-for parameters are divided into locals $\mathbf{l}^i$, pertaining only to arc i , and globals $\mathbf{g}$. Local parameters, such as the spacecraft position, velocity, and velocity increments from maneuvers, are estimated independently for each data arc. In contrast, global parameters, like the harmonic coefficients of the gravity field, remain constant across all arcs. For each set of observables $\mathbf{y}^i$ belonging to the arcs, the design matrix $\mathbf{H}$ that links the observable residuals to the vector of solved-for parameters correction is expressed by distinguishing between the contributions of local and global parameters. At iteration k :

$$\mathbf{y}_k^i = \begin{bmatrix} \mathbf{H}_{k,l^i}^i & \mathbf{H}_{k,g}^i \end{bmatrix} \begin{pmatrix} \hat{\mathbf{l}}_k^i \\ \hat{\mathbf{g}}_k \end{pmatrix} \quad (2.2)$$

where $\hat{\mathbf{l}}_k^i$, $\hat{\mathbf{g}}_k$ are the differential corrections to the local and global parameters, respectively. If the trajectory is divided into r arcs, the matrix $\mathbf{H}_k$ of Eq. (2.1) assumes a block structure, where each local block $\mathbf{H}_{k,l^i}^i$ is independent of those of other arcs:

$$\begin{pmatrix} \mathbf{y}_k^1 \\ \mathbf{y}_k^2 \\ \vdots \\ \mathbf{y}_k^r \end{pmatrix} = \begin{bmatrix} \mathbf{H}_{k,l^1}^1 & 0 & \cdots & 0 & \mathbf{H}_{k,g}^1 \\ 0 & \mathbf{H}_{k,l^2}^2 & \cdots & 0 & \mathbf{H}_{k,g}^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{H}_{k,l^r}^r & \mathbf{H}_{k,g}^r \end{bmatrix} \begin{pmatrix} \hat{\mathbf{l}}_k^1 \\ \hat{\mathbf{l}}_k^2 \\ \vdots \\ \hat{\mathbf{l}}_k^r \\ \hat{\mathbf{g}}_k \end{pmatrix} \quad (2.3)$$

The technique is called “pure” multi-arc strategy when the arcs are handled as completely independent from each other. This means that the trajectories corresponding to different arcs are effectively treated as they would belong to distinct objects.

2.3 Inter-arc constraints

In a “pure” multi-arc strategy the trajectory is discontinuous in phase space, with discontinuities occurring at the transition points between arcs. However, a constrained multi-arc approach can be applied, where the differences in position and velocity between consecutive arcs are constrained within predefined thresholds (Milani et al., 1995). Let $\mathbf{r}_T^i$ and $\mathbf{r}_T^{i+1}$ the spacecraft positions at the transition time T of two subsequent arcs, the continuity condition is imposed by inter-arcs constraints as:

$$\mathbf{r}_T^i - \mathbf{r}_T^{i+1} = (\mathbf{r}_T^i - \bar{\mathbf{r}}_T^i) - (\mathbf{r}_T^{i+1} - \bar{\mathbf{r}}_T^{i+1}) \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_i) \quad (2.4)$$

where $\bar{\mathbf{r}}_T^i, \bar{\mathbf{r}}_T^{i+1}$ are a priori values that satisfy the continuity condition. The residual is modelled as a zero-mean multivariate normal distribution with covariance matrix $\boldsymbol{\Sigma}_i$. Assuming independent components with equal variance σ_i^2 , the covariance matrix becomes $\boldsymbol{\Sigma}_i = \sigma_i^2 \mathbf{I}$. The left-hand side of Eq. (2.4) is written making use of the partial derivatives of the position with respect to the state vector parameters, collected in a state transition matrix:

$$\begin{aligned}
\frac{\partial \mathbf{r}_T^i}{\partial \mathbf{x}^i} \partial \bar{\mathbf{x}}^i - \frac{\partial \mathbf{r}_T^{i+1}}{\partial \mathbf{x}^{i+1}} \partial \bar{\mathbf{x}}^{i+1} &= \frac{\partial \mathbf{r}_T^i}{\partial \bar{\mathbf{l}}^i} \partial \bar{\mathbf{l}}^i + \frac{\partial \mathbf{r}_T^i}{\partial \bar{\mathbf{g}}} \partial \bar{\mathbf{g}} - \left(\frac{\partial \mathbf{r}_T^{i+1}}{\partial \bar{\mathbf{l}}^{i+1}} \partial \bar{\mathbf{l}}^{i+1} + \frac{\partial \mathbf{r}_T^{i+1}}{\partial \bar{\mathbf{g}}} \partial \bar{\mathbf{g}} \right) = \\
&= \frac{\partial \mathbf{r}_T^i}{\partial \bar{\mathbf{l}}^i} \partial \bar{\mathbf{l}}^i - \frac{\partial \mathbf{r}_T^{i+1}}{\partial \bar{\mathbf{l}}^{i+1}} \partial \bar{\mathbf{l}}^{i+1} + \left(\frac{\partial \mathbf{r}_T^i}{\partial \bar{\mathbf{g}}} - \frac{\partial \mathbf{r}_T^{i+1}}{\partial \bar{\mathbf{g}}} \right) \partial \bar{\mathbf{g}}
\end{aligned}$$

Analogous relations stand for the velocity components. The inclusion of the inter-arcs constraints for each pair of subsequent arcs allows to obtain the continuity of the trajectory to a given tolerance. The optimal constraint values are determined through a trade-off between maintaining solution stability and ensuring that the constraints effectively reduce discontinuities. This strategy will be used in Section 5.3 for the OD of BepiColombo to estimate the wheel off-loading maneuvers not covered by tracking data.

2.4 Observable quantities

Spacecraft tracking with microwave links provides the fundamental observables used for navigation and OD, primarily range and range-rate (Doppler). These observables are obtained by transmitting and receiving radio signals between the spacecraft and ground stations, which encode information on the distance and relative radial velocity of the spacecraft. Depending on how the reference frequency is generated and how many stations are involved, different tracking configurations can be employed. In one-way tracking, the downlink carrier is generated by the spacecraft onboard frequency standard, typically an Ultra-Stable Oscillator (USO). In two-way coherent tracking, a signal generated at the ground station is sent to the spacecraft, coherently retransmitted back, and measured again at the same station, ensuring that both uplink and downlink are tied to the same high-performance ground frequency standard (usually a hydrogen maser). This configuration does not require a stable oscillator onboard (as in GNSS), but only a transponder characterized by a small Allan deviation over the relevant observation times (see Section 2.4.3). This device applies a coherent shift of the signal phase by a fixed turn-around frequency ratio to avoid interference between the uplink and downlink carriers.

In three-way tracking, the uplink and downlink involve different ground stations, which is sometimes necessary for distant probes since the spacecraft may go out of sight due to Earth's rotation. Deep space tracking relies almost exclusively on the two-way coherent configuration, since it provides the highest accuracy in both range and Doppler observables. For this reason, all analyses presented in this thesis are based exclusively on two-way radiometric measurements. Details on the hardware implementation, signal processing chain, and advanced techniques such as Delta-DOR and VLBI are beyond the scope of this work (see Asmar et al., 2005; Iess et al., 2014; Thornton & Border, 2000).

2.4.1 Range

The spacecraft range is obtained from the measurement of the round-trip light time (RTLT) between transmission and reception of a known signal modulated onto the carrier. Past ranging systems used a sequence of tones of increasing wavelengths as modulating signal for ground delay evaluation (Thornton & Border, 2000). In the coherent two-way configuration, a ranging signal generated at the ground station modulates the uplink carrier. The spacecraft transponder demodulates the carrier, recovers the uplink ranging signal and retransmits it to ground, phase modulating the downlink carrier. The time delay taken by the electromagnetic signal to travel from Earth to the spacecraft and back again represents the range measurement. Due to the periodicity of the ranging signal, an ambiguity shall be resolved. The tone at highest frequency defines the accuracy of the measurement, while the other tones are sequentially applied for ambiguity resolution. Current state of the art system, instead, are based on Pseudo-Noise (PN) codes, enabling a more efficient use of the signal power (CCSDS, 2014). This approach consists in modulating onto the carrier a random-like binary code. The code is repeated after a predefined number of bits (code length). The signal is received and demodulated by the ground station and the phase shifting is determined by correlating the received code with a replica of the uplink code. The chip length provides the accuracy in the range determination while ambiguities are resolved by the code length. BepiColombo is the first ESA mission to routinely use PN range for spacecraft navigation (Chapter 5).

The accuracy of range measurements is inherently impinged by systematic errors referred as range bias, which can be estimated within the OD. The error contributions are mainly due to delays from the propagation media (plasma, ionosphere and troposphere) and from the ground stations and onboard electronics.

2.4.2 Range-rate

Two-way range-rate is obtained by measuring the Doppler shift of a radio carrier between transmission and reception from and to a ground station over a count interval. The interval is usually in the range 5-1000 seconds depending on the setup. Longer count times improve precision but reduce spatial resolution, since the spacecraft can travel a long distance during this period in the case of strong dynamical environment. Here, a simplified derivation is reported. Considering, a one-way link sent from a source S to a receiver A , the signal can be described as a sinusoidal tone:

$$s(t) = \text{Re}[s_0 \exp[j(\omega_0 t + \phi_0)]] \quad (2.5)$$

The received signal will be delayed, attenuated and Doppler shifted:

$$s_A(t) = \text{Re} \left[s_{0A} \exp \left[j \left(\omega_0 t + \phi_0 - \mathbf{k}_A \cdot (\mathbf{r}_A(t) - \mathbf{r}_S(t - t_A)) \right) \right] \right] \quad (2.6)$$

where s_{0A} is the attenuated amplitude of the received signal; $\mathbf{k}_A$ is the wave vector of the electromagnetic signal at the receiver which takes into account also light-time and aberration; $\mathbf{r}_A$, $\mathbf{r}_S$ are the positions vectors of the receiver and the transmitter, respectively; finally, t_A is the light-time between the transmitter and the receiver. The frequency of the received signal is computed as the time derivative of its phase:

$$\phi_A(t) = \omega_0 t + \phi_0 - \mathbf{k}_A \cdot (\mathbf{r}_A(t) - \mathbf{r}_S(t - t_A)) \quad (2.7)$$

$$f = \frac{1}{2\pi} \frac{d}{dt} \phi_A(t) = \frac{1}{2\pi} \frac{d}{dt} \left(\omega_0 t + \phi_0 - \mathbf{k}_A \cdot (\mathbf{r}_A(t) - \mathbf{r}_S(t - t_A)) \right) = f_0 - \frac{\|\mathbf{k}_A\|}{2\pi} \frac{d}{dt} \rho \quad (2.8)$$

where $\mathbf{k}_A$ is assumed to be constant and colinear with $\mathbf{r}_A(t) - \mathbf{r}_S(t - t_A)$. The term ρ is the range, while its time derivative is the range-rate. Since $\|\mathbf{k}_A\| = \omega_0/c = 2\pi f_0/c$, where c is the speed of light in vacuum:

$$\frac{\Delta f}{f_0} = -\frac{\dot{\rho}}{c} \quad (2.9)$$

We can see that the range-rate is mapped into a frequency shift of the carrier. The measurement of this frequency shift provides a valuable information for the orbit determination process. For the sake of simplicity we have analysed a one-way link, but for the two-way configuration it is sufficient to sum up the uplink and downlink paths. In the simplest two-way coherent implementation, a carrier of very stable frequency f_0 is transmitted from ground toward the spacecraft. The onboard transponder receives the signal and generates a cleaned, coherent replica by means of a phase-locked loop (PLL). To avoid interference between incoming and outgoing signals, the replica is frequency-shifted by an amount K (the turn-around frequency ratio) before amplification and retransmission to ground. The ground station receives the carrier from the spacecraft and beats it against a signal of frequency $\sim Kf_0$, measuring the Doppler shift.

The high frequency stability of the hydrogen maser and of the onboard transponder, measured by their Allan deviation (see next section), are the key factors making the two-way mode superior to one-way mode. The main error sources in two-way range-rate data arise from media propagation. Dispersive delays in the solar plasma and Earth's ionosphere dominate, particularly when signals pass close to the solar corona. These effects are mitigated by using higher carrier frequencies, such as Ka-band (32–34 GHz), reduced by up to 75% with dual-link systems (Bertotti et al., 1993), and almost completely eliminated with triple-link systems combining X/X, X/Ka, and Ka/Ka measurements (less et al., 2014). Tropospheric effects are the second most significant source of error, while thermal noise becomes relevant at shorter integration times (~ 1 s) but decreases as count interval increases. A detailed discussion of Doppler error sources is given by Asmar et al. (2005) and less et al. (2014).

2.4.3 Allan Variance

The accepted measure of stability of frequency standards in the time domain is the two-sample Allan Variance (AVAR) with no dead time (Thornton & Border, 2000). This quantity is intended to estimate frequency stability in clocks, oscillators, and radio links due to noise processes or imperfections such as frequency drift or temperature effects. The AVAR is defined as follows:

$$\sigma_y^2(\tau) = \frac{1}{2} \langle (\bar{y}_{k+1} - \bar{y}_k)^2 \rangle \quad (2.10)$$

where the brackets denote ensemble averaging and $\bar{y}_{k+1}, \bar{y}_k$ are adjacent fractional frequency deviations averaged over the observation time τ :

$$\bar{y}(t, \tau) = \frac{1}{\tau} \int_0^\tau y(t + t_v) dt_v \quad (2.11)$$

In the above equation $y(t)$ is the fractional-frequency error at time t . It is defined as:

$$y(t) = \frac{f(t) - f_n}{f_n} \quad (2.12)$$

where $f(t)$ is the actual frequency of the oscillator and f_n is the nominal frequency.

The Allan deviation (ADEV) is the square root of the AVAR and it is generally indicated as $\sigma_y(\tau)$. All the error sources (clock instability, instrumental effects, transmission media) can be characterized in terms of the ADEV and the total budget gives the precision of the tracking system. A small ADEV is a characteristic of a tracking system with good stability over the measured period and translates into a small error contribution in the estimation of the spacecraft radial velocity. For example, the DSN hydrogen masers exhibit white frequency noise over time scales of approximately 60–1000 s and flicker frequency noise at longer time scales, from about 1000 s up to 12 hours (Kuhnle, 1989). For a Doppler count time, τ , which is less than the RTLT, which is itself less than 1000 s, the range-rate error contribution due to frequency instability is given by $\sigma_{vr}(\tau) \sim \sqrt{2} c \sigma_y(\tau)$, where c is the speed of light (Thornton & Border, 2000). Data analysis of Cassini (Asmar et al., 2005), Juno (Iess et al., 2018), BepiColombo (Cappuccio et al., 2020) and JUICE (Cappuccio et al.,

2025) showed that with state of the art tracking systems, proper correction of tropospheric effects (particularly those due to water vapor variability through dedicated calibrations), or favourable weather conditions, ADEV of $3\text{-}7\cdot 10^{-14}$ at 60 seconds count time are possible, corresponding to range-rate measurements accuracy at the level of $10\text{-}20\text{ }\mu\text{m/s}$.

2.5 Data analysis and simulations

The works presented in this thesis both address real data analysis and numerical simulations. The MESSENGER reanalysis (Section 4.3) and the one-month OD of the Lunar Reconnaissance Orbiter (Section 6.4) are based on the processing of available radiometric data. These represent real OD cases (see Figure 2.1), where the true spacecraft trajectory and model parameters are not known. In contrast, the BepiColombo analysis (Section 5.3 and 5.4) rely on simulated data, as radiometric observations are not yet available at the time of writing. This type of analysis can be decomposed in 2 main steps (see Figure 2.2). In the first step, synthetic observables are generated by numerically integrating the spacecraft trajectory with a chosen dynamical and observation model, which is assumed to represent the “truth”. This model defines the true trajectory and model parameter values. The simulated measurements are then perturbed with white Gaussian noise consistent with the expected tracking accuracy. The second step is the least-squares estimation, where the simulated radio observables are compared with predictions computed with a reference (“best-guess”) model. The estimation process provides reconstructed trajectories, estimated parameters and their formal uncertainties.

The BepiColombo simulations presented in Chapter 5 include two approaches:

- A perturbative analysis to simulate a realistic scenario and provide a robust assessment of the OD performance in presence of modelling errors (Section 5.3).
- A covariance analysis, which only aims at assessing the formal uncertainties on the estimated parameters (Section 5.4).

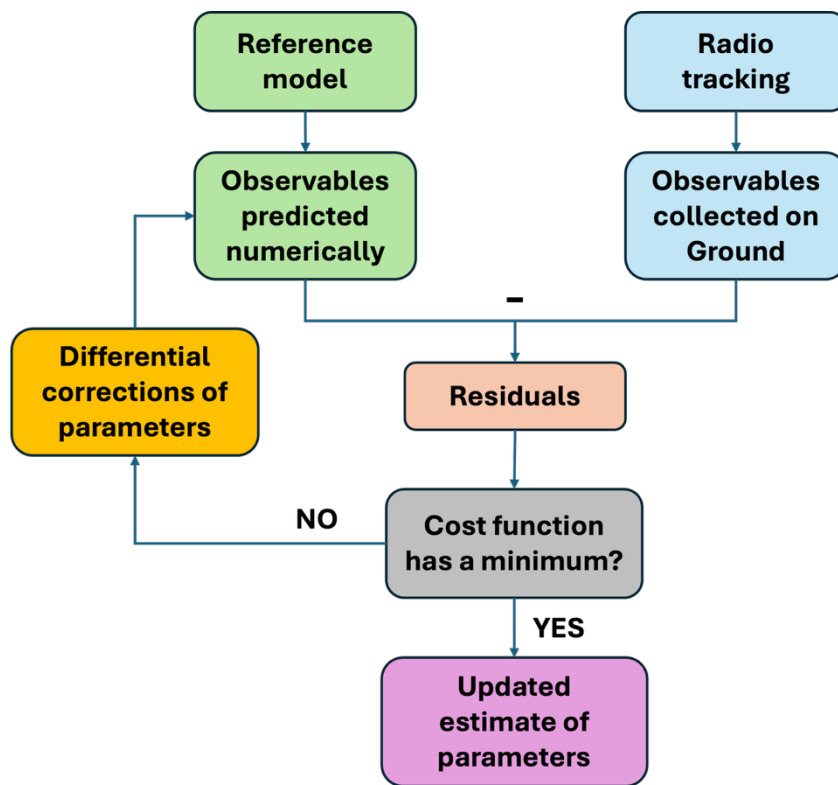


Figure 2.1. Block diagram for OD using real tracking data. Observables collected at ground stations are compared with predictions. A least-squares cost function, defined on the residuals, is minimised by iteratively correcting the spacecraft state and the other solved-for parameters.

A perturbative analysis most closely reproduces a real OD process. In this case, discrepancies are deliberately introduced between the model parameters used to generate synthetic radio observables and the ones adopted in the estimation step. This means that the computed observables are derived from a different trajectory with respect to the one used to produce the synthetic dataset. This analysis allows to simulate a realistic scenario and detect possible bias in the solution in the presence of modelling errors. On the other hand, in a covariance analysis the estimation model is identical to the “true” model used to produce the synthetic dataset. As a result, a single iteration of the OD filter is sufficient, since the a priori parameters coincide with the true ones. The objective is only to quantify the formal statistical uncertainty on the estimated parameters, given the chosen dynamical model and observation dataset.

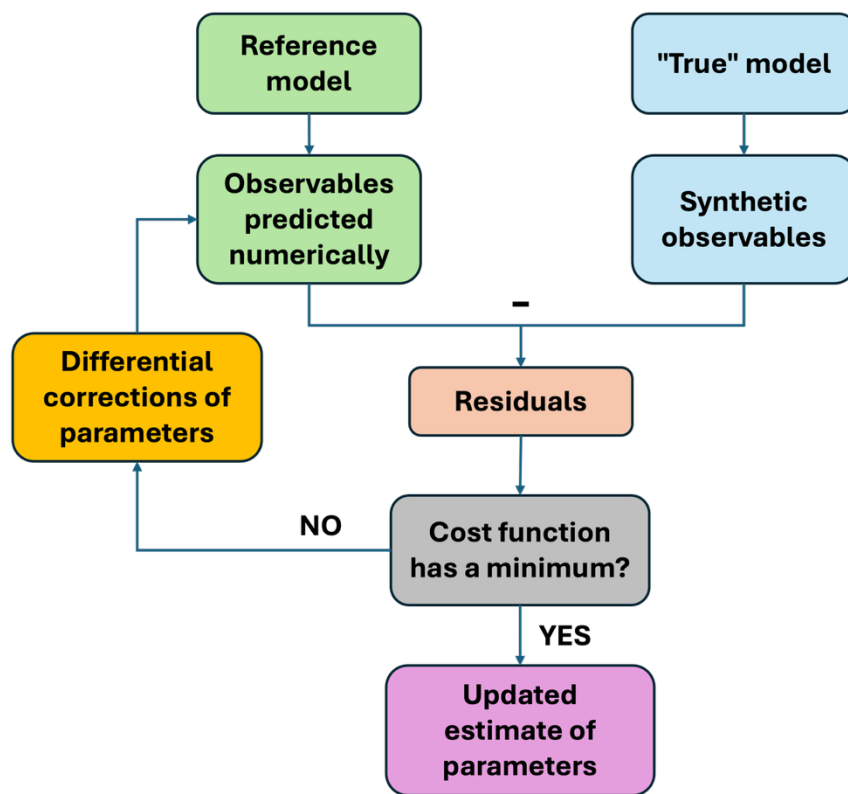


Figure 2.2. Block diagram for OD with simulated data. Synthetic observables are generated from a “true” model and compared with predictions from the estimation model. A least-squares cost function, defined on the residuals, is minimised by iteratively correcting the spacecraft state and the other solved-for parameters.

Chapter 3

The dynamical environment at Mercury

A comprehensive and reliable dynamical model is essential to accurately integrate a spacecraft trajectory over time. This model must account for all significant forces acting on the spacecraft, which can broadly be divided into conservative (gravitational) and non-conservative forces, often referred to as non-gravitational accelerations (NGAs). NGAs at Mercury mainly arise from the interaction between the spacecraft and radiative fluxes from the Sun and the planet. The intrinsic challenge in the characterization of these accelerations makes them one of the primary source of error in the orbit determination (OD). This is especially true for MESSENGER and BepiColombo, where the proximity with the Sun makes NGAs a critical contributor to the spacecraft dynamics.

This chapter reviews the forces acting on a spacecraft orbiting Mercury and their mathematical formulations. Sections 3.1 and 3.2 describe the static and dynamic components of Mercury's gravity field, respectively. Section 3.3 outlines the planet's rotational state, while Section 3.4 presents methods that combine gravity, rotation, and topographic data to infer properties of the core and the crust. Finally, Section 3.5 provides the mathematical formulation of non-gravitational accelerations.

3.1 Gravity

The gravity field of a planetary body can be represented through a spherical harmonics expansion, which allows a compact and useful description of its gravitational potential. This formalism describes the potential in terms of spherical coordinates (r, θ, α) , where r is the radial distance from the planet's center of mass, and (θ, α) are the colatitude and longitude in a body-fixed reference frame (Kaula, 1966). The potential outside a body of mass M and mean radius R is commonly expressed as:

$$U(r, \alpha, \theta) = \frac{GM}{r} \left\{ 1 + \sum_{l=2}^{\infty} \sum_{m=0}^l \left(\frac{R}{r} \right)^l \bar{P}_{lm}(\cos \theta) [\bar{C}_{lm} \cos m\alpha + \bar{S}_{lm} \sin m\alpha] \right\} \quad (3.1)$$

where $\bar{P}_{lm}$ are the normalised associated Legendre polynomials and l, m are the degree and order of the expansion. The task of measuring the gravity field of a body consists in determining the value of the normalised coefficients $\bar{C}_{lm}, \bar{S}_{lm}$. The normalisation factor commonly adopted in space geodesy is:

$$N_{lm} = \sqrt{(2l+1)(2-\delta_{m0}) \frac{(l-m)!}{(l+m)!}} \rightarrow \begin{pmatrix} \bar{C}_{lm} \\ \bar{S}_{lm} \end{pmatrix} = \begin{pmatrix} N_{lm} C_{lm} \\ N_{lm} S_{lm} \end{pmatrix} \quad (3.2)$$

The degree l controls the wavelength λ of the surface feature being represented, with higher degrees capturing finer structures:

$$\lambda = \frac{\pi R}{l} \quad (3.3)$$

Eq. (3.3) provides an estimate of the spatial resolution achievable at a given harmonic degree. The amplitude associated with each degree is typically described by the degree RMS spectrum of the gravity coefficients C_l , defined as:

$$C_l = \sqrt{\frac{1}{2l+1} \sum_{m=0}^l (\bar{C}_{lm}^2 + \bar{S}_{lm}^2)} \quad (3.4)$$

This quantity provides a global value for the gravity signal strength at each spatial scale, allowing a direct comparison with the uncertainties provided by the model's covariance matrix. As a good approximation, the decay of the magnitude spectrum of terrestrial bodies with increasing degree follows the empirical power law of Eq. (3.5), known as Kaula rule (Kaula, 1966):

$$C_l \sim \frac{A_k \cdot 10^{-5}}{l^2} \quad (3.5)$$

where the Kaula factor A_k is a constant for the planet. Through scaling relationships from other bodies (Konopliv et al., 2014), we would expect $A_k \sim 4\text{-}5$ for Mercury.

The degree at which the spectrum of signal amplitudes intersects the uncertainty spectrum is typically interpreted as the global resolution of the estimated gravity field. However, this global criterion does not reflect the non-uniform sensitivity of the inversion process. In particular, spacecraft with eccentric orbits, such as MESSENGER, sample the planetary gravity field unevenly, with enhanced sensitivity near pericenter. As a consequence, the gravity signal can be resolved to higher degrees locally than what is inferred globally. Following the methodology reported in Konopliv et al., (1999), one way to evaluate the local resolution is to identify the degree at which the modelled radial acceleration falls below its uncertainty at each surface location. This comparison provides a degree strength map, which highlights local differences in terms of sensitivity to the gravity signal. The first step is to compute the expected radial acceleration at the surface from all coefficients of degree l , given by:

$$a_l = \frac{GM}{R^2} (l+1) \sum_{m=0}^l \bar{P}_{lm}(\cos \theta) [\bar{C}_{lm} \cos m\alpha + \bar{S}_{lm} \sin m\alpha] \quad (3.6)$$

Consequently, its RMS magnitude spectrum for $l \gg 1$ is:

$$(a_l)_{RMS} = \frac{GM}{R^2} \sqrt{\frac{2}{l}} A_k \cdot 10^{-5} \quad (3.7)$$

This quantity is compared to the acceleration uncertainty for the coefficients of degree l only, referred as $\sigma(a_l)$. This is given by the sum total uncertainty from degree 2 up to degree l subtracting the contribution due to the lower degrees (Konopliv et al., 1999):

$$\sigma(a_l) = \sigma(a_{2,l}) - \sigma(a_{2,l-1}) \quad (3.8)$$

where $\sigma(a_{2,l})$ is the acceleration uncertainty from the summed contributions of all coefficients from degree 2 to l . This quantity is retrieved from the gravity field's covariance matrix as follows:

$$\sigma(a_{2,l}) = \frac{\partial a_{2,l}}{\partial \mathbf{G}_{2,l}}^T \mathbf{P}_{2,l} \frac{\partial a_{2,l}}{\partial \mathbf{G}_{2,l}} \quad (3.9)$$

where $\mathbf{G}_{2,l}$ is the vector of all normalised gravity coefficients from degree 2 to l and $\mathbf{P}_{2,l}$ is the corresponding covariance.

The degree l at which the Kaula signal $(a_l)_{RMS}$ equals the acceleration uncertainty $\sigma(a_l)$ is called the degree strength of the gravity field for that particular latitude and longitude. For degrees greater than the degree strength, the noise in the data exceeds the signal. For example, because of the large altitude variations of MESSENGER's orbit, the resolution exceeds degree 100 over regions of the northern hemisphere, where the spacecraft pericenter reached altitudes as low as 5–20 km. In contrast, the equatorial zone and the southern hemisphere were limited to lower resolutions, around degree 15, which is equivalent to spatial scales of roughly 500 km. This uneven coverage of Mercury's surface requires the use of a priori information to enable a smooth inversion of the normal equations of the batch-least square method used to process the MESSENGER radiotracking data. Usually, the gravity harmonic coefficients are constrained through the Kaula rule, which sets the a priori value of the gravity coefficients of degree l to zero with an a priori uncertainty given by Eq. (3.5). However, to compute a degree strength map that reflects the actual data resolution, the covariance matrix in Eq. (3.9) should be retrieved without assuming any regularization.

Short wavelength features are typically investigated through maps of gravity anomalies, which are deviations between the measured gravity field of a planet and the

value predicted by a reference model. The latter typically accounts for the planet's spherical shape and its oblateness through the degree 2 coefficients. These anomalies arise from lateral variations in subsurface density, such as dense volcanic provinces or low-density sedimentary basins. To isolate these effects, measurements are corrected for altitude and topography. The free-air anomaly accounts for the elevation of the measurement point above a reference surface, while the Bouguer anomaly further removes the gravitational contribution of the topographic mass between the observation point and the reference level (Wahr, 1996). Together, these corrections allow gravity data to reveal hidden structures in a planet's crust and mantle.

Updated version of the Hermean gravity field obtained by processing MESSENGER radio science data have been published during the mission (Mazarico, Genova, et al., 2014; Smith et al., 2012; Verma & Margot, 2016). The first global solution based on the complete dataset was HgM007 (Mazarico et al., 2016), estimated up to degree and order 100 (corresponding to a spatial resolution of ~ 80 km), with a Kaula constraint of $A_k = 3$ applied beyond degree 10. This was later refined into the HgM008 model (Genova et al., 2019), maintaining the 100x100 resolution but adopting a slightly higher Kaula factor of $A_k = 4$. More recently, two higher-resolution models, MESS160A (Konopliv et al., 2020) and HgM009 (Genova et al., 2023) were produced by reprocessing the full dataset, extending the expansion to degree and order 160. The first one adopts a Kaula rule with $A_k = 5$ and discusses also alternative constraint methods using surface accelerations measurements or the harmonic spectrum of gravity derived from topography. The second one is instead retrieved using a looser Kaula constraint with $A_k = 10$, which lead an improved correlation with topography. These solutions achieved resolutions beyond degree 100 in the northern hemisphere (spatial scales better than 80 km). However, coverage over the equator and southern hemisphere remained limited to spatial scales of about 500 km.

3.2 Tides

In addition to the static (time-invariant) component of the gravity field, the dynamical contributions to the gravitational potential must be taken into account when modelling spacecraft motion with high precision. A primary source of this dynamical contribution is the tidal deformation induced by external bodies. For Mercury, the primary tide-raising body is the Sun, whose gradient of gravitational force across the body causes a deformation of the planet and a corresponding modification of the gravity field. This change in the potential, referred to as the tidal potential V , is proportional to the external perturbing potential W , which can be expressed in spherical harmonics. The proportionality constants are the Love numbers, denoted as k_l . In the Mercury body-fixed reference frame, the perturbing potential at a surface point identified by the vector $\mathbf{r}$, can be written as (Van Hoolst & Jacobs, 2003):

$$W(\mathbf{r}) = \sum_{l=2}^{\infty} W_l = \frac{GM_{\odot}}{d} \sum_{l=2}^{\infty} \left(\frac{R}{d}\right)^l P_l(\cos \psi) \quad (3.10)$$

where d is the distance between the mass centers of the Sun and Mercury, P_l the Legendre polynomial of degree l and ψ is the angle between the directions from Mercury to the Sun and the point considered. Due to the tidally-induced mass shift, the planet's potential changes by (Wahr, 2013):

$$V_l = k_l \left(\frac{R}{r}\right)^{l+1} W_l(R) \quad (3.11)$$

where R is the radius of Mercury and k_l is the Love number of degree l . This quantity is a complex number, where the real part k_l^r corresponds to the elastic response, while the imaginary part k_l^i accounts for internal dissipative processes. For practical purposes, the complex Love number representation (k^r, k^i) is often re-expressed in amplitude-phase form (k, δ_k) :

$$k = \sqrt{(k^r)^2 + (k^i)^2} \quad \delta_k = -\frac{k^i}{2k} = -\frac{1}{2Q} \quad (3.12)$$

where k is the amplitude and δ_k the tidal phase lag, which describes the misalignment between the tidal bulge and the tide-raising body due to the non-instantaneous response of the perturbed body. The lag angle is related to the quality factor Q , a measure of tidal dissipation, defined as:

$$Q = \frac{2\pi E_0}{\Delta E} \quad (3.13)$$

with E_0 representing the maximum stored energy during a tidal cycle and ΔE the dissipated energy. The effect of tides manifests as time-dependent modifications to the planet's gravitational potential coefficients. These modifications can be written as additional contributions to the spherical harmonic coefficients, linearly dependent on the Love numbers and the position of the perturbing body (Murray & Dermott, 2000). Among the different degrees, the degree-2 Love number k_2 plays a dominant role in Mercury's case, due to the smallness of the ratio R/d (planetary radius to Sun-Mercury distance). The value of k_2 provides key insights into Mercury's internal structure: a high value indicates a more uniform density distribution, while a lower value points to strong central condensation, typically associated with a large, dense core. For reference, a body with constant density would yield the theoretical maximum $k_2 = 1.5$, whereas real rocky planets show smaller values due to internal layering and rigidity. Accurate estimation of k_2 can constrain models of Mercury's interior, including the core size, mantle thickness, and the mechanical properties of the lithosphere.

The analysis of MESSENGER radio science data enabled the first reliable measurement of Mercury's tidal response, despite limitations imposed by non-gravitational perturbations (see Section 3.5). The estimated real part of k_2 is in the range 0.53–0.569 (Genova et al., 2019; Konopliv et al., 2020), consistent with the existence of a liquid outer core and providing initial constraints on mantle rheology. These estimates are compatible with a warm and weak mantle (Padovan et al., 2014). However, key properties such as mantle temperature, iron content, and grain size remain unconstrained. Future

determinations of both the amplitude and phase lag of k_2 with BepiColombo will significantly improve our understanding of Mercury's interior.

3.3 Rotation

In gravity field modelling, the representation of Mercury's gravitational potential of Eq. (3.1) is expressed in a body-fixed frame, which depends on the planet's rotational parameters. The orientation parameters can be considered dynamical quantities, as they affect the inertial trajectory of the satellite. Hence, the orbit of a satellite also allows gathering information on the rotational state of the planet. A precise representation is required to translate the equations to the inertial frame where the dynamical equations are integrated.

Mercury's rotational state is characterized by a unique 3:2 spin-orbit resonance (Colombo & Shapiro, 1966). The planet completes one rotation every ~ 58 days and one orbit every ~ 88 days, resulting in three rotations for every two revolutions around the Sun. Under the tidal influence of the Sun and other planets, Mercury's spin axis undergoes precession, a slow retrograde motion around the orbital angular momentum, and nutation, short-period oscillations caused mainly by the varying solar torque along its elliptical orbit.

Mercury is also expected to occupy the Cassini state 1, a dynamical equilibrium in which the spin axis, the orbit normal, and the normal to the Laplace plane remain coplanar as the orbit precesses. The geometry for the Cassini state 1 is shown in Figure 3.1. The Laplace plane is defined as the mean reference plane about which the inclination of the orbital plane experiences minimal variations over long timescales. In this configuration, the obliquity (angle between the spin axis $\hat{\mathbf{s}}$ and the orbital normal $\hat{\mathbf{h}}$) remains approximately constant, and the spin axis precesses synchronously with the orbital plane around the Laplace pole $\hat{\mathbf{n}}_L$.

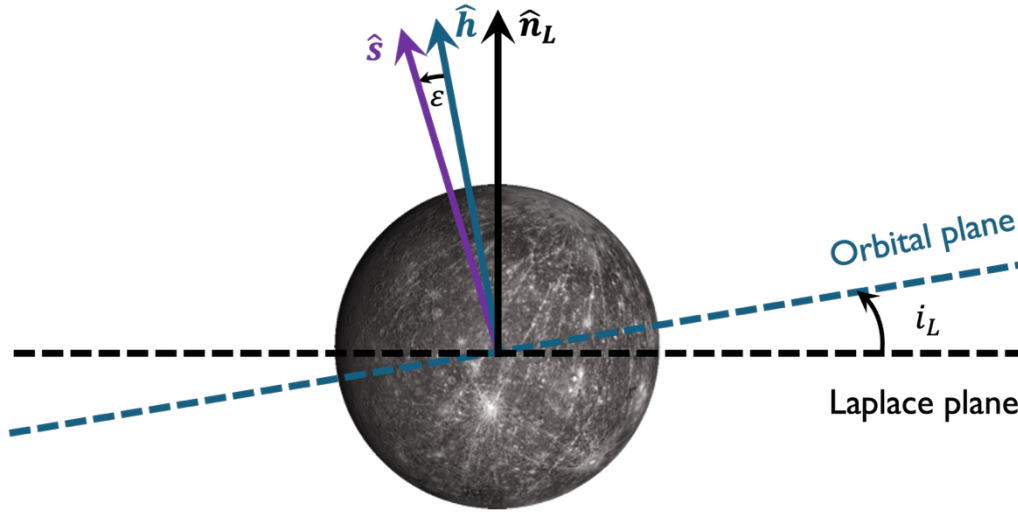


Figure 3.1. The geometry of a Cassini state of type 1. The orbit normal $\hat{h}$ lies between the spin axis $\hat{s}$ and the normal to the Laplace plane $\hat{n}_L$. The obliquity ϵ remains constant during the precession around the normal to the Laplace plane, expected to have a period of $\sim 300,000$ years.

The spin–orbit resonance also produces longitudinal librations due to the planet’s eccentric orbit and the periodic solar torque acting on its mass distribution. The main libration has a period of 88 days, equal to Mercury’s orbital period. In addition, there are smaller amplitude librations at harmonic frequencies. Additional forced librations, induced by planetary perturbations, have small amplitudes unless their periods approach that of a free libration, leading to possible resonant amplification (Yseboodt et al., 2013). Depending on their magnitude and on the sensitivity of the tracking system, librations can be detected in tracking data as periodic variations in the spacecraft orbit. These signals are directly linked to Mercury’s internal mass distribution, providing valuable constraints on the state and size of its core.

All the effects mentioned above must be included in the rotation model of the planet. In general, this is characterized by three time-dependent angles: right ascension $\alpha(t)$, declination $\delta(t)$, and prime meridian angle $W(t)$ (Archinal et al., 2018). The right ascension and declination describe the direction of the spin axis with respect to the International Celestial Reference Frame (ICRF), and account for its precession and nutation. The prime meridian angle represents the rotation around the spin axis. Using these three parameters, it is possible to convert coordinates between the ICRF and a

body-fixed frame at any given time. All three angles are typically expressed in the form of polynomial terms (secular components) and a summation of trigonometric functions (periodic oscillations due to the librations in longitude at different frequencies f_i):

$$\begin{aligned}\alpha(t) &= \alpha_0 + \dot{\alpha}t \\ \delta(t) &= \delta_0 + \dot{\delta}t \\ W(t) &= W_0 + \dot{W}t + \sum_i \phi_i \sin(2\pi f_i t + \varphi_i)\end{aligned}\tag{3.14}$$

where the initial conditions are typically referred to the J2000 epoch.

A summary of the rotation models for Mercury can be found in (Stark et al., 2018). The latest IAU model for Mercury adopts $\alpha_0 = 281.0103^\circ$, $\delta_0 = 61.4155^\circ$, and $W_0 = 329.5988^\circ$ (Archinal et al., 2018), where the 20° meridian of Mercury is defined by the crater Hun Kal. The values for the rotation axis orientation at J2000 and the libration amplitude are based on 10 years of Earth-based radar observations of Mercury (J. Margot et al., 2012). These observations infer the value of $\phi_0 = 38.5 \pm 1.6$ arcsec for the 88 days librations in longitude and $\varepsilon = 2.04 \pm 0.08$ arcmin for the obliquity. These measurements confirm that Mercury is in or near a Cassini state and that the mantle is decoupled from a large core, that is at least partially molten (J. L. Margot et al., 2007).

A joint analysis of laser altimeter profiles from 3 years of MESSENGER orbital observations with stereo digital terrain models confirmed radar-based estimates for the spin axis, obliquity and mean rotation rate of the planet (Stark et al., 2015). They provided also the first estimate of Mercury's librations from orbit, which is $\phi_0 = 38.9 \pm 1.3$ arcsec. The spin axis orientations of the latest gravity solutions obtained from MESSENGER (Genova et al., 2019; Konopliv et al., 2020) show small but significant offsets compared to surface-based estimates, although the solution from Genova et al. (2019) is strongly consistent with the Cassini state (Baland et al., 2017). This discrepancy may arise because surface observations constrain only the orientation of Mercury's outer solid shell (crust and mantle). The orientation of the core may be slightly misaligned, since the fluid core is mechanically decoupled, leading to the observed difference.

3.4 From gravity to interior

Gravity measurements, when combined with complementary measurements, provide a powerful tool to probe the planet's interior. In particular, the joint analysis of gravity and rotation data constrains the size and state of Mercury's core, while the combination of gravity and topography allows estimates of crustal thickness and lithospheric properties. Together, these approaches offer complementary perspectives on the planet's formation and evolution.

3.4.1 The Peale's procedure

Combining gravity and rotation measurements can provide key insights into Mercury's core structure. To investigate the size and state of the core, it is important to obtain information about the ratio between the principal moment of inertia of the mantle (plus crust) and that of the entire planet, represented as C_m/C . This quantity can be estimated using the Peale's procedure (Peale, 1976; Peale et al., 2002). The procedure starts from a homogenous multi-layer model of Mercury, which includes a core and a mantle (plus crust). The ratio can be written as:

$$\frac{C_m}{C} = \left(\frac{C_m}{B - A} \right) x \left(\frac{MR^2}{C} \right) x \left(\frac{B - A}{MR^2} \right) \quad (3.15)$$

where $A < B < C$ and $A_m < B_m < C_m$ are the principal moments of the inertia of the entire planet and of the mantle plus crust respectively. This method relies on the assumption that the core does not experience the 88-day physical libration of the mantle but instead follows the mantle over the timescale of the $\sim 300,000$ year precession of the spin in Cassini state 1. Under these hypotheses it is possible to write (Peale, 2005):

$$\phi_0 = \frac{3}{2} \left(\frac{B - A}{C_m} \right) \left(1 - 11e^2 + \frac{959}{28}e^4 + O(e^6) \right) \quad \left(\frac{B - A}{MR^2} \right) = 4C_{22} \quad (3.16)$$

$$\frac{C}{MR^2} = \frac{n\varepsilon \left[J_2(1 - e^2)^{-\frac{3}{2}} + 2C_{22} \left(\frac{7}{2}e - \frac{123}{16}e^3 + O(e^5) \right) \right]}{\omega_p(\sin(i_L) + \varepsilon \cos(i_L))} \quad (3.17)$$

where ϕ_0 is the amplitude of the 88 days librations in longitude, ε is the obliquity of the Cassini state 1, which Mercury is expected to occupy, i_L is the inclination of the orbit plane to the Laplace plane on which Mercury precesses at angular rate $-\omega_p$, n is the mean orbital motion and e the eccentricity of Mercury's orbit. Thus, estimating the right ascension and declination of the spin axis allows to determine the obliquity. The combination with the gravity quadrupole coefficient provides insights into the normalised moment of inertia C/MR^2 , also known as the concentration factor. This factor is expected to be 0.4 for a homogeneous body, and substantially lower for a differentiated body like Mercury. By estimating the physical librations in longitude, the concentration factor, and the degree 2 gravity field, we can obtain the ratio C_m/C . This ratio equals one for a rigid body and approximately 0.5 for a body with a liquid layer or core.

For Mercury, current estimates of the obliquity yield values of C/MR^2 in the range 0.318-0.358, indicating that Mercury is a differentiated body with a large iron core. The presence of a solid inner core, however, depends on the adopted obliquity estimate. Following Dumberry & Rivoldini (2015), Rivoldini & Van Hoolst (2013), the presence of an inner core is not confirmed if the obliquity from radar measurements is used (J. Margot et al., 2012). The largest inner core compatible with geodetic observations is 1325 ± 250 km. Conversely, adopting the obliquity from Genova et al. (2019) implies an inner core with a radius greater than 600 km (Steinbrügge et al., 2021). A sufficiently large inner core (> 1000 km) would significantly affect both the short-term and long-term rotational behaviour. For example, an inner core with a radius of 1500 km could alter the amplitude of the 88-day libration by up to 1 arcsec (Van Hoolst et al., 2012), which is equivalent to ~ 10 m at the equator, a variation comparable to current measurement accuracy and potentially detectable by MORE. Moreover, the presence of a substantial solid inner core would require a revision of the derivation of the concentrator factor from obliquity (Peale et al., 2016).

3.4.2 Crustal thickness

The crustal thickness of a planet is an essential clue to its geophysical and geological history. For Mercury, constraining this parameter offers insights into its formation, differentiation, and thermal history. The MESSENGER mission greatly advanced our knowledge by combining radio tracking and topographic data to map crustal thickness in the northern hemisphere. Estimates of Mercury's average crustal thickness range between 20 and 50 km (Genova et al., 2019; James et al., 2015; Mazarico, Genova, et al., 2014; Padovan et al., 2015; Sori, 2018). More recent gravity field models, expanded up to degree and order 160 (Genova et al., 2023; Konopliv et al., 2020), enabled improved studies of Mercury's crust and lithosphere, although coverage remains limited to the northern hemisphere due to MESSENGER's eccentric orbit.

Global crustal thickness can be computed following the approach outlined by Wieczorek & Phillips (1998), implemented through the SHTOOLS software (Wieczorek & Meschede, 2018). In brief, free-air potential anomalies are converted into Bouguer anomalies by removing the gravity contribution of surface topography. Contributions from lateral density variations, if present, are also subtracted. The residual anomaly is then downward-continued through the crust to relate it to the relief of the mantle-crust boundary (MCB). Because of finite-amplitude effects, the resulting nonlinear equation is solved iteratively for the bottom crustal relief for assumed MCB density contrasts. The effective resolution of the crustal thickness map is limited by the gravity field resolution, which is lower than the maximum spherical harmonic degree l_{max} would suggest.

The computation of a crustal thickness map relies on both physical (average crustal thickness, mantle density and crustal density, which is characterized by its grain density and porosity) and design parameters to suppress noise. Noise can be reduced either by applying a low-pass filter to the downward-continued gravity, defined by the degree $l_{1/2}$ where the filter amplitude drops to 0.5, or by truncating the input gravity field coefficients at a degree $l_{cut} < l_{max}$ (see Wieczorek & Phillips (1998) for more details).

3.5 Non-gravitational accelerations

Objects in space experience a momentum transfer induced by the absorption and reflection of photons or atmospheric particles. The reflection can be either in a single mirror-like (specular) direction or scattered in multiple directions (diffuse). This process produces small but continuous non-gravitational accelerations (NGAs) that can significantly affect long-term orbit predictions (di Stefano et al., 2023; Fliegel & Gallini, 1996; Floberghagen et al., 1999; Kato et al., 2012; Kato & van der Ha, 2012; Marshall et al., 1992). Since this work focuses on the Mercury environment, we consider only accelerations arising from the interactions between photons and surfaces, neglecting atmospheric drag (Mostaza Prieto et al., 2014). The primary sources of such radiation are shown in Figure 3.2. They include direct solar flux (solar radiation pressure), sunlight reflected by the planet (albedo pressure), thermal emission from the planet (infrared radiation pressure) and from the spacecraft (thermal recoil pressure). Secondary effects, such as accelerations from spacecraft antenna radio-frequency emissions and solar wind (di Stefano et al., 2023), are not considered here, as their influence on the overall dynamics is negligible for the purposes of this work. The accelerations due to solar and planetary radiation can be represented as a function of an incoming input radiation flux and a “geometry vector” $\mathbb{L}$:

$$\mathbf{a} = \frac{\Phi_{in}}{mc} \mathbb{L}(\alpha, \delta) \quad (3.18)$$

where Φ_{in} is the input flux in W/m^2 , which depends on the considered physical phenomenon responsible for the acceleration, m is the spacecraft mass, c is the speed of light and $\mathbb{L}$ is the “generalized geometry vector”. This vector is representative of the interaction of the spacecraft surface and a radiation of unitary intensity, as a function of its direction of arrival. This is identified by its right ascension α and declination δ with respect to the spacecraft body-fixed frame. In more general terms, $\mathbb{L}$ can be seen as the transfer function between the input flux and the output acceleration.

Due to their subtle nature and sensitivity to spacecraft geometry, surface properties, and environmental conditions, these perturbations are a major source of uncertainty in OD. An incorrect modelling of non-gravitational effects can introduce errors in the reconstructed trajectory that build up over time, potentially resulting in incorrect interpretations of scientific results.

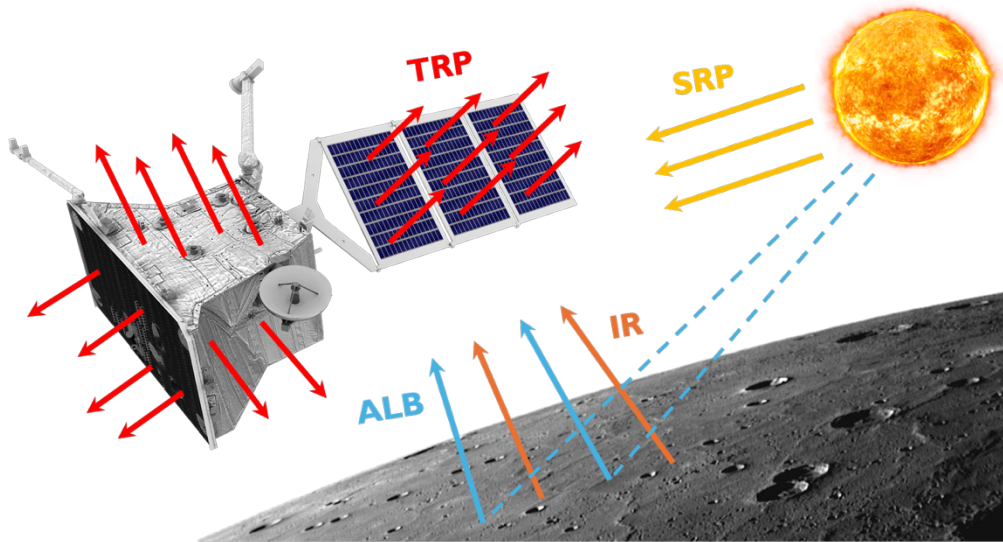


Figure 3.2. Schematic representation of non-gravitational accelerations. The dominant contributors are direct radiation from the Sun (SRP), radiation from the Sun reflected by the planet (ALB), thermal infrared radiation from the planet (IR) and from the spacecraft (TRP).

3.5.1 Generalized geometry vector

The computation of the geometry vector $\mathbb{L}$ in Eq. (3.18) depends on knowledge of the spacecraft geometry, attitude and thermo-optical properties of its surface materials. However, these material properties are not always available. Some values can be found in literature, while others are inferred based on standard spacecraft materials and estimated through OD.

In this work we adopt two modelling strategies for the computation of $\mathbb{L}$. For the data analysis of MESSENGER and for the numerical simulations of BepiColombo we adopt the classical approach used by OD software, known as plate model. This method consists in the discretization of the probe as a set of plates or 2D elementary shapes. In the plate

model, determining whether a surface is impinged by the radiation relies entirely on calculating the scalar product between the surface normal and the incoming ray direction. The orientation of the surface normal in time is typically inferred from attitude kernels supplied by the mission. This approach has been widely used for the data analysis of almost all the past planetary probes (e.g. Durante et al., 2019; Genova et al., 2016, 2023; Goossens et al., 2020, 2024; Konopliv et al., 1999, 2020; Park et al., 2016). This method entails an approximation with respect to the true shape of the spacecraft, but has been more than adequate until now in providing accurate force models for the purposes of radio science experiments. Following this approach, the geometry vector is computed by summing the individual contributions from every spacecraft illuminated surface element k . The geometry is shown in Figure 3.3. If the hypothesis of a Lambert surface is made, meaning that the intensity I of the diffusely reflected radiation as a function of the incidence angle θ follows the Lambert's cosine distribution $I = I_{MAX}\cos(\theta)$, the geometry vector can be computed as follows:

$$\mathbb{L}(\alpha, \delta) = \sum_{k=1}^{Ne} A_k \cos(\theta_k) \left[(1 - \rho_{sk}) \hat{\mathbf{s}}_k - 2 \left(\rho_{sk} \cos(\theta_k) + \frac{\rho_{dk}}{3} \right) \hat{\mathbf{n}}_k \right] \quad (3.19)$$

where $\hat{\mathbf{n}}_k$ is the normal direction to the exposed surface; A_k is the surface area; ρ_{sk} and ρ_{dk} are its specular and diffuse coefficients; $\hat{\mathbf{s}}_k$ is the incidence direction and $\cos(\theta_k) = -(\hat{\mathbf{s}}_k \cdot \hat{\mathbf{n}}_k)$. A surface element is illuminated if $\theta_k < \pi/2$, otherwise it does not contribute to the geometry vector.

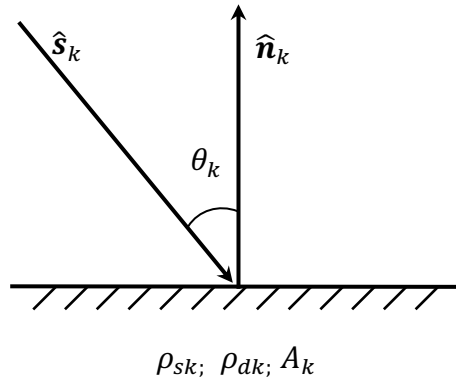


Figure 3.3. Geometry of incident radiation for a spacecraft surface element. $\hat{\mathbf{n}}_k$ is the normal direction to the exposed surface; A_k is the surface area; ρ_{sk} and ρ_{dk} are its specular and diffuse coefficients; $\hat{\mathbf{s}}_k$ is the incidence direction.

Note that the dependency of $\mathbb{L}$ on α, δ is implicit in $\hat{\mathbf{s}}_k$. Let $\{x_s, y_s, z_s\}$ be the coordinates of the unit vector $\hat{\mathbf{s}}_k$ in the spacecraft body-fixed frame, the two angles can be derived as:

$$\delta = \arcsin(-z_s); \quad \alpha = \arctan\left(\frac{y_s}{x_s}\right) \quad (3.20)$$

Although computationally efficient, this model cannot account for self-shadowing (when parts of the spacecraft block radiation from reaching other surfaces) or multiple reflections of photons (see Section 6.1).

The second approach, presented in Chapter 6, offers the capability to overcome the limitations of the plate model by employing ray tracing techniques and 3D modelling of the spacecraft. Following this methodology, radiation is modelled as a collection of rays. Each ray is tracked until it interacts with a detailed 3D representation of the spacecraft, where each element has specific thermo-optical properties. While more computationally intensive, ray tracing has the potential to significantly improve dynamical modelling for applications where accuracy is paramount.

3.5.2 Solar radiation pressure

The dominant non-gravitational force in the Mercury environment is solar radiation pressure (SRP), whose intensity increases as the spacecraft gets closer to the Sun. In the general formulation of SRP, the term Φ_{in} of Eq. (3.18) becomes the solar radiation flux $\Phi_{\odot}$ at the spacecraft location, while α, δ are the right ascension and declination of the Sun direction in the spacecraft body-fixed frame:

$$\mathbf{a}_{SRP} = f_s \frac{\Phi_{\odot}}{mc} \mathbb{L}(\alpha, \delta) \quad (3.21)$$

where f_s is a scale factor between 0 and 1 for the sunlit/penumbra/eclipse conditions (Vokrouhlicky et al., 1993). The radiation flux at the spacecraft location is calculated by scaling the radiation flux at 1 astronomical unit (AU):

$$\Phi_{\odot} = \Phi_{1\text{AU}} \left(\frac{R_{1\text{AU}}}{D_{SC}} \right)^2 \quad (3.22)$$

where D_{SC} is the distance between the spacecraft and the Sun and $R_{1\text{AU}}$ is equal to 1 AU.

The solar flux exhibits random fluctuations at the level of 0.01-0.1% of its mean value over a few hours, mainly due to the combined effects of faculae and sunspots on the solar surface (Kopp, 2016). These fluctuations can be measured by space-based radiometers on board of satellites orbiting in the Earth's (Kopp et al., 2004). While the faint accelerations induced by these fluctuations are relevant for general relativity tests during BepiColombo's cruise phase (di Stefano et al., 2023), they are negligible compared to other NGAs for a spacecraft in planetary orbit. Therefore, for most applications, the solar flux at 1 AU can be fixed to its average value ($\sim 1367 \text{ W/m}^2$). Any residual effects can be absorbed in the OD process, either by estimating scale factors for the solar radiation pressure acceleration or by refining the spacecraft thermo-optical coefficients.

3.5.3 Planetary radiation

A spacecraft orbiting a planet is also affected by radiation pressure from the planet itself. This planetary radiation consists of two components: reflected solar radiation (albedo) and thermal infrared emission (IR) from the heated planetary surface.

When solar photons hit the planet, a fraction is reflected and the rest is absorbed, contributing to the planet's thermal budget. The reflected flux is characterized by the Bond albedo a , defined as the fraction of total solar energy reflected over all directions and wavelengths:

$$a = \frac{F_r}{F_{in}} \quad (3.23)$$

where F_r is the reflected flux and F_{in} is the total incident flux. Unlike the geometric albedo, which depends on the observation angle, the Bond albedo accounts for the total energy reflected in all directions and is therefore the appropriate quantity when

computing the resulting radiation pressure on an orbiter. Unless given for a specific wavelength (spectral albedo), albedo refers to the entire spectrum of solar radiation. However, albedo is more accurately defined as the integrated reflectance over the range of wavelengths where most solar energy is present, which is between 0.2 μm and 4.0 μm (Knocke et al., 1988).

Assuming that the planet is in thermal equilibrium (i.e. the flux emitted by the planet is equal to the absorbed flux), the remaining absorbed fraction $(1 - a)F_{in}$ is re-emitted as thermal infrared radiation, which operates at wavelengths above 4 μm . This emission is characterized by the emissivity ϵ , defined as:

$$\epsilon = \frac{F_{emi}}{\sigma T^4} \quad (3.24)$$

where F_{emi} is the emitted thermal flux, T the surface temperature and σ is the Stephan-Boltzmann constant. Here, $\epsilon = 1$ corresponds to a perfect black body.

Modelling planetary radiation is a relatively complex task, especially for bodies like Mercury where surface reflectivity and emissivity vary significantly with location. This asymmetry introduces long-term perturbations in the spacecraft trajectory, which can accumulate over time with a significant impact on precise OD. The most straightforward way to compute the overall planetary radiation acceleration would consist in a fine discretization of the planetary surface (for example using a grid model with equally spaced cells). Following this approach, the planetary surface is divided in a set of cells $\mathbb{S}$. The geometry is shown in Figure 3.4. Each surface cell ij has assigned an albedo value a_{ij} , an emissivity ϵ_{ij} , a temperature T_{ij} , a surface area A_{ij} and a surface normal $\hat{\mathbf{n}}_{ij}$. The accelerations due to a surface element can be computed as follows:

$$\mathbf{a}_{ALB,ij} = \frac{a_{ij}\Phi_{\odot}A_{ij}\cos(\varphi_{ij})\cos(\psi_{ij})}{mc\pi r_{sc,ij}^2}\mathbb{L}(\alpha_{ij},\delta_{ij}) \quad (3.25)$$

$$\mathbf{a}_{IR,ij} = \frac{\epsilon_{ij}\sigma T_{ij}^4 A_{ij}\cos(\psi_{ij})}{mc\pi r_{sc,ij}^2}\mathbb{L}(\alpha_{ij},\delta_{ij}) \quad (3.26)$$

where σ is the Stephan-Boltzmann constant; $\hat{\mathbf{r}}_{sc,ij}$ is the spacecraft position with respect to the center of the element; φ_{ij} is the angle between the Sun direction $\hat{\mathbf{r}}_{sun}$ and $\hat{\mathbf{n}}_{ij}$; ψ_{ij} is the angle between $\hat{\mathbf{r}}_{sc,ij}$ and $\hat{\mathbf{n}}_{ij}$. Note that in this case, the right ascension and declination α_{ij}, δ_{ij} are related to the incoming radiation from the surface element ij , while in Eq. (3.21) they referred to the Sun direction.

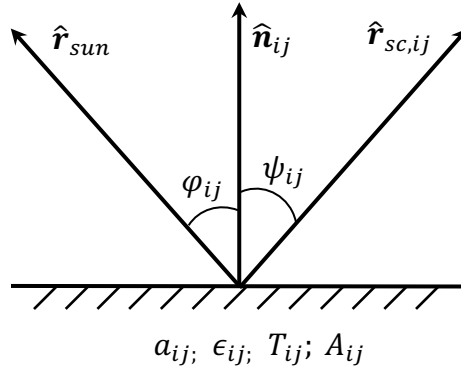


Figure 3.4. Geometry of incident radiation for a planetary surface element.

$\hat{\mathbf{n}}_{ij}$ is the normal direction; A_{ij} is the area; a_{ij}, ϵ_{ij} and T_{ij} are the albedo, emissivity and temperature values; $\hat{\mathbf{r}}_{sun}, \hat{\mathbf{r}}_{sc,ij}$ are the directions of the Sun and the spacecraft with respect to the grid cell, respectively.

The total acceleration generated by the whole planet is obtained by summation on all the planetary surface elements that are in visibility of the spacecraft. The albedo considers only sunlit regions of the planet, while thermal infrared also accounts for radiation from the night side. Let the set of cells that are sunlit at a specified epoch be $\mathbb{A}_s$ and the set of cells that are visible to the spacecraft $\mathbb{A}_v$:

$$\begin{aligned}\mathbb{A}_s &= \{ij \in \mathbb{S} \mid \varphi_{ij} < \pi/2\} \\ \mathbb{A}_v &= \{ij \in \mathbb{S} \mid \psi_{ij} < \pi/2\}\end{aligned}\tag{3.27}$$

The total accelerations can be expressed as:

$$\mathbf{a}_{ALB} = \sum_{ij \in \mathbb{A}_s \cap \mathbb{A}_v} \mathbf{a}_{ALB,ij}; \quad \mathbf{a}_{IR} = \sum_{ij \in \mathbb{A}_v} \mathbf{a}_{IR,ij}\tag{3.28}$$

However, this procedure is computationally very expensive. A more efficient approach employed by traditional OD software, such as MONTE (Evans et al., 2018) and GEODYN (Nicholas et al., 2024), calculates these forces by dividing the visible portion of the central body's surface into a central polar cap and n_R concentric rings. Each ring r is further divided into $6r$ sections. Counting the cap, the number of sections is (Moyer, 2003):

$$N_s = 1 + 3n_R(1 + n_R) \quad (3.29)$$

The total accelerations are computed as the sum of the contributions from all the sections and all spacecraft surfaces impinged by the planetary radiation:

$$\mathbf{a}_{ALB} = \sum_{s=1}^{N_s} \frac{2(1 - \cos(\gamma))}{N} \frac{\Phi_{\odot}}{mc} a_s \cos(\varphi_s) \mathbb{L}(\alpha_s, \delta_s) \quad (3.30)$$

$$\mathbf{a}_{IR} = \sum_{s=1}^{N_s} \frac{(1 - \cos(\gamma))}{2N} \frac{\Phi_{\odot}}{mc} \tilde{\epsilon}_s \mathbb{L}(\alpha_s, \delta_s) \quad (3.31)$$

where γ is the angular semidiameter of the source body as seen from the spacecraft; $a_s, \tilde{\epsilon}_s$ are the albedo and emissivity values at section s . It is worth remarking that the surface emissivity $\tilde{\epsilon}$ in Eq. (3.31) is different from the surface emissivity ϵ of (3.26). The relation between the two quantities is:

$$\tilde{\epsilon} = \frac{4\sigma T^4}{\Phi_{\odot}} \epsilon \quad (3.32)$$

The methods presented above requires to define global maps of albedo and temperature to set the properties of each surface element. The emissivity $\tilde{\epsilon}$ can be retrieved from temperature values using Eq. (3.32). When such data are not available, temperature at each grid point can be computed with a simplified approach that assumes that the planet is in thermal equilibrium. Under this hypothesis, the temperature value T_{ij} at the surface element ij can be calculated as:

$$T_{ij} = \begin{cases} \left(\frac{(1 - a_{ij}) \Phi_{\odot} \cos \varphi_{ij}}{\sigma \epsilon_{ij}} \right)^{0.25} & \text{if } \varphi_{ij} < \pi/2 \\ T_{night} & \text{otherwise} \end{cases} \quad (3.33)$$

where T_{night} is the surface temperature at the night side of the planet. Once the maps are available (as gridded data), they can be interpolated to retrieve the value at the center point of each surface element and used for the computation. The procedure followed by MONTE and GEODYN requires to expand the grids with a spherical harmonics representation.

Albedo maps must always be referenced in the planetary body-fixed frame. Emissivity maps, however, can be defined in a Sun-fixed frame, where the X-axis points from the planet to the Sun, requiring the sub-solar point to be aligned at 0° latitude and longitude. For short propagation intervals, the emissivity map can alternatively be defined in the planetary body-fixed frame, as long as the temperature distribution properly reflects the Sun's position at the epoch of interest.

3.5.4 Thermal recoil pressure

TRP primarily arises from the anisotropic re-emission of absorbed solar radiation by the spacecraft external surfaces. For planetary orbiters, albedo and thermal infrared emission from the planet can contribute significantly to the net radiation on the spacecraft. This effect is particularly pronounced in the harsh thermal environment of Mercury, where TRP levels can reach ~20% of SRP (see Figure 4.5). The formulation of this acceleration is slightly different with respect to solar and planetary radiation pressure, since the radiation flux is not incident but emitted by the spacecraft. Considering every spacecraft element k as a Lambertian surface, characterized by an emissivity ϵ_k and a temperature T_k , the total acceleration is:

$$\mathbf{a}_{TRP} = - \sum_{k=1}^{Ne} \frac{2}{3} \frac{\epsilon_k \sigma}{mc} A_k T_k^4 \hat{\mathbf{n}}_k \quad (3.34)$$

TRP is one of the most important source of error in the Mercury environment due to the difficulty in predicting the temperature of the spacecraft elements. The temperature of each surface element can be calculated through the differential heat balance equation, accounting for the heat power inputs from the Sun and the planet, which depend on the orbital position and the attitude orientation of the spacecraft elements relative to the central body.

Considering the generic spacecraft element k , the total heat power input is:

$$Q_{in,k} = Q_{sun,k} + Q_{alb,k} + Q_{ir,k} \quad (3.35)$$

On the other hand, the radiated power output is:

$$Q_{out,k} = A_k \epsilon_k \sigma T_k^4 \quad (3.36)$$

The input heat power from the Sun depends on the absorbance of the spacecraft element α_k , the incidence angle $\theta_{sun,k}$ (angle between the normal to the spacecraft element and the spacecraft-Sun direction), the surface area A_k and the illumination condition:

$$Q_{sun,k} = \begin{cases} \alpha_k \Phi_{\odot} A_k \cos(\theta_{sun,k}), & \text{if } \theta_{sun,k} < \pi/2 \text{ and no eclipse} \\ \alpha_k f_{PE} \Phi_{\odot} A_k \cos(\theta_{sun,k}), & \text{if } \theta_{sun,k} < \pi/2 \text{ and partial eclipse} \\ 0, & \text{if } \theta_{sun,k} \geq \pi/2 \text{ or eclipse} \end{cases} \quad (3.37)$$

where f_{PE} is a scale factor for the partial eclipse condition (Vokrouhlicky et al., 1993).

To compute the energy input from the planet, a planet surface model with corresponding view factors is needed. As for the previous section, we introduce a grid model of the planet surface discretized in equally spaced cells. The geometry is shown in Figure 3.5. The ratio of the energy received by the shape element k of the orbiter compared to the energy that is radiated by the generic grid cell ij can be obtained by using the angles ψ_{ij} and $\theta_{ij,k}$. In addition, we assume that both albedo and infrared radiations represent hemispherically diffuse emissions.

The view factor $f_{ij,k}$ at each grid point can be defined as:

$$f_{ij,k} = \begin{cases} \frac{\cos(\psi_{ij}) \cos(\theta_{ij,k}) A_k}{\pi r_{sc,ij}^2} & \text{if } \psi_{ij} < \pi/2 \text{ and } \theta_{ij,k} < \pi/2 \\ 0 & \text{otherwise} \end{cases} \quad (3.38)$$

where $r_{sc,ij}$ is the distance between the spacecraft and the grid point ij . The element is in visibility if both ψ_{ij} and $\theta_{ij,k}$ are below $\pi/2$. Otherwise, the view factor is set to zero.

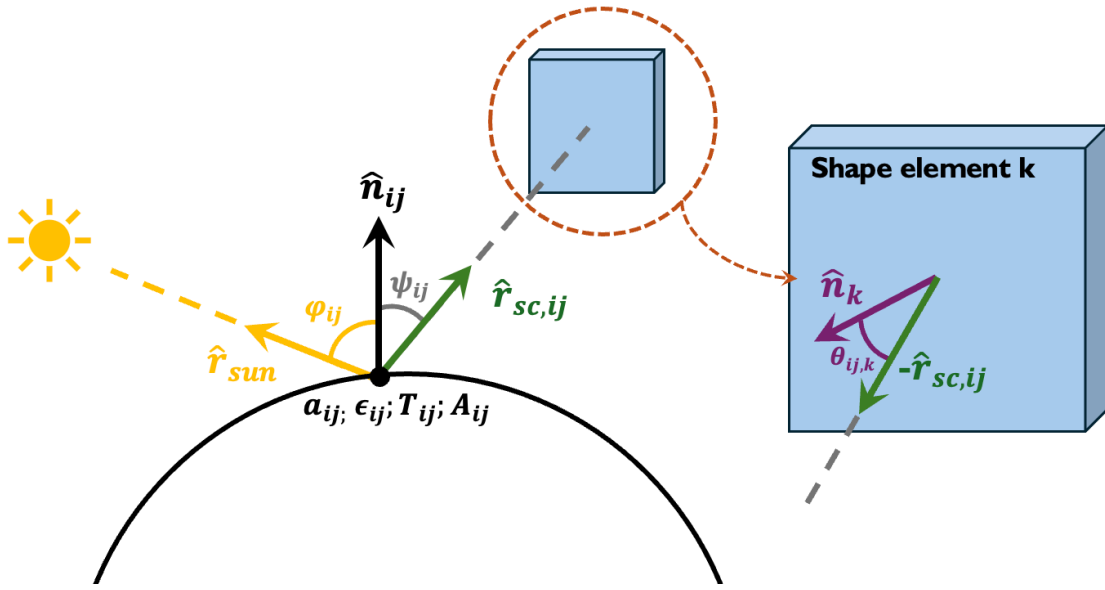


Figure 3.5. Geometry for the radiative flux emitted by a planetary surface cell and received by a spacecraft surface element. $\hat{n}_{ij}$ is the normal to the surface cell; A_{ij} is the area; a_{ij} , ϵ_{ij} and T_{ij} are the albedo, emissivity and temperature values; $\hat{r}_{sun}$, $\hat{r}_{sc,ij}$ are the directions of the Sun and the spacecraft with respect to the grid cell, respectively; $\hat{n}_k$ is the normal to the spacecraft surface element.

The albedo energy is obtained by summing the contribution to every sunlit grid point, while infrared radiation also accounts for the night side. Let the set of cells that are sunlit at a specified epoch be $\mathbb{A}_s$ (see Eq. (3.27), the planetary inputs are:

$$Q_{alb,k} = \sum_{ij \in \mathbb{A}_s} f_{ij,k} a_{ij} \Phi_{\odot} A_{ij} \cos(\varphi_{ij}) \quad (3.39)$$

$$Q_{ir,k} = \sum_{ij} f_{ij,k} \sigma \epsilon_{ij} T_{ij}^4 A_{ij} \quad (3.40)$$

Note that in this case the visibility condition is implicit in $f_{ij,k}$. For the solar panels, the last term to consider is the heat power $Q_{trans,k}$ transferred from the warmer front side to the colder back side. This quantity can be obtained by using the procedure established in the Rosetta study, which expresses the heat transfer through the solar panel in terms of a conducted part and a radiated part (Kato & van der Ha, 2012). Let the front and backside temperature of the panel be T_f and T_b , the internal power transferred is:

$$Q_{trans,k} = \mathcal{K}(T) \frac{0.95S_{Al}}{h} (T_f - T_b)A_k + \sigma \frac{(1 - S_{Al})}{2 - \epsilon_i} (T_f^4 - T_b^4)A_k \quad (3.41)$$

where $\mathcal{K}(T)$ is the conductivity; h is the solar panel thickness; S_{Al} is the net aluminum cross-section per square meter of solar panel; and ϵ_i is the inner core surface emissivity.

The temperature of the front and back side can be retrieved by solving the differential heat balance equations:

$$\begin{cases} C_{k,f} \frac{dT_{k,f}}{dt} = Q_{in,k,f} - Q_{out,k,f} - Q_{trans,k} \\ C_{k,b} \frac{dT_{k,b}}{dt} = Q_{in,k,b} - Q_{out,k,b} + Q_{trans,k} \end{cases} \quad (3.42)$$

where C_k is the thermal capacity of the k-th element. Note that if one of the side of the shape element is thermally isolated, just one differential equation needs to be solved and $Q_{trans,k}$ is set to zero.

Chapter 4

MESSENGER data analysis

This chapter presents a complete reanalysis of the entire MESSENGER Doppler dataset, yielding independent estimates of Mercury’s gravity field, spin-axis orientation, physical librations in longitude, and tidal Love number. The aim is not to provide another gravity solution, but rather to establish a reliable baseline for a future joint analysis with the dataset from MORE. One possible strategy is a sequential approach, in which MORE uses MESSENGER-derived gravity models as a priori. However, MESSENGER solutions rely on strong Kaula regularization due to the spacecraft uneven spatial coverage, which may bias the solution. A unique, simultaneous inversion of MESSENGER and BepiColombo data is instead preferable: it allows the full set of radiometric observables to be processed with the same software and consistent models. Also, the low eccentric orbit of the MPO relaxes the use of Kaula constraint, reducing the risk of regularization artifacts.

This chapter is organized as follows: an overview of the MESSENGER mission is provided in Section 4.1; a description of the radio science experiment follows in Section 4.2; the data and methods for the data analysis are presented in Section 4.3, together with the results of the gravity field inversion; finally, concluding remarks are given in Section 4.4.

4.1 Mission overview

The MErcury Surface, Space ENvironment, GEochemistry, and Ranging (MESSENGER) mission, part of NASA's Discovery Program and developed by the Johns Hopkins University Applied Physics Laboratory (APL), was the first spacecraft to orbit Mercury. Launched on August 3, 2004 it performed a 6.6-year cruise phase that included one Earth flyby, two Venus flybys, and three Mercury flybys, as well as multiple orbit correction maneuvers (OCMs). Orbit insertion occurred on March 18, 2011. The spacecraft was scheduled for a one-year mission but ultimately operated until April 30, 2015, when it impacted Mercury's surface. The evolution of the orbit during the mission is shown in Figure 4.1. The spacecraft was initially placed into a nearly polar, 12-h period orbit with pericenter altitude of ~200-km and ~60°N latitude. The first extended mission (XM1) began in March 2012, with the orbital period reduced to 8 hours the following month.

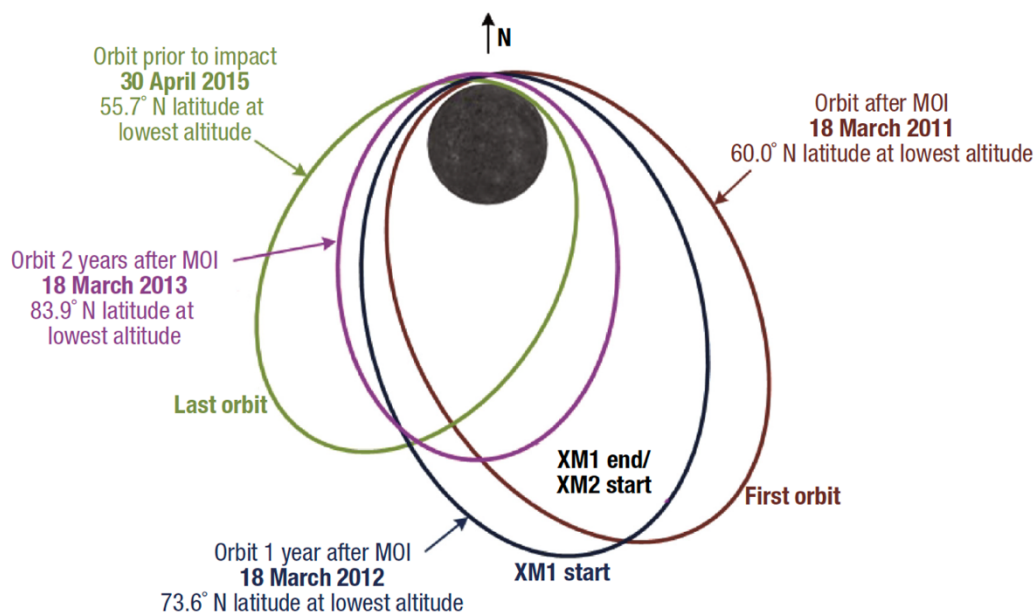


Figure 4.1. MESSENGER orbit evolution, from (Bedini, 2017).

The second extended mission (XM2) started in March 2013 and continued until the mission's end. For the first three years, the pericenter altitude was maintained between 200 and 500 km with regular OCMs, before being progressively lowered to enable close-

range observations of the surface. During its final year, MESSENGER carried out the “Low Altitude Campaign,” operating at altitudes lower than 25 km. This phase provided unprecedented observations of the northern hemisphere and radiometric data highly sensitive to the short-wavelength components of Mercury’s gravity field, enabling spatial resolutions finer than 80 km. At such low altitudes, the spacecraft orbit was also strongly perturbed by the planet’s radiation, making the modelling of non-gravitational accelerations (NGAs) a crucial aspect for the data analysis (Section 3.5). Figure 4.2 shows the altitude of the pericenters and the corresponding latitude for different phases of the mission. As can be seen, no low altitude data were collected in the southern hemisphere.

Instrument	Description
Mercury Dual Imaging System (MDIS)	Narrow-angle (NAC) and wide-angle (WAC) cameras for mapping surface morphology and landforms (Hawkins et al., 2007).
Mercury Laser Altimeter (MLA)	Solid-state laser transmitter and receiver for measuring surface topography, enabling detailed studies of Mercury’s shape (Cavanaugh et al., 2007).
Gamma-Ray and Neutron Spectrometer (GRNS)	Measures elemental abundances in the crust. Its neutron sensor is also sensitive to hydrogen, helping to detect water ice at the poles (Goldsten et al., 2007).
Mercury Atmospheric and Surface Composition Spectrometer (MASCS)	Combines ultraviolet, visible, and infrared spectrometers to analyse Mercury’s exosphere composition and surface mineralogy (McClintock & Lankton, 2007).
Magnetometer (MAG)	Mounted on a 3.6 m boom, maps Mercury’s magnetic field and looks for magnetized crustal rocks (Anderson et al., 2007).
X-Ray Spectrometer (XRS)	Detects X-ray fluorescence from the surface induced by solar X-rays, providing elemental composition of Mercury’s crust (Schlemm et al., 2007).
Energetic Particle and Plasma Spectrometer (EPPS)	Measures composition, energy, and spatial distribution of ions and electrons in and around Mercury’s magnetosphere (Andrews et al., 2007).

Radio Science (RS)

Investigates Mercury’s interior and mass distribution by determining its gravity field and rotational state (Srinivasan et al., 2007).

Table 4.1. MESSENGER scientific instruments.

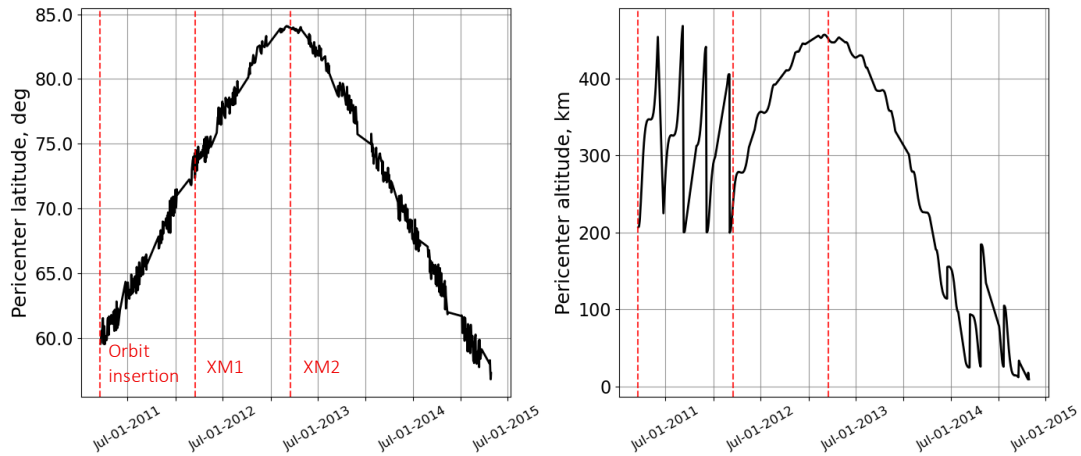


Figure 4.2. Evolution of MESSENGER’s pericenter latitude (left) and altitude (right) throughout the orbital phase of the mission. Vertical red lines indicate orbit insertion and the start of the first (XM1) and second (XM2) extended missions.

The mission aimed to advance our understanding of Mercury by investigating its interior, surface composition and geological evolution, probing the materials at its poles, exploring the nature of its magnetic field, and studying the processes shaping its exosphere, magnetosphere, and volatile environment. To fulfil its science objectives, MESSENGER’s payload was equipped with a suite of eight instruments, which are reported in Table 4.1. All instruments interfaced through redundant Data Processing Units (DPUs).

The large amount of data collected by MESSENGER during more than four years in orbit around Mercury radically reshaped our knowledge of the planet. MESSENGER enabled the development of the first detailed gravity field models of Mercury and produced the first global topographic map (Becker et al., 2016), thanks to measurements from MLA (Cavanaugh et al., 2007) and MDIS (Hawkins et al., 2007). Its observations revealed key aspects of Mercury’s interior (Genova et al., 2019) and geological evolution,

confirmed the presence of water ice in polar deposits (Lawrence et al., 2013), and provided new insights into the magnetic field (Johnson et al., 2012) as well as the abundance of volatile materials (Blewett et al., 2011).

4.2 Radio science investigation

The MESSENGER radio science (RS) investigation relies on the spacecraft Radio Frequency (RF) telecommunication system, described in the next section. The system featured redundant transponders, solid-state power amplifiers (SSAs), and was equipped with a suite of eight antennas. By processing Doppler and range tracking data through orbit determination (OD), the experiment enables the inversion of Mercury's ephemeris, gravity field, and rotational state, including obliquity and forced longitudinal librations. Doppler measurements are most sensitive to the gravity field, while range data improves Mercury's ephemeris (Konopliv et al., 2020). The determination of the spacecraft orbit is also essential for MLA to produce a topographic map. Since the southern hemisphere remained outside MLA's coverage, spacecraft occultations by the planet provided local measurements of Mercury's radius, particularly valuable for those areas. Finally, MESSENGER radiometric data allowed dedicated analysis of relativistic parameters (Genova et al., 2018; Park et al., 2017).

4.2.1 RF telecommunication subsystem

The antenna suite of the RF subsystem was made of eight non-steerable antennas mounted on the spacecraft and is shown in Figure 4.3, together with the orientation of the spacecraft body-fixed frame. Two phased-array antennas (PAAs) were mounted on opposite sides of the spacecraft. Each PAA could electronically steer $\pm 60^\circ$ in the XY-plane, and combined with spacecraft rotation about the Y-axis, provided nearly omnidirectional high-gain coverage. Collocated with the PAAs were two fan-beam antennas (FBAs), which offered medium-gain capabilities, while four low-gain antennas (LGAs) provided

additional coverage. Redundant X-band Small Deep Space Transponders (SDSTs) received uplink signals and coherently retransmitted them to Earth after being amplified by the internal SSAs, enabling two- and three-way radio tracking (Srinivasan et al., 2007).

Depending on orbital geometry, spacecraft attitude, time of day, and scheduling constraints, MESSENGER operations relied on different combinations of these antennas. While the highest quality observables were obtained with the high- and medium-gain radio links, many pericenter passes were tracked using LGAs due to attitude constraints.

Communications were maintained with NASA's Deep Space Network (DSN) 34-m and 70-m antennas at Goldstone (California), Madrid (Spain), and Canberra (Australia), operating at 7.2 GHz (uplink) and 8.4 GHz (downlink). The DSN tracked MESSENGER for about 16 h/day during early commissioning, 8 h/day during the first year and ~6 h/day after the 2012 mission extension.

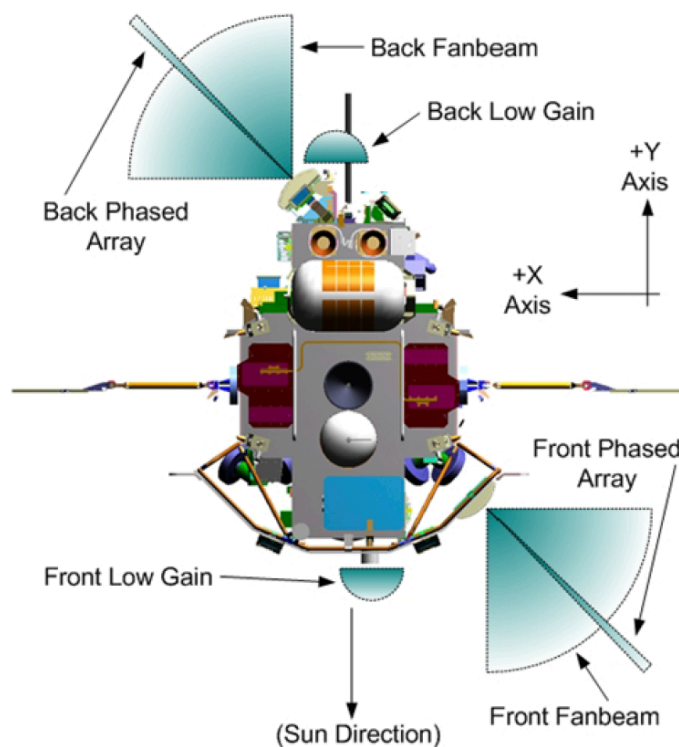


Figure 4.3. MESSENGER body-fixed frame and antenna suite, from (Srinivasan et al., 2007).

4.2.2 Geodesy and geophysics investigation

The geodesy and geophysical objectives of the RS investigation focus on three main areas: the determination of Mercury's static gravity field, the estimation of the Love number k_2 , and the measurement of the planet's rotational state, including spin axis orientation and physical librations in longitude. Together, these measurements provide key constraints on the structure and composition of Mercury's interior.

Gravity and rotation measurements are especially important for probing Mercury's interior. Following Peale's procedure (Section 3.4.1), measurements of longitudinal librations, degree-2 gravity, and obliquity constrain the normalised moment of inertia C/MR^2 , and consequently the polar moment of inertia ratio C_m/C . These quantities are extremely important to provide information on the size and state of the core, discriminating between a rigid and a partially molten interior.

By precisely reconstructing the spacecraft orbit, the radio science experiment enables the estimation of the real part of the Love number k_2 , which quantifies the elastic response of the planet to solar tides (Van Hoolst & Jacobs, 2003). This quantity provides additional constraints on the rheology of the mantle and outer core, although the quality of the radiometric observables and lack of accelerometer data to model non-gravitational effects prevented the determination of the phase lag.

Finally, gravity and topographic measurements provide insights into Mercury's crustal structure. The RS investigation delivers accurate gravity anomalies, while the MLA altimeter maps surface topography. Combining the two datasets allows to derive Bouguer anomalies, which are gravity anomalies corrected from the contribution of spacecraft altitude and surface topography. Comparison of measured and expected anomalies enables the recovery of a global map of crustal thickness (Wieczorek & Phillips, 1998). Further, the high-resolution data collected during the low-altitude campaign over the northern hemisphere allows detailed local admittance and gravity analyses, constraining the density and elastic thickness of the outer layers (Genova et al., 2023). These investigations offer valuable insights into Mercury's thermal and geological evolution.

4.3 Data analysis

The OD of MESSENGER in this work is performed with the Jet Propulsion Laboratory's MONTE software (Evans et al., 2018). MONTE relies on a number of models to integrate the spacecraft trajectory and to compute a measurement to be compared with the actual observable. This section describes the data and models used for the complete reanalysis of the two-way Doppler dataset. A critical aspect is the accurate modelling of non-gravitational forces (see Section 3.5), which strongly affect MESSENGER's dynamics due to its proximity to the Sun. To prevent the accumulation of errors from dynamical mismodelling, the dataset is processed using a multi-arc approach (Section 2.2), dividing the mission timeline into shorter arcs.

4.3.1 Observable quantities

The radiometric observables are archived in the Planetary Data System (PDS) and include one-way, two-way, and three-way Doppler, as well as two-way range (Verma et al., 2020). In this study, only two-way coherent Doppler data are used. One-way Doppler is excluded because of its lower accuracy and its small contribution (<6%) to the dataset (Verma & Margot, 2016). Similarly, range is not included, since it represents less than 1% of the dataset and is mainly relevant for improving Mercury's ephemeris rather than estimating the gravity field (Konopliv et al., 2020).

The specified accuracy of the two-way Doppler measurement measurements (X band, uplink at 7.2 GHz, downlink at 8.4 GHz) for the RF subsystem is within ± 0.1 mm/s over a 60-s integration period (Srinivasan et al., 2007). The typical precision of the two-way Doppler as measured during the first 400 days of cruise was 0.03 mm/s with an integration time of 60 s (Mazarico, Genova, et al., 2014). Longer integration times improve precision but reduce spatial resolution, since the spacecraft can move approximately 200 km during 60 s at the pericenter. To balance thermal noise reduction

with sensitivity to short-wavelength gravity signals, we process data compressed at 10 s integration time.

The precision of two-way X-band Doppler measurements depends on the antennas configuration and is primarily limited by solar plasma noise, which varies with the Sun–probe–Earth (SPE) angle. Additional contributions come from Earth media effects and the received signal-to-noise ratio (SNR; (Asmar et al., 2005). Doppler precision improves when using the HGA/MGA antennas due to their higher SNR (Mazarico, Genova, et al., 2014). As shown in (Mazarico, Genova, et al., 2014), the noise level increases significantly for SPE angles $<40^\circ$, and such data are excluded in this analysis. Future missions such as BepiColombo are expected to achieve unprecedented accuracy in radiometric data by calibrating plasma noise with the triple-link (Iess et al., 2021).

Antenna	Offset [m]
FRONT LGA	[-0.1270, -1.0348, -1.9939]
BACK LGA	[0.1095, 1.2753, -1.4262]
AFT LGA	[-0.2794, -0.8593, 0.3686]
FWD LGA	[0.2794, -0.8593, -2.2291]
FRONT FBA	[-0.7272, -0.6487, -1.6162]
BACK FBA	[0.2743, 1.1394, -1.2603]
FRONT PAA	[-0.7272, -0.6487, -1.6162]
BACK PAA	[0.2743, 1.1394, -1.2603]

Table 4.2. Antenna coordinates in the spacecraft body-fixed frame with origin at the adaptator ring.

Earth-based DSN antenna locations are modelled to centimeter level accuracy in the 1993 realization of the International Earth Rotation Service (IERS) Terrestrial Reference Frame and are corrected to account for ocean loading effects (Folkner, 1997; NASA DSN, 2014). Antenna positions and motions are modelled relative to the spacecraft center of mass to account for their signatures in the Doppler data. The coordinates in the spacecraft body-fixed frame can be found from the PDS archive (PDS, 2008; Perry et al.,

2022) and are reported in Table 4.2. The center of mass coordinates in the spacecraft body-fixed frame, as provided by the navigation team, are also incorporated in our model (PDS, 2015b; Perry et al., 2022).

Finally, media corrections are applied using troposphere and ionosphere calibrations obtained from in situ meteorological data at the DSN sites and available in the PDS archive (Perry et al., 2022).

4.3.2 Spacecraft model

To model the effect of NGAs, a detailed representation of the spacecraft geometry is required. The spacecraft structure is shown in Figure 4.4. It was primarily built from lightweight composite materials to withstand the extreme thermal variations encountered during the mission (Leary et al., 2007). A ceramic-cloth sunshade ($\sim 2 \text{ m} \times 2.5 \text{ m}$) shielded the bus, while power was provided by two solar arrays ($1.54 \text{ m} \times 1.75 \text{ m}$ each) made of aluminium honeycomb with composite face sheets. The arrays were designed to tolerate high temperatures and adjust their orientation for thermal control.

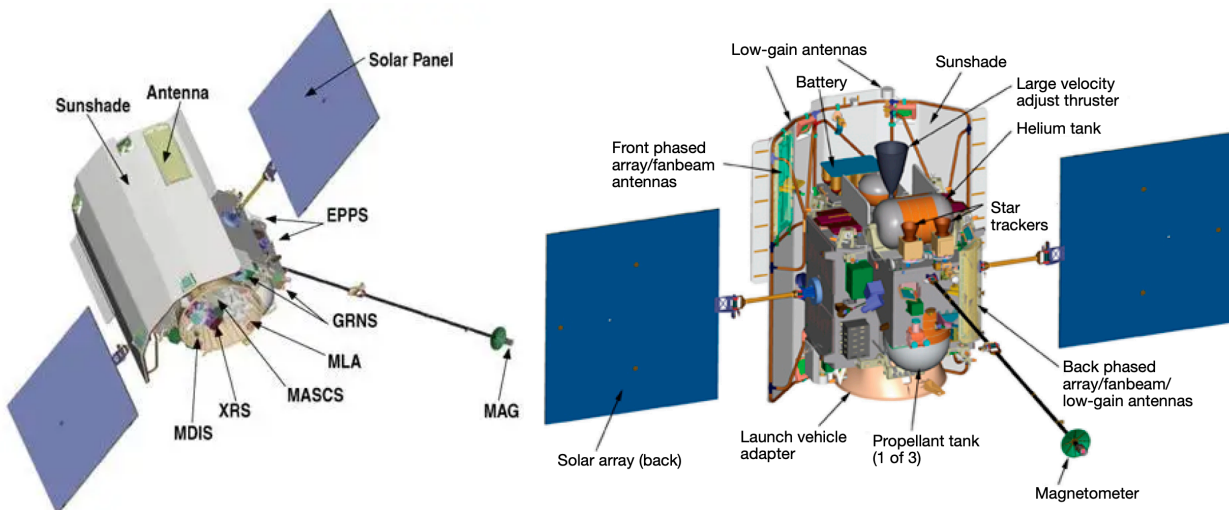


Figure 4.4. The MESSENGER spacecraft. Credit: NASA.

For this analysis, the spacecraft is modelled as a set of plates, listed in Table 4.3. Since the bus was nominally protected from direct solar radiation by the sunshade, only the

backside plate of the bus is accounted for solar pressure and albedo. For thermal infrared radiation, also the other plates of the bus are included. Secondary structures, such as the boom and antennas, are neglected as their contribution to the overall acceleration is negligible. The surface areas and thermo-optical properties are found from the literature (Dakermanji et al., 2006; Page et al., 2014; Stanbridge et al., 2012).

The effective area exposed to solar and planetary radiation depends on the orientation of the spacecraft bus and solar arrays. MESSENGER was three-axis stabilized and momentum-biased, maintaining the sunshade pointing to the Sun while allowing instrument targeting through rotations about the Sun–spacecraft direction. Spacecraft angular momentum was controlled by reaction wheels, with periodic commanded momentum desaturations (CMDs). The attitude profile is inferred from the attitude kernels provided by the mission (PDS, 2015a).

Plate	Area (m ²)	ρ_S, ρ_D (visible)	ρ_S, ρ_D (IR)
Sunshade central	1.264	0.13, 0.24	0.0, 0.1
Sunshade lateral	2.122	0.13, 0.24	0.0, 0.1
Solar arrays front	2.695	0.20, 0.07	0.0, 0.2
Solar arrays back	2.695	0.11, 0.25	0.0, 0.3
Bus panel +Y	2.627	0.07, 0.26	0.0, 0.3
Bus panels $\pm X$	2.350	0.07, 0.26	0.0, 0.3
Bus panels $\pm Z$	1.800	0.07, 0.26	0.0, 0.3

Table 4.3. MESSENGER plate model representation (Dakermanji et al., 2006; Page et al., 2014; Stanbridge et al., 2012). See Figure 4.3 for the orientation of the spacecraft body-fixed frame.

4.3.3 Dynamical model

The dynamical model accounts for the gravitational monopole accelerations due to Solar System planets, the spherical harmonics coefficients of the Sun up to degree and order 2 and the spherical harmonics coefficients of Mercury up to degree and order 100.

We use the DE440 planetary ephemeris from JPL (Park et al., 2021), which provides a refined orbit of Mercury thanks to the MESSENGER ranging data. The starting values of Mercury’s gravity coefficients are initialized from the HgM008 model (Genova et al., 2019).

Due to the proximity between the spacecraft and the Sun, MESSENGER experienced significant NGAs (see Section 3.5). We model the acceleration due to direct sunlight (SRP), sunlight reflected by the planet (albedo), infrared radiation emitted by the planet (IR) and by the spacecraft (TRP).

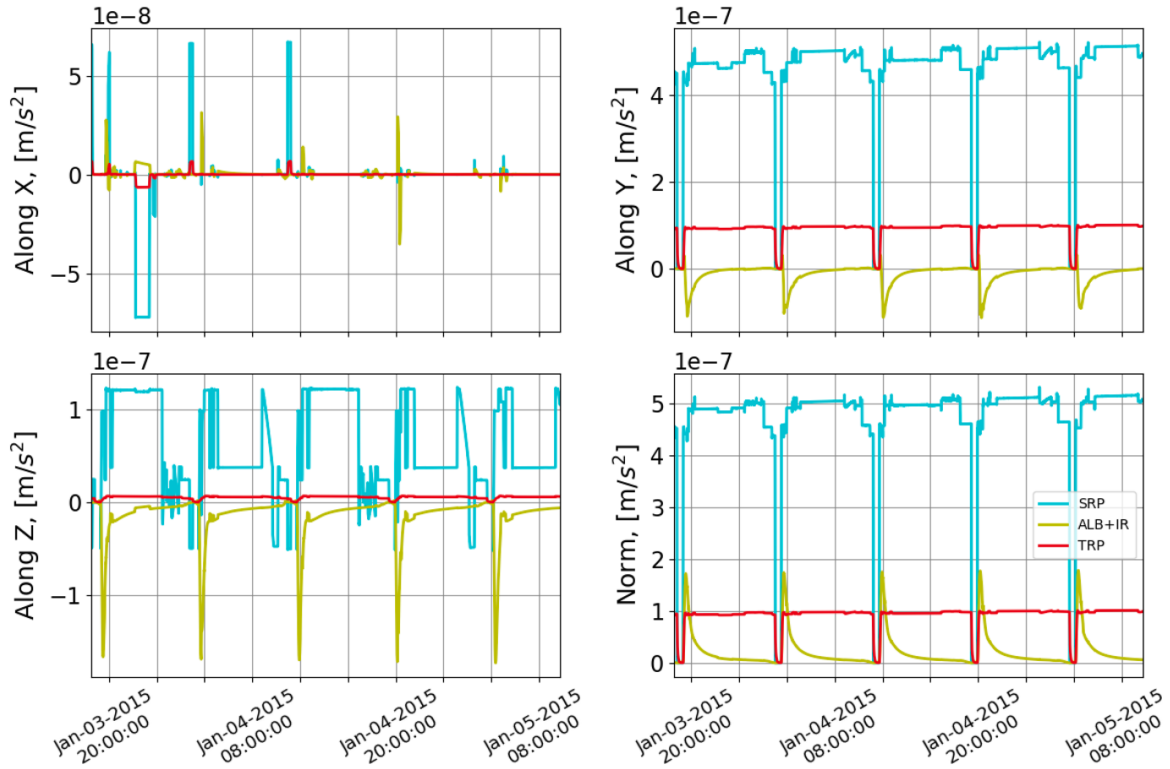


Figure 4.5. Non-gravitational accelerations components in the spacecraft body frame during one arc of the low altitude campaign. The plot shows the contributions from solar radiation pressure (SRP), planetary radiation pressure (albedo + infrared), and thermal recoil pressure (TRP).

Figure 4.5 shows the magnitude of each component of modelled NGAs in the spacecraft reference frame, during one arc of the low altitude campaign. The dominant non-gravitational force acting on the spacecraft is SRP (Section 3.5.2), which produces an acceleration at the level of $\sim 5 \cdot 10^{-7} \text{ m/s}^2$.

The most significant effect after SRP is the sum of planetary albedo and IR (Section 3.5.3). MONTE discretizes the visible portion of Mercury’s surface using a central cap and concentric rings. This methodology requires an albedo and emissivity map to be represented in spherical harmonics. The albedo map, shown in Figure 4.6, is derived from MDIS (Hawkins et al., 2007) low-incidence angle (LOI) global map, which reveals the varying reflectance properties of Mercury’s surface. The thermal infrared emissivity is computed from a temperature map, also shown in Figure 4.6. This map is generated by discretizing Mercury’s surface into a $1^\circ \times 1^\circ$ grid in longitude and latitude. At each grid point, the temperature is determined by Eq. (3.33). Emissivity is then calculated at each surface point using Eq. (3.32). The resulting albedo and emissivity distributions, are then expanded into spherical harmonics up to degree and order 6 using the SHTOOLS open-source library (Wieczorek & Meschede, 2018).

Finally, TRP is computed following the approach reported in Section 3.5.4. Each spacecraft plate is assigned an emissivity value, thermal capacity, surface area, and normal vector. In the absence of specific design data, these properties are inferred from standard spacecraft materials and from Rosetta (Kato & van der Ha, 2012). As can be seen in Figure 4.5, TRP magnitude reaches $\sim 20\%$ of the SRP, making it a critical component of the dynamical model.

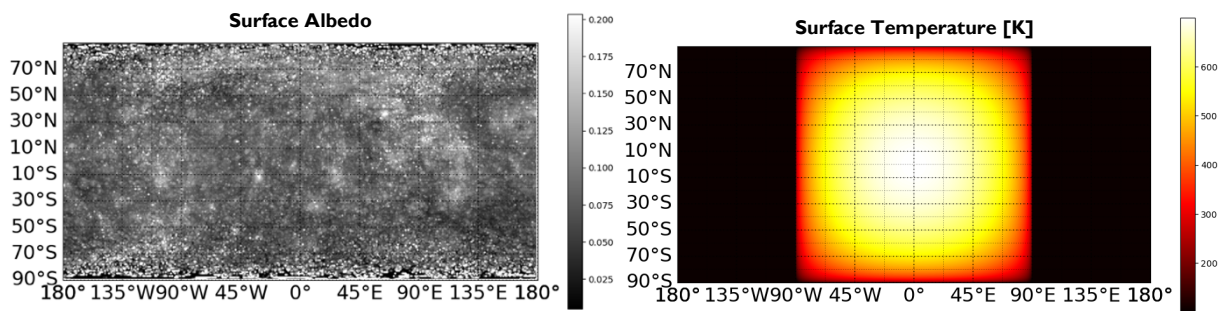


Figure 4.6. Mercury surface albedo and temperature maps.

4.3.4 Filter setup

The mission timeline is divided in 1064 arcs. The subdivision of the timespan in arcs is done by attributing to each arc a duration of ~ 24 hours to include at least ~ 6 hours of

tracking from DSN. We exclude arcs with SPE angles below $<40^\circ$ and arcs affected by the performance of any thrusted maneuver, including CDMs and OCMs. After this data removal, ~ 750 arcs remained.

The set of local parameters, solved independently for each arc, with their a priori uncertainty, is provided in Table 4.4. The state vector is determined at the central time of each arc, integrating forward and backward. The thermo-optical coefficients are adjusted to absorb mismodelling, together with a single scale factor for planetary radiation and three scale factors (one for each spacecraft body-fixed axis) for spacecraft radiation. Finally, a constant empirical acceleration in the along-track direction is included as a local parameter to absorb residual errors.

Local parameter	A priori uncertainty
S/C Position (J2000)	300 m
S/C Velocity (J2000)	3 cm/s
Optical coefficients (visible and IR)	20% ref. value
Planetary radiation scale factor	0.2
Spacecraft radiation scale factors	0.2
Constant along-track acceleration	10^{-12} km/s ²
Global parameter	A priori uncertainty
Mercury's GM	Unconstrained
Gravity field coefficients 100x100	Unconstrained
Mercury's k_2	Unconstrained
Spin axis α, δ	Unconstrained
Mercury angular velocity ω	Unconstrained
Libration amplitude ϕ_0	Unconstrained
Phase center offsets	10 cm

Table 4.4. Set of local and global parameters.

The estimated global parameters are also shown in Table 4.4. Antenna phase center locations with respect to the center of mass are adjusted in the global solution together with gravity and orientation parameters. Refinement of planetary ephemerides is not addressed in this work. This assumption is supported by two main reasons. First, we process only Doppler measurements, which exhibit minimal sensitivity to inaccuracies in planetary ephemerides. Second, we use DE440 ephemeris, which already fitted MESSENGER data to improve Mercury’s orbit.

Although latest MESSENGER derived gravity fields are extended up to degree and order 160 (Genova et al., 2023; Konopliv et al., 2020), we solve for the 100x100 gravity field as in earlier solutions (Genova et al., 2019; Mazarico et al., 2016) to reduce computational effort. This choice is supported by the fact that the aim of this work is only to provide a reliable basis for a future joint analysis with BepiColombo. Future works can improve the solution by increasing the maximum estimated degree and can incorporate ranging data to refine Mercury’s ephemeris. To mitigate numerical instability, we apply a Kaula-type constraint starting from degree 10 onward (Kaula, 1966). Specifically, we adopt a Kaula factor $A_k = 4$, equal to the value used by Genova et al. (2019). All the other estimated geophysical parameters are unconstrained.

4.3.5 Results

This section presents the results of the MESSENGER gravity and rotation reanalysis. Our solution is compared against previous gravity and surface-based estimates to evaluate its consistency. The focus here is on validating the quality of the recovered gravity and rotation model, rather than on geophysical interpretation. The aim is to establish a reliable basis for a future joint analysis with the forthcoming BepiColombo dataset. The simulation of a joint inversion is discussed later in Section 5.4.

Gravity

The amplitude spectrum (CI) of our gravity solution is shown in Figure 4.7. The trend matches the latest gravity models expanded to the same degree (Genova et al., 2019; Mazarico et al., 2016) and remains comparable with more recent solutions extended to degree 160 (Genova et al., 2023; Konopliv et al., 2020). The aliasing at high degrees is caused by residual signal originating from the very low-altitude data acquired during the final phase of the MESSENGER mission. Since the spherical harmonic expansion is truncated, this short-wavelength signal is mapped into lower degrees, producing the observed artifacts. This effect can be mitigated by extending the expansion to higher spherical harmonic degrees, allowing the residual signal to be absorbed at shorter wavelengths. Figure 4.8 presents the free-air gravity anomalies from our solution over shaded topography. The high-resolution features enabled by the low-altitude campaign in the northern hemisphere are clearly visible and are consistent with those in recent models .

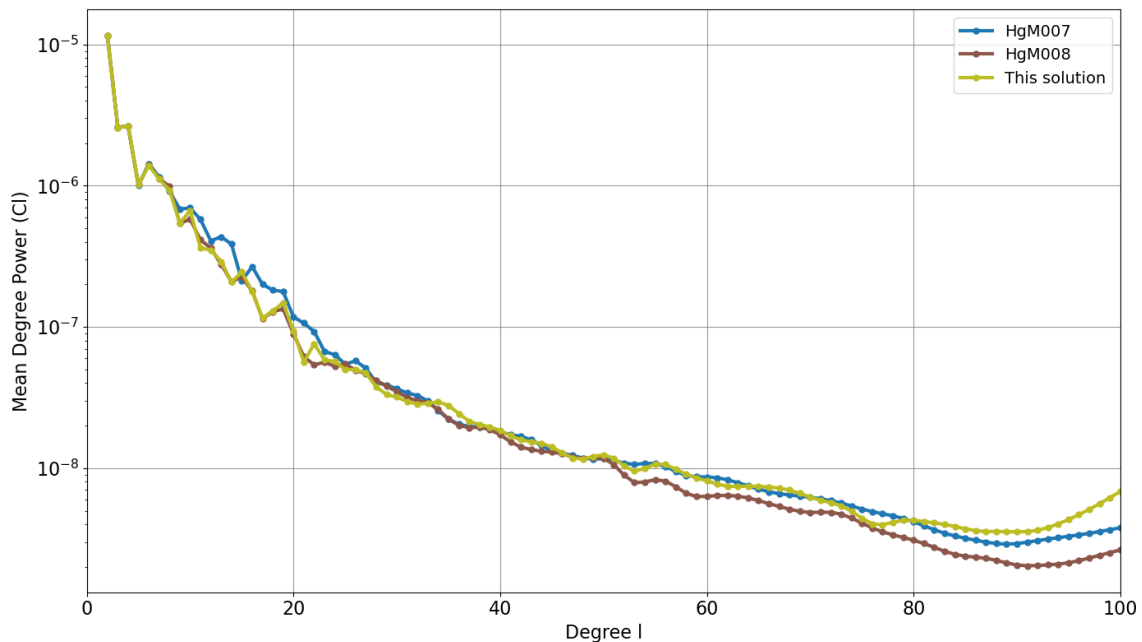


Figure 4.7. Amplitude spectrum of the estimated gravity field, compared with earlier solutions expanded at the same degree.

The gravitational Love number k_2 derived from this solution is 0.570 ± 0.011 (the uncertainty is ten times the formal value, as in Mazarico, et al., 2014). This result is in strong agreement with Genova et al. (2019) and higher than earlier estimates (Mazarico, Genova, et al., 2014; Verma & Margot, 2016), suggesting either a warm and weak mantle or the presence of a solid FeS layer at the top of the core (Hauck et al., 2013; Padovan et al., 2014). The inferred k_2 value is compatible with the existence of a molten core, which supports decoupling between the silicate shell and fluid core on short timescales (~ 88 days).

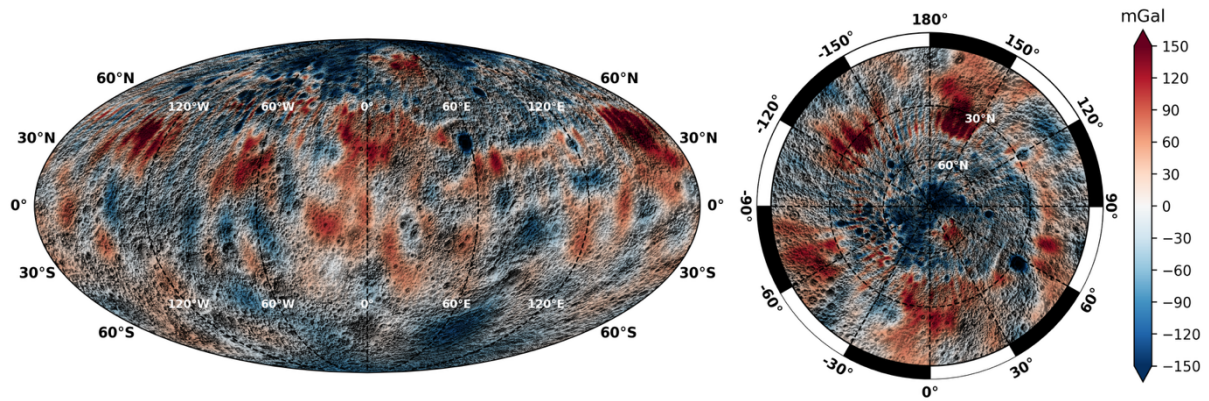


Figure 4.8. Free-air gravity anomalies on Mercury's surface over shaded topographic relief in a Mollweide projection for the entire planet (left) and in a polar stereographic projection for the northern hemisphere (right).

Parameter	(Genova et al., 2019)	(Konopliv et al., 2020)	This solution
$\bar{J}_2$	$2.24910 \cdot 10^{-5} \pm 1.1 \cdot 10^{-9}$	$2.2503 \cdot 10^{-5} \pm 1.7 \cdot 10^{-8}$	$2.25103 \cdot 10^{-5} \pm 1.4 \cdot 10^{-9}$
$\bar{C}_{22}$	$1.245786 \cdot 10^{-5} \pm 6.6 \cdot 10^{-10}$	$1.2455 \cdot 10^{-5} \pm 2.4 \cdot 10^{-8}$	$1.246532 \cdot 10^{-5} \pm 9.4 \cdot 10^{-10}$
k_{2r}	$0.569 \pm 2.5 \cdot 10^{-2}$	$0.53 \pm 3.0 \cdot 10^{-2}$	$0.570 \pm 1.1 \cdot 10^{-2}$
α [deg]	$281.0082 \pm 9.0 \cdot 10^{-4}$	$281.0138 \pm 2.5 \cdot 10^{-3}$	$281.0100 \pm 1.1 \cdot 10^{-3}$
δ [deg]	$61.4164 \pm 3.0 \cdot 10^{-4}$	$61.4161 \pm 1.7 \cdot 10^{-3}$	$61.41527 \pm 5.5 \cdot 10^{-4}$
ϕ_0 [arcsec]	40.0 ± 8.7	/	43.9 ± 4.1
ω [deg/day]	$6.1385054 \pm 1.3 \cdot 10^{-6}$	$6.138514 \pm 6.0 \cdot 10^{-6}$	$6.1385068 \pm 1.8 \cdot 10^{-6}$
ε [arcmin]	$1.968 \pm 2.7 \cdot 10^{-2}$	$1.99 \pm 1.2 \cdot 10^{-1}$	$2.048 \pm 3.4 \cdot 10^{-2}$
C/MR^2	$0.333 \pm 5.0 \cdot 10^{-3}$	$0.337 \pm 2.0 \cdot 10^{-2}$	$0.347 \pm 5.6 \cdot 10^{-3}$
C_m/C	$0.443 \pm 1.9 \cdot 10^{-2}$	$0.437 \pm 3.2 \cdot 10^{-2}$	$0.425 \pm 1.9 \cdot 10^{-2}$

Table 4.5. Uncertainties on geophysical parameters compared to latest gravity solutions. All the uncertainties are $3\text{-}\sigma$ except for the k_2 , which is $10\text{-}\sigma$. The reported values of C/MR^2 and C_m/C for Konopliv et al. (2020) are calculated following the Peale's procedure (Section 3.4.1).

Rotation

The rotational parameters are reported in Table 4.5. The joint estimation of Mercury's rotational state and gravity field allows sensitivity to both the interior structure and the shape of the surface, offering a way to probe the deep interior beyond surface observations (see Section 3.4.1). We estimate a libration amplitude of $\phi_0 = 43.9 \pm 4.1$ arcsec (3σ), consistent with Genova et al. (2019). This relatively large value is supported by the decoupling between the outer silicate shell and the fluid core on short timescales (~ 88 days).

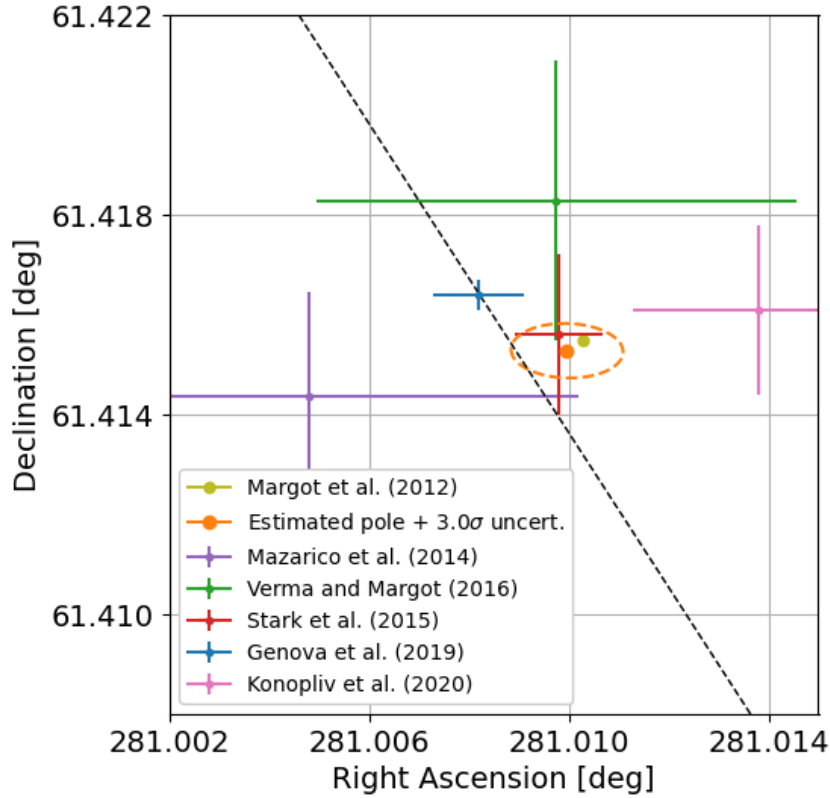


Figure 4.9. Comparison of different estimates of the pole position. The orange ellipse represents the 3σ uncertainty ellipse of this solution. The black dashed line represents the Cassini state 1.

Our derived spin-axis orientation is strongly in agreement with Earth-based radar (J. Margot et al., 2012) and MESSENGER altimetric and imaging measurements (Stark et al., 2015). The Cassini state remains within the 3σ uncertainty ellipse, as shown in Figure 4.9. Our estimated obliquity $\varepsilon = 2.048 \pm 3.3 \cdot 10^{-2}$ arcmin translates into a normalised polar

moment of inertia $C/MR^2 = 0.347 \pm 5.6 \cdot 10^{-3}$, suggesting limited interior differentiation. These values are both fully consistent with constraints derived from surface-based measurements.

Finally, the fractional polar moment of inertia of the solid outer shell (crust + mantle) provides an additional constraint on interior models. Combining our obliquity estimate with the libration amplitude of J. Margot et al. (2012) yields $C_m/C = 0.425 \pm 1.9 \cdot 10^{-2}$, consistent with latest gravity and surface-based estimates. This inference implicitly assumes that the libration amplitude measured by radar observations, which only constrain the rotation of the solid outer shell (crust+mantle), is representative of the libration of the entire planet. Using instead our estimated value for the libration gives $C_m/C = 0.373 \pm 3.5 \cdot 10^{-2}$.

Crustal Thickness

The free-air gravity anomaly map of Figure 4.8 is used to derive Mercury's crustal thickness variations (see Section 3.4.2), using the SHTOOLS software package (Wieczorek & Meschede, 2018). The model parameters include mantle and crustal density, surface porosity, compaction scale depth (e-folding depth), and average crustal thickness, along with a filtering degree to suppress short-wavelength gravity noise at the mantle-crust boundary (MCB). For consistency with previous work (Beuthe et al., 2020; Genova et al., 2019), we adopt an average crustal thickness of 35 km. The surface grain density, estimated from XRS and mineralogical data, averages 2957 kg/m³, but the bulk crustal density is lower due to porosity from impact cratering. We apply a compaction model with porosity decreasing exponentially with depth, assuming a uniform surface porosity of 24% (middle value for the farside highlands of the Moon) and an e-folding depth of 8 km (Beuthe et al., 2020). Noise is reduced using a low-pass filter with cutoff degree $l_{1/2} = 40$.

Two cases are considered: (1) constant surface density (2957 kg/m³), and (2) lateral variations in surface density derived from mineralogy. In the second case, lateral density contrasts are smoothed with depth, as crustal intrusions and flows are unlikely to

propagate radially down to the MCB. The mantle density at the MCB is set to 3200 kg/m^3 (Beuthe et al., 2020). The results are shown in Figure 4.10. The resulting thickness ranges from 10–67 km in case (1) and 8–95 km in case (2), consistent with previous maps (Beuthe et al., 2020). Notably, the crust is thickest in the high-Mg terrane in the northern hemisphere (approximately between 120°W and 40°W longitude), previously thought to be thinner. As highlighted by Beuthe et al., (2020), the thickness variations in the northern hemisphere suggest a correlation between crustal thickness and mantle melt production. This link will be further tested once BepiColombo provides global coverage, enabling extension of the analysis to the southern hemisphere.

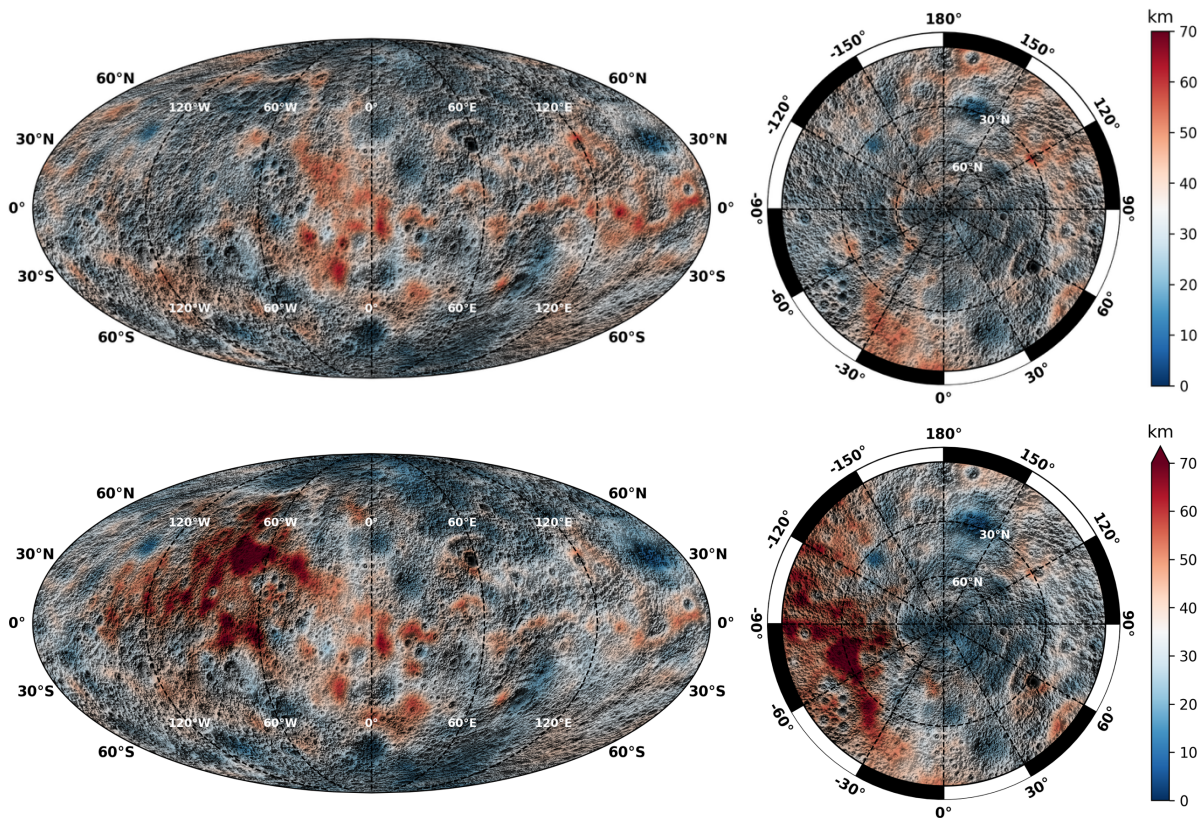


Figure 4.10. Crustal thickness map with constant surface density (top) and lateral variations of surface density (bottom). The maps are shown over shaded topographic relief in a Mollweide projection for the entire planet (left) and in a polar stereographic projection for the northern hemisphere (right).

4.4 Summary

In this chapter, we presented an independent reanalysis of the MESSENGER two-way Doppler dataset using NASA JPL’s MONTE software. The derived gravity field, rotational parameters, and crustal thickness maps were compared with the most recent solutions in the literature. Our gravity field exhibits a degree RMS spectrum consistent with both the degree-160 solutions and earlier fields, confirming the robustness of the estimation at long and short wavelengths. The recovered tidal Love number k_2 strongly agrees with one of the most recent estimates and supports the presence of a molten core, while the libration amplitude, although larger than surface-based measurements, remains compatible within uncertainties. The spin pole orientation is in closer agreement with Earth-based radar, altimetric, and imaging measurements than previous global gravity solutions, still compatible with the Cassini state. The crustal thickness maps confirm global variations comparable with earlier studies but also highlight important regional features when lateral variations in surface density are included, such as a thickened crust in the high-Mg terrane.

Overall, these results validate our independent solution and indicate a reliable basis for future improvements. They provide a solid foundation for extending the analysis to the upcoming BepiColombo radio science dataset, enabling a more comprehensive global mapping of Mercury’s gravity, rotation, and interior structure.

Chapter 5

BepiColombo: orbital phase simulations

The BepiColombo mission, jointly developed by ESA and JAXA, is currently in its cruise phase toward Mercury. Among its scientific investigations is the Mercury Orbiter Radio Science Experiment (MORE), which will employ a multi-frequency microwave tracking system with an advanced Ka-band transponder to investigate Mercury's geodesy and test fundamental physics. Leveraging precise measurements from this novel technology, MORE will refine and extend the results of the MESSENGER mission, providing new insights into Mercury's interior. This chapter introduces two simulations focused on the geodesy experiment, which is decoupled from the relativity test (Imperi et al., 2018): a contingency analysis without accelerometer data to assess performance considering dynamical mismodelling, and a covariance analysis combining MESSENGER and BepiColombo datasets. Science operations are expected to begin in April 2027.

This chapter is organized as follows: an overview of the BepiColombo mission is provided in Section 5.1; a description of the radio science experiment follows in Section 5.2; the numerical simulation with dynamical mismodelling is presented in Section 5.3; the simulation of a joint analysis with MESSENGER is reported in Section 5.4; finally, concluding remarks are given in Section 5.5.

5.1 Mission overview

BepiColombo is the third spacecraft mission to Mercury and is designed to extend MESSENGER's results by delivering a comprehensive observation of the planet (Benkhoff et al., 2021). Jointly developed by the European Space Agency (ESA) and the Japan Aerospace Exploration Agency (JAXA), the mission aims to characterize Mercury's interior, surface, exosphere, magnetosphere, and conduct tests of fundamental physics.

The mission was named after Giuseppe (Bepi) Colombo (1920–1984), the Italian mathematician, physicist, astronomer, and engineer best known for explaining Mercury's 3:2 spin–orbit resonance (Colombo & Shapiro, 1966). BepiColombo was approved in 2007 under ESA's Cosmic Vision program and successfully launched on 20 October 2018 from Kourou, French Guiana, aboard an Ariane 5. The eight-year cruise phase is shown in Figure 5.1 and involves nine planetary flybys. At the time of writing, arrival at Mercury is scheduled for November 2026, with nominal science operations to begin the following April. The nominal mission foresees one year of orbital operations, with a probable one-year extension.

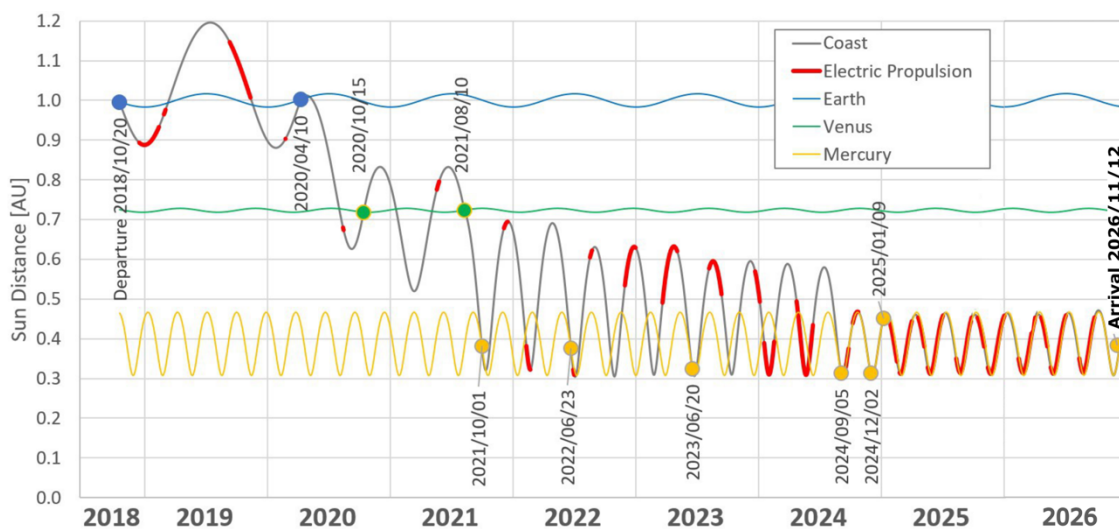


Figure 5.1. Eight-year interplanetary cruise of BepiColombo, involving nine planetary flybys (credit: ESA). The trajectory also shows the departure from Earth and the arrival at Mercury, which are represented by additional markers. At the time of writing, BepiColombo has successfully completed its sixth Mercury flyby and is expected to enter orbit around the planet in November 2026.

The mission consists of two orbiters delivered into Mercury orbit. The Mercury Magnetospheric Orbiter (MMO, developed by JAXA), also known as Mio, will operate in a highly elliptical 9.2 hr period polar orbit (590×11600 km) to investigate the exosphere and magnetosphere. The Mercury Planetary Orbiter (MPO, developed by ESA) will follow a near-circular 2.3 hr period polar orbit (480×1500 km) optimised for surface geology and interior structure studies.

5.1.1 Mercury Composite Spacecraft

During launch and the interplanetary cruise, both Mio and MPO are housed within the Mercury Composite Spacecraft (MCS), together with the Mio Sunshield and Interface Structure (MOSIF), the Mercury Transfer Module (MTM), and the solar arrays (Figure 5.2).

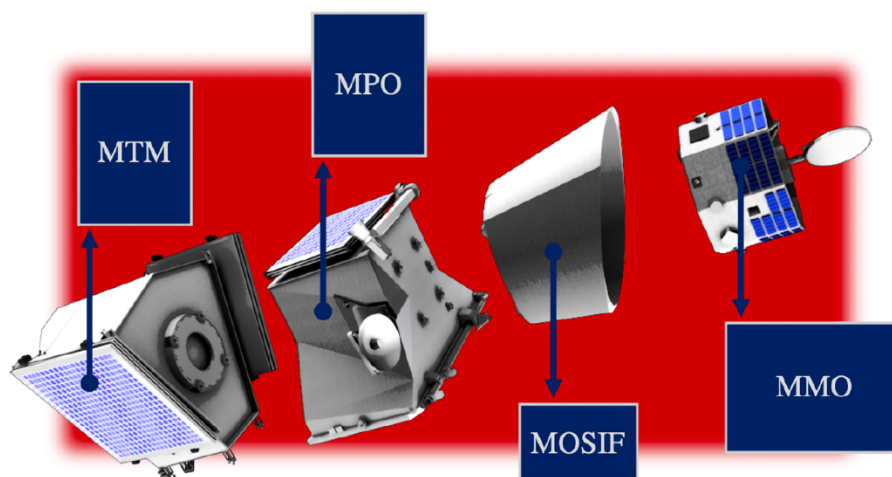


Figure 5.2. Overview of the Mercury Composite Spacecraft (MCS). The spacecraft hosts both Mio and MPO, together with the Mio Sunshield and Interface Structure (MOSIF), the Mercury Transfer Module (MTM), and the solar arrays.

The MOSIF serves as the interface between MPO and Mio, shielding Mio from direct solar radiation until orbital insertion. The MTM, built from carbon fiber–reinforced plastic, provides both propulsion and power during cruise. It delivers the required acceleration and braking to achieve Mercury orbit insertion, while supplying the large power demand of the Solar Electric Propulsion System (SEPS). The SEPS, which accounts

for the majority of the cruise ΔV , operates four 145 mN ion thrusters. For attitude control, the MTM also carries a bi-propellant system with 10 N thrusters. Its solar arrays, built with the same high-temperature technology as those of the MPO, produce up to 14 kW of power, of which 10.6 kW is allocated to SEPS. After an eight-year cruise, including one Earth, two Venus, and six Mercury flybys, the MCS will arrive at Mercury in November 2026. A series of maneuvers will then deploy Mio and MPO into their respective science orbits (see Figure 5.3).

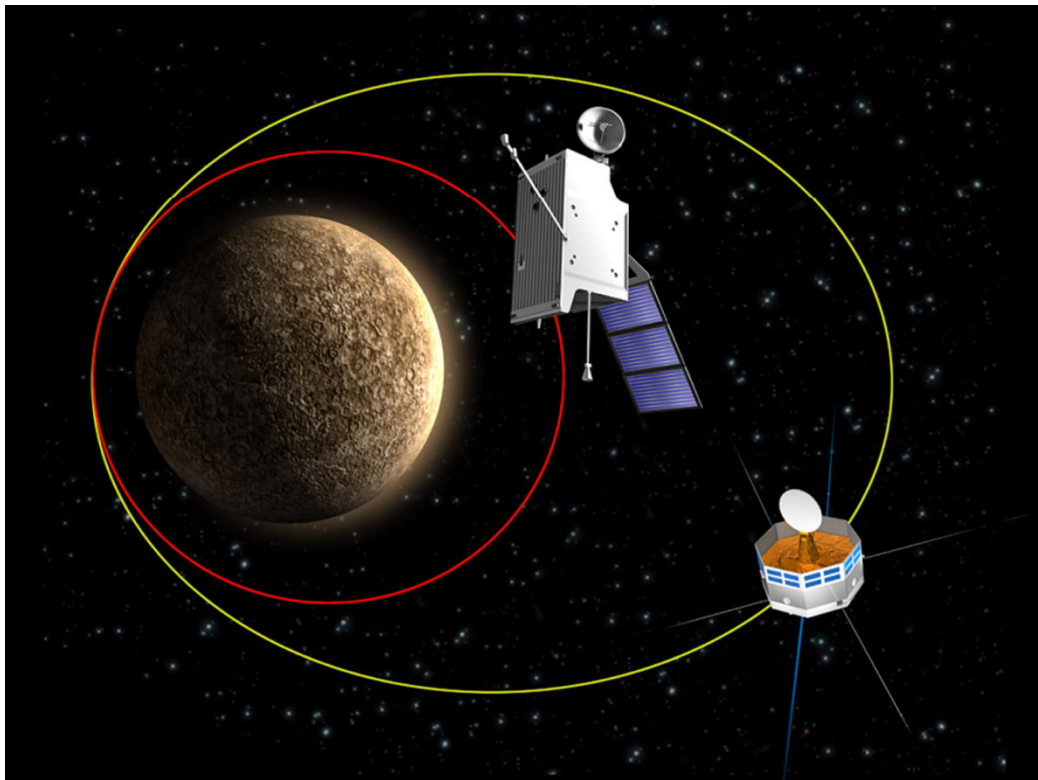


Figure 5.3. Orbits of MPO and MMO, from Benkhoff et al., (2021).

5.1.2 Mercury Magnetospheric Orbiter

BepiColombo MMO, also known as Mio, was built and is operated by JAXA. Its highly eccentric orbit, with the major axis lying in Mercury's equatorial plane, was chosen to enable detailed investigations of the planet's magnetic field and plasma environment. Mio performs in situ measurements of charged and energetic neutral particles, magnetic and electric fields, plasma waves, and dust, while also conducting remote sensing of radio waves and exospheric emissions.

The spacecraft is spin-stabilized at 15 rpm, with attitude determined by a pair of Sun sensors and a star scanner, and controlled by a cold-gas propulsion system. Its instruments are mounted on a primary structure shaped as an octagonal prism, 1.8 m across and 0.9 m tall. Mio has a mass of 250 kg and is powered by solar cells on each side, providing about 350 W. Communications are supported by an 80-cm high-gain antenna (HGA) operating in X band, complemented by a medium-gain antenna (MGA) for backup. Telemetry and tracking are primarily conducted through JAXA's 64-m Usuda antenna in Japan. The spacecraft carries five scientific experiments, summarised in Table 5.1.

Instrument	Description
Mercury Dust Monitor (MDM)	An in-situ cosmic dust instrument aimed to clarify the dust environment around Mercury (Kobayashi et al., 2020).
MMO MAGnetometer (MMO/MAG)	The instrument consists of two sets of fluxgate magnetometers. It will work in synergy with MPO/MAG onboard MPO to collect magnetic field measurements (Baumjohann et al., 2020).
Mercury Plasma Particle Experiment (MPPE)	Consists of seven sensors, which constitute a comprehensive instrument package for plasma, high energy particles and energetic neutral atom measurements to investigate the detailed plasma structure and dynamics of Mercury (Saito et al., 2021).
Mercury Sodium Atmosphere Spectral Imager (MSASI)	A high-spectral, high-temporal resolution, and wide-FOV imager devoted to address a range of fundamental scientific questions pertaining to Mercury's exosphere. The measurements will provide new information on regolith-exosphere-magnetosphere coupling as well as new understanding of the dynamics governing the exosphere bounded by the planetary surface, the solar wind and interplanetary space (Yoshioka et al., 2008).
Plasma Wave Investigation (PWI)	Enables in-situ and remote-sensing observations of electric fields, plasma waves, and radio waves in the Hermean magnetosphere and exosphere (Kasaba et al., 2020).

Table 5.1. Scientific instruments onboard the MMO.

5.1.3 Mercury Planetary Orbiter

The MPO, represented in Figure 5.4, is a three-axis stabilized spacecraft, designed and operated by ESA. The spacecraft is dedicated to investigating Mercury’s interior, surface, and exosphere and accommodates 11 scientific instruments, summarised in Table 5.2. The spacecraft measures $3.9 \times 2.2 \times 1.7$ m, with a dry mass of 1075 kg. Power is supplied by a three-panel solar array with a total area of 8 m², supplemented by a 96 Ah lithium-ion battery for solar eclipses. To maintain its temperature below 200 °C, the array is tilted up to 75° from the Sun and features a mix of solar cells and optical solar reflectors (OSRs).

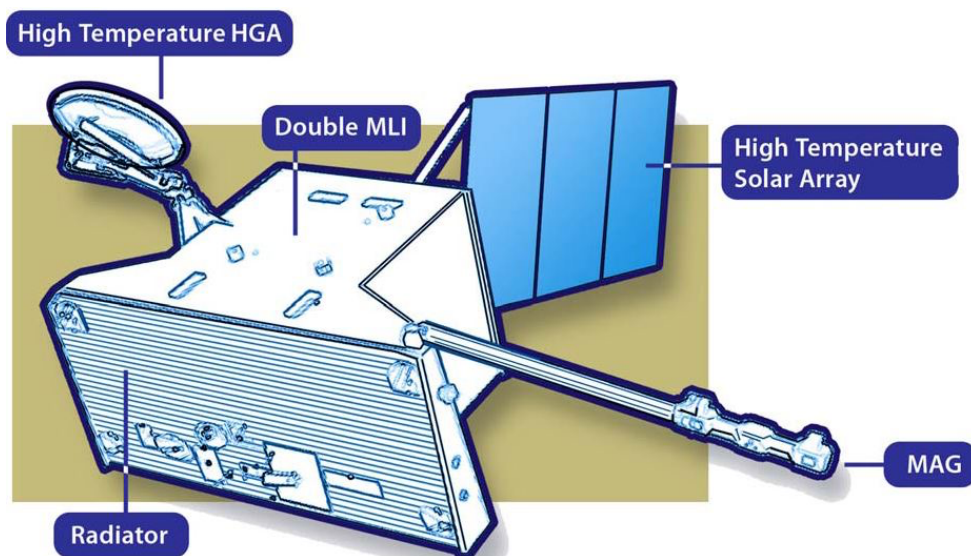


Figure 5.4. The Mercury Planetary Orbiter. Credit: ESA.

Instrument	Description
BEpiColombo Laser Altimeter (BELA)	The altimeter will characterize and measure the figure, topography, and surface morphology of Mercury. It will provide absolute topographic height and position with respect to a Mercury centered coordinate system (Thomas et al., 2021).
Italian Spring Accelerometer (ISA)	A three-axis accelerometer devoted to the measurement of the non-gravitational perturbations acting on the spacecraft, whose knowledge is important in order to

	fully exploit the quality of the tracking data for the MORE investigation (Iafolla et al., 2010). Consists of a low-noise, triaxial, dual-sensor, digital fluxgate instrument. The primary science objectives are to determine the spatial and temporal structure of the magnetic field in the Hermean system. The cooperation with the MMO/MGF will allow to distinguish between temporal and spatial magnetic field variations in the magnetically closely coupled Hermean system (Glassmeier et al., 2010).
MPO MAGnetometer (MPO/MAG)	
MERcury Radiometer and Thermal Infrared Spectrometer (MERTIS)	Is aimed to characterize the surface composition, identify rock-forming minerals, map the surface mineralogy and study the temperature variations and thermal inertia (Hiesinger & Helbert, 2010).
Mercury Gamma and Neutron Spectrometer (MGNS)	Its main goals are to measure the elemental composition of the shallow subsurface and test the presence of water ice deposits on the permanently shadowed polar craters of the planet (Mitrofanov et al., 2021).
Mercury Imaging X-ray Spectrometer (MIXS)	It will measure fluorescent X-ray emission from the surface of Mercury induced by incident solar X-rays and solar wind electrons and protons. These X-rays will reveal the elemental composition of the surface of Mercury and aid the determination of the planet's evolution. MIXS measurements of surface elemental composition will help determine rock types, the evolution of the surface and ultimately a probable formation process for the planet (Fraser et al., 2010).
Mercury Orbiter Radio science Experiment (MORE)	It will characterize Mercury's gravity field, tides and rotation and conduct a test of general relativity (Iess et al., 2021).
Probing of Hermean Exosphere By Ultraviolet Spectroscopy (PHEBUS)	A double spectrometer for the extreme ultraviolet range and the far ultraviolet range devoted to the characterization of Mercury's exosphere composition and dynamics, and surface-exosphere connections (Chassefière et al., 2010).
Search for Exospheric Refilling and Emitted Natural Abundances (SERENA)	It will investigate Mercury's complex particle environment that is composed of thermal and directional neutral atoms (exosphere) caused by surface release and charge-exchange processes, and of ionized particles caused by photo-ionization of neutrals as well

	by charge exchange and surface release processes (Orsini et al., 2021).
Spectrometer and Imagers for MPO BepiColombo-Integrated Observatory SYStem (SIMBIO-SYS)	The integrated suite of instruments, which includes a stereo imaging system (STC), a high-resolution imager (HRIC) and a visible-near-infrared imaging spectrometer (VIHI), will scan the Hermean surface with the three channels and map the physical, morphological, tectonic and compositional properties of the entire planet (Flamini et al., 2010).
Solar Intensity X-ray and particle Spectrometer (SIXS)	It will investigate the direct solar X-rays, and energetic protons and electrons which pass the spacecraft on their way to the surface of Mercury (Huovelin et al., 2010).

Table 5.2. Scientific instruments onboard the MPO.

MPO operates in a near-circular polar orbit (480×1500 km) with an orbital period of 2.3 hours. Table 5.3 shows the orbital parameters of the reference trajectory at the beginning of the scientific phase. As can be seen in Figure 5.5, the pericenter progressively moves toward the south pole while decreasing in altitude due to perturbations from gravitational harmonics (Genova et al., 2013).

MPO reference trajectory	
Semi-major axis	3430 km
Pericenter radius	2905 km
Apocenter radius	3955 km
Eccentricity	0.153
Inclination	90 deg
Right ascension of the ascending node	111 deg
Argument of pericenter	13 deg

Table 5.3. Orbital parameters of the MPO reference trajectory.

The spacecraft maintains nadir pointing to ensure global coverage and maximise science return. This configuration exposes nearly all spacecraft surfaces to solar illumination, leaving only one face available for the radiator. To keep it oriented away from the Sun, the spacecraft performs 180° flip-over maneuvers at each perihelion and aphelion. The design accounts for extreme thermal loads, including direct solar flux (up to 14 kW/m²), planetary albedo (up to 6 kW/m²), and infrared emission (~5.4 kW/m² at MPO altitude).

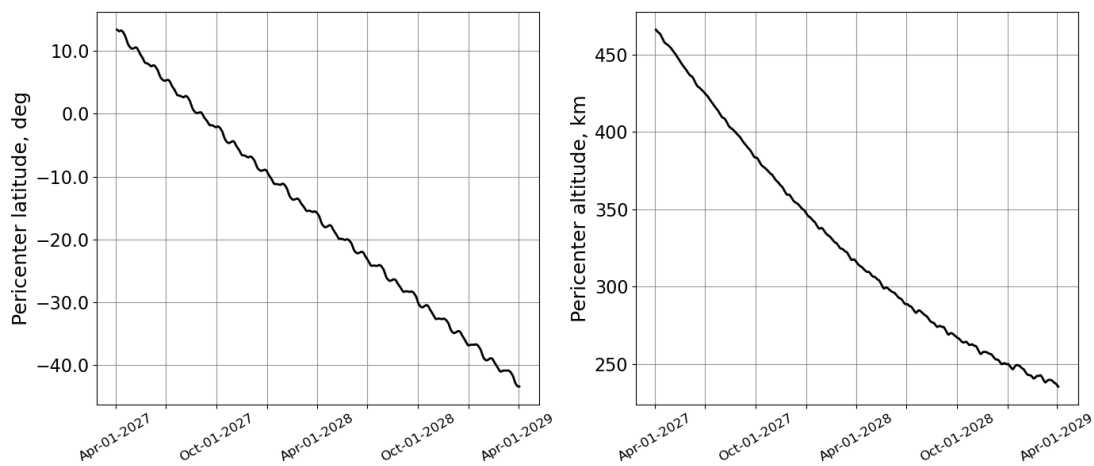


Figure 5.5. Evolution of MPO's pericenter latitude (left) and altitude (right) throughout the extended orbital phase of the mission.

To cope with the extreme environment, the spacecraft is covered in multi-layer insulation (MLI), while thermal control system regulates onboard equipment temperatures and directs excess heat to the radiator. Attitude is determined using three star trackers, Sun sensors, and a high-precision gyroscope package, providing accuracy of a few arcseconds, which is crucial for many instrument observations. Control is provided by four reaction wheels in tetrahedral configuration, supported by two redundant sets of 10 N thrusters for wheel off-loading maneuvers (WOLs). For communications with Earth, the spacecraft is equipped with a two-axis steerable HGA, a steerable MGA, and two LGAs. The primary ground station is ESA's 35-m Deep Space Antenna (DSA-3) in Malargue, Argentina, with additional support from DSA-2 in Cebreros, Spain, as needed to handle the mission's data volume.

5.2 Mercury Orbiter Radio science

Experiment

MORE is the radio science experiment (RS) onboard the MPO. For an exhaustive description of the experiment, refer to less et al. (2021). By leveraging its novel tracking technology for deep-space navigation, the MORE investigation is expected to drive new discoveries in space geodesy and to estimate parameters of relativistic gravity with unprecedented accuracy. As the non-gravitational accelerations (NGAs) on MPO are quite large and difficult to model in the harsh Hermean environment, a fundamental support for the MORE investigation will be given by ISA (Iafolla et al., 2010). The instrument, devoted to the measurement of non-gravitational perturbations, consists of three linear sensing elements, arranged along three orthogonal directions. The ISA readouts will be sent to ground in telemetry stream and used to replace NGAs in the dynamical model of MPO. Here the main features of the experiment are addressed, followed by an overview of the scientific investigations in fundamental physics and space geodesy.

5.2.1 RF telecommunication subsystem

The required accuracy of MORE relies on a suitable radio link configuration, shown in Figure 5.6. All measurements are carried out in a coherent two-way mode, in which the frequency reference is generated at the ground station and all onboard transponders are commanded in a coherent mode. The key aspect is the use of a multi-frequency link in X/X (7.2 GHz uplink / 8.4 GHz downlink), X/Ka (7.2 / 32.2 GHz) and Ka/Ka (34.4 / 32.1 GHz) (less & Boscagli, 2001; Simone et al., 2008). This scheme, successfully used by the NASA's Cassini spacecraft, allows to completely remove the noise due to solar plasma at almost all solar elongation angles (Bertotti et al., 1993). Indeed, charged particles generate phase scintillation, a phenomenon which adds a significant amount of noise in the observations.

This is especially true for tests of general relativity during the superior solar conjunctions (SSCs), in which the radio signal passes near the solar corona.

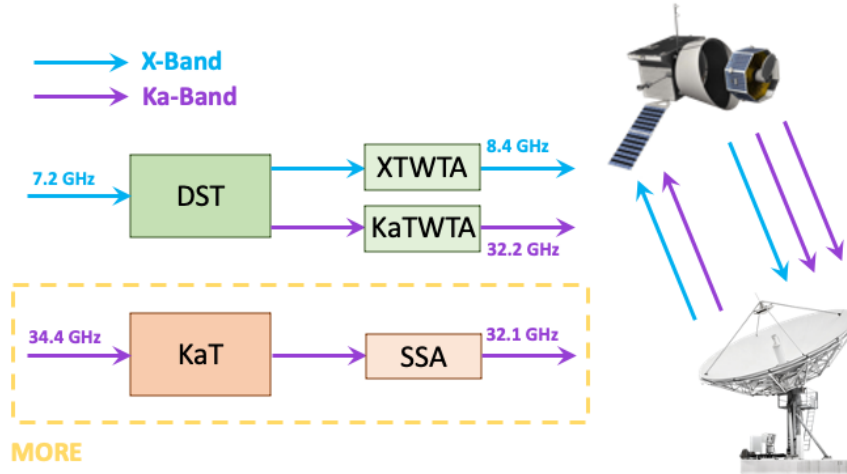


Figure 5.6. Multi-frequency link configuration of MORE. All measurements are carried out in a coherent two-way mode. The use of a multi-frequency link (X/X, X/Ka, Ka/Ka) allows to completely remove the noise due to solar plasma at almost all solar elongation angles.

The core of MORE is an advanced fully digital Ka-band transponder (KaT), designed to receive a Ka-band radio signal from Earth and retransmit it coherently in phase (Figure 5.7). It has been funded by the Italian Space Agency and built by Thales Alenia Space Italia, according to the specifications of the science team.



Figure 5.7. KaT flight model, from less et al., (2021).

The Ka/Ka link provided by the KaT will be the primary link for the RS experiment. The KaT uses an internal solid state amplifier (SSA), providing an output power of 1.9 W, and

operates in synergy with the Deep Space Transponder (DST). The DST, also onboard the MPO, is used for Telemetry Tracking and Commands systems (TTC), and provides two-way coherent radio links at X/X and X/Ka bands for plasma noise cancellation. These carriers are routed to ground after being amplified by two Travelling Wave Tube Amplifier (TWTA) with an output power of 35 W at X and Ka band. Being critical units, both the DST and the TWTA are duplicated for redundancy. All carriers may be phase modulated by the standard PN (pseudo noise) ranging to a chip rate of 3 million chips per second (Mcps). The KaT enables also range measurements in Ka-band with a 24 Mcps PN modulation, which represents a custom implementation outside the commonly adopted specifications (CCSDS, 2014). It is worth remarking that BepiColombo is the first ESA mission to use PN ranging for spacecraft navigation. For TTC, the MPO is endowed with 2 LGAs, a 2-axis steerable MGA and a 1 m 2-axis steerable HGA, capable of withstanding the extreme temperature variations of the Hermean environment.

The primary ground facility for TTC is the 35-m DSA-3 station of ESA's ESTRACK network, located in Malargüe, Argentina (Figure 5.8). At the time of writing, this station is the only ESTRACK station capable of Ka-band uplink. This facility is also equipped with a water vapor radiometer for wet tropospheric delay calibrations. Additional support will be provided by DSA-2 ground station in Cebreros (Spain), which will provide X-band links in some periods due to data volume constraints.



Figure 5.8. Deep Space Antenna 3 in Malargüe. Credit: ESA.

5.2.2 Fundamental physics tests

The aim of the fundamental physics experiment conducted by MORE is to test general relativity (GR) and alternative theories of gravity by accurately tracking the orbit of the spacecraft via range and Doppler measurements. These data will be used to estimate parameters that characterize deviations from GR within the Parameterized Post-Newtonian (PPN) formalism, a framework commonly adopted to evaluate relativistic theories under weak-field and slow-motion conditions (Will, 2018).

Parameter	Current limit	Method
$\gamma - 1$	$2.3 \cdot 10^{-5}$	Cassini SCE (Bertotti et al., 2003)
$\beta - 1$	$7.0 \cdot 10^{-5}$	INPOP global planetary fit (Fienga et al., 2015)
	$[1.8, 3.9] \cdot 10^{-5}$	MESSENGER range data (Genova et al., 2018; Park et al., 2017)
η	$3.0 \cdot 10^{-4}$	Lunar Laser Ranging (Müller et al., 2014)
	$7.2 \cdot 10^{-5}$	MESSENGER range data (Genova et al., 2018)
α_1	$6.0 \cdot 10^{-6}$	Solar System precession (Iorio, 2014)
α_2	$1.9 \cdot 10^{-9}$	Milliseconds pulsars (Shao et al., 2013)
$\mu_\odot$	$0.14 \text{ km}^3/\text{s}^2$	INPOP global planetary fit (Fienga et al., 2015)
ζ	$2.9 \cdot 10^{-14} \text{ yr}^{-1}$	INPOP global planetary fit (Fienga et al., 2015)
	$1.47 \cdot 10^{-14} \text{ yr}^{-1}$	MESSENGER range data (Genova et al., 2018)
$J_{2\odot}$	$6.0 \cdot 10^{-9}$	Helioseismology (Pijpers, 1998)
	$1.2 \cdot 10^{-8}$	INPOP global planetary fit (Fienga et al., 2015)
	$[2.2, 9.0] \cdot 10^{-9}$	MESSENGER range data (Genova et al., 2018; Park et al., 2017)
λ_g	$> 1.6 \cdot 10^{13} \text{ km}$	Gravitational waves (Bernus et al., 2019)

Table 5.4. State of the art accuracies for PPN parameters.

MORE will focus on refining the estimates of PPN parameters and seek for potential deviations from the values predicted by GR. The PPN parameters are γ and β , related to the curvature of spacetime and non-linearity of gravity, respectively, as well as α_1 , α_2 , and η , which quantify gravitationally preferred-frame effects and possible violations of

the strong equivalence principle. In addition to these dimensionless parameters, the experiment will constrain other key quantities such as the solar gravitational parameter $\mu_{\odot}$, its temporal variation $\zeta = \dot{\mu}_{\odot}/\mu_{\odot}$, the solar quadrupole moment $J_{2\odot}$, and the Compton wavelength λ_g . The current estimates are shown in Table 5.4.

Detailed orbital simulations of the fundamental physics tests using realistic error models can be found in Iess et al. (2021). In general, all PN parameters can be determined by MORE with considerable improvement over present accuracies (De Marchi & Cascioli, 2020). Whether confirming GR with unprecedented precision or detecting deviations, the experiment is poised to play a decisive role in shaping our understanding of gravitation.

5.2.3 Geodesy and geophysics investigation

The geodesy and geophysics experiment of MORE focuses on the same objectives as the MESSENGER radio science investigation (Section 4.2). However, MORE will conduct its measurements with an advanced tracking system and from a low-eccentricity orbit (Section 5.2.1). This configuration provides more uniform global coverage and enhanced sensitivity in the southern hemisphere, complementing MESSENGER's high-eccentricity orbit with pericenter in the north (Section 4.1). The enhanced performance in the southern hemisphere results from the drift of the spacecraft pericenter, which progressively moves toward the south pole while decreasing in altitude (see Figure 5.5).

In the MESSENGER-derived gravity fields, resolution exceeds degree 100 over regions of the northern hemisphere, where the spacecraft pericenter reached altitudes lower than 25 km. In contrast, the equatorial zone and the southern hemisphere were limited to lower resolutions, around degree 15, which is equivalent to spatial scales of roughly 500 km. As shown in previous analyses (De Filippis et al., 2024; Imperi et al., 2018), MORE can solve the gravity field up to about degree 40 in the southern hemisphere, corresponding to a spatial resolution near 180 km, without assuming any regularization. This refined gravity field of Mercury will allow revealing the properties of geological units in the southern hemisphere.

The precise determination of the spacecraft orbit using MORE radio tracking data will also improve the determination of k_2 and provide an initial estimate of its phase lag (see Section 3.2). These measurements will help constrain the rheological properties of the mantle and outer core. The measurement of Love number h_2 , which measures the physical displacement of the planetary surface, will be obtained thanks to BELA laser altimetric data processed with the accurate orbit products coming from MORE (Thor et al., 2020). A combined knowledge of k_2 and h_2 will result in outstanding refinements of the interior modelling (Steinbrügge et al., 2018) and composition (Van Hoolst & Jacobs, 2003). The determination of the lag angle and its uncertainty also holds significance as it provides an upper bound to the value of the quality factor Q of Mercury's mantle (Iess et al., 2021). By jointly analysing moments of inertia and Love numbers, it is possible to obtain comprehensive geophysical constraints that facilitate a better understanding of Mercury's composition, size, and interior structure. These constraints are essential for refining interior modelling solutions and improving our knowledge of the planet (Padovan et al., 2014).

Finally, a better coverage of the equatorial regions will improve the characterization of the spin axis orientation, obliquity and librations in longitude. The amplitude of the 88-day libration may change up to 1 arcsec (or 10 m on the equator) if the inner core radius is 1500 km (Van Hoolst et al., 2012). This is beyond the level of the current accuracy, but potentially detectable by MORE. The improved characterization of Mercury's rotation, combined with an improved estimate of the degree 2 coefficients, will result in an enhanced estimation of the dimensionless moment of inertia C/MR^2 and fractional polar moment of inertia of the solid crust and mantle C_m/C (Section 3.4.1). The expected level of accuracy on these parameters will result in sophisticated modelling of Mercury's deep interior including size and status of its core.

5.3 Simulation with dynamical mismodelling

During real mission operations, data coming from ISA will be included in the orbit determination code to avoid significant mismodelling due to the strong and difficult to model NGAs. However, thermal drifts and other systematic effects affecting the accelerometer's measurements could introduce errors in reconstructing the spacecraft trajectory. To compensate for these issues, a dedicated calibration strategy is required. Other works have investigated the impact of accelerometer readout errors and calibration (De Filippis et al., 2024; Imperi et al., 2018), showing that adding the accelerometer bias and bias rate in the fit, degrade the uncertainties in rotational parameters, the Love number k_2 , and low-degree gravity coefficients by a factor 2–3. However, considering the accuracy declared after ground tests (Santoli et al., 2020), the use of accelerometer data would be very important to reduce errors on the orbital solution coming from dynamical mismodelling. Furthermore, the tool would become fundamental for the detection of unexpected accelerations such as outgassing that could degrade the orbital reconstruction (De Filippis et al., 2025; Magnafico et al., 2025).

Here we address a contingency analysis which relies on a comprehensive prediction of NGAs achievable through accurate numerical models, without ISA accelerometer measurements. We focus on the geodesy investigation, which is decoupled from the relativity test. Unlike previous analyses (Imperi et al., 2018), this study incorporates also various sources of mismodelling to simulate a realistic scenario and dynamical state of the probe. These include errors in the spacecraft reference state and optical properties, unbalanced ΔV s due to wheel off-loading maneuvers, solar irradiance fluctuations, residual effects due to unmodelled NGAs and not perfectly known gravity field.

The methodology behind this work consists of an OD process, implemented through the Jet Propulsion Laboratory's MONTE OD code (Evans et al., 2018). The procedure is outlined in Section 2.5. First of all, synthetic radiometric observables are generated using

a dynamical and observation model that we assume as the truth. Then, during the least-squares estimation, the simulated radio observables are compared with predictions computed using a best-guess reference model. We introduce discrepancies between the dynamical model used for the generation of synthetic radio observables and the one used in the estimation process. This approach not only allows to simulate a realistic OD process (where mismodelling is unavoidable), but also enables assessment of whether the recovered parameters are biased. To address this, we compute the difference between the estimated and true values, and compare it to the associated 3- σ uncertainties.

5.3.1 Observable quantities

Since the focus of this analysis is the gravity experiment, which is uncoupled from the relativity test, we opt for a more conservative and robust approach by initially excluding range data. Although we found that range observables could, in principle, enhance the recovery of the geodesy parameters by $\sim 30\%$, they also introduce a coupling with planetary ephemeris errors and may introduce instabilities in the solution. We simulate data acquired at elevation angles above 15° to prevent degradation caused by tropospheric effects. Preliminary inflight performances of the Ka-band two-way Doppler were in good agreement with the predictions made in the design phase of the radio science experiment ($0.024 \text{ mm/s @ } 30 \text{ s}$; Cappuccio et al., 2020). Table 5.5 shows the accuracies used in this work and the related integration time.

Measurement type	Accuracy	Integration time
Two-way coherent Ka-band doppler	$24 \mu\text{s}$	30 s
Two-way coherent X-band doppler	$87 \mu\text{s}$	30 s

Table 5.5. Measurements accuracy and integration time.

The observables are filtered to account for occultations by Mercury and outages of the HGA, which occur when the antenna is blocked by other parts of the spacecraft.

Additionally, when the radio signal passes near the Sun’s surface, solar plasma noise significantly degrades the quality of the radiometric observables. Although data acquired at very small Sun-Probe-Earth (SPE) angles can be used in the OD process (Di Stefano et al., 2024), we exclude all observables obtained with $\text{SPE} < 5^\circ$ (corresponding to a signal path within ~ 7 solar radii), where the noise level may deviate significantly from the values assumed in this analysis. Finally, we discard data collected before and after each perihelion when Mercury's mean anomaly is $\pm 40^\circ$, as the KaT will be switched off due to power constraints. For the purpose of this analysis, we choose not to use X-band data alone during these periods. Table 5.6 summarises the percentage of Ka-band data lost due to each of these factors, treating them as independent. However, there are occurrences of two or more conditions simultaneously. For example, data lost due to HGA outages may already be excluded due to occultations. Therefore, the total percentage of discarded data is $\sim 48\%$, lower than the sum of individual contributions.

Filter type	%
HGA outages	25
Perihelia	21
Mercury occultation	9
Low SPE	3

Table 5.6. Percentage of data cut.

5.3.2 Spacecraft model

The spacecraft is represented through the commonly used plate model (see Section 3.5.1). The bus consists of six plates, while the solar array is modelled as a plate with a surface area of 8 m^2 . The HGA is approximated as a dish with a radius of 0.55 m and a depth of 0.47 m . Secondary surfaces, such as the other antennas and the two extendable booms, are not considered, as their contribution to the overall acceleration is negligible. The thermo-optical coefficients of the surfaces in the visible and infrared wavelengths

have been provided by the spacecraft prime contractor AIRBUS. The effective area exposed to solar and planetary radiation is determined by the orientation of the spacecraft bus and solar panel, which is inferred from preliminary attitude kernels supplied by ESA (ESA, 2025).

5.3.3 Dynamical model

The dynamical model accounts for the gravitational monopole accelerations due to Solar System planets, the spherical harmonics coefficients of the Sun up to degree and order 2 and the spherical harmonics coefficients of Mercury, provided by the MESSENGER HgM009 reference gravity field (Genova et al., 2023). The field is truncated at degree and order 100, although the sensitivity is well below those harmonics because of the orbital altitude of the MPO, which stays above 200 km at pericenter during the entire extended mission (see Figure 5.5).

Similarly to MESSENGER, BepiColombo will experience significant NGAs. Although the actual NGAs experienced during the orbital phase may differ from the values presented here due to potential deviations in the spacecraft real attitude, this discrepancy is not the primary focus of this study. Instead, the objective is to assess the impact of inconsistencies between the model used to simulate the tracking data and the one adopted during the parameter estimation phase, as detailed in the following section. The dynamical modelling of NGAs is consistent with Section 4.3.3.

Figure 5.9 shows the magnitude of each component of the modelled NGAs for the nominal mission, which spans a one-year orbital phase. The dominant non-gravitational force acting on the MPO is solar radiation pressure (SRP), followed by planetary (ALB and IR) and spacecraft (TRP) radiation. During solar eclipses, SRP and TRP drop sharply, creating visible discontinuities. Similarly, ALB and IR exhibit a sharp peak when the spacecraft passes over the sunlit region and drop rapidly afterwards in the night side. Due to the plot's scale and resolution, these drops appear as filled segments along the curve.

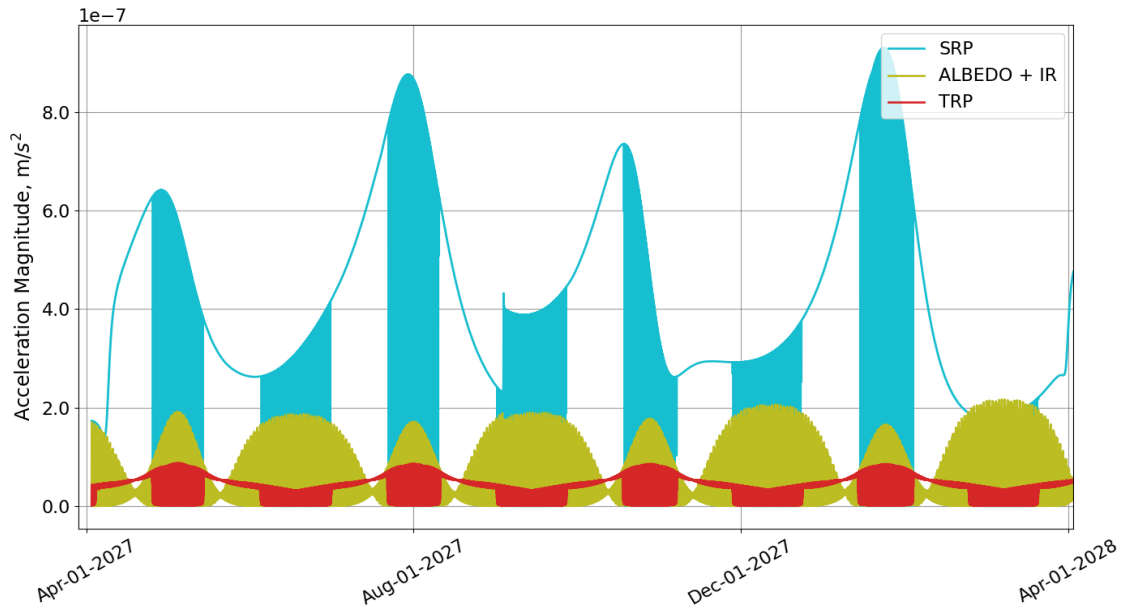


Figure 5.9. Non-gravitational accelerations magnitude during the nominal mission. The plot shows the contributions from solar radiation pressure (SRP), planetary radiation pressure (albedo + infrared), and thermal recoil pressure (TRP). The sharp acceleration drops caused by geometric conditions appear as filled segments due to the plot's scale.

The presence of a single solar panel and the asymmetric shape of the spacecraft generate large torques. Reaction wheels are used to counteract these effects and maintain the spacecraft attitude. While the original mission plan foresaw one WOL per day to dump excess angular momentum, this was later updated to two WOLs per day: the first one occurs at apoherm during the tracking pass and the other after the pass, with a time interval of ~ 12 hours between them. This operational configuration is adopted as the nominal scenario in our analysis. However, due to an unexpected performance issue with the reaction wheels, mission planners are also evaluating alternative configurations that involve different WOL strategies. These may affect both the frequency and timing of WOLs, potentially influencing the overall trajectory reconstruction accuracy. An alternative scenario with one WOL every three orbits at apoherm (3 or 4 WOLs per day) is discussed later in the results. The residual ΔV s due to thruster's misalignment are included in the dynamical model to account for their impact on the spacecraft motion.

5.3.4 Dynamical perturbations

The inherent uncertainty in precisely modelling NGAs on the spacecraft can generate significant errors in OD, introducing biases in scientific measurements and potentially leading to incorrect conclusions. To simulate a realistic operational context, we deliberately introduce mismatches between the force models employed for generating synthetic radio tracking data and those used within the OD process. As a result, at iteration zero, before any parameter corrections are applied, the integration of the spacecraft equations of motion yields a trajectory that differs from the simulated “true” path, with position discrepancies reaching up to 2–3 km after the out-of-pass maneuvers.

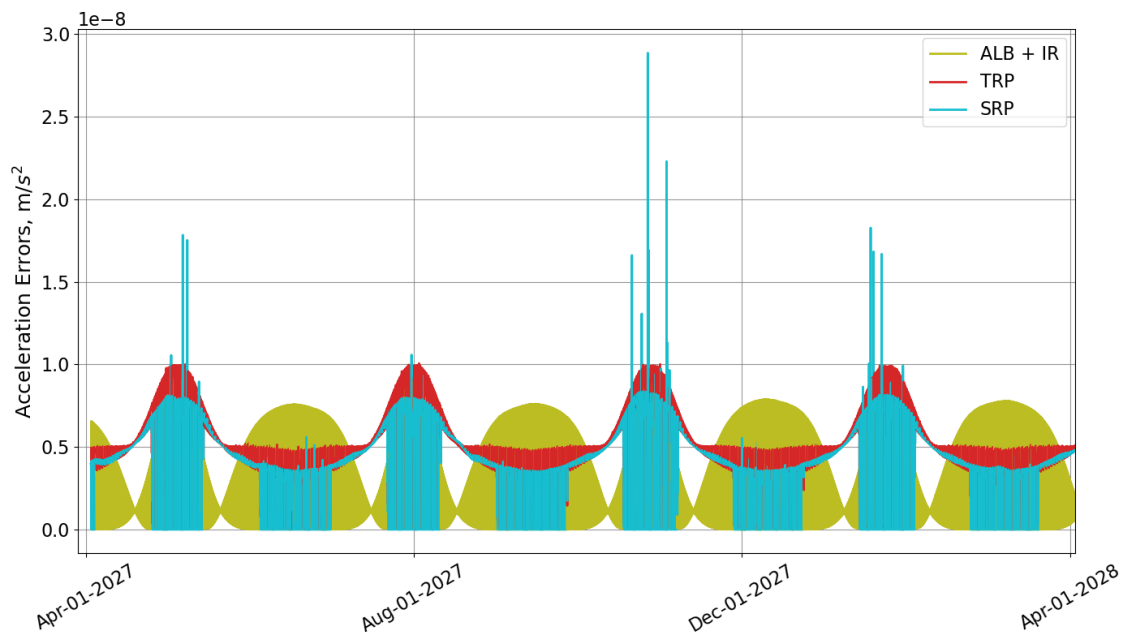


Figure 5.10. Magnitude of non-gravitational acceleration pre-fit errors during the nominal mission. The plot shows the contributions from solar radiation pressure (SRP), planetary radiation pressure (albedo + infrared), and thermal recoil pressure (TRP).

Figure 5.10 shows the pre-fit errors on the predicted NGAs during the nominal mission. It is important to note that there are some assumptions behind this approach. First, we are using the same software to simulate observables and to estimate the gravity field. This means that while we introduce mismatches between the force models, they rely on the same mathematical formulation of the accelerations. These models, in

addition, do not account for secondary effects such as self-shadowing that may be relevant at Mercury. Second, each type of perturbations is assumed to be compensated through a dedicated set of estimated parameters. In this way, we implicitly do not account for the possible presence of unmodelled accelerations or unknown error sources. Despite these limitations, the perturbation levels introduced are chosen to reflect realistic acceleration errors, based on current knowledge of the spacecraft physical and optical properties. This methodology allows us to evaluate the stability of the orbital solution and the reliability of the recovered geophysical parameters under controlled mismodelling conditions.

Reference state

The initial conditions of position and velocity to integrate the spacecraft in the estimation process deviate from the “true” initial state. These initial perturbations are drawn from a uniform random distribution. The perturbation ranges are set to [10 m, 150 m, 50 m] for the position components in the radial, transverse, and normal (RTN) frame. For velocity, the random deviations are below 1 cm/s for every component in the J2000 inertial frame. These thresholds are derived from the formal uncertainties obtained from a pure covariance analyses, accounting for large safety margins.

Wheel off-loading maneuvers

WOLs introduce non-negligible velocity increments ΔV s, due to thrusters’ misalignment, that affect the orbital motion. The estimated maximum magnitude of the uncompensated ΔV s is 17 mm/s, 0.2 mm/s and 42 mm/s in the RTN orbital reference frame (Imperi et al., 2018). These velocity shifts are expected to be known with a confidence level of 5% (Iafolla et al., 2011). For this reason, we model the residual errors by sampling random values from a uniform distribution with extremes equal to 5% of the nominal values.

Thermo-optical properties

Ground-measured values of reflectivity, emissivity, and absorptivity deviate from their actual in-flight values and continue to degrade over time due to exposure to the space environment (Ding et al., 2009). In this study, the spacecraft shape model used for the generation of synthetic observables assumes the expected end-of-life values for each surface. In the dynamical model of the estimation step we include errors equal to 10% of their predicted end-of-life values on both specular and diffuse coefficients of the exposed sides of the array and the bus. Variations in these coefficients may partially compensate for each other, reducing their combined effect. The corresponding residual acceleration mismodelling reaches up to $8 \cdot 10^{-9} \text{ m} \cdot \text{s}^{-2}$ for solar radiation and a few parts in $10^{-9} \text{ m} \cdot \text{s}^{-2}$ for albedo and thermal infrared acceleration.

Albedo & Infrared scale factors

Beyond thermo-optical properties, the computation of non-gravitational accelerations due to planetary radiation is subject to several additional error sources. These include simplifications in the surface temperature model, truncation effects in the spherical harmonics expansion of albedo and emissivity, and the discretization of the visible cap in concentric rings. We assume that, by the end of the orbital phase, temperature and albedo maps of Mercury will be available with sufficient accuracy to constrain the total mismodelling of albedo and IR accelerations to be below $10^{-8} \text{ m} \cdot \text{s}^{-2}$. To account for this effect, in the estimation process we increase both the albedo and IR accelerations magnitude by 5%. For some elements of the spacecraft shape, this increment partially compensates the mismodelling introduced through errors in the thermo-optical coefficients, resulting in a maximum residual mismatch of approximately $8 \cdot 10^{-9} \text{ m} \cdot \text{s}^{-2}$.

Thermal recoil pressure

The acceleration induced by thermal re-radiation from the spacecraft surfaces depends on their temperature distribution, which is influenced by variations in emissivity, absorptivity, and heat capacity. We expect this acceleration to be the dominant

mismodelling due to the difficulty in predicting the temperature of the spacecraft elements. The dynamical model used to simulate synthetic observables incorporates the TRP acceleration calculated with the predicted end-of-life values of the spacecraft thermo-optical properties. On the other hand, the acceleration in the estimation step is computed with the perturbed coefficients. These discrepancies propagate into residual acceleration errors of $\sim 10\%$ of the nominal thermal recoil pressure. The resulting perturbation levels are up to $10^{-8} \text{ m}\cdot\text{s}^{-2}$.

Solar irradiance fluctuations

The value of solar flux shows temporal random fluctuations ranging at the level of $[0.01, 0.1]\%$ of the central value over a few hours. These oscillations of the solar irradiance are due to the simultaneous effect of faculae and sunspots on the solar surface (Kopp, 2016). Space-based radiometers on board of satellites orbiting in the Earth's can measure these irradiance fluctuations with excellent accuracy (Kopp et al., 2004).

To generate the observed observables, we use a constant value of the solar flux, while in the estimation phase we include the irradiance fluctuations measurements of the SORCE mission in the dynamical model (Figure 5.11). The dataset provides values of the Total Solar Irradiance (TSI) at 1 AU from the Sun, which are time-shifted by approximately two solar cycles to align them with the BepiColombo mission timeline and resampled at 6-hour intervals. The resulting faint signal induced by SRP fluctuations is up to $10^{-10} \text{ m}\cdot\text{s}^{-2}$, well below the level induced by the other perturbations.

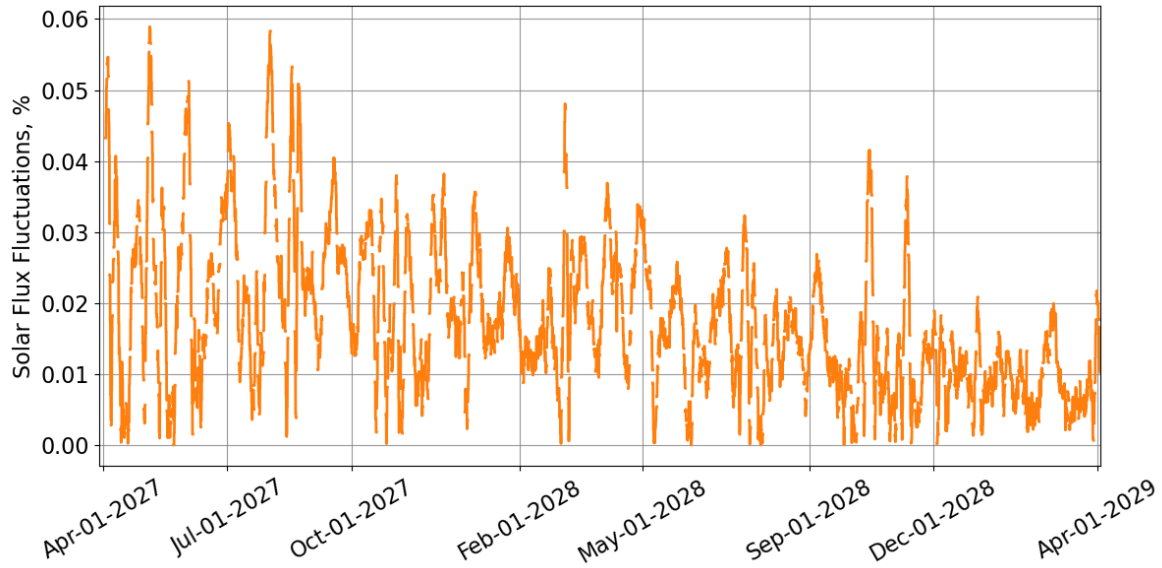


Figure 5.11. Fluctuations of solar flux from SORCE mission (Kopp et al., 2004). The dataset provides values of the Total Solar Irradiance (TSI) at 1 AU from the Sun, which are time-shifted by approximately two solar cycles to align them with the BepiColombo mission timeline and resampled at 6-hour intervals.

Gravity

The Mercury gravity field used for trajectory integration is derived from the latest MESSENGER gravity solution (Genova et al., 2023). Due to the highly eccentric orbit of the spacecraft, MESSENGER gravity measurements provide high-resolution coverage of the northern hemisphere but significantly lower accuracy in southern latitudes.

For synthetic data generation, we adopt the full HgM009 gravity field truncated at degree and order 100. The uncertainties in Mercury's gravity field spherical harmonic coefficients are incorporated into the estimation process by adopting a perturbed realization of the HgM009 gravity solution. This is obtained by varying the coefficients of the reference field with deviations compliant with its covariance matrix, taking into account also the correlations. This approach guarantees that the introduced errors remain statistically consistent with the formal uncertainties and correlations in the HgM009 gravity solution. The amplitude spectrum curves (CI) of the reference and perturbed gravity field are shown in Figure 5.12.

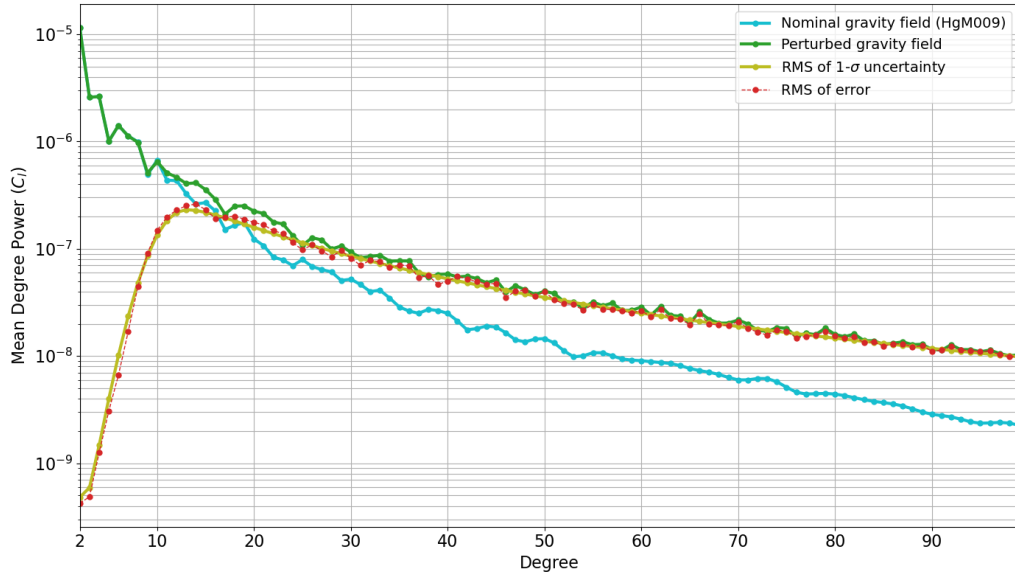


Figure 5.12. Amplitude associated with each degree for the reference and perturbed gravity field, with the corresponding error and uncertainty spectrum.

5.3.5 Filter setup

Following the up-to-date mission scenario, we simulate the extended two-year orbital phase, with the scientific operations beginning in April 2027. The subdivision of the timespan in arcs is done by attributing to each arc a duration of 24 hours. A daily tracking window from DSA-3 is included in each arc, except for June 2027 when X-band tracking may be provided by DSA-2. The state vector is determined at the central time of each arc, integrating forward and backward (Imperi et al., 2018). Figure 5.13 shows a typical arc configuration.

The set of local parameters, solved independently for each arc, with their a priori uncertainties, are provided in Table 5.7. This set includes the spacecraft state at the central epoch of the arc, its thermo-optical coefficients and scale factors to absorb non-gravitational accelerations mismodelling and the uncompensated ΔV s related to the two daily WOLs.

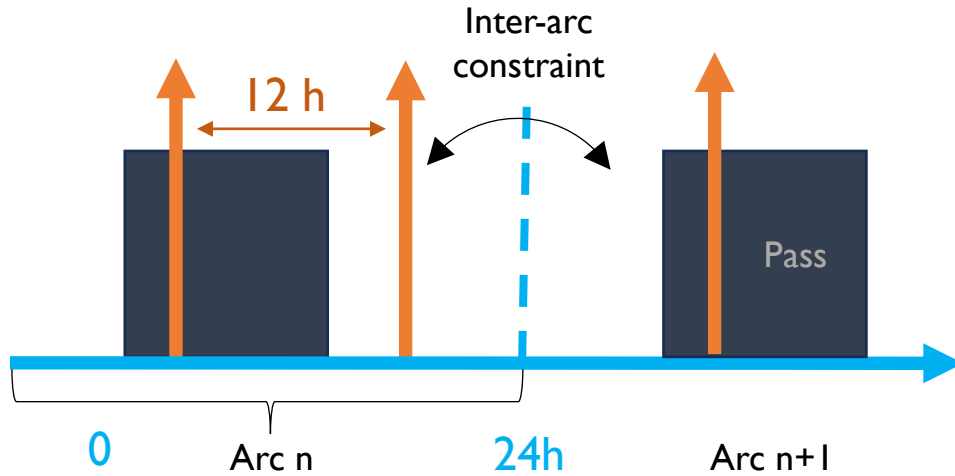


Figure 5.13. Arcs configuration. Each arc includes a daily tracking window (blue square) and two wheel off-loading maneuvers (orange arrows). Inter-arc constraints are applied to estimate the out-of-pass maneuvers.

In a pure multi-arc strategy, trajectory discontinuities naturally arise at the transition points between arcs. These discontinuities are greater due to the presence of WOLs that occur outside the tracking passes. However, having a continuous accurately reconstructed trajectory, without gaps induced by unmodelled maneuvers, is essential to ensure reliable scientific results from the other instruments, especially the BELA laser altimeter (Thor et al., 2020). For this reason, we include inter-arc constraints to allow estimating the out-of-pass maneuvers, which would otherwise remain unobservable, making the trajectory largely uncertain (see Section 2.3). If the constraint values are chosen properly, these continuity conditions generally lead to reduced formal uncertainties on position and velocity compared to an unconstrained, or pure, multi-arc solution. However, the tightness of the constraints must be carefully selected, as it may introduce bias and compromise the robustness of the solution. In this analysis, we impose position continuity constraints at the 1-meter level in the J2000 reference frame. By multiplying this spatial tolerance by the orbital angular frequency ($7.3 \cdot 10^{-4}$ Hz), we derive the corresponding velocity constraint, which amounts to 0.73 mm/s. The same values were adopted also in Imperi et al. (2018) and reflect a balanced trade-off between constraint effectiveness and the solution stability. An alternative case with no inter-arc constraints and one WOL every three orbits is discussed later in the results section.

The estimated global parameters are shown in Table 5.7. Given that the geodesy investigation is conceptually independent from the relativity experiment, we assume the framework of general relativity to be exact and exclude any relativistic parameters from the estimation process. We adopt the DE432 planetary ephemerides from JPL, an update of the DE430 solution (Folkner et al., 2014), as they are consistent with the ephemeris kernel delivered by ESA for the BepiColombo mission. No errors are applied to the state vectors of Mercury or the Earth at the reference epoch. This assumption is supported by the fact that our analysis is based exclusively on Doppler measurements, which exhibit minimal sensitivity to inaccuracies in planetary ephemerides. Refinement of planetary ephemerides can be performed separately using also range data, within the framework of the relativistic analysis.

Local parameter	A priori uncertainty
S/C Position (J2000)	300 m
S/C Velocity (J2000)	3 cm/s
Uncompensated ΔV s (J2000)	[4, 2, 2] mm/s
Optical coefficients (visible and IR)	20% ref value
IR scale factor	0.2
TRP scale factors	0.2
Constant along-track acceleration	10^{-12} m/s ²
Global parameter	A priori uncertainty
Mercury's gravitational constant	Unconstrained
Gravity field coefficients 100x100	$3 \cdot 10^{-4}/l^2$ for $l > 30$
Mercury's k_2	Unconstrained
Spin axis α, δ	Unconstrained
Mercury angular velocity ω	Unconstrained
Libration amplitude ϕ_0	Unconstrained

Table 5.7. Set of estimated parameters with associated a priori uncertainty.

In contrast to previous studies that solved for the gravity field up to degree 50 without regularization, we extend the estimation to degree 70. To mitigate numerical instability, we apply a Kaula-type constraint beyond degree 30 (Kaula, 1966). Specifically, we adopt a Kaula factor $A_k = 30$, which is three times larger than the value used in the reference HgM009 field (Genova et al., 2023). This more conservative regularization scheme is justified by the fact that gravity field coefficients were perturbed up to degree 100 in our simulations and including higher-degree terms in the estimation mitigates the risk of introducing biases in the lower-degree coefficients due to correlations. The choice of degree 30 as the transition point for regularization reflects the global resolution attainable by the experiment (see Figure 5.20). All the other estimated global parameters are unconstrained.

5.3.6 Results

This paragraph presents the expected performance of the MORE gravity and rotation experiment and spacecraft orbit reconstruction, accounting for the dynamical modelling errors listed in Section 5.3.4. The impact of an increment of WOL frequency is also addressed.

Gravity

A comparison of the degree RMS spectrum curves (C_l) of the estimated errors and uncertainty for the retrieved gravity is shown in Figure 5.14.

The majority of the errors remain within the $3\text{-}\sigma$ uncertainty envelope, demonstrating that the estimation process effectively compensates for dynamical mismodelling without introducing significant biases. The 99.72 % of the errors in the coefficients are within $3\text{-}\sigma$ and only 0.28 are between 3- and $5\text{-}\sigma$, likely due to residual mismodelling in the force model. Despite these minor deviations, the recovered gravity field is statistically consistent with the reference solution.

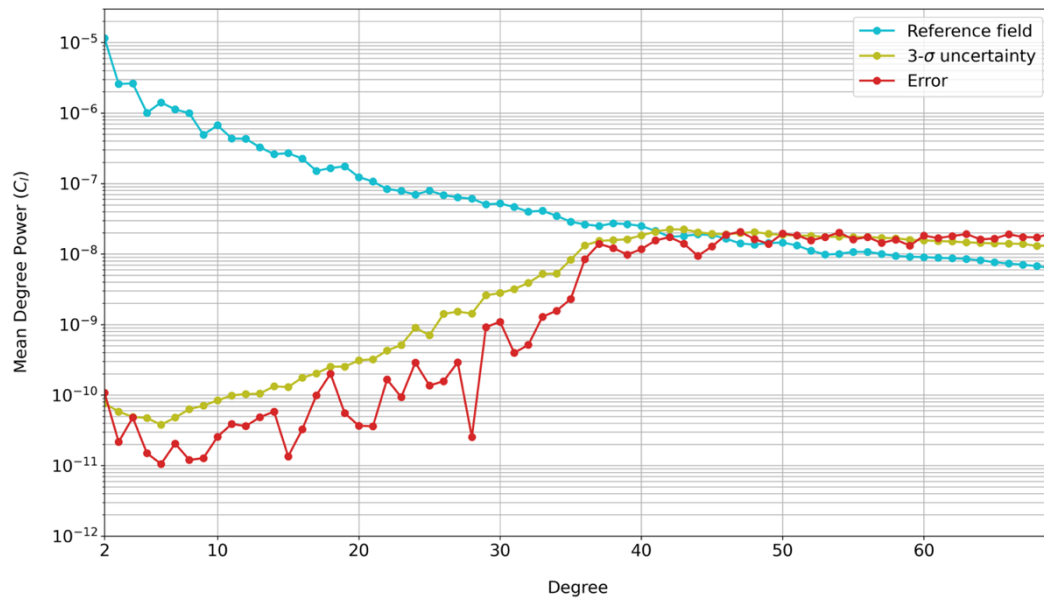


Figure 5.14. Amplitude associated with each degree for the estimated gravity field, with the corresponding error and uncertainty spectrum.

As shown in Figure 5.15, the uncertainties in gravity anomalies in the majority of the southern regions are at the level of ~ 20 mGal, indicating a promising opportunity for improved measurements of internal mass distributions and comparisons with surface topography.

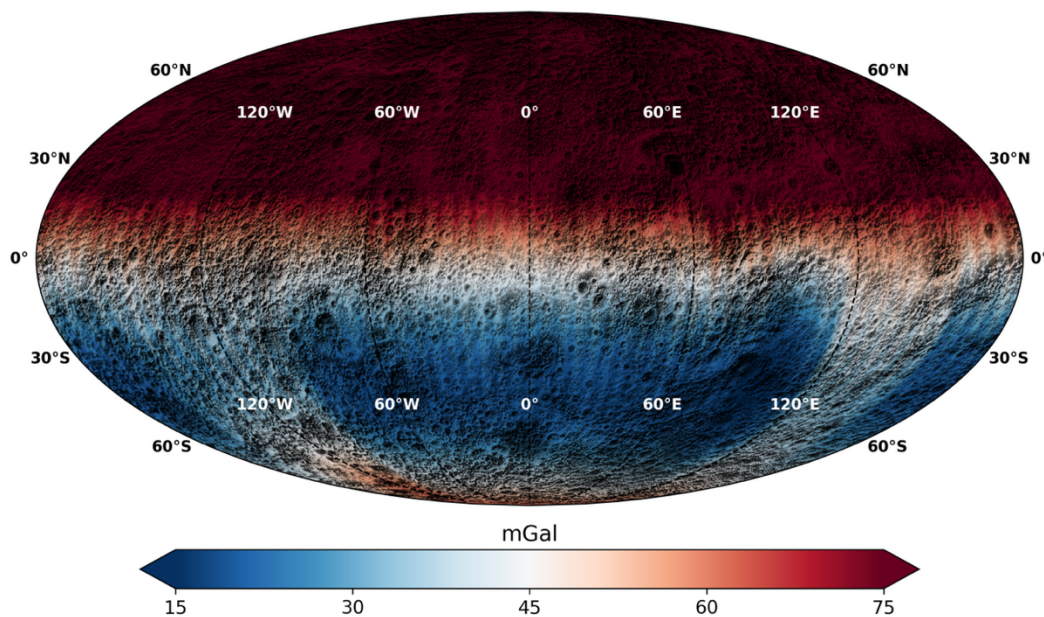


Figure 5.15. Mercury gravity anomalies $1\text{-}\sigma$ uncertainty when using a loose Kaula constraint ($A_k = 30$) applied for degrees greater than 30.

These uncertainties are larger with respect to the ones shown in previous works (less et al., 2021), primarily due to the significant data loss caused by HGA outages and perihelia, the choice to evaluate a first-case scenario using Doppler-only tracking data, and the estimation of higher-degree gravity coefficients under a relatively loose Kaula regularization. In this analysis, a Kaula constraint with $A_k = 30$ is applied for degrees greater than 30, three times the value used in the reference field (Genova et al., 2023). This regularization helps to absorb part of the mismodelling of the higher degrees while stabilizing the solution, and ensures conservative uncertainty estimates at the lower degrees. While it does not improve the intrinsic spatial resolution of the solution, we found that this approach enables a more robust recovery of the gravity field in the presence of dynamical mismodelling. The errors in the gravity anomalies, calculated with respect to the reference field, are inside the $3\text{-}\sigma$ uncertainty, as shown in Figure 5.16.

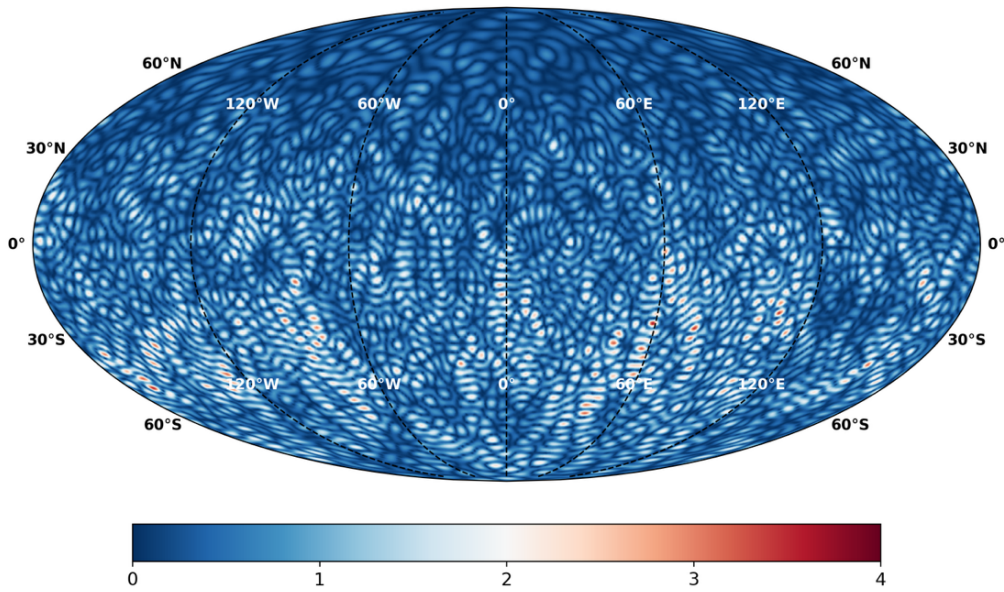


Figure 5.16. Mercury gravity anomalies error over $1\text{-}\sigma$ uncertainty.

Table 5.8 presents the accuracy achieved by MORE in estimating the static degree-2 gravity coefficients and the real and imaginary components of the tidal Love number k_2 . All the reported uncertainties are $3\text{-}\sigma$. The k_2 , which is crucial for understanding Mercury's deformation under the Sun's gravitational influence, can be determined with an uncertainty of $1.8 \cdot 10^{-3}$. These refined estimate will strongly constrain Mercury's

internal structure, the rheological properties of Mercury's mantle and the range of interior models compatible with rotational measurements (Padovan et al., 2014). These uncertainties can be further improved by $\sim 30\%$ including range data in the OD.

Rotation

Table 5.8 summarises the expected $3\text{-}\sigma$ uncertainty in determining Mercury's rotational parameters, compared with the MESSENGER gravity solution from Section 4.3. The right ascension and declination of the rotational pole can be estimated with uncertainties of 0.48 and 0.36 arcseconds, respectively. Consequently, Mercury's obliquity can be determined at the level of 0.3 arcsec. The libration amplitude in longitude is expected to be resolved within 0.63 arcseconds, corresponding to a few meters at the equator. This enhanced accuracy could constrain the size of the solid inner core, if present, with respect to Mercury's liquid outer core (Van Hoolst et al., 2012). The resulting uncertainty on C/MR^2 is $8.8 \cdot 10^{-4}$, while C_m/C can be determined at the level of $8.5 \cdot 10^{-3}$ by following Peale's procedure (Section 3.4.1).

Parameter	MESSENGER	MORE
$\bar{J}_2$	$1.4 \cdot 10^{-9}$	$1.8 \cdot 10^{-10}$
$\bar{C}_{22}$	$9.4 \cdot 10^{-10}$	$2.4 \cdot 10^{-10}$
k_{2r}	$3.3 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$
k_{2i}	/	$1.7 \cdot 10^{-3}$
α [deg]	$1.1 \cdot 10^{-3}$	$1.3 \cdot 10^{-4}$
δ [deg]	$5.5 \cdot 10^{-4}$	$9.9 \cdot 10^{-5}$
ϕ [arcsec]	4.1	$6.3 \cdot 10^{-1}$
ω [deg/day]	$1.8 \cdot 10^{-6}$	$6.6 \cdot 10^{-7}$
ε [arcmin]	$3.4 \cdot 10^{-2}$	$5.3 \cdot 10^{-3}$
C/MR^2	$5.6 \cdot 10^{-3}$	$8.8 \cdot 10^{-4}$
C_m/C	$1.9 \cdot 10^{-2}$	$8.5 \cdot 10^{-3}$

Table 5.8. Comparison of $3\text{-}\sigma$ uncertainties between MORE and the MESSENGER solution from Section 4.3.

Positioning

The positioning of the MPO with respect to Mercury is a crucial task to ensure reliable scientific results from the other instruments. The advanced tracking system onboard the MPO will allow a very accurate trajectory reconstruction. Figure 5.17 shows the propagation of the MPO 3- σ position uncertainty during the first 100 days of the orbital phase. For each day the uncertainty is reported at the beginning, center and end of the arc. Yellow and red bars point out the superior solar conjunctions and perihelia periods, respectively. The radial positioning of the spacecraft is the best determined due to the gravity acceleration which constrains the MPO position along the spacecraft–planet direction, enabling accuracies at the level of few tens of centimeters inside the tracking pass. The position in the along-track and transverse directions are more poorly determined, still with 3- σ accuracies at the level of ten meters. It is worth remarking that these uncertainties can be further reduced by incorporating range measurements into the analysis. The error, calculated as the difference between the estimated trajectory and the “true” one, is always inside the 3- σ uncertainty, indicating a successful trajectory reconstruction process.

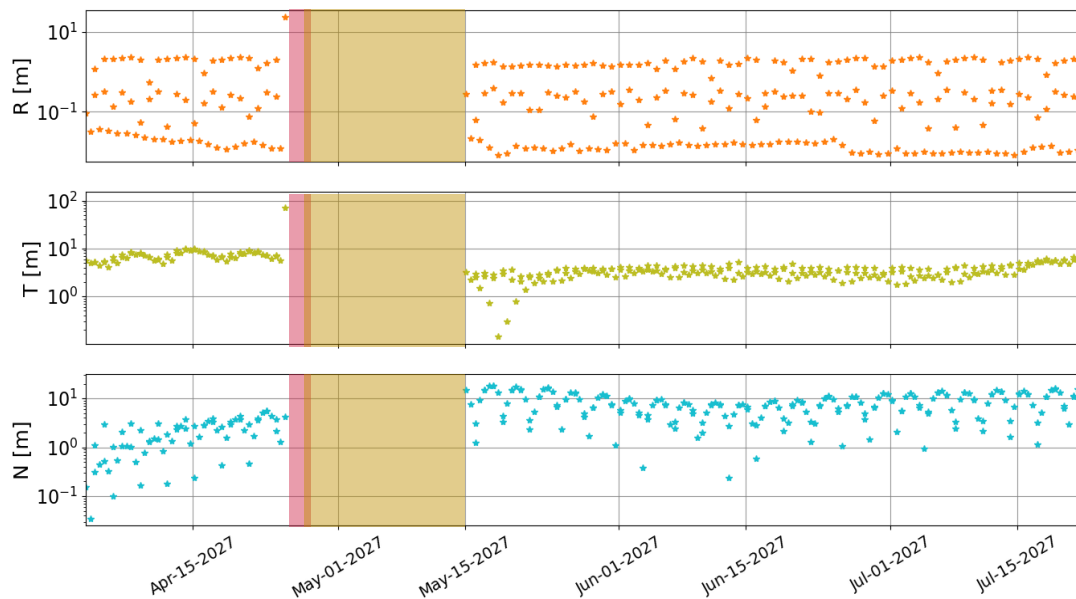


Figure 5.17. Position uncertainty components (3- σ) over the first 100 days of the orbital phase. For each day the uncertainty is reported at the beginning, center and end of the arc. Yellow and red bars point out the superior solar conjunctions and perihelia periods, respectively.

Figure 5.18 shows the propagation of the MPO 3- σ position uncertainty during a 1-day observation arc together with the time of the WOLs execution, considering the nominal scenario with two WOLs per day. The first maneuver, carried out in the tracking period, does not impact the spacecraft positioning accuracy since it can be determined better than the mm/s level through radio-tracking data. The second maneuver, carried out when no tracking is available before the end of the arc, is instead constrained only by imposing matching conditions among subsequent arcs through the inter-arc constraint (Section 2.3). The constraint value of 0.73 mm/s for the matching of velocities among subsequent arcs impose a lower bound for the absolute sensitivity of the second maneuver. The uncertainty in the radial components increases soon after the out-of-pass desaturation maneuver reaching values compatible with the threshold imposed by the inter-arc constraints (1 meter 1- σ). An alternative strategy to define the duration of each arc is to end an arc at the out of pass WOL, making the time with increased uncertainty virtually equal to 0.

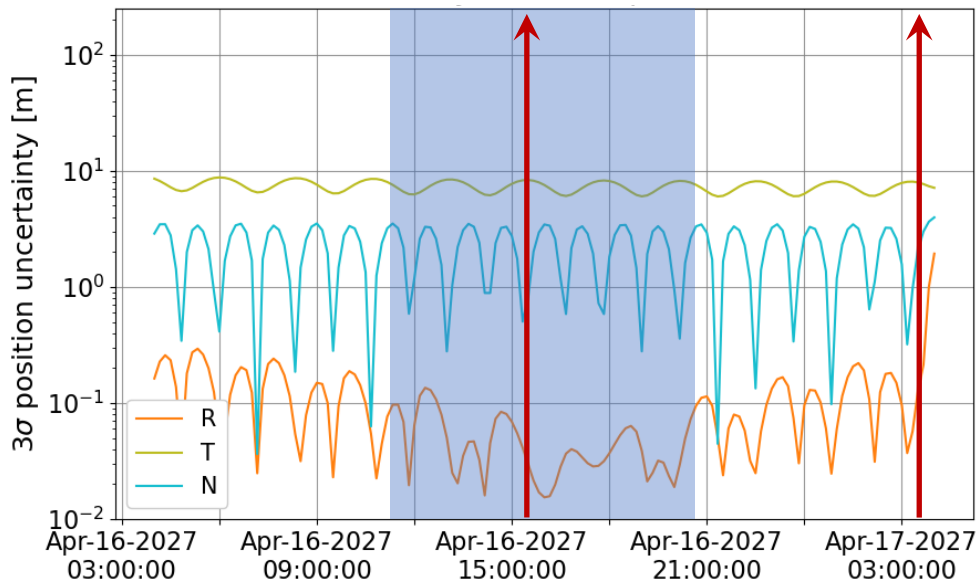


Figure 5.18. Position uncertainty (3- σ) for one arc (radial, transverse and normal components) with two daily maneuvers. The red arrows represent the wheel off-loading maneuvers. The blue block is the tracking pass.

An alternative contingency scenario involves the use of one maneuver every three orbits (3 or 4 maneuvers per day). In this case, the occurrence of multiple maneuvers outside the tracking pass does not allow to distinguish their individual contributions to the spacecraft dynamics through inter-arc constraints. The removal of inter-arc constraints has a minor impact on the geodesy experiment, leading to a degradation of 10-20 % in the set of estimated parameters. On the other hand, it has a strong impact on the trajectory reconstruction performance. In the absence of such constraints, the positioning accuracy strongly degrades after the out-of-pass desaturation maneuvers, as shown in Figure 5.19, reaching tens of meters in the radial and normal direction and hundreds of meters in the transverse direction.

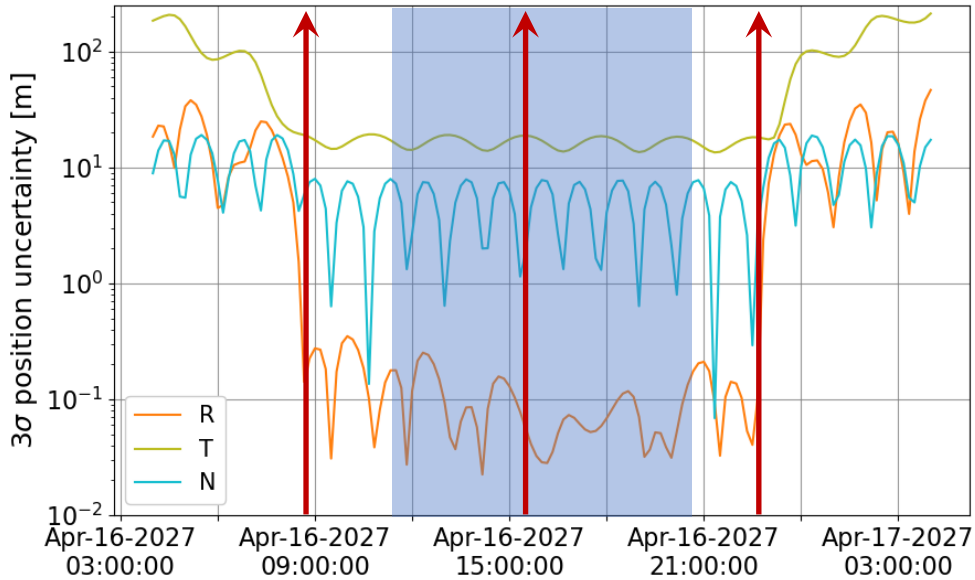


Figure 5.19. Position uncertainty (3σ) for one arc (radial, transverse and normal components) with 3 daily maneuvers. The red arrows represent the wheel off-loading maneuvers. The blue block is the tracking pass.

5.4 Joint analysis with MESSENGER

The complementary coverages of BepiColombo and MESSENGER, with pericenters located in the southern and northern hemisphere, respectively, offer a unique opportunity to obtain a comprehensive global mapping of the gravity field of Mercury.

One possible strategy is a sequential approach, in which the MORE inversion uses MESSENGER-derived gravity models as a priori. However, MESSENGER solutions rely on strong Kaula regularization due to the spacecraft highly eccentric orbit and uneven spatial coverage, which may bias the solution. A unique, simultaneous inversion of MESSENGER and BepiColombo data is instead preferable: it allows the full set of radiometric observables to be processed with the same software and a consistent dynamical model. Last but not least, enables more flexibility in choosing the Kaula constraint, avoiding the risk of regularization artifacts. Indeed, the low eccentric orbit of the MPO helps to decorrelate the gravity coefficients, leading to a smoother and more reliable recovery of the field. In this section we combine the results of the MESSENGER reanalysis (Section 4.3) and MORE simulation to provide the expected performance of a joint solution.

5.4.1 Simulation setup

The methodology relies on a covariance analysis implemented with the MONTE OD code (Evans et al., 2018). The procedure is outlined in Section 2.5. The synthetic two-way Doppler dataset for MORE is generated by integrating the MPO trajectory using the gravity and rotation model resulting from the MESSENGER reanalysis (Section 4.3). The dynamical modelling of NGAs is consistent with Section 4.3.3 and Section 5.3.3, incorporating solar, planetary and spacecraft radiation pressure. The radiometric data for the extended orbital phase are simulated with white Gaussian noise compatible with the expected measurement accuracy (see Table 5.5). Approximately 50% of data are discarded to account for HGA outages, low SPE angles, Mercury occultations, and perihelia (Table 5.6).

The estimation is performed with a single filter iteration, combining the entire MESSENGER tracking data with the simulated MORE dataset. After data editing, about 1500 arcs remain. The sets of local parameters for MESSENGER and BepiColombo are listed in Table 4.4 and Table 5.7, while the solved global parameters follow those of the MESSENGER reanalysis (Table 4.4). To enable direct comparison of gravity anomaly uncertainties, the same Kaula factor $A_k = 4$ is applied to the gravity coefficients, even if a

looser Kaula constraint can be used (see Section 5.3.5). The constraint is imposed from degree 10 for MESSENGER only, and beyond degree 30 when the MORE dataset is included. The choice of degree 30 as the transition point for regularization reflects the global resolution attainable by the MORE experiment (see Figure 5.20).

5.4.2 Results

The amplitude spectra (C_l) of the gravity field coefficients are shown in Figure 5.20 for the different solutions. The degree at which the signal spectrum intersects the uncertainty spectrum is generally taken as the global resolution of the gravity field. For MORE, this occurs around degree 35 without assuming any regularization. For MESSENGER, the spectra is affected by the applied Kaula constraint beyond degree 10, owing to the highly non-homogeneous spatial coverage of the spacecraft. As summarised in Table 5.9, combining the two datasets yields uncertainties on the low-degree field, the tidal Love number k_2 , and the rotational parameters comparable to the case using MORE data alone. The inclusion of MESSENGER data does not significantly enhance sensitivity to the global geophysical parameters, except for the spin rate and the polar moment of inertia ratio (C_m/C).

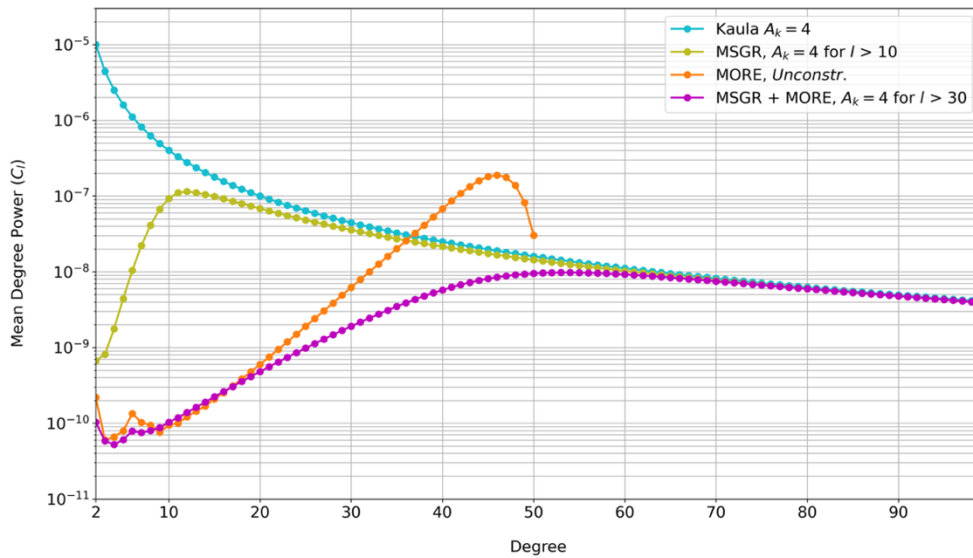


Figure 5.20. Expected amplitude spectrum from the Kaula rule (cyan line) and degree RMS of $1\text{-}\sigma$ uncertainties for different solutions. The green line represents the MESSENGER solution alone, the orange line the MORE solution alone (without regularization), the purple line the joint solution of

Parameter	MESSENGER	MORE	MESSENGER + MORE
$\bar{J}_2$	$1.4 \cdot 10^{-9}$	$1.8 \cdot 10^{-10}$	$1.6 \cdot 10^{-10}$
$\bar{C}_{22}$	$9.4 \cdot 10^{-10}$	$2.4 \cdot 10^{-10}$	$2.2 \cdot 10^{-10}$
k_{2r}	$3.3 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
α [deg]	$1.1 \cdot 10^{-3}$	$1.3 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$
δ [deg]	$5.5 \cdot 10^{-4}$	$9.9 \cdot 10^{-5}$	$9.7 \cdot 10^{-5}$
ϕ [arcsec]	4.1	$6.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$
ω [deg/day]	$1.8 \cdot 10^{-6}$	$6.6 \cdot 10^{-7}$	$1.4 \cdot 10^{-7}$
ε [arcmin]	$3.4 \cdot 10^{-2}$	$5.3 \cdot 10^{-3}$	$5.0 \cdot 10^{-3}$
C/MR^2	$5.6 \cdot 10^{-3}$	$8.8 \cdot 10^{-4}$	$8.3 \cdot 10^{-4}$
C_m/C	$1.9 \cdot 10^{-2}$	$8.5 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$

Table 5.9. Comparison of 3- σ uncertainties on the set of geophysical parameters.

The main advantage of the combined inversion lies in the complementary coverage provided by the two spacecraft. MESSENGER provided high-resolution coverage of the northern hemisphere, while MORE ensures a more homogeneous global mapping with enhanced coverage of the southern hemisphere. Combining the two datasets therefore results in an enhanced gravity field, maintaining the high-resolution mapping of the north while improving the sensitivity in the south. Figure 5.21 illustrates this effect, comparing the gravity anomaly 1- σ uncertainties for the different solutions. While MESSENGER alone achieves uncertainties of about 2 mGal in the north but exceeds 20 mGal in the south, MORE alone enables 15–20 mGal in the south with resolution up to degree ~ 40 . The joint solution merges these complementary performances, yielding uncertainties below 20 mGal globally.

The extended time span and diverse orbital geometries leads to more stable and reliable estimates of Mercury’s gravity field, spin state, and internal structure, reducing the need of strong Kaula regularization. This improved global solution provides an excellent basis for geophysical interpretation, refining estimates of crustal thickness variations, planet’s moment of inertia, and consequently offering tighter constraints on Mercury’s internal differentiation and thermal evolution.

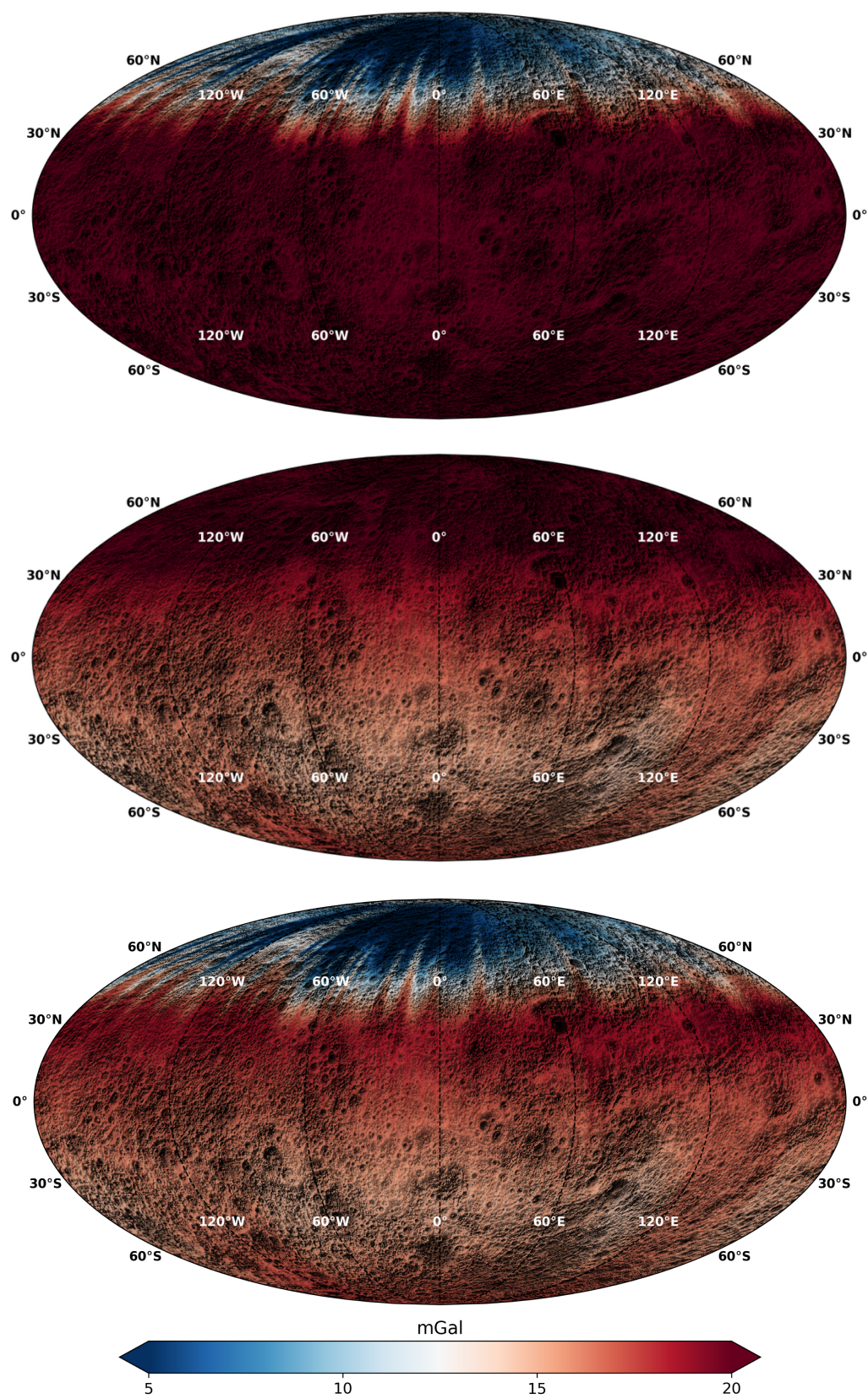


Figure 5.21. Gravity anomalies $1\text{-}\sigma$ uncertainty for MESSENGER (top), MORE (center) and MESSENGER + MORE (bottom). The joint solution merges the complementary coverages of the missions, yielding uncertainties below 20 mGal globally.

5.5 Summary

This chapter investigated the performance of the geodesy experiment onboard the BepiColombo mission with two different works: i) a contingency analysis with dynamical mismodelling and ii) a covariance analysis for the joint solution with MESSENGER.

The first work is a contingency analysis without accelerometer data, based on the prediction of NGAs through numerical models. To reproduce a realistic scenario, this study accounted for the effect of inconsistencies in the force models. We introduced errors in the spacecraft reference state and optical properties, unbalanced ΔV s due to wheel off-loading maneuvers, solar irradiance fluctuations, residual effects due to unmodelled non-gravitational accelerations and not perfectly known gravity field. We solved for the set of geophysical parameters and the gravity field coefficients up to degree and order 70, constrained with a loose Kaula rule ($A_k = 30$) for degrees greater than 30. The analysis is subject to two main limitations (i) the force models used for simulating the observables and for estimation share the same mathematical formulation and (ii) each class of perturbation is compensated by a corresponding set of estimated parameters, implicitly neglecting unknown unmodelled effects. Under these assumptions and without including ISA data, our results have shown that the OD filter effectively compensates for the introduced mismodelling without significant biases. We provided updated accuracy for the gravity and rotation experiment after the extended orbital phase using only Doppler data. The uncertainties derived from our analysis confirmed that MORE is expected to provide a reliable, high-resolution gravity field at spatial scales of ~ 180 km in the southern hemisphere, revealing associations between gravity anomalies and geological features such as impact craters, rift zones, and lobate scarps. This opportunity will enhance our understanding of the processes that shaped Mercury's surface, offering valuable insights into the planet's geological history. The estimation of the Love number k_2 could be improved at the level of $1.8 \cdot 10^{-3}$ (3- σ value), allowing to set tighter constraints to the interior structure of Mercury. The pole's right ascension and declination could be estimated better than half arcsecond. The amplitude of Mercury's physical librations in longitude could be determined with an accuracy of a few meters at

the equator. Mercury's obliquity could be retrieved with an accuracy of ~ 0.3 arcsec, improving the determination of the dimensionless moment of inertia and the ratio between the principal moment of inertia of the mantle and that of the entire planet. Furthermore, the precise radio-tracking enables unprecedented accuracy in the spacecraft positioning. With two daily wheel off-loading maneuvers, the expected orbit reconstruction accuracy is at the level of a few tens of centimeters in the radial direction, and a few meters in the normal and transverse directions ($3\text{-}\sigma$ values). These uncertainties can be further reduced by including range data. With an increment in maneuver frequency, this accuracy is expected to degrade to several tens of meters in the radial and normal directions, and to hundreds of meters in the transverse direction for a significant percentage of the day.

Finally, a joint analysis with MESSENGER does not enhance the sensitivity on the set of geophysical parameters with respect to MORE alone, except for the rotation rate ω and the polar moment of inertia ratio C_m/C , which show noticeable improvements. However, the joint solution enables an outstanding high resolution global mapping of the planet. Merging the two datasets allows to detect fine spatial features in the north with MESSENGER while improving the sensitivity in the south with MORE, enabling uncertainties on gravity anomalies below 20 mGal over the entire planet.

Chapter 6

Dynamical modelling with ray tracing

Accurate computation of non-gravitational accelerations is essential for reliable orbit determination, as even small mismodellings can bias scientific results. With modern tracking systems now capable of measuring line-of-sight velocities at the level of tens of $\mu\text{m/s}$ and relative distances at the level of few centimetres (Cappuccio et al., 2020, 2025; less et al., 2018), radiometric data are sensitive to perturbations that were previously negligible. This growing precision exposes the limitations of traditional force models and highlights the need for a new generation of numerical methods. This chapter introduces and validates a novel Python ray tracing software, named pyRTX (Cascioli, 2021), enabling advanced computation of non-gravitational accelerations. The tool can be broadly applicable to missions where non-gravitational effects are dominant and critical to the science objectives, including those to Mercury, Venus, Earth, the Moon, and Mars.

This chapter is organized as follows: the limitations of the traditional plate model are listed in Section 6.1; the mathematical force models based on ray tracing are presented in Section 6.2; the structure of the code is summarised in Section 6.3; followed by an application in the context of the Lunar Reconnaissance Orbiter in Section 6.4; finally, concluding remarks are given in Section 6.5.

6.1 Limitations of the plate model

The recent advance in spacecraft tracking technology will lead new insights in planetary sciences, as well as measurements of relativistic gravity to an enhanced level of accuracy compared to past outcomes. However, while the accuracy of radio observables improved over the years, the determination of the spacecraft orbits still relies on traditional methods with limitations.

The spacecraft modelling for the MESSENGER reanalysis in Section 4.3.2 and for the numerical simulations of BepiColombo in Section 5.3.2 is based on the classical approach used by orbit determination (OD) software. This method, known as plate model, consists in the discretization of the probe as a set of plates or 2D elementary shapes (see Section 3.5.1). In its basic formulation, however, this model exhibits limitations that become noteworthy when precision is paramount. The first and most important limitation is related to the self-shadowing effect, which occurs when parts of the spacecraft are shielded from the incoming flux by other structural elements (Balmino & Marty, 2018; Mazarico et al., 2009). As an example, consider the case in Figure 6.1, in which part of the spacecraft bus is shadowed by the large sunshade.

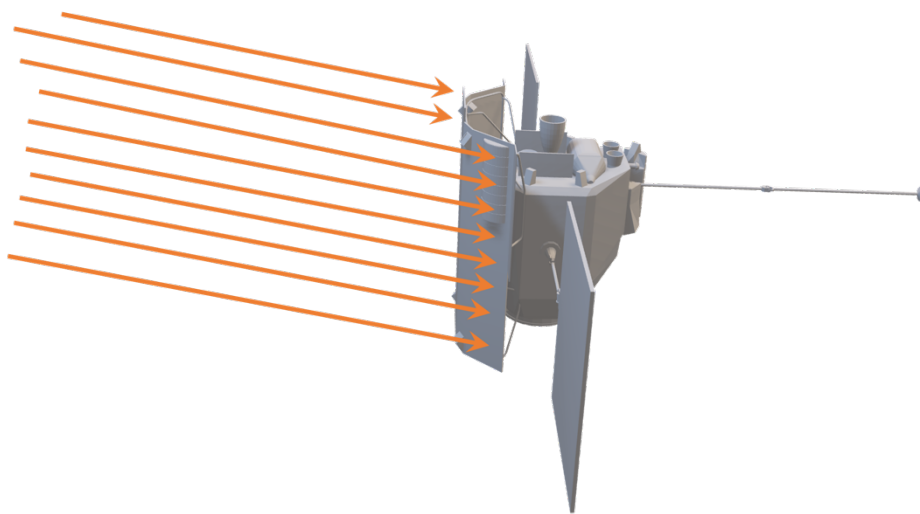


Figure 6.1. Example of self-shadowing. The bus is hidden by the sunshade and thus, it is not impacted by the incoming radiation.

In the plate model, the determination of whether a surface is sunlit relies only on the scalar product between the surface normal and the incoming ray direction. In the case presented in Figure 6.1, using this approach would mean accounting for the contribution of a portion of the spacecraft bus, indeed overestimating the total acceleration. The model lacks the capability to differentiate surfaces directly exposed to incoming flux from those shielded by other spacecraft components. This effect becomes even more important when dealing with more complex (typically convex) shapes, in which self-shadowing occurs more often. Another limitation arises in scenarios involving multiple reflections of photons on the spacecraft surface, where reflected photons can impact subsequent surfaces, leading to a repetitive process. As highlighted by several authors (Z. Li et al., 2018; Mazarico et al., 2009), these effects cannot be neglected for certain classes of missions, since they significantly impact the momentum exchange and, thus, the overall acceleration.

Addressing the limitations of the plate model would provide significant benefits to future space applications, especially the ones relying on state-of-the-art tracking systems for meter-level determination of the spacecraft position. This is especially true for the next generation missions to the inner Solar System, such as ESA-JAXA's BepiColombo (Section 5.1), ESA's EnVision (Straume-Lindner et al., 2025) and NASA's VERITAS missions (Cascioli et al., 2021; Giuliani et al., 2025; Smrekar et al., 2022). In the next decades, these missions will exploit their advanced tracking systems to provide accurate measurements from the harsh thermal environments of Mercury and Venus. Due to their proximity to the Sun, these spacecraft experience strong accelerations from radiation pressure (both solar and planetary). Improved non-gravitational force modelling is also relevant for missions to Earth, the Moon, and Mars, where solar radiation pressure (SRP) remains a major contributor to spacecraft dynamics (Fliegel & Gallini, 1996; Marshall et al., 1992). Although planetary radiation effects are less extreme in these environments, they must be correctly modelled for accurate gravity field recovery (Floborghagen et al., 1999). For low-altitude orbiters on Venus, the Earth, and Mars, atmospheric drag from the upper layers of the atmosphere is also very important, as it is the main non-gravitational effect acting on the probe.

6.2 Non-gravitational accelerations with ray tracing

The limitations inherent in the plate model, as outlined earlier, provided the motivations to increase the modelling accuracy of non-gravitational effects. The mathematical description of NGAs has already been addressed in 3.5. Recalling Eq. (3.18), the non-gravitational acceleration $\mathbf{a}$ can be written in the general form:

$$\mathbf{a} = \frac{\Phi_{in}}{mc} \mathbb{L}_{\mathbf{b}}(\alpha, \delta) \quad (6.1)$$

where Φ_{in} is the input flux in W/m^2 , which depends on the considered physical phenomenon responsible for the acceleration; m is the spacecraft mass; c is the speed of light and $\mathbb{L}_{\mathbf{b}}$ is the “generalized geometry vector”. Note that in this case, the subscript “b” indicates that this quantity is computed accounting for b reflections. $\mathbb{L}_{\mathbf{b}}$ is representative of the interaction of the spacecraft surface and a radiation of unitary intensity, as a function of its direction of arrival. This is identified by its right ascension α and declination δ with respect to the spacecraft body-fixed frame.

The following sections provide a rigorous and flexible way of evaluating $\mathbb{L}_{\mathbf{b}}$ by means of geometrical ray tracing (Darugna et al., 2018; Kenneally & Schaub, 2020; Z. Li et al., 2018), which simulates the paths of photons emitted by a source as they encounter a surface (Whitted, 1979). Although more computationally demanding than macro-models such as the plate model, advances in hardware and algorithms have significantly reduced its computational cost. Ray tracing provides the essential geometric information required by high-fidelity force models, including the precise locations of photon or particle impacts. This enables accurate determination of impinging spacecraft surfaces (including self-shadowing), the paths of reflected or diffused radiation, and the effective cross section relevant for atmospheric drag. The spacecraft shape can be represented with a 3D engineering model, discretized as an ensemble of facets. As discussed in the following

sections, the applicability of ray tracing extends beyond these examples and includes, for instance, the accurate computation of eclipse transitions.

6.2.1 The pixel plane approach

Following the approach of Ziebart (2004), the incident radiation or atmospheric particle flux is represented by a pixel plane: a rectangular grid of ray source points. Each pixel is the origin of a ray and the total flux intensity is equally subdivided into each ray. The size of the plane and the ray density are design parameters. The incoming flux direction is specified by the right ascension α and declination δ in the spacecraft body-fixed frame. The geometry is shown in Figure 6.2. The pixel plane reference frame $\mathbf{X}_P = \{\mathbf{x}_P, \mathbf{y}_P, \mathbf{z}_P\}$ is initially coincident with the spacecraft body frame $\mathbf{X}_B = \{\mathbf{x}_B, \mathbf{y}_B, \mathbf{z}_B\}$. To orient the plane so that its normal points toward the origin of the body-fixed frame, two rotations are applied: (1) a rotation of α around $\mathbf{z}_B$, (2) a rotation of $3\pi/2 - \delta$ around $\mathbf{y}_P$. The plane is then translated along its normal to a distance d_0 from the origin. The transformation between the two coordinate frames is then (Z. Li et al., 2018):

$$\mathbf{r}_P = \mathbf{R}_y\left(\frac{3\pi}{2} - \delta\right) \mathbf{R}_z(\alpha) \mathbf{r}_B - \mathbf{x}_0 \quad (6.2)$$

$$\mathbf{x}_0 = -d_0[\cos(\alpha)\cos(\delta), \sin(\alpha)\cos(\delta), \sin(\delta)]$$

where $\mathbf{R}_a(\theta)$ denotes a rotation of θ around axis $\mathbf{a}$; $\mathbf{r}_B$ represents the pixel coordinates in the spacecraft body-fixed frame before rotation and translation, and $\mathbf{r}_P$ the corresponding pixel coordinates after the transformation, still expressed in the body-fixed frame. The distance d_0 and the dimensions of the pixel plane must be selected according to the size of the target object. The plane should lie fully outside the body, and its width and height must be large enough for the emitted rays to cover the entire geometry.

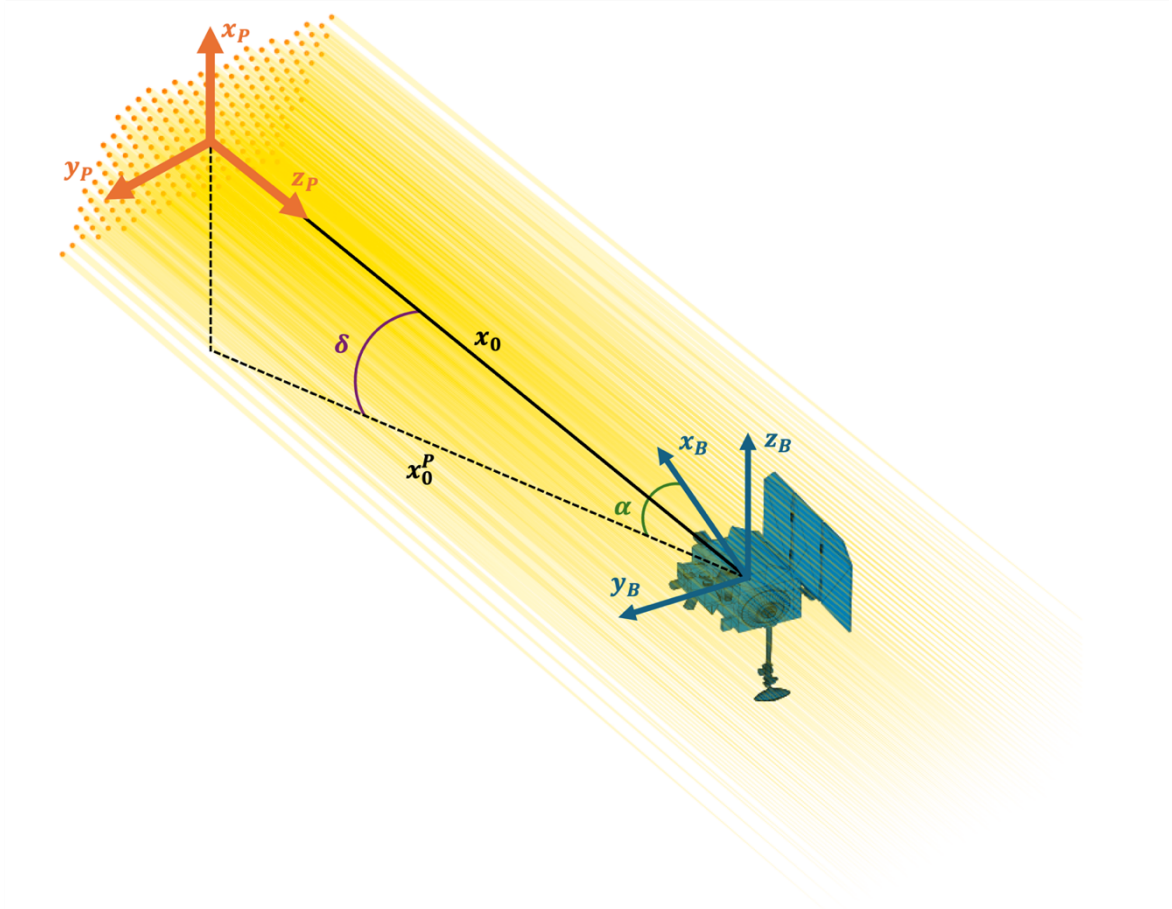


Figure 6.2. Ray tracing geometry. The pixel plane is represented as the orange square. The spacecraft and the pixel plane reference frames are $\{x_B, y_B, z_B\}$ and $\{x_P, y_P, z_P\}$, respectively. The position vector of the pixel plane origin is x_0 , while x_0^P is its projection on the (x_B, y_B) plane. The right ascension and declination of the incoming ray directions are α, δ .

By discretizing the incoming flux into a finite set of rays defined on this plane, the origin and direction of each ray can be computed. Defining the plane's width w (along x_P), the height h (along y_P), and the discretization step of the plane d_s (the ray spacing), the ray origins are first generated in the unrotated plane and subsequently transformed to their final positions using Eq. (6.2).

$$\mathbf{M}(i, j) = \mathbf{R}_y\left(\frac{3\pi}{2} - \delta\right) \mathbf{R}_z(\alpha) \left[-\frac{w}{2} + (i-1)d_s, -\frac{h}{2} + (j-1)d_s, 0 \right] - \mathbf{x}_0 \quad (6.3)$$

where $\mathbf{M}$ is the multi-dimensional matrix containing the pixels origins, expressed in the $\mathbf{x}_B$ frame, and i, j are the row, column indexes of the matrix. The $\mathbf{M}$ matrix has dimensions

$([w/d_s], [h/d_s], \times 3)$, where $[\cdot]$ denotes the integer obtained by excess approximation. The direction of the rays are all the same and equal to the $\hat{\mathbf{x}}_0$ versor.

Once the ray origins and directions are defined, they are input to the ray tracing kernel (or ray tracer), which propagates each ray until it intersects the spacecraft surface. The kernel returns the intersection points, making it possible to identify the specific mesh elements impacted by the rays. This naturally accounts for self-shadowing, an essential capability for modelling the interaction of radiation with complex geometries. A second key feature is the possibility to account for multiple reflections, which will be described later in the next section.

6.2.2 Generalized geometry vector

Considering only the first bounce (i.e. neglecting multiple reflections of the incoming rays) the geometry vector is obtained by performing ray tracing from the pixel plane up to the spacecraft. The spacecraft can be represented with a realistic 3D shape, discretized as an ensemble of triangular faces (e.g., Wavefront .obj format), each with specific thermo-optical properties assigned. If the number of rays is sufficiently high, the impacted area by each ray will be $dA = 1/r_d$, where r_d is the ray density in $rays/m^2$ (Z. Li et al., 2018). The geometry vector is computed by summing the contributions of the N_i rays that have impacted the shape elements:

$$\mathbb{L}_1(\alpha, \delta) = r_d^{-1} \sum_{i=1}^{N_i} \left[\hat{\mathbf{s}}_i - \rho_{si} \hat{\mathbf{r}}_i - \frac{2}{3} \rho_{di} \hat{\mathbf{n}}_i \right] \quad (6.4)$$

where i denotes the i -th ray; $\hat{\mathbf{s}}_i, \hat{\mathbf{r}}_i$ are the directions of arrival and reflection; $\hat{\mathbf{n}}_i$ is the normal to the facet of the 3D spacecraft model which is impacted by the ray and ρ_{si}, ρ_{di} are the specular and diffuse coefficients associated to the facet (see Figure 6.3). Considering only the first bounce inherently accounts for self-shadowing, since any surface shielded by other parts of the spacecraft does not receive any rays (Mazarico et al., 2009). As shown in Eq. (3.20), the dependency on α, δ is implicit in $\hat{\mathbf{s}}_i$.

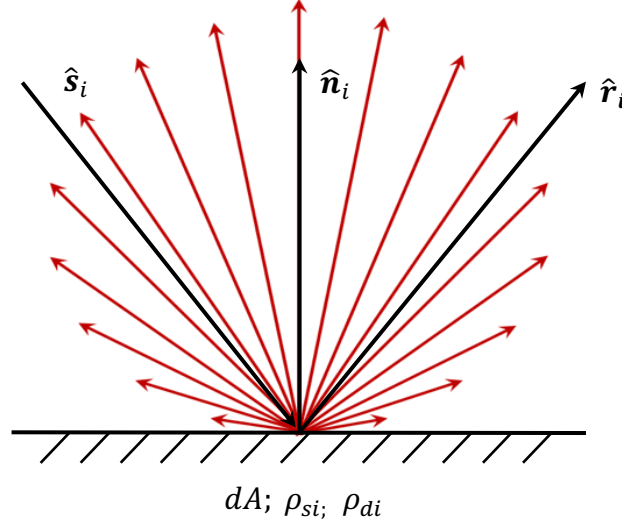


Figure 6.3. Geometry of incident radiation for a spacecraft surface element. The diffused rays in the hypothesis of a Lambertian surface are shown in red. $\hat{n}_i$ is the normal direction to the exposed surface; dA is the surface area; ρ_{si} and ρ_{di} are its specular and diffuse coefficients; $\hat{s}_i, \hat{r}_i$ are the directions of arrival and specular reflection.

6.2.3 Multiple reflections

To account for successive bounces, the procedure is repeated recursively by updating the ray properties. Consider, for now, that the reflected radiation is only of specular nature (see Figure 6.3). When a ray with initial direction $\hat{s}_i$ hits a surface at point $\mathbf{p}_i$, and the surface normal is $\hat{n}_i$, the reflected ray originates at $\mathbf{p}_i$ and propagates in the direction:

$$\hat{r}_i = \hat{s}_i - 2(\hat{s}_i \cdot \hat{n}_i)\hat{n}_i \quad (6.5)$$

Once $\mathbf{p}_i$ and the reflected direction $\hat{r}_i$ are known for all the rays that reach the surface, these reflected rays are submitted back to the ray tracer to determine secondary intersections. The new origin is the impact point, the new direction is the reflected direction, and the new intensity is the reflected fraction of the incoming radiation (the previous intensity multiplied by ρ_{si}). This process can be repeated for as many bounces as required. Figure 6.4 shows the norm of the geometry vector as a function of the number of reflections for LRO. Each value is normalised by the magnitude of the

geometry vector of the first bounce. As can be seen, including at least two bounces may be relevant in the computation of the overall accelerations.

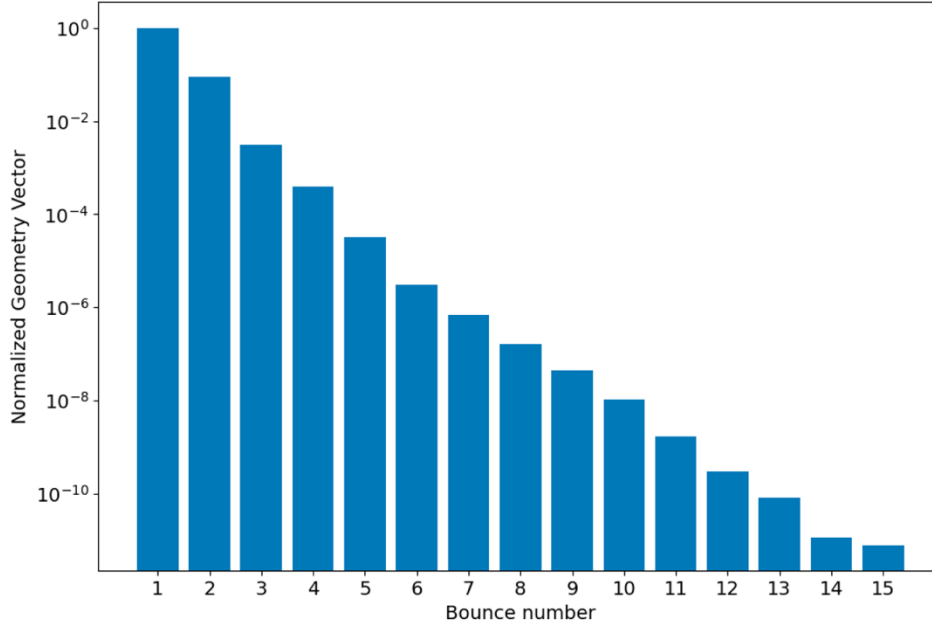


Figure 6.4. Norm of the geometry vector as a function of the number of reflections for LRO. Each value is normalised by the magnitude of the geometry vector of the first bounce. The plot refers to the epoch 09:25:00 ET on 10 May 2010.

As shown in Eq. (6.4), the geometry vector depends not only on the reflected portion of the incoming radiation but also on its diffuse component. Handling diffusion is numerically more demanding because each incoming ray produces not a single outgoing ray, but an entire distribution of them. We model the diffusion as Lambertian, so each incoming ray generates a Lambertian distribution of outgoing rays (see Figure 6.3). The number of rays n_d used to discretize this distribution is a design choice that strongly affects the computational cost. The intensity of each diffuse ray is determined by the incoming flux Φ_{in} and the diffuse reflectivity of the emitting surface ρ_{di} :

$$\Phi_d = \frac{\Phi_{in}\rho_{di}}{n_d} \quad (6.6)$$

The generalized geometry vector accounting for b bounces can be computed recursively as follows:

$$\begin{aligned}
\mathbb{L}_1 &= r_d^{-1} \sum_{i=1}^{N_i} \left[\hat{\mathbf{s}}_i - \rho_{si} \hat{\mathbf{r}}_i - \frac{2}{3} \rho_{di} \hat{\mathbf{n}}_i \right] \\
\mathbb{L}_2 &= \mathbb{L}_1 + r_d^{-1} \sum_{i=1}^{N_i} \rho_{si_0} \left[\hat{\mathbf{s}}_i - \rho_{si} \hat{\mathbf{r}}_i - \frac{2}{3} \rho_{di} \hat{\mathbf{n}}_i \right] + r_d^{-1} \sum_{i=1}^{N_d} \frac{\rho_{di_0}}{n_d} \left[\hat{\mathbf{s}}_i - \rho_{si} \hat{\mathbf{r}}_i - \frac{2}{3} \rho_{di} \hat{\mathbf{n}}_i \right] \\
&\dots \\
\mathbb{L}_b &= \mathbb{L}_{b-1} + r_d^{-1} \sum_{i=1}^{N_i} \Phi_{i,b-1} \rho_{si_0} \left[\hat{\mathbf{s}}_i - \rho_{si} \hat{\mathbf{r}}_i - \frac{2}{3} \rho_{di} \hat{\mathbf{n}}_i \right] + r_d^{-1} \sum_{i=1}^{N_d} \Phi_{i,b-1} \frac{\rho_{di_0}}{n_d} \left[\hat{\mathbf{s}}_i - \rho_{si} \hat{\mathbf{r}}_i - \frac{2}{3} \rho_{di} \hat{\mathbf{n}}_i \right]
\end{aligned} \tag{6.7}$$

where N_i and N_d are the number of impacting rays for bounce b due to specular and diffuse reflectivity, respectively; ρ_{si_0} and ρ_{di_0} are the specular and diffuse coefficients associated with the origin surface of the ray i ; $\Phi_{i,b-1}$ is the fraction of the external flux arriving at the surface that contains the origin of ray i . This last quantity carries the cumulative effect of all previous interactions along the ray's path. For example, suppose a ray from the original external radiation hits a surface with specular reflectivity ρ_{s1} , the reflected ray then hits a second surface with specular reflectivity ρ_{s2} , and the resulting reflected ray reaches a third surface with diffuse reflectivity ρ_{d3} . The flux fraction associated with one of the diffusely reflected rays from the third surface is $\Phi = \rho_{s1} \rho_{s2} \rho_{d3} / n_d$. Figure 6.5 shows the percentage of the full geometry vector (specular + diffuse) of LRO that is missed when diffusion is neglected. It displays the percent difference between the geometry vector computed with both specular and diffuse reflection and the one obtained using only specular reflection. As can be seen, ignoring diffusion can introduce a mismodelling of up to 10% in the computation of the total acceleration.

The recursion to account for multiple reflections continues until a stopping criterion is met. In practice, a maximum number of bounces is chosen a priori based on preliminary numerical tests that establish an acceptable acceleration tolerance.

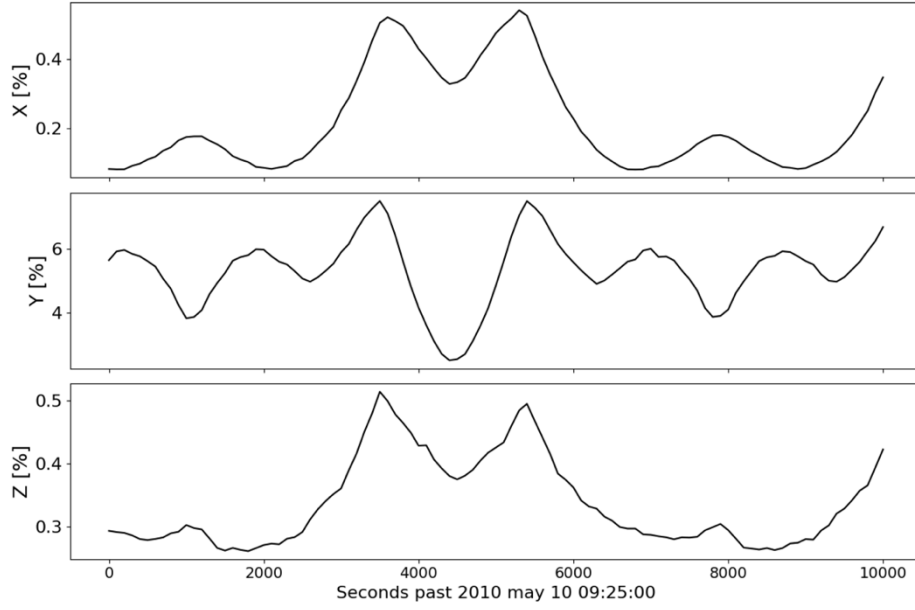


Figure 6.5. Percent differences between the LRO geometry vector computed with both specular and diffuse reflection and the one obtained using only specular reflection. The differences are reported in the spacecraft body-fixed frame.

6.2.4 Refined input fluxes

The ray tracing approach has the capability to overcome the limitations of the plate model, capturing both self-shadowing and multiple reflections and ensuring accurate computation of $\mathbb{L}_b$. Moreover, it allows to improve the evaluation of the input fluxes Φ_{in} (see Section 3.5 for the mathematical formulation). The most notable refinement concerns the modelling of eclipse transitions. Recalling Eq. (3.21), the acceleration due to solar radiation pressure (SRP) can be described as:

$$\mathbf{a}_{SRP} = f_s \frac{\Phi_{\odot}}{mc} \mathbb{L}_b(\alpha, \delta) \quad (6.8)$$

where f_s is the shadow function, which assumes values between 0 and 1 to scale the solar flux depending on the location of the spacecraft and its illumination conditions. Many geometric models have been developed for the precise calculation of sunlit/penumbral/eclipse conditions. Here we leverage the ray tracing capability also for this task. A square pixel plane with side equal to the diameter of the Sun is defined. Then, only the circular portion corresponding to the effective Sun disk is selected, from which

a finite number of rays are cast (i.e. defining a ‘circular’ pixel plane). Ray tracing is performed with these rays against a 3D shape of the planet and assessing the fraction of rays that impact the planet. The planet shape can be as simple as a sphere or a more complex model from measured high-resolution topography, as in the case of lunar digital elevation models (Barker et al., 2016, 2025). Especially for spacecraft operating in the inner solar system, the limb darkening effect of the Sun, during eclipse transitions (umbra-penumbra) can be important. Given the finite dimension of the Sun, not all the regions of its apparent disk have the same luminosity. A limb darkening function S_d is defined to model the intensity associated to each pixel as a function of the angle of emission β . Here we use the Eddington’s approximation (Vokrouhlicky et al., 1993):

$$S_d(\beta) = \frac{3}{4} \left[\frac{7}{12} + \frac{1}{2} \cos(\beta) + \cos(\beta) \left(\frac{1}{3} + \frac{1}{2} \cos(\beta) \right) \ln \left(\frac{1 + \cos(\beta)}{\cos(\beta)} \right) \right] \quad (6.9)$$

where β is the angle of emission. The geometry is shown in Figure 6.6.

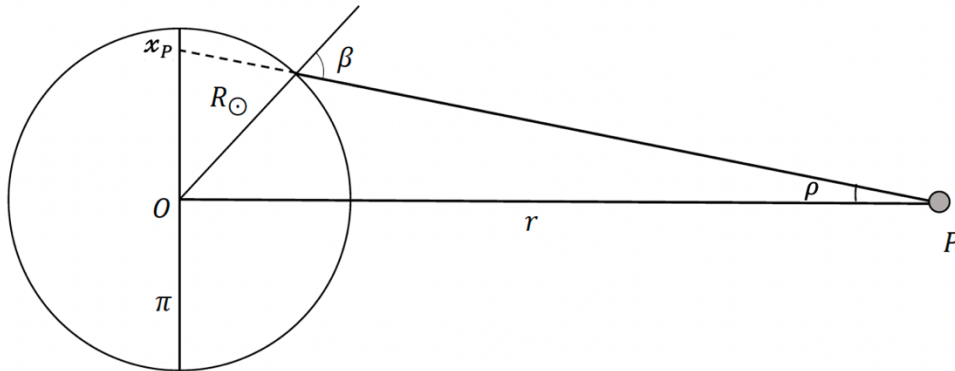


Figure 6.6. Geometry for the computation of the angle of emission β . π is the pixel plane with origin in O ; $\mathbf{x}_p$ is the position vector of a generic pixel; r is the spacecraft-Sun distance; P is the position of the spacecraft; $R_\odot$ is the Sun’s radius.

Said $\mathbf{x}_p$ the position vector of a generic pixel, the angle between the pixel plane origin O and the pixel, as seen by the spacecraft can be easily computed as:

$$\begin{aligned} \rho &= \arccos(\hat{\mathbf{x}}_p \cdot \hat{\mathbf{r}}) \\ \beta &= \arcsin \left(\frac{r}{R_\odot} \sin \rho \right) \end{aligned} \quad (6.10)$$

where r is the spacecraft-Sun distance and $R_{\odot}$ is the radius of the Sun. The relative weight of each ray can be defined as:

$$\omega_i = \frac{1}{n_p} S_d(\beta_i) \quad (6.11)$$

where ω_i is the weight of the i -th pixel, β_i the emission angle associated to the i -th pixel and n_p is the number of ray source points the pixel plane has been subdivided into. Defining the number of rays that effectively impact the planet as N_i and the total number of rays as N_t , the shadow function can be computed as follows:

$$f_s = 1 - \frac{\sum_{i=1}^{N_i} \omega_i}{\sum_{i=1}^{N_t} \omega_i} \quad (6.12)$$

Figure 6.7 shows the shadow function for a typical eclipse transition computed on LRO. The figure highlights the difference that exists when accounting for solar limb darkening.

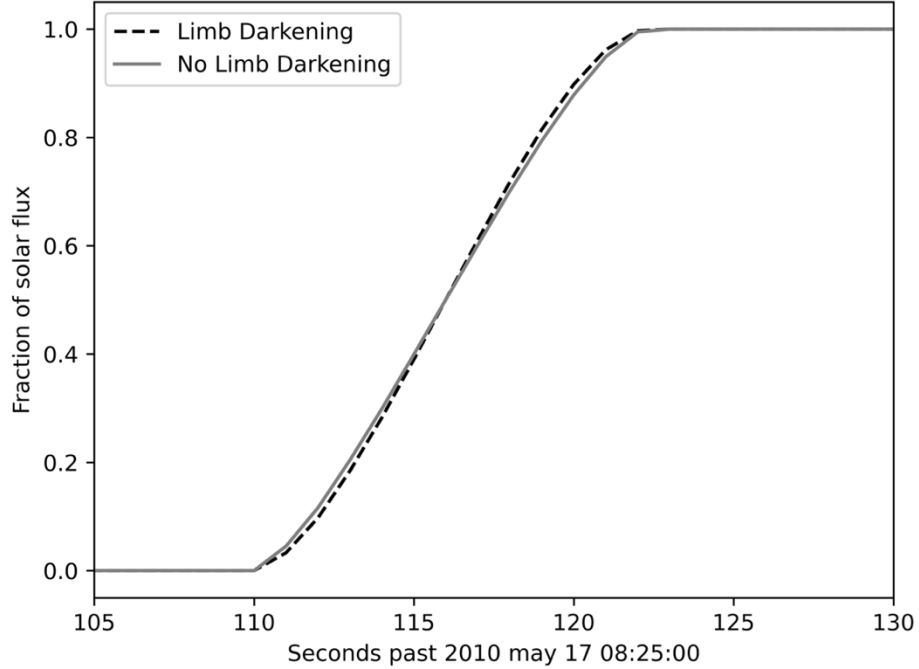


Figure 6.7. Shadow function (i.e., fraction of solar flux) computed for an eclipse transition of LRO. The dashed and solid lines represent the shadow function when the limb darkening is accounted for and not accounted for, respectively. Adapted from (Cascioli, 2021).

Using complex shapes not only refines the computation of eclipse transitions but also improves the calculation of planetary radiation fluxes (albedo and infrared) by capturing the effects of fine-scale surface features (see Section 6.4.4). Figure 6.8 shows the difference in the total planetary radiation acceleration when using different digital elevation models of the lunar topography with an increasing number of nodes (i.e., with an increasing resolution). The differences are calculated with respect to the baseline case with the Moon represented as a simple sphere. These signatures, on the order of 10^{-10} m/s^2 , are due to local differences in the Moon's radius. These discrepancies are negligible in the LRO case since they are well below the noise level of the S-band data, but can be relevant for applications involving state-of-the-art tracking systems like the forthcoming BepiColombo mission (see Section 5.2.1).

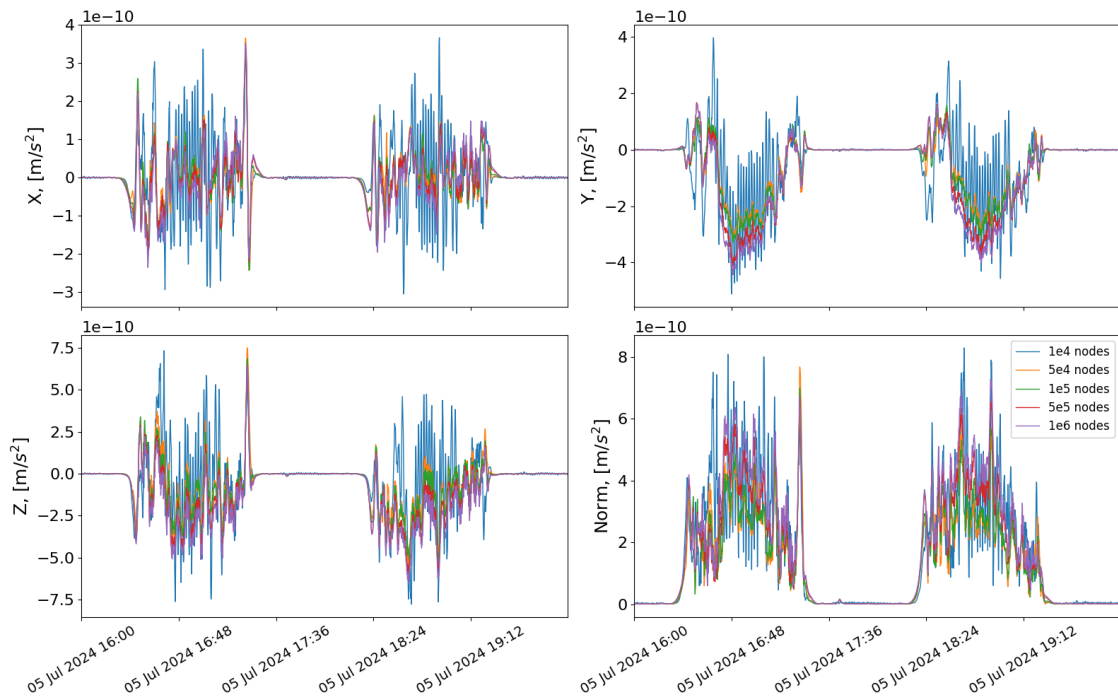


Figure 6.8. Differences in planetary radiation acceleration when using a different number of nodes to represent the Moon's shape, with respect to the simple sphere model. The differences are reported in the spacecraft body frame.

While the plate model remains a preferred choice for numerical simulations, due to its significantly reduced computational demands, the ray tracing approach proves powerful for missions equipped with the new generations of tracking systems, requiring high precision in the evaluation of accelerations.

6.3 The pyRTX software

The methodology described in the previous section constitutes the foundation of a dedicated Python software package, named pyRTX, developed to support the numerical calculations. The tool was conceived at NASA's Goddard Space Flight Center (GSFC) in Greenbelt, Maryland (Cascioli, 2021). The idea is to compute NGAs by leveraging ray tracing techniques. The package is divided into three sub-modules which can be imported by the user and used to perform high-fidelity calculations:

- Classes (classes to define the spacecraft, planets, ray tracer and NGAs)
- Core (physical/analytical functions to support calculations)
- Visual (plotting tools)

The software library is currently being documented and expanded and is publicly available on GitHub (<https://github.com/gaelccc/pyRTX>). A basic flowchart for the computation of the accelerations is presented in Figure 6.9.

The ray tracer (or ray tracing kernel) is the fundamental building block for all the algorithms that need to know the locations of the intersections between the rays and the spacecraft. The software exploits a ray tracing kernel based on the Intel® Embree open source library (Wald et al., 2014).

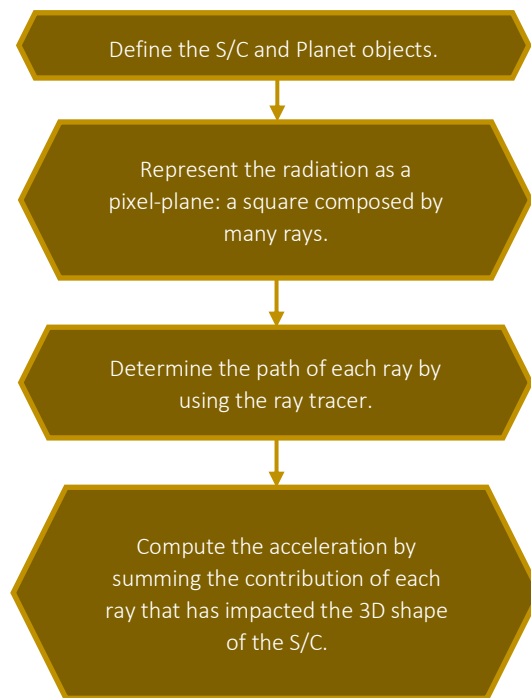


Figure 6.9. Basic flowchart for the computation of non-gravitational accelerations using pyRTX.

For some applications, such as those requiring very fine time sampling or involving planetary radiation, computing the geometry vectors directly at each epoch of interest is prohibitive in terms of computational effort. A more efficient alternative is to precompute the generalized geometry vectors over a grid of right ascensions and declinations that spans all expected directions of the incoming rays. When the spacecraft has moving components (e.g., solar arrays or steerable antennas), their possible orientations must also be included in the grid. The resulting values are stored in a lookup table (LUT) and can later be retrieved as needed. Because the requested input direction will rarely coincide exactly with a sampled grid point, interpolation is used to obtain the geometry vector at intermediate directions.

Generating the LUT is typically the most computationally demanding step, as it requires evaluating all relevant combinations of angles. While optional for solar radiation pressure, the LUT is essential for computing planetary radiation acceleration due to the significantly higher computational cost. The total time needed to generate the LUT depends on the number of moving components, the range of orientations considered,

and the resolution chosen for each angular dimension. Figure 6.10 shows the components of the generalized geometry vector of LRO in the spacecraft body-fixed frame as a function of the right ascension and declination of the incoming ray. The values are sampled from a LUT computed on a grid with 10° angular resolution, assuming the spacecraft moving parts are frozen at a fixed orientation (09:25:00 ET on 10 May 2010). In this configuration, the LUT computation required approximately 10 minutes using 10 CPU cores, including diffusive effects. However, accounting for the possible orientations of moving appendages can dramatically increase the computational effort. For the application described in Section 6.4, which includes multiple solar array orientations, the LUT generation required about two days of computation on roughly 100 CPU cores.

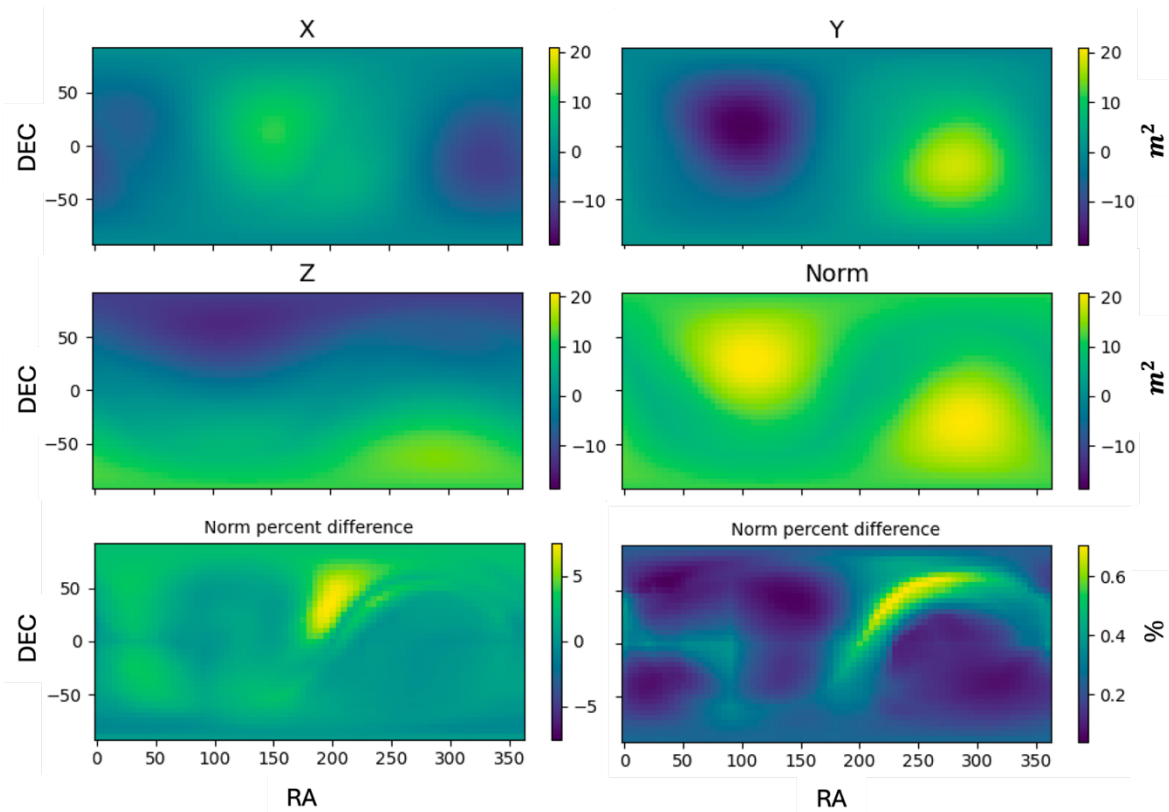


Figure 6.10. Generalized geometry vector as a function of the right ascension and declination of the incoming rays in the spacecraft body-fixed frame. The top and central rows show the X, Y, and Z components and the norm of the geometry vector considering only the first bounce. The bottom-left panel shows the relative change in the norm introduced by including a second specular reflection. The bottom-right panel shows the additional relative change obtained when diffuse reflection is also included in the second bounce.

Although computationally demanding, the pyRTX package offers substantial advantages for high-precision dynamical modelling of space probes. It enables the use of realistic 3D shapes for both spacecraft and planetary bodies, moving beyond the simplified plate or spherical models typically employed in traditional force modelling. Built around a SPICE (Spacecraft, Planet, Instrument, C-matrix, and Events; Acton et al., 2018) interface, pyRTX allows the user to load the standard SPICE kernels (e.g. SPK, CK, and FK files) to access trajectory, attitude, and frame orientation data, facilitating direct integration with other astrodynamics pipelines.

The software currently supports detailed modelling of solar radiation pressure, planetary albedo, thermal infrared emission, and atmospheric drag. The ray tracing approach allows to capture self-shadowing, multiple reflections, penumbra and umbra transitions, and accurate computation of planetary radiative fluxes. Although atmospheric drag is not relevant for Mercury, it becomes a critical perturbation for missions such as NASA's VERITAS, which will orbit Venus at altitudes down to ~ 180 km, where drag from the upper atmosphere plays a major dynamical role. For this purpose, pyRTX employs high-fidelity ray tracing techniques to evaluate the cross section area in the spacecraft velocity direction, ensuring accurate force computation.

6.4 Application to the Lunar Reconnaissance Orbiter

Since 2009, the Lunar Reconnaissance Orbiter (LRO) has been mapping the Moon to unprecedented detail, capturing, among others, high-resolution images and altimetric profiles to acquire invaluable datasets for understanding its evolution. Transforming this wealth of data into detailed maps and terrain models depends on the accurate determination of the spacecraft trajectory. Furthermore, the OD of LRO can allow scientists to refine estimates of the Moon's geophysical parameters (e.g., gravity field, tidal response, topography), advancing our understanding of its internal structure and

history (Goossens et al., 2024; Mazarico, Barker, et al., 2014). An incorrect modelling of non-gravitational effects can introduce errors in OD that build up over time (Floberghagen et al., 1999), leading to biases in scientific measurements and potentially resulting in incorrect interpretations of the Moon's geophysical properties.

Although the S-band tracking data of LRO is relatively noisy, which may partially hide the improvements gained through a refined modelling of NGAs, the Moon nevertheless provides an excellent test case for testing the ray tracing approach for several reasons. First, the lunar gravity field is exceptionally well characterized thanks to the GRAIL mission (Goossens et al., 2020; Park et al., 2025), which removes the need to correct for the gravity field coefficients and allows to focus mainly on NGAs. Second, high-resolution digital elevation models at different spatial resolutions make it possible to evaluate how the representation of lunar topography affects the modelling of planetary radiation (see Figure 6.8). Finally, LRO experienced long intervals without the performance of any thrusted maneuver, which perturb the spacecraft dynamics making the trajectory reconstruction more complex; it thus provides a rare extended unperturbed radiometric dataset (Table 6.1). Validating the software in this favourable scenario thus provides confidence in its application to other missions (e.g. BepiColombo and VERITAS), where similar techniques can be employed.

Batch	Timespan
#1	02 May 2023 00:00 to 07 Aug 2023 08:30
#2	08 Feb 2024 16:00 to 10 Jul 2024 12:00

Table 6.1. Two long batches of radiometric data of LRO not affected by wheel-off loading maneuvers.

To mitigate errors introduced by mismodelling, a multi-arc approach is typically employed within the OD (Section 2.2), dividing the mission timeline into shorter arcs. The standard length of the arcs for LRO is 2.5 days (Cascioli et al., 2025; Mazarico et al., 2018). However, this approach reduces sensitivity to long-term signals, such as those originating from the Moon’s interior (Park et al., 2025; Wieczorek et al., 2013; J. G. Williams, 2007)

and topography (Barker et al., 2025). As an example, Figure 6.11 shows the recovery uncertainty of the time-variable gravity coefficients induced by the lunar inner core, when the data set of Table 6.1 is processed with different arc durations. The gravitational signal of the lunar inner core can be simplified as $\Delta C_{21} = C_{21C} \cos(\omega t) + C_{21S} \sin(\omega t)$, $\Delta S_{21} = S_{21C} \cos(\omega t) + S_{21S} \sin(\omega t)$, see Equation 1 of J. G. Williams (2007). As can be seen, extending the arc duration improves the sensitivity of the long-term signals by factors of 3-10 $\times$. By dividing the data set with the typical 2.5-day arcs, $C_{21} \sin(\omega t)$ is determined to $\sim 10^{-10}$, while with 30-day arcs it is reduced to $\sim 10^{-11}$. An improved spacecraft dynamical model allows to extend the duration of the arcs, ensuring a continuous estimated trajectory, and thus potentially enabling the recovery of previously undetected signals and a better understanding of the Moon's interior properties. As shown by J. G. Williams (2007), the detection of a lunar inner core of 190 or 300 km radius would require the determination of these degree-2 time-variable gravity coefficients to 10^{-11} or 10^{-10} , respectively. Thus, with several unbroken LRO arcs of 30 days or longer, it may become possible to constrain the lunar inner core size. This study shows the first steps in reaching this goal.

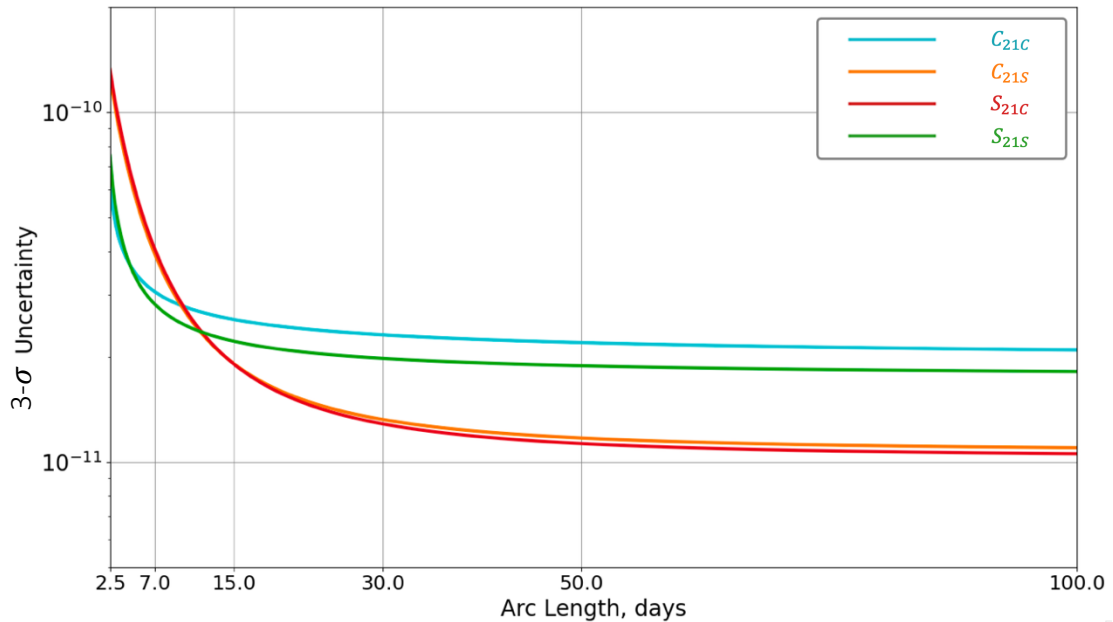


Figure 6.11. 3- σ uncertainty on the amplitudes of the periodic signal induced by the Moon solid inner core as a function of the arc length. The total amount of data is always the same and reported in Table 6.1. By processing the data set with the typical 2.5-day arcs, $C_{21} \sin(\omega t)$ is determined to $\sim 10^{-10}$, while with 30-day arcs it is reduced to $\sim 10^{-11}$.

This section analyses a 30-day tracking arc from 08 February to 09 March 2024. The aim is to evaluate the performance of the precise NGA modelling technique discussed in this chapter by directly comparing it with the standard plate-model approach. This case study provides a validation of the dedicated pyRTX software package, developed to support the required numerical calculations. To this end, two independent OD solutions are carried out using the Jet Propulsion Laboratory’s MONTE software (Evans et al., 2018). In the first solution, NGAs computed with the pyRTX tool are ingested as tabulated accelerations in the LRO dynamical model. In the second, NGAs are calculated using MONTE's built-in models based on the classical plate formulation.

6.4.1 Data and calibrations

The OD of LRO relies on radiometric Doppler and range measurements acquired primarily in S-band (2.1 GHz uplink / 2.3 GHz downlink). Although the spacecraft has a Ka-band downlink, it is reserved for telemetry and science data, and all tracking data are archived at the PDS Geosciences Node at 5 seconds integration time (*LRO RS DATA*, 2025). In this work, we process only two-way S-band Doppler data collected between 08 February and 09 March 2024 in a single arc. This is the first month of the second long batch unaffected by wheel-off loading maneuvers (Table 6.1).

The primary tracking station is the White Sands station in New Mexico, which observes LRO daily for 8–10 hours. Despite the inherent limitations of S-band, White Sands delivers good-quality range-rate data (0.2–0.3 mm/s), significantly better than the original mission requirements. Its continuous support enables consistent monitoring of LRO OD performance. Additional support is provided by the commercial Universal Space Network (USN) stations in Hawaii, Australia and Europe and NASA’s DSN. USN Doppler data are slightly noisier (0.5–0.7 mm/s) and affected by larger timing biases, but remain well within mission requirements (Mazarico et al., 2018).

Given most of the tracking for LRO, especially during its primary mission and first extended missions, was provided by USN, typical NASA’s Deep Space Network (DSN)

calibration files for ionosphere and troposphere are not available. For this reason, the LRO LOLA team, responsible for the precise OD of the probe, developed in-house algorithms to produce calibration products. The ionospheric calibrations are calculated based on a global ionospheric model, which relies on GPS data, maintained and distributed by the International GNSS Service (Teunissen & Montenbruck, 2017). The tropospheric delay calibrations are based on numerical weather models, enhanced by local weather readings at the ground stations (distributed together with the tracking data). The tropospheric calibrations are computed by ray tracing through the numerical weather model using the RADIATE software developed and maintained by the University of Wien (Hofmeister & Böhm, 2017; *VMF Data Server*, 2024).

6.4.2 Spacecraft model

An accurate modelling of NGAs requires a detailed representation of the spacecraft geometry. LRO is a 3-axis stabilized spacecraft designed to provide near continuous nadir viewing for its seven science instruments. It is asymmetrical, with a single solar array pointed sunward and a 2-axis steerable high-gain antenna (HGA). In this study we compare two modelling strategies: the traditional plate model, widely used in OD, with the novel approach based on ray tracing techniques.

Adopting the plate model, LRO is represented by a set of 2D elements: the spacecraft bus is modelled as six flat plates, the solar array as a double-sided surface with a total area of 11 m², and the HGA is approximated as a 40 cm radius dish. Secondary surfaces are neglected due to their minimal contribution to the overall acceleration. Thermo-optical properties for each surface, in both visible and infrared wavelengths, are set to reference values from the original mission design. To perform ray tracing, a realistic 3D engineering shape is adopted, available on (*NASA Science - 3D Resources*, 2025) and shown in Figure 6.12.

The surfaces exposed to solar and planetary radiation depend on the time-varying orientation of the bus and solar panels. Inaccurate modelling of the spacecraft attitude

can introduce time-dependent modelling errors that degrade OD accuracy. To mitigate this, we use the latest LRO SPICE kernels (*LRO SPICE Data Archive*, 2025), which provide precise attitude data for the solar array, the steerable HGA, and the spacecraft body-fixed frame. The 2-axis steerable HGA position is modelled in the spacecraft body frame, while the motion of the center of mass accounts for the attitude variations of the solar array, the HGA, and all other moving appendages.

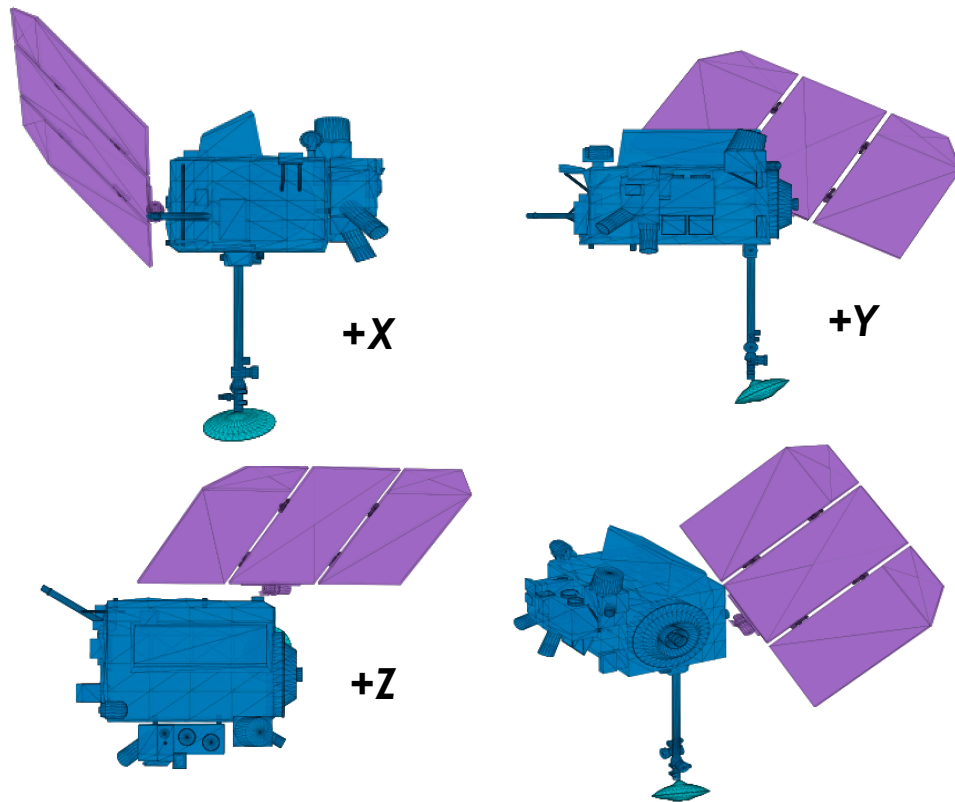


Figure 6.12. Detailed 3D model of LRO.

6.4.3 Dynamical model

The accuracy of lunar OD depends strongly on the gravity field model and the degree to which it is expanded. Prior to the GRAIL mission, our understanding of the Moon's gravity field was limited due to the lack of accurate tracking data from the far side. The GRAIL mission (Zuber et al., 2013) dramatically improved this situation by providing high-precision intersatellite tracking data, leading to gravity field solutions with

unprecedented resolution. The final GRAIL models reach degree and order 1800 (Park et al., 2025), resolving features down to ~ 3 km, and include key geophysical parameters such as the lunar GM, Love numbers, and tidal dissipation. In this work, we adopt the GRGM1200B model (Goossens et al., 2020), truncated at degree 400. Solid body tides are modelled using the associated Love number $k_2 = 0.0239$. While higher-degree expansions generally yield better results, previous studies have shown that for LRO OD, using degrees above 250–300 offers diminishing returns (Mazarico et al., 2018), as non-gravitational and other modelling errors begin to dominate.

Our dynamical models accounts for all the relevant NGAs, including SRP, albedo and IR. The albedo and temperature maps for the moon are shown in Figure 6.13. The lunar albedo map used in this study is derived from LOLA measurements at 1,064 nm, which were acquired at zero phase angle, minimising topographic shadowing artifacts (Lucey et al., 2014). The map is scaled with a factor of 0.39 to obtain the bolometric Bond albedo (Feng et al., 2020). For thermal infrared acceleration, an emissivity map is required, derived from nearly six years of calibrated radiance data collected by the Diviner instrument on LRO (J.-P. Williams et al., 2017). This dataset provides global surface temperature measurements at a resolution of 0.5° , which are converted into emissivity using equation Eq. (3.32). Both the albedo and emissivity maps are expanded into spherical harmonic series up to degree and order 10 for use within MONTE. Tests with higher and lower truncation levels showed negligible impact on the final solution for the LRO case, indicating that degree 10 provides a sufficient representation for this application.

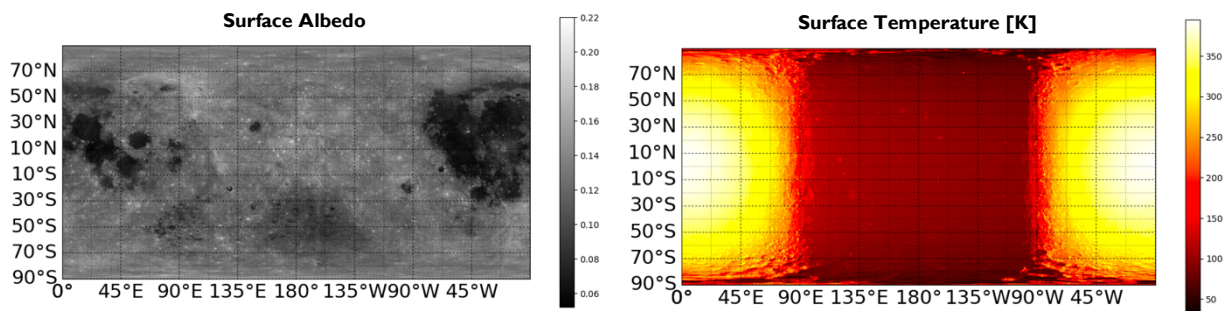


Figure 6.13. Albedo and temperature maps of the Moon.

Our model uses the DE440 JPL Developmental Ephemerides for the planetary positions and lunar orientation model (Park et al., 2021), which provides the updated orientation of the principal axes of the Moon's body-fixed frame.

6.4.4 Models comparison

Figure 6.14 shows a comparison between the SRP acceleration calculated with MONTE built-in methods based on the plate model and with pyRTX.

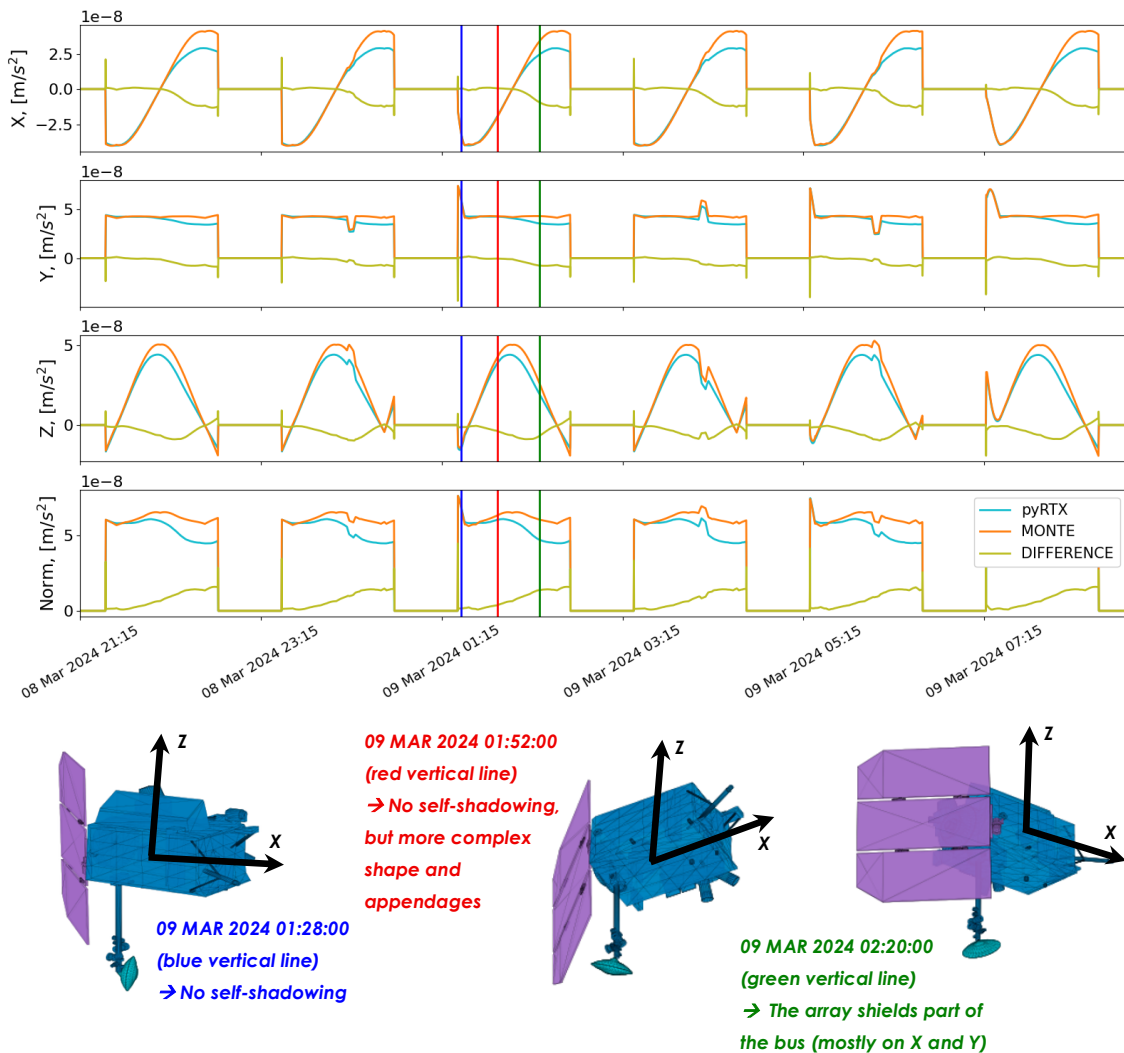


Figure 6.14. SRP acceleration differences in the spacecraft body frame. Three different orientation of the spacecraft are indicated with a blue, red and green vertical bar, corresponding to different self-shadowing configurations.

The accelerations are represented in the spacecraft body-fixed reference frame for a time interval of ~ 12 hours. As can be seen from the attitude of the spacecraft, also shown in Figure 6.14, there is a drop in the pyRTX acceleration when the bus is occulted by the array, demonstrating that the code is able to capture self-shadowing effects. When there is no self-shadowing, the two accelerations match better than 10^{-9} m/s^2 on average. The main discrepancies besides self-shadowing appear as sharp spikes, caused by differences in eclipse-time determination. pyRTX computes penumbra transitions using direct ray tracing and limb-darkening, whereas MONTE relies on a purely geometrical formulation (Moyer, 2003). The two approaches occasionally diverge in resolving the exact eclipse time, producing the observed peaks. The enhanced accuracy of ray tracing, however, comes at the cost of higher computational times, with pyRTX requiring roughly 50 times more than the standard plate model implementation.

The accelerations due to planetary radiation (albedo and infrared) match very closely, as can be seen in Figure 6.15. The differences are on the order of 10^{-9} m/s^2 , mainly due to self-shadowing, the different spacecraft shape and computational approach. Indeed, MONTE uses the spherical harmonics expansion of the emissivity and albedo maps, while pyRTX uses directly the gridded values.

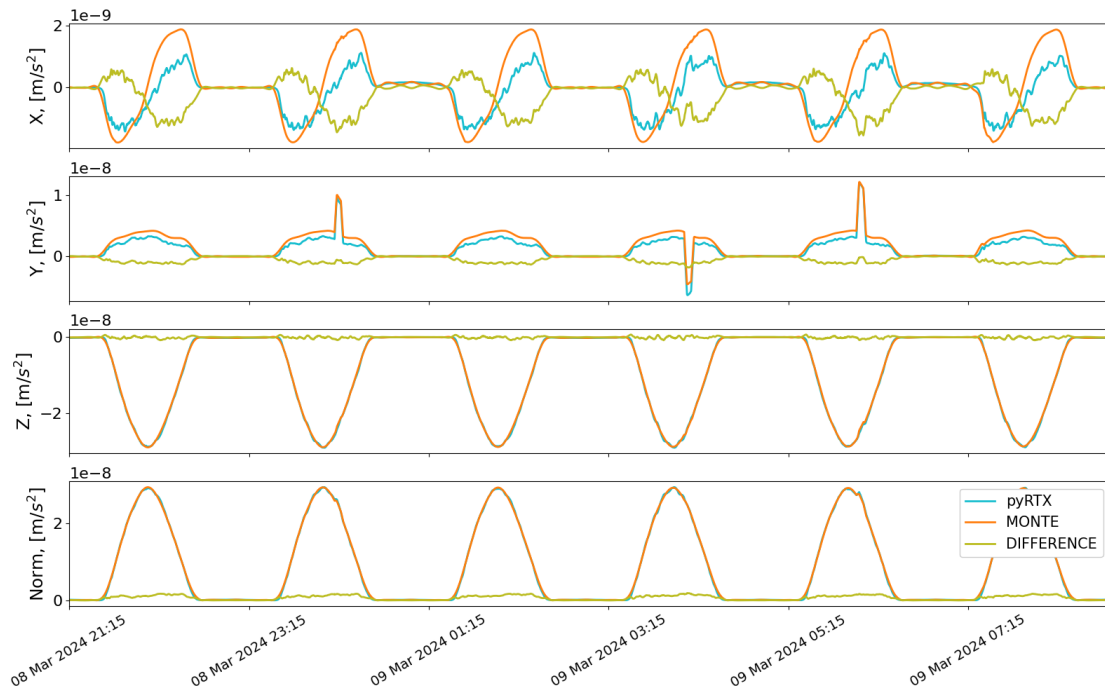


Figure 6.15. Albedo + IR differences in the spacecraft body frame.

6.4.5 Filter setup

The set of estimated parameters with the associated a priori uncertainty is shown in Table 6.2. Since just a single arc is processed, there is no need to distinguish between local and global parameters. This set includes the spacecraft state at the initial epoch of the arc, a station bias for each tracking pass, a scale factor for the SRP and a constant along-track acceleration to absorb residual mismodelling. The scale factor and constant acceleration are updated every two days to reduce the build-up of mismodelling errors over time.

Local parameter	A priori uncertainty
S/C Position (J2000)	300 m
S/C Velocity (J2000)	1 cm/s
SRP scale factor	0.2
Constant along-track acceleration	$8.0 \cdot 10^{-13} \text{ km/s}^2$
2-Way Doppler bias	1 Hz

Table 6.2. Set of estimated parameters.

6.4.6 Results

The post-fit range-rate residuals obtained over the 30-day arc period are shown in Figure 6.16 for both the solution with the standard plate model and that with the ray tracing approach. As can be seen, the pyRTX solution leads to a better fit and a clear improvement in the post-fit range-rate residuals, testified by the lower RMS (Table 6.3). In both solutions, residuals are consistent with the intrinsic noise level of the S-band radiometric link, although the arc duration is 12 times the commonly used arc length of LRO.

A close-up of the final two days, shown in Figure 6.17, highlights a visible reduction in systematic residuals signatures when using pyRTX, indicating an enhanced trajectory reconstruction and dynamical modelling. The improvement is more consistent in the final

days because of large self-shadowing effects (see Figure 6.14) and the build-up of modelling errors during the 30-day arc.

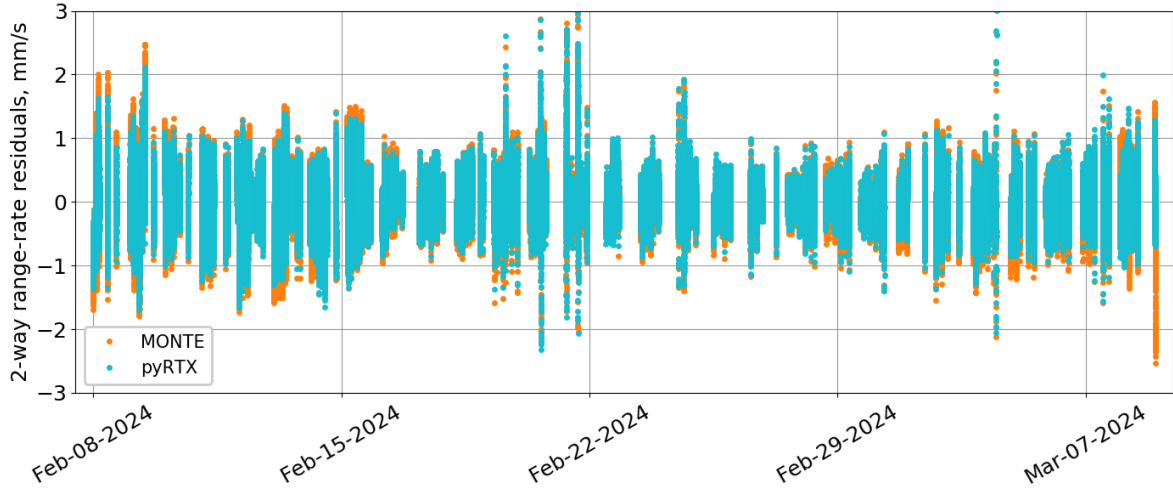


Figure 6.16. MONTE (orange) and pyRTX (cyan) post-fit two-way range-rate residuals.

Table 6.3 summarises the residuals statistics. The pyRTX-based solution reduces the RMS of the residuals by approximately 15% relative to the solution using the plate model. White Sands station is the dominant contributor, with an overall RMS of 0.37 mm/s. Comparable performance is obtained at the Dongara USN station, while the USN Hawaii station shows a slightly higher RMS of 0.43 mm/s.

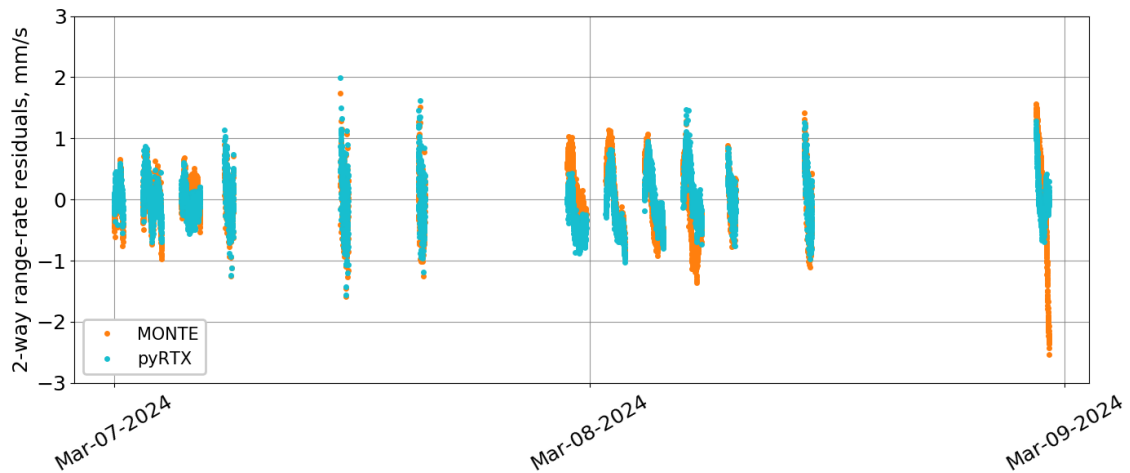


Figure 6.17. MONTE (orange) and pyRTX (cyan) post-fit residuals at the end of the long arc.

	MONTE		pyRTX	
Station	MEAN [mm/s]	RMS [mm/s]	MEAN [mm/s]	RMS [mm/s]
White Sands	-0.008	0.428	0.009	0.368
USN Dongara	-0.029	0.392	0.002	0.371
USN Hawaii	-0.011	0.481	0.021	0.427
Total	-0.010	0.429	0.010	0.373

Table 6.3. Statistics on post-fit two-way range rate residuals.

Inspecting the residuals is particularly insightful, as they directly highlight the improvement achieved through enhanced modelling. However, since the primary goal of OD is to generate and deliver accurately georeferenced scientific products, it is equally important to evaluate how these modelling improvements affect the reconstructed spacecraft trajectory. As can be seen in Figure 6.18, the difference between the two trajectories is at the level of a few meters, demonstrating the good consistency between the solutions.

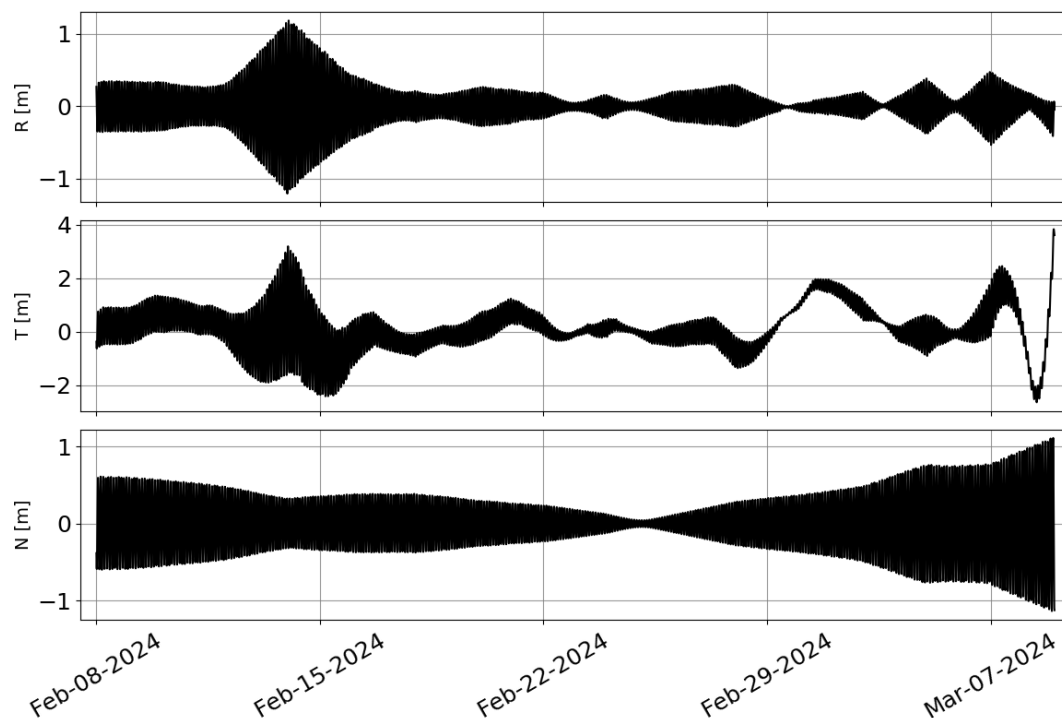


Figure 6.18. Differences in the estimated trajectories when using pyRTX and MONTE acceleration models. The components are shown in the RTN frame.

The estimated dynamical parameters reveal systematic differences. The estimated SRP scale factor averaged over the full 30-day arc is 0.99 for MONTE models and 1.07 for pyRTX. More importantly, the empirical along-track acceleration, typically used to absorb dynamical mismodelling, is significantly reduced when using pyRTX, with an average value of $2.1 \cdot 10^{-10} \text{ m/s}^2$, about one third of that obtained with MONTE (see Figure 6.19). This is a good indicator that pyRTX allows to improve the modelling of non-gravitational effects.

In summary, pyRTX provides a measurable improvement in residual statistics and a significant reduction on empirical accelerations, validating the proposed software tool. While the differences are modest due to the relatively noisy S-band tracking data of LRO, the method is expected to yield substantially larger benefits for future missions equipped with higher-precision tracking systems, such as the forthcoming missions to Mercury and Venus.

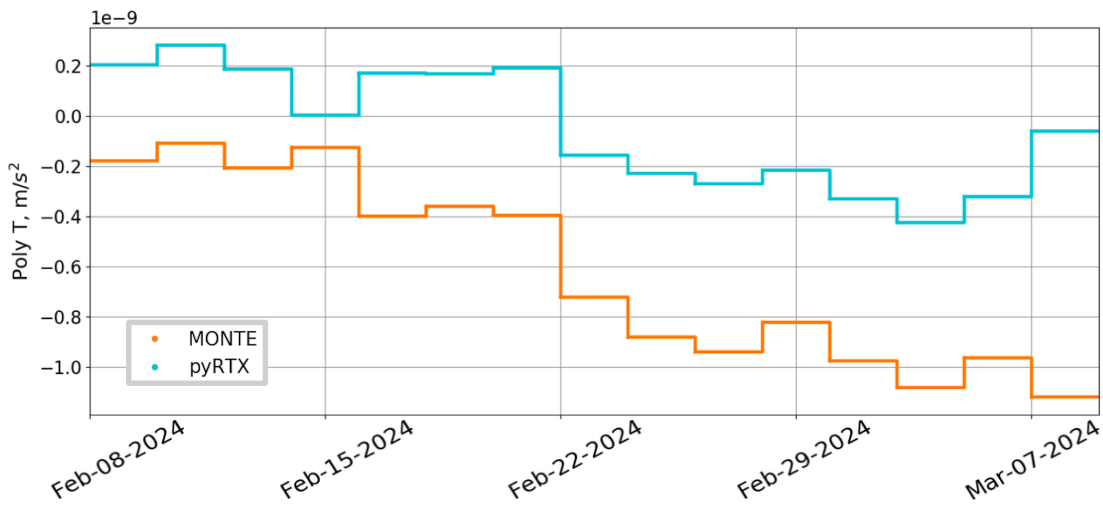


Figure 6.19. MONTE (orange) and pyRTX (cyan) estimated along-track empirical acceleration (updated every 2 days).

6.5 Summary

In this chapter we validated the pyRTX software tool for the precise computation of NGAs using ray tracing. We applied the ray tracing technique on the OD of LRO. By processing a single 30-day arc, we obtained residuals consistent with the expected noise

level of the S-band radiometric link. We compared the ray tracing solution with the one obtained with the traditional plate model approach. By using pyRTX accelerations, we saw a decrease in both the RMS in the residuals and in the magnitude of the along-track empirical accelerations. This is a good indicator that the ray tracing approach allows to improve the modelling of non-gravitational accelerations. The differences between the two trajectories are at the level of 1-5 m, demonstrating a trajectory reconstruction level comparable to standard OD tools. This framework not only helps obtaining an accurate trajectory reconstruction but also provides a means for gaining deeper insights into the Moon's internal dynamics, contributing to a more comprehensive understanding of its geophysical and evolutionary processes. Besides the Moon, our results show that ray tracing is a powerful tool to refine the dynamical models of BepiColombo and the next generation of spacecraft. Further work is ongoing to ameliorate the computational efficiency of the code and to model additional types of NGAs.

Chapter 7

Conclusion

Summary

This thesis contributed to the preparation for the orbital phase of BepiColombo through three complementary efforts: (i) an independent reanalysis of the complete MESSENGER radiometric dataset, (ii) numerical simulations of the MORE radio science experiment, and (iii) the validation of a ray tracing software tool for precise modelling of non-gravitational accelerations.

In Chapter 4, the reanalysis of MESSENGER two-way Doppler data produced independent estimates of Mercury's gravity field, spin axis orientation, and tidal response. The recovered gravity field, tidal Love number and libration amplitude are comparable with previous solutions. The spin pole orientation agrees closely with surface-based measurements, remaining compatible with the Cassini state. Crustal thickness maps reproduce global trends but also reveal important regional variations when accounting for lateral density contrasts. These results show that our solution provides a reliable basis for a future joint analysis with BepiColombo.

In Chapter 5, two numerical simulations of MORE are presented. The first one investigated the possibility to use numerical models for non-gravitational accelerations instead of accelerometer data. Dynamical errors are introduced in the integration of

spacecraft trajectory to reproduce a realistic scenario. Our results show that the OD filter effectively absorbs modelling errors without introducing significant biases, enabling a high-resolution gravity field solution with ~ 180 km spatial scales in the southern hemisphere. Expected accuracies include: a Love number k_2 at the 10^{-3} level, spin axis orientation within 0.5 arcsec, libration amplitude within a few meters at the equator, and obliquity at ~ 0.3 arcsec. Orbit reconstruction is expected at the level of decimeters radially and below ten meters in the along and cross-track directions, degrading with more frequent wheel off-loading.

The second simulation shows the expected performance of a combined analysis of MESSENGER and MORE. Merging the two datasets does not significantly improve global geophysical parameters, but enables exceptional global mapping of the gravity field with gravity anomaly uncertainties on the order of few mGal in the north and below 20 mGal in the south.

Finally, the ray tracing software package pyRTX was validated with LRO radiometric data. By modelling self-shadowing and radiative fluxes with a realistic 3D shape of the spacecraft, pyRTX achieved residuals comparable to nominal S-band noise levels processing a single 30-day arc. The comparison with the plate model shows improvements in the residuals and a reduction of the empirical accelerations, confirming its effectiveness for modelling non-gravitational effects. Trajectory differences remain at the meter level. These results demonstrate that ray tracing provides a powerful approach to refine dynamical modelling for planetary geodesy missions, with clear applicability to BepiColombo, where it may play an important role when the accelerometer data become noisy or are not available.

Overall, this work establishes a solid foundation for the analysis of BepiColombo radio science data. The MESSENGER reanalysis provides the starting point for a combined solution with MORE. The simulations of the MORE experiment confirm the expected performance to achieve its geodesy objectives, and the ray tracing approach offers advanced tools to model non-gravitational accelerations. Together, these efforts pave the way to the success of the MORE investigation of BepiColombo and, in a wider perspective, will hopefully contribute to an improved knowledge of the planet Mercury.

Future works

The results presented in this thesis provide a solid baseline for the analysis of BepiColombo radio science data during the orbital phase. In preparation for Mercury orbit insertion, the procedures developed here will be further refined to ensure reliable and efficient real data processing.

The reanalysis of the MESSENGER dataset will be extended to assess the robustness of the recovered solution. This will include generating additional solutions using different Kaula constraints and increasing the maximum degree of the estimated gravity field, in line with more recent models. These efforts will lead to a more consolidated reference solution suitable for a combined analysis with BepiColombo.

The perturbative simulations of the MORE experiment can be further improved by relaxing simplifying assumptions adopted in this work. In particular, future analyses will consider mismatched physical-numerical models between synthetic data generation and parameter estimation to better represent realistic dynamical modelling errors and unmodelled effects. For example, observables may be generated using ray tracing-based non-gravitational accelerations, while the estimation process relies on the simplified plate model.

Finally, the ray tracing approach validated in this thesis can be applied to BepiColombo using a detailed 3D mesh of the spacecraft. This technique may prove valuable in cases of degraded or unavailable accelerometer data or could also serve as a reference for calibrating the instrument. Further optimisation of the computational performance of the pyRTX routines will be essential to enable its use within the BepiColombo orbit determination framework.

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