A wireless method to obtain the impedance from scattering parameters

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Abstract

The coaxial wire method is a common and appreciated choice to assess the beam coupling impedance (BCI) of an accelerator element. Nevertheless, the results obtained from wire measurements could be inaccurate due to the presence of the stretched conductive. The aim of this work is to establish a solid technique to obtain the BCI from electromagnetic simulations, without modifications of the device under test. In this framework, we identified a new relation to get the resistive wall beam coupling impedance of a circular chamber directly from the scattering parameters. Furthermore, a possible generalization of the method to arbitrary cross section geometries has been studied and validated with numerical simulations.

Wireless method

- BCI describes the electromagnetic interaction between the particle beam and the accelerating structure.
- It is essentially related to the energy loss of the electromagnetic wave propagating in the structure -> intrinsically linked to the S_{21}.
- The proposed relation to evaluate the impedance, without modifications of the DUT, has the following form [1]:

$$Z = -\frac{1}{2\pi} Z_{mode} \ln\frac{|S_{21DUT}|}{|S_{21REF}|} \qquad \text{Eq. (1)}$$

- Z_{mode} : characteristic wave impedance of the TM propagating mode
- S_{21DUT}: transmission scattering parameter of the 2-Port DUT
- S_{21REF}: refers to the related reference structure (chamber with PEC walls).

Analytical validation

- BCI of the circular resistive chamber (radius b, wall conductivity σ and length L) can be analytically calculated by using the following well-known equation [2]:

$$Z^{Theory} = -\frac{\zeta_s L}{2\pi\, b}, \qquad \text{Eq. (2)}$$

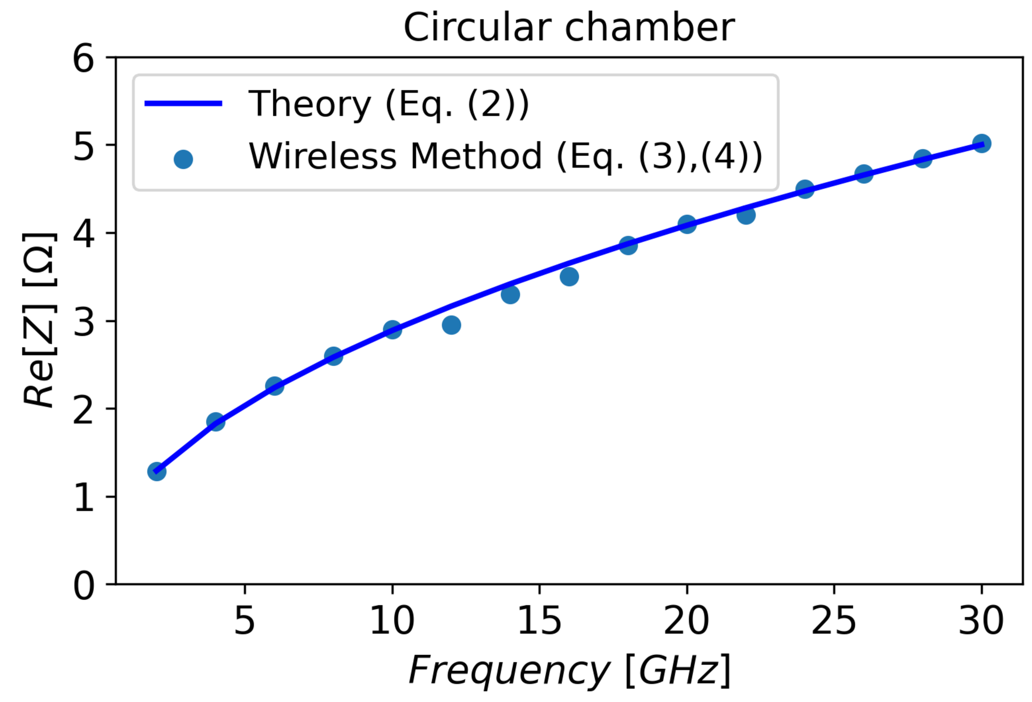
where in the classical thick wall regime $\zeta_s = \zeta(1+j) = \sqrt{\frac{\omega\mu_0}{2\sigma}}(1+j)$

- It has been demonstrated, that Eq. (1) can be written with the following expressions [1]:

Below cut-off $$Z^{below} = \frac{1}{2\pi} Z_{mode}\left(\sqrt{A} - \sqrt[4]{B^2 + C^2}\right)\text{L}, \qquad \text{Eq. (3)}$$

Above cut-off $$Z^{above} = \frac{1}{2\pi} Z_{mode}\left(\sqrt[4]{B^2 + C^2}\frac{-C}{2B}\right)\text{L}. \qquad \text{Eq. (4)}$$

- Under the assumption that b $\gg \delta = \sqrt{\frac{\omega}{2\mu_0\sigma}}$, the impedance of Eqs. (3), (4) reduces to Eq. (2). It means that Eq. (1) reduces to Eq. (2).



- $A = ({k_0}^2 - \frac{u^2}{b^2})$
- $B = ({k_0}^2 - \frac{u^2}{b^2} + 2\omega\varepsilon_0\frac{\zeta}{b})$
- $C = 2\omega\varepsilon_0\frac{\zeta}{b}$

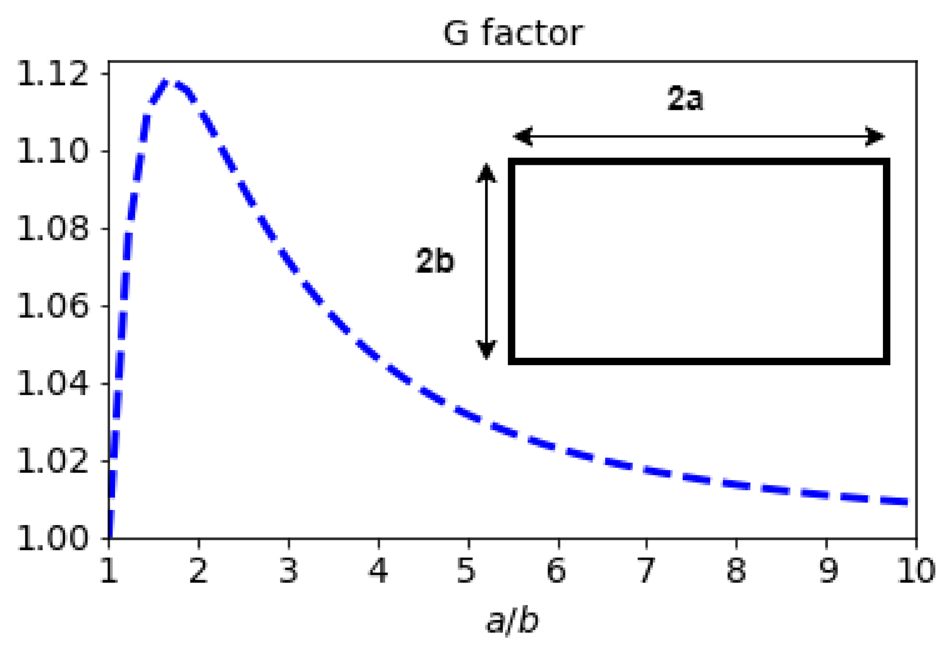
- k_0: wave number of free space
- u_{nm}: m^{th} zero of the Bessel function

Generalization to arbitrary cross section geometries

- Above the cut-off frequency of the chamber, the impedance can be generalized as follows, as shown in [1]:

$$Z = -\frac{G\,F}{2\pi} Z_{mode} \ln\frac{|S_{21DUT}|}{|S_{21REF}|}. \qquad \text{Eq. (7)}$$

- F is the longitudinal form factor (see [3], [4]).
- Analytical expressions of F could be computed for any geometry with simulations.
- G turns out from the analytical derivation.

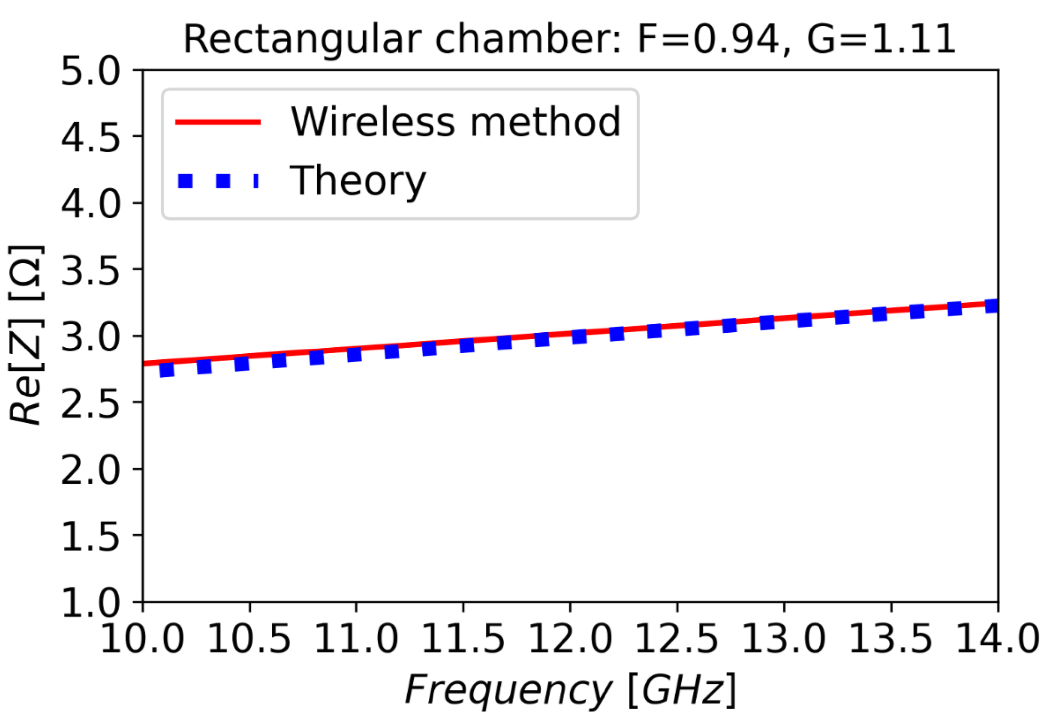


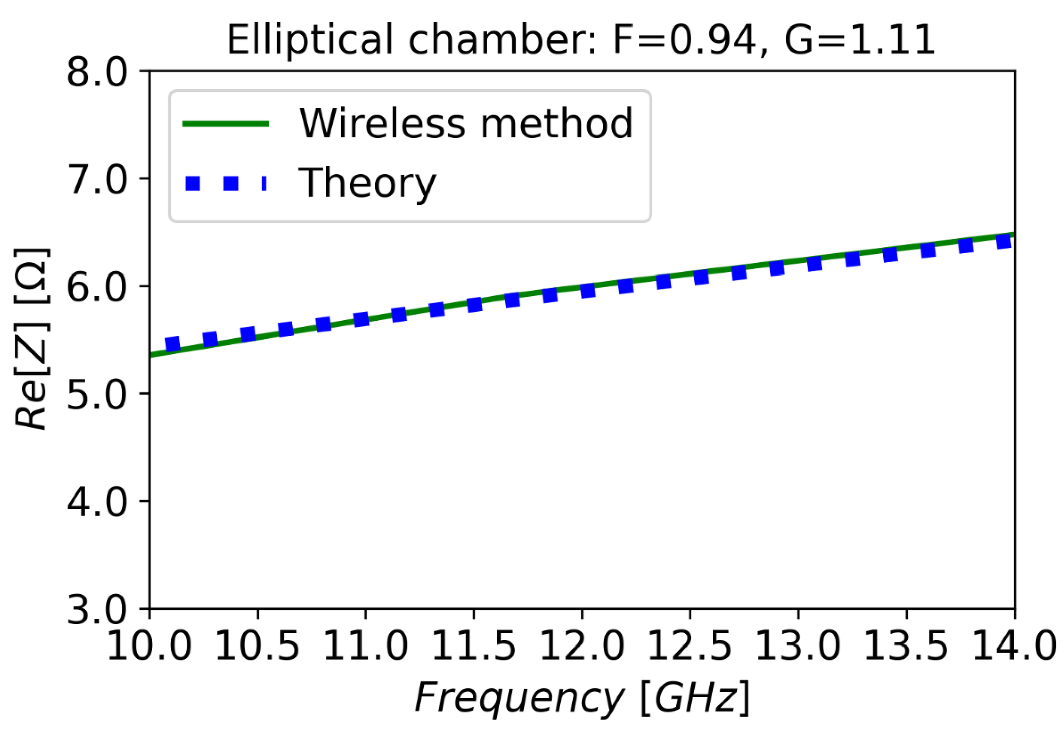
$$F = \frac{w(z)^{DUT}}{w(z)^{CIR}}; \quad G = \frac{(b^2+a^2)a}{b^3+a^3}$$

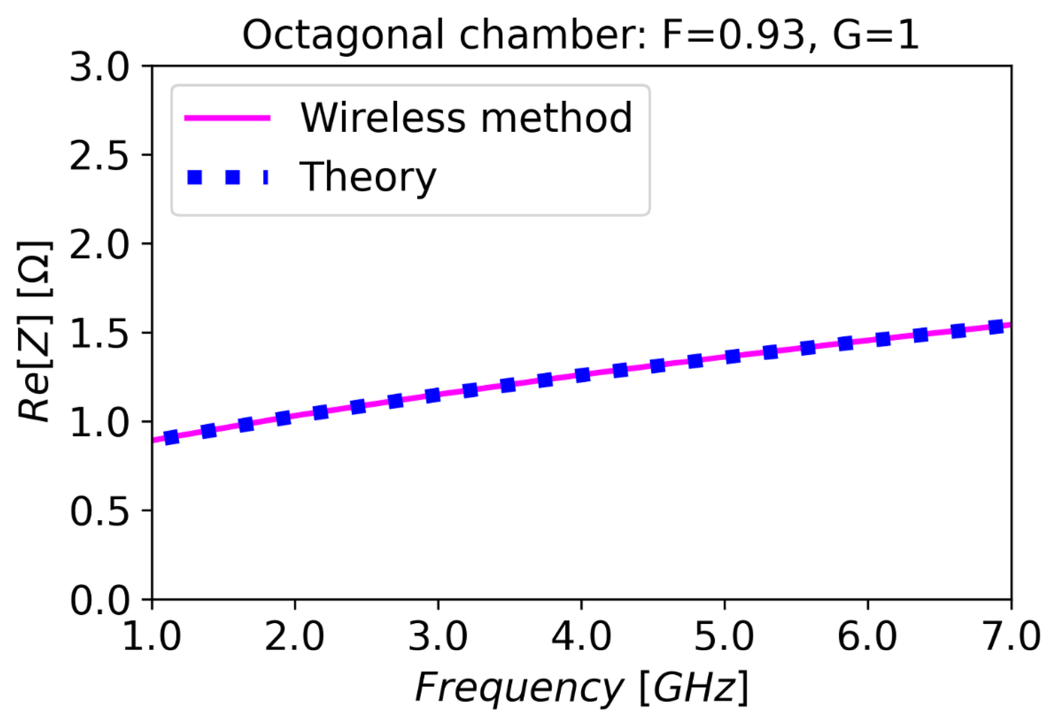
- w(z): wake function
- F, G are geometrical factors

Simulation results and comparison with theory

- Simulation studies are carried out using CST Studio Suite.
- DUT is excited using the Waveguide Ports -> only the TM mode is launched.
- The longitudinal impedance computed from frequency domain simulations by using Eq. (7), has been compared with the exact theoretical evaluation in the following figure:







- Almost perfect agreement between the two curves suggesting -> the proposed simulation approach is a suitable and accurate method to compute the beam coupling impedance of arbitrary shaped chambers.

Conclusion and outlook

- A Logarithmic formula that relates the longitudinal beam coupling impedance and the transmission scattering parameter without modification of the DUT has been proposed.
- The new formula has been analytically validated for a resistive circular chamber.
- The generalization to arbitrary chamber shapes by means of appropriate geometrical factors, F and G has been studied.
- This very promising method could pave the way to develop a bench measurement technique -> engineering of an excitation able to excite the first TM mode.
- Possible extension to resonant structures is ongoing.

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[4] C. Zannini, "Electromagnetic Simulation of Cern Accelerator Components and Experimental Applications", Ph.D. thesis, Phys. Dept., École polytechnique fédérale de Lausanne, Lausanne, Switzerland, 2013.