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Public Debt, Inequality and Economic Growth

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INTRODUCTION

In recent decades, economic disparities across the globe have become more marked, leading to growing divisions between and within countries. This division has serious consequences for the stability of societies and goes against the principles of fairness and justice established by many developed countries and by the UN[1].

The research has helped identify the possible causes and effects of these inequalities, but many factors come into play in a complex system. The relationship between inequality and governments is particularly important, as governments are the main actors in the fight against poverty, but they are also the most influenced by it, especially in democratic countries where people can exercise pressure on political choices.

Most studies focus on income distribution, giving little attention to other types of economic inequality, such as wealth inequality, which is of crucial importance. There are very few studies that deal with wealth inequality from an empirical point of view, partly due to the lack of data availability. But recently, institutions such as the Luxembourg Wealth Study[2] and the World Inequality Database[3] have taken steps to make such data easily accessible and comparable.

The problem with wealth is that it is formed over time and can suffer from endogeneity in empirical analyses. But it proves to be more relevant in defining an individual's well-being through its multiple functions such as an emergency resource fund, a mechanism of consumption distribution along the lifespan, and a

[1] https://www.un.org/sustainabledevelopment/poverty/
[2] https://www.lisdatacenter.org/our-data/lws-database/
[3] https://wid.world/

legacy for future generations.

Moreover, income cannot be considered as a proxy for wealth, as recent studies have shown that the correlation between the two phenomena is only partial. Therefore, transposing the results of one form of inequality onto another may lead to wrong conclusions if not first supported by an adequate empirical analysis.

Another phenomenon that is always much discussed is public debt. In recent years, interest has risen again following new peaks reached due to global crises such as the COVID-19 pandemic and the emergence of conflicts in crucial economic areas. However, a study of the trends has noted that the levels globally as well as locally have grown even in periods of relative stability. In this case, too, the literature has studied the possible determinants of this growth and numerous empirical works have suggested a contribution of income inequality to this growth.

In democratic countries, high levels of inequality can push governments to implement more pro-poor policies. However, they are also pressured by an economically relevant segment of the population to finance such policies with public debt rather than adjusting the tax system.

The empirical analyses on public debt are limited to dealing only with income inequality. The link between income and public debt is narrower, as income contributes to the latter directly only through the interest paid on government bonds. Debt, on the other hand, contributes to changing the wealth of an individual through changes in the value and interest paid on securities. The interest rate can also influence other aspects of the economic system such as mortgage rates and inflation. Therefore, there is a need for a study on the interdependence between public debt and inequality.

Dealing with relationships between economic variables involves numerous difficulties due to the intrinsic characteristics of time series. Most of the economic variables are non-stationary, leading to the failure of many fundamental assumptions for the estimation of econometric models. Another problem is the endogenous nature of the variables which produces distortion in the estimates, leading to incorrect conclusions.

A very useful tool in such a situation is the multivariate model capable of capturing a system as a whole and therefore solving the endogeneity problem. The Vector Autoregressive (VAR) model is fundamental, a multivariate version of the autoregressive model for univariate variables. This model does not resolve the non-stationarity of the data. A possible solution is to move to the first differences, but this would lose important information on the long-run relationship of the variables. For this reason, it is common to use the error correction model, an evolution of the differentiated VAR model in that an error term is added to the latter as an exogenous variable that describes deviations from one or more long-term equilibria. Each equilibrium is expressed in the form of a linear regression whose estimated residuals are used as a proxy for the deviations. Although many paid software products provide numerous specific tools for the analysis of this model, the R software lacks an adequate package capable of adequately estimating the model and subsequently implementing adequate analyses of model robustness and causality between the variables of interest

Furthermore, a crucial choice in the study of inequality is the measure chosen to capture it. Empirical works mainly rely on a single statistic to capture the entire profile of inequality, such as the Gini index or ratios between percentiles. However, it has been proven that such indices calculated on different distributions can produce the same results. Thus, important information is lost - the shape of the distribution. Inequalities within narrow sections of the general distribution can interact differently on the economic variable of interest, even producing new results of great interest.

The aims of this project are multiple and heterogeneous. Mainly it aims to contribute to the state of the art of literature regarding the relationship of wealth inequality within the economic system, while simultaneously providing a useful tool for studying internal relationships within the economic system. The project is built as follows. Chapter 1 describes the state of the art of analysing the relationship between public debt and inequality. Chapter 2 describes the R package built for estimating the VEC model. 3 applies the model described in the previous chapter to study the interrelation between public debt and wealth inequality. Chapter 4 makes a study of alternative inequality measures capable of capturing the entire distribution profile. The final chapter draws the conclusions.

CHAPTER

1 WEALTH INEQUALITY AND PUBLIC DEBT: A LITERATURE REVIEW

1.1 Introduction

There has been a long-standing interest among policymakers and researchers in the extent of the impact of economic inequality. Recent economic research has delved into the reciprocal relationship between income disparity and various economic variables, examining how inequality affects these variables and how these variables, in turn, impact inequality. While a focus on income is reasonable, there has been recently increased interest in wealth inequality. Most of the findings related to income are now discussed in terms of wealth. Many key mechanisms may not hold or may even be reversed.

This is the case of the link between public debt and inequality, a topic treated in empirical analysis from an income perspective, but not from a wealth perspective. This new interest requires a systematic review of the previous theoretical literature to identify old and new transmission mechanisms and detect possible critical issues arising from wealth inequality studies.

This chapter examines recent literature on the connections between public debt and wealth inequality, outlining various theoretical models that link debt to inequality in both income and wealth and vice versa. We begin by describing the trends of the two variables over the past decades based on international reports and work carried out in the literature. Public debt and economic inequality have increased in recent decades, alternating only briefly with moments of fragile stability.

Next, a brief literature review is conducted to identify possible determinants and impacts of public debt and inequality. What came out is the endogenous nature of both variables within the economic system. The literature on public debt and inequality is extensive, yet their ascendance in recent decades has intensified scholarly interest. In the context of inequality, it is not simply seen as an outcome but also as a key factor influencing many economic processes.

We present an overview of the existing theories concerning the relationship between the two variables of interest. The research highlights a bidirectional relationship between public debt and inequality, where each variable positively reinforces the other, leading to an upward spiral of both variables. The study of the link between debt and inequality focuses on wealth. It is based on the discussed theory on considering government bonds as net worth (Modigliani, 1961; Tobin, 1987; Barro, 1974; Diamond, 1965; Mankiw, 2000). The opposite link focuses on the impact of income inequality on public debt, based on the pressure of the poor on governments for redistributive policies and the opposition of the rich to increase tax burden (Meltzer and Richard, 1981; Dixit and Londregan, 1996).

A review of the empirical literature reveals models that examine the impact of income inequality on public debt and provide evidence of a growth effect resulting from economic disparities, as the theory suggests. Nonetheless, the findings are not consistently significant and may vary depending on the chosen country or panel tested. Additionally, the empirical analysis does not check the validity of the inverse relationship between debt and inequality, and focuses on income inequality, with no analysis made for the wealth distribution. This lack of literature is because data on wealth may not always be available for a wide range of times and in many countries.

In summary, the topic we have discussed is significant to academic research, and we can find a vast amount of theoretical and empirical literature dedicated to its study. However, there is room for improvement in the prior literature with a two-way analysis of wealth inequality, thanks to the great efforts made by scholars who have enhanced the availability of data on this topic.

1.2 Inequality

There has been a well-documented rise in economic inequality within countries across the globe over the past four decades (Chancel et al., 2022). In 2022, the net worth of the top 1 per cent of the global richest population reached 2.7 million euros. The global bottom 50 percent of the population owns only 2% of the total wealth, in contrast with the top 10 which possesses 75%, and the top 1 holds 38%. Latin American countries have the highest concentration of wealth, with the top 10% of the wealthiest population holding approximately 77% of the total wealth.

The disparities are smaller in Europe, where the top 10 account for just below 60% of wealth, while the middle class accounts for almost 40%. However, there is heterogeneity among the European countries. Northern Europe had a much lower level of wealth concentration than Western Europe in the 1980s. The gap has narrowed over time as wealth concentration has increased faster in Northern Europe than in Western Europe. Wealth concentration in Southern Europe was lower than in Western Europe and slightly higher than in Northern Europe in the 1990s and 2000s, but in the 2010s wealth concentration levels have converged with those of Western Europe. In contrast, Eastern Europe emerged as the region with the highest average wealth concentration in the 2010s(Blanchet and Martínez-Toledano, 2023).

In their research, Saez and Zucman (2016, 2020) outlined the progression of wealth concentration in the United States between 1980 and 2020. They found that the richest top 10 per cent of the population owned about 77-78 per cent of the national wealth in 2018. Their wealth had risen by 10 percentage points since 1989. The concentration of wealth is more pronounced when examining the top 1 percentile, which possesses 38 per cent of the total wealth, indicating a 10-point rise from 1989 to 2018. Moreover, the top 1 wealthiest population owns 38 times the average amount of national wealth. Extending the examination towards the higher end of the distribution, the data for the 0.00025% richest households, as estimated by the Forbes 400 ranking, show even faster growth of net worth than the top 0.1%.

The swift accumulation of wealth over the four decades from the 1980s to 2020 was also supported by faster growth of wealth relative to income and output. Total household wealth increased from three times the national income in 1980 to almost six times in 2020, implying that the former grew almost twice as fast as the latter. The primary drivers of wealth accumulation are interest-bearing assets, which account for almost a quarter of net wealth for the top 1 percentile, and pension assets for the top 10 percentile.

The faster growth of wealth relative to income is at similar levels and trends in the most advanced economies (Piketty and Zucman, 2014). Although, wealth inequality has grown faster in the US than in the European countries. The ratio of wealth to income is slightly higher in Europe than in the US, due to the decline in wealth in the US as a result of the financial crisis and the subsequent fall in house prices. Moreover, although inequality has risen in both the US and the European countries, the increase in the latter has been more moderate and more related to price effects, as opposed to the predominant volume effects in the US (Blanchet and Martínez-Toledano, 2023).

The concern over the increase of economic disparities has inspired research contributions to seek the determinants of inequality. Income and wealth inequalities are closely linked and positively correlated. This is because part of income is often saved, which in turn increases wealth. Additionally, wealth can generate income through returns. Still, the correlation is only partial as proved by empirical analysis (Berman et al., 2016). The differentiation between the two is of vital relevance since influencing one may not necessarily impact the other, and vice versa. Recent studies have expanded knowledge on wealth inequality trends in the US (Saez and Zucman, 2016) and Europe (Blanchet and Martínez-Toledano, 2023), allowing us to perform new research even for the distribution of net worth.

While income measures the flow of financial resources available to a person or household in a particular period, wealth is a cumulative measure of an individual or household's assets (Killewald et al., 2017) that may play a role in the lives of individuals, such as by facilitating household consumption smoothing during periods of economic insecurity (Davies et al., 2006), providing a source of retirement income or even creating a bequest to future generations. For this reason, wealth better reflects the social and economic differences within a society (Berman et al., 2016; Keister and Moller, 2000). The distribution of household wealth is even found to be significantly more uneven than the distribution of income in many rich countries Chancel et al. (2022).

To understand the determinants of wealth inequality, we must first identify the mechanisms of wealth accumulation. Berman et al. (2016), implementing an evolution of the decomposition used by Piketty and Zucman (2014) between capital gains - change of and volumes. They identify two main sources of wealth decomposition, that is, savings from post-tax labour and capital income; and the change in the value of capital.

Based on the work of Blanco et al. (2021), Blanchet and Martínez-Toledano (2023) implement a further decomposition in which the two driving forces of wealth accumulation, the change in asset prices and savings, are differentiated by asset class, namely, net housing, business assets, and financial assets. This decomposition provides a better understanding of the link between accumulation and wealth inequality.

The reason for differences in household portfolios across the wealth distribution is due to their composition and leverage. Wealthy households tend to have portfolios dominated by business and financial assets, while middle and poor households have portfolios with high leverage and concentration in residential real estate. The heterogeneity of portfolios across the wealth distribution is persistent, as confirmed by research.

Changes in equity and house prices have different effects along the distribution.

Housing booms result in significant gains for leveraged middle and bottom-class households and generally decrease wealth inequality, assuming all else is equal. Stock market booms primarily increase the wealth of households at the top of the wealth distribution, whose portfolios consist mostly of listed and unlisted business equity. The heterogeneity of portfolios creates competition between the housing market and the stock market in shaping the distribution of wealth.

Kuhn et al. (2020) demonstrate that from 1971 to 2007 the wealth of the bottom 50% grew almost entirely through price effects. Price-induced wealth growth is high for the bottom 50%, despite below-average home-ownership rates, because virtually all of this group's existing wealth is invested in highly leveraged housing wealth.

Blanchet and Martínez-Toledano (2023) show that volume effects were greater in the US, primarily because of stronger growth in business investments. Capital gains were more pronounced in Europe due to rapid price growth since the early 2000s and a less severe drop in real estate values during the global financial crisis. In contrast, financial assets saw greater value appreciation in the US.

In literature, we can identify three main determinants of wealth concentration (Kuhn et al., 2020; Blanchet and Martínez-Toledano, 2023), that is, labour income inequality, saving rate inequality, and capital gain inequality. While many countries around the globe have experienced one force more than the others, in the US the three determinants have impacted wealth concentration in equal measure.

High levels of savings allow for middle-class households to narrow their net-worth differences with the richest ones, but only if income(and saving as well) and wealth are not identically distributed and highly correlated. Since the 1980s, wealth inequality has been steadily rising in most developed countries. This increase has been accompanied by a decline in personal savings, like in the US and Japan.

Capital gains inequality is closely linked to the differences in portfolio composition across the wealth distribution, which in turn reflect the varying price changes of different assets.

This is a summary of the factors that may affect wealth inequality. However, in this last period, economic inequality is no longer described just as a simple outcome, but even as a key mechanism to explain social and economic dynamics. Once more, previous research has primarily focused on the effects of income inequality due to the longstanding disposable of data on income compared to wealth.

Numerous studies have established a correlation between income inequality and various issues, including health and social problems as well as economic growth. In their literature review, Rowlingson (2011) and Pickett and Wilkinson (2015) found evidence that a decrease in individual income or an increase in income inequality generally leads to an increase in health and social problems. Fajnzylber et al. (2002) analysed the determinants of national crime rates both across countries and over time by constructing a panel dataset of intentional homicide and robbery rates for a sample of developed and developing countries covering the period 1970-1994. The study demonstrates that crime rates increase with rising income inequality.

The empirical analysis provided by Atems and Jones (2015) suggests that shocks to various measures of income inequality significantly reduce the level of per capita income, although historical events may influence the relationship between the two variables.

The evidence presented underscores the significant and escalating issue of economic inequality globally and within specific regions. Understanding the intricate mechanisms of wealth accumulation, including the interplay between capital gains, savings, and income, is vital in addressing this complex issue. Wealth may profoundly impact socio-economic outcomes more than income. As we move forward, efforts to mitigate wealth inequality and its negative repercussions require a comprehensive and multi-faceted approach.

1.3 Public Debt

The recent global events have considerably increased the amount of public debt on a global scale. The financial crisis that began in 2007 and exacerbated in 2008, along with the consequent economic recession, had a very serious negative impact on the public finances of most advanced economies. As a result, several countries observed a substantial rise in debt ratio, ultimately culminating in the European sovereign debt crisis in 2011.

From 2007 to 2011, the average debt ratio in the Euro area increased from 66% in 2007 to nearly 87% in 2011. Ireland experienced the largest increase at almost 90 percentage points. During the same period, the United States' debt ratio increased from just over 60% to almost 100% of GDP. Emerging and developing countries with low public debt ratios at the start of the crisis had not seen a significant rise in their debt ratios.

In recent years, debt ratios have sharply risen worldwide during the COVID-19 pandemic, to the extent that the global level exceeded 100% in 2020. However, between 2020 and 2022, the world's public debt decreased by almost ten percentage points, dropping from 100% to 92%. This reduction can be attributed to a robust expansion of real GDP, inflation shocks, and the reversal of fiscal support measures relating to the impact of the COVID-19 pandemic. Nonetheless, global public debt is expected to increase again from 2023 onwards (IMF, 2020).

As in the previous century, the sharp rise of public debt led to the question of how it may impact the economic system. In the realm of economics and finance, the topic of public debt has been a subject of extensive analysis and discussion for many years. Scholars have delved into the complexities of public debt, exploring its implications, consequences, and management strategies. This body of literature offers valuable insights into the dynamics of government borrowing, fiscal policy, and their impact on both national and global economies. In this section will be given only a glimpse of this wide literature.

Starting from its definition, Government debt is the financial contractual obligation of the central government to repay creditors at a future date, including both the principal amount and accumulated interest (IMF, 2003). Government liabilities predominantly constitute public debt and consist mostly of debt securities and loans. It is vital to categorize public debt into two primary types: domestic debt and foreign debt. The debt classification depends on the residency of the debt holders, the currency in which the debt is denominated, and whether the debt was issued on the international or domestic debt market (Elmendorf and Mankiw, 1999). However, the two types of public debt affect the economy differently: external debt places a burden on the community by requiring transfers from domestic debtors to foreign lenders; domestic debt entails a transfer of wealth from taxpayers to property owners, resulting in no net loss within the borders, but with a possible impact on wealth distribution over time.

Debt is a crucial tool in developmental and fiscal stabilization policies. It acts as a replacement for tax financing government expenditure. The attractiveness of debt as a financing mechanism lies in its capacity to delay taxes and distribute their obligation across several years and generations while offering a well-balanced approach to the funding of crucial public projects.

Debt financing also aids in stabilizing output, enabling governments to adopt counter-cyclical fiscal policies, such as tax cuts in times of economic recession. This, in turn, enhances the ability to curtail the negative effects of economic downturns, delivering much-needed relief to businesses and individuals in times of need. By avoiding excessive distortionary effects produced by tax increases during recessions, debt financing can even promote economic recovery and growth, thereby reducing the negative impact and duration of crises(Barro, 1974).

Debt financing should respect the principle of inter-generational equity (Catrina, 2013). The assumption is that future generations should contribute to the costs of capital accumulation, infrastructure development, and education. By spreading the financial burden across generations, this approach aims to ensure fairness and minimize the immediate financial pressure on current taxpayers. However, it also raises questions about the extent to which future generations should be held accountable for decisions made by contemporaries, particularly in debt accumulation. Political opportunism plays a relevant role in the preference for debt financing. Shifting the tax burden to future generations may have short-term electoral benefits given that future generations cannot vote yet.

Indeed, opting for public debt instead of adjusting the tax system helps avoid distributional conflicts. While the poor press for a stronger welfare system, the rich want a reduction of the tax burden. Hence, debt financing may reduce the social and political tensions that often arise when certain groups feel disproportionately burdened by taxation or when their economic support from the government is drastically reduced.

In addition, it was much discussed in literature about the inflationary pressure that a constant and growing indebtedness produces, regardless of the policies followed by the central bank (Sims, 1980). Studying the impact between public debt and inflation means studying the interaction between monetary and fiscal policy. Two opposing views can be identified. The Ricardian view sees fiscal policy as passive, supporting the assumption that government bonds are not net wealth (Barro, 1974), and monetary policy works through interest rates to determine prices.

The second view is defined in a non-Ricardian environment where the price level is a function only of fiscal policy variables. According to Nastansky and Strohe (2015), the relationship between public debt can be either direct or indirect. In the first case, the central bank buys public bonds, in the second, demand derives from the private and banking sectors, as well as from inflationary expectations of economic agents arising from high levels of public debt. According to Kwon et al. (2009), the two transmission mechanisms by which public debt impacts inflation are monetization and the wealth effect.

Empirical findings on the relationship between public debt and inflation are mixed in many countries. Wheeler (2004) finds that government debt in the US affects not only prices but also interest rates and output. Taghavi (2001), who has investigated the relationships between public debt, growth and inflation in Germany, Italy, France and the UK, suggests that debt influences inflation in the long run.

Other articles did not find evidence of a significant relationship between the two variables as in the case of Janssen et al. (2002) who focused on the UK with a sample spanning from 1702 to 1996. Similar results are found in Castro's work where the interdependence between fiscal and monetary policies is examined. The paper found no evidence of a significant role of government debt in determining price levels in the OECD countries studied.

Kwon et al. (2009) suggests that debt affects price levels if debt levels are high. Karakaplan (2009) tested the hypothesis that external debt is less inflationary in a well-developed financial market.

The literature supports that public debt affects the economic system as in the case of economic growth and inflation. The results on the significance of these effects as well as on the direction of these relationships do not always agree. However, seems that the advantages of debt financing come with trade-offs. While it allows the burden of financing to be shared over many generations, it also reduces capital accumulation and future income. High levels of debt can place a significant financial burden on future generations, potentially limiting their economic opportunities and prosperity. Moreover, debt can be a powerful tool for stimulating the economy, particularly during recessions, boosting growth and employment, and improving well-being for the less affluent segments of society. However, accumulating high levels of debt can increase the risk of a fiscal crisis which can lead to economic turmoil and harm the poor.

So what are the conditions that drive governments to finance their spending through public debt under the majority rule? As explained before, one possible motivation is to avoid possible conflicts in a society with high economic disparities. Financing public expenditure through debt allows the demands of the population to be met without the loss of popularity of the party or government.

Cukierman and Meltzer (1989) suggest that in a society where some voters are bequest-constrained, a replacement of taxes by debt that preserves present value increases the consumption of bequest-bound individuals. By voting for deficits, they increase their consumption, crowding out capital, but reducing the severity of the bequest constraint.

These can be seen as the main reasons for increasing levels of public debt in the context of the topic of this literature search, but more can be identified. For example, population ageing is likely to contribute to that deterioration by putting additional pressure on public expenditure on health care and pensions (Nautet and Van Meensel, 2011).

1.4 Theoretical framework

Influential theoretical literature explores the connection between national debt levels and inequality. Inequality might impact public debt, leading to consequential outcomes that could influence the government's expenditure decisions due to voter pressure. Alternatively, issuing bonds might augment households' wealth, thus altering inequality based on the quantity and distribution of this newfound wealth among the populace. The effects of taxation can either be mitigated or heightened depending on the type of taxation system adopted by the government.

The literature yields a substantial number of studies investigating the relationship between public debt and inequality, precisely the inter-dependency of these two factors in either direction. Interdependence is a topic that is seldom explored, resulting in the existing theoretical literature being divided into two distinct branches, each focusing on a specific unidirectional relationship: the effect of public debt on inequality and the influence of inequality on public debt.

Finally, numerous studies have examined the topic empirically, yet research has often emphasized a particular one-way relationship, neglecting the alternative and potential interdependent factors. The methodologies adopted may be characterized by possible weaknesses.

1.4.1 Debt as a determinant of inequality

We start by studying debt as a determinant of inequality. The first studies on this topic were carried out by Barro (1974) and Diamond (1965): even if they did not study directly the relationships between debt and inequality, they provided a study on the effects of public debt on fiscal policies overtimes with their models of overlapping generations of finite agents.

The work of Barro (1974), based on the Ricardian equivalence, suggests that public debt is completely neutral in the economy: fiscal policy redistributes tax burden among generations through the issue of bonds and increasing debt stock, but families, that have a higher income due to less taxes, do not spend these new resources expecting to repay this debt in the future, and so they smooth consumption for all their lifespan and over, taking care of next generations too through bequests. This process brings a reversing of the positive effects of the initial redistribution policy.

Of opposite thinking, we have the study of Diamond (1965) which developed a new model and pointed out that people smooth consumption over their lifetimes to cover themselves against possible increases in taxes, but they do not leave any bequest to the next generations. Therefore, following the author's model, national debt issued has an impact over time, enriching the actual generation at the expense of the next ones, reducing private investment and steady-state living standards, and producing a possible effect on wealth distribution on long-horizons.

In his critique of the works of Barro (1974) and Diamond (1965), Mankiw (2000) questions the strong assumption of homogeneity of consumer behaviour. In his paper on the "savers-spenders" theory. Michel and Pestieau (1998) and Mankiw (2000) reported on a new model to interconnect those two views, introducing two types of agents with different time horizons: they identify an altruistic agent with infinite horizons who provide inter-generational transfers to its children, acting as described by Barro (1974), and life cycles agent with finite horizons, consuming all its savings during its life spans and leaving no bequest, as described by Diamond (1965). These different perspectives brought to the existence and the sizes of bequests. As a consequence, some consumers save part of their incomes planning for themselves and their descendants, while others make decisions based only on their life span, leaving no bequests to the next generations. The assumption behind the studies of Mankiw and Michel and Pestieau seem to be more realistic and have been widely used in various subsequent works. Moreover, it provides a better view of the effects of fiscal policies on inequality. It shows that public burrowing redistributes income from the poorer non-altruists to the richer altruists, under the assumption that the entire capital stock and public debt is owned by the rich altruists, and this implies that their wealth increases by exactly the amount of increase in public debt.

Empirical analysis on saving and bequest motives carried out by Dynan et al. (2004), Bozio et al. (2013), Alan et al. (2015), and Gandelman (2017) highlights that the rich have a higher savings rate than the poor, supporting the assumption of a link of poor with non-altruist and rich with the altruists, and the existence of a linkage between bequests and inequality.

Further improvements have been carried out by Maebayashi and Konishi (2019) and Borissov and Kalk (2020). In their paper, Maebayashi and Konishi (2019) have studied the relationship between public debt and inequality in an endogenous growth model with heterogeneous agents. They improve the prior literature taking into consideration not a simple steady value of debt which may result at the end of a convergence process, but its sustainability in a general equilibrium framework. For this purpose, they built a two-period OLG model following the one proposed by Diamond (1965), introducing two groups of agents, rich and poor, with similar characteristics of whom described by Mankiw (2000).

They defined a threshold of public debt that guarantees the government to sustain the fiscal policy. This threshold is positively linked to inequality levels: an increase of the latter would bring a new threshold and a more sustainable debt level. Countries with debt levels under the threshold will see their inequality converge to a stable value, whereas in countries with high debt economic system would be unstable and both public debt and inequality would continue to increase.

However, even in a stable framework, it is suggested a positive dependence of inequality on public debt exists and increments of debt can bring a rise in inequality level. Moreover, countries would be discouraged from reducing inequality, because re-distributive policies would bring an increment of debt and instability in the economic system in exchange for a relatively small decrease in inequality. This is due to higher savings rates of the rich than the poor, that is, taxing rich bequests would bring to a more significant reduction of rich savings than gains of poor ones.

Following the study of Maebayashi and Konishi (2019), Borissov and Kalk (2020) proposed an AK endogenous-growth model. Under the same assumption of public debt financed by distortionary taxes, authors suggest that agents' decisions depend not only on their absolute level of consumption but also on the perception of their social status and, following this idea, they introduce a measure of positional concerns in their model: if these concerns are strong enough poor dynasties can get to consume all their income and more through indebtedness to the point that they become even poorer and leave no bequest to future generations. Maebayashi and Konishi's use of this measure of positional concern can be seen as a threshold that splits the population between altruistic and non-altruistic, overcoming the simplified link between altruism and wealth. An increasing number of papers have studied the implication of this idea for inequality and public policy choice (see Wendner and Goulder (2008), Dioikitopoulos et al. (2019), Alvarez-Cuadrado and Long (2012), Mino and Nakamoto (2016), García-Peñalosa and Turnovsky (2008)).

Theoretical results highlighted by the study of Maebayashi and Konishi (2019) suggest that positional concerns work in favour of inequality, reducing the savings of poor dynasties and making the rich dynasties the only ones to own bonds and capital.

Like in the work of Maebayashi and Konishi (2019), Borissov and Kalk (2020) define a threshold of the debt-to-GDP ratio which works as a crossroad between two different paths to balanced-growth equilibrium: an egalitarian regime for countries with debt ratios below the threshold value and a two-class regime for ratios above. An economy in a two-class regime can increase its growth rate and switch to the egalitarian regime with a long-run reduction in public debt, whereas a policy aimed at reducing inequality by increasing public debt may bring the opposite effect of increasing wealth inequality in the long run.

Hence, after an initial discussion about the existence or less of causality of public debt in inequality, subsequent studies have come to sustain more the former case than the latter. The level of savings and bequest motive have to be considered two of the key mechanisms. Still, another relevant factor to take into consideration, that can strengthen or weaken this link is the initial levels of national debt (Maebayashi and Konishi, 2019; Borissov and Kalk, 2020). High levels of debt can bring opposite effects of re-distributive policies.

1.4.2 The pressure of inequality on debt

The previous section proposes inequality as an outcome of the economic system, based on the nature of debt as a wealth and tax burden transfer over time and generations through the bequest motive of agents. However, literature studies inequality even as a determinant of public debt. In this section, we are going to explore the opposite link that occurs between the two variables.

One of the key mechanisms that link inequality with debt, whether income or wealth, is the choices of fiscal policies. Many authors studying this relationship have focused on the hypothesis of the median voter: hypothesized by Harold (1929) first, the theorem states that in a representative democracy, politicians will converge towards the point of view of the median voter. In a society with high levels of inequality, the preferences of the population would be very heterogeneous due to the different preferences of rich and poor households: the rich population would rather rely on private consumption and will be more in favour of a lower marginal tax rate; instead, the poor population will push for more social transfers and a higher marginal tax rate. The greater the inequality, the more it will tend to increase the power of the poor sections of the population, and therefore the pressure on politicians to adopt re-distributive policies.

These results are based on the works of Meltzer and Richard (1981) and Dixit and Londregan (1996), which suggest that more unequal income distribution leads to greater re-distributive spending. This is further supported by the articles of Alesina and Rodrik (1994) and Persson and Tabellini (1991), which argue that higher inequality increases the demand for re-distributive policies. In times of economic expansion, the increase in income may compensate for most of the social transfers thanks to greater public revenues.

In times of recession, countries with high-income inequality may experience a conflict between groups with different incomes, leading the government to use a greater collection of resources. Tax deficits and public debt are created when earnings are less than expenses, and governments choose to pay social transfers to the poor without taxing the wealthy (Milanovic, 1999).

Azzimonti et al. (2014) and Arawatari and Ono (2017) built models to study the impact of income inequality on debt with cross-country differences in inequality and fiscal policy. The model developed by Azzimonti et al. (2014) is a multi-country model with income risk and incomplete markets. The authors do not focus directly on the link between public debt and inequality but assume that the increase in income inequality is associated with an increase in income risk which, in turn, impacts public debt growth. However, they cannot establish unambiguously whether government debt increases in response to a rise in risk, even with the help of simulations and empirical evidence. Their paper is relevant because it relates public debt and inequality through financial stability and international financial integration.

Arawatari and Ono (2017) focus their paper more directly on the dynamics that occur between public debt and inequality. They improve the multi-country politico-economic model of public debt proposed by Song et al. (2012) introducing a measure of inequality. The findings of the authors, supported by simulations, arrive at the same conclusions of Azzimonti et al. (2014). They suggest that public debt responds positively to income inequality, with the distributional inequity rising within a country that leads to an increase in its public debt and a decrease in the public debt of foreign countries.

Karayalçin and McCollister (2005) show that redistribution policies are partially financed by external loans, which can expose them to an increased risk of default. Kim (2013), who studied the relationship among financial development, economic growth, and income inequality using cross-country panel VAR models, also shows that unequal economies present a higher probability of default.

The work of Kumhof et al. (2015) supports empirically how an increasing inequality can bring an excessive private debt of lower-income classes and a possible burst of a financial crisis. The government may use public debt to crowd out private capital to stabilize markets. Although, some authors connect inequality with sovereign debt defaults.

The theoretical literature suggests the existence of an interrelationship between public debt and inequality due to various mechanisms of the economic system like policy decisions, voter pressure, bequests, and cross-countries differences.

1.5 Empirical evidences

While the theoretical literature examines the relationship between public debt and both income and wealth inequality in both directions, the empirical literature is limited to a one-way analysis and focuses solely on income inequality.

Within the theoretical study of Azzimonti et al. (2014) can be found an empirical application. Using a fixed effect regression equation, the authors analyze the relationship between the growth rate of real government debt and changes in capital mobility and income inequality. Income inequality is measured using the share of income earned by the top 1% and the averages of gross Gini coefficients. The analysis is based on a panel of 22 OECD countries and encompasses data from 1995 to 2010. The results support the hypothesis that income inequality leads to an increase in public debt. The study focus is on the short-run dynamics, without providing any insight into a possible long-run relationship.

The study conducted by Jabłoński et al. (2015) was based on an econometric model of public debt, similar to the one put forth by Azzimonti et al. (2014). The study used the same data range but employed a different inequality index that referred to both the upper and lower parts of the income distribution. The empirical analysis was conducted in support of the author's theoretical arguments on the relationship between growing income inequality and increasing public debt. The conclusions drawn from the study appear to be similar to those reached by Azzimonti et al. (2014). However, a key issue with both studies is the oversimplified model that was adopted. This lack of complexity can be justified since the main focus of both studies was on theory, and empirical analysis was used only as a tool to validate the thesis.

The paper of Bittencourt (2015) centres on empirical analysis, examining the determinants of external debt in South America from 1970 to 2007. Although the results demonstrate economic growth as the primary factor that significantly reduces debt, the author also recognizes income inequality as a relevant variable capable of producing positive causal effects on the dependent variable, although the results are not always statistically significant. Nevertheless, as described in Section 1.7, many scholars found evidence of a relationship between income inequality and economic growth, even with mixed results. These results suggest that inequality may have both direct and indirect impacts on public debt.

Carrera and de la Vega (2021) focuses on the empirical analysis of the relationship between income inequality and public debt. They used a panel made of 158 countries, 35 advanced and 123 developing countries. The period covers 20 years and goes from 2000 to 2019. The dependent variable is the government debt as a percentage of GDP, while the independent variable is the Gini index computed by the distribution of disposable income. The results indicate a positive causal effect of inequality on debt. The study even proposes a test for a two-way relationship between debt and inequality. However, the preliminary test results find no evidence of a bi-directional link.

Variables levels, rather than changes, are used in the analysis conducted by Bittencourt (2015) and Carrera and de la Vega (2021), following the assumption made by Bohn (1998) regarding the stationarity of variables that are ratios or bounded within closed intervals. This assumption seems to be too strong, given the wide range of non-stationary economic variables, and if not supported by specific tests can bring estimation bias.

According to Woo (2003), who studied the determinants of public deficits using panel data from 57 developed and developing countries between 1970 and 1990, the results suggest that inequality is associated with larger public deficits. Similar findings can be found in the Larch paper, which emphasizes that countries with higher levels of inequality experience large deficits and tend to accumulate significant levels of government debt.

In contrast, Aksman, using a dynamic panel data model, presents diverse findings that indicate there is no evidence to suggest that poverty and income inequality contribute to public debt since countries with higher absolute poverty or income inequality tend to spend less on social benefits.

The work of Obiero and Topuz (2021), using an ARDL model, shows that there is a causal relationship between domestic debt, consisting of the stock of public securities held by domestic entities, and inequality in Kenya over the period from 1970 to 2018. Moreover, the authors suggest the existence of a positive long-term interaction between domestic and public debt, and inequality.

All these empirical studies that are carried out show similar features and weaknesses: they have mainly focused, both directly and indirectly, on the unidirectional link that goes from inequality to debt, ignoring the opposite causal effect. However, the existent theoretical literature supports a two-way relationship, even if empirical analysis neglects to test it.

Another critical issue in these empirical studies is the type of inequality for which data are collected. Income inequality, as well as wage inequality, are more readily available and serve the majority of the goals outlined in these studies.

However, this analysis considers the potential impact of debt on income solely from the perspective of policymakers' decisions, without accounting for its significant effect on individuals' overall wealth. As such, an index of wealth inequality may lead to a different conclusion.

Additionally, a crucial issue with much of the empirical literature is that it is vulnerable to endogeneity caused by simultaneous and reverse causation, which could result in biases and erroneous inferences about theory validity.

The deficiencies in previous empirical research and the backing of theoretical literature indicate the necessity for a fresh analysis. Such an analysis would address concerns of endogeneity and stationarity while also exploring the potential bidirectional association between public debt and wealth inequality. It is necessary to verify the effectiveness of transmission mechanisms previously identified and tested for income inequality and to explore additional ones that link public debt to wealth inequality.

1.6 Mechanisms of transmission

In this section, we will be explaining the potential ways in which transmission can occur between public debt and wealth inequality.

As discussed in section 1.5, the empirical analysis examined various channels through which income inequality affects the public debt. These include fiscal dynamics (Woo, 2003; Larch, 2012; Jabłoński et al., 2015; Carrera and de la Vega, 2021), financial market integration and stabilization (Kim, 2013; Azzimonti et al., 2014), and economic growth (Carrera and de la Vega, 2021).

However, these works focus on the one-way link described in the previous sections. The findings of this empirical analysis may not be valid for wealth inequality due to the partial correlation between the two types of inequality. Moreover, there is no research about possible transmission mechanisms for the opposite link that goes from inequality to public debt.

As there is this lack of empirical research on the inverse relation, this study examines theoretical studies already mentioned in section 1.4. Additionally, this study will explore both theoretical and empirical studies that have investigated the connections between government debt/inequality and other economic variables.

From the previous sections, we have seen that one of the potential transmission mechanisms is certainly fiscal policy. When a country has high levels of debt, the government may choose to implement restrictive fiscal policies to reduce the debt and make it more sustainable. This can lead to decreased welfare expenditure and increased tax burden. On the other hand, if there are significant economic disparities, social groups may pressure the government to implement redistributive policies. Conflicts between social groups may lead to political instability and more opportunistic behaviour by governments to please the electorate. Governments will thus opt for expansive fiscal policies financed through debt (see Sections 1.4.2 and 1.4.1).

Another possible mechanism could be attributed to the various aspects of public debt. The debt composition, that is, the share of external is a factor of great relevance in developed and developing countries. However, the relationship between public external debt and inequality, both empirically and theoretically, is examined in only a few studies.

External debt is an outflow of wealth to foreign countries, so it does not modify the already existing wealth distribution. In addition, the government obtains resources for re-distributive policies. From this point of view, external debt may be a functional tool for reducing inequality. Nevertheless, external debt impacts many aspects of the economic systems – inflation and economic growth – and of debt itself – debt services payments –, producing negative outcomes capable of reducing or even reversing positive effects on inequality.

Research suggests that an excessive external debt tends to crowd out social spending, reducing those public goods and services that most interest the poorest sections of the population. This is due to increased pressure on fiscal consolidation policies of foreign investors (Loko et al., 2003).

As far as we know, few studies address this issue; the works of Bittencourt (2015), Chowdhury (1994), and Akram (2013) are among them. Bittencourt (2015) argues that inequality is a determinant of external debts since his empirical results show a positive effect of inequality on external debt. Nevertheless, estimates are not always statistically significant.

Chowdhury (1994) estimates a structural simultaneous equation model to capture the interrelationships between public and private external debt, capital accumulation, and production, taking into account, along with other variables, a measure of inequality. The findings show a positive relationship between income inequality and external debt.

Akram (2013), with a focus on Pakistan, investigates the contrary link between the two variables with the help of a VEC model. The results indicate that the size of foreign debt has a strong and positive relationship with income inequality.

The study conducted by Salti (2015) analyzes the impact of domestic public debt on income redistribution. The author argues that those who lend money to the government tend to belong to the higher-income group, while the cost of debt service is shared across the entire tax base. By using cross-country panel data, the author runs regressions to examine the relationship between income inequality and the share of domestic public debt. The study reveals that the composition of public debt significantly influences income inequality. Specifically, the findings suggest that the burden of debt financing falls on the broader public, and not just on the lenders to the government.

The stability of the economic system may be another key mechanism that links wealth inequality with public debt. Various authors suggest strong interrelationships between increasing inequality, financial crisis and public debt (Kumhof et al., 2015; Bohoslavsky, 2016).

There is a wide range of literature that explores the connection between inequality and financial variables. The already cited paper of Azzimonti et al. (2014) proposes a theoretical model that suggests that the increase in income inequality is at least in part associated with an increase in income risk. Public debt rises following the greater demand for safe assets.

The model presented by Kumhof et al. (2015) suggests that higher leverage and crises are the endogenous result of a growing income share of high-income households. This thesis is supported by the works of Rajan (2010) and Galbraith (2012) which argue that rising income inequality forced low and middle-income households to increase their indebtedness to maintain their consumption levels. The increased volume of credit results in excess leverage, which eventually degenerates into a financial crisis.

The upward pressure on interest rates resulting from public debt makes it more difficult and expensive for households to obtain loans. As a result, households may be forced to increase their leverage, which reduces their net worth and savings. Domestic savings may contribute to reducing the differences between wealth classes (Berman et al., 2016; Blanchet and Martínez-Toledano, 2023). Furthermore, the decline in house prices may cause the wealth of the lower and middle classes to decrease even more due to reduced demand caused by the increased difficulty in obtaining a mortgage.

In their paper, Cardaci (2018) examine the impact of rising income inequality on the economy, taking into account the influence of peer effects and home equity borrowing. The study finds that a surge in consumption fuelled by debt could endanger the stability of the financial system, possibly resulting in a financial crisis.

Inequality may even be affected by financial variables. de Haan and Sturm (2017) analyse the relationship between financial development, financial liberalization, banking crises, and income inequality. They use a panel fixed-effects model for a sample of 121 countries covering from 1975 to 2005. The results suggest that all finance variables increase income inequality.

Another possible transmission mechanism, previously mentioned in Section 1.4.1, is inheritance. The link between public debt and inheritance primarily lies in the differing wealth compositions between the top 10% and the bottom 90%. For the top 10%, financial assets and bonds, including government bonds, constitute the main form of wealth, which is passed from one generation to the next through inheritance. The literature has extensively addressed the connection between inheritance and inequality. The study by Adermon et al. (2018) highlights the importance of inheritance in transmitting wealth across generations, showing that most wealth is passed down from grandparents and parents rather than generated through investments. The thesis is supported by the empirical analysis conducted by Fessler and Schürz (2018)

Elinder et al. (2018) identify two effects of inheritance on wealth distribution. Their findings are based on a Swedish dataset, but the work of Boserup et al. (2016) on Danish data confirms these results. The overall effect is a balance of two causal mechanisms. The first, known as the direct mechanical effect (DME), captures the immediate impact of inheritances before any behavioural responses occur. The second is the behaviour-adjusted effect (BAE), which suggests that heirs may alter their behaviours in response to inheritances, such as by consuming or investing part of their wealth or reducing their work effort. While the direct effects tend to reduce relative inequality, they simultaneously increase absolute inequality. This discrepancy arises because wealthier heirs receive larger inheritances, but less wealthy heirs inherit significantly larger amounts relative to their pre-inheritance wealth.

Behavioural adjustments tend to dilute the equalizing effects of inheritances, as the BAE is generally smaller than the DME. This equality-reducing effect is consistent with previous research indicating that less wealthy heirs spend a larger portion of their inherited wealth compared to wealthier heirs, as suggested by Mankiw (2000) and supported by Druedahl and Martinello (2022).

In conclusion, the relationship between public debt and wealth inequality is complex and multifaceted, with numerous potential transmission mechanisms and feedback loops. Fiscal policy, debt composition, economic stability, and financial crisis are just a few of the many factors that can impact the relationship between public debt and wealth inequality. While more research is needed to fully understand this relationship, the studies discussed in this section suggest that policymakers should consider the potential impacts of public debt on wealth inequality when making economic decisions. By understanding the complex interplay between these two variables, policymakers can work towards creating a more equitable and sustainable economic system for all.

1.7 Inequality, Public Debt, and Economic Growth

Extensive literature explores the relationship between inequality and growth, as well as the connection between development and public debt. To gain a deeper understanding of how these three variables interact, it would be beneficial to examine the individual links connecting them.

An old but still enduring topic of discussion is the connection between inequality and economic growth. The study of this relationship dates back to Kuznets (1955), who developed a theory on how national income per capita correlates with inequality in the income distribution. This relation takes the form of an inverted U curve. As a country's per capita national income increases, income inequality initially rises. However, once income reaches its intermediate-level peak, inequality starts to decline. The idea behind this theory is that in the early stages of development, there is a transition from a rural to an industrial organization, in the late stages the modern structure has penetrated the entire socio-economic texture. Although the hypothesis was later attacked, especially in the wake of rising levels of inequality in the U.S. and other developed countries(Saez and Zucman, 2020), Kuznets' research is credited with starting a line of work that is still thriving today. Current literature on this topic tends to concentrate on the relationship between growth and inequality, rather than on the relationship between inequality and the level of economic output as discussed by Kuznets in his ground-breaking work.

A significant paper by Perotti (1996) explores the topic through four key approaches: fiscal policy, socio-political instability, borrowing constraints, and education/fertility interaction. The resulting conclusions are that societies with greater equality tend to have lower fertility rates and higher investments in education, both of which contribute to increased economic growth. In contrast, highly unequal societies often experience political and social instability, leading to reduced investment and slower growth. Education and fertility are found to be two crucial factors that can impact growth in contrasting ways. Investments in education generally promote economic growth, whereas societies with high fertility rates often experience lower economic growth. Instead, the data does not support the notion that more equal societies, especially those with democratic institutions, grow faster due to fewer demands for redistribution and the associated economic distortions.

The work of Gründler and Scheuermeyer (2018) emphasizes the fertility approach. The author suggests that the negative impact of inequality on economic growth is due to its negative effect on education and a positive one on fertility rates, especially in regions with limited access to capital. Public investment in education has the potential to mitigate these negative effects. In contrast, re-distributive policies hamper economic growth by reducing investment and increasing fertility. It's important to note that the impact of these factors can vary according to a country's level of economic development.

Aghion et al. (1999) summarize three reasons why inequality might stimulate growth. First, they argue that the wealthy have a higher marginal propensity to save, leading to more investment and faster growth. Second, wealth concentration facilitates investment in new activities due to indivisibilities and sunk costs. Third, they suggest that inequality can improve efficiency by providing incentives for workers, as an egalitarian wage distribution might reduce effort and production efficiency.

De Dominicis et al. (2008) produced a meta-analysis to find an explanation for the mixed outcomes observed in empirical studies. The authors suggest that for most of the sample examined the relationship between economic growth and inequality is negative, positive for the remaining countries. The mixed results are mainly due to different estimation methods, data quality and sample coverage.

The empirical analysis of Barro (1974) brings evidence supporting the Kuznets' curve. He found that inequality harms economic growth in poor countries, while positive in countries with high income. Benhabib et al. (2003) shows similar conclusions: while an initial increase in inequality positively affects economic growth, when it reaches high levels there is a reversal of the impact.

The study by Bagchi and Svejnar (2015) explores the impact of wealth inequality on economic growth using a global measure of wealth inequality from Forbes magazine's listing of billionaires. The results suggest that wealth inequality is negatively related to economic growth. Controlling for the fact that some billionaires acquired their wealth through political connections, the relationship between politically connected wealth inequality and economic growth is negative. Instead, politically unconnected wealth inequality has no significant relationship.

Islam and McGillivray (2020) investigate the effects of wealth inequality on economic growth using a panel data set from Credit Suisse for 45 sample countries from 2000-2012. Their results align with the ones brought by Bagchi and Svejnar, who found that wealth inequality is negatively associated with cross-country economic growth. Particularly, the authors suggest that better governance may mitigate the negative impact of inequality on economic growth.

An even wider literature then investigated the relationship between economic growth and public debt, a topic of great interest even before the 21st-century crises, visible with the works of Aschauer (2000), Bernheim (1987), Modigliani (1961) and Seater (1993).

When choosing debt financing, one cannot disregard its impact on the economic system. Although some consensus has been reached on the existence of such a relationship, there are still mixed views on the direction of this link. The work of Panizza and Presbitero (2013) makes a systematic review of this topic.

Several papers suggest that debt harms economic growth through a crowding-out effect. An increase in public sector spending pushes the government to raise more revenue, which it does by raising taxes or selling government bonds, thereby reducing private-sector spending. However, the results are often contradictory, and empirical analysis suggests that any effects are quantitatively small.

Theoretical work suggests that, in the short run, the output is demand-determined and, consequently, budget deficits have a positive effect on disposable income, aggregate demand, and total output (Elmendorf and Mankiw, 1999). In the long run, the reduction in public savings caused by an increased budget deficit may not be fully offset by an increase in private savings. As a result, a decrease in national savings will result in reduced total investment. This, in turn, will negatively impact GDP due to smaller capital stock, higher interest rates, lower labour productivity, and wages.

The negative effect of public debt could be much larger if high public debt increases uncertainty or leads to expectations of future confiscation, possibly through inflation and financial repression (Cochrane, 2011). In this case, higher debt could have negative effects even in the short run. While uncertainty and policy credibility may amplify the negative effects of crowding out, DeLong et al. (2012) find that protracted recessions may lead to situations where expansionary fiscal policy produces a positive effect even on long-run growth.

1.8 Conclusion

In this chapter, we surveyed the recent theoretical and empirical literature on the potential linkages between public debt and wealth inequality. Strong evidence supports the idea of an interrelationship between the two variables. The exact mechanisms of transmission are more difficult to define.

From a theoretical point of view, scholars support the thesis of a link that goes from public debt to wealth disparities, but the theorized link is currently not supported by an empirical analysis.

Inequalities have a direct and indirect impact on public debt. Several theoretical studies based on the median voter hypothesis suggest that governments may increase public debt levels in response to rising inequalities for redistributive policies. On the other hand, inequality can cause financial instability due to a rise in income risk and excessive household debt, leading governments to opt for expansive fiscal policies to stabilize the financial markets.

Public debt has an impact on wealth inequality primarily through the uneven distribution of sovereign treasuries and savings. This leads to a transfer of wealth from the tax-paying population to the owners of the treasuries, equal to the service payments made on them. In addition, public debt can lead to instability and economic crises, which affect the poorer segments of society more severely. Furthermore, it can also affect interest rates, making it difficult and expensive for low- and middle-class classes to obtain loans.

The primary reason for this is the limited availability of data on wealth distribution, in contrast to the greater availability of data on other distributions such as income, wages, and earnings.

It is important to note that there is only a partial correlation between wealth and income distributions. Therefore, some of the mechanisms that have been verified for income may not apply to wealth.

This highlights the necessity of performing fresh empirical analyses using data on wealth inequality to examine mechanisms that have already been confirmed with income, as well as exploring mechanisms that have yet to be tested. Fortunately, there has been significant progress in expanding the availability of wealth data, particularly for the United States and European countries.

CHAPTER

2

AN R-PACKAGE FOR ESTIMATING THE VECTOR ERROR CORRECTION MODELS

2.1 Introduction

The Vector Error Correction Model (VECM) is a statistical method used in time series analysis to model and analyze relationships between multiple non-stationary variables. It is an extension of the Vector Autoregressive Model (VAR). While VAR models assume the stationarity of all variables and focus on short-term dynamics, VECM is specifically designed to handle non-stationary variables with cointegrated relationships.

Co-integration is a concept introduced by Granger (1981) and indicates the existence of a long-run equilibrium - a stationary linear combination - of a collection of no stationary time series. As mentioned by Maysami and Koh (2000), an advantage of co-integration analysis is that the short-term dynamics between the variables and the adjustment process towards long-term equilibrium - described by one or more linear combinations - can be studied together.

Compared to similar models such as the dynamic stochastic general equilibrium model or the structural equation model, the VECM enables solving economic questions, without the need to specify the entire economic structure, while applying fewer and weaker restrictions.

The VEC model, proposed by Engle and Granger (1987), is frequently employed in econometrics to examine the effects of economic policies or external shocks on a set of correlated variables. The model has been widely used to estimate the impact of the stock market on macroeconomic variables (Maysami and Koh, 2000; Mukherjee and Naka, 1995; Suharsono et al., 2017). The work of Asari et al. (2011) uses a VEC model to explain the relationship between interest rate and inflation towards exchange rate. Ghali (1998) investigates the presence of a cointegrating relationship between public and private investment, and economic growth, using Tunisian data. A similar paper of Akomolafe et al. (2015) examines the nexus between public debt and domestic investment in Nigeria.

Another relevant application of this model is the one carried out by Bildirici and Ersin (2007) which investigates the economic relationship between inflation and domestic debt, taking in exam three panels made each one by three countries with similar levels of inflation and borrowing costs. A further study on the nexus between public debt and inflation is the one of Nastansky and Strohe (2015) with a focus on the German case.

Some applications of the model have been made for inequality as well. Berisha et al. (2015) wanted to determine the existence of a cointegrating relationship between household debt and income inequality using a VECM on US data. The subsequent work of Berisha and Meszaros (2018), still on US data, reinforces the study carried out years earlier, this time adding the level of consumption to the analysis. The paper of Taghizadeh-Hesary et al. (2020) uses a VECM model to study the impact of monetary and tax policy on income inequality in Japan.

Recent works include those by Giudici and Pagnottoni (2020) and Ren et al. (2020), the former of which examines return connectivity across eight of the major bitcoin exchanges from both a static and dynamic perspective; and the latter of which analyses the relationships between the level of development of green finance, non-fossil energy consumption and carbon intensity using data from 2000 to 2018 with a focus on China.

At the time of writing, two R packages for the VEC model application are already available from the Comprehensive R Archive Network (CRAN), although they have some limitations. The package **urca** (Pfaff, 2008) includes functions to test for cointegration and to estimate the number of long-run relationships, but it is limited to three model specifications, i.e. no intercept, constant term, and trend variable in the error correction term. The package **tsDyn** (Stigler, 2019) contains functions for estimating the model, allowing one to choose the deterministic components for both the error correction term and the VAR part. It even allows one to choose the type of estimator between the two-step and Johansen

MLE approaches. However, it lacks functions for post-estimation analysis, which are partly compensated by its interactions with the **vars** package.

Outside the R environment, several software applications have been developed to estimate the model. **JMulTi** (Lütkepohl and Krätzig, 2004) is a free interactive software designed for univariate and multivariate time series analysis, but compared to other software, it lacks different tools. **Stata** adapts to the VEC model and allows several post-estimation analyses. **Eviews** is also a paid online application like Stata and it is the one that produces the most complete instrument set. The main shortcoming of **Stata** and **Eviews** is that they are limited by using only the maximum likelihood method of Johansen (1995). They are also both paid applications. With this package, we want to provide a free and as complete a tool as possible to use the VEC model.

In this chapter, we present the **VECM** package (version 1.0) for R. It includes functions to estimate the Vector Error Correction model and to realize pre and post-estimation analyses. The function allows to estimate of the model not only by using the maximum likelihood (ML) method of Johansen (1995), but also the estimated generalized least squares (EGLS) method described in Lütkepohl (2005). This approach is proved to be more robust in small samples compared to the ML estimator (Brüggemann and Lütkepohl, 2005). The package includes multiple functions to test the robustness of the estimated model, like the normality and uncorrelation of the residuals, using several tests for a more clear result. It even provides useful instruments to test the causality between two or more variables like a Granger-causality test, impulse-response functions and variance decomposition. Summary and print functions have been created to provide an easy-to-understand and user-friendly output.

The following section of this paper briefly reviews the theoretical background of the Vector Error Correction model. Next, Section 2.3 describes the software architecture, presenting the main functions of the package. Section 2.4 shows an application of the package using data presented in Lütkepohl (2005). Finally, Section 2.5 presents some final remarks, describing the next implementations of new functions and the process of submission to CRAN.

2.2 Theory

We begin by describing its connection to the VAR model and then move on to describe its basic formulation. After that, the main tests associated with the model will be described, both in defining the specifics of the model - the number of cointegrating relationships, and deterministic components - and the robustness of the model - residual normality and the correlation between residuals. Finally, Granger-causality will be defined and how to test it through the VEC model.

2.2.1 The Vector Error Correction Model

Consider a p-dimensional vector autoregressive model with Gaussian errors:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + u_t \tag{2.1}$$

where y_t is the k-vector with the variables of our interest, and p is the number of lags. A VAR is a basic econometric tool in econometric analysis with a wide range of applications. The model typically treats all variables as endogenous. However, the VAR system works only if all the variables are stationary, namely if they do not have trends or seasonality. Trends are quite common in many economic series and they can lead to spurious regression and misleading conclusions.

A possible solution to this problem is to work with the rates of change of our variables of interest, or first difference. Even if the original process is not stationary, the sequence of its changes can be. If the series of first differences of a process is stationary, it is defined as integrated ($I(1)$). With first differences, $\Delta y_t = y_t - y_{t-1}$, the Eq.2.1 can be rewritten as follows

$$\Delta y_t = A_1 \Delta y_{t-1} + A_2 \Delta y_{t-2} + \ldots + A_p \Delta y_{t-p} + u_t \tag{2.2}$$

Even this solution is not always appropriate: stationary processes cannot capture some main features of many economic time series. Moreover, if interest centres on analyzing the original variables rather than the rates of change, it is necessary to have models that accommodate the nonstationary features of the data.

These problems are related to the possible presence of common trends in the variables so that they move together to some extent. Suppose the variables of interest are summarised in the vector $y_t = (y_{1t}, \ldots, y_{Kt})'$ and their long-run equilibrium relation is $\beta' y_t = \beta_1 y_{1t} + \cdots + \beta_K y_{Kt} = 0$, where $\beta = (\beta_1, ..., \beta_K)'$. In any given period, this relationship may not be exactly satisfied, but we may have $\beta' y_t = z_t$, where z_t is a stochastic variable representing the deviations from equilibrium. If there is an equilibrium, it seems plausible to assume that the y_t variables move together and that z_t is stable. Integrated variables with this property are called cointegrated. A process consisting of cointegrated variables is called a cointegrated process. These processes were introduced by Granger (1981) and Engle and Granger (1987). Since then they have become popular in theoretical and applied econometric work.

Equilibrium relationships are suspected between many economic variables. It is shown that vector error correction models (VECMs) offer a convenient way to parameterize and specify them. A VEC model is defined as

$$\Delta y_t = v_0 + v_1(t-1) + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + u_t \tag{2.3}$$

$$\Pi = \alpha\beta'$$

The VECM is obtained from the levels VAR form 2.1 by subtracting y_{t-1} from both sides and rearranging terms. The lag decreased by one. $\Pi = -(I_K - A_1 - \cdots - A_p)$ and $\Gamma_i = -(A_{i+1} + \cdots + A_p)$ for $i = 1, \ldots, p-1$. The term Πy_{t-1}, usually called *Error Correction Term* (ECT) or *long-term* part, describes the long-run effects, that is, the adjustments of a variable due to deviations from its long-run equilibrium. The $\Gamma_i s$ are often referred to as the *short-run* or *short-term* parameters. They are the coefficients related to the first differences and describe the effects in the short run. v_0 and v_1 are the constant and the trend. The restrictions on v_0 and v_1 allow in some cases the absorption of the two terms into the cointegrating relations. It will be examined in more detail later.

The model can be written in a reduced form

$$\Delta Y_t = \Pi Y_{-1} + \Gamma \Delta X + U$$

where

- $\Delta Y := [\Delta y_1, \ldots \Delta y_T]$
- $Y_{-1} := [y_0, \ldots y_{T-1}]$
- $\Delta X := [\Delta X_0, \ldots, \Delta X_{T-1}]$, $\Delta X_{t-1} = \begin{bmatrix} \Delta Y_{t-1} \\ \vdots \\ \Delta Y_{t-p} \end{bmatrix}$
- $\Gamma = \begin{bmatrix} \Gamma_1 : \cdots : \Gamma_p \end{bmatrix}$

It is also possible to determine the A_j levels VAR parameter matrices from the coefficients of the VECM. More precisely, $A_1 = \Pi + I_K + \Gamma_1$, $A_i = \Gamma_i - \Gamma_{i-1}$ for $i = 2, \ldots, p-1$, and $A_p = -\Gamma_{p-1}$.

If the levels VAR(p) process has unit roots, the matrix Π is singular, that is, $\text{rk}(\Pi) = \text{rk}(\alpha) = \text{rk}(\beta) = r$, with r indicating the number of relations. This is why r is referred to as the *cointegrating rank* of the system. It has to be less than the number of the variables inserted into the model and at least equal to one. If r is equal to zero, it means that $\Pi = 0$ and so there is no long-run equilibrium. As a consequence Δy_t is a stable VAR system. If $r = K$, the matrix Π has full rank and the original series, y_t, can be represented with a stable VAR process. β, usually called *cointegrating vector* or *cointegrating matrix*, describes the r long-run relations. α, which takes the name of *loading matrix(vector)*, contains the weights attached to the cointegrating relations in the individual equations of the model. It describes the rate at which the processes return to equilibrium.

The matrices α and β are not identified, that is, not unique. With specific identifying assumptions, we obtain unique parameter values and estimators. Some a priori identification restrictions may be applied based on the existing theory. If the restrictions exactly identify or overidentify β, then the estimates of the unconstrained parameters in β will be superconsistent, i.e. the estimates of the free parameters in beta will converge faster than the estimates of the short-run parameters in alpha and gamma. The estimator of the short-run parameters can be derived conditionally on the estimated beta (Johansen, 1995; Boswijk et al., 1995). Lütkepohl (2005) has proposed a normalization method, widely adopted by researchers, for use when theory does not provide sufficient a priori restrictions to identify the cointegrating vector. The normalization is as follows.

$$\beta = \begin{bmatrix} I_r \\ \beta_{(k-r)} \end{bmatrix}$$

With this normalization, the relations may be read as r regressions where the first r variables introduced into the model are the dependent variable and the other their covariates. Be that as it may, despite this similarity to the regression model, no conclusions can be drawn on possible causal effects between variables, only to demonstrate the presence of long-run relationships between variables. Furthermore, if the variables under consideration are in logarithmic form, the β coefficients can be seen as long-run elasticities.

To most used estimator of the VEC model is the maximum likelihood (ML) estimator proposed by Johansen (1995). If the process y_t is Gaussian or, equivalently, $u_t \sim N(0, \Sigma_u)$, the loglikelihood function for a sample of size T is

$$\ln \ell = -\frac{T}{2} \ln |\Sigma_u| - \frac{1}{2} \sum_{t=1}^{T} \left[(\Delta Y - \alpha\beta' Y_{-1} - \Gamma \Delta X)' \, \Sigma_u^{-1} \left(\Delta Y - \alpha\beta' Y_{-1} - \Gamma \Delta X \right) \right]$$

Let $M := I_T - \Delta X'(\Delta X \Delta X')^{-1} \Delta X$, $R_0 := \Delta Y M$ and $R_1 := Y_{-1} M$, and define the following matrix

$$S_{11}^{-1/2} S_{10} S_{00}^{-1} S_{01} S_{11}^{-1/2}$$
$$\text{with} \quad S_{ij} = R_i R_j{}'/T; \quad i, j = [0, 1]$$

From this matrix we calculate the eigenvalue $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K$ and the corresponding eigenvectors $v_1, v_2, \ldots, v_K$. We use them to estimate the cointegrating matrix, the loading matrix and the VAR components:

- $\hat{\beta}_{ML} = [v_1, \ldots, v_r]' \, S_{11}^{-1/2}$
- $\hat{\alpha}_{ML} = S_{01} \hat{\beta}_{ML} \left(\hat{\beta}'_{ML} S_{11} \hat{\beta}_{ML} \right)^{-1}$

- $\Gamma_{ML} = \left(\Delta Y - \hat{\alpha}\hat{\beta}' Y_{-1}\right) \Delta X' \left(\Delta X \Delta X'\right)$
- $\Sigma_u = (\Delta Y - \alpha_{ML}\beta'_{ML} Y_{-1} - \Gamma_{ML}\Delta X)(\Delta Y - \alpha_{ML}\beta'_{ML} Y_{-1} - \Gamma_{ML}\Delta X)/T$

The ML β estimator is not normalized a priori. The ML estimator of $\beta_{(k-r)}$ may be obtained from the ML estimator of β by denoting the first r rows of β by $\beta_{(r)}$ and letting $\beta_{(k-r)}$ consist of the last $K-r$ rows of $\beta\beta_{(r)}^{-1}$.

The ML estimator of $\beta_{(K-r)}$ has the following asymptotic distribution

$$\text{vec}\left[(\hat{\beta}'_{(K-r)} - \beta'_{(K-r)}) \left(R_1^{(2)} R_1^{(2)'}\right)^{1/2}\right] \xrightarrow{d} \mathcal{N}\left(0, I_{K-r} \otimes (\alpha' \Sigma_u^{-1} \alpha)^{-1}\right)$$

with vec, a linear operation that transforms a matrix into a column vector by stacking its columns on top of each other, and $\otimes$, the Kronecker product. The asymptotic distribution of the ML estimator of $\hat{\alpha}$ and $\hat{\Gamma}$ is the following

$$\sqrt{T}\text{vec}\left(\left[\hat{\alpha}_{ML} : \hat{\Gamma}_{ML}\right] - [\alpha : \Gamma]\right) \xrightarrow{d} \mathcal{N}\left(0, \Omega^{-1} \otimes \Sigma_u\right)$$

where

$$\Omega = \text{plim}T \begin{bmatrix} \beta' Y_{-1} Y'_{-1} \beta & \beta' Y_{-1} \Delta X' \\ \Delta X Y'_{-1} \beta & \Delta X \Delta X \end{bmatrix}$$

The normality of the process is not essential for the asymptotic properties of the ML estimators. Much of the asymptotic properties described above hold under weaker conditions when quasi-ML estimators based on the Gaussian likelihood function are considered.

The ML estimator has been widely used in literature. However, some studies have found that it may have some undesirable properties in small samples. In particular, it may produce occasional outlying estimates that are far away from the true parameter values due to the lack of finite sample moments(Lütkepohl, 2005).

An alternative estimator is the estimated generalized least squares (EGLS) estimator, proposed by Ahn and Reinsel (1990) and Saikkonen (1992). This estimator has been proven to be more robust than the ML estimator when the sample size is small (Breitung et al., 2004). For EGLS estimation we assume that β is already normalized. We first estimate Π and Γ through the Least Squares estimator.

$$\left[\hat{\Pi}_{LS} : \hat{\Gamma}_{LS}\right] = \left[\Delta Y Y'_{-1} : \Delta Y \Delta X'\right] \begin{bmatrix} Y_{-1} Y_{-1} & Y_{-1} \Delta X' \\ \Delta X Y_{-1} & \Delta X \Delta X' \end{bmatrix}^{-1}$$

We then take the first r columns of $\hat{\Pi}_{LS}$ as an estimator of $\hat{\alpha}_{LS}$. Finally, we get the EGLS estimator of $\hat{\beta}_{(k-r)}$.

$$\hat{\beta}_{(k-r)} = (\hat{\alpha}' \hat{\Sigma}_u^{-1} \hat{\alpha})^{-1} \hat{\alpha}' \hat{\Sigma}_u^{-1} (R_0 - \hat{\alpha} R_1^{(1)}) R_1^{(2)'} (R_1^{(2)} R_1^{(2)'})^{-1}$$

with $R_1^{(1)}$ and $R_1^{(2)}$ respectively the first r and last $K-r$ rows of R_1, which together with R_0, has been already defined for the ML estimator. This estimator was proposed by Ahn and Reinsel (1990) and Saikkonen (1992). The EGLS estimators have the same asymptotic properties as the ML estimators.

The current model excludes any deterministic terms to simplify the exposition. However, these terms can be easily accommodated in the estimation procedures for VECMs discussed so far. The computation of the estimators is equally straightforward as in the case without deterministic terms. Additionally, the asymptotic properties of the parameter estimators remain essentially unchanged if the VECM is specified properly, including the cointegrating rank r, and if EGLS or ML methods are used. Deterministic terms will be better addressed in the following section due to their relevance to the distribution associated with the cointegration test.

2.2.2 Pre-estimation Analysis and Specification of VECMs

Before estimating the models, some questions need to be answered: the order of integration of the variables, the optimal lag, the presence and number of cointegrating relationships and the nature of the deterministic component, if there is one.

Let's answer the first dilemma. To apply the VEC approach, the variables of interest must be integrated. This requires checking for the presence of unit roots. The empirical literature focuses on three different tests: the augmented Dickey-Fuller (ADF) test (Said and Dickey, 1984), the Phillips-Peron (PP) test (Phillips and Perron, 1988), and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et al., 1992). Both the ADF and PP tests involve estimating a regression model with lagged differences in the time series. There are three versions of the tests given the presence of deterministic components. The null hypothesis of both tests is that there is at least one unit root in the time series that is the same for all three tests. Unlike the ADF and PP tests that test for the presence of a unit root, the KPSS test assesses whether the series is stationary in a different way. This involves estimating a model with a deterministic trend and comparing the squared residuals to critical values. The null hypothesis for the KPSS test is that a time series is stationary around a deterministic trend.

Once the non-stationarity of the variables has been tested, the tests must be repeated for the first-difference variables. If they are found to be stationary, this indicates that the variables are integrated of order one.

Before testing the presence of cointegrating relationships, the optimal lag must be found. The number of lags is investigated through the multivariate forms of information criteria. The most popular and used are the Akaike information criterion (AIC), the Hannan–Quinn information criterion (HQ), the Schwarz Criterion (SC) and the Final Prediction Error (FPE). They are calculated for different VAR(p)

models with the first-differences variables. The corresponding lag that minimizes the information criteria is then reduced to one to find the optimal lag of the VEC model.

The FPE criterion was originally introduced for stationary and stable processes to minimise forecast MSE, making it a justifiable criterion for forecasting objectives. However, the forecast MSE correction used for estimated stationary processes is difficult to justify in the cointegrated case. It is worth noting that AIC is asymptotically equivalent to the FPE criterion, and therefore similar comments apply to AIC. These criteria were chosen for their ability to correctly order large samples, making them consistent criteria. The consistency property of the criteria HQ and SC is maintained for integrated processes (Tsay, 1984; Paulsen, 1984).

To define r, we use the cointegrations tests proposed by Johansen (1988, 1995). Both the tests are based on likelihood ratio ratios. Let's resume the following matrix

$$S_{11}^{-1/2} S_{10} S_{00}^{-1} S_{01} S_{11}^{-1/2}$$

already define in Section 2.2.1 with $S_{ij} = R_i R_j'/T$, $i = 0, 1$. The eigenvalues of this matrix are calculated and used to perform tests to estimate the number of distinct cointegrating vectors. There are two types of statistics proposed by Johansen (1988), both based on a likelihood ratio test.

$$\lambda_{\text{trace}}(r) = -T \sum_{i=r+1}^{p} ln(1 - \hat{\lambda}_i)$$

$$\lambda_{\max}(r, r+1) = -T ln(1 - \hat{\lambda}_{r+1})$$

They have different hypothesis systems: the former statistic tests the null hypothesis that the number of distinct cointegrating vectors is less than or equal to r, and the latter statistic tests the null that the number of distinct cointegrating vectors is r against the alternative hypothesis of *r+1* cointegrating vectors. Trace and maximum eigenvalue tests have very similar local power in many situations, whereas each test version has its relative advantages in small samples, depending on the criterion for comparison. Thus, neither of the tests is generally preferable in practice.

For determining the cointegrating rank of a given system of K variables, we test a sequence of null hypotheses, from $r = 0$ to $r = K - 1$. We stop our tests at the first null hypothesis that is not rejected. If we stop at the first null hypothesis, that is, we cannot reject $r = 0$, so there is no evidence of cointegration and a model in first differences has to be considered. If we do not reject any hypotheses made then r may be greater than $K - 1$ and we can hypothesize a stationary VAR model for the levels of the variables.

Johansen (1988, 1995) shows that the asymptotic distributions of these LR statistics under the null hypothesis are

$$\lambda_{trace}(r_0, K) \xrightarrow{d} tr(\mathcal{D})$$

$$\lambda_{max}(r_0, r_0 + 1) \xrightarrow{d} \lambda_{max}(\mathcal{D})$$

where $\mathcal{D}$ denotes a matrix defined as

$$\mathcal{D} := \left(\int_0^1 W dW'\right)' \left(\int_0^1 WW' ds\right)^{-1} \left(\int_0^1 W dW'\right)$$

and $W := W_{(K-r_0)}(s)$ stands for a $(K - r_0)$ – dimensional standard Wiener process. Percentage points of the asymptotic distributions and, thus, critical values for the LR tests can be generated by replacing the Brownian motion with a Gaussian random walk (for more information, see Johansen (1988)) or found in tables collected from the literature[1]. The test is sensible to the specification of the model. Both time series and cointegrating relations may have nonzero means and deterministic trends. There are main five cases:

1. **No constant or trend**: ΔX, Y_{-1}, Γ and β as already defined above
2. **Restricted constant**: $Y_{-1}^{+} = [Y_{-1}, 1]$, $\beta^{+} = [\beta', v_0]'$, and $v_0 = -\beta\mu_0$
3. **Unrestricted constant**: $\Delta X^{+} = [1, \Delta X']'$, $\Gamma^{+} = [v_0, \Gamma]$ and $v_0 \neq 0$
4. **Restricted trend**: $Y_{-1}^{+} = [Y_{-1}, t - 1]$, $\beta^{+} = [\beta', v_1]'$, $\Delta X^{+} = [1, \Delta X']'$, and $\Gamma^{+} = [v_0, \Gamma]$ with $v_0 \neq 0$ and $v_1 = -\beta\mu_t$
5. **Unrestricted trend**: $\Delta X^{+} = [1, (t - 1), \Delta X']'$ and $\Gamma^{+} = [v_0, v_1, \Gamma]$ with $v_0 \neq 0$ and $v_1 \neq 0$

These five cases are the most common but not the only ones. Seasonal dummy variables are another type of deterministic term used to account for seasonal fluctuations in variables. If seasonal dummy variables are centred (orthogonalized), they will not impact the asymptotic distributions of the LR statistics for the cointegration rank (Lütkepohl, 2005).

A distinct scenario emerges when the deterministic term incorporates a shift dummy variable, denoted as $I(t > T_B)$, which remains zero until time T_B and then jumps to one. This variable impacts the asymptotic distributions of the LR test statistics. Notably, Johansen et al. (2000) demonstrated that in such instances,

[1] MacKinnon et al. (1999) provide the most reliable empirical critical values.

the asymptotic distributions are contingent on the temporal placement of the shift within the sample. More specifically, they are contingent on the proportion of the sample preceding the break-point. In contrast, impulse dummy variables, characterized by a constant value of zero except for a single designated period, do not affect the asymptotic properties of the LR tests.

Even the introduction of one or more exogenous variables affects the distribution. With the software that computes the Johansen test, if the user chooses to include exogenous variables, the critical values reported do not account for these variables.

Consequently, the choice of the deterministic term is relevant. If a linear time trend may be possible, one could include such a term in the process in a fully general form. However, this can result in a significant loss of power if the time trend is not needed in the model. Doornik and Hansen (2008) provide small-sample and asymptotic evidence that ignoring a deterministic trend presented in the data generation process can lead to large-size biases. However, even including an unnecessary trend term may be harmful due to a loss of power of the test (Hubrich et al., 2001).

Therefore, it is helpful to have statistical methods available to consider which terms to include. One way is based on subject matter considerations or visual inspection of the time series plots under consideration. Alternatively, Johansen (1994, 1995) suggested LR tests for hypotheses about the deterministic terms. These tests are an obvious choice because the maxima of the likelihood functions are easy to compute for different deterministic terms.

The test is nested. The test compares the performance of the model with constrained deterministic terms and the model with unrestricted terms. The null hypothesis (H0) is that deterministic components are not necessary, i.e., that the deterministic component under test is zero. The distribution of the test statistic is a chi-square with degrees of freedom equal to the number of parameters introduced into the model by the deterministic component.

To identify the cointegrating rank, another method is to use information criteria. We estimate models by imposing a rank from zero to $K-1$ and choose the model that minimizes the selected information criterion. This method avoids the problem that can arise by introducing any deterministic components, seasons, or shift dummies.

2.2.3 Post-estimation Analyses

Model Diagnostics

Diagnostic checking constitutes a crucial stage in the modelling process for VECMs. The identification of model defects, such as residual autocorrelation or Autoregressive Conditional Heteroskedasticity (ARCH) effects during this phase, typically signals that the model poorly captures the underlying Data Generating Process.

A useful way to begin checking for specification issues is to conduct a graphical analysis. This can involve creating plots of roots of the characteristic polynomial, standardized residual series, squared residuals, empirical distribution compared to the normal, and the residuals correlogram.

However, it is important to note that graphical analysis alone cannot fully replace formal misspecification tests. For the VEC model, there are several tests available that are based on those developed for the VAR model. This section will discuss both multivariate and univariate residual tests, which are included in the proposed R package. The most relevant diagnostic tests are about autocorrelation, normality and heteroskedasticity.

Testing for Residual Autocorrelation

We begin by defining autocovariance as follows

$$\hat{C}_i = \frac{1}{T}\sum_{t=i+1}^{T} \hat{u}_t \hat{u}'_{t-i}, \quad i = 0, 1, \ldots, h$$

with $\hat{C}_0$, the estimated residual variance, $\hat{\mathrm{C}}_h = (\hat{C}_1, \ldots, \hat{C}_h)$, the autocovariance function and $\hat{\mathrm{c}}_h = \mathrm{vec}(\hat{\mathrm{C}}_h)$, the vectorized autocovariance function.

A formal test for residual autocorrelation may be based on the portmanteau or adjusted portmanteau statistic. The test checks the null hypothesis that

$$H_0 : E(u_t, u_{t-i}) = 0 \quad \text{with} \quad i = 1, \ldots, h, \quad h > p$$

against the alternative that at least one autocovariance is nonzero. The test statistic has the form.

$$Q_h = T\sum_{i=1}^{h} \mathrm{tr}\left(\hat{C}'_i \hat{C}_0^{-1} \hat{C}_i \hat{C}_0^{-1}\right)$$

with an approximate $\chi^2(hK^2 - K^2p - Kr)$-distribution. The degrees of freedom are given by subtracting from the number of autocovariances included in the statistic (hK^2), the number of estimated parameters (K^2p) and the elements of the cointegration matrix (Kr). The approximate χ^2- distribution is obtained under the assumption that h goes to infinity with the sample size. Thus, the portmanteau test is not suitable for testing for residual autocorrelation of low order. A modified statistic with potentially superior small sample properties is the adjusted portmanteau statistic

$$Q_h^{ad} = T\sum_{i=1}^{h} (T-i)^{-1} \mathrm{tr}\left(\hat{C}'_i \hat{C}_0^{-1} \hat{C}_i \hat{C}_0^{-1}\right)$$

If h is chosen too small, the χ^2-approximation to the null distribution may be very poor, whereas a large h may result in a loss of power. Consequently, it is suggested to try different values of h before concluding.

Another possibility to test for residual autocorrelation is obtained by fitting a VEC model to the residuals. This variant is sometimes referred to as *Breusch–Godfrey test* (Godfrey, 1988). The test for h-th order residual autocorrelation assumes a model

$$u_t = B_1 u_{t-1} + \cdots + B_h u_{t-h} + \text{error}_t$$

with null hypothesis $H_0 : B_1 = \cdots = B_h = 0$ and alternative hypothesis $\exists\, i \in \{1, 2, \ldots, h\} : B_i \neq 0$. An auxiliary model is built by running a regression of the residuals on the original VEC model described in 2.3 and the lagged residual, where the missing first values are filled with zeros. The corresponding model is

$$\hat{u}_t = v_0 + v_1(t-1) + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \sum_{i=1}^{h} B_i \hat{u}_{t-i} + e_t$$

No exogenous variables are considered and the models are estimated by OLS. Denoting the estimated residuals by $e^t (t = 1, \ldots, T)$, we obtain the following residual covariance matrix estimator from the auxiliary models:

$$\hat{\Sigma}_e = \frac{1}{T} \sum_{t=1}^{T} \hat{e}_t \hat{e}_t'$$

Moreover, if we estimate once more the relevant auxiliary model without the lagged residuals $u_{t-i} (i = 1, \ldots, h)$, that is, impose the restrictions $B_1 = \cdots = B_h = 0$ and denote the resulting residuals by e_t^R, the corresponding covariance matrix estimator is

$$\hat{\Sigma}_R = \frac{1}{T} \sum_{t=1}^{T} \hat{e}_t^R \hat{e}_t^{R\prime}$$

There are four versions of the test, that is, LM, LR(Johansen, 1995) and Rao(Edgerton and Shukur, 1999).

- $\lambda_h^{LR} = T \log \frac{|\hat{\Sigma}_e|}{|\hat{\Sigma}_R|}$
- $\lambda_h^{LM} = T \left[K - \operatorname{tr}(\hat{\Sigma}_R \hat{\Sigma}_e^{-1}) \right]$
- $\lambda_h^{R} = \left[\left(\frac{|\hat{\Sigma}_e|}{|\hat{\Sigma}_R|} \right)^{1/s} - 1 \right] \frac{Ns-1}{Km}$
  with $\quad s = \left(\frac{K^2 m^2 - 4}{K^2 + m^2 - 5} \right)^{1/2}$, $q = \frac{1}{2} Km - 1$, $N = T - n - m - \frac{1}{2}(K - m + 1)$

where n is the number of regressors in each equation of the original system and $m = Kh$ is the number of additional regressors in the auxiliary system.

The test statistics produced by the former two versions are asymptotically distributed as χ^2-distributions with hK^2 degrees of freedom under the null hypothesis, while the last test distribution under H_0 can be approximated well by an $F(hK^2, Ns - q)$-distribution. The Breusch–Godfrey test is effective in detecting low-order residual autocorrelation when dealing with small values of h, while a portmanteau test is recommended for larger h.

Testing for Normality

The second thing to test is the normality of the residual distribution. Multivariate tests for non-normality can be constructed by generalising the Lomnicki-Jarque-Bera tests of univariate time series. The joint normal distribution is transformed to obtain the independent components. Then the tests will be applied to the independent components like for the univariate series. The tests consist of comparing the estimated third and fourth moments with those of a standard normal. We will look at the deviation to the right or left of the distribution (symmetry) and the concentration around the mean (kurtosis).

Given the residuals $\hat{u}_t$ $(t = 1, ..., T)$ of an estimated VECM, the residual covariance matrix is estimated as

$$\tilde{\Sigma}_u = \frac{1}{T}\sum_{t=1}^{T}(\hat{u}_t - \overline{\hat{u}})(\hat{u}_t - \overline{\hat{u}})'$$

and the square root matrix $\tilde{\Sigma}_u^{1/2} = \tilde{P}$ is computed. The third and fourth sample moments are obtained from the standardized residuals $\hat{u}_t^s = \tilde{P}^{-1}(\hat{u}_t - \overline{\hat{u}})$. Then $b_{1i} = T^{-1}\sum_{t=1}^{T}(\hat{u}_{it}^s)^3$ and $b_{2i} = T^{-1}\sum_{t=1}^{T}(\hat{u}_{it}^s)^4$ and the corresponding vectors $b_1 = (b_{11}, \ldots, b_{1K})$ and $b_2 = (b_{21}, \ldots, b_{2K})$. We can now compute *skewness* $s_3^2 = Tb_1'b_1/6$ and *kurtosis* $s_4^2 = T(b - 3_K)'(b - 3_K)/24$. Both statistics have asymptotic χ^2-distributions with K degrees of freedom under the null hypothesis of normality. Moreover, under the null, $LJBK = s_3^2 + s_4^2$ has a χ^2 limiting distribution with $2K$ degrees of freedom.

There are different types of residual standardization based on the factorization technique that can be applied on $\tilde{\Sigma}_u$. The most used are three:

- Lütkepohl (2005): it is based on the Cholesky decomposition. we have that $\tilde{\Sigma}_u = \tilde{P}\tilde{P}'$ with $\tilde{P}$ the inverse of the lower triangular Cholesky factor of $\tilde{\Sigma}_u$. The ordering of variables may affect the test result because the normalization of residuals is not invariant to variable ordering(Doornik and Hansen, 2008)

- Doornik and Hansen (2008): it starts by computing $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_K)$, a diagonal matrix containing the eigenvalues of the residual correlation matrix; Q, a matrix containing the corresponding eigenvectors in columns; and V, a diagonal matrix containing the inverse square root of the residual variances. We can now define $\tilde{P} = Q\Lambda^{-1/2}Q'V$. The test is invariant to the ordering of the variables in the VECM. Moreover, using the correlation matrix, rather than the covariance, even makes the test scale invariant. Doornik and Hansen (2008) propose a small sample correction: for skewness

$$\beta = \frac{3(T+1)(T+3)(T^2+27T-70)}{(T-2)(T+5)(T+7)(T+9)}, \quad w = \sqrt{2(\beta-1)} - 1$$

$$\delta = \frac{1}{\sqrt{\log(\sqrt{w})}}, \quad y = b_1\sqrt{\frac{(w-1)(T+1)(T+3)}{12(T-2)}}, \quad s_3^2 = \left[\delta \log(y + \sqrt{y^2+1})\right]^2$$

while for kurtosis we have

$$\delta = (T-3)(T+1)(T^2+15T-4), \quad a = \frac{(T-2)(T+5)(T+7)(T^2+27T-70)}{6\delta}$$

$$c = \frac{(T^2-49)(T+5)(T^2+2T-5)}{6\delta}, \quad k = \frac{(T+5)(T+7)(T^3+37T^2+11T-313)}{12\delta}$$

$$\alpha = a + b_1^2 c, \quad \gamma = (b_2 - b_1^2 - 1)\cdot 2k, \quad s_4^2 = \left[\left(\frac{\left(\frac{\gamma}{2\alpha}\right)^{1/3} + \frac{1}{9\alpha} - 1}{\sqrt{9\alpha}}\right)\cdot\sqrt{9\alpha}\right]^2$$

- Urzúa (1996) $\tilde{P} = GD^{-1/2}G'$whit D, the diagonal matrix containing the eigenvalues of the residual covariance matrix and G, a matrix containing the corresponding eigenvectors in columns. Even in this case, a small sample correction is needed. The correction differs from the one proposed by Doornik and Hansen (2008).

$$V_{b_1} = \frac{6(T-2)}{(T+1)(T+3)}, \quad E_{b_2} = \frac{3(T-1)}{T+1}, \quad V_{b_2} = \frac{24T(T-2)(T-3)}{(T+1)^2(T+3)(T+5)}$$

from which we have that $s_3^2 = b_1^2/V_{b_1}$ and $(b_2 - E_{b_2})^2/V_{b_2}$.

ARCH-LM Test

Finally, a scholar may want to check if ARCH effects are present in the residuals. A multivariate extension of the univariate ARCH-LM test may work for this purpose. Consider the multivariate regression model

$$\text{vech}(\hat{u}_t\hat{u}_t') = \beta_0 + B_1\text{vech}(\hat{u}_{t-1}\hat{u}_{t-1}') + \cdots + B_q\text{vech}(\hat{u}_{t-q}\hat{u}_{t-q}') + \text{error}_t$$

where vech is the column-stacking operator for symmetric matrices that stacks the columns from the main diagonal downwards, β_0 is a vector of length $\frac{1}{2}K(K+1)$, and the B_j s are squared matrix with dimensions equal to $(\frac{1}{2}K(K+1))x(\frac{1}{2}K(K+1))$ and $j = 1, \ldots, q$. There is no ARCH in the residuals if all the B_j matrices are zero, that is, $H_0 : B_1 = \cdots = B_q = 0$. The LM statistic is computed as

$$\text{MARCH}_{LM}(q) = \frac{1}{2}TK(K+1)R_m^2,$$

where

$$R_m^2 = 1 - \frac{2}{K(K+1)}\text{tr}(\hat{\Omega}\hat{\Omega}_0^{-1})$$

with $\hat{\Omega}$, the residual covariance matrix of the multivariate regression model described above and $\hat{\Omega}_0$, the residual covariance matrix with $q = 0$. Under the null hypothesis, the test statistic is distributed as a χ^2-distribution with $qK^2(K+1)^2/4$ degrees of freedom.

Chow Structural Break Test

Events such as wars or new tax legislation that cause turbulence in economic systems during specific periods are significant sources of nonstationarity. These events may lead to structural changes in economic systems. Stability and stationarity are crucial conditions for the validity of estimator properties and computing forecasts and forecast intervals.

Tools exist for checking whether there has been a structural shift in the data generation process. The first two tests check whether there has been a change in the parameters at some point in time. This is done by comparing the estimated parameters before and after the possible break date. These are known as Chow tests. The last test is based on comparing forecasts with actual observed values. Forecasts are made before a period of possible structural change and compared to the values observed during that period. The stability or stationarity hypothesis is rejected if the forecasts differ significantly from the observed values.

In deriving the Chow tests, it is assumed that a change in the parameters of the VECM is suspected after period $T_B < T$. For a sample $y_1, \ldots, y_T$ plus the required pre-sample values, The model can be divided into two parts:

$$\Delta Y_{(1)} = \alpha_{(1)}\beta_{(1)}Y_{-1(1)} + \Gamma_{(1)}\Delta X_{(1)} + U^{(1)}$$

and

$$\Delta Y_{(2)} = \alpha_{(2)}\beta_{(2)}Y_{-1(2)} + \Gamma_{(2)}\Delta X_{(2)} + U_{(2)}$$

with $\Delta Y_{(1)} := [\Delta y_1, \ldots, \Delta y_{T_B}]$, $\Delta Y_{(2)} := [\Delta y_{T_B+1}, \ldots, \Delta y_T]$, and the other data matrices are partitioned accordingly. The parameter matrices $\alpha_{(i)}$, $\beta_{(i)}$, and $\Gamma_{(i)} := [\Gamma_{1(i)}, \ldots, \Gamma_{p-1(i)}]$ contain the values for the i-th subperiod, where $i = 1, 2$. The ML estimators for these parameter matrices can be determined by two separate reduced rank regressions, one for each of the two models. To avoid an overlap of the pre-sample values with the last observations of the first subperiod, the start of the second subsample is commonly set off with observation y_{T_1+p+1}, with T_1 and T_2 the sample size of the two subsamples. Denoting the resulting residuals by $\hat{u}_t$, $\hat{u}_{t(1)}$, and $\hat{u}_{t(2)}$, respectively of the full-sample, pre-sample and post-sample models, we define

$$\hat{\sigma}_u^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_t\hat{u}_t',$$

$$\hat{\sigma}_{1,2}^2 = (T_1+T_2)^{-1}\left(\sum_{t=1}^{T_1}\hat{u}_t\hat{u}_t' + \sum_{t=T-T_2+1}^{T}\hat{u}_t\hat{u}_t'\right),$$

$$\hat{\sigma}_{(1,2)}^2 = T_1^{-1}\sum_{t=1}^{T_1}\hat{u}_t\hat{u}_t' + T_2^{-1}\sum_{t=T-T_2+1}^{T}\hat{u}_t\hat{u}_t',$$

$$\hat{\sigma}_{(1)}^2 = T_1^{-1}\sum_{t=1}^{T_1}\hat{u}_{t(1)}\hat{u}_{t(1)}',$$

and

$$\hat{\sigma}_{(2)}^2 = T_2^{-1}\sum_{t=T-T_2+1}^{T}\hat{u}_{t(2)}\hat{u}_{t(2)}'.$$

With this notation, the sample-split test statistic becomes

$$\lambda_{SS} = (T_1+T_2)\left[\log|\hat{\Sigma}_{1,2}| - \log\{(T_1+T_2)^{-1}(T_1|\hat{\Sigma}_{(1)}| + T_2|\hat{\Sigma}_{(2)}|)\}\right],$$

and the break-point test statistic is

$$\lambda_{BP} = (T_1+T_2)\log|\hat{\Sigma}_{(1,2)}| - T_1\log|\hat{\Sigma}_{(1)}| - T_2\log|\hat{\Sigma}_{(2)}|.$$

The sample-split test examines the null hypothesis that the AR coefficients and deterministic terms remain constant throughout the sample period. The break-point test, on the other hand, examines the constancy of the white noise variance in addition to this. Under parameter constancy, they have limiting χ^2-distributions with $Kr + (p-1)K^2 + K$ and $Kr + (p-1)K^2 + K + K(K+1)/2$ degrees of freedom, respectively. The degrees of freedom are determined by the number of additional parameters estimated for the two subsamples compared to those for the full sample. In the case of the BP test, $K(K+1)/2$ possibly different parameters of the white noise covariance matrix are to be added.

Doornik (1997) have proposed a multivariate version of the Chow forecast test statistic based on approximations proposed by Rao et al. (1973). The test statistic can be written as follows

$$\lambda_{CF} = 1 - \left(1 - R_r^2\right)^{\frac{1}{s}} \left(1 - R_r^2\right)^{\frac{1}{s}} \frac{N_s - q}{Kk^*},$$

where

- $R_r^2 = 1 - \left(\frac{T_1}{T}\right)^K \left|\hat{\Sigma}_{(1)}\right| \left(\left|\hat{\Sigma}_u\right|\right)^{-1}$
- $s = \sqrt{\frac{K^2 k^{*2} - 4}{K^2 + k^{*2} - 5}}$
- $q = Kk^{*2} - 1$
- $N = T - k_1 - k^* - \frac{K - k^* + 1}{2}$

and with k_1 being the number of regressors in the restricted, time-invariant model; k^* the number of forecast periods considered by the test, that is, $k^* = T - T_1$. The test statistic is approximately distributed as an F-distribution with Kk^* and $Ns - q$ degrees of freedom.

Causality Analysis

Once the fitted model is found to be robust, the next step is to test the presence of causality among the variables. Three main instruments can be applied: the Granger-causality test, the impulse-response functions (IRF) and the forecast error variance decomposition (FEVD).

Granger-causality test

Granger (1981) has proposed a causality concept. Formally, a variable x_t causes in the sense of Granger a variable y_t if conditioning with respect to the past values of x_t the mean square error of prediction of y_t is reduced with respect to the case in which the information about the past values of x_t is ignored. This definition works even with groups of variables.

To test if a group of M variables, y_t^2, Granger-cause a group of $M-K$ variables, y_t^1, in a VECM contest, suppose that the vector of variables of interest is partitioned into the two subgroups, x_t and y_t.

$$y_t = \begin{bmatrix} y_t^1 \\ y_t^2 \end{bmatrix}$$

As a consequence, Π and Γs matrices are partitioned in accordance with the partitioning of y_t, and the new formulation of the model described by Eq.2.3 is

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{bmatrix} = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} \Delta y_{1t-1} \\ \Delta y_{2t-1} \end{bmatrix} + \sum_{i=1}^{p-1} \begin{bmatrix} \Gamma_{11,i} & \Gamma_{12,i} \\ \Gamma_{21,i} & \Gamma_{22,i} \end{bmatrix} \begin{bmatrix} \Delta y_{1t-i} \\ \Delta y_{2t-i} \end{bmatrix} + u_t$$

y_t^2 does not Granger-cause y_t^1 if and only if we do not reject the following null hypothesis

$$H_0 : \Pi_{12} \quad \text{and} \quad \Gamma_{12,i} \quad \text{for} \quad i = 1, \ldots, p-1$$

To check Granger-causality, we test a set of linear hypotheses. A Wald test is a standard choice for this purpose. Let be $\hat{\epsilon}$ the estimator of K^2p - vector $\epsilon = [\Pi, \Gamma_1, \ldots, \Gamma_{p-1}]$ and $\hat{\Sigma}_\epsilon$ the estimator of the corresponding covariance matrix Σ_ϵ, then a Wald test can be conducted for the pair of hypotheses

$$H_0 : C\epsilon = 0 \quad \text{against} \quad H_1 : C\epsilon \neq 0$$

with C is an N x K^2p matrix of rank N and N the number of restricted coefficients. The Wald statistic is then defined as follows

$$\lambda_W = T\hat{\epsilon}'C'(C\hat{\Sigma}_\epsilon C')^{-1}C\hat{\epsilon}$$

The statistic λ_W has an asymptotic χ^2(N)-distribution, provided the null hypothesis is true and $\text{rk}(C\Sigma_\epsilon C') = \text{rk}(C\hat{\Sigma}_\epsilon C') = N$. The latter condition is satisfied for for stable, full VAR process, but in the case of a VEC model, the matrix Σ_ϵ may be singular if the cointegration rank r is less than K. As a consequence, it may be that $\text{rk}(C\Sigma_\epsilon C') < N$, even if C has full row rank N. A possible solution is to replace the inverse $C\Sigma_\epsilon C'$ with a generalized inverse, with the asymptotic distribution of λ_W that becomes $\chi^2(\text{rk}(C\Sigma_\epsilon C'))$. Even in this case there is the condition that $\text{rk}(C\Sigma_\epsilon C') = \text{rk}(C\hat{\Sigma}_\epsilon C')$ with probability one (Andrews, 1987)[2].

Impulse-response Function

[2]For further information about the problem to test Granger-causality in cointegrated analysis see Toda and Phillips (1993)

The IRF measures the effect of a sudden and temporary change in one variable on the future values of another variable in a system. It provides insights into the dynamic interactions and propagation of shocks within a system over time. Impulse response matrices can be computed starting from the VAR transposition of the VEC model already described in Section 2.2.1. Although the Wold representation does not exist for nonstationary cointegrated processes, the matrices can still be computed but they do not tend to zero as the horizon tends to infinity. Consequently, some shocks may produce permanent effects. The process can be represented as a moving average, MA(∞)

$$y_t = \Phi_0 u_t + \Phi_1 u_{t-1} + \Phi_2 u_{t-2} + \dots$$

where $\Phi_0 = I_K$ and

$$\Phi_s = \sum_{z=1}^{s} \Phi_{s-z} A_z, \quad s = 1, 2, \dots$$

The (i, j)th elements of the matrices Φ_s trace out the expected response of $y_{i,t+s}$ to a unit change in $y_{j,t}$, holding constant all past values of y_t. Since the change in $y_{i,t}$ is measured by the innovation $u_{i,t}$, the elements of ϕ_s represent the impulse responses of the components of y_t with respect to the u_t innovations. The accumulated effects can be easily calculated by adding up the Φ_s matrices. If the components of u_t are instantaneously correlated, meaning that Σ_u is not diagonal, it is unlikely that the underlying shocks will occur in isolation. Therefore, it is preferable to use orthogonal innovations in an impulse response analysis. A Choleski decomposition of the covariance matrix is usually used to compute them, even if the choice is arbitrary. The orthogonalized shocks are given by $e_t = B^{-1} u_t$ with B, a lower triangular matrix such that $\Sigma_u = BB'$. The new moving average process can now be written as

$$y_t = \Psi_0 e_t + \Psi_1 e_{t-1} + \Psi_2 e_{t-2} + \dots$$

where $\Psi_i = \Phi_i B$. The corresponding impulse responses are often referred to as orthogonalized impulse responses.

Forecast Error Variance Decomposition

Starting from Eq.2.2.3, the MA representation of our model with orthogonal white noise innovations, a further instrument can be built for interpreting the results. We define the error of the optimal h-step forecast as

$$y_{t+h} - y_t(h) = \sum_{i=0}^{h-1} \Psi_i e_{t+h-i}$$

with y_{t+h} the observed value and $y_t(h)$ the expected value. Denoting the (m,n)-th element of Ψ_i by $\psi_{mn,i}$, the h-step forecast error of the j-th component of y_t is

$$y_{j,t+h} - y_{j,t}(h) = \sum_{k=1}^{K}\sum_{i=0}^{h-1}\psi_{jk,i}e_{jk,t+h-i}$$

The forecast error of the j-th component potentially consists of all the innovations $e_{1t},\dots,e_{Kt}$, and given that the innovations are uncorrelated and have unit variances, the MSE of $y_{j,t}(h)$ is

$$MSE\left[y_{j,t}(h)\right] = E\left[y_{j,t+h} - y_{j,t}(h)\right]^2 = \sum_{k=1}^{K}(\psi_{jk,0}^2 + \cdots + \psi_{jk,h-1}^2)$$

with $(\psi_{jk,0}^2 + \cdots + \psi_{jk,h-1}^2)$, the contribution of variable k to the h-step forecast error variance of variable j. Dividing it by $MSE\left[y_{j,t}(h)\right]$ gives the percentage contribution of variable j to the h-step forecast error variance of variable j.

$$\omega_{jk}(h) = \frac{\psi_{jk,0}^2 + \cdots + \psi_{jk,h-1}^2}{MSE\left[y_{j,t}(h)\right]}$$

Forecasting

Forecasting is conveniently made using the VAR representation as discussed in Section 2.2.1 and already used for the estimation of the IRFs described in Section 2.2.3. We consider a levels VAR(p) model

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t$$

Deterministic terms are left out for reading convenience, but they can be easily added. We replace the coefficients A_i $(i = 1,\dots,p)$ and the covariance matrix Σ_u with one of the estimators described in Section2.2.1. Forecasting can be of two types in the case of multivariate models - VEC and VAR models-, that is, *static* and *dynamic*. The former consists of forecasting one step ahead within the sample range, while the latter is out of sample.

Therefore, the static version can be computed as follows

$$\hat{y}_t(1) = \hat{A}_1 y_t + \cdots + \hat{A}_p y_{t+1-p}$$

with y_i that is available for $i = t,\dots,t+1-p$. If we want to go beyond the one-step horizon, that is, $h > 1$, then we have

$$\hat{y}_t(h) = \hat{A}_1\hat{y}_t(h-1) + \cdots + \hat{A}_p\hat{y}_t(h-p)$$

For this predictor, the forecast error becomes

$$y_{t+h} - \hat{y}_t(h) = \hat{u}_{t+h} + \Phi_1 \hat{u}_{t+h-1} + \cdots + \Phi_{h-1} \hat{u}_{t+1}$$

with Φ_i $(i = 1, \ldots, h-1)$ as described for the IRFs. The MSE matrix of an h-step forecast is

$$\Sigma_y(h) = E\left[y_{t+h} - \hat{y}_t(h))(y_{t+h} - \hat{y}_t(h))'\right] = \sum_{j=0}^{h-1} \Phi_j \Sigma_j \Phi_j'$$

The following forecast intervals can be established:

$$\left[y_{kt}(h) - \omega_{1-\alpha/2}\sigma_k(h)\,,\, y_k t(h) + \omega_{1-\alpha/2}\sigma_k(h)\right].$$

with $\omega_{1-\alpha/2}$, the $1 - \frac{\alpha}{2}$ percentile of the standard normal distribution, $y_{kt}(h)$, the k-th component of $y_t(h)$, and $\sigma_k(h)$, the square root of the k-th diagonal element of $\Sigma_y(h)$. If there are deterministic terms, it is easily possible to extend the formula as their future development is known by definition. Exogenous variables may be more challenging to deal with in some respects, but they can also be handled if their future development is known. Otherwise, a model for the exogenous variables is required.

2.3 Description of the VECM Package

2.3.1 Installation

The development version of the package can be installed from GitHub using **devtools** (Wickham et al., 2022) with the following command:

```
R> devtools::install_github("gianlucacarpigo/VECM")
```

These commands also install the dependencies of **VECM**. The data visualizations are made using the packages **graphics**(R Core Team, 2022) and **viridis**(Garnier et al., 2024) for colourblind colours. From **MASS**(Venables et al., 2002) is taken the multivariate normal distribution. The packages **expm**(Maechler et al., 2023) and **matrixcalc**(Novomestky, 2022) are used for matrix operations. The functions are thoroughly documented using **roxygen2** (Wickham et al., 2022).

2.3.2 Software Architecture and Main Function

The **VECM** package follows a software architecture based on functional programming. Each main function in the package returns an instance of an S3 class that implements relevant methods for reading results, such as `print()`, `summary()`, and `plot()`. Table 2.1 displays the main functions implemented in the package, along with their associated `S3` classes. The three groups are estimation, pre-estimation, and post-estimation. Post-estimation can be further divided into model diagnostics, causality analysis, and forecasting.

Function name	S3 class	Description
`VECM()`	`'VECM'`	Fit the VECM
`Cointegration_test()`	`'Cointegration _test'`	Cointegration test
`port_test()`	`'port_test'`	Portmanteau test for residual autocorrelation
`BG_test()`	`'BG_test'`	Breusch-Godfrey test for serial correlation
`norm_test()`	`'norm_test'`	Nonnormality test
`ARCH_test()`	`'ARCH_test'`	ARCH-LM test
`chow_test ()`	`'ARCH_test'`	Chow structural break test
`granger_test()`	`'granger_test'`	Granger-causality test
`VECM_IR()`	`'VECM_IR'`	Impuls-response functions
`VECM_FEVD()`	`'VECM_FEVD'`	Forecast error variance decomposition
`VECM_forecast()`	`'VECM_forecast'`	Forecasts

Table 2.1: Summary of main functions and associated `S3` classes in **VECM**.

2.3.3 The Fit of the VEC Model

The VECM package provides various options for estimating VEC models, similar to those offered by other software such as Eviews and Stata. One unique feature of the function `VECM()` is the ability to estimate the model not only with the ML method but also with the EGLS method, which, as described in Section 2.2.1, is more powerful than ML in small samples. The function allows the user to select from the five specifications of the deterministic component as described in Lütkepohl (2005). The available options are

- `spec = "none"`: Neither constant nor trend, neither inside nor outside the ECT,
- `spec = "rconst"`: A constant term inside the ECT, but no constant or trend outside the ECT,

- `spec = "uconst"`: A constant term outside the ECT, but no constant or trend inside the ECT,
- `spec = "rtrend"`: A constant term and a trend term outside the ECT, with a trend inside the ECT,
- `spec = "utrend"`: One constant term and one trend term outside the ECT, but no trend inside the ECT,

The function enables the incorporation of exogenous variables into the model, both inside and outside the ECT, with the option to select a lag order for them.

The user can include seasonal dummy variables in the model and choose the reference season. However, to insert dummies, a spec with a constant outside the ECT must be selected. Therefore, `"none"` and `"rconst"` are not available options.

Additionally, the function enables the insertion of a shift dummy variable in the presence of a structural break. The user can even manually construct other relevant dummy variables and input them as exogenous variables.

The function implements an `S3` method for `print()` that provides the loading matrix, the cointegrating matrix and the short-run parameters matrix. It also implements an `S3` method for `summary()` which not only gives various information and statistics of the estimated model but also provides estimates and significance for the `r` long run equations and the `K` equations of the system. It is associated even with a function `plot()` which when loaded, the user can select a number from one to ten in the dialogue box that appears. The available options are:

1. Error correction terms
2. Raw residuals
3. Raw residuals standardized
4. Squared residuals
5. Squared residuals standardized
6. Autocorrelation functions of raw residuals
7. Autocorrelation functions of squared residuals
8. Partial autocorrelation functions of raw residuals
9. Partial autocorrelation functions of squared residuals
10. Kernel density estimations of raw residuals

If the error correction term is chosen to be printed out, four additional options are available.

1. Representation without deterministic terms
2. Representation with deterministic terms (if any)
3. Representation without deterministic terms and short-run dynamics
4. Representation with deterministic terms (if any) but without short-run dynamics

The plot function can receive arguments such as the level for estimating confidence intervals (`alpha`), the maximum lag used in the calculation of the (partial) autocorrelation function (`p_max`), a logical value indicating whether cross-correlation is desired or not (`cross`), and the smoothing kernel used in the calculation of the kernel density estimates (`kernel`).

The `VECM` package also includes the function `coef.VECM()` to extract parameter estimates from an object of class `'VECM'` and the function `VECM_to_VAR()` to switch from the VEC model to the corresponding levels VAR model.

2.3.4 Methods for Specifying the Model

The function `cointegration_test()` provides two methods for estimating the cointegration test. By setting `method = "info_criteria"`, the information criteria of a series of models with increasing rank are estimated. The cointegration rank corresponding to the lowest value of information criteria is the optimal rank. The function estimates three information criteria are estimated, namely AIC, BIC and HQC.

By setting `method = "johansen"` the function will perform the Johansen test. This method uses two other functions, `johansen_distr()` which estimates the Johansen distribution and `johansen_stat()` which estimates the percentiles of the trace and the maximum eigenvalue test statistic from the estimated distribution. The distribution is affected by the choice of lag order and deterministic components.

The main function is associated with a print function that displays a table on the screen. The table contains the eigenvalues and test statistics, quantiles, and p-values of the trace and maximum eigenvalue tests. These values correspond to the different cointegrating ranks indicated in the first column.

The package includes a function, `VECM_spec()`, which identifies the most suitable specification. The user must specify the lag and cointegration rank, as well as a null hypothesis model (either `"none"`, `"rconst"`, `"uconst"`, or `"rtrend"`) and an alternative hypothesis model. The test works with nested models, allowing for testing of `"uconst"` against `"rtrend"`, but not the reverse. The printout provides information on the two tested models, the test statistic, degrees of freedom, and p-value.

2.3.5 Methods for Diagnostics

The package provides several functions to assess the goodness of fit of the model. The function `stability_check()` plots the real and the complex parts of the roots of the characteristic polynomial, plus a circle of unit radius within which the roots must stay. The VEC model has at least one unit root; if other roots are close to the circumference, it could be a symptom of misspecification of the model.

To detect autocorrelation in errors, two tools are available: the functions `BG_test()` and `port_test()`. The former performs the Breusch-Godfrey test and it is recommended for testing a low lag number, preferably less than or equal to that of the estimated VEC model. The latter function performs a portmanteau test that is used for larger horizons. The `BG_test()` function also allows alternative versions to be used for testing. In addition to the likelihood ratio (`"LR"`), the two Lagrange multiplier (`"LM"`) and Rao (`"Rao"`) versions can be selected. The correct statistics for the number of parameters are also calculated for the first two.

The normality of the residuals is tested using the Jarque-Bera test in the function `normality_test()`. The command `type` allows you to select the type of factorisation of the covariance matrix of the residuals. The available options are `"Lutkepohl"`, `"Doornik-Hansen"`, and `"Urzua"`. The function outputs three tables. The text provides skewness and kurtosis estimates for individual equations and the entire system, along with the corresponding test statistics. Additionally, it reproduces the results of the Jarque-Bera test.

The function `ARCH_test()` tests the null hypothesis of homoschedasticity of the residuals by an LM test. As with the `BG_test()` function, the lag of the auxiliary regression can be chosen. It also implements an `S3` method for `print()` similar to the other tests in the package.

If a structural shift is suspected, it can be tested using the function `chow_test()`. This function tests for the presence of a structural shift by implementing the sample-split (`"SS"`), break-point (`"BP"`) and forecast (`"F"`) versions of the Chow test. The user must indicate the location of the suspected shift by entering a value between one and the sample size. In the future, the function will allow the user to specifically select the period of the time series.

2.3.6 Methods for Causality Analysis

Causality between variables can be studied with the help of three main tools provided within the package: a Wald test for Granger-causality, impulse-response functions, and forecast error variance decomposition. The Granger-causality test is performed by the function `granger_test()`. When an object of class `'VEC'` is provided, the function enables causality testing for both individuals and groups of variables. This is achieved by entering their names into a character vector in the arguments `dip` and `indip`.

The function `VECM_IR()` allows the estimation of pulse-response functions. By setting `transform = "orthogonal"`, orthogonal IRFs will be estimated. In addition, the upper and lower extremes will also be calculated via the covariance matrix estimated by setting `method = "analytic"`. In a later version of the package, the opportunity to estimate the confidence intervals by the bootstrap method will be made available as well. In addition to the ability to display IRFs in tabular format via the `print()` function, it is also possible to represent them in graphical format via the implemented `S3` method `plot()`.

Finally, the function `VECM_FEVD()` can be used to estimate the forecast error variance decompositions up to a given horizon. The function returns the forecast standard errors and the percentage of the forecast variance due to each random innovation. The function implements an `S3` method for `print()` which displays FEVDs in tabular format. It also implements an `S3` method for `plot()` which plots FEVDs in stacked columns.

2.4 Applications

The literature extensively discusses the link between public debt, income inequality, and economic growth. It suggests that an increase in income inequality leads to a growth in public debt and a reduction in economic development. Regarding the link between public debt and economic growth, empirical analyses have repeatedly demonstrated the existence of, even if no firm conclusions have been reached on this effect, nor the direction of the causal link.

To illustrate the package **VECM**, we use the data on the three variables and apply the several functions of the package. The United States is selected as a case study due to the abundance of available data. we use the Gini index as a measure of inequality and GDP per capita as a measure of economic growth, both extrapolated from the WID[3] database. Additionally, we obtain data on public debt to GDP from FRED. We test that our variables are integrated of order one using the function `kpss.test()` of the package **tseries**.

```
R> require(readxl)
R> data <- read_excel("DebtIneqGDP.xlsx")
```

[3] https://wid.world/

```
R> set.seed(2)
R> data <- log(data[,c("debt", "gini", "percapgdp")])
R> require(tseries)

R> kpss.test(data$gini)

KPSS Test for Level Stationarity

data:  data$gini
KPSS Level = 1.2828, Truncation lag parameter = 3, p-value = 0.01

Warning message:
In kpss.test(data$gini) :  p-value smaller than printed p-value

R> kpss.test(data$percapgdp)

KPSS Test for Level Stationarity

data:  data$percapgdp
KPSS Level = 1.5177, Truncation lag parameter = 3, p-value = 0.01

Warning message:
In kpss.test(data$percapgdp) :  p-value smaller than printed p-value

R> kpss.test(data$debt)

KPSS Test for Level Stationarity

data:  data$debt
KPSS Level = 1.3746, Truncation lag parameter = 3, p-value = 0.01

Warning message:
In kpss.test(data$debt) :  p-value smaller than printed p-value
```

The KPSS tests indicate that the variables have at least one unit root. We compute the first difference and apply once again the test.

```
R> kpss.test(diff(data$gini))

KPSS Test for Level Stationarity
```

```
data:  diff(data$gini)
KPSS Level = 0.17105, Truncation lag parameter = 3, p-value = 0.1

Warning message:
In kpss.test(diff(data$gini)) :  p-value greater than printed p-value

R> kpss.test(diff(data$percapgdp))

KPSS Test for Level Stationarity

data:  diff(data$percapgdp)
KPSS Level = 0.081482, Truncation lag parameter = 3, p-value = 0.1

Warning message:
In kpss.test(diff(data$percapgdp)) :  p-value greater than printed p-value

R> kpss.test(diff(data$debt))

KPSS Test for Level Stationarity

data:  diff(data$debt)
KPSS Level = 0.27307, Truncation lag parameter = 3, p-value = 0.1

Warning message:
In kpss.test(diff(data$debt)) :  p-value greater than printed p-value
```

We do not reject the null hypothesis of stationarity, so we can assume that the variables are integrated of order one. We now pass to test the presence of cointegration, starting with the estimation of the lag order with the help of the function `VARselect()` of the package **vars**.

```
R> VARselect(data, type = "both", lag.max = 5)

$selection
AIC(n)       HQ(n)       SC(n)       FPE(n)
2            2           2            2

$criteria
```

```
IC      1             2             3             4             5
AIC(n) -2.204631e+01 -2.257085e+01 -2.248465e+01 -2.231919e+01 -2.224941e+01
HQ(n) -2.183052e+01 -2.222559e+01 -2.200992e+01 -2.171499e+01 -2.151573e+01
SC(n) -2.148345e+01 -2.167027e+01 -2.124636e+01 -2.074319e+01 -2.033569e+01
FPE(n) 2.668008e-10 1.587866e-10 1.751825e-10 2.111668e-10 2.342408e-10
```

Given the uniform lag order of the VAR given by the information criteria, we choose to take one as the lag order of our VEC model. The KPSS test suggests that there is no trend. We test the Johansen cointegration test with spec = "none".

```
R> cointegration_test(data, 1, "none", "johansen")

COINTEGRATION TEST

Method:  Johansen
Model specification:  none

Number of observations:  55
Lag order:  1

TRACE TEST

 rank  param  eigenvalue  stat      quantile  p-value
 0     9      NA          38.62238  24.24335  0.00013
 1     14     0.41922     8.73620   12.35196  0.18280
 2     17     0.09917     2.99236   4.17476   0.10087
 3     18     0.05295     NA        NA        NA

MAXIMUM EIGENVALUE TEST

 rank  param  eigenvalue  stat      quantile  p-value
 0     9      NA          29.88618  17.69638  0.00073
 1     14     0.41922     5.74384   11.27296  0.37733
 2     17     0.09917     2.99236   4.17476   0.10087
 3     18     0.05295     NA        NA        NA

- quantiles are at the 0.95 level
- quantiles and p-values computed by sampling from the distributions of
the test statistics
- tests do not take exogenous variables into account
```

Both the trace and the maximum eigenvalue tests are performed and printed. The print shows the quantiles at the given alpha. In this case, we choose to take the default value of 0.05 which corresponds to a 0.95 level. The test suggests the presence of a cointegrating relation both with no deterministic terms and with a restricted trend.

```
R> cointegration_test(data = data, p = 1, spec = "none",
method = "info_criteria")

COINTEGRATION TEST

Method:  information criteria
Model specification:  none

Number of observations:  55
Lag order:  1

 rank  param  eigen    log-lik    AIC       BIC       HQC
 0     9      NA       379.29136  -13.46514  -13.13667  -13.33812
 1     14     0.41922  394.23445  -13.82671  -13.31575  -13.62912
 2     17     0.09917  397.10637  -13.82205  -13.20160  -13.58212
 3     18     0.05295  398.60255  -13.84009  -13.18315  -13.58605

- chosen rank:  r = 3 (AIC) r = 1 (BIC) r = 1 (HQC)
- the test takes exogenous variables into account
```

The result is confirmed even by the information criteria test with BIC and HQC.

We then estimate our VEC model with lag order and cointegrating rank equal to one, and no deterministic terms. We use the EGLS estimator given the relatively small sample. The other arguments are left as the default.

```
R> vec_egls <- VECM(data = data, p = 1, r = 1, method = "EGLS", spec
= "none")
R> summary(vec_egls)

---------------
VECTOR ERROR CORRECTION MODEL
---------------
```

```
Estimation method:  EGLS
Deterministic terms:  none

Number of observations after adjustments:  55
Number of estimated parameters:  14

Lag order:  1
Cointegration rank:  1

log-likelihood:  398.60255
det(sigma_u):  1.01774e-10

AIC: -13.94918
BIC: -13.40173
HQC: -13.73748

Significance levels:  *** 0.01 ** 0.05 * 0.1

COINTEGRATING VECTORS

Normalization with respect to:  debt_L
```

Variable	Estimate	S.E.	T-Value	P-Value	
gini_L	-3.01890	0.44393	-6.80036	0.00000	***
percapgdp_L	-0.60463	0.03435	-17.60055	0.00000	***

```
DIFFERENCED VAR EQUATIONS

Equation 1:  debt_D
```

Variable	Estimate	S.E.	T-Value	P-Value	
ECT_1	-0.09172	0.01944	-4.71800	0.00002	***
debt_LD1	0.56087	0.08684	6.45840	0.00000	***
gini_LD1	-0.29684	0.22009	-1.34872	0.18339	
percapgdp_LD1	-1.08202	0.21487	-5.03560	0.00001	***

```
--
E(debt_D): 0.02056
SD(debt_D): 0.05016
Multiple R-squared:  0.58388
Adjusted R-squared:  0.55941
Residual standard error:  0.03330
```

```
Durbin-Watson statistic:  2.35317

Equation 2:  gini_D

 Variable       Estimate  S.E.     T-Value   P-Value
 ECT_1          -0.00247  0.01074  -0.23026  0.81881
 debt_LD1       0.15694   0.04797  3.27167   0.00192 ***
 gini_LD1       0.15192   0.12157  1.24965   0.21713
 percapgdp_LD1  -0.14706  0.11869  -1.23906  0.22100

--
E(gini_D): 0.00267
SD(gini_D): 0.02018
Multiple R-squared:  0.21586
Adjusted R-squared:  0.16973
Residual standard error:  0.01839
Durbin-Watson statistic:  1.94251

Equation 3:  percapgdp_D

 Variable       Estimate  S.E.     T-Value  P-Value
 ECT_1          0.01592   0.01165  1.36571  0.17802
 debt_LD1       0.10316   0.05206  1.98140  0.05295 *
 gini_LD1       0.11413   0.13195  0.86501  0.39109
 percapgdp_LD1  0.09407   0.12882  0.73029  0.46855
--
E(percapgdp_D): 0.01389
SD(percapgdp_D): 0.02055
Multiple R-squared:  0.10912
Adjusted R-squared:  0.05671
Residual standard error:  0.01996
Durbin-Watson statistic:  1.96986

F-TEST TABLE (H0:  all parameters = 0)

 Equation  TSS      RSS      F-Value   P-Value
 1         0.13588  0.05654  23.85391  0.00000 ***
 2         0.02200  0.01725  4.67968   0.00580 ***
 3         0.02281  0.02032  2.08220   0.11410
--
DF numerator:  3
DF denominator:  51
```

```
RESIDUALS OF DIFFERENCED VAR EQUATIONS

 Equation  Minimum   Q1        Median    Mean      Q3       Maximum
 1         -0.05441  -0.02372  -0.00157  0.00001   0.01324  0.10708
 2         -0.06430  -0.00760  0.00070   -0.00002  0.01372  0.02730
 3         -0.04869  -0.00672  0.00164   0.00003   0.01271  0.04768
RESIDUAL COVARIANCE MATRIX BETWEEN DIFFERENCED VAR EQUATIONS

             debt_D   gini_D    percapgdp_D
 debt_D      0.00103  -0.00009  -0.00009
 gini_D               0.00031   0.00011
 percapgdp_D                    0.00037
MOVING-AVERAGE IMPACT MATRIX

             debt_D    gini_D   percapgdp_D
 debt_L      -0.17757  9.31727  0.42417
 gini_L      -0.08870  3.00586  -0.04421
 percapgdp_L 0.14921   0.40170  0.92225
```

The summary starts by giving general information about the fitted model like the final sample size, the number of the estimated parameters, the log-likelihood and the information criteria. It then passes to show the estimated cointegrating equations. In this case, there is only one long-run equilibrium. The equation is normalized with respect to the first variable of the dataset, in this case, public debt. The resulting estimates for a correct interpretation are to be ridden with opposite signs. The results seem to confirm the results of prior empirical analysis about the relation of public debt with inequality and economic growth. However, the cointegrating equation does not give information about the direction of the causal link.

After the cointegrating equations, the summary displays separately the first difference VAR equations, giving each some statistics as an initial goodness-of-fit analysis of the model. The ECT is the parameter associated with the error correction term and gives information on the speed of the come-back of the variable to the long-run equilibrium. The ECT is significant only for the debt equation, giving further information about possible causal effects produced by the other two variables. Moreover, inequality and economic growth are suspected weak exogenous variables. The parameters that are identified with LD give information on the effects of short-run dynamics.

After the first-difference equations the summary prints further information to support the user. For each equation, an F-test is made to verify the goodness-of-fit. Some descriptive statistics of the residuals are given, as well as the residual covariance matrix between the equations and the moving-average impact matrix.

Using the function `plot()` with an object of class VECM, the following menu is printed. The user can then select the graph of interest.

```
R> plot(vec_egls)

Make a plot selection or press ESC to exit.  Options available:

1:  error correction terms
2:  raw residuals
3:  raw residuals standardized
4:  squared residuals
5:  squared residuals standardized
6:  autocorrelation functions of raw residuals
7:  autocorrelation functions of squared residuals
8:  partial autocorrelation functions of raw residuals
9:  partial autocorrelation functions of squared residuals
10:  kernel density estimations of raw residuals

Enter your selection:
```

By selecting the first option, the user has four further options to choose from.

```
Which representation of the error correction terms do you want to plot?

1:  representation without deterministic terms
2:  representation with deterministic terms (if any)
3:  representation without deterministic terms and short-run dynamics
4:  representation with deterministic terms (if any) but without short-run
dynamics

Enter your selection:
```

Figure 2.1 shows the error correction term representation without deterministic terms - in this case is the same as choosing the option with data spec = 'none' in the build - and without short-run dynamics. If the user plots the raw residuals standardized or the autocorrelation functions, a further argument (`alpha`) may be given to change the range of the bands of confidence. Figure 2.2 shows the raw residuals and the ACFS.

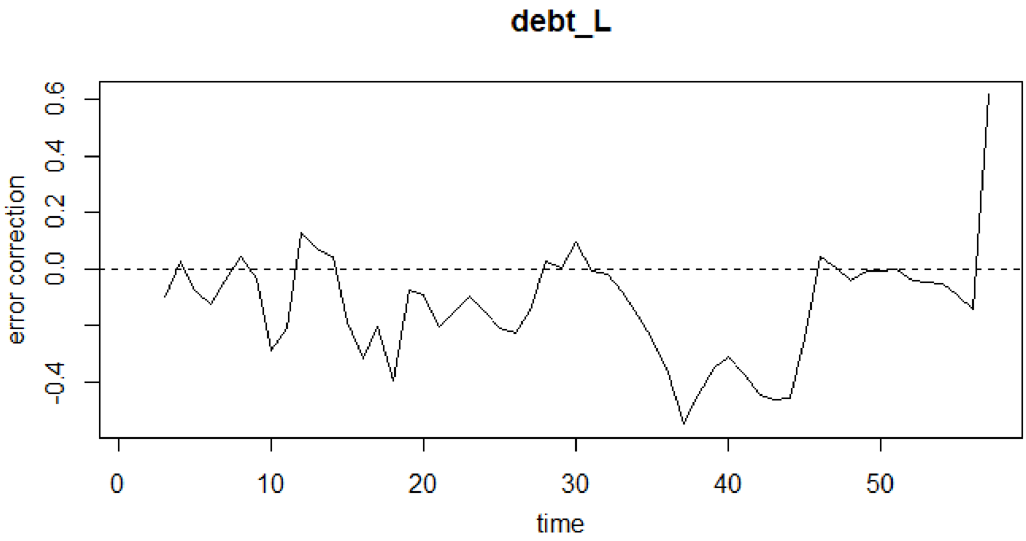


Figure 2.1: Error correction term

```
R> plot(vec_egls, alpha = 0.10)
```

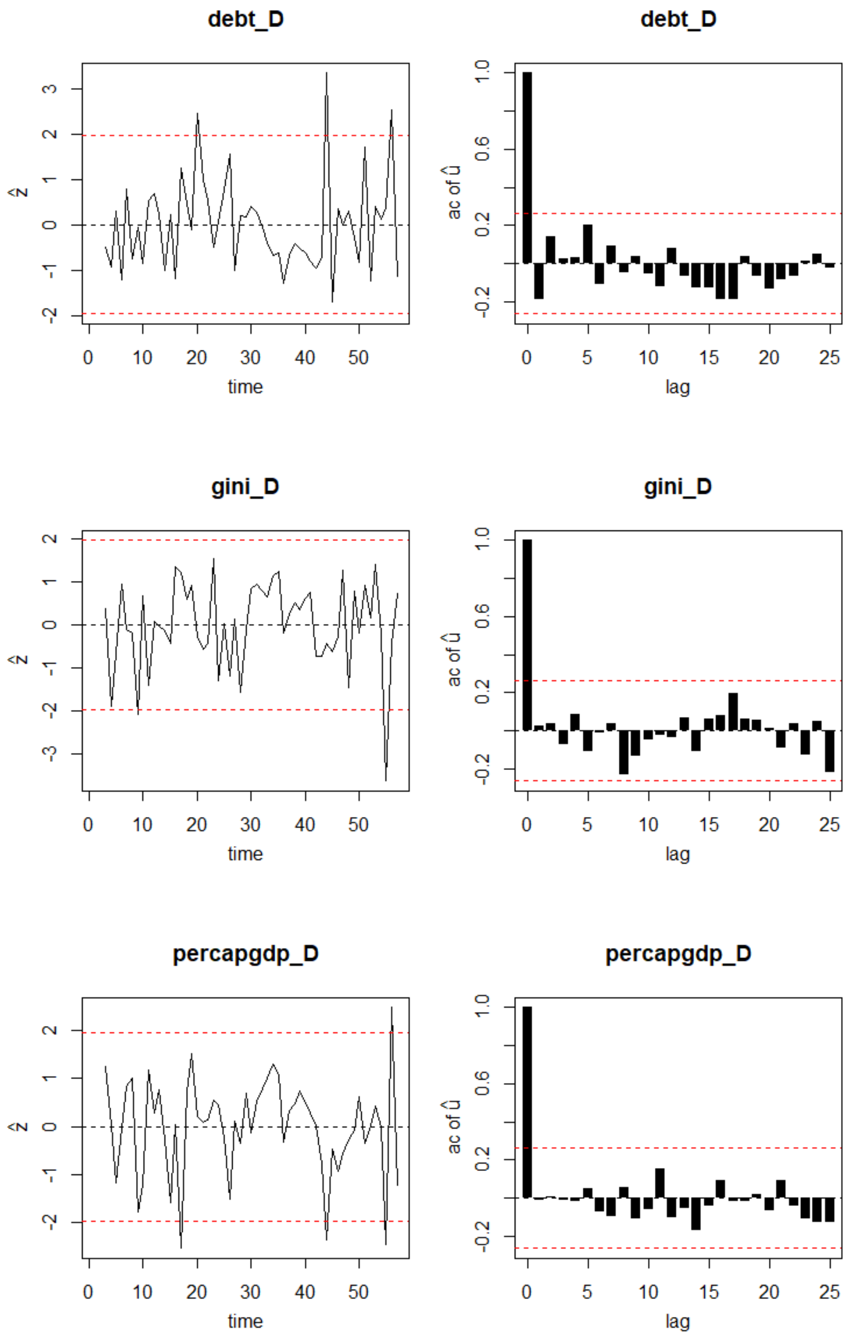
Once the model is estimated, we pass to perform some diagnostic tests. We start by performing the function `VECM_spec()` to test the presence of a restricted constant versus the absence of any deterministic term. The result of the test confirms the choice of the deterministic component.

```
R> VECM_spec(data = data, r = 1, p = 1, spec_H0 = "none", spec_H1 = "rconst")

LIKELIHOOD-RATIO TEST FOR DETERMINISTIC SPECIFICATION

H0:  spec = none
H1:  spec = rconst

Chi-squared:  4.05795 df:  2 p-value:  0.13147
```



(a) Raw residuals standardized (b) Autocorrelation functions

Figure 2.2: Residuals and residual autocorrelations of VECM of public debt to GDP, income Gini index and GDP per capita of the United States from 1966 to 2022

We then pass to test the presence of serial correlation of the residuals using the Breusch-Godfrey test and the portmanteau test respectively with lag equal to two and ten. For the former, we even choose to perform the Rao version of the test. Both the tests indicate that the residuals are not correlated.

```
R> BG_test(object = vec_egls, lag = 2, type = "Rao")

BREUSCH-GODFREY TEST

Null hypothesis:  residuals are not autocorrelated up to lag 2

Test version:  Rao

F-value:  0.71305, df:  (18, 122), p-value:  0.79236


R> port_test(object = vec_egls, lag = 10)

PORTMANTEAU TEST

Null hypothesis:  residuals are not autocorrelated up to lag 10

Chi-squared:  53.27291, df:  87, p-value:  0.99835

Adjusted Chi-squared:  58.92847, df:  87, p-value:  0.99088
```

The residuals' normality is another test to verify the correct model specification, although it is not essential since most results are asymptotically normally distributed. The function `normality_test()` is used with the Urzua factorisation of the covariance matrix. It was found that the residuals of the first two equations, which are associated with debt and inequality, are not normally distributed. However, the residuals associated with GDP per capita do not reject the null hypothesis. There may be several reasons for this, such as a small sample size, omitted variables, or an incorrect specification.

```
R> normality_test(object = vec_egls, type = "Urzua")


VEC residual normality tests
Orthogonalization:  Urzua
```

```
Null hypothesis:  residuals are multivariate Normal

 Variable     Skewness  Chi-squared  df  P-Value
 debt_D       1.06917   11.67560     1   0.0006333 ***
 gini_D       -0.79449  6.44707      1   0.0111135 *
 percapgdp_D  -0.33202  1.12593      1   0.2886445
 joint                  19.24861     3   0.0002429 ***

 Variable     Kurtosis  Chi-squared  df  P-Value
 debt_D       4.4447    7.2239       1   0.007194 **
 gini_D       3.9988    3.6694       1   0.055420 .
 percapgdp_D  3.7796    2.3587       1   0.124585
 joint                  13.2520      3   0.004122 **

 Variable     Jarque-Bera  df  P-Value
 debt_D       18.8995      3   7.871e-05 ***
 gini_D       10.1165      3   0.006357 **
 percapgdp_D  3.4846       3   0.175113
 joint        32.5006      6   1.308e-05 ***
```

Another possible reason is the display of unit reductions provided by the `stability_check()` function. The number of unit roots must be equal to $K - r$. In this case, however, there is a third value close to the unit circle, suggesting further instability in the data.

```
R> stability_check(vec_egls)

$Re
[1] 1.00000000 1.00000000 0.93958699 0.34149089 0.34149089 0.09041631

$Im
[1] 0.0000000 0.0000000 0.0000000 0.4057227 -0.4057227 0.0000000

$Mod
[1] 1.00000000 1.00000000 0.93958699 0.53030836 0.53030836 0.09041631
```

We now move on to investigate possible causal effects between the variables under consideration. We start by performing the Granger-causality test. Below is the typical print of the function `granger_test()`, setting `dip = "debt"` as the dependent variable and `indip = "gini"` as the independent variable. The null hypothesis that income inequality does not Granger-cause public debt is rejected with a p-value lesser than 0.01.

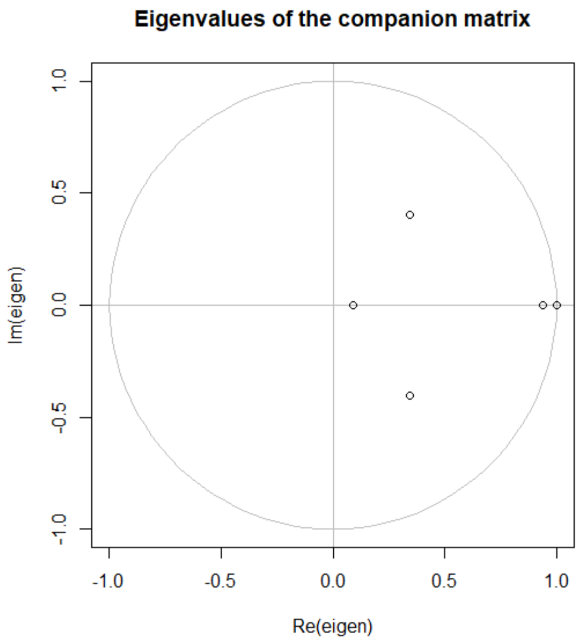


Figure 2.3: The eigenvalues of the companion matrix.

```
R> granger_test(object = vec_egls, dip = "debt", indip = "gini")

GRANGER-CAUSALITY TEST

Null hypothesis:  gini does not Granger-cause debt

Chi-squared:  10.59083, df:  2, p-value:  0.00501
```

Table 2.2 presents the results of the Granger-causality tests performed on the estimated VEC model. Both income inequality and GDP per capita have an impact on public debt, as well as the level of public debt itself. At the same time, public debt affects both the inequality and the economic measures.

The second tool to test causality is the forecast error variance decomposition. The function `VECM_FEVD` has a horizon equal to 10 as the default but it can be changed by setting the argument `horizon` with the desired horizon. Figure 2.4 shows the plot of the three FEVDs. We can see how the share of debt variance explained by inequality and GDP tends to grow over time. The same is visible

	debt		gini		percapgdp	
	stat	p-val	stat	p-val	stat	p-val
debt	33.60	<0.01	8.48	0.01	4.90	0.08
gini	10.59	<0.01	1.34	0.51	1.37	0.50
percapgdp	33.21	<0.01	1.25	0.53	1.31	0.51

Table 2.2: Results of the Granger-causality test with dependent variables in column and independent variables in row.

with economic growth where the contribution of inequality tends to rise.

```
R> fevd <- VECM_FEVD(vec_egls)
R> par(mfrow = c(1,3), pty = "s")
R> plot(fevd)
```

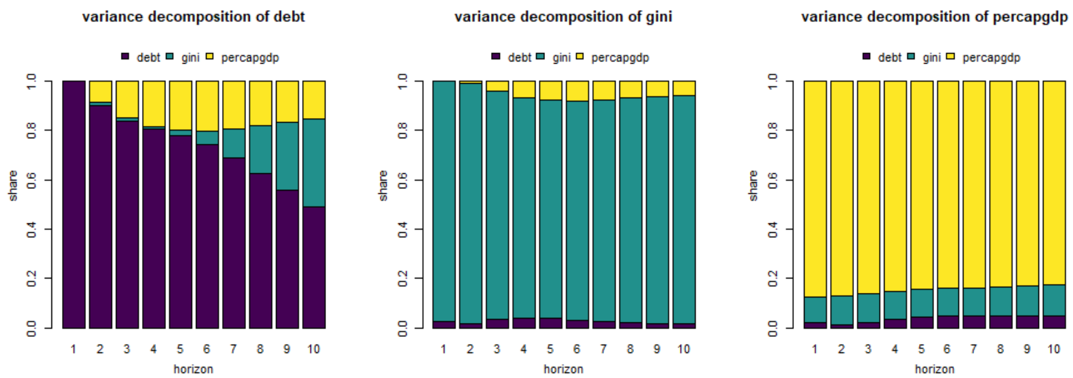


Figure 2.4: Forecast error variance decomposition with horizon equals to 10

The third and final tool for analysing causality is the impulse response function. The user can choose different options of settings as seen in the previous section.

In this example, we have chosen to calculate orthogonal IRFs and estimate the corresponding standard errors. The latter setting is necessary in order to be able to plot the confidence intervals later. The final result can be seen in figure 2.5.

```
R> ir <- VECM_IR(object = vec_egls, transform = "orthogonal", method
= "analytic")
plot(ir)
```

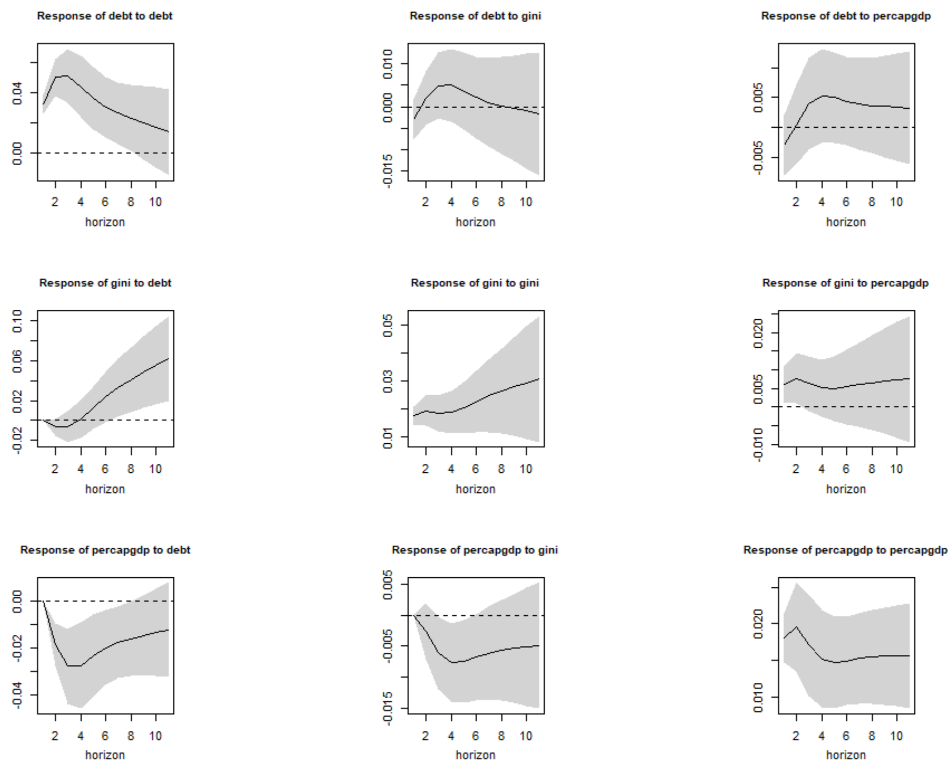


Figure 2.5: Impulse-response functions with horizon equals to 10

Although the three tools provide results that do not always agree, the user must take into account when drawing conclusions that these tools deal with different aspects of relationships.

Finally, the user can make predictions on the future trends of the variables. Here too, the user has various settings at his disposal, depending on his needs. We decided to opt for the default settings in this example. Figure 2.6 shows the estimated forecasts. Government debt and inequality look set to decline over time, but the confidence interval is quite wide especially for the latter. This is probably due to the small sample. GDP per capita, on the other hand, seems set to grow steadily.

```
R> forecast <- VECM_forecast(object = vec_egls, type = "dynamic")
R> par(mfrow = c(3,1))
R> plot(forecast, type = "area", observed = F)
```

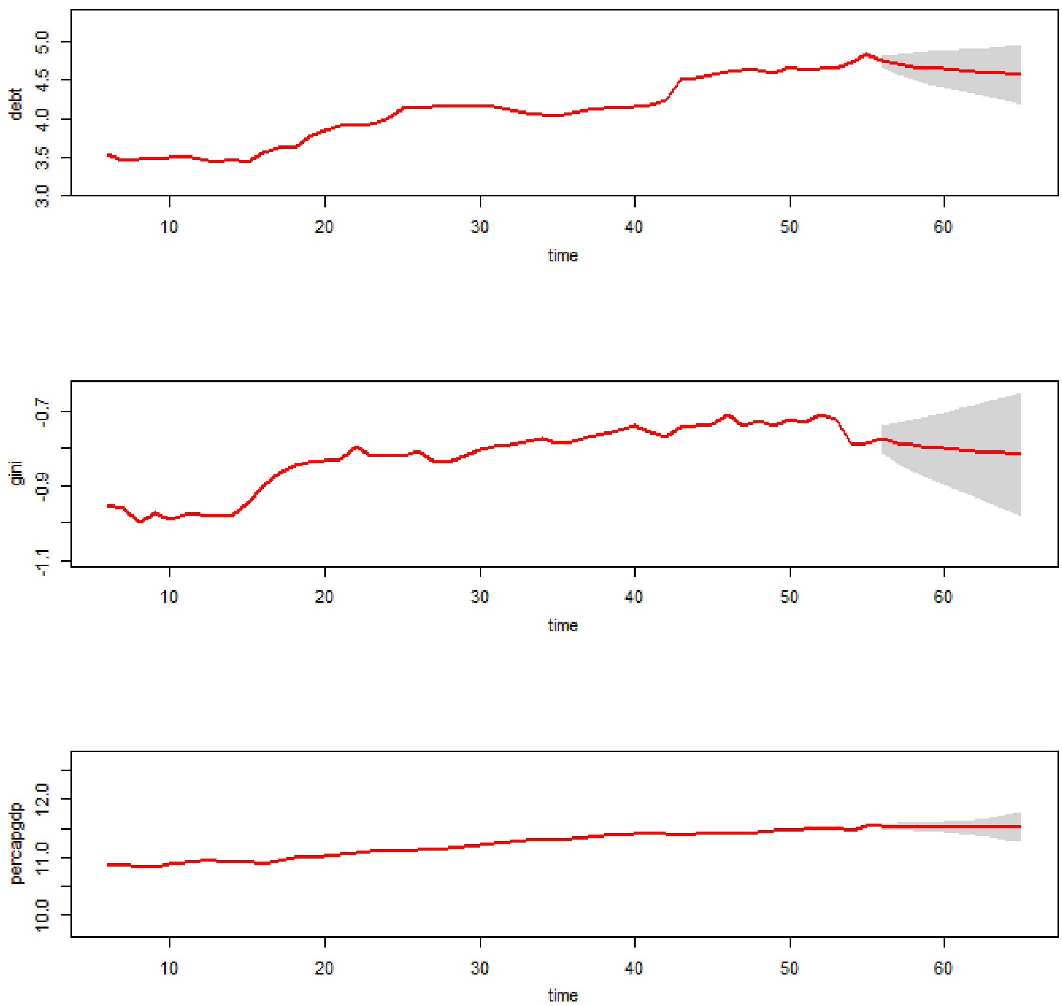


Figure 2.6: Forecasts of the endogenous variables with horizon equals to 10

2.5 Closing remarks

The prevalence of variables being interconnected leads most classical models to produce biased results. Models such as VAR and VEC make it possible to overcome these problems. Numerous software packages allow a relatively practical use of these approaches, but they are mostly software for which one has to pay or with limited methods and tools.

R software, through its free and open-source nature, allows these limitations to be overcome, aiming to produce good and reliable output. Some packages for VAR and VECM model estimation are already available in the CRAN environment. For the latter, however, the functions are quite limited and in some cases do not seem to be supported by any literature. Furthermore, for more meticulous analyses they must rely on other packages, making it complicated for the user to find the correct tools for an adequate analysis of the data.

The package presented in this chapter aims to overcome the aforementioned limitations by providing as complete an environment as possible of tools and settings on a par with paid software. Although this objective has mostly been achieved, further work is required to introduce new functions and to extend and enhance the performance of old ones.

In particular, one wants to create a function that allows one to introduce parameter constraints for more meaningful estimates or to test theories proposed in the literature; one wants to make the function `cointegration_test()` faster by supplementing the code with links to **C++**.

In addition, the package is already downloadable from github and it is intended to propose the use of the package to an audience of researchers in order to identify critical issues and gather suggestions.

CHAPTER

3

PUBLIC DEBT AND WEALTH INEQUALITY: AN EMPIRICAL INVESTIGATION

3.1 Introduction

Recent reports from international institutions highlight that levels of inequality (Chancel et al., 2022) and public debt (IMF, 2020) have risen in both advanced and developing economies in recent decades.

Several national and global events have played a significant role in the rise of public debt. In response to the 2008 financial crisis, governments were compelled to introduce substantial spending programs, largely financed through borrowing. The COVID-19 pandemic further strained public finances, as countries implemented massive fiscal measures to mitigate the economic fallout of lockdowns and support households. As a result, the global average public debt-to-GDP ratio surpassed 100% in 2020. Though it dipped below this threshold the following year, the surge in sovereign debt levels has sparked renewed concerns about long-term fiscal sustainability.

Economic inequalities have also widened in many countries alongside rising public debt. Notably, wealth distribution has shifted markedly toward the upper end, with a significant and persistent concentration of wealth at the top. However, the pace and magnitude of this inequality growth have differed between regions. In Europe, inequality has increased more gradually compared to the United States, where the concentration of wealth has accelerated more sharply.

The United States presents an interesting case for studying the relationship between public debt and inequality. Before Ronald Reagan's presidency in 1981, the U.S. economy struggled with stagnant economic growth, high unemployment, and soaring inflation, which defined much of the 1970s. The traditional Keynesian policies were ineffective, prompting a shift in economic thinking. When Reagan took office, he implemented a series of economic measures known as Reaganomics, rooted in supply-side economics. The core components of his policies included significant tax cuts, particularly for the wealthy and businesses, aimed at stimulating investment and economic growth. Reagan also sought to reduce government regulation, cut social spending, and increase defence expenditures.

These policies had profound consequences. Initially, the U.S. experienced a sharp recession in the early 1980s, but by mid-decade, the economy rebounded, leading to robust growth, lower inflation, and increased employment. However, public debt soared due to increased defence spending and reduced government revenue from tax cuts. Social programs were scaled back, leading to greater hardship for lower-income families. While Reagan's policies boosted economic growth and reshaped the U.S. economic landscape, they also contributed to long-term fiscal imbalances and heightened economic disparities. These shifts marked a turning point, driving the U.S. toward a more pronounced wealth gap than many European countries. This gap has continued to widen over time.

We chose the United States as the case study given its background and the data available. The United States has extensive data availability for a range of economic variables. Furthermore, empirical analysis leads the United States to be among the countries with the longest and most consistent time series on wealth inequality.

The relationship between public debt and economic inequality has been extensively explored in theoretical literature, examining potential connections between debt, income, and wealth distribution. However, a key challenge has been the scarcity of reliable data on wealth distribution, which has caused most empirical studies to focus solely on income. Consequently, researchers have primarily concentrated on studying the effects of income inequality on public debt, leaving the impact of wealth distribution under-explored. This narrow scope has limited the practical relevance of many theories.

This paper explores the dynamics between public debt and wealth inequality from an empirical perspective. To this end, we apply a Vector Error Correction (VEC) model, which allows for two-way causality and both long and short-run analysis, thereby solving the problems of stationarity and endogeneity. Focusing on the wealth share held by the richest 10 per cent of the population, we find some results in contrast with the prior literature, that is, the transmission mechanisms for income inequality may not work for wealth.

The rest of this paper is organized as follows. The next section discusses the literature review and previous empirical evidence. Section 3.3 presents the applied strategy. Section 3.4 analyzes the results, and Section 3.5 states the conclusions.

3.2 Background Literature

Recently, with the global rise in inequality levels, there has been an intensified research effort to study the impact of economic inequality on the economic system. Nowadays, scholars no longer treat inequality as a mere outcome variable, but in many cases even as an input (Benhabib et al., 2003; De Dominicis et al., 2008; Alesina and Perotti, 1996; Albanesi, 2007; Siami-Namini and Hudson, 2019).

Most works treated the distribution of income, as well as the distributions of earnings and wages, thanks to the large amount of data available for these distributions, leaving less space for wealth.

Income is an essential element for explaining wealth disparities, but income cannot be considered as a proxy for wealth. Wealth is proved to be much more concentrated than income, and the literature suggests only a partial correlation between the two economic distributions. Hence, many results linked to income may not be valid for wealth.

When we choose to study wealth inequality, several concerns arise. The main one is the limited availability of data on the variable. However, efforts have been made in the last few years to build an adequate cross-country database. We have the pioneering work of Piketty et al. (2018) for the US and, following it, that of Blanchet and Martínez-Toledano (2023) for the European countries.

Despite the methodological concerns that may arise in studying wealth inequality, the rationale for pursuing this study is twofold: (i) to add a new perspective to research, and (ii) to test whether the results found for income are also valid for wealth. Moreover, wealth inequality better reflects the social and economic gaps within a society and the economic interactions between public debt and income may not hold for the wealthy counterpart. Public debt impacts directly on income only with interest on government bonds.

We have divided the literature review into two subsections: the first explores the topic from a theoretical standpoint, while the second collects relevant empirical studies. The theoretical basis identifies two possible paths, but for data availability, empirical analysis has largely been constrained to one of these, leading to partial conclusions that may not always apply to wealth. A third subsection describes the possible transmission mechanisms.

3.2.1 Theoretical framework

The relationship between public debt and economic inequality has been widely explored in theoretical literature. Scholars have examined the potential connection in both directions; however, the focus has been mostly on one-way causality. As a result, theoretical studies can be divided into two main branches, each addressing one side of this relationship.

In discussing the impact of national debt on inequality, the foundational works of Barro (1974) and Diamond (1965) are notable. Although they do not directly address the relationship between these two variables, their models offer insights into how public debt affects fiscal policies and economic systems over time.

Barro's work, based on Ricardian equivalence, supports the thesis that public debt is neutral in the economy. Fiscal policies redistribute tax burdens across generations through bonds and increasing debt. However, households smooth out consumption over their lifetimes and the ones of their descendants, anticipating future repayment and nullifying the effects of initial redistribution. Diamond (1965), while agreeing on consumption smoothing, contends that no inheritance is left to future generations, making current generations wealthier at the expense of the next. Both these theories treat the impacts of public debt on inter-generational iniquities, giving no information about intra-generational inequality.

Mankiw (2000) criticizes the assumption of homogeneous consumer behaviour in these models, introducing altruistic and non-altruistic agents with different time horizons, the former following the theory proposed by Barro (1974), the latter following the theory proposed by Diamond (1965). His model links wealth with altruism, supported by empirical studies showing that the rich save more than the poor.

Subsequent work by Maebayashi and Konishi (2019) and Borissov and Kalk (2020) builds on Mankiw's model, identifying a debt threshold that determines whether inequality stabilizes or escalates, showing that public debt correlates positively with inequality. Countries with debt levels below a threshold may stabilize inequality, but above this threshold, both public debt and inequality rise unchecked. Borissov and Kalk (2020) suggest that positional concerns exacerbate inequality, reducing savings among poor households while allowing the rich to accumulate wealth. These positional concerns may affect the threshold.

Turning now to the other direction of the link, a central mechanism linking inequality to public debt is fiscal policy decisions. In highly unequal societies, preferences for fiscal policies vary significantly between rich and poor households. The rich prefer lower taxes and rely on private consumption, while the poor push for higher marginal tax rates and increased social transfers. As inequality rises, the political pressure for redistributive policies intensifies. According to Harold (1929)'s median voter theorem, in representative democracies, politicians are incentivized to align their policies with the preferences of the median voter, who, in a highly unequal society, may represent the poorer sections of the population. This dynamic is supported by the theoretical works of Meltzer and Richard (1981) and Dixit and Londregan (1996), which argue that greater income inequality leads to higher redistributive spending as poorer households gain more political influence.

During periods of economic growth, increased public revenues may cover social transfers and store resources, but in times of recession, the government faces more challenges. In countries with high-income inequality, economic downturns can heighten conflicts between income groups. Governments may finance social transfers through public debt rather than increasing taxes on the wealthy, leading to deficits and rising debt levels (Milanovic, 1999). This approach avoids taxing the wealthy but also accumulates national debt, crowding out private investment and potentially harming long-term economic stability.

Other studies, like those of Azzimonti et al. (2014) and Arawatari and Ono (2017), investigate how cross-country differences in inequality affect public debt, linking higher inequality with increased debt. Azzimonti et al. (2014) suggest that income inequality is associated with an increase in income risk which, in turn, impacts public debt growth. Arawatari and Ono (2017) theorize that the rise of distributional inequality within a country leads to increased public debt and a reduction of foreign debt.

The instability due to inequality is supported even by other authors, like Kumhof et al. (2015) and Karayalçin and McCollister (2005), that connect inequality to sovereign debt defaults, suggesting that rising inequality can lead to excessive private debt and increased default risk.

In conclusion, theoretical literature describes public debt as a source of iniquities through agents' legacies and behaviours, while inequality affects public debt through fiscal policy pressures and economic instability. Although interdependence has not been studied directly, the study of one-to-one relationships suggests a bi-directional link. For further literature review, you can consult Chapter 1.

3.2.2 Empirical evidences

This paper has so far focused on the abundant theoretical evidence for the interdependence between national debt and wealth inequality. The following section will discuss empirical analyses and highlight potential weaknesses.

While theoretical literature explores the bidirectional relationship between public debt and inequality, the empirical literature is more limited, focusing predominantly on the unidirectional impact of income inequality on public debt.

Azzimonti et al. (2014) offer an empirical application within their theoretical framework. Using fixed-effect regression, they examine how income inequality and capital mobility affect the growth rate of real government debt. Income inequality is measured by the top 1% income share and average gross Gini coefficients, with data from 22 OECD countries covering 1995–2010. Their results suggest that higher income inequality correlates with increased public debt on short-run dynamics.

Similarly, Jabłoński et al. (2015) adopt an econometric model of public debt akin to Azzimonti et al. (2014), analyzing the same data period but with a broader inequality index reflecting both upper and lower income distribution. The results align with those of Azzimonti et al. (2014), indicating a positive relationship between income inequality and public debt. However, both studies rely on simplified models, as their primary focus is on theoretical validation rather than comprehensive empirical exploration.

Bittencourt (2015) conducts an empirical analysis of external debt in South America from 1970 to 2007. While economic growth emerges as the main factor in reducing debt, income inequality also shows some positive, even if not consistently significant, effects. This aligns with broader studies linking inequality and economic growth, hinting at indirect influences on public debt. Carrera and de la Vega (2021) provides a more comprehensive empirical analysis using a panel of 158 countries from 2000 to 2019. The study finds a significant positive relationship between income inequality, measured by the Gini index, and public debt as a percentage of GDP. Although Carrera and de la Vega (2021) attempts to test for a bidirectional relationship, the results show no evidence of debt affecting inequality.

Further empirical studies, such as those by Woo (2003) and Larch (2012), corroborate the link between inequality and public deficits, suggesting that countries with higher inequality tend to run larger deficits and accumulate more public debt. However, contrary findings from Aksman (2017), who use a dynamic panel data model, indicate no significant relationship between poverty, income inequality, and public debt, as countries with higher inequality appear to spend less on social benefits.

In a more localized study, Obiero and Topuz (2021) applies an ARDL model to examine Kenya from 1970 to 2018, showing a positive long-term interaction between domestic debt and inequality, with domestic debt contributing to greater inequality over time.

Despite their contributions, these empirical studies share certain limitations. They predominantly examine the impact of inequality on debt while neglecting the potential reverse causal effect, which is supported by theoretical literature. Moreover, the income focus is a critical oversight. While income data are more accessible, wealth inequality may offer a more comprehensive understanding of how public debt influences overall economic disparities, as wealth has a stronger long-term impact on household finances.

Finally, a significant issue in the empirical analysis is the risk of endogeneity, caused by the possible presence of reverse causality. This can lead to incorrect inferences about the relationship between inequality and debt.

3.2.3 Mechanisms of Transmission

While the empirical evidence highlights a clear relationship between public debt and inequality even with mixed results, it is crucial to delve deeper into the mechanisms through which these effects are transmitted. Understanding these channels provides insight into how public debt influences inequality and vice versa, as well as the specific factors that drive these dynamics. In this section, we describe the possible mechanisms of transmission between public debt and wealth inequality.

As we have already seen in Section 3.2.2, the empirical analysis tested several mechanisms of transmissions that go from income inequality to public debt like fiscal dynamics (Woo, 2003; Larch, 2012; Jabłoński et al., 2015; Carrera and de la Vega, 2021), financial market integration and stabilization (Kim, 2013; Azzimonti et al., 2014) and economic growth (Carrera and de la Vega, 2021).

For the opposite link, the scarcity of empirical research leads us to turn to theoretical studies already described in Section 3.2.1 as well as to theoretical and empirical studies which have explored the interactions between government debt/inequality, and other variables.

Fiscal policy may be a mechanism even in the opposite direction. An excessive share of external debt may lead to pressure on debt sustainability by international investors. As a consequence, the government adopts restrictive policies that reduce welfare and increase taxes, to the detriment of the poor.

External debt is an outflow of wealth to foreign countries and does not directly modify the wealth distribution within the country. On the other hand, the government gets new resources that can be applied to finance redistribution programs. From this point of view, external debt seems to be a valuable tool for reducing inequality. However, foreign debt impacts many aspects of the economic systems (e.g. inflation and economic growth) and of debt itself (e.g. debt services payments), producing harmful outcomes capable of mitigating or even reversing the positive effects. For example, the argument of Loko et al. (2003) is that an external debt overhang tends to crowd out social spending through increased pressure on fiscal consolidation policies to attract foreign investors.

As far as we know, few studies address this issue; the works of Bittencourt (2015), Chowdhury (1994) and Akram (2013) are among them. Bittencourt (2015) argues that inequality is a determinant of external debts since his empirical results show a positive effect of inequality on external debt. Nevertheless, estimates are not always statistically significant.

Chowdhury (1994) estimates a structural simultaneous equation model to capture the interrelationships between public and private external debt, capital accumulation, and production, taking into account, along with other variables, a measure of inequality. The findings show a positive relationship between income inequality and external debt. Akram (2013), with a focus on Pakistan, investigates the contrary link between the two variables with the help of a VEC model. The results indicate that the size of foreign debt has a strong and positive relationship with income inequality.

The stability of the economic system may be a key mechanism that links wealth inequality with public debt. Various authors propose a significant link between growing inequality, excessive private debt, financial crises, and public debt (Kumhof et al., 2015; Bohoslavsky, 2016). This is because the rise in inequality generates an uneconomical surge in private debt due to uninsurable idiosyncratic risk. Therefore, public deficits are implemented by the government to crowd out private capital.

Regarding the relationship between external debt and economic growth, there are mixed opinions (Chowdhury, 1994). Sachs (1990) and Kenen (1990) argue that excessive external debt brings weak growth in high-indebted countries by increasing debt service payments and crowding out the effects of private investment. Of the opposite view is Bulow and Rogoff (1990) who argued that excessive external debt, or more generally public debt, is more a consequence of mismanagement than a cause of slow economic growth. Most of the studies, e.g. Lin and Sosin (2001), Kumar and Woo (2010) and Elmeskov and Sutherland (2012), align with the first view, finding that foreign debt negatively impacts economic growth. The work of Chowdhury (1994) investigated which theory is closer to reality through a VAR application. However, mixed results lead the author to no conclusion.

This study evaluates the effectiveness of these mechanisms on wealth inequality, and the final model considered only those that were found to be valid. Further studies would be useful to ascertain the validity or otherwise of the excluded mechanisms, perhaps with other countries or a panel.

In summary, the mechanisms of transmission that we test are two: the fiscal policy for a bi-directional link, and the economic stability for the link that goes from inequality to public debt.

3.3 Methodology and Data Analysis

To effectively explore the relationship between debt and inequality, we use a model that emphasizes multivariate analysis.

Co-integration is a concept introduced by Granger (1981) and describes the existence of a stationary linear combination of a collection of no stationary time series. As mentioned by Maysami and Koh (2000), an advantage of co-integration analysis is that the short-term dynamics between the variables and the adjustment process towards long-term equilibrium - described by the linear combination - can be studied together. We believe the project would be lacking if we omit a co-integration analysis.

The VECM, proposed by Engle and Granger (1987), has been widely used to study the long-term equilibrium relationships between the stock market and macroeconomic variables (Maysami and Koh, 2000; Mukherjee and Naka, 1995; Suharsono et al., 2017). Asari et al. (2011), in their works, used a VECM approach to explain the relationship between interest rate and inflation towards exchange rate.

Some applications considered public debt. Ghali (1998) have investigated the presence of a co-integrating relationship between public and private investment and economic growth, using Tunisian data. Akomolafe et al. (2015) have examined the nexus between public debt and domestic investment taking Nigeria as the study case.

Another relevant application of this model is the one carried out by Bildirici and Ersin (2007) which investigates the economic relationship between inflation and domestic debt, taking in exam three panels made each by three countries with similar inflation levels and borrowing costs. A further study on the nexus between public debt and inflation is the one of Nastansky and Strohe (2015), focusing on the German case.

Regarding inequality investigation, VEC models have been used to investigate its relationship with household debt: Berisha et al. (2015) wanted to determine the existence of a co-integrating relationship between household debt and income inequality using a VECM on US data. The subsequent work of Berisha and Meszaros (2018), still on US data, reinforces the study carried out years earlier, this time adding the level of consumption to the analysis. Both papers suggest a co-integrating relationship between the variables. Furthermore, households' debt seems to have a positive impact on inequality in the short and long run. The initial positive effects of the debt on consumption may reverse in the long run.

The paper of Taghizadeh-Hesary et al. (2020) is the work most in line with our project where authors used a VECM model to study the impact of monetary and tax policy on income inequality in Japan. The results show that whereas monetary policy increases inequality, tax policy negatively affects inequality and, even in the long run, tax policy seems to direct income distribution toward equity. Our project focuses on the second face of fiscal policy, a possible complement to the research carried out by the authors. However, as far as we know, this project will be the first to use this model to study the nexus between public debt and inequality.

This study focuses on the United States and analyzes annual data from 1950 to 2019. The variables under consideration are wealth inequality, public debt, personal taxation, government revenue, and household debt. Wealth inequality is measured as the wealth share of the richest 10 per cent of the population, taken from the World Inequality Database.

The Top 10% share is a commonly used measure in economic inequality studies. It represents the proportion of total wealth owned by the richest 10% of a population. While it provides insight into wealth concentration, it has several limitations, especially when compared to more comprehensive inequality measures like the Gini index.

The share focuses on one specific segment of the population, providing only a partial view of inequality. By focusing on the richest segment, the measure ignores changes in the distribution among the remaining 90%.

Even within the top 10%, there may be vast differences in wealth. For instance, the top 1% may be accumulating wealth faster than the 9% below them, yet the Top 10% share metric aggregates these groups, potentially hiding extreme wealth concentration at the very top.

On the other hand, the Top 10% share highlights wealth concentration among the richest, which is critical for understanding the capture of economic resources by a small elite. While the Gini index reflects overall inequality, it can sometimes obscure the specific dynamics of wealth concentration at the top.

The measure is easy to understand, communicate, and track over time. It provides a clear snapshot of inequality that resonates with the public and policymakers, facilitating discourse on the concentration of wealth and its socio-economic implications.

Policy interventions often target wealth concentration at the top. The Top 10% share helps in justifying progressive taxation, wealth redistribution, and social welfare policies aimed at reducing concentration at the top, which could otherwise go unnoticed by more diffuse measures like the Gini index.

Given the big share of financial assets owned by the very rich, this wealth inequality measure may be associated with dynamics more linked to the business cycle than to inequality dynamics in the whole population. However, Piketty and Zucman (2014) demonstrate that the wealth composition of the top 10% of the population is heterogeneous, increasing the risk distribution among the various sources of wealth. Stock and bond markets often serve as alternative investment options. When government bond yields are low, investors typically shift capital into the stock market in pursuit of higher returns. Conversely, during periods of economic uncertainty or heightened stock market volatility, investors tend to move into government bonds, considered safer assets, creating a 'flight-to-quality' effect.

As a result, the top 10% are generally more resilient to economic crises, as losses in one asset class can be offset by gains in another, or they can reallocate their wealth between markets, adjusting their portfolios to preserve or even increase their overall wealth.

In conclusion, while the Top 10% share has notable limitations, such as its inability to capture full income or wealth distribution and its simplification of complex inequalities, it remains a valuable measure. Its strength lies in its focus on wealth concentration and ease of communication.

Further analysis has been conducted to verify the validity of the inequality measure. Figure 3.1 presents a scatter plot of the top 10% wealth share against the Gini index for the U.S. over the period 1962 to 2019[1]. The plot suggests a strong relationship between the two measures, supported by a correlation coefficient of 0.95.

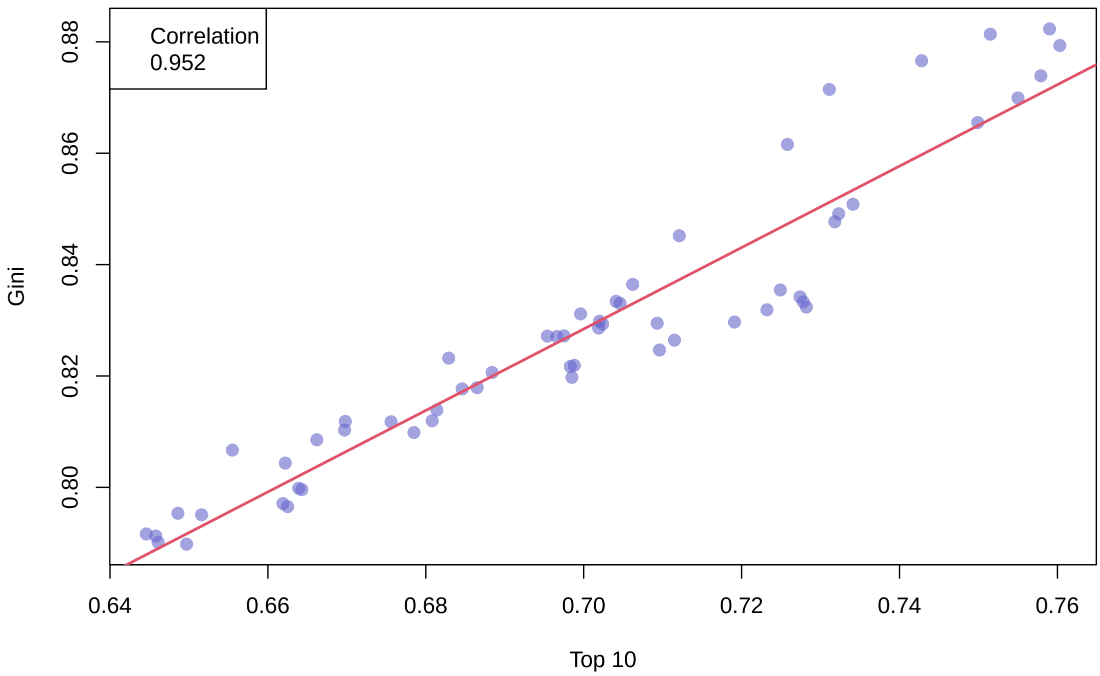


Figure 3.1: Scatter plot of top 10 wealth share and Gini index for a sub-sample from 1962 to 2019

The two measures of inequality thus appear to be closely related. This could be due to the high concentration of wealth in the better-off population and the zero or in some cases negative wealth of the poorer population. Further studies could be carried out to validate the link between the two measures and to investigate possible links between the wealth share of the top 10 and the business cycle.

[1]Gini index data are only available starting from 1962, which limits the ability to estimate the model due to the large number of parameters involved. While applying certain restrictions could reduce the number of parameters, the sample size remains small.

Public debt, expressed as a percentage of GDP, is sourced from the Federal Reserve Economic Data. The ratio of personal taxes to personal income, along with government social expenditures and receipts (both measured as a percentage of GDP), is obtained from the Bureau of Economic Analysis (BEA) to examine fiscal policy as a transmission mechanism. Additionally, the household debt-to-GDP ratio, sourced from the International Monetary Fund, is utilized to assess the transmission mechanism related to economic instability.

All the variables are taken in logarithmic form. Table 3.1 provides the definitions, the sources, and the labels of the variables used in our analysis. The summary statistics are collected in Table 3.2.

Variable	Label	Description	Source
Wealth inequality			
Top 10 wealth share	T10	Net personal wealth share held by the p90p100 group.	WID
Public Debt			
Public debt (% to GDP)	debt	Total public debt as a share of GDP	US Department of treasury
Transm. mechanisms			
Social expenditure (% of GDP)	SocExp	Gov. social benefits as percent of GDP	Bureau of Economic Analysis
Household debt (% of GDP)	HouDebt	Household debt as percent of GDP	Federal Reserve Bank of St. Louis
Government receipts (% of GDP)	Rec	Gov. receipts as percent of GDP	Bureau of Economic Analysis
Personal tax (% of income)	Tax	Personal tax as percent of personal income	Bureau of Economic Analysis

Table 3.1: Variable definitions.

	Mean	SD	Min	Max
T10	.70	.03	.64	.76
debt	57.21	22.91	31.02	106.03
SocExp	10.25	2.94	4.63	15.27
HouDebt	63.66	17.44	43.26	99.15
Rec	24.54	2.11	20.42	29.08
Tax	11.82	0.98	9.54	14.28

Table 3.2: Descriptive statistics of data.

Figure 3.2 shows the scatter plot between public debt and the wealth inequality measure. The tendency line shows an upward trend for both variables. Even the correlation is positive and significant. This prior analysis supports the thesis of a positive relation between the two variables.

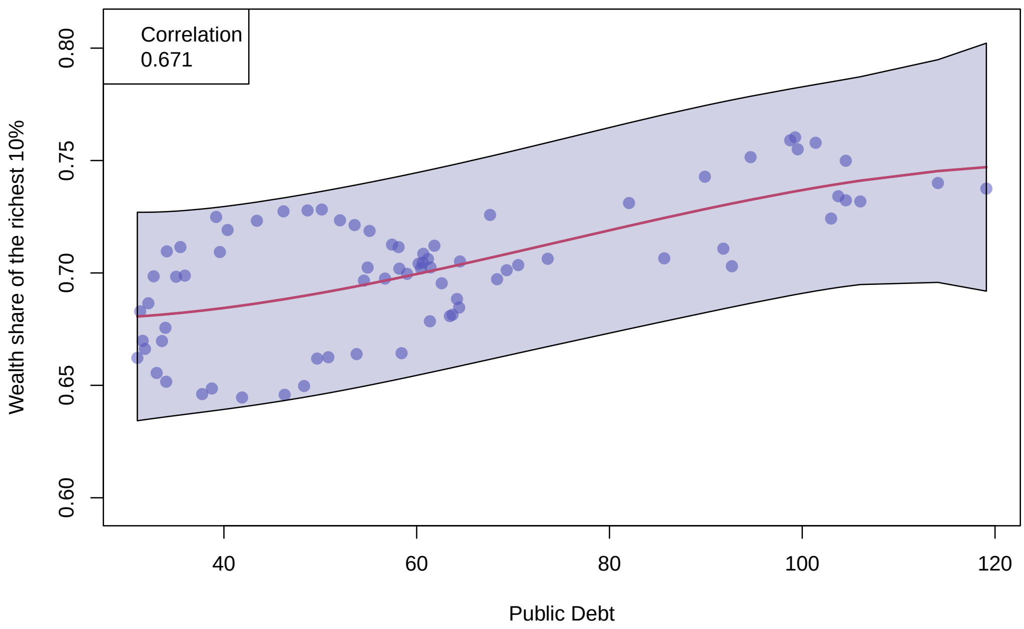


Figure 3.2: Scatter plot of public debt and top 10 wealth share with an approximation of the relation with a third-grade polynomial

The VEC model requires some preliminary tests. Variables must be integrated of order one, that is, the levels variables must be no-stationary, while the first-order differences of the variables must be stationary. For this purpose, I applied three different tests: i) the augmented Dickey-Fuller (ADF); ii) Phillips-Peron (PP); and iii) Kwiatkowski–Phillips–Schmidt–Shin (KPSS). The first two tests have the null hypothesis that at least a unit root is present in the time series, whereas the KPSS test has the null hypothesis that time series are stationary around a deterministic trend. Our processes are non-stationary if the null hypothesis is not rejected - the null hypothesis of the KPSS test has to be rejected.

The multivariate forms of information criteria give the choice of the lags. In this project, we apply the Akaike information criterion (AIC), the Hannan–Quinn information criterion (HQ), the Schwarz Criterion (SC), and the Final Prediction Error (FPE) for different VAR(p). Given the small sample size, we restrict p to a maximum of four. The resulting lag is equal to one.

To test the existence and define the number of long-run equilibria, we use the cointegration tests proposed by Johansen (1988, 1995). There are two types of statistics proposed by Johansen (1988), both LR tests. These tests have different hypothesis systems: the former statistic tests the null hypothesis that the number of distinct cointegrating vectors is less than or equal to r, and the latter statistic tests the null that the number of distinct cointegrating vectors is r against the alternative hypothesis of $r{+}1$ cointegrating vectors. Their asymptotic distributions are linked to a $(K - r)$-dimensional standard Wiener process, whose critical values can be estimated or found in tables collected by Johansen (1988). We cannot reject the null hypothesis of at least a long-run equilibrium.

For determining the cointegrating rank of a given system of K variables, we test a sequence of null hypotheses, from $r = 0$ to $r = K - 1$. We stop our tests at the first null hypothesis that is not rejected. If we stop at the first null hypothesis and so we cannot reject $r = 0$, then a model in first differences has to be considered. If we do not reject any hypotheses made then r may be greater than $K - 1$ and we can hypothesize on a stationary VAR model for the levels of the variables.

Some tests are used to prove the robustness of the model. First of all, we test that the resulting residuals are not auto-correlated and normally distributed. To verify the former hypothesis we use two types of tests: one is the Breusch-Godfrey test and the other is the Portmanteau test. We use the Jarque-Bera test to verify normality. We also check the stability of the model by computing the roots of the characteristic polynomial and testing the presence of ARCH effects.

Given the estimates of our parameters, we move to test the presence of effects among the variables. In addition to t-ratios, we will use the Granger-causality test. Finally, We employ the impulse-response function (IRF) and the Forecast Error Variance Decomposition (FEVD). The former traces the incremental effect of a unit of exogenous shock in one of the variables on the future values of the others ceteris paribus, the latter is a technique that reveals how much each variable contributes to the other variables in the model[2].

We started with some preliminary tests. The results of the unit-roots test for all the variables at their levels and first-order difference are summarized respectively in Table 3.3 and Table 3.4. Even with mixed results among the three tests, we generally reject the null hypothesis of stationarity of the KPSS test and do not reject the unit root hypothesis of the ADF and PP tests. The processes are thus found to be nonstationary. At the first-order differentiation of the time series, the ADF test still indicates the presence of unit roots for some variables, whereas the PP and KPSS tests suggest that all the variables are stationary. We conclude that the variables are integrated of order one and so we proceed with the application of the VEC model.

	ADF test	PP test	KPSS test
T10	-2.062	-3.407	0.365***
debt	-2.889**	-5.677	0.250***
SocExp	-2.176	-4.988	0.245***
HouDebt	-2.386	-5.882	0.144*
Rec	-2.941	-18.545*	0.195**
Tax	-3.287*	-13.596	0.155**

Table 3.3: Unit root test for log data. Significance levels: *** 0.01 ** 0.05 * 0.1

	ADF test	PP test	KPSS test
d1.T10	-2.840	-97.25***	0.334
d1.debt	-2.774	-24.15**	0.501*
d1.SocExp	-3.92**	-26.45***	0.202
d1.HouDebt	-2.847	-26.05***	0.193
d1.Rec	-6.34***	-26.03***	0.050
d1.Tax	-5.61***	-33.97***	0.045

Table 3.4: Unit root test for log data after the First Differencing. Significance levels: *** 0.01 ** 0.05 * 0.1

We then pass to find the optimal lag, p, with the help of information criteria. Table 3.5 collects the lowest result of the four ICs among the different VARs estimated on the variable levels with lags up to 4. Since the order of lags for the

[2]These three instruments capture different aspects of the possible interconnections among the variables, and they may seem to lead the reader to mixed conclusions. For further information about the described tests and instruments, you are invited to read Chapter 2

VEC model is equal to *p-1* of the VAR counterpart, we take at least *p=2* as the lower bound. In parenthesis, we indicate the lag values associated with the VEC model.

Ineq	Top 10
AIC(p)	-45.0(3)
HQ(p)	-42.8(1*)
SC(p)	-41.0(1*)
FPE(p)	4.5e-20(3)

Table 3.5: VEC lag order selection. * corresponds to the choice of p

After finding the optimal lag length, we use the Johansen cointegration test to determine whether there is a long-term and stable relationship between wealth inequality, public debt, and the other variables of interest. The results of the maximal eigenvalue and trace test statistics were presented in Table 3.6. The hypothesis of no cointegration among the variables is rejected. In the following tests, Trace and Maximum Eigenvalue statistics indicate the presence of at least two cointegrating relationships. The relevant relation is the first one, associated with the highest eigenvalue. Since the variables are cointegrated, it is concluded that a long-run equilibrium relationship is present.

	eigen	trace
r = 0	58.7***	160.7***
r ≤ 1	44.3***	101.9***
r ≤ 2	27.2	57.6
r ≤ 3	16.2	30.4
r ≤ 4	8.29	14.3
r ≤ 5	5.96	5.96

Table 3.6: Johansen cointegration tests. Significance levels: *** 0.01 ** 0.05 * 0.1

With the optimal number of lags and cointegrating relationships, the equations of our VEC model can be written as follows:

$$
\begin{aligned}
d.y_t^j = \alpha_{j0} + \alpha_j(&T10_{t-1} - \beta_1 debt_{t-1} - \beta_2 SocExp_{t-1} + \\
&- \beta_3 Tax_{t-1} - \beta_4 HouDebt_{t-1} - \beta_5 Rec_{t-1} - \delta_t) + \\
&+ \Gamma_{j1} d.T10_{t-1} + \Gamma_{j2} d.debt_{t-1} + \Gamma_{j3} d.SocExp_{t-1} + \\
&+ \Gamma_{j4} d.Tax_{t-1} + \Gamma_{j5} d.HouDebt_{t-1} + \Gamma_{j6} d.Rec_{t-1} + u_{1t}
\end{aligned}
\tag{3.1}
$$

with δ_t, a deterministic linear trend inside the error correction term, α_{j0}, a constant term and u_{jt}, an error term. The $d.$ term before the name of variables is the first-difference operator ($d.x_t = x_t - x_{t-1}$).

3.4 Empirical Results

Table 3.7 summarizes the estimates of the cointegrating vector of the estimated model. This yields the following cointegrating relationship:

$$\begin{aligned} T10_{t-1} = .18 * debt_{t-1} + .07 * SocExp_{t-1} + .30 * HouDebt_{t-1} + \\ -.49 * Rec_{t-1} + .25 * Tax_{t-1} - .01 * (t-1) \end{aligned}$$

Given the log transformation, the resulting estimates are long-run elasticities. The relation between the wealth inequality measure and the public debt-to-GDP ratio is positive, as hypothesized. The same positive sign is found for the social expenditures and the household debt. These results seem to confirm the fiscal policy and the economic stability mechanisms. The positive sign for the estimate of the *Tax* parameter suggests that an increase in pressure for restrictive policies maintains debt sustainability at the expense of the poor. Moreover, the positive elasticity between the inequality measure and the tax pressure may support the thesis of a no-progressive tax system in the US. Instead, the negative value of the coefficient of government receipts may support the hypothesis of a lower tax base due to higher wealth inequality. All the estimates are significant, as well as the one associated with the trend supporting the choice of the deterministic components.

Table 3.8 presents the estimated short-run dynamics from Eq.3.1. Starting with the estimates of α_j, which capture deviations from the long-run equilibrium, we observe significant values for both the inequality measure and government receipts. In contrast, the estimate of α_j in the Debt equation is not significant, suggesting potential weak exogeneity. In the empirical analysis of income outlined in Section 3.2.2, inequality was treated as an exogenous variable, with public debt as the dependent variable. Our findings align with the hypothesis that wealth and income inequality interact in distinct ways within the economic system.

	Top 10
debt	-.1848***
	(.0239)
SocExp	-.0664*
	(.036)
HouDebt	-.3031***
	(.0399)
Rec	.4949***
	(.0954)
Tax	-.2596***
	(.0915)
Trend	.0060***
	(.0014)

Table 3.7: Estimates for cointegrating vectors are presented along with their standard errors in parentheses. The equation is normalized for the inequality measure associated with the value of 1. Significance levels: *** 0.01 ** 0.05 * 0.1

	T10	Debt	SocExp	HouDebt	Rec	Tax
α_j	**-.293***	-.085	.021	-.123	**-.665***	-.361
	(.043)	(.173)	(.224)	(.116)	(.206)	(.273)
Const	-.659***	.532**	.778***	.346**	.671**	.993***
	(.056)	(.224)	(.291)	(.151)	(.269)	(.354)
d1.T10	-.510***	.013	-.206	**.431****	**.662***	.379
	(.080)	(.317)	(.412)	(.214)	(.382)	(.501)
d1.Debt	-.037	.457***	**-.214***	**-.126***	-.145	**-.371****
	(.024)	(.096)	(.124)	(.064)	(.115)	(.151)
d1.SocExp	.012	.037	.347***	-.163***	.045	.046
	(.016)	(.063)	(.081)	(.042)	(.075)	(.099)
d1.HouDebt	**-.074****	-.209	-.468**	.486***	.050	.033
	(.036)	(.141)	(.184)	(.096)	(.171)	(.224)
d1.Rec	.010	**-.616*****	-.526***	-.296***	.791***	.956***
	(.034)	(.134)	(.173)	(.090)	(.161)	(.211)
d1.Tax	-.003	**.378*****	.421**	.058	-.240**	-.294*
	(.024)	(.097)	(.127)	(.065)	(.112)	(.153)
R^2	.614	.674	.613	.661	.506	.483
Adj R^2	.569	.636	.560	.621	.449	.422
Dur-Wat stat	1.929	1.692	1.720	1.946	1.819	1.930

Table 3.8: Short-run dynamics estimates estimates. The values in parentheses underneath parameter estimates are the standard errors. Significance levels: *** 0.01 ** 0.05 * 0.1.

In the short run, inequality and public debt do not directly influence each other. However, inequality is negatively impacted by private debt, while inequality itself drives an increase in both private debt and government revenues. In the first case, a rise in household debt may be associated with real estate purchases, which constitute the primary asset for middle- and lower-income groups, thereby increasing their wealth. In the second case, growing wealth inequality leads to a larger proportion of the population being debt-constrained. This segment is forced to rely on highly leveraged debt to boost their spending capacity. Additionally, the link between wealth inequality and government revenues can be explained by increased investments, which result in higher capital gains tax collections.

Government debt is negatively impacted by government receipts and positively by the tax burden. The first link is straightforward: an increase in receipts may lead to a surplus that can be used to lower debt levels. To understand the second relationship, we observe that in the short run, higher tax pressure harms government receipts, supporting the Laffer curve's argument that excessive tax pressure can reduce tax revenues. Thus, an increase in the tax burden may lead to a decline in tax revenues and to a rise in public debt.

The estimated model demonstrates robustness based on various applied tests. The Breusch-Godfrey test with a lag of two and the portmanteau test across multiple lags confirm that the residuals are not autocorrelated, while the Jarque-Bera test indicates normality in their distribution. Additionally, a further test indicates that there are no ARCH effects. The model's estimates also remain consistent when using alternative inequality measures, such as the top 1% and top 5%. The model is also re-estimated with a reduced sample size, reaffirming the most significant results[3].

The Granger-causality test results, shown in Tables 3.9 and 3.10, reveal that public debt Granger-causes wealth inequality, but there is no evidence of reverse causality. Furthermore, the findings suggest a bi-directional Granger causality of public debt with government receipts and social expenditures. While inequality only exerts influence over government receipts, it is significantly impacted by household private debt and social expenditures. These results suggest that social spending may be a key transmission mechanism in the relationship between public debt and inequality, even supported by the long-run relationship and the short-run dynamics. However, even if other variables give no proof of Granger-causality, they always produced an impact on inequality and public debt as shown by the estimates of the long-run equilibrium and the short-run dynamics.

Further analyses are conducted on the impulse-response functions and the forecast error variance decompositions. Starting with the IRFs, Figure 3.3 presents

[3]Due to the small sample, the reliability of this re-estimation is limited, prompting us to place greater emphasis on the other test results

the impulse-response functions with the alternation of public debt and wealth inequality as impulse and response variables. The diagonal panels capture the persistence of shocks within each variable. The top-right panel illustrates the response of public debt to a shock in inequality, while the bottom-left panel shows the impact of a public debt shock on inequality. Focusing on these two panels, we observe that both shocks generate a positive effect. However, the response of public debt to an inequality shock is small. In contrast, the response of inequality to a public debt shock is more pronounced, increasing by almost 2%, with persistent effects over time.

Figure 3.4 collects the IRF with the Top 10 wealth share and public debt as response variables, while Figure 3.5 as shock sources. Among these IRFs, some are notable. In Figure 3.4, we observe a negative response of inequality to a tax pressure shock, approximately 2%. Instead, public debt shows a negative reaction to both government revenue and tax pressure shocks in the short run, with both responses around 4%. In Figure 3.5, Government revenues negatively respond to inequality shocks by 0.8%, with this effect persisting over time, while the response to debt shocks is around 2%, oscillating around zero.

Lastly, Figure 3.6 presents the Forecast Error Variance Decomposition (FEVD) of wealth inequality and public debt. The results show that public debt explains a substantial and increasing share of the variance in wealth inequality, though the most prominent factor is government receipts. In contrast, the FEVD of public debt reveals a more diverse set of influences, with inequality accounting for a significant and growing share over time.

Dip|Indip	T10	Debt
T10	62.6***	31.8***
Debt	0.16	20.3***
SocExp	0.22	16.2***
HouDebt	4.28	4.02
Rec	9.30***	7.43**
Tax	1.64	3.96

Table 3.9: Granger causality test with Top 10 wealth share and public debt-to-GDP as independent variables. Significance levels: *** 0.01 ** 0.05 * 0.1

3.5 Conclusions

In this paper, we explored the relationship between public debt and wealth inequality, a topic that has been extensively addressed in the literature, though with some

Indip|Dip	T10	Debt
T10	62.6***	0.16
Debt	31.8***	20.3***
SocExp	4.83*	9.39***
HouDebt	41.7***	16.1***
Rec	3.32	20.3***
Tax	2.85	15.8***

Table 3.10: Granger causality test with Top 10 wealth share and public debt-to-GDP as dependent variables. Significance levels: *** 0.01 ** 0.05 * 0.1

gaps. Theoretical frameworks broadly suggest an interdependence between these two variables. Even if the theory is partially supported by empirical studies, most applied models — whether on individual countries or panels — have primarily focused on income inequality due to data availability. This focus has limited the analysis to the cause-effect relationship originating from wealth inequality, leaving other potential dynamics unexplored.

This article aimed to address the lack of literature regarding the economic repercussions of wealth disparity and the interrelationship between public debt and economic inequality in the empirical literature. We considered economic stability and fiscal policy choice as key mechanisms of transmission, following the empirical analysis conducted on income inequality and the theoretical literature on the topic.

We applied the VEC model to test the transmission mechanisms avoiding bias due to non-stationarity and endogeneity. At the same time, the model will capture the long-run relationship between public debt and wealth inequality through the error correction term. The analyses were carried out with an R package built for the occasion, given the lack of an existing package on the open-source software.

We identified both direct and indirect relationships between public debt and inequality. These relationships are positive and observed over the long run, indicating that an increase in public debt corresponds with a rise in wealth inequality over time. The application of the VEC model proved particularly effective in capturing these long-term dynamics, which are crucial for understanding trends in wealth inequality.

Our findings elucidate the mechanisms at play within the U.S. economic system in particular during the Reaganomics: an increase in public debt, driven by a reduction in the tax burden while maintaining government spending levels — except for social spending — has contributed to a rise in inequality over the long term.

Further analysis is necessary to verify the validity of these results. The US is a special case given the politics adopted by the Reagan mandate. This work may be further improved by repeating these analyses with different countries or with panel data. The use of panel data would allow the use of other measures of wealth (e.g. wealth percentiles ratio or Gini index) that would otherwise cover an insufficient range of years to allow an adequate analysis. Another way to improve this work is suggested in the next chapter.

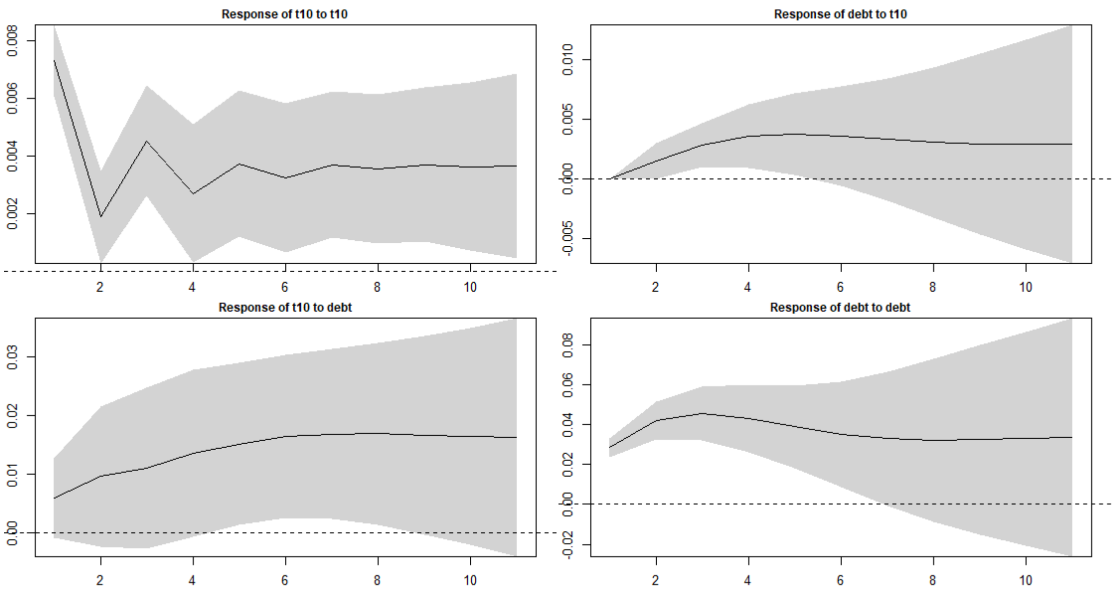


Figure 3.3: IRF with the alternate of impulse and response between public debt and wealth inequality

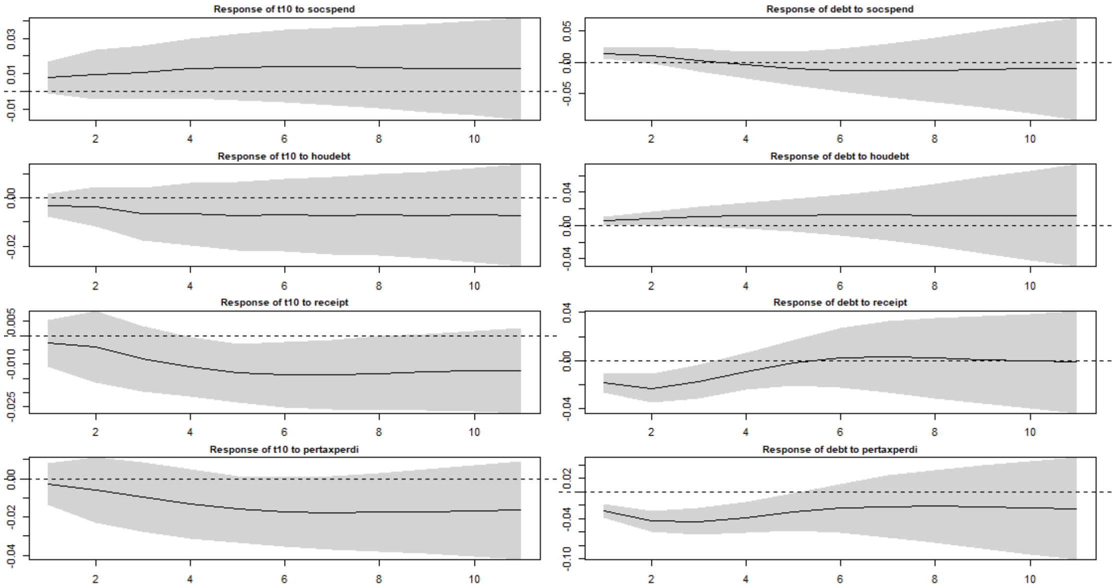


Figure 3.4: IRF with wealth inequality (left) and public debt (right) responding to shocks of social spending, household debt, government receipts and tax pressure

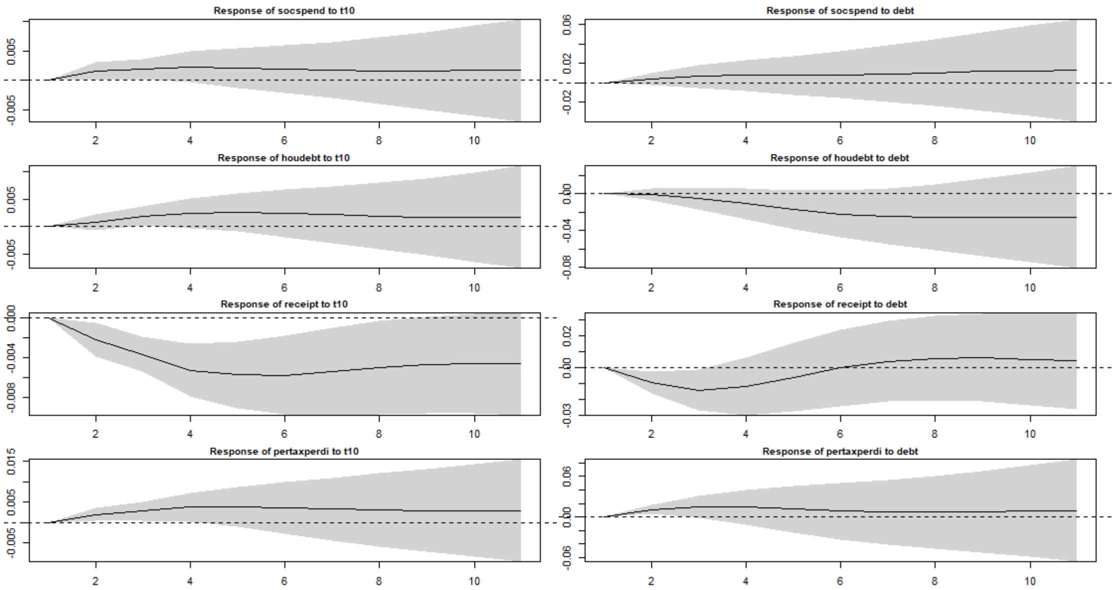


Figure 3.5: IRF with social spending, household debt, government receipts and tax pressure responding to shocks of wealth inequality (left) and public debt (right)

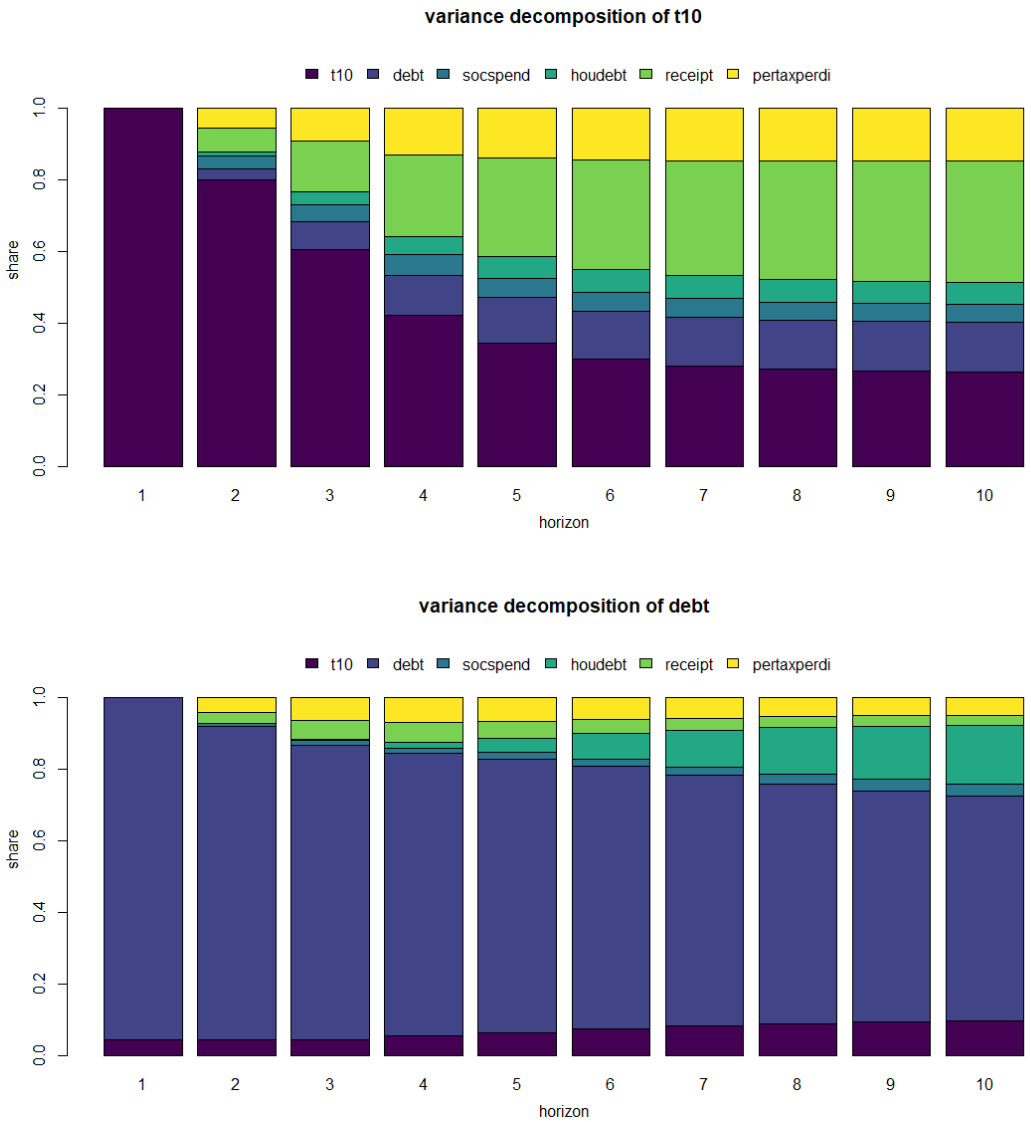


Figure 3.6: FEVD of wealth inequality (top) and public debt (bottom)

CHAPTER

4

THE PROFILE OF INCOME INEQUALITY AND ECONOMIC GROWTH: AN EMPIRICAL INVESTIGATION

4.1 Introduction

In recent years, there has been a significant effort to expand the availability of data on inequality (Saez and Zucman, 2020; Piketty et al., 2018; Blanchet and Martínez-Toledano, 2023). This has resulted in a growing empirical literature on the potential links between income inequality and economic variables. Past works are updated with longer time series, while issues that were previously treated only from a theoretical point of view can now be verified through appropriate empirical analyses.

The relationship between economic growth and income inequality has been a topic of evergreen interest among researchers. Kuznets (1955) may be considered one of the fathers of this branch of literature, hypothesizing a link between economic development and inequality. According to the hypothesis, economic development initially led to an increase in economic inequality, but later on, development slowed down in favour of a more equal society. Although the hypothesis was later attacked, especially in the wake of rising levels of inequality in the U.S. and other developed countries(Saez and Zucman, 2020), Kuznets' work is credited with starting a line of work that is still thriving today.

Currently, scholars have produced a significant amount of literature investigating the impact of income inequality on economic development. Theoretical papers and empirical applications have produced mixed results regarding the relationship between inequality and growth. While a considerable portion of the literature suggests that inequality is detrimental to growth, more recent studies have questioned this finding, suggesting a positive effect of inequality on growth. De Dominicis et al. (2008), by using meta-analysis on a sample of empirical studies, find that estimation methods, data quality and sample coverage systematically affect the results.

According to Voitchovsky (2005), the theoretical literature suggests that inequality in various sections of the income distribution affects economic growth in several ways. Some key mechanisms suggest that a high level of top-income inequality positively affects economic growth, while high levels of bottom-income inequality are detrimental to growth. The author found empirical evidence of these effects due to the shape of income distribution. However, the study focuses only on a small sample of countries and years and has never been replicated since its publication.

Van der Weide and Milanovic (2018) find that the measures used to analyse economic growth, such as GDP per capita or average incomes, have limitations. For example, if economic growth is reduced, does it affect the entire population in the same way, or do the poor suffer more? Also, does an increase in economic growth reduce income disparities?

The authors' empirical analysis indicates that while an increase in income inequality harms the income growth of the poor, it leads to an increase in economic growth for the rich. The study focuses on the US states, and no further research has been conducted to replicate these findings in other countries or using a cross-country panel.

This paper aims to combine the works brought on by Voitchovsky (2005) and Van der Weide and Milanovic (2018) on the impact of inequality on economic growth. The main contributions of this research are to replicate the previous works with an updated sample compared to the study of Voitchovsky (2005) and with a cross-country panel compared to the study of Van der Weide and Milanovic (2018). Additionally, the studies are replicated using personal income to measure economic growth and Gini coefficients to capture inequality across different percentile groups of the distribution. In fact, Voitchovsky (2005) used heterogeneous measures of income inequality across the distribution, a choice that may affect the final results.

A fixed-effects OLS estimate is used on the entire sample provided by WIID. The model is re-estimated even for two subsamples, i.e., for high-income countries -includes some additional countries to those used by Voitchovsky (2005) -, and middle- and low-income countries. We use individual growth as the dependent variable following the results of Van der Weide and Milanovic (2018). The results are partially in contrast to the previous findings. Inequality in the three groups significantly and negatively impacts economic growth. The only positive effect is given by inequality between. The result thus seems to only partially confirm the old results, i.e., the profile of inequality is a relevant determinant of economic growth in that within inequality impacts negatively while between inequality impacts positively.

Reproducing the work of Van der Weide and Milanovic (2018), the results are similar to the ones shown by the two authors. Inequality negatively affects the income growth of the bottom and middle-income groups, while increasing the one of the top income groups.

The chapter is structured as follows: Section 4.2 presents the main findings from the literature on how the shape of the income distribution impacts economic growth, theoretically and empirically. Section 4.3 outlines the sample constructed and the variables examined. Moreover, it describes the model and the estimator used. Section 4.4 presents the results obtained. Finally, the conclusions and possible future developments of the research are discussed.

4.2 Literature

The theoretical literature has identified multiple and interrelated mechanisms by which income inequality can influence economic growth, either by strengthening or weakening it. Some of these mechanisms arise from specific segments of the income distribution, while others derive from the potential social conflicts that may arise in a society deeply polarized between the extremes of wealth and poverty. This section will explore and summarize the most widely debated mechanisms in the literature.

Kaldor (1955) argues that inequality is necessary for growth because it increases the savings crucial for capital accumulation and economic growth. Savings and the ability to access resources are essential elements of economic growth. The wealthiest are facilitated to invest, either through access to capital markets or through the use of savings. Theory and empirical analyses have shown that the wealthy have a high marginal propensity to save, mainly due to higher income from capital rather than wages. Moreover, the greater the resources, the more diversified the risk, the greater the return. An increase in inequality at the top of the distribution therefore means an increase in private resources and consequently in investment (Aghion et al., 1999).

Conversely, an increase in inequality at the bottom of the distribution means less access to capital markets due to credit constraints, or the creation of highly leveraged household debt, which over time further reduces savings. Consumption then tends to decline, as does aggregate demand and future investment as a result. Moreover, without sufficient resources, the poor cannot invest in developing their skills and talents, also reducing human capital over time (Galor and Tsiddon, 1997; Aghion et al., 1999).

Another relevant key mechanism of transmission is the surge of anti-social behaviours. Economic inequality and poverty are significant factors in the occurrence of crime and social conflict. The widening economic disparities force the poorest members of society into criminal activities and even rebellion (Kelly, 2000; Fajnzylber et al., 2002). This, in turn, creates uncertainty and mistrust towards the economic system, discouraging investment and capital accumulation. Ultimately, this slows down the process of economic growth (Alesina and Perotti, 1996). According to Meltzer and Richard (1981), when there is a high level of inequality, the median voter who belongs to the poor section of society tends to push for two things. Firstly, they demand higher taxes that discourage investment. Secondly, they demand redistributive policies that take resources away from public investment but can help the poor enhance their competencies through investments in education (Persson and Tabellini, 1991; Alesina and Rodrik, 1994; Perotti, 1996).

Even the far-right end of the distribution can engage in anti-social and criminal behaviour. The wealthy can push back pro-poor policies, appropriate or divert the country's resources, or even subvert legal and political institutions through corruption and political pressure (Easterly, 2001; Glaeser et al., 2003).

Moreover, high inequality can lead to almost complete political polarisation and possible social conflicts, in which each group aims to prevail over the other (Benabou, 1996; Larch, 2012). In this conflict, governments can attempt to satisfy both sides by increasing welfare spending for the poor and reducing taxes for the rich. However, this approach may lead to reduced current and future public investments due to debt service payments. Additionally, there is a risk of delaying essential reforms for the country's development (Alesina and Rodrik, 1994).

In a meritocratic economic system, there are incentives for effort, productivity, and risks, which leads to higher growth rates and income inequality in the form of a higher level of income mobility. Moreover, skilled individuals are supposed to be concentrated within the upper-income brackets, especially in advanced technology sectors. This leads to a boost in technological progress and growth. Positive incentives can motivate greater effort along the entire distribution (Galor and Zeira, 1993; Hassler and Mora, 2000).

Recently, empirical analysis has endeavoured to broaden the study of the relationship between inequality and economic growth, supported by a major effort to expand the available data on incomes with increasing details, for more and more countries and periods. This availability of data has enabled work such as that of Voitchovsky (2005), Marrero and Rodríguez (2012, 2013, 2023) and Van der Weide and Milanovic (2018). Voitchovsky (2005) is credited with showing how previous studies are limited in that they only consider inequality at a global level, while the theory suggests that different portions of the distribution can lead to different effects on growth. The empirical findings of the author confirm this thesis.

Instead, Marrero and Rodríguez (2012, 2013) implemented a decomposition of inequality into two components, one of opportunity due to several characteristics out of control of the individual such as race or parental education, and one of residual due to the individual's efforts and luck. In the two papers produced by the authors, evidence was found that the first component acts negatively on growth while the second component acts positively. The two authors have updated their work (Marrero and Rodríguez, 2023) with a new paper, strengthening their findings and adding that the higher the poverty, the smaller the effect produced by the two inequalities. Results are further supported by the work of Aiyar and Ebeke (2020) in which inequality of opportunity is identified with intergenerational mobility.

Finally, Van der Weide and Milanovic (2018) examine how household income growth varies across income levels in relation to income inequality. After analyzing the data, the authors discovered that the impact of inequality tends to be negative for the poor and positive for the rich as the percentile group increases.

4.3 Data and Methodology

To explore the relationship between income inequality and economic growth, we extract data from two main sources, the World Income Inequality Database (WIID) and the World Bank Development Indicators (WDI). The WIID offers information on per capita income distributions at the percentile level in developed, developing, and transition countries. The database covers 200 countries and contains data from as early as 1945. However, the most complete series starts from the 1990s. The USA has the most comprehensive series, with 62 observations.

We compute the Gini coefficients using the standardised version of the WIID (UNU-WIDER 2021), which provides us with information on the average income of each percentile of the income distribution. These data allow us to estimate the full profile of inequality.

We estimate the Gini index for inequality across three distinct segments of the income distribution: the bottom 40%, the middle 40% (between the 41st and 80th percentiles), and the top 20%. This specific partition was chosen due to its clear social interpretation and strong foundation in the literature on poverty and inequality.

The first segment, representing the bottom 40% of the distribution, is significant for measuring shared prosperity, a concept promoted by the World Bank since the 1970s and actively monitored since 2014 (McNamara, 1972; World Bank, 2015). This group broadly captures the most disadvantaged individuals worldwide.

The second segment, encompassing the 41st to 80th percentiles, corresponds to the global middle class. For instance, (Easterly, 2001) classifies the middle class as those between the third and eighth income deciles.

Finally, the top 20% of the income distribution is critical for assessing the evolution of top incomes and inequality. This segment can be interpreted as representing the wealthy class (see Dabla-Norris et al., 2015).

These three Gini indexes provide a measure of inequality within each income group. Using the Gini index, we have also estimated inequality between these three income groups. Therefore, we have a complete profile of inequality that includes four indicators: three indicators of within-income-group inequality and one indicator of between-group inequality. This is an improvement of the work of Voitchovsky (2005) which uses the 90/75 percentile ratio or the 50/10 percentile ratio to capture respectively the inequality at the bottom and the top of the distribution.

We also use the percentiles of the income distribution as an alternative measure of economic growth to study the impact of inequality on the shape of the distribution.

In addition, we collect data on country characteristics from the WDI, which serve as controls in our empirical analysis. While data on civil liberties are taken from the Freedom House database[1].

By merging the databases, we obtain a final sample of 82 countries, including developed, developing and transition countries, observed from 1992 to 2021.

We use the fixed-effects OLS estimator to address possible bias due to unobserved country-specific influences. We want to estimate two linear probability models. The first one is the following

$$y_{i,t} = \alpha Y_{i,t-1} + \beta Ineq_{i,t} + \gamma X_{i,t} + \mu_i + \tau_t + \epsilon_{i,t} \tag{4.1}$$

where $y_{i,t}$ is the measure of the economic growth measured by the percentage change in per capita mean income compared to the previous year. $Y_{i,t-1}$ is the log per capita mean income.

$Ineq_{i,t}$ is an income statistic or a list of income statistics. In the former case, we have the income Gini index for the whole distribution ($Gini_{1:100}$). In the latter case, inequality Gini indexes within percentiles 1–40 ($Gini_{1:40}$), 41–80 ($Gini_{41:80}$), and 81–100 ($Gini_{81:100}$), and between these three income groups ($GiniBet$). A well-known characteristic of the Gini index is that it cannot generally be perfectly

[1] https://freedomhouse.org/

decomposed into within-group and between-group inequality. This limitation arises when group distributions overlap, leading to a residual in the decomposition. In our analysis, however, we focus on income percentile distributions where there is no overlap among the three income classes, resulting in a zero residual term. Consequently, the indexes we have listed represent a complete decomposition of total income inequality, enabling us to capture the various dimensions of inequality in the same income distribution.

The table 4.1 presents the main statistics of the Gini coefficient statistics across different income percentile groups and the overall distribution. The means indicate that the bottom 40% and top 20% income groups exhibit similar Gini coefficient values, although the top 20% group demonstrates greater variability. The middle 40% group shows the lowest level of inequality and minimal variability. The total inequality is the most stable yet also the most variable within the panel. Notably, Azerbaijan reports the lowest inequality values for both the bottom 40% and overall groups, while the United Arab Emirates shows the lowest inequality for the top 20%. Conversely, South Africa has the highest inequality levels across the middle, top, and overall groups, while Guinea-Bissau ranks highest for the bottom 40%.

Table 4.1: Gini Coefficient Statistics

	gini_1_40	**gini_41_80**	**gini_81_100**	**gini_1_100**
Mean	20.8	12.32	23.95	40.29
Var	29.92	11.48	56.87	99.72
Min	5.96 (AZE-15)	4.76 (AZE-15)	9.25 (ARE-15)	15.16 (AZE-15)
Max	48.48 (GNB-91)	28.56 (ZAF-08)	51.97 (ZAF-05)	74.24 (ZAF-05)

X_{it} is a set of individual control variables. When choosing control variables we rely on past empirical analyses and the data availability (Barro, 2003; Shen and Zhao, 2023; Voitchovsky, 2005). We choose to take investment (*invest*) measured by the average share of gross fixed capital formation in GDP, trade openness (*trade*) measured by exports plus imports in GDP, life expectancy (*life*), unemployment index (*unem*) and civil liberties (*civil*) measured by an index with values between one and seven.

Finally, we include country- (μ_i) and time-fixed effects (τ_t) to control for country-specific unobserved factors and common shocks; $\epsilon_{i,t}$ is the error term.

The second equation that we estimate is the following

$$y^p_{i,t} = \alpha Y^p_{i,t-1} + \beta Ineq_{i,t} + \gamma X_{i,t} + \mu_c + \tau_t + \epsilon_{i,t} \tag{4.2}$$

compared to Eq. 4.1, the dependent variable is the growth rate of the mean income of the percentile p in state i between time $t-1$ and t while the covariate is the corresponding log levels at time $t-1$. With this equation, we want to reproduce the work of Van der Weide and Milanovic (2018). The other equation components are not changed.

4.4 Results

Compared to other empirical studies, the sample used is larger. This may lead to a greater degree of heterogeneity. To solve this problem, we use several control variables to capture the characteristics of the countries. Moreover, we choose to repeat the analysis with two subsamples made by including only high-income countries and excluding the high-income countries from the other. With the former subsample, we can estimate the model with a homogenous countries group and test if the results change with the development degree.

All estimations are obtained using the package **plm** for R provided by Millo (2017). The results for the growth models estimations, using the pooling (columns 1-2) and fixed effects (columns 3-6) OLS estimators, are reported in table 4.2. Inequality measured either by the Gini index for the entire distribution or subgroups appears to be significant and with constant signs with both the adopted approaches. Moreover, the results remain robust even when excluding the control variables. The coefficients associated with the Gini index for the entire distribution are negative, supporting the thesis of a negative general impact of inequality on economic growth. However, when we look at the inequality profile, we find two opposite forces: inequality within the subgroups negatively affects economic growth, while inequality between the subgroups positively affects economic growth.

The results do not change when we repeat the estimation with the subsample made by the high-income inequality (tab.4.3), but even increasing the magnitude of the inequality effects. When we look at the second subsample made by no high-income countries (tab.4.4), the signs of the effects remain invariant, but the size and the significativity fall. In the case of $\text{Gini}_{41:80}$, the estimate is no more significant. This may be due to heterogeneity among countries and further studies are required.

In the complex, these results contrast with the ones of Voitchovsky (2005), supporting the idea that the choice of inequality measure, estimation methods, and sample coverage affect results (De Dominicis et al., 2008).

The results obtained from the study seem to both support and contradict several proposed theories. The positive effect of inequality among subgroups can support the theory of increased efforts at work or aggregate savings, but it contradicts the theory of possible social conflicts.

On the other hand, the harmful effects of inequality in the top 10 percentile group negate the theory of increased investment. A possible explanation is that as the

	(1)	(2)	(3)	(4)	(5)	(6)
Y_{-1}	-2.174^{***}	-2.090^{***}	-2.271^{***}	-2.286^{***}	-2.274^{***}	-2.288^{***}
	(0.036)	(0.037)	(0.046)	(0.046)	(0.046)	(0.046)
$\text{Gini}_{1:100}$	-0.124^{***}		-0.161^{***}		-0.211^{***}	
	(0.005)		(0.017)		(0.018)	
$\text{Gini}_{1:40}$		-0.098^{***}		-0.544^{***}		-0.536^{***}
		(0.017)		(0.034)		(0.034)
$\text{Gini}_{41:80}$		-0.315^{***}		-0.881^{***}		-0.653^{***}
		(0.089)		(0.137)		(0.138)
$\text{Gini}_{81:100}$		-0.259^{***}		-0.464^{***}		-0.481^{***}
		(0.028)		(0.053)		(0.053)
GiniBet		0.234^{***}		0.920^{***}		0.790^{***}
		(0.058)		(0.105)		(0.105)
invest					-0.006	0.0004
					(0.011)	(0.011)
trade					0.046^{***}	0.049^{***}
					(0.003)	(0.003)
life					-0.462^{***}	-0.430^{***}
					(0.057)	(0.057)
unem					-0.240^{***}	-0.215^{***}
					(0.018)	(0.018)
civil					-0.814^{***}	-0.901^{***}
					(0.097)	(0.098)
FE	N	N	Y	Y	Y	Y
Obs	104,500	104,500	104,500	104,500	104,500	104,500
R^2	0.034	0.035	0.106	0.108	0.112	0.113
Adj R^2	0.034	0.035	0.105	0.107	0.111	0.112

Table 4.2: Fixed effects OLS estimation with the full panel

distance within the wealthiest population rises, competitiveness - and investments - decline.

In contrast, the impact of inequality in the bottom 40 supports various theories, such as an increase in criminal behaviour, reduced accessibility to loans, and a push for pro-poor fiscal policies.

To strengthen our analysis, we perform jackknife tests, where the sample of each specific country is successively dropped from the estimations. As reported in Figure 4.1, our main results hold, with the coefficients on inequality stable and statistically significant across the series of countries dropping, apart from very few

	(1)	(2)	(3)	(4)	(5)	(6)
Y_{-1}	-1.014^{***}	-0.954^{***}	-0.693^{***}	-0.695^{***}	-0.744^{***}	-0.748^{***}
	(0.033)	(0.033)	(0.032)	(0.032)	(0.031)	(0.031)
$\text{Gini}_{1:100}$	0.016^{***}		-0.380^{***}		-0.355^{***}	
	(0.005)		(0.016)		(0.016)	
$\text{Gini}_{1:40}$		-0.237^{***}		-0.854^{***}		-0.713^{***}
		(0.018)		(0.028)		(0.028)
$\text{Gini}_{41:80}$		-0.644^{***}		-1.905^{***}		-1.799^{***}
		(0.072)		(0.098)		(0.099)
$\text{Gini}_{81:100}$		-0.509^{***}		-0.901^{***}		-0.904^{***}
		(0.025)		(0.034)		(0.033)
GiniBet		0.724^{***}		1.577^{***}		1.498^{***}
		(0.049)		(0.075)		(0.076)
invest					0.110^{***}	0.089^{***}
					(0.008)	(0.008)
trade					0.095^{***}	0.091^{***}
					(0.002)	(0.002)
life					-0.697^{***}	-0.525^{***}
					(0.048)	(0.048)
unem					-0.230^{***}	-0.262^{***}
					(0.011)	(0.011)
civil					-0.087	0.189^{**}
					(0.084)	(0.085)
FE	N	N	Y	Y	Y	Y
Obs	50,600	50,600	50,600	50,600	50,600	50,600
R^2	0.022	0.031	0.290	0.304	0.330	0.341
Adj R^2	0.022	0.031	0.289	0.303	0.329	0.340

Table 4.3: Fixed effects OLS estimation with the high-income countries

exceptions.

The estimation results of equation 4.2 are presented in table 4.5. Our analysis was limited to the global inequality index and we used the same percentiles selected by Van der Weide and Milanovic (2018). The obtained results are similar to theirs. Our findings reveal that Initial inequality is inversely related to income growth for the population below the median, and positively related to income growth for the population in the top percentile. The coefficient turns positive and increases in size and significance as we move towards the richer parts of the top decile.

Similarly, the absolute size of the negative coefficient increases as we move

	(1)	(2)	(3)	(4)	(5)	(6)
Y_{-1}	-2.895^{***}	-2.888^{***}	-3.107^{***}	-3.124^{***}	-3.101^{***}	-3.118^{***}
	(0.067)	(0.067)	(0.077)	(0.077)	(0.077)	(0.077)
$\text{Gini}_{1:100}$	-0.167^{***}		-0.188^{***}		-0.217^{***}	
	(0.008)		(0.028)		(0.028)	
$\text{Gini}_{1:40}$		-0.059^{**}		-0.511^{***}		-0.475^{***}
		(0.028)		(0.054)		(0.055)
$\text{Gini}_{41:80}$		-0.285^{*}		-0.296		-0.179
		(0.155)		(0.237)		(0.240)
$\text{Gini}_{81:100}$		-0.128^{**}		-0.217^{**}		-0.238^{**}
		(0.056)		(0.098)		(0.099)
GiniBet		0.057		0.439^{**}		0.361^{**}
		(0.109)		(0.183)		(0.184)
invest					-0.106^{***}	-0.086^{***}
					(0.020)	(0.020)
trade					-0.003	0.005
					(0.006)	(0.006)
life					-0.311^{***}	-0.261^{***}
					(0.095)	(0.095)
unem					-0.251^{***}	-0.197^{***}
					(0.033)	(0.034)
civil					-0.792^{***}	-0.925^{***}
					(0.151)	(0.153)
FE	N	N	Y	Y	Y	Y
Obs	53,900	53,900	53,900	53,900	53,900	53,900
R^2	0.036	0.036	0.094	0.096	0.096	0.098
Adj R^2	0.035	0.035	0.093	0.095	0.095	0.096

Table 4.4: Fixed effects OLS estimation with the exclusion of high-income countries

towards the poorest parts of the distribution. In other words, total inequality seems to have a negative association with growth among the poor and those around the median, and a positive association among the top 1 per cent. These results are confirmed even in both subsamples (tables 4.6 and 4.7).

Figure 4.2 shows the jackknife resampling technique used to check the robustness of the estimated models. The results hold for all the equations except for the one with y^90 as the dependent variable.

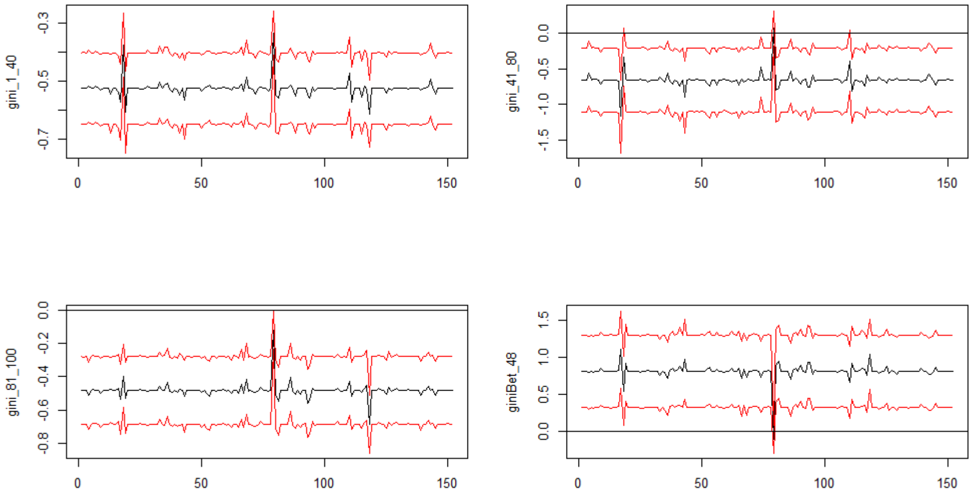


Figure 4.1: Jackknife robustness check for coefficients associated with inequality measures

4.5 Conclusions

Studies analyzing the impact of inequality on economic variables, particularly economic growth, have remained relevant over the years. However, recent works by Voitchovsky (2005) and Van der Weide and Milanovic (2018) have pointed out a potential flaw in these studies. Most rely on a single index to summarize income inequality, which fails to capture the complete distribution. Even the dependent variable is often limited to a percentage variation of a heterogeneous measure of a country's development such as per capita income.

In previous studies, Voitchovsky proposed using the Gini index and percentile ratio as covariates to capture the profile of inequality. On the other hand, Van der Weide and Milanovic used the average incomes of different income percentiles as dependent variables. Both studies yielded significant results and evidence of the limitations of the previous empirical analysis.

This chapter aims to improve previous works using a more up-to-date and broader panel. The study takes into account the two innovations by considering individual growth as the dependent variable and the entire inequality profile as the covariate.

Compared to Voitchovsky's work, the focus was on the calculation of Gini indices only. This approach not only enabled the researchers to cover the entire income distribution but also to analyze inequality between subgroups. This decision

	y^5	y^{10}	y^{25}	y^{50}	y^{75}	y^{90}	y^{95}	y^{99}
Y^p_{-1}	-91.5***	-51.2***	-27.7***	-19.0***	-14.5***	-12.4***	-15.0***	-29.7***
	(28.46)	(8.713)	(4.519)	(2.661)	(1.675)	(1.608)	(1.987)	(3.361)
$\text{Gini}_{1:100}$	-5.346***	-2.766***	-1.189***	-0.675***	-0.315***	0.048	0.342***	1.359***
	(1.318)	(0.589)	(0.224)	(0.141)	(0.102)	(0.081)	(0.092)	(0.172)
invest	0.016	0.251***	0.230**	0.168**	0.183***	0.191***	0.227***	0.361***
	(0.421)	(0.195)	(0.114)	(0.080)	(0.062)	(0.054)	(0.072)	(0.125)
trade	-0.147	-0.005	0.025*	0.034***	0.026***	0.017*	0.013	-0.003
	(0.167)	(0.049)	(0.025)	(0.019)	(0.016)	(0.014)	(0.015)	(0.026)
life	2.579	0.932	0.446	0.148	0.141	0.208	0.215	0.670*
	(2.411)	(0.845)	(0.445)	(0.329)	(0.267)	(0.227)	(0.199)	(0.378)
unem	-2.634**	-0.922***	-0.445***	-0.315***	-0.242***	-0.206***	-0.224***	-0.461***
	(1.274)	(0.337)	(0.172)	(0.121)	(0.092)	(0.074)	(0.075)	(0.122)
civil	-3.621	-0.517	-0.351	-0.317	0.133	0.318	0.276	0.837
	(4.536)	(1.565)	(0.753)	(0.526)	(0.373)	(0.360)	(0.462)	(0.820)
Obs	1,045	1,045	1,045	1,045	1,045	1,045	1,045	1,045
R^2	0.358	0.390	0.269	0.215	0.167	0.134	0.157	0.278
Adj R^2	0.277	0.313	0.177	0.116	0.062	0.024	0.051	0.187
F	73.9***	84.8***	48.8***	36.2***	26.5***	20.5***	24.7***	51.1***

Table 4.5: Fixed effects OLS estimates with different percentiles income as the dependent variable with the full panel

proved to be the right one, as the results varied from those of the author. The findings indicated negative effects for inequality within the bottom 40, middle 41-80, and top 20 on economic growth. However, positive effects were observed for inequality between the three subgroups.

These results therefore supported some theories on inequality such as debt constraints and the median voter. The positive effect of inequality between the subgroups may also support the link of inequality with savings or work incentives.

The work of Van der Weide and Milanovic has been replicated with a larger panel, which confirms the previous findings. The study shows that inequality harms the incomes of the poor and the middle class, while it has a positive effect on the incomes of the richer population. We also attempted to estimate the model again by considering the entire inequality profile, and although some results are significant, more time is required to study them.

The research findings have been validated through model estimation using two different sub-samples. The first sub-sample consists of only high-income countries, similar to the panel used by Voitchovsky. The second sub-sample, on the other hand, includes the entire sample except for high-income countries. The results from

	y^5	y^{10}	y^{25}	y^{50}	y^{75}	y^{90}	y^{95}	y^{99}
Y^p_{-1}	-44.69***	-35.30***	-21.29***	-15.27***	-13.08***	-12.80***	-13.18***	-31.92***
	(6.917)	(5.367)	(3.128)	(2.468)	(1.630)	(1.975)	(2.276)	(4.142)
$Gini_{1:100}$	-3.260***	-2.206***	-1.100***	-0.677***	-0.285***	-0.003	0.158	1.674***
	(0.889)	(0.583)	(0.212)	(0.144)	(0.106)	(0.110)	(0.147)	(0.439)
invest	0.665**	0.426**	0.294**	0.197*	0.204**	0.186**	0.178**	0.371**
	(0.268)	(0.205)	(0.120)	(0.106)	(0.095)	(0.086)	(0.089)	(0.173)
trade	0.151***	0.105***	0.075***	0.079***	0.070***	0.061***	0.060***	0.044
	(0.035)	(0.021)	(0.017)	(0.016)	(0.014)	(0.018)	(0.020)	(0.045)
life	1.208	0.790	0.399	0.045	0.023	0.135	0.031	1.560***
	(1.730)	(0.949)	(0.462)	(0.391)	(0.312)	(0.329)	(0.343)	(0.584)
unem	-0.935**	-0.846***	-0.545***	-0.389***	-0.361***	-0.361***	-0.360***	-0.833***
	(0.372)	(0.207)	(0.148)	(0.120)	(0.121)	(0.115)	(0.109)	(0.212)
civil	1.965	1.599	0.953	0.229	0.173	0.328	0.867*	3.243***
	(2.382)	(1.348)	(0.856)	(0.640)	(0.592)	(0.564)	(0.489)	(1.147)
Obs	506	506	506	506	506	506	506	506
R^2	0.353	0.368	0.320	0.253	0.226	0.180	0.158	0.290
Adj R^2	0.242	0.260	0.203	0.124	0.094	0.039	0.014	0.168

Table 4.6: Fixed effects OLS estimates with different percentiles income as dependent variable with the high-income countries

both sub-samples confirm the initial findings, although some parameters in the study on the inequality profile, despite retaining their sign, lose their significance in the second sub-sample.

While other forms of inequality could be considered—such as those driven by different factors, across various dimensions of well-being, or from alternative income distribution partitions—the current analysis focuses on specific profiles. These extensions would be valuable for developing a broader framework to better understand the complex relationship between inequality and economic growth.

The estimations produced significant results, aligning with the findings of the two studies that inspired this work. However, this analysis for now is mainly descriptive, as additional tests are needed to confirm the model's validity, particularly considering the potential for reverse causality. Ultimately, the goal is to replicate the analysis using wealth data from the WID database, which was utilized in the previous chapter.

	y^5	y^{10}	y^{25}	y^{50}	y^{75}	y^{90}	y^{95}	y^{99}
Y^p_{-1}	-108.2***	-62.42***	-37.36***	-25.61***	-18.43***	-15.96***	-20.89***	-37.22***
	(34.87)	(11.14)	(7.01)	(4.153)	(2.579)	(2.416)	(3.136)	(4.892)
$\text{Gini}_{1:100}$	-5.804***	-2.976***	-1.268***	-0.658***	-0.272**	0.151*	0.567***	1.740***
	(1.597)	(0.674)	(0.273)	(0.169)	(0.131)	(0.092)	(0.098)	(0.229)
invest	-0.428	0.070	0.170	0.123	0.136	0.156**	0.219**	0.334**
	(0.459)	(0.240)	(0.162)	(0.122)	(0.095)	(0.079)	(0.109)	(0.157)
trade	-0.198	-0.018	0.012	0.009	-0.003	-0.008	-0.008	0.012
	(0.208)	(0.072)	(0.039)	(0.026)	(0.022)	(0.018)	(0.024)	(0.042)
life	1.476	1.112	0.964	0.473	0.383	0.414	0.514	0.845
	(2.899)	(1.263)	(0.711)	(0.520)	(0.420)	(0.366)	(0.350)	(0.532)
unem	-3.719*	-0.899*	-0.378	-0.280*	-0.201*	-0.144**	-0.167**	-0.358*
	(1.900)	(0.504)	(0.262)	(0.165)	(0.111)	(0.073)	(0.084)	(0.197)
civil	-6.077	-1.630	-0.946	-0.509	0.183	0.322	-0.084	-0.202
	(6.533)	(2.395)	(1.120)	(0.790)	(0.544)	(0.515)	(0.649)	(1.013)
Obs	539	539	539	539	539	539	539	539
R^2	0.407	0.449	0.299	0.232	0.167	0.147	0.211	0.372
Adj R^2	0.306	0.356	0.180	0.101	0.026	0.002	0.077	0.265

Table 4.7: Fixed effects OLS estimates with different percentiles income as dependent variable with the high-income countries excluded from the full panel

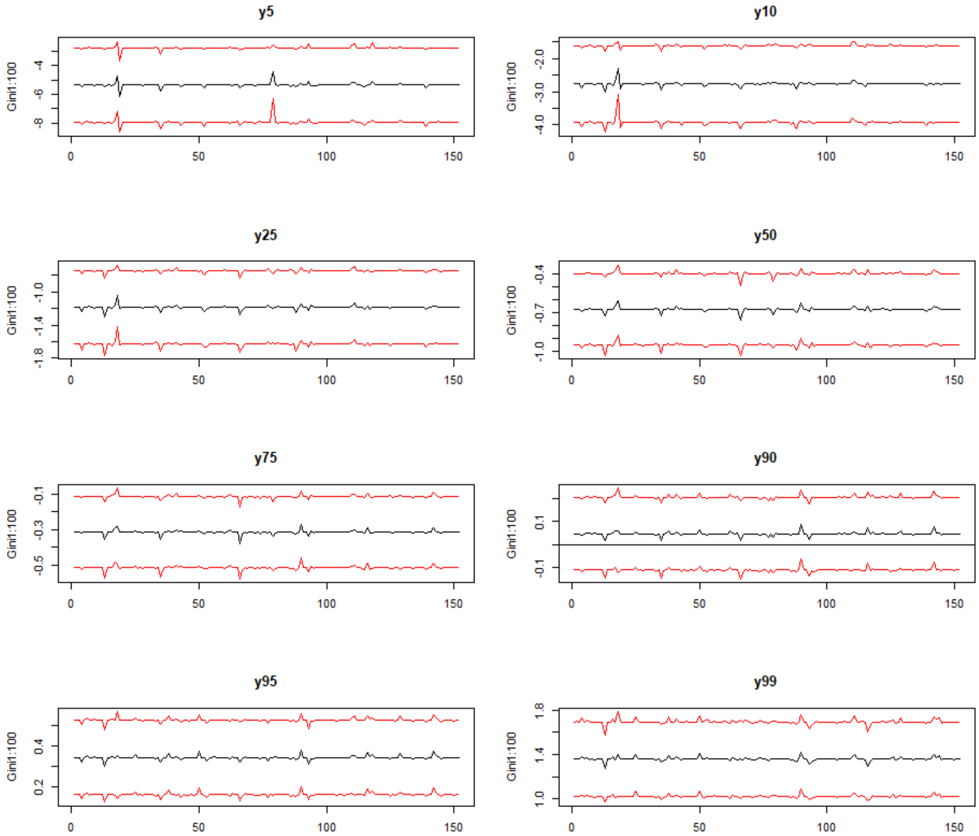


Figure 4.2: Jackknife robustness check for coefficients associated with inequality measures

CONCLUSION

Inequality in all its forms has grown globally in recent decades. Therefore, there has been a growing interest in understanding its relationship with the economic system. To support this renewed interest, there are now more extensive, updated, and accurate databases available, enabling the use of large panels for research purposes.

This project aimed to contribute to the literature by exploring wealth inequality, an area often overlooked in favour of income inequality.

The analysis presented in Chapter 2 of the literature has highlighted the limitation of focusing solely on income inequality. This approach not only produces results that are difficult to apply to wealth but also tends to reduce the analysis to a simple cause-and-effect relationship from inequality.

Inequality is no longer just a byproduct of the economic system and society, but rather an important player that interacts with many others. For instance, it has a significant impact on public debt. Inequality affects public debt by pressing governments to implement policies more favourable to the poor financed by debt to avoid conflict with the wealthy. Conversely, debt is influenced by inequality because it involves transferring resources from taxpayers to government bondholders, who are typically wealthy.

There are other ways in which inequality affects the economy. Firstly, high levels of inequality can lead to reduced economic stability, which in turn can result in increased public interventions and public debt. Secondly, public debt is linked to various economic variables that have a varying impact on different segments of the population.

Given the lack of empirical analysis on the relationship between public debt and wealth inequality, this project aimed to fill this gap. To do this, the construction of an R package for the estimation and analysis of VEC models was required. The adopted model is a very useful approach for the study of systems with endogenous and nonstationary variables. This package was built to overcome the lack of an adequate tool in the widely used statistical software. This package, described in Chapter 2, has already been made available on GitHub and will be submitted to CRAN in the future. We believe that this package will help scholars in their studies, given the creation of outputs and options that are well contrapose to those of paid software, like Eviews and Stata.

The study of the relationship between public debt and wealth inequality was focused on the US. This choice is due not only to the data availability. The US are a special case given the policies adopted during the Reagan mandate. This event produced a shock of public debt that was interesting to study. We found evidence of a bi-directional link between the public debt-to-GDP ratio and the Top 10 wealth share through the Granger-causality test and the impulse-response functions. The results show a long-run relationship between public debt and wealth inequality, whose deviations impact only the latter. Among the key mechanisms, the most relevant is found to be economic instability. We found evidence that household debt is influenced by inequality, and in turn influences public debt.

The results obtained through wealth inequality measurements differ from those obtained through income measurements, which implies that more research is needed in this area. Furthermore, these results indicate that new measures should be developed in addition to classic ones such as the Gini index or percentile ratios. The fourth chapter of this project aimed to test new measures of inequality and measures of economic development, and it invites replication with wealth percentiles, which are available for many countries in the WID portal. The income inequality profile produced results that contradict those highlighted by other empirical work on the same topic. Re-testing with the wealth inequality profile could lead to further findings.

The main conclusion that this project brings is therefore that wealth deserves to be investigated, even by repurposing old research conducted with other forms of inequality, even in the face of increasingly available data.

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