LEARNING FROM INTERACTIONS: TAX EVASION IN AN EVOLUTIONARY PERSPECTIVE

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Chapter 1: Introduction. On evolutionary games and their use in studying tax evasion

This work studies the effects of interactions between fiscal authorities and taxpayers on income tax evasion, both in a theoretical and empirical perspective.

Particularly, an evolutionary game is used to investigate the role of repeated interactions on taxpayers' choice to be honest, cheating or ghost with respect to income taxation. We formalize both the individual and population perspective, without following the assumption of common knowledge of rationality (Fudenberg and Tirole, 1991), and by focusing on the role of learning from interactions instead of on the utility maximization problem. We assume a large population, divided into two subpopulations, taxpayers and fiscal agencies, where individuals are characterized by heterogeneous intrinsic characteristics (the propensity to declare their income to fiscal authorities varies from one individual to another), but by homogeneous decision making processes. In other words, although individuals' parameters may vary within the subpopulation, the taxpayers' and fiscal agencies' decision making processes are the same. We will use the replicator dynamic equation, representing both the individual and population perspective, to answer the following questions: when individuals are exposed to a certain behavior spreading in the population, how is their original choice affected? Do the proportion of honest/cheating increase when individuals are exposed to other honest/cheating peers, or to different forms of pressure from tax authority? How (more or less) effective control affects the distribution of honest/cheating taxpayers in the population? According to evolutionary models, individuals decide which strategy to choose and when to switch from a strategy to another one, by comparing the expected payoff of that particular strategy with the expected payoff of the overall population (Friedman, 1998). We refer to the replicator dynamic equation with the purpose of studying the role of individual learning from interactions with other individuals, all sharing the same "rules" (including the decision making and the learning processes themselves): individuals do not maximize their utility function, they rather decide which strategy to choose, and when to switch from a strategy to another by comparing the expected payoff of a given strategy with the expected payoff of the overall population. Within this context, we formalize different features of both taxpayers (i.e., their attitude to tax compliance) and fiscal agencies (i.e., both effort and effectiveness of costly inspecting activities) under the following assumptions: *(i)* individuals belonging to the two subpopulations - fiscal agencies and taxpayers - randomly and repeatedly match with each other over time, having each a different set of strategies (or characters); *(ii)* subjects' influence on the other members of the subpopulation is indirect, depending on both the random matching between individuals, and on the memory, formed at individual level, of the others' responses over time. An equilibrium solution – i.e., an evolutionary stable strategy (ESS) - is found during the repeated matching process when the distribution of characters/strategies in the subpopulations stops varying over time.

An important feature of the considered setting is the relation between individual choices and collective outcomes. Individuals – repeatedly and randomly matched pairwise - choose strategies (or characters) within their strategy set, given by the distribution of all individual strategies within the population over time. After each interaction, individuals revise their subjective probabilities on the distribution of characters/strategies within the population, choosing the strategy associated to the highest expected payoff at the previous interaction, and the revised probabilities, in turn, determine the new expected payoffs after each interaction. This form of indirect learning is a characteristic of each player, and thus the equilibrium solution is given by the (un-voluntarily) coordination of players towards refinements in the probabilities they assign to each character (strategy). The approach allows to identify the distribution of the strategies "evade" or "not evade", and "inspect" or "not inspect" within the subpopulations of taxpayers and tax authorities, respectively, according to the best performing payoffs over time. Moreover, by analysing tax compliance not only within taxpayers' perspective, but also in fiscal institutions' perspective, we identify the drivers of the equilibrium distribution of characters, as well as how such distributions evolve over time.

The results of the model presented in Chapter 2 may be summarized as follows.

Taxpayers' perception of costs associated with inspecting activities and fiscal agencies' level of effort in inspections play a relevant role in individuals' compliance, in addition to tax and penalty rate. In fact, results suggest that compliance level increases as taxpayers perceive that costs associated with inspecting activities are low, or the level of effort in enforcement is high. As long as the average effort in inspections is higher than the ratio between the costs and the amount recovered, the equilibrium solution is represented by a positive proportion of honest individuals within the subpopulation of taxpayers, $\overline{q_H} > 0$. On the other side, tax authorities' higher levels of effort are associated with a lower proportion of inspecting agencies, as both the direct and indirect (through taxpayers' perception) effect of effort lead to the need of a lower level of auditing activities (as the perception of effort leads to a higher level of compliance). Higher tax rates are therefore associated with a higher proportion of both cheating taxpayers and inspecting agencies. Higher penalty rates induce a higher proportion of both compliant taxpayers and non inspecting fiscal agencies. Higher levels of effort in enforcement and inspecting activities induce a decrease in the proportion of cheating individuals within the subpopulation of taxpayers through perception, which in turn is associated with an indirect effect on the proportion of inspecting fiscal agencies, and a decrease in the proportion of inspecting agencies (direct effect on the proportion of inspecting fiscal agencies) due to a better allocation of resources. Effort put in inspections and enforcement is therefore crucial, together with tax and penalty rate, to enhance compliance.

This work, thus, approaches individuals' choices about tax compliance under two new points of view. First, players are not assumed to maximize utility, they rather compare the expected payoff of a strategy with the average expected payoff of the entire set of strategies of the population. Second, the effects of repeated interactions and the mechanism of learning are studied both from an individual and population perspective. In this respect, the model encompasses those aspects emerging by empirical investigations in tax evasion literature difficult to be reconciled with the formalization of individuals' behavioural decision making process under expected utility theory.

The work is organized as follows: in Chapter 2, we will present a model where fiscal agencies' effort and costs in inspections are modelled, whereas in Chapter 3 opportunity costs fiscal agencies face when non inspecting a cheating individual are presented, as well as costs faced by taxpayers once they are controlled by fiscal authorities. Then, Chapter 4 presents an experiment conducted with students at the Faculty of Economics of the University "La Sapienza" in Rome, which aims at verifying whether learning affects individuals' propensity to be honest, as well as some of the results presented in the previous chapters, and specifically with respect to tax and penalty rate. Particularly, in the experimental design, participants were asked to declare their endowment in several rounds, subject to several different combinations of tax and penalty rate. Participants had to take decisions within several different settings with respect to fiscal parameters, whereas the probability of inspection was held constant at 0.5. For each different setting of the experiment, several rounds were played, and after each round participants were told whether they were inspected or not by fiscal authorities, and how much of their endowment remained after the payment of taxes due and on the (eventual) penalty. The interactions presented in the model of Chapter 2 were simplified in the experimental design, and particularly, the inspections were represented by a probability of control equal to 0.5, with a message displayed to participants saying whether they were inspected or not, and how much penalty was due accordingly.

Allingham and Sandmo (1972) approach tax evasion choice as a decision under uncertainty, combined with the basic assumption that "*failure to report one's full income to the tax authorities does not automatically provoke a reaction in form of a penalty*" (Allingham and Sandmo, 1972, p. 306). Individuals' behaviour is channelled through the Von-Neumann and Morgenstern framework, where individuals' utility function is characterized by an always positive and strictly decreasing marginal utility, assuming risk averse individuals. Government can affect individuals' compliance choice through three variables under its control: tax rate, penalty rate and probability of detection. The model predicts a positive relation between penalty rate (and probability of detection) and the level of compliance, whereas with respect to the effect of tax rate, the model appears to be counter-intuitive. More generally, the expected utility theory (EUT) model of tax evasion predicts a negative relationship between tax rates and evasion, whenever fines are imposed on the evaded tax and taxpayers exhibit decreasing absolute risk aversion (Yitzhaki 1974). The negative relationship between tax rates and evasion predicted by the EUT model has sometimes been called "Yitzhaki paradox" according to which at the interior optimum, tax evasion is strictly decreasing in penalty rate and strictly increasing in marginal tax rate.[1]

To exit the paradox, some formalizations within the EUT setting were focused on the existence of interdependency between taxpayers. Geeroms and Wilmots (1985) explore the interdependency of taxpayers' behaviour, when other individuals (i.e. peers) are believed to evade, referring to the existence of a mixture of moral effects due to imitation, even in tax evasion behaviour[2]. Within a similar conceptual framework, Bebchuk and Kaplow (1992) study the optimal level of penalty on tax evasion, by assuming imperfect information about the probability of apprehension. Their results suggest a lower optimal level of penalty with respect to the optimal level of penalty in contexts where perfect information is assumed.

Empirical evidence suggests that not only fiscal variables such as tax and penalty rate chosen by the government are relevant when studying individuals' compliance decision, but also interdependency among individuals belonging to the same population, and consequently social norms, as well as individuals' perception, moral suasion, peer effects and the presence of obligatory advance payments of taxes.

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[1] A setting that seems to reconcile theoretical and empirical features of tax evasion is due to the studies within the conceptual framework of prospect theory, and the reconsideration of the Yitzhaki puzzle in this respect (Piolatto and Rablen, 2017).

[2] Kleven, Kreiner and Saez (2016), using an agency model where firms' development leads to an increasing number of employees, study the effect of third-party information, and internal organization, on the performance of tax enforcement. Their result show that firms' organization leads to a slight decrease in tax evasion, as a result of the pressure exercised on the firm by employees and third parties.

With respect to peer effect, social norms and interdependency among individuals pertaining to the same population, Hallsworth et al. (2017), within two large natural field experiments in the United Kingdom, found that social norms are relevant drivers of tax compliance. Messages linking tax payments to their use, as well as reminder letters containing public goods messages, play an important role in enhancing individuals' compliance decision. As Polinsky and Shavell (2000) underline, in fact, individuals are induced to comply to law either through enforcement acts and strategies, or with the help of social norms, which can also be used for assessing strategies. The degree of internalization of social norms into the enforcement strategy may be key in improving their effectiveness, as well as their ability to prevent crimes instead of intervening *ex post*[3]. Kleven (2014), by comparing Scandinavian countries with the U.S., in light of their different marginal tax rates and audit systems, investigates the main determinants of Scandinavian redistribution model and its effectiveness. His results show that policy makers' decisions may have played a relevant role in shaping individuals' social norms, and in turn, their high rate of compliance to tax standards. This result is particularly interesting when addressing refinements to public policies on taxation, which should be considered not only in terms of variables directly controlled by the government, but also in light of the role played by moral suasion and other social norms on individuals' compliance decision. In fact, the latter can be enhanced by targeted policy measures related to social norms[4]. Individuals' willingness to comply with taxation might be also channelled through advance

[3] In this same line of research, Dulleck et al. (2016), within an in-lab experimental setting where psychic stress was measured through heart rate variability, suggest a positive and statistically significant relation between tax compliance and psychic stress related to the tax declaration process. Moreover, their experimental evidence allows them to distinguish between three types of taxpayers, with respect to their features related to psychic stress, tax morale and level of compliance. Nonetheless, as psychic stress is a negative feature of taxpayers' compliance, it might not be a powerful tool to be enhanced by policy measures of taxation, whereas a positive status such as tax morale may be, by contrast, a beneficial feature to be addressed by fiscal authorities.

[4] Kleven et al. (2011) find a high level of compliance in many modern tax systems, even in presence of low audit rates and fairly modest penalties. Moreover, Dhami and Al-Nowaihi (2007) underline two other relevant aspects in the contrast between theoretical paradigm and empirical studies. First of all, assuming a positive expected return to tax evasion, all taxpayers should hide a certain proportion of their income, whereas data show the percentage of evaders is lower than the one representing the majority of eligible taxpayers. Additionally, as regards obligatory

payments of taxes, as well as services offered, financed through taxes[5]. Alm, Blomqvist and McKee (2017), stress the importance of peer effects, therefore the effect of neighbours' choices on individuals' tax compliance level. Their results suggest that the provision of information on whether one's neighbours are filing returns and/or reporting income has a statistically significant and large impact on subjects' reporting decisions. Alm, Bruner and McKee (2016) examine the priming role of honesty messages sent by individuals to their peers, regarding their audit outcome and compliance behaviour. Results indicate the positive effect of accurate messages regarding audit outcomes and compliance behaviours on individuals' choices, as well as the fact that many individuals are also systematically dishonest about being audited[6]. Moral suasion has also been investigated as a potential driver of individuals' compliance, although empirical evidence did not confirm such relationship[7].

Another line of theoretical research in individuals' tax evasion decision is related to the agency costs and bargaining, and repeated dynamic games. In this respect, Reinganum and Wilde (1985) use an agency-based paradigm to compare alternative audit policies, and specifically, policies based on "audit cut-offs" with standard random audit, finding the latter being less efficient than the audit policy based on the cut-off, in both settings based on lump-sum and proportional taxation. Moreover, Greenberg (1984) uses a dynamic, multiple stage game with utility maximizing individuals to investigate optimal audit activities,

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advance tax payments, the latter should not affect tax evasion decision under expected utility theory, whereas empirically a negative effect on tax evasion has been demonstrated (Yaniv, 1999).

[5] Alm, Cherry and McKee (2010) use laboratory experiments to test whether taxpayer services enhance the level of compliance. Then, they extend the baseline experimental setting allowing to the existence of services helping subjects to compute more easily their tax liabilities. Their findings suggest that uncertainty reduces individuals' compliance, showing that agency-provided information has a positive impact on declaring their income to authorities. Behavioural economics studies, additionally, try to attack the topic with respect to emotions and reciprocity.

[6] They also observe a strong interaction between individuals' audit outcomes and their compliance behaviours, so that individuals who engaged in tax evasion and who were audited were more truthful in their communications than the uninspected ones.

[7] With data gained from a controlled field experiment in Switzerland, Torgler (2004) tests the effects of moral suasion on tax compliance. His results suggest moral suasion doesn't act as driver in taxpayers' compliance behaviour. The strongest effect can be observed for the variable tax payments.

starting from a classification of players into groups according to the probability of being audited, and the decision each individual takes between being honest or cheat. Additional evidence with respect to interactions within dynamic repeated games have been set up in the study by Fortin et al. (2007), in an extension of the standard utility maximization model of tax evasion allowing for social interactions. Their results suggest that the latter are relevant in tax evasion decision when measuring fairness and imitative pressures[8].

Theoretical research was also focused on the cost - benefit analysis of the optimal tax agency behaviour, and the optimal degree of enforcement. Particularly, Sandmo (1981) indicates an optimal degree of enforcement corresponding to the point where the marginal cost of controls is larger than the marginal tax revenue. Yitzhaki and Slemrod (1985) reconcile this result with previous literature by assuming that Sandmo's result also includes the compensation for maintaining the same level of utility. Assuming utility maximizing individuals, both studies evaluate the two influencing factors of tax decision separately (mainly tax parameters and detection cost-benefit considerations and strategies), without taking into

[8] Dynamic models have also been widely used for determining the optimal level of taxation in light of economic growth. Macho-Stadler et al. (1999) show the effects of tax amnesty policies on compliance, with a dynamic model of tax evasion, where the probability of inspection decreases with both present and past evaded taxes, finding that extensive tax amnesties only temporarily improve compliance. Dalamagas (2011) uses a comparative dynamics framework in the context of a neoclassical growth model, allowing taxation burden affect taxpayer's decision making process, as well as time. His results suggest that detection probability and penalty rate play an important role in compliance decision, as well as learning, as "*taxpayers adjust their behavior over time in the direc- tion of reporting increasing proportions of their income as the fine rate rises*" (Dalamagas, 2011, p. 322). Bethencourt and Kunze (2019), within a dynamic overlapping generations model of tax evasion where also tax morale is taken into account and explicitly modelled, investigate the negative relationship between tax evasion and economic development, showing that moral costs are behind that negative relationship, as tax morale is positively correlated with GDP per capita. Chen (2003) formalizes tax evasion into a standard AK model, where the government aims at finding the optimal level of tax rate, whereas individuals choose how much to evade by maximizing their utility function. His findings suggest that an increase in tax auditing affects negatively tax evasion only in presence of low costs of enforcement. Finally, Roubini and Sala-i-Martin (1995) investigate within a dynamic model the effects of financial repression policies on long-term growth, showing the crowding out effect of financial repression in countries where tax evasion is large, as governments choose to address tax repression policies towards the financial sector, through a reduction of investments. It should be noticed that, when studying the optimal social level of taxation within growth and dynamic OLG models, individuals are assumed to be utility maximizing, and fully rational.

consideration the role played by the interactions between the two. In a later study, Slemrod and Yitzhaki (2002) underline the role of policy instruments affecting the magnitude of tax avoidance and evasion response, as well as of the appropriate level of resources to be devoted to enforcement, and pointing out that the "elasticity of behavioural response" is a policy instrument, to be optimally chosen, together with relevant "classical" parameters such as tax and penalty rate. Tax authorities should therefore choose the optimal level of resources to be devoted to inspections and enforcement, taking into account individuals' response to policy measures, not only induced by variables controlled by fiscal authorities, but also by individuals' perception about audit activities. The evolutionary model that will be presented in the following Chapter 2 starts from the assumption that fiscal agencies are active parts not only in taxpayers' decision making process about compliance, but also in determining the equilibrium level of compliance, addressing relevant fiscal parameters, and setting up their auditing strategies also on the basis of taxpayers' behavioural response to perceptions about the strategy chosen.

Moreover, Slemrod (1990) points out that, as alternative tax systems show significant differences in operational costs, such differences are critical determinants in the choice of an "optimal" tax policy. A theory addressing optimally fiscal institutions' auditing strategy should therefore take into account the "technology" related to inspections, the related costs, and the main features of such technology.

Additionally, Alm (2019), in his recollection of tax compliance and administration studies, underlines the importance of studying the effect of enforcement actions on tax returns. In fact, even though tax administrations' primary aim is to enforce compliance, regardless of whether costs are covered by tax returns, restraints in resources might force authorities to perform inspections only when overall benefits from the audits performed overcome the related costs. The role of administration in this latter setting is therefore to guarantee a certain level of compliance, given the disposable resources devoted by the government.

Empirical investigations have shown that interactions play a significant role on individuals' compliance decisions also through moral suasion and peer pressures (including the role of tax administration). Moreover, results may be read also in terms of effectiveness of tax authorities' policy actions, for which enforcement - joint with the role of tax administration as a facilitator and a provider of services to taxpayers/citizens - seems to be of paramount importance.[9] Nevertheless, empirical evidence does not help to uniquely solve the Yitzhaki paradox, as it provides opposite evidence and explanations with respect to the effects of variables controlled by government (audit probability, tax and penalty rate) on individuals' tax evasion decision. In fact, empirical investigations suggest the existence of individuals that, even when facing the same gamble as others, openly choose to behave differently. These findings suggest a wide range of individual attitudes towards tax planning, with the implication that individuals' perception of

[9] Casaburi and Troiano (2016), studying policy measures implemented in Italy against "ghost buildings", and including benefits for honest taxpayers, find a positive communities' reaction (in terms of incumbent politicians' re-election) to such tax evasion policies.

government policies against tax evasion plays an important role in taxpayers compliance.[10] Such findings, in turn, have led to depart from the assumption of taxpayers as fully rational agents.[11]

To sum up, Allingham and Sandmo (1972) assume rational, utility maximizing, and risk averse taxpayers, whose marginal utility is always positive and decreasing along the utility curve. Tax planning is studied in an individual perspective, taking the role of fiscal institutions and their policy choice as given. Unlike suggestions from empirical evidence, effects of individual behavior on the entire population, as well as individuals' repeated interactions with tax authorities, are not considered, being individuals not considered as part of a population or community. By contrast, with evolutionary models, we can study tax compliance both from the individual and collective perspective, as repeated interactions among individuals are included in the model. Removing the assumption of common knowledge of rationality, evolutionary games

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[10] Alm et al. (1990) find that both detection tools (penalties and audits) and tax rate have a significant impact on compliance decision, with tax revenues increasing when decreasing marginal tax rates, and counter-intuitively, less compliance associated with higher penalties and audit probabilities. Ali et al. (2001) find a significant effect of both tax and penalty rate on the level of compliance, suggesting that they both play an important role in the tendency of individuals towards noncompliance. Baldry (1987) finds that the assumption of no effects of a change in tax rates on individuals' compliance decision is strongly rejected by evidence. Also considering the effects of audits and of audit strategies on compliance, Pommerehne and Weck-Hannemann (1995), in light of the relationship between the taxes paid and the public good to be financed with taxes, suggest the positive effects of audits on tax compliance decision, whereas penalty rate appears to be less significant in terms of effectiveness. Similar evidence is suggested by the studies conducted by Ali, Cecil, and Knoblett (2001). In addition, the role of an audit strategy is often a key element in enhancing compliance. Alm, Cronshaw, and McKee (1993) experimentally compare different tax enforcement settings, showing that they induce a higher compliance level with respect to purely random audit.

On the other hand, Alm et al. (1992) outline that one relevant feature is the perception individuals have of the critical variables rather than the variable themselves. In this same line of research, experimental results by Torgler (2003), further qualify those factors, like moral suasion, affecting positively compliance decision. Dwenger at al. (2016) find intrinsic motivations quite relevant for compliance decision. In a recent field experiment conducted within bakery shops in Italy, Battiston and Gamba (2016), investigate the effect of potential peer pressure of buyers in sellers' tax declaration decisions, showing a positive and persistent effect of clients' pressure on sellers' propensity to compliance. Alm, Blomqvist and McKee (2016), within an experimental setting, stress the importance of peer effects, i.e. if tax compliance behaviours are affected by the behaviour of their neighbours. They also examine the issue of the honesty of messages that an individual chooses to send to his/her peers regarding their own audit outcome and their own compliance behaviour.

[11] In this respect, an interesting extension has focused on ghosts, an increasing empirically relevant phenomenon of individuals deliberately fully omitting to declare their income status to tax authorities under unclear/unknown motivations (Erard and Ho, 2001).

have shown the existence of a stable equilibrium in the long run, depending on "evolutionary pressures", which in economic terms may be translated as the best performing strategy at a population level.[12]

Evolutionary games' potential in economic applications has been identified in light of two research goals. First, with the purpose of explaining the concept of Nash Equilibrium and its meaning within a context that does not necessarily imply fully rational individuals. Second, with the purpose of explaining economic choices in light of aspects that classical game theory failed to understand, or did not take into consideration. As regards the first goal, evolutionary games revealed to be valid tools in explaining solutions in classical game theory within a different logical framework, as selection among Nash equilibria or the Nash equilibrium itself[13]. With respect to the concept of Nash Equilibrium, Fudenberg and Kreps (1993) explore further the differences between repeated games' framework and the so called "fictitious play" framework. In the first context, as the game is played repeatedly, individuals' guesses about the dominant strategy are based on learning from past experience. In the second one, by contrast, individuals choose the strategies associated to the highest expected payoff, assuming opponents will choose strategies with a probability derived from the past choices, incorporating historical frequency in their expected payoffs[14]. As expected

[12] "Evolutionary pressures" are characterized by a different meaning with respect to biological sciences, as they can be thought as an implicit mechanism of learning from experience and from the influence exercised by other members of the population.

[13] Aumann and Brandenburger (1995), and Bhaskar (1995), within their works on equilibrium selection and dynamics in non-cooperative games, show that equilibria derived from the mechanism of strategies' adaptive behaviour leads to similar results to the optimizing behaviour setting. As Binmore and Samuelson (1999) point out, stationary states derived from the formalization of learning from interactions appear to be similar to the equilibria found by repeatedly playing non-cooperative classical games, where stability can be studied through small deviations from the stationary state, as in the theory of growth. In evolutionary games, in fact, individuals' decision making processes are formalized through systems of differential equations, representing the dynamics of each character over time as the result of interactions between members pertaining to the population of players. The equilibrium solution of the game corresponds then to the point at which all characters stop varying over the entire population, the same way as in economic growth models the steady state is found as the point at which all variables stop varying over time (Barro and Sala-i-Martin, 2004).

Mailath (1998) compares solutions of repeated games with monotone dynamics with evolutionary game theory results, which are based on class aggregate dynamics, concluding a substantial similarity of the two approaches in terms of results, confirming the usefulness of the evolutionary setting in explaining from an epistemic point of view classical game theory's solutions, without implying fully rational, utility maximizing agents.

[14] This latter concept is key in understanding the reasoning behind evolutionary game dynamics also within the game that will be described in Chapter 2, as it is the mechanism through which characters spread over time within

payoffs change over time due to the refinement of probabilities on the basis of experience, characters evolve following this mechanism of refinement, until their growth rate goes to zero. A remarkable feature of the mechanism underlying evolutionary games equilibrium is indirect influence. The refinement process resulting from the enrichment of historical data on the distribution of characters within the population starts from the assumption that each individual influences others through experience[15].

As regards the second goal, another remarkable contribution in highlighting the explanatory usefulness of the evolutionary games' features with respect to classical game theory results has been given by Samuelson

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the population, as a result of the interactions between members taking different characters. By including historical probabilities derived from past experience within the expected payoffs, over time, each individual faces an enrichment of historical data, as experience increases. As a consequence, probabilities are refined and updated with experience, and also expected payoffs are adjusted accordingly.

[15] With the purpose of investigating and better qualifying the mechanism of strategy selection, in this direction, other lines of research resorted to the so-called "*heuristic learning models*", which assume myopic individuals using payoffs gained in the past as the main driver of decision making, or "coordinated Bayesian learning" setting, where coordination appears to be a feature of ex ante predictions rather than of best response. Examples of these models are represented by Erev and Roth (1998), where individuals use their own personal "historical" data set of payoffs gained to predict their best strategy in the future, and Fudenberg and Levine (1997). In this setting, Kalai and Lehrer (1993), demonstrate that, with infinitely repeated games, predictions made with the formalization of learning processes converge to the Nash Equilibrium derived from the classical repeated games, both in case of complete and incomplete information. As Fudenberg and Kreps (1993) results show that fictitious play and repeated games logics lead to similar results, other refined and more recent studies, as the ones presented by Berger (2004), Leslie and Collins (2006) and Han and Hu (2020), further investigate these two frameworks, basically deriving similar conceptual results. Moreover, further studies investigated convergence with respect to the equilibrium, leading to different results, depending on the mathematical formalization of the game. Sandholm (2007) and Hofbauer and Schlag (2000) derived the result of the substantial convergence of games based on evolutionary concepts to an equilibrium which is stable "*à la Nash*". Particularly, Hofbauer and Schlag (2000), suggest that the Nash Equilibrium corresponding to the equilibrium in a large population game of the Matching Pennies is stable over the economic cycle, depending on the sample size of the population studied. Moreover, Hart and Mas-Colell (2003) refine the understanding of convergence to the equilibrium by viewing the switch from a strategy to another one within their strategy set, depending on their measure of regret of their choices, which is called the adaptive rule of ''*regret-matching*'' by the authors. Other formalizations (Berger and Hofbauer, 2006) explain the dominance of pure strategies, and contextual survival of the dominated one, or a certain degree of parallelism between the replicator dynamics and the projection dynamic, which appear to show similar features (Sandholm, Dokumaci and Lahkar, 2008). Graphically, the issue has been investigated by Young (1993), including refinements as Markov processes in the formalization and computation. Thus, a wide part of literature was substantially devoted to the explanation of Nash Equilibrium, and its stability features, in the context of non-rational individuals, to derive a conceptual framework that explains learning and imitative processes, as well as the reach of equilibria without using the basic assumptions of classical game theory.

and Zhang (1992). Schlag (1998) describes analytically the logic behind the replicator dynamics specification within the context of individuals' imitation, by comparing different rules of payoff selection within a population oriented approach. Results demonstrate that replicator equations represent a situation where all individuals choose their strategy according to a mechanism allowing the prevailing strategy to be the one associated with the imitation of the individual with the highest payoff, with a probability that is proportional to the gains obtained with the specific strategy[16].

Therefore, stable strategy coming from the solution of the system of replicator equations can be explained by both individual learning from experience and imitation. Nonetheless, interaction plays an indirect role in the evolution of characters. The conceptual setting of evolutionary games is in fact based on modelling a population with a high number of individuals, each of them having the same set of strategies, where each member of the population matched randomly with another one, and at each interaction individuals play a game choosing the strategy that gives them the highest expected payoff - which can be renamed as fitness functions within this setting – where "*fitness (payoff) of a given behaviour (strategy) depends in general on the composition of behaviors in the current population*" (Weibull, 1992)[17].

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[16] Börgers and Rajiv Sarin (1997) examine the effectiveness of the replicator equation in explaining evolutionary game dynamics by looking at the differences with games characterized by a stochastic learning approach, to qualify the learning process behind the replicator dynamics. Their results show that differences between the stochastic learning approach and the biological replicator dynamics arise from the fact that the first is a stochastic process, whereas the second one is purely deterministic. When a time limit is set, individuals' learning process based on stochastic assumption may be used to qualify the individuals' dynamics behind the aggregate result coming from the solution of the replicator equation, as the two approaches tend to converge (Börgers and Rajiv Sarin, 1997, p. 15).

[17] Weibull (1992) qualifies two different paradigms for theories based on evolutionary pressures, one based on the fact that individuals' set of choices include both pure and mixed strategies, which has been first investigated by Maynard Smith (1974), and the other one, by Taylor and Jonker (1978), in which only pure strategies are disposable to individuals. In both settings, as Weibull points out, "*forces of evolutionary selection produce a tendency towards Nash equilibrium play; in the long run, individuals behave as if they met the stringent rationality (and coordination) conditions of non-cooperative game theory*" (Weibull, 1992, p. 2). In other words, the mechanism of selection based on evolutionary pressures, where strategies are chosen on the basis of the comparison between the expected payoff associated to a strategy and the average expected payoff of the population, produces similar results with respect to a fully rational agents' setting.

Since the introduction of evolutionary games in economics, due to the pioneer work by Axelrod (1984)[18], with infinitely played prisoner dilemma's tournaments, the main issue with respect to the use of these models in economics was the translation of the concept of "evolutionary pressures" within the ESS. The highest proportion in the population of players is thus associated with the *tit-for-tat* strategy, the latter being an ESS (Evolutionary Stable Strategy). By the latter, we mean an equilibrium characterized by stability over time, where the distribution of characters stops varying for any additional (new) character introduced in the population (Friedman, 1998). As mentioned, it is associated with results similar to Nash Equilibrium in fully rational agents games (Samuelson, 1997)[19]. However, what distinguishes evolutionary games from repeated games in economics is what has been called "*Game against the Nature*" condition (Friedman, 1998), which can be explained as the influence individuals have on the other members of the population, derived from the random matching of them, through a mechanism of learning from interactions[20].

Another important point when coming to the application of evolutionary games to economic contexts has been underlined by Weibull (1992), for which "*the distinction between "fitness" and utility or profit needs to be carefully treated in applications to the social sciences*". Fitness can be basically associated to the level of utility individuals take from a specific strategy (Friedman, 1991). Therefore, from an analytical

[18] Sandholm (2020) offers an analytical review of evolutionary games' structure, explaining their main features also in light of their applications to economic contexts, which broadly include works on technological change and policies.

[19] Other contributions, focused on cooperative outcomes related to non-cooperative games (Linster, 1992 and Nachbar, 1992) have shown, instead, that an infinitely repeated prisoner dilemma does not always coincide with an ESS, as the dominant character in the population is subject to perturbations, depending on the specific set of choices the player faces. Nonetheless, a wide part of Nash Equilibrium features has been explained through the decision making mechanisms of evolutionary games, and the approach of this work relies on these main results.

[20] Cheung (2016) further defines the concept of "*imitative dynamics*", as the one caracterized by the "*imitative property and payoff monotonicity*" (Cheung, 2016). According to the imitative property, the choice of a strategy by an individual depends on the distribution of strategies within the population, and therefore "*the revising agent randomly chooses an opponent from the population and imitates the opponent with a probability depending on the revision protocol*" (Cheung, 2016).

point of view, fitness functions can be treated as expected payoffs, where the probabilities associated to each strategy correspond to the distribution of each character within the population[21]. This prompts to consider the aspect of strategic interaction as strictly related to the concept of "*prevalence in the current population of the behavior and the prevalence of alternative behaviors*" (Friedman, 1998). Evolutionary models applied to economics refer to both the definition of prevalence of a strategy (behaviour) over the set of all possible choices, and to the criteria under which subjects may choose to dynamically change their strategies. In those context where preferences are influenced indirectly by interactions, evolutionary games may be an alternative to the classical game theory and EUT for investigating tax planning choices, mainly because the concept of Evolutionary Stable Strategy (ESS) does not imply rational agents.[22] In other words, the potential in using this class of models resides in their view of individuals not as optimizing agents but rather driven by context, history and perception, leading to results that can be mathematically reconciled with the ones derived from full rationality (i.e. Nash Equilibrium). Together with prevalence, it is important to qualify the concept of indirect influence. As explained by Schuster and Sigmund (1983), in case of replicator dynamics, characters prevail over time on the basis of the overall fit within the population, and evolve according to their expected payoff, when the expected payoff associated to them is higher than the average expected payoff of the entire population[23]. Several other formalizations refined the concept of evolutionary processes, to explain why certain characters (strategies) survive, and others

[21] Friedman (1991) translates the fitness functions and evolutionary equilibria into the concept of steady state. As the latter in growth economics represents the point at which a variable stops varying over time (in this sense, see Barro and Sala-i-Martin, 2004), the evolutionary stable equilibrium may be seen as the steady state, representing the point at which the proportion of individuals playing a certain strategy within the population stops varying, even in presence of continuing interactions (Friedman, 1991).

[22] As explained by Schuster and Sigmund (1983), in case of replicator dynamics, characters become prevalent over time on the basis of the overall fit within the population, which means that characters grow according to their expected payoff, and particularly, when the expected payoff associated to them is higher than the average expected payoff of the entire population.

[23] In the game of Hawks and Doves described by Maynard Smith and Price (1973), lately refined to represent other aspects of conflict as in the formalization by McNamara, Merad and Collins (1991), and recently revised with respect to territorial power competition by Kokko, López-Sepulcre, and Morrell (2006), the logic of survival is explained following the concept of best fit, which in turn relies on best response.

not[24]. In the formalization proposed by Joosten (1996), relative fitness functions are given by the difference between each subgroup fitness (expected payoff) and the average fitness of the entire population. Joosten (1996) analytically demonstrates that the equilibrium derived from the formalization proposed in his work is similar to the Nash equilibrium (Nash, 1951), being a fixed point (which, in turn, is a Nash Equilibrium according to Nash, 1951) for "*weakly sign-compatible dynamics*". Furthermore, defining "*a strict saturated equilibrium is a saturated equilibrium where precisely one subgroup has maximum (relative) fitness*", the author demonstrates that the latter is "*asymptotically stable for weakly sign-compatible dynamics*" (Joosten, 1996).

Evolutionary games have been used in economics mainly for the study of technology changes, in the context of economic growth with technological innovation, and in explaining path dependence in industrial evolution (Dosi and Nelson, 1994). Moreover, several investigations applied evolutionary games to the economy of sustainable resources (Zhao et al., 2016), poverty traps (Accinelli and Sanchez Carrera, 2012), intellectual property rights (Yang et al., 2018), and the dynamics of the labour market (Araujo and De Souza, 2010).

Very little of these models has been dedicated to the study of social policies and tax detection. Nonetheless, recent research has given attention to the potential of using behavioural models, and particularly evolutionary games, in solving social policy puzzles.

In our context of randomly matched taxpayers and tax agencies, the evolution of characters within the two subpopulations will be modelled as a comparison between the payoff associated to each character (or strategy) and the average payoff, and not as a utility maximizing problem. The evolution of characters is modelled with the replicator equation, borrowed by the biological sciences. The replicator equation implies

[24] The situation where strategies associated to lower expected payoffs tend to disappear (the proportion of individuals taking that strategy within the population goes to zero) has been analytically studied by Hofbauer and Sigmund (1989).

that, statistically, each individual compares the payoff associated to each character with the average payoff of all characters, therefore choosing the strategy for which the difference between the expected payoff and the average population payoff is positive. Assuming all individuals behave this way, the replicator equations represent both the individual and collective evolution of characters: the equilibrium of the model is the solution of a system of differential equations, representing the evolution of characters within the population, occurring when all characters stop simultaneously changing over time and across the population.

Our approach is a relatively new one, considering that, despite the promising features of these models, very little has been done to apply them to the theory of tax planning. Chica et al. (2021) use an evolutionary computational model to investigate the features of frauds with respect to consumption taxes, finding that audit probabilities are efficient drivers of compliance in case of low numbers of transactions, as well as the effectiveness of policies enhancing peer pressure as rewards for individuals that decide to cooperate. Carboni and Russu (2020), by using an evolutionary model to evaluate optimal fiscal policies of the social planner, model also a certain degree of corruption level. These studies, although looking at taxation policy measures from the perspective of individuals' interaction, start from a very different and disruptive perspective with respect to the standard classical theory of taxation, as agents are not rational utility maximizing individuals. Studies modelling tax evasion and avoidance over time, used to consider probability of detection as an exogenous and fixed variable, while evolutionary models treat it as an endogenous, time-varying variable, depending also on the strategy chosen by fiscal authorities, which therefore enters into the taxpayer decision making process. This approach appears consistent with empirical evidence, which suggests audit probability to be endogenous. According to Alm (2019), as information about tax revenues is taken into consideration by fiscal authorities in many tax systems, audit probabilities are endogenous, depending on both taxpayers' and fiscal authorities' choices.

Our model is also an attempt to reconcile the main conceptual features of game theory applied to taxation, with evolutionary games characteristics and assumptions, to explain other drivers of tax compliance decision making processes, without loosing tractability.

With respect to the role played by interactions on audit strategies and probability, as Alm (2019) points out, two approaches appear to be relevant in literature over time. Principal-agent models, within a EUT multi-period repeated game-theoretic approach, broadly used to study the effects of interactions between taxpayers and fiscal authorities on tax agencies' audit strategies. They identify a so-called "*cutoff rule*" (Reinganum and Wilde, 1985), according to which, fiscal authorities inspect certainly all taxpayers reporting less than a certain level of income, while all taxpayers reporting more than the pre-determined threshold are not inspected. Alternatively, games modelling the interactions between fiscal authorities and taxpayers, where multiple equilibria arise, do not point a unique audit strategy, but rather "*conditional audit*" rules that allow tax agencies to include information about taxpayer's history when choosing their audit strategy. Also this latter set of models, although including the presence of a continuum of taxpayers showing different endowments (therefore, different levels of income), formalizes the decision making process of the *n-th* taxpayer, assuming all taxpayers behave the same way, as a utility maximization problem.

On the side of games attempting to model the interactions between fiscal authorities and taxpayers, in Graetz, Reinganum and Wilde's (1986) set up, law enforcement is explicitly included within the taxpayers' decision-making process. The latter does not only depend on penalties and detection, but also on the presence of fiscal authorities, which base their behaviour on information gathered by taxpayers. Erard and Feinstein (1994a), by including both fully compliant and cheating taxpayers within a classic tax compliance game similar to the one presented by Reinganum and Wilde (1986), where a continuum of taxpayers represents a continuous income distribution, show that, when considering also inherently honest individuals, the income distribution impacts compliance level, the level of tax evasion varying with true income.

Cronshaw and Alm (1995), within a similar setting, model the differences in the information set between taxpayers and fiscal authorities. In their study, they assume that the taxpayer knows only his/her income, not knowing fiscal agencies' audit capability, whereas on the contrary, the tax agency knows the cost of an audit or the probability that an audit will discover unreported income, not knowing the taxpayer's true income. Their results show that higher penalty rates decrease both the probability of cheating for the taxpayer, and the probability of performing inspections by fiscal authorities. These findings appear to be similar to the ones found in the model of Chapter 2, where the proportion of inspecting agencies and of cheating taxpayers decrease as penalty rate increases, although the setting is different (here we talk about utility maximizing individuals, whereas in the model that will be presented in the following pages individuals are not utility maximizing agents). Particularly, in classic games describing the interaction between fiscal authorities and taxpayers, as the ones mentioned above, not only it is assumed that players are rational individuals, but also fiscal agencies are not considered as part of the population, being only subjects that enter into taxpayers' decision making process. Therefore, the interactions between fiscal authorities and taxpayers are modelled through the circumstance that probability of audit is included within the utility maximization problem of the taxpayer. Since individuals are part of the population, and social norms - as well as imitation - drive their behaviour towards tax compliance or evasion (in addition to incentives given by audit probability, level of penalty and tax rates), evolutionary games' perspective might prove useful to understand the effect of interactions over time between taxpayers (viewed as members of the population of taxpayers) and fiscal agencies. The latter are not indifferent between inspecting or non inspecting, as it happens for the models cited above, but become active subjects, and the process of choosing their audit strategy is explicit within the structure of the game, by incorporating past experience and adaptation of behaviour to the response of the members of the taxpayers' sub-population.

Their behaviour over time affects the compliance behaviour in equilibrium. Therefore, the main difference and advantage between classic models, adapted for the presence of fiscal authorities in the game, and the evolutionary approach is that the latter models the presence of fiscal authorities as active individuals being part of the population to which also taxpayers belong. Fiscal authorities choose whether to inspect or not, depending on the response of taxpayers over time to the choice of inspecting/non inspecting, and therefore on the revision of their choices over time based on the interactions with the members of taxpayers' population. By contrast, classical games attempting to include fiscal authorities' behaviour within the fully rational utility maximizing taxpayer do not consider fiscal agencies' adaptation to taxpayers' behaviour, but instead assume a fixed strategy based on the information gathered from taxpayers.

Moreover, as fiscal agencies within the models that will be presented in the following pages are active individuals, and part of the population, they become part of the equilibrium, and thus, the approach allows also to investigate the role played by the behaviour of taxpayers on the equilibrium solution of fiscal agencies. Modelling the interdependence between taxpayers and fiscal authorities in a way such that both types of players adapt their behaviour to the response of the opponent over time, allows to identify an equilibrium solution which is not only taxpayers' optimal level of compliance, but instead it describes the distribution of fully honest and cheating individuals (by the latter meaning all taxpayers who do not declare their entire true income to fiscal authorities), and of inspecting and non inspecting fiscal agencies (thus, fiscal authorities audit strategy), both as the result of the adaptation over time to the opponents' behaviour, up to the point at which characters do not change any longer.

As we assume that individuals behave this way, the replicator equations presented in the following pages represent both the individual and collective evolution of characters, therefore leading to the fact that the model can be thought both in a static and in a temporal-dynamic perspective. The equilibrium of the model is not the result of an individual utility maximization, but it is the solution of the system of four differential equations, representing the evolution of characters within the population.

It appears clear that both the determinants of compliance and non-compliance decisions shall be taken into consideration when examining the effectiveness of taxation systems, as both questions "*why do people pay taxes*" and "*why do people evade taxes*" are relevant in fine-tuning policy decisions. As we have recalled in the previous pages, social norms shape populations' tax declaration attitudes, which, in turn, may be shaped by policy decisions. Therefore, both sides of the story (motivations and drivers for compliance and non compliance) are relevant in enhancing the performance of fiscal systems.

Additionally, as past investigations found significant effects of learning mechanisms, social norms and peer pressures on tax declaration decision, and such aspects have not been taken into consideration by standard, classical game theory, the use of a conceptual framework as the one provided by evolutionary games may address some of them, as for example learning and peer effects, modelled taking into account interactions between fiscal authorities and taxpayers, where both groups refine their choices through a learning process driven by experience[25].

[25] In this sense, Alm (2012) suggests that future research on the topic of tax compliance will be "*largely outside the mainstream of economics and indeed will move beyond psychology to sociology, anthropology, and other social sciences in order to understand better which features of naturally occurring settings are likely to affect individual and group decisions. For example, the notion of "reciprocity" arises in large part from anthropology, and that of "adherence to group norms" from sociology*" (Alm, 2012, p. 75).

Chapter 2: a model formalizing the interactions between fiscal authorities and taxpayers[26]

1. The Model – an Economy with honest and cheating taxpayers

In order to formalize the interactions between fiscal authorities and taxpayers, the model in this Chapter, and in the following Chapter 3, follows an evolutionary game approach. The starting point of the evolutionary approach, borrowed from the biological sciences, is given by the interactions among members of a society: individuals may be divided into groups and act according to the strategies individually chosen that shape their behaviour, depending on incentives related to individual expected payoffs.

The game, describing the interactions between fiscal authorities and taxpayers, is characterized by two subpopulations, denoted with k=*A*, *T*, where k=*A* indicates the population of fiscal agencies, whereas k=*T* the population of taxpayers.

For each subpopulation, we identify two sets of characters, each one corresponding to a strategy for the member. Given individuals' expected payoff, a character indicates the strategy chosen by each individual of the subpopulation before each round of the game starts, within the set of possible characters of its subpopulation. In this game, we assume risk-neutral individuals, therefore only net income after taxes and net tax revenues matter, respectively, for taxpayers and fiscal agencies.

As we will see in the following pages, the game takes into consideration both the individual and collective perspective. Before each round of the game, each individual chooses its strategy, taking into account the fitness functions (9.1) - (10.2) represented in the following pages, which represent individuals' expected

[26] The model presented in this chapter has been developed together with Professor Silvia Fedeli. The paper "*Learning from Interactions: Income Tax Evasion Decisions in an Evolutionary Perspective*", based on this essay, has been accepted for presentation at the 2022 European Public Choice Society Conference to be held in Braga (Portugal).

payoffs related to a particular strategy. According to the concept of dynamic replicator equation, representing individuals' decision making process with respect to the strategy "evade" or "being honest", "inspect" or "not inspect", individuals choose a certain strategy if the fitness function (expected payoff) associated to this strategy is higher than the average payoff across the overall subpopulation. As an example, an individual within the subpopulation of taxpayers will choose the strategy "honest" if the expected payoff for the character honest is higher than the average expected payoff related to the subpopulation of taxpayers. The concept of replicator equation departs from the concept of common knowledge of rationality within the game (Fudenberg and Levine, 1998). It represents how characters evolve during the repeated interactions of the game within each subpopulation, assuming all individuals within the subpopulation behave the same way (Friedman, 1998). Thus, in this setting, individuals are characterized by heterogeneous intrinsic characteristics - the relevant parameters such as the individual propensity to declare his/her entire income are different from one player to another - but by homogeneous behaviours, subject to "survival pressures" (in a very broadly sense, as it is intended in all economic applications of evolutionary games). In other words, all individuals behave the same way, choosing the strategy associated to a payoff which is higher than the overall population payoff, although with slightly different intrinsic characteristics. A key concept in this dynamic setting is that individuals revise their expected probabilities of finding certain specific characters, as a result of the interactions occurred with members of the other subpopulation. In fact, as expected payoffs' weighting factors are the probabilities of finding characters in the subpopulation, individuals adjust their choices according to previous experience, by revising the subjective probabilities of finding characters within the population. As all individuals in this large population behave the same way, the laws of motion of each character, represented by (11)-(14) below, approximate the decision making processes at the individual level.

In this setting, we assume that fiscal agencies can take two characteristics (their set is formed by two strategies), indicated with *a*= *i*, *ni*, where *a*=*i* “inspect” and *a*=*ni* “don’t inspect”, whereas taxpayers can take two characteristics indicated with *b*=*h*, *c*, where *b*=*h* “honest”, and *b*=*c* “cheating”.

Thus, we can write the set of individuals’ possible matches as follows

$$S = ((s_A^i, s_T^h), (s_A^i, s_T^c), (s_A^{ni}, s_T^h), (s_A^{ni}, s_T^c))$$

where

s_A^i is the character “*inspect*” for a fiscal agency,

s_A^{ni} is the character “*don’t inspect*” for a fiscal agency,

s_T^h is the character “*honest*” for a taxpayer,

s_T^c is the character “*cheat*” for a taxpayer.

Therefore, the game can be described as the structure $G = \{S, F\}$, where S is the set of strategy profiles, and F is a *n-tuple* of payoff functions where *n=4*, with the payoff matrix represented by

Strategy	**“honest”**	**“cheat”**
“inspect”	$\pi(s_A^i, s_T^h)$	$\pi(s_A^i, s_T^c)$
“don’t inspect”	$\pi(s_A^{ni}, s_T^h)$	$\pi(s_A^{ni}, s_T^c)$

We shall denote with the following

- Y is the true income earned by taxpayers, and unknown to tax authorities. We assume that the level of income is the same for all individuals although not known to tax authorities, therefore it does not affect the results.
- $\beta \in [0; 1]$ is the proportion of income declared at individual level in the subpopulation of taxpayers. The parameter β captures the propensity of an individual to declare his/her entire income or only a part of it. Taxpayers characterized by $\beta = 0$ are "*ghosts*", since the proportion of income declared to tax authorities is zero, and they evade their entire income. By contrast, subjects with $\beta = 1$ are fully compliant individuals, declaring their entire income. There are also individuals showing $0 < \beta < 1$, declaring only a certain part of their income. Thus, a subset of the subpopulation of taxpayers – characterized by $\beta = 1$ – declares the entire income to fiscal authorities, and a subset declares less, or nothing, showing $0 \leq \beta < 1$. The true value of β chosen by each individual inspected is not known by fiscal agencies before each inspection takes place. We denote with $\bar{\beta}$ the (arithmetic) average proportion of income declared within the subpopulation of taxpayers. When $\bar{\beta} = 0$ or $\bar{\beta} = 1$, the subpopulation of taxpayers is formed only by ghost or honest individuals, respectively. Nonetheless, when the initial distribution of characters within the subpopulation of taxpayers is such that $\bar{\beta} = 0$ – all individuals are ghosts – the interactions between taxpayers and fiscal authorities may lead to a different distribution of characters at the end of the game, as the payoff associated to cheating taxpayers highly depends on whether they are controlled or not. Finally, $\bar{\beta} = 1$ refers to a subpopulation of taxpayers formed by all fully honest individuals.
- τ is the tax rate applied on the income declared βY – as we will focus on the effect of interactions on income tax compliance - such that $0 < \tau < 1$ and proportional to βY.
- $C\tau Y$ is the total cost of inspections for the tax agency, which we assume to be linear on tax revenue τY, where parameter $C > 0$ is the (marginal and average) cost of inspections. We assume that C is unknown to taxpayers, and – by contrast - well known to fiscal agencies.

- α is the proportion of tax and fine recovered from auditing activity. It measures the effort of fiscal authorities in their inspecting activities. It takes values above zero, and such that $\alpha \in (0; 1]$, as $\alpha = 0$ corresponds to the case of non-inspecting fiscal agencies. When $\alpha = 1$, the fiscal agency recovers the entire amount of taxes and fines applied, and the audit is considered successful. When, by contrast, $0 < \alpha < 1$, tax agencies' inspections are partial. The latter might be due to a lack of effort, in case of one taxpayer inspected at each time, or to a selection of individuals to be inspected, in case of insufficient resources.
- φ is the penalty rate applied on the evaded income, when a cheater is caught by an inspecting tax agency, such that $\varphi > 0$, assuming $\varphi > \tau$.

We now consider the specific payoffs' structure of the game.

When an inspecting agency is matched with a honest taxpayer, the inspecting agency payoff $\pi_A\left(s_A^i, s_T^h\right)$ is given by the tax revenue on the entire income (τY), less the costs of control ($C\tau Y$). The taxpayer payoff $\pi_T\left(s_A^i, s_T^h\right)$ is given by his/her disposable income after taxes $(1-\tau)Y$, as described in (1)-(2). The parameter α is not relevant in this case, as the honest taxpayer declares his/her entire income, and the amount of taxes due is fully paid.

(1) $\pi_A\left(s_A^i, s_T^h\right) = \tau Y - C\tau Y$

(2) $\pi_T\left(s_A^i, s_T^h\right) = (1-\tau)Y$.

Suppose then an inspecting agency is matched with a cheating taxpayer. The payoff related to the tax agency $\pi_A\left(s_A^i, s_T^c\right)$ is given by the recovered amount of taxes applied on the true income earned τY, and of fines applied on the evaded income, $\varphi(1-\beta)Y$, multiplied by α, indicating the effort in recovering the entire amount, less the tax agency's costs of detection, $C\tau Y,$ as it can be noticed in (3). The cheating taxpayer's payoff once discovered, $\pi_T(s_A^i, s_T^c)$, is given by the disposable income after taxes $(1-\tau)Y$ less the fine on the evaded part of income, $\varphi(1-\beta)Y$ linear in the true income (4), multiplied by $1-\alpha$, indicating the part not recovered during the inspection.

(3) $\pi_A(s_A^i, s_T^c) = \left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha - C\tau Y$

(4) $\pi_T(s_A^i, s_T^c) = [(1-\tau)Y \underbrace{- \varphi(1-\beta)Y}] (1-\alpha)$

In case a <u>non inspecting agency</u> is matched with an honest taxpayer, (5) shows that tax agencies obtain the amount of taxes due, without facing any cost, whereas the taxpayer gets his/her disposable income (6). The case of non inspecting fiscal agencies includes also the circumstance under which there might be no "one-to-one" matching between taxpayers and fiscal agencies, as the number of taxpayers might be higher than the number of fiscal agencies in the population, as it happens in most real contexts.

(5) $\pi_A(s_A^{ni}, s_T^h) = \tau Y$

(6) $\pi_T(s_A^{ni}, s_T^h) = (1-\tau)Y$

When a matching between <u>a non inspecting agency and a cheater</u> occurs, fiscal agency's payoff $\pi_A(s_A^{ni}, s_T^c)$ is the revenue coming from the under-declared income, $\tau\beta Y$, less $\underbrace{\tau(1-\beta)Y}$ which is the amount of taxes applied on the undiscovered part of income, as in (7), where $0 \leq \beta < 1$. The taxpayer, which remains undiscovered, gets $\pi_T(s_A^{ni}, s_T^c)$, given by the disposable income Y after taxation on the under-declared income $\tau\beta Y$, where the tax rate is applied on the amount of declared outcome. It should be noticed here that equation (7) is negative for $\beta < 0.5$. Therefore, in case a fiscal agency does not inspect, and the taxpayer declares less than 50% of his/her income, fiscal agencies' payoff becomes negative.

$$(7)\ \pi_A\left(s_A^{ni}, s_T^{c}\right) = \underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}$$

$$(8)\ \pi_T\left(s_A^{ni}, s_T^{c}\right) = Y(1 - \underbrace{\tau\beta)}$$

The game is a continuous interaction of individuals pertaining to the two subpopulations, where each member of the subpopulation of taxpayers is randomly matched with a member of tax agencies' subpopulation, over a continuous time-horizon. Each individual pertaining to the taxpayers' or tax agencies' subpopulation takes a character within the subset, before the actual matching takes place. Each tax agency is then matched with a taxpayer. Neither of them knows *ex ante* which character is taken by the opponent. Thus, a non-inspecting agency may be matched either with a honest or a cheating taxpayer, without knowing it in advance. Moreover, β and Y are known only to the subpopulation of taxpayers, and unknown to fiscal authorities *ex ante*.

In evolutionary game literature, the expected payoff for each player, associated to the distribution of characters for each subpopulation, is denoted with the expression "fitness function" (Sandholm, 2010). Fitness functions represent therefore the expected payoffs for each taxpayer being honest or cheating, and for each tax agency being inspecting or non inspecting, respectively. Fitness functions are given by the weighted average payoffs associated with a character, where weights are represented by the probabilities of finding a given character in the subpopulation. Going into detail, we indicate with $(\boldsymbol{e^a}, \boldsymbol{T})$ the state of the world for each tax agency, given the behaviour of the subpopulation of taxpayers, where *a*= *i* (inspect), *ni* (not inspect), and with $(\boldsymbol{e^b}, \boldsymbol{A})$ the state of the world for each taxpayer, given the behaviour of the subpopulation of tax agencies, where *b*=*h* (honest), *c* (cheating).

We denote with p_I the proportion of inspecting agencies in the population, and with p_{NI} the proportion of non-inspecting agencies. By denoting with q_H the proportion of honest, and with q_C the proportion of cheating within the subpopulation of taxpayers, we can write the generic fitness functions associated to each strategy (or character) pertaining to each subpopulation under examination (using the notation by Friedman, 1991) as:

$$\boldsymbol{f_A(e^a, T) = q_H[\pi_A(s_A^a, s_T^h)] + q_C[\pi_A(s_A^a, s_T^c)]} \quad \textbf{(9)}$$

$$\boldsymbol{f_T(e^b, A) = p_I[\pi_T(s_A^i, s_T^b)] + p_{NI}[\pi_T(s_A^{ni}, s_T^b)]} \quad \textbf{(10)}$$

which are displayed below:

(9.1) $\boldsymbol{f_A(e^i, T) = q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]}$

(9.2) $\boldsymbol{f_A(e^{ni}, T) = q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]}$

(10.1) $\boldsymbol{f_T(e^h, A) = p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]}$

(10.2) $\boldsymbol{f_T(e^c, A) = p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]}$

Therefore, the fitness function for an inspecting tax agency – eq. (9.1) - is obtained by multiplying the probability of being matched with a honest taxpayer by the payoff obtained by an inspecting agency matched with a honest taxpayer ($\boldsymbol{q_H[\pi_A(s_A^i, s_T^h)]}$**),** plus the probability of being matched with a cheating taxpayer multiplied by the payoff obtained by an inspecting agency matched with a cheating taxpayer ($\boldsymbol{q_C[\pi_A(s_A^i, s_T^c)]}$**).** As regards eq. (9.2), the expected payoff of a non inspecting fiscal agency $\boldsymbol{f_A(e^{ni}, T)}$ is given by the payoff associated to the situation where a non inspecting fiscal agency is paired with a honest taxpayer, multiplied by the proportion of honest taxpayers in the population (hence, the probability of being matched with a honest taxpayer), $\boldsymbol{q_H}\left[\boldsymbol{\pi_A}\left(\boldsymbol{s_A^{ni}}, \boldsymbol{s_T^h}\right)\right]$, plus the payoff related to the case where an inspecting agency is matched with a cheating taxpayer, multiplied by the proportion of cheating taxpayers within the subpopulation, $\boldsymbol{q_C}\left[\boldsymbol{\pi_A}\left(\boldsymbol{s_A^{ni}}, \boldsymbol{s_T^c}\right)\right]$.

In eq. (10), $f_T(e^b, \boldsymbol{A})$ is the expected payoff for the taxpayer associated with each state "*b*=honest" or "*b*=cheating", where *k*=A, T. This expected payoff is obtained by taking payoffs associated with a matching of a *b*-type individual in the population of the taxpayers, and multiplying them by the proportion of inspecting or non inspecting tax agencies, respectively. Equation (10.1) above is therefore obtained by multiplying the payoffs associated to honest taxpayers, under the circumstance of meeting an inspecting or non inspecting tax agency, denoted by $\pi_T\left(s_A^i, s_T^h\right)$ and $\pi_A\left(s_A^{ni}, s_T^h\right)$ respectively, by the proportion of inspecting and non inspecting fiscal agencies, p_I and p_{NI}. The same happens for (10.2), which is obtained as the payoffs of cheating taxpayers, associated to the situation of meeting an inspecting or non inspecting tax agency, denoted by $\pi_T\left(s_A^i, s_T^c\right)$ and $\pi_A\left(s_A^{ni}, s_T^c\right)$ respectively, multiplied by the proportion of inspecting and non inspecting fiscal agencies, p_I and p_{NI}.

Each individual's fitness function depends on the distribution of characters within the other subpopulation, which is not known to subjects. The distribution of characters among subpopulations is thus a subjective probability assigned by each individual to the event of being paired with a specific character within subpopulations. In other words, each taxpayer assigns a probability to the event "matching with an inspecting agency" and "matching with a non-inspecting agency", and each fiscal agency assigns a probability to the circumstance of matching with a honest or a cheating taxpayer. At each time *t*, a member of taxpayers' subpopulation calculates his/her expected payoff by assuming a certain distribution of characters within the subpopulation of fiscal agencies. The same, in turn, occurs for tax agencies. The latter, at each *t*, calculate their expected payoff taking the distribution of the honest and cheating subjects as a given. At each time *t*, interactions take place between each member of the subpopulation of taxpayers with a member of the fiscal agencies population. At time *t+1,* a memory is formed on the distribution of the characters with respect to the other subpopulation (both collective, at a subpopulation level, and individual), on the basis of overall interactions that took place at time *t*. In other words, at each time, a taxpayer is paired with a fiscal agency. The following four matches are possible: a honest taxpayer meets an inspecting agency, a honest taxpayer meets a non-inspecting agency, a cheating taxpayer meets an inspecting agency, a cheating taxpayer meets an inspecting agency. In all cases displayed, each taxpayer or tax agency may decide to change its strategy in the subsequent period, as a result of the payoff resulting from the previous time *t* interaction, following the so-called "*adaptive dynamics*" (Hofbauer, 2003). The *adaptive dynamics* is a key concept for evolutionary game theory: individuals' choice between characters is not the result of a maximization process, but, by contrast, it follows a learning process on how the characters are distributed within the population. The learning process considers the distribution of characters within the subpopulation, as all individuals choose their strategy according to a mechanism allowing the prevailing strategy to be the one associated with the imitation of the individual with the highest payoff (Schlag, 1998). Thus, interactions at each time *t* lead to a re-shape in the distribution of characters, as individuals adjust their subjective probabilities on both the proportion of cheating and honest individuals, and on inspecting and non inspecting agencies in (9) and (10). Therefore, taxpayers' and tax agencies' expected payoffs (fitness functions) change over time as a result of the adjustment in the probabilities of finding characters within each subpopulation. Cheating taxpayers, once inspected at time *t*, might choose not to be cheating at time *t+1*. By contrast, honest taxpayers may choose to become cheating as a result of an encounter with a non-inspecting fiscal agency. At the end of the game, all these small changes produced at each time *t* – when interactions occur between members of the two subpopulations – reshape the subpopulations' distributions.

As an example, consider (10), which displays taxpayers' expected payoff. At the beginning of the game, each taxpayer calculates his/her expected payoff on the basis of his/her subjective probabilities on p_I and p_{NI}.

After each interaction at time t, the information set on which the taxpayer builds subjective probabilities on the distribution of characters is enriched by experience, and, in turn, expected payoffs as p_I and p_{NI} change, entering also into fitness functions (10.1) and (10.2). These changes in expected payoffs lead to the reshape of the proportion of honest and cheating taxpayers, since q_H and q_C dynamic evolution depends on taxpayers' expected payoff, as their expected payoffs change according to the enriched information set gained with experience, and the same occurs for fiscal agencies.

Over time, the distribution of characters within the taxpayers' and tax agencies' subpopulations is adjusted on the basis of experience. At a population level, after all interactions at each time t, the distribution of characters changes. The spread of a character depends therefore on how the subpopulation as a whole reacts to the matching occurring between each pair of individuals pertaining to the two subpopulations, in terms of speed of reproduction.

We shall refer to Taylor and Yonker's dynamic replicator law[27] (Sandholm, 2010; Taylor and Yonker, 1978), already used in economic applications, for example in studying the labour market dynamics within the shadow economy (Araujo and Almeida de Souza, 2010), and behaviour towards innovation (Mahmoudi and Rasti-Barzoki, 2018), or shadow economy and tax evasion (Bloomquist et al., 2016)[28].

As previously explained, we assume that individuals share the same behaviour, although they show heterogeneous intrinsic characteristics. Thus, as all individuals behave the same way, and their adaptive

[27] The replicator equation, defined for the first time in literature by Schuster and Sigmund (1983), is a mathematical description of the evolutionary game dynamics. It explains the redistribution of characters within a population over time as a results of a mechanism of learning from the payoffs obtained. As well described by Cressmann and Tao (2014) "*With payoff translated as fitness (i.e., repro- ductive success), the frequency of a strategy in a large, well-mixed single species changes under the (continuous-time) replicator equation at a per capita rate equal to the difference between its expected payoff and the average payoff of the population*".

[28] Particularly, within an evolutionary model with a population of business owners characterized by four different characters describing their "*tax morale and compliance propensity*", strategic, defiant, and random, results suggest that - over time - honest individuals within the population are replaced by defiant or strategic individuals, which appears to be in deep contrast with the findings of the model presented in this pages. Nonetheless, it should be considered that in the model presented by Bloomquist (2011), no evidence of tax authorities is modelled, as it is taken as given, whereas in the model presented here the interaction between taxpayers and fiscal agencies plays a key role in explaining taxpayers' subpopulation dynamics over time.

behaviour can be described as a comparison between the payoff associated with a strategy and the overall payoff of the subpopulation, the law of motion of each character well approximates individual's behaviour. The law of motion displays therefore both the individual perspective and collective perspective, although the relevant parameters differ from one individual to another within subpopulations.

The evolutionary approach applied to economics explains the change in the distribution of characters within a population as the collective reaction to a specific input - as it happens in biological sciences - treating behavioural features as any other biological feature, shaped by adaptive mechanisms. In this study, a key role in taxpayers' subpopulation dynamics is played by interactions with fiscal authorities. Particularly, taxpayers' changes in characters are due to experience, which is given by either meeting an inspecting, or a non inspecting fiscal agency. By contrast, changes in characters within the subpopulation of fiscal agencies are due to the circumstance of being matched with a honest or a cheating taxpayer.

Tax evasion, and particularly the distribution of cheating individuals within a population, can be explained by modelling several periods interactions between members of the two subpopulations (taxpayers and fiscal agencies) up to a condition where both the distributions of cheating and honest taxpayers, and of inspecting and non inspecting fiscal agencies, stop changing (the rate of change associated with each character goes to zero).

According to the approach by Taylor and Yonker (1978), the dynamics of the game associated with the long run distribution of characters within each subpopulation (which we can identify as long run equilibrium), can be described by the rate of growth of each character pertaining to each subpopulation, up to the point at which the distribution stops changing, and therefore the growth rate of the character goes to zero.

Moving to the description of the dynamics, we let $q_H(t)$, $q_C(t)$, $p_I(t)$ and $p_{NI}(t)$ be the proportion of honest and cheating taxpayers, inspecting and non inspecting fiscal agencies respectively at time *t*.

Assuming continuous time, the growth rate of each variable in a continuous time setting is calculated as the derivative of the logarithm with respect to time, following Taylor and Yonker (1978), Friedman (1991) and Barro and Sala-i-Martin (2004) as, in the baseline case, it is assumed a logarithmic growth of the characters. Therefore, by taking the logarithm of $q_H(t)$, $\ln(q_H)$, and differentiating with respect to time, we get $\frac{dlnq_H(t)}{dt} = \frac{1}{q_H}\frac{dq_H(t)}{dt} = \frac{\dot{q_H}}{q_H}$, where we then denote $\dot{q_H} = \frac{dq_H(t)}{dt}$, where the dot over q_H, q_C, p_I and p_{NI}, indicate their change over time. The same can be derived for all the remaining variables $q_C(t)$, $p_I(t)$ and $p_{NI}(t)$.

Starting from these definitions, we can set up the replicator equations for the game as in Taylor and Yonker (1978) and Sandholm (2010), as follows:

(11) $\frac{\dot{q_H}}{q_H} = f_T(e^1, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\left(f_T(e^2, s)\right)\right]$

(12) $\frac{\dot{q_C}}{q_C} = f_T(e^2, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\left(f_T(e^2, s)\right)\right]$

(13) $\frac{\dot{p_I}}{p_I} = f_A(e^1, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}\left(f_A(e^2, s)\right)\right]$

(14) $\frac{\dot{p_{NI}}}{p_{NI}} = f_A(e^2, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}\left(f_A(e^2, s)\right)\right]$

The dynamic replicator law equations (11)-(14) represent the law of motion[29] of each character within each subpopulation over time. In other words, (11)-(14) represent how characters change over time for each subpopulation, related to taxpayers and fiscal agencies. Particularly, dynamic replicator equations

[29] The law of motion for a character describes the evolution of a character over a continuous time, based on a set of relevant variables, as payoffs associated to each character, and the proportion of each character within the subpopulation. In the context of the replicator equation, the law of motion of each character is given by the difference between the expected payoff associated to a character, and the average expected payoff the subpopulation (Friedman, 1998; Sandholm, 2010). For example, the replicator equation explaining the evolution over time of the honest taxpayers is given by the expected payoff of being a honest taxpayer, less the average expected payoff of the entire subpopulation of taxpayers.

describe the growth rate of each character as given by its excess payoff with respect to the average expected payoff within the population (also defined as average fitness function). The speed at which each character grows depends on the difference between the payoff associated with the specific character, and the average payoff of the subpopulation (Cressman and Tao, 2014). The latter is the average fitness function over all characters within the subpopulation, weighted for the distribution of characters within each subpopulation. In economic terms, in this game, the dynamics of a character does not depend on individuals' payoff maximization, but it depends instead on the difference between the payoff associated to the character itself and the average expected payoff of all characters. If this difference is positive (if the payoff associated to the character is higher than the average expected payoff among all characters), the proportion of individuals choosing the character tends to grow. By contrast, if the difference is negative, the proportion of individuals choosing the character will decrease.

The growth rate of each character is thus given by the difference between the fitness function related to the character - the expected payoffs, as described above - and the average payoff related to the subpopulation (where each expected payoff associated to each character is weighted for the probability of finding such character within the subpopulation). For example, the dynamic replicator equation for the character "honest" in the subpopulation of taxpayers (equation 11) is given by the fitness function associated with the character "honest" (expected payoff associated with the character "honest") less the average expected payoff within the subpopulation of taxpayers, with the probabilities of finding honest and cheating individuals as weighting factors. In economic terms, at the subpopulation level, characters grow according to the difference between the expected payoff associated to that character and the average payoff across all other characters within the subpopulation.

Two levels of analysis are therefore relevant within this model. First, individuals' level is modelled with fitness functions, which in turn depend on subjective probabilities related to the distribution of characters. As described by (9.1), (9.2), (10.1) and (10.2), taxpayers and fiscal agencies calculate their expected payoffs assuming a certain distribution of characters within the subpopulation of fiscal agencies and taxpayers, respectively, adjusting these subjective probabilities in light of experience, after each interaction in each time *t*.

Second, population level is modelled through (11)-(14) (as an approximation of the individual level behaviour, assuming homogeneity among members of a subpopulation). Equations (11)-(14) describe how the subpopulations' dynamics is shaped over time, thus how characters' distributions change over time.

By making equations (11) - (14) explicit, the dynamics related to the distribution of characters for each subpopulation depends also on the distribution of characters within the other subpopulation, which represents strategic interactions among individuals, as explained in the equations (11a) – (14a) below

(11a) $$\frac{\dot{q_H}}{q_H} = \{p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]\} - \{q_H[p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]] + q_C[p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]]\}$$

(12a) $$\frac{\dot{q_C}}{q_C} = \{p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]\} - \{q_H[p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]] + q_C[p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]]\}$$

(13a) $$\frac{\dot{p_I}}{p_I} = \{q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]\} - \left\{p_I\left[q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]\right] + p_{NI}\left[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]\right]\right\}$$

(14a) $$\frac{\dot{p_{NI}}}{p_{NI}} = \{q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]\} - \left\{p_I\left[q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]\right] + p_{NI}\left[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]\right]\right\}$$

Therefore, not only the distribution over time affects the evolution of characters "honest" and "cheating" within the subpopulation of taxpayers, but also distribution of characters "inspecting" and "not inspecting" within the subpopulation of fiscal agencies, as shown by (11a) and (12a).

Conversely, the growth rate of the two characters of the subpopulation of fiscal agencies evolves according to both the distribution of "cheating" and "honest" taxpayers and "inspecting" and "non inspecting" tax agencies, as represented in (13a) and (14a).

Taxpayers, as described by (11) and (12) choose whether to be compliant or cheat by comparing the fitness function of a strategy with the average fitness functions associated to all strategies disposable within the strategy set. The same happens for fiscal agencies, choosing whether to inspect or not by comparing the fitness functions (expected payoffs) associated to these characters with average fitness functions of all characters within their set. At each time t, interactions take place between members of the subpopulation of taxpayers and fiscal agencies. After each interaction between members of the two subpopulations, the distribution given by q_H, q_C, p_I and p_{NI} is fine-tuned in light of population experience[30]. It occurs because, at each instant t, interactions among members of the two subpopulations take place, and a review of the distribution of characters for each subpopulation takes place afterwards, at a subpopulation level. Then, taxpayers and fiscal agencies choose a character for the subsequent matching by comparing the payoff associated to a character (being honest or cheating, being inspecting or non inspecting) with the average payoff of the subpopulation they belong to (the average payoff associated to the subpopulation of taxpayers or fiscal agencies). The decision between a character and another one is thus driven by the comparison between the expected payoff associated to a character, and the average payoff of the overall population (Cressman and Tao, 2014).

[30] In fact, as each individual belonging to the subpopulation of taxpayers or fiscal agencies, after each interaction, revises at time t its strategy as a result of the interaction at the previous time *t-1*, the distribution of characters changes at each time of the game, as a result of the previous time, driven by experience.

Moreover, at each time t, members of the two subpopulations form their subjective probabilities on the distribution of characters within the other subpopulation, as suggested by (9) and (10). The ESS[31] is the equilibrium solution given by the fine-tuning in the distribution of characters happening at the end of each time *t*, after several interactions between the two subpopulation members.

In the ESS equilibrium, the rate of growth of each character is zero, meaning that the distribution of each character stops varying over time, and thus that individuals stop changing their strategy (Friedman, 1991). Therefore, as $\frac{\dot{q_H}}{q_H}, \frac{\dot{q_C}}{q_C}, \frac{\dot{p_I}}{p_I}$ and $\frac{\dot{p_{NI}}}{p_{NI}}$ represent the rates of growth of the proportion of characters within the subpopulations over time, we find the long run solution values of q_H, q_C, $p_I$and p_{NI} by solving the system (11)- (14) above for $\frac{\dot{q_H}}{q_H} = 0, \frac{\dot{q_C}}{q_C} = 0, \frac{\dot{p_I}}{p_I} = 0$ and $\frac{\dot{p_{NI}}}{p_{NI}} = 0$. We will define the equilibrium solutions as $\overline{q_H}$, $\overline{q_C}$, $\overline{p_I}$and $\overline{p_{NI}}$. The values $\overline{q_H}$, $\overline{q_C}$, $\overline{p_I}$and $\overline{p_{NI}}$ represent the distribution of characters within each subpopulation after all interactions, at the ESS. The equilibrium related to the solution of the system of differential equations presented above may approximate a concept of equilibrium defined as evolutionary stable strategy (ESS), characterized by the fact that any "perturbation" over time (in other words, any exogenous change) does not change the distribution of characters over the population. The ESS has proven to be both locally and asymptotically stable under certain conditions (Hines, 1980)[32], as the Nash Equilibrium. The ESS is thus reached either from a starting point within its neighbourhood, or starting from a state farther from the neighbourhood of the equilibrium. Nonetheless, one of the conditions for attaining the ESS equilibrium is the fact that the game is played within an infinite population, which *per se* does not depict a real world situation (Fogel et al. 1997). We then assume for our game a very large

[31] According to Maynard Smith (1982), an evolutionary stable strategy is a "*strategy such that, if all members of a population adopt it, then no mutant strategy could invade the population under the influence of natural selection.*". In economic terms, a strategy is evolutionary stable if there is no other strategy associated with a higher or equal expected payoff. This latter concept associates the ESS equilibrium to the Nash Equilibrium, as already explained in the introductory Chapter 1.

[32] Particularly, according to Takada and Kigami (1990), the stable solution coinciding with the Nash solution is always attainable when interactions among members of the populations are purely competitive and in case of a population formed by a large number of individuals.

population formed by taxpayers and fiscal agencies, where characters evolve according to the laws of motion described above.

2. Results

The solution of the game at the equilibrium, found by solving the system (11)-(14), is given by the following values $\overline{q_H}$, $\overline{q_C}$, $\overline{p_I}$ and $\overline{p_{NI}}$ in (17)-(20):

(17) $\overline{q_H} = 1 - \frac{\tau C}{\bar{\alpha}(\tau+\varphi)-\bar{\beta}(\tau+\bar{\alpha}\varphi)}$

(18) $\overline{q_C} = \frac{\tau C}{\bar{\alpha}(\tau+\varphi)-\bar{\beta}(\tau+\bar{\alpha}\varphi)}$

(19) $\overline{p_I} = \frac{\tau(1-\bar{\beta})}{\varphi(1-\bar{\alpha}-\bar{\beta}+\bar{\alpha}\bar{\beta})+\tau(1-\bar{\alpha}-\bar{\beta})+\bar{\alpha}}$

(20) $\overline{p_{NI}} = \frac{\varphi(1-\bar{\alpha}-\bar{\beta})+\bar{\alpha}(1-\tau+\bar{\beta}\varphi)}{\varphi(1-\bar{\alpha}-\bar{\beta}+\bar{\alpha}\bar{\beta})+\tau(1-\bar{\alpha}-\bar{\beta})+\bar{\alpha}}$

where $\bar{\beta} \in [0,1]$ is the *ex-ante* average proportion of income declared by individuals pertaining to the subpopulation of taxpayers, unknown to fiscal authorities, and $\bar{\alpha}$ is the average effort within the subpopulation of fiscal agencies[33].

The equilibrium proportion of honest, cheating, inspecting and non inspecting individuals is studied considering individuals' propensity to declare their income to fiscal authorities, as well as fiscal agencies' effort with respect to inspections. As shown in (17)-(20), the level of income is not relevant for the equilibrium of the game, whereas tax rate and penalty rate are. In future developments of this model,

[33] For the calculations and methodology for the solution of the model, see Annex I.

income will be treated as an endogenous variable, to study the effect of different levels of income on the proportion of honest and cheating taxpayers after interactions with fiscal authorities.

The first thing to be noted is that, as shown in (17) and (18) - representing the distribution of characters within the subpopulation of taxpayers in equilibrium - $\overline{q_H}$ and $\overline{q_C}$ depend on $\varphi, \tau, C\ \bar{\alpha}$ and $\bar{\beta}$. Particularly, as shown in (18), the proportion of cheating taxpayers is given by the ratio between τC and $\bar{\alpha}(\tau + \varphi) - \bar{\beta}(\tau + \bar{\alpha}\varphi)$. The term $\bar{\alpha}(\tau + \varphi) - \bar{\beta}(\tau + \bar{\alpha}\varphi)$ is given by fiscal agencies effort in inspection activities, multiplied by the amount of taxes and penalty recovered from inspections, less the average compliance level of taxpayers multiplied by tax rate plus the penalty applied once discovered cheating.

As the distribution of characters adjusted at a population level (the fine-tuning based on experience happening for the overall population) is an approximation of individual behaviour, which is characterized by homogeneity in behaviour but intrinsic heterogeneity (β varies among members), the equilibrium at the population level, $\overline{q_H}$ and $\overline{q_C}$, depends on $\bar{\beta}$, the latter being the arithmetic average proportion of income declared within the population. In the analysis of the results presented in the following pages, honest individuals are the ones who declare their entire income, and characterized by $\beta = 1$, whereas cheating taxpayers declare less than their actual income, $0 \leq \beta < 1$, individuals showing $\beta = 0$ being the "ghosts". Additionally, the average $\bar{\beta}$ should be distinguished by the proportion of honest and cheating individuals, as $\overline{q_H}$ and $\overline{q_C}$ represent all individuals having $\beta = 1$ and $0 \leq \beta < 1$, respectively.

Before studying the effects of tax and penalty rate on the proportion of honest and cheating taxpayers, we should specify the role of $\bar{\alpha}$ (average effort of fiscal agencies in inspecting activities) and $\bar{\beta}$ (average population preferences - or attitudes - with respect to tax compliance) in the solution of the model. The parameter β (individual preference towards evasion or compliance) is taken as given, and it is different for each individual. Also, neither tax agencies nor taxpayers do actually know *ex ante* the exact β of each member of the subpopulation of taxpayers (taxpayers are aware only of their own level of β). Fiscal agencies' choices about audits are taken assuming that β is a constant out of control, or alternatively explained, guessing the value of β. Therefore, in fiscal agencies' perspective, $\bar{\beta}$ is the average guess made by fiscal authorities on the subpopulation of taxpayers. The same reasoning applies to α and $\overline{\alpha}$.

Further specifying the role of $\bar{\beta}$ as distinct from β, the latter is taken as given, being different for each individual.

Finally, equilibrium values of $\overline{q_H}$, $\overline{q_C}$, $\overline{p_I}$ and $\overline{p_{NI}}$ are strictly interrelated, as they represent the distribution of characters, reached by the matching occurred between tax agencies and taxpayers. This happens because the equilibrium is the result of the reactions to the interactions between the two subpopulations, represented by the encounters between members of the other subpopulation. Thus, the equilibrium distribution of characters within the subpopulation of taxpayers strictly depends on the distribution of characters within the subpopulation of fiscal agencies, and vice-versa.

We then summarize in the following table I the direction of the effect of *ceteris paribus* changes of parameters on the equilibrium results. In Annex I, we shall report the exercises of comparative statics carried out at equilibrium values.

Table I: Direction of the effects on the equilibrium values of the ceteris paribus change of the relevant parameters

	Parameters affecting the equilibrium results and direction of the effects on the equilibrium values of the ceteris paribus change of the relevant parameter				
Equilibrium values	$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\boldsymbol{C}$	$\boldsymbol{\bar{\alpha}}$	$\boldsymbol{\bar{\beta}}$
$\overline{\boldsymbol{q_H}}$	>0	<0	<0	>0	<0
$\overline{\boldsymbol{q_C}}$	<0	>0	>0	<0	>0
$\overline{\boldsymbol{p_I}}$	<0	>0	-	<0	<0
$\overline{\boldsymbol{p_{NI}}}$	>0	<0	-	>0	>0

When $0 < \bar{\beta} < 1$ and $0 < \bar{\alpha} < 1$, we see then from **(17)** above that equilibrium results for the proportion of honest and cheating taxpayers depend on $1 - \frac{\tau C}{\bar{\alpha}(\tau+\varphi)-\bar{\beta}(\tau+\bar{\alpha}\varphi)} \geq 0$, therefore on

$\frac{\tau C}{\bar{\alpha}(\tau+\varphi)-\bar{\beta}(\tau+\bar{\alpha}\varphi)} \leq 1$ (21).

Looking at (21), we shall distinguish between two cases, *i)* $\frac{\tau C}{\bar{\alpha}(\tau+\varphi)-\bar{\beta}(\tau+\bar{\alpha}\varphi)} < 1$ in which case $\overline{q_H} > 0$ and $\overline{q_C} > 0$; and *ii)* $\frac{\tau C}{\bar{\alpha}(\tau+\varphi)-\bar{\beta}(\tau+\bar{\alpha}\varphi)} = 1$, in which case $\overline{q_H} = 0$ and $\overline{q_C} = 1$.

In the first case *i)*, the long run equilibrium value for "honest" $\overline{q_H}$ is strictly positive: honest individuals can be found as long as $\tau C < \bar{\alpha}(\tau+\varphi) - \bar{\beta}(\tau+\bar{\alpha}\varphi)$, thus as long as the cost of control remains strictly less than the amount of fine and taxes recovered from inspections, $\bar{\alpha}(\tau+\varphi)$, less $\bar{\beta}(\tau+\bar{\alpha}\varphi)$, which represents the amount of taxes and fines actually recovered (considering the income actually declared $\bar{\beta}$, which is less than 1).

Honest taxpayers can be found as long as the cost of control remains strictly less than the net amount of fine and taxes recovered from inspections. By contrast, if $\tau C > \bar{\alpha}(\tau+\varphi)Y - \bar{\beta}(\tau+\bar{\alpha}\varphi)$ no honest individual can be found within the population, as inspection activities become unprofitable to fiscal agencies.

By writing (21) with respect to $\bar{\alpha}$, we find another remarkable result in terms of policy, expressed in (22.B) below:

$$\bar{\alpha} > \frac{\tau C + \bar{\beta}\tau}{[\tau + \varphi(1-\bar{\beta})]} \qquad (22.B)$$

for which in order to find a positive proportion of honest individuals in equilibrium, the level of effort must be perceived to be higher than the ratio between the costs of inspections plus the amount of taxes already paid by the taxpayers, $\tau C + \bar{\beta}\tau$, and the amount recovered from inspections $[\tau + \varphi(1-\bar{\beta})]$.

The second case *ii)* of $\frac{\tau C}{\bar{\alpha}(\tau+\varphi)-\bar{\beta}(\tau+\bar{\alpha}\varphi)} = 1$ determines $\overline{q_H} = 0$ and and $\overline{q_C}$ =1, i.e. all taxpayers are cheating in equilibrium. This requires $\tau CY = \bar{\alpha}(\tau + \varphi)Y - \bar{\beta}(\tau + \bar{\alpha}\varphi)Y$, or $\bar{\alpha} = \frac{\tau C + \bar{\beta}\tau}{[\tau+\varphi(1-\bar{\beta})]}$ by re-arranging the terms, which means that, when the cost of controlling exactly equals the net amount of fine and taxes recovered from inspections, or the perceived effectiveness of control is equal to the ratio between the costs of inspections plus the amount of taxes paid over and the amount recovered from inspections $[\tau + \varphi(1 - \bar{\beta})]$, it is worthy for taxpayers to be cheating (and thus the probability of cheating among the population increases).

As taxpayers perceive that costs of inspections remain lower than the amount of taxes and penalties paid by the non-compliant taxpayer, the proportion of honest individuals is always strictly positive in equilibrium. The latter depends on the costs of control, tax and penalty rates, and on the average preferences towards compliance/evasion within the subpopulation of taxpayers.

Taxpayers' perception of $\overline{\alpha}$ and of the cost of inspection C determines their compliance level, with higher levels of fiscal agencies' efficiency in inspections, or lower costs of inspections, being associated with a higher level of compliance.

Therefore, taxpayers' behaviour is primarily affected by the perception they have on fiscal agencies' strategy. They adjust their behaviour according to their perception about fiscal agencies' inspecting activities, only in case the costs of inspections are fully covered by the amount recovered, regardless of whether tax agencies' actually behave in this way or not.

Within National tax systems, inspections and enforcement are pursued either when $\tau CY \leq \left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha$ (total cost of control is lower than, or equal to the recovered tax revenue, plus the revenue coming from penalty), or when $\tau CY > \left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha$, thus when the total cost of control is higher than the recovered tax revenue, plus the revenue for penalty. Enforcement activities are therefore carried out either when they are economically convenient (hence, when $\tau CY \leq$

$\left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha)$, or when they are not economically convenient for fiscal authorities ($\tau CY > \left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha)$, as enforcement is mandatory. Nonetheless, from the solutions of the model, we see that honest individuals can be found in equilibrium only in case $\tau CY \leq \left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha$. The exclusion of the case $\tau CY > \bar{\alpha}(\varphi + \tau)Y$ can be reconciled with the fact that fiscal agencies – by strategically organizing their inspecting activities - might only be oriented to the recovery of the amount due by cheating taxpayers. This behaviour, although not ethically correct, might occur especially in case of very limited resources allocated to auditing activities. In fact, we see that solutions for the proportion of honest and cheating individuals make sense under the condition $\tau CY \leq \bar{\alpha}(\tau + \varphi)Y - \bar{\beta}(\tau + \bar{\alpha}\varphi)Y$, which does not contemplate the situation where the amount of fines and taxes recovered from inspections does not cover the costs of inspections.

Moreover, looking at whether $\overline{q_h} \gtreqless \overline{q_C}$ depending on the parameters, $\overline{q_h} > \overline{q_C}$ when $\tau < \frac{\bar{\alpha}\varphi(1-\bar{\beta})}{2C-(\bar{\alpha}-\bar{\beta})}$ and $\varphi > \frac{\tau(2C-\bar{\alpha}+\bar{\beta})}{\bar{\alpha}(1-\bar{\beta})}$. For example, considering a penalty rate equal to 0.6, a level of effort equal to 0.5, $\bar{\beta} = 0.6$, and $C = 0.4$, tax rate should be equal to 0.26 in order to have $\overline{q_h} > \overline{q_C}$. Please refer to the simulations presented in the following pages and in Annex I for further numerical examples.

Looking at (19) and (20), $\overline{p_I} \gtreqless \overline{p_{NI}}$, depending on whether $\tau(1-\bar{\beta}) \gtreqless \varphi(1-\bar{\alpha}-\bar{\beta}) + \bar{\alpha}(1-\tau+\bar{\beta}\varphi)$. As we can see from simulations in Annex I, for low values of τ and φ, $\overline{p_I} < \overline{p_{NI}}$ for both low and high values of $\bar{\alpha}$.

As for the role of $\bar{\alpha}$, when $\bar{\alpha} \to 1$, and $0 < \bar{\beta} < 1$, (19) and (20) become $\overline{p_I} = \frac{\tau(1-\bar{\beta})}{1-\bar{\beta}\tau}$ and $\overline{p_{NI}} = \frac{1-\tau}{1-\bar{\beta}\tau}$, which implies that $\overline{p_I} \gtreqless \overline{p_{NI}}$ depending on whether $\tau \gtreqless \frac{1}{2-\bar{\beta}}$. In this case, (17) and (18) become $\overline{q_H} = 1 - \frac{\tau C}{(\varphi+\tau)(1-\bar{\beta})}$ and $\overline{q_C} = \frac{\tau C}{(\varphi+\tau)(1-\bar{\beta})}$, and as $\overline{q_C}$ is always positive (unless either tax rate, or cost of control, or both are equal to zero), the proportion of honest individuals $\overline{q_H}$ will always be less than 1.

Moreover, $\overline{q_H} > \overline{q_C}$ as long as $\tau < \frac{\varphi(1-\bar{\beta})}{2C-(1-\bar{\beta})}$. Therefore, in case *ex ante* $0 < \bar{\beta} < 1$ and the level of effort $\bar{\alpha} \to 1$, both $\overline{p_I}$ and $\overline{p_{NI}}$ are positive, with $\overline{p_I} > \overline{p_{NI}}$ depending on whether $\tau > \frac{1}{2-\bar{\beta}}$, and also both $\overline{q_H}$ and $\overline{q_C}$ positive, with $\overline{q_H} > \overline{q_C}$ as long as $\tau < \frac{\varphi(1-\bar{\beta})}{2C-(1-\bar{\beta})}$.

In (17)-(20), when costs are lower than benefits related to tax inspections, the perception of higher effectiveness in fiscal authorities' activities, and of higher effort in conducting inspections, leads to an increase in the proportion of compliant taxpayers, with a positive effect on individuals' tax compliance decision.

By examining the derivatives of (17)-(18) with respect to tax and penalty rates, to check the effects of parameters on the proportion of honest and cheating individuals *ex post*, and *ceteris paribus*, we see that, for higher levels of tax rates, the proportion of honest individuals within the subpopulation of taxpayers decreases, whereas the proportion of cheating increases. By taking derivatives of (19) and (20) with respect to tax rate, moreover, we can see that higher tax rates are associated with a higher proportion of inspecting, and a lower proportion of non inspecting fiscal agencies.

A higher level of penalty rate, instead, increases the proportion of honest taxpayers, lowering the level of cheating ones, *ceteris paribus*. This latter result seems to be aligned with Allingham and Sandmo's (1972), as their results – within a different theoretical setting – suggest a positive effect of penalty rate on individuals' compliance decision (which is represented within their model by the amount of endowment declared). From the derivatives of (19) and (20), we can see that for higher penalty rates, the proportion of non-inspecting fiscal agencies increases, whereas the proportion of inspecting decreases (for the calculation of derivatives, please refer to Annex I).

Thus, higher levels of tax rates induce a decrease in the proportion of honest taxpayers, and an increase in the proportion of cheating ones, *ceteris paribus* (individuals tend to declare less, as tax rate increases, and thus the proportion of taxpayers declaring their entire income decreases). As higher tax rates are associated with a lower level of taxpayers' compliance, they also induce an increase in the proportion of inspecting fiscal agencies, as fiscal authorities perceive that taxpayers will tend to declare less, and thus their inspections are more likely to find non-compliant individuals.

By contrast, higher penalty rates induce an increase in the proportion of honest taxpayers, decreasing the proportion of cheating ones (individuals tend to evade less, as penalties increase, and thus, the proportion of taxpayers declaring their entire income increases). The proportion of inspecting fiscal agencies tends to decrease, as fiscal authorities perceive taxpayers will tend to evade less, and their inspections will be less likely to find non-compliant individuals.

Looking at the role played by the guessed average proportion of income declared, $\bar{\beta}$, as fiscal agencies perceive an increase in the average proportion of income declared within the subpopulation of taxpayers, they are induced to decrease their inspecting activities, as shown in table I. With respect to taxpayers, table I shows that a higher $\bar{\beta}$ is associated with a higher proportion of cheating and a lower proportion of honest, as $\overline{q_C}$ represents individuals having $0 \leq \beta < 1$.

Unexpectedly, thus, the proportion of fully honest taxpayers is reduced when the guess about the average level of compliance, $\bar{\beta}$, increases. This means that, on one side, the number of cheating taxpayers increases as fully honest taxpayers switch their character, and become cheating. The change from honest to cheating is due to the fact that the increase of $\bar{\beta}$ also reduces the probability of control, by negatively affecting $\overline{p_I}$, and this, in turn, reduces the amount of fully honest taxpayers, although cheating taxpayers declare a higher level of income, on the other side. In case of an increase of $\bar{\beta}$, thus, fiscal agencies are induced to reduce their inspecting activities, and the proportion of cheating increases whereas the proportion of fully honest individuals (showing $\beta = 1$) declines. However, although the proportion of cheating taxpayers increases, the latter tend to declare more of their income, and thus to evade less. The subpopulation of taxpayers, in this case, is therefore characterized by a higher number of cheating individuals, who nonetheless declare on average a higher proportion of their income.

As we can see in table I, also the level of effort in inspections increases the level of compliance, as taxpayers perceive fiscal agencies' higher effort in inspections. By contrast, costs of inspection lead to an increase in the proportion of cheating taxpayers, as the latter perceive fiscal authorities would tend to inspect less when costs related to inspections increase.

The proportion of inspecting agencies is decreasing in the level of $\overline{\alpha}$, since higher effort would require a lower proportion of inspecting fiscal agencies.

Therefore, results suggest that fiscal authorities can induce taxpayers towards compliance both with modifications in fiscal parameters such as tax and penalty rate, as well as through enhancing individuals' perception of effort put in enforcement, which, in turn, leads to the need of a lower proportion of inspecting agencies.

2.1 Equilibrium results for $\overline{\beta} = 0$ and $\overline{\beta} = 1$

We shall comment the other results referring to table II below that summarizes the game equilibria for different levels of $\overline{\alpha}$ and $\overline{\beta}$. The case $\overline{\alpha} = 0$ is not relevant, as it corresponds to the case when all fiscal agencies do not inspect.

Table II: equilibrium results of the model, for different combinations of parameters

	$\bar{\beta} = 0$	$\bar{\beta} = 1$
$0 < \bar{\alpha} < 1$	**(A)** $(17.a)\quad \overline{q_H} = 1 - \frac{\tau C}{\bar{\alpha}(\varphi + \tau)}$ $(18.a)\quad \overline{q_C} = \frac{\tau C}{\bar{\alpha}(\varphi + \tau)}$ $(19.a)\quad \overline{p_I} = \frac{\tau}{(\varphi + \tau)(1 - \bar{\alpha}) + \bar{\alpha}}$ $(20.a)\quad \overline{p_{NI}} = \frac{\varphi(1 - \bar{\alpha}) + \bar{\alpha}(1 - \tau)}{(\varphi + \tau)(1 - \bar{\alpha}) + \bar{\alpha}}$	**(C)** (17.b) $\overline{q_H} = 1 - \frac{\tau C}{\bar{\alpha}(\tau+\varphi)-(\tau+\bar{\alpha}\varphi)}$ (18.b) $\overline{q_C} = \frac{\tau C}{\bar{\alpha}(\tau+\varphi)-(\tau+\bar{\alpha}\varphi)}$ (19.b) $\overline{p_I} = 0$ (20.b) $\overline{p_{NI}} = 1$
$\bar{\alpha} \rightarrow 1$	**(D)** (17.d) $\overline{q_H} = 1 - \frac{\tau C}{(\varphi+\tau)}$ (18.d) $\overline{q_C} = \frac{\tau C}{(\varphi+\tau)}$ (19.d) $\overline{p_I} = \tau$ (20.d) $\overline{p_{NI}} = (1 - \tau)$	**(F)** (17.c) $\overline{q_H} = 1$ (18.c) $\overline{q_C} = 0$ (19.c) $\overline{p_I} = 0$ (20.c) $\overline{p_{NI}} = 1$

When $\bar{\beta} = 1$ all individuals within the subpopulation of taxpayers are honest (they declare the entire amount of their income). This case will be studied in the following pages for the situation where $0 < \bar{\alpha} < 1$. The case where the level of effort tends to 1 and $\bar{\beta} = 1$ (thus, $\bar{\beta} = 1$ joint with $\bar{\alpha} \rightarrow 1$) corresponds to the solution where $\overline{q_H} = +\infty$, $\overline{q_C} = -\infty$, $\overline{p_I} = 0$ and $\overline{p_{NI}} = 1$. As $\overline{q_H}$ and $\overline{q_C}$ are outside of the space of the game, they converge to 1 and 0, respectively, and they are characterized by instability (they are non-stable solutions). **Case F)** in table II above is therefore the corner solution, where in the presence of perceived fully effective control and, on average, fully honest taxpayers, the proportion of honest/cheating is at its maximum/minimum, and again no inspections are needed.

When $\overline{\beta} = 1$, thus when all individuals within the subpopulation of taxpayers are honest (they declare the entire amount of their income), **the equilibrium C** ($\overline{\beta} = 1$ joint with $0 < \bar{\alpha} < 1$) is quite interesting. It is a case of fully compliant taxpayers joint with not fully effective tax agencies. Depending on the level of $\bar{\alpha}$ (i.e. the perceived effectiveness of tax control) and in spite of $\overline{\beta} = 1$, the proportion of cheating can be positive $\overline{q_C} > 0$, whereas the proportion of inspecting (non inspecting) tax agencies tends to 0 (tends to 1) being affected by the initial $\overline{\beta} = 1$, i.e., all individuals declare their entire income and pay the amount of taxes due.

When $\overline{\beta} = 0$, all individuals within the subpopulation of taxpayers are ghosts (they evade the entire amount of their income). In **case A**, when $\bar{\beta} = 0$ and $0 < \bar{\alpha} < 1$, the equilibrium values for taxpayers, $\overline{q_C}$ and $\overline{q_H}$ depend on $1 - \frac{\tau C}{\bar{\alpha}(\varphi+\tau)} \geq 0$ starting from (17.a), therefore on

(21.A) $\frac{\tau C}{\bar{\alpha}(\varphi+\tau)} \leq 1,$

which shows that, when taxpayers are perceived as fully cheating, the level of $\bar{\alpha}$ determines whether the proportion of honest taxpayers in the population is a positive quantity. Also in this case, we shall distinguish between $\frac{\tau C}{\bar{\alpha}(\varphi+\tau)} < 1$ and $\frac{\tau C}{\bar{\alpha}(\varphi+\tau)} = 1$. The first case, $\frac{\tau C}{\bar{\alpha}(\varphi+\tau)} < 1$, qualifies the long run equilibrium value for "honest" $\overline{q_H}$ as stricly positive. This requires $\tau C < \bar{\alpha}(\varphi + \tau)$, and it means that honest individuals can be found in the population when $\overline{\beta} = 0$, as long as the cost of control remains strictly less than the amount of tax and fines recovered (at a rate $\bar{\alpha}$). In this case, the condition for a positive proportion of honest taxpayers requires $\bar{\alpha} > \frac{\tau C}{(\varphi+\tau)}$, i.e. the average perceived level of fiscal authorities' effort in inspections should be higher than the ratio between the costs of control and the amount recovered from inspections.

The second case mentioned, $\frac{\tau C}{\bar{\alpha}(\varphi+\tau)} = 1$, qualifyes $\overline{q_H} = 0$, $\overline{q_C} = 1$, where it is worthy for taxpayers to be cheating (and the probability of honest among the population tends to zero). This requires $\tau C = \bar{\alpha}(\varphi + \tau)$ or $\bar{\alpha} = \frac{\tau C}{(\varphi+\tau)}$, thus that the effort characterizing inspections is exactly equal to the ratio between costs and amounts recovered from controls.

This suggests that if taxpayers perceive the level of effort in inspections as equal to the ratio between costs and revenues of inspections, honest individuals disappear, and only cheating taxpayers are found within the subpopulation.

As for tax agencies, we can see that $\overline{p_I} > 0$ as long as $\tau > 0$ (tax rate is higher than zero), as the denominator is always positive, for each value of $0 < \bar{\alpha} < 1$, τ and φ. Moreover, $\overline{p_I} = \overline{p_{NI}}$ when $\tau = \frac{\varphi(1-\bar{\alpha})+\bar{\alpha}}{1+\bar{\alpha}}$, whereas $\overline{p_I} > \overline{p_{NI}}$ as $\tau > \frac{\varphi(1-\bar{\alpha})+\bar{\alpha}}{1+\bar{\alpha}}$, which is verified, assuming $\varphi > \tau$, for $\tau \geq 0.5$, and for $\bar{\alpha} > 0.1$. By contrast, for $\varphi \geq \tau$, $\tau < 0.5$ and $\bar{\alpha} < 0.4$, $\tau < \frac{\varphi(1-\bar{\alpha})+\bar{\alpha}}{1+\bar{\alpha}}$ and $\overline{p_I} < \overline{p_{NI}}$.

According to the results, the equilibrium proportion of honest taxpayers is affected by the quality and quantity of inspections, and particularly, by how much effort fiscal authorities put in inspecting activities. Lower levels of effort in inspections are associated with a lower proportion of honest individuals. Audit activities, therefore, have a positive impact on the proportion of honest individuals only when the latter perceive that benefits of inspections overcome their costs.

Inspecting activities on tax declarations are usually carried out by fiscal authorities regardless of whether the amount recovered from inspections is higher or lower than their costs. Fiscal controls are in fact aimed at both pursuing illegal behaviours and recovering the amount due by cheating taxpayers. In this model, instead, we focus on the case where fiscal agencies are mainly concentrated on the activity of recovering the amount of taxes evaded, also in light cheating taxpayers' education.

Activities carried out by public administrations, including inspections and enforcement pursued by fiscal agencies, affect the proportion of honest and cheating taxpayers if they are economically convenient. This would suggest that inspections shall be strategically organized such that the costs of inspection are fully covered by the amounts recovered from inspections. The circumstance of excluding the case when costs of enforcement overcome its benefits ($\tau CY > \bar{\alpha}(\varphi + \tau)Y$) coincides with the assumption that fiscal agencies are governed by the un-ethical principle of efficiency, binding enforcement to the extent where its benefits are higher than its costs.

For the case presented above, honest individuals can be found within the subpopulation of taxpayers, in case of $\bar{\beta} = 0$, as long as taxpayers perceive that costs of inspections are lower than the amount of taxes and penalties recovered, therefore as long as $\tau C < \bar{\alpha}(\varphi + \tau)$, which happens for $\bar{\alpha} > \frac{\tau C}{(\varphi+\tau)}$, therefore for $\bar{\alpha} > 0.5$, for all values $\varphi \geq \tau$. The proportion of honest, in case of $\bar{\beta} = 0$, will be therefore higher for a higher level of $\bar{\alpha}$, and particularly for $\bar{\alpha}$ above 0.5 (fiscal authorities putting a high level of effort in fiscal inspections), as in this case $\overline{p_I} > \overline{p_{NI}}$, for penalty rates higher than tax rates.

We see that, in addition to the level of tax and penalty rate, an important feature in fiscal authorities' inspecting activities is the level of effort that characterizes inspections. When the level of effort is high (above 0.5), a higher proportion of honest individuals with respect to the cheating ones is reached for both higher and lower levels of tax and penalty rate.

Looking then at the case of a population formed by ghosts, where fiscal authorities put all effort in inspecting activities (*ex ante* $\bar{\beta}$ is equal to 0 and $\bar{\alpha}$ tends to 1), **case (D)** in the table above, $\overline{q_H}$ and $\overline{q_C}$ depend on the cost of control, tax and penalty rate. In this case, $\overline{q_H}$ is positive when $\tau C < \varphi + \tau$. Moreover, in this case $\overline{q_H} > \overline{q_C}$ only when $C < \frac{\varphi+\tau}{2\tau}$ for $\varphi \geq \tau$. As an example, when $\tau = 0.45$ and $\varphi = 0.6$, $\overline{q_H} > \overline{q_C}$ only for values of $C < 0.24$.

The equilibrium proportions of inspecting and non inspecting fiscal agencies, $\overline{p_I}$ and $\overline{p_{NI}}$, depend only on the tax rate, and particularly $\overline{p_I}$ equals the tax rate. Therefore, in case $\bar{\beta}$ tends to 0 and $\bar{\alpha}$ tends to 1 *ex ante*, $\overline{p_I} \gtreqless \overline{p_{NI}}$ depends on the level of tax rate, and thus for higher values of tax rate the $\overline{p_I}$ tends to increase, whereas $\overline{p_{NI}}$ tends to decrease. According to the results presented above, therefore, $\overline{p_I} = \overline{p_{NI}}$ when $\tau = 0.5$. For values of $\tau > 0.5$, $\overline{p_I} > \overline{p_{NI}}$, whereas for values of $\tau < 0.5$, $\overline{p_I} < \overline{p_{NI}}$. In a context where the effort put by fiscal authorities in inspections tends to 1, and the population *ex ante* is formed only by ghost individuals, the proportion of inspecting agencies increases with the potential revenue loss determined by the level of taxation.

2.2 Simulations

By summing up the results of the game in the long run, as long as taxpayers perceive that costs associated with inspecting activities are lower than the amount recovered from inspections, the proportion of honest individuals is higher than zero in equilibrium. Moreover, as auditing costs increase, *ceteris paribus*, the equilibrium proportion of honest taxpayers decreases, whereas the equilibrium proportion of cheating individuals increases.

Higher levels of penalty lead to an increase in the proportion of honest individuals – *ceteris paribus* – and to a decrease in the proportion of cheating, whereas, by contrast, higher levels of tax rate – *ceteris paribus* – induce an increase in the proportion of cheating taxpayers, lowering the proportion of honest ones.

The level of effort characterizing fiscal agencies' inspections affects the equilibrium proportion of honest individuals, as well as costs. As long as individuals' perception about the level of effort put in inspections by fiscal authorities exceeds the ratio between costs and amounts recovered, the equilibrium solution will lead to a positive proportion of honest individuals within the subpopulation of taxpayers, $\overline{q_H} > 0$, which increases as the level of effort perceived increases.

As regards fiscal authorities, higher levels of effort are associated with a lower proportion of inspecting agencies. It happens as both the direct effect and the indirect effect of effort (through taxpayers' perception) lead to the need of a lower proportion of inspecting fiscal agencies (as the perception of effort is *per se* a policy tool to increase the proportion of fully honest taxpayers).

Higher tax rates induce therefore a higher proportion of both cheating taxpayers, and consequently, of inspecting agencies, whereas, on the contrary, higher penalty rates induce – *ceteris paribus* – a higher proportion of fully honest taxpayers, and of non inspecting fiscal agencies. Higher levels of effort in enforcement and audits induce a decrease in the proportion of cheating taxpayers through their perception, which in turn is associated with an indirect effect on the proportion of inspecting fiscal agencies. Higher levels of effort in inspections also induce a decrease in the proportion of inspecting agencies (direct effect on the proportion of inspecting fiscal agencies) due to a better allocation of resources. Effort in inspections and enforcement is therefore crucial, together with tax and penalty rate, to enhance compliance.

In this setting, we have seen that a crucial condition to find honest individuals is that their perception of effort in inspections is such that benefits related to audits overcome their costs. Under this circumstance, in fact, taxpayers perceive that fiscal agencies pursue non-compliant taxpayers by organizing inspections as long as costs are lower than the amount of fines and taxes recovered (less the amount already paid by cheaters).

As a further implication, the condition below is verified when average *ex ante* β is low, and average α is high:

$$\tau C < \bar{\alpha}(\tau + \varphi) - \bar{\beta}(\tau + \bar{\alpha}\varphi)$$

which implies that the right hand side will be higher than its left hand side as $\bar{\alpha}$ grows and as $\bar{\beta}$ declines, and thus for higher values of the average *ex ante* effort and lower values of average *ex ante* income declared.

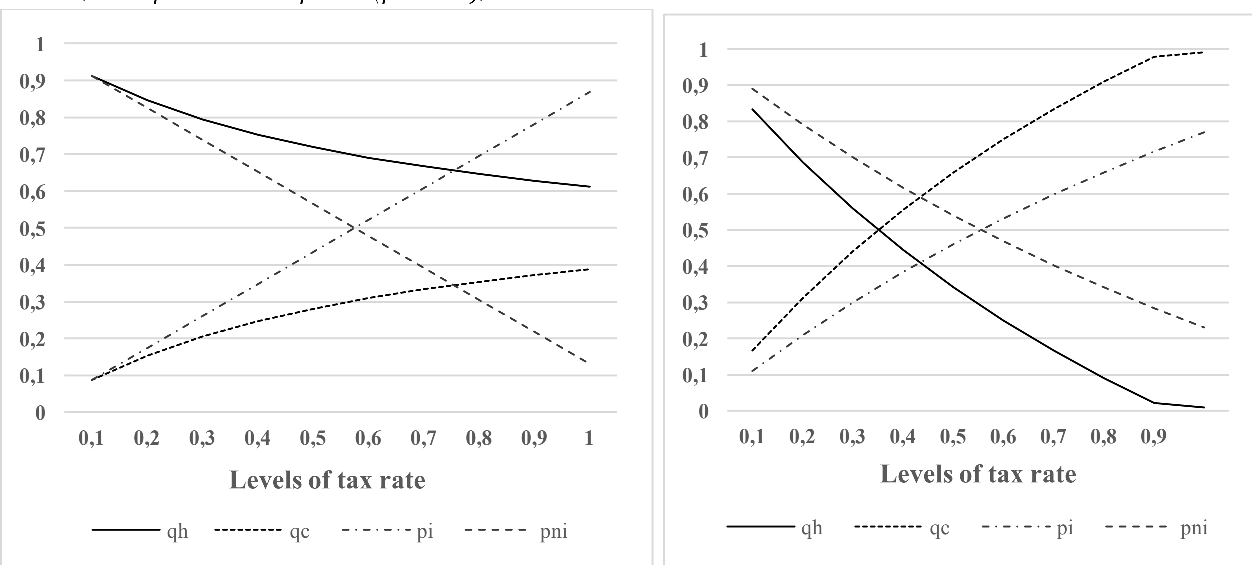
Therefore, fiscal agencies would find honest individuals in the long run when inspections are characterized by higher levels of effort, in case of taxpayers with a low propensity to declare their income, by letting taxpayers perceive the increased level of effort put in auditing activities. In fact, when costs associated with inspecting activities are perceived as equal to the average amount of tax revenues potentially recovered, the long run equilibrium is characterized by only cheating taxpayers. All taxpayers would hide a certain part of their income, some of them can be ghosts, as they perceive audits as a costly activity. As long as the average amount of tax revenues recovered is perceived as higher than costs of inspections, honest taxpayers can be found in the long run, as benefits related to auditing practices are higher than their costs.

Taylor and Jonker (1978), and Hines (1980), proved the local stability and asymptotically stability of the ESS equilibrium with replicator dynamics, under certain conditions, one of the latter being the population being formed by a very large number of individuals. Both solutions presented above are therefore locally stable. The latter circumstance implies that, starting from each point in its neighbourhood, the solution after a high number of interactions always ends at that point. Additionally, the asymptotic stability allows the solutions above to be attractive also from starting points not falling within the neighbourhood of the ESS equilibrium itself.

Recall that, according to its definition in Chapter 1, an evolutionary stable state (ESS) is a set of solutions $x^* \in X$ such that for each set $y \in X$, $(y - x^*)'F(x^*) < 0$. This condition implies that an ESS equilibrium is a Nash equilibrium (Sandholm, 2010).

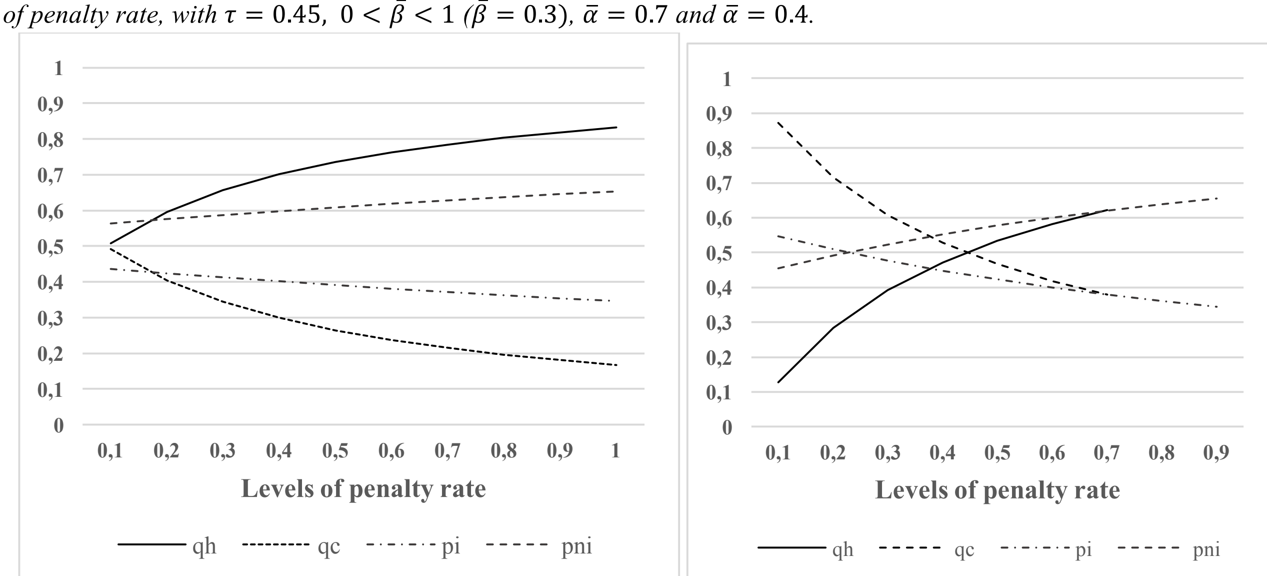
Charts 1.a-1.b and 2.a-2.b below show the different simulations of equilibrium solutions for different levels of tax rate and penalty rate, in case where $0 < \bar{\beta} < 1$. Charts 1.a, 1.b and 2.a and 2.b in Annex I, Section III, represent the simulations for the case $\bar{\beta} = 0$.

Chart 1.a and 1.b: distribution of characters within the subpopulations of taxpayers and fiscal agencies for different levels of tax rate, with $\varphi = 0.5$ $0 < \bar{\beta} < 1$ $(\bar{\beta} = 0.3)$, $\bar{\alpha} = 0.7$ *and* $\bar{\alpha} = 0.4$.



Distribution of characters within the subpopulation of tax agencies and taxpayers, for different levels of tax rate, **with $\bar{\alpha} = 0.7$ (left) and $\bar{\alpha} = 0.4$ (right)**.

Chart 2.a and 2.b: distribution of characters within the subpopulations of taxpayers and fiscal agencies for different levels of penalty rate, with $\tau = 0.45$, $0 < \bar{\beta} < 1$ $(\bar{\beta} = 0.3)$, $\bar{\alpha} = 0.7$ *and* $\bar{\alpha} = 0.4$.



Distribution of characters within the subpopulation of tax agencies and taxpayers, for different levels of penalty rate, **with $\bar{\alpha} = 0.7$ (left) and $\bar{\alpha} = 0.4$ (right).**

As we can see in figures 1 and 2, when $0 < \overline{\beta} < 1$ and specifically $\overline{\beta} = 0.3$, the proportion of honest individuals is higher for higher levels of penalty rate, and lower for higher levels of tax rate. By contrast, the equilibrium proportion of cheating taxpayers increases for higher levels of tax rate, and decreases for higher levels of penalty rate, for both levels of $\overline{\alpha} = 0.7$ and $\overline{\alpha} = 0.4$. Moreover, for high levels of $\overline{\alpha}$, and specifically $\overline{\alpha} = 0.7$, the proportion of honest taxpayers is always higher than the proportion of cheating individuals, whereas when $\overline{\alpha}$ is low, for high levels of tax rate (holding penalty rate constant) and for low levels of penalty rate (holding tax rate constant), the equilibrium proportion of cheating is higher than the equilibrium proportion of honest individuals.

As regards the subpopulation of fiscal agencies, when $0 < \overline{\beta} < 1$ and specifically $\overline{\beta} = 0.3$, figures 1 and 2 show that the equilibrium proportion of inspecting agencies is higher for high levels of tax rate and lower for high levels of penalty rate, for both high and low levels of $\bar{\alpha}$. Moreover, when tax rate is high (holding constant the penalty rate) the proportion of inspecting fiscal agencies is higher than the proportion of non inspecting agencies, whereas for high levels of penalty rate (holding by contrast tax rate constant) the proportion of non-inspecting fiscal agencies exceeds the proportion of inspecting ones.

Table III below therefore summarizes the considerations above about the effect of different levels of tax and penalty rates - in both scenarios considered for $\overline{\alpha}$ - on equilibrium solutions for both taxpayers' and fiscal agencies' subpopulation.

Table III: summary of the results of simulations performed on the model results for $0 < \bar{\beta} < 1$.

	Levels of τ, holding $\varphi = 0.5$ (constant)	Levels of φ, holding $\tau =$ 0.5 (constant)	$\overline{\alpha}$=0.7	$\overline{\alpha}$=0.4
$\overline{q_H}$	Decreasing (for both low and high levels of $\overline{\alpha}$)	Increasing (for both low and high levels of $\overline{\alpha}$)	$\overline{q_H} > \overline{q_C}$	

$\overline{q_C}$	Increasing (for both low and high levels of $\overline{\alpha}$)	Decreasing (for both low and high levels of $\overline{\alpha}$)		$\overline{q_H} < \overline{q_C}$ for high levels of φ and low levels of τ
$\overline{p_I}$	Increasing $\overline{p_I} > \overline{p_{NI}}$ for high levels of τ	Decreasing $\overline{p_I} < \overline{p_{NI}}$ for high levels of φ	–	–
$\overline{p_{NI}}$	Decreasing	Increasing	–	–

Alla in all, simulations show, in line with the *ceteris paribus* analysis, that an increase in tax rate decreases the equilibrium proportion of honest taxpayers and of non inspecting fiscal agencies, increasing the proportion of cheating individuals, and consequently of inspecting fiscal agencies. By contrast, a higher level of penalty rate has a positive effect on honest taxpayers, decreasing the proportion of cheating ones, the latter result being aligned with A-S model's results.

3. Concluding remarks

In this Chapter, we presented an evolutionary game with a population divided into two subpopulations, taxpayers and fiscal agencies, each of them with a different set of strategies.
Individuals choose the strategy offering the highest payoff according to a process of learning from their peers and context, thus in light of the interactions between a member of the subpopulation of taxpayers and a member of the subpopulation of fiscal agencies. The aim of the study was then to identify how the strategy "evade" or "not evade" spread within the population, according to the better performing payoffs over time, to identify the main drivers of income tax evasion and compliance decision among individuals. We have shown a long run equilibrium where the proportion of characters chosen by the members of each subpopulation does not change over time. Particularly - in equilibrium – the proportion of honest individuals is positive as long as the costs of inspections are less than the amount of taxes and fines recovered from inspecting activities, and the effort put by fiscal agencies is higher than the ratio between

the costs and the amount recovered from inspections. In case the cost is equal to the amount recovered with inspections, the subpopulation of taxpayers is formed only by cheating individuals, which may include also cases of ghost individuals.

As this equilibrium approximates the ESS, it is both locally and asymptotically stable, and it is reached regardless of whether the starting state is close or far from it. When one or more parameters are changed, another stable equilibrium is reached, with a different distribution of characters within the population. Particularly, higher tax rates are associated with a distribution of characters oriented to a higher proportion of cheating individuals (which in turn, hide a certain proportion or their income), but also a higher proportion of inspecting tax agencies with respect to non inspecting ones.

Higher penalty rates are instead associated with a higher proportion of honest individuals with respect to cheating taxpayers, and with a lower proportion of inspecting fiscal agencies. As lower costs of controls induce a higher proportion of honest with respect to cheating, it seems the effect of penalty and the perception of penalty leads to an adjustment in taxpayers' behaviour towards honesty. Moreover, as long as individuals perceive that costs related to audits are lower than the amount of revenues and penalty recovered through inspections, they choose the strategy honest, as they know fiscal agencies have a strong cost-benefit imbalance towards inspecting activities. In other words, in this case, inspecting activities are economically reasonable for tax authorities. As costs of audits become equal to the amount of taxes recovered through inspections, although there is still a proportion of inspecting tax agencies, the latter are in lower number, and some tax agencies would decide to become non inspecting.

In conclusion, costs of detection, perception of effort in inspections, and the level of penalty rate are important parameters in explaining the distribution of honest and cheating individuals within the subpopulation of taxpayers, as well as tax rates. Moreover, it seems that the perception of the imbalance between the costs of inspecting activities and the amount of taxes recovered through them is the main driver of adjustments in individuals' behaviour, both with respect to the subpopulation of taxpayers and of fiscal agencies.

The next Chapter 3 will present a model similar to the one presented in this chapter, within the same conceptual framework, but in which the role of both opportunity costs faced by fiscal agencies and inspection costs faced by inspected taxpayers are investigated.

Chapter 3: exploring the role of fiscal agencies' opportunity costs and taxpayers' costs of being subject to inspections

1. An Economy with honest and cheating taxpayers facing costs associated with inspections, and fiscal agencies facing opportunity costs

The model presented in this chapter will follow the same approach as the one presented in the previous Chapter 2. Also in this case, the game describing the interactions between fiscal authorities and taxpayers is characterized by two subpopulations, denoted with k=*A*,*T*, where k=*A* indicates the population of fiscal agencies, whereas k=*T* the population of taxpayers.

In this model, we introduce two types of costs. The first one is an opportunity cost occurring when fiscal agencies do not perform audit activities on cheating individuals (thus, an opportunity cost associated with the circumstance of missing the right target in inspections). The second type of costs (for taxpayers) is associated with the circumstance of being inspected. It therefore captures the costs faced by taxpayers when audited by fiscal authorities.

With the introduction of opportunity costs for tax agencies, and inspection costs for taxpayers, we can study a different attitude of both subpopulations with respect to the effectiveness of fiscal auditing. In Chapter 2, in fact, we have shown that taxpayers' perception of fiscal authorities' effectiveness in auditing activities is an important driver of tax compliance, assuming that fiscal agencies do not evaluate negatively the circumstance of missing their right target in audits, and taxpayers do not face costs associated with fiscal agencies' auditing activities. Here, instead, we have a subpopulation of fiscal agencies highly concerned about the circumstance of missing the right target, and particularly of non inspecting a cheating individual, whereas taxpayers face a cost when inspected.

Unlike in Chapter 2, penalty rate is applied on the evaded tax, and not on the evaded income, as in the Yitzhaki's conceptual framework (Yitzhaki, 1974), and in general, in line with many theoretical models addressing tax evasion from an individual's choice perspective.

Both these criteria are nonetheless taken into consideration by legal systems of most developed countries, as deterrent for tax evasion and avoidance.

Also in this chapter, we identify a set of characters for each subpopulation, each character corresponding to a strategy for the member. Given individuals' expected payoff, a character indicates the strategy chosen by each individual of the population before each round of the game starts, within the set of possible characters of its subpopulation. Recalling Chapter 2, we assume risk-neutral individuals, therefore only net income after taxes and net tax revenues matter, respectively, for taxpayers and fiscal agencies.

We assume fiscal agencies taking two characteristics (their set is formed by two strategies), indicated with *a*= *i*, *ni*, where *a*=*i* "inspect" and *a*=*ni* "don't inspect", whereas taxpayers can take two characteristics, indicated with *b*=*h*, *c*, where *b*=*h* "honest", and *b*=*c* "cheating".

We can write the set of individuals' possible matches as follows

$$S = ((s_A^i, s_T^h), (s_A^i, s_T^c), (s_A^{ni}, s_T^h), (s_A^{ni}, s_T^c))$$

where

s_A^i is the character "*inspect*" for a fiscal agency

s_A^{ni} is the character "*don't inspect*" for a fiscal agency

s_T^h is the character "*honest*" for a taxpayer

s_T^c is the character "*cheat*" for a taxpayer

Thus, the game can be described as the structure $G = \{S, F\}$, where S is the set of strategy profiles, and F is a n-tuple of payoff functions where *n=4*, with the payoff matrix represented by

Strategy	**“honest”**	**“cheat”**
“inspect”	$\pi\left(s_A^i, s_T^h\right)$	$\pi\left(s_A^i, s_T^c\right)$
“don’t inspect”	$\pi\left(s_A^{ni}, s_T^h\right)$	$\pi\left(s_A^{ni}, s_T^c\right)$

By using the same notation of Chapter 2:

- Y is the true income earned by taxpayers, not known to fiscal authorities. As in the model presented in the previous chapter, thus, the level of income is the same for all individuals, unknown to tax agencies, and not affecting the results.
- $\beta \in [0; 1]$ is the proportion of income declared at individual level in the subpopulation of taxpayers. As in Chapter 2, β captures the propensity of an individual to declare his/her entire income, or only a part of it. Taxpayers characterized by $\beta = 0$ are “ghosts”, since the proportion of income declared to tax authorities is zero, evading their entire income. By contrast, subjects with $\beta = 1$ are fully compliant individuals, declaring their entire income. There are also individuals showing $0 < \beta < 1$, thus declaring only a certain part of their income. Thus, within the subpopulation of taxpayers there is a part of individuals declaring their entire income to fiscal authorities, taking the value $\beta = 1$, and a part declaring less, or nothing, showing $0 \leq \beta < 1$. The true value of β chosen by each individual inspected is not known by fiscal agencies before each inspection takes place. We denote with $\bar{\beta}$ the arithmetic average proportion of income declared within the subpopulation of taxpayers. The cases where $\bar{\beta} = 0$ or $\bar{\beta} = 1$, correspond to a subpopulation of taxpayers formed only by ghost or honest

individuals, respectively. We should notice here that, even when the initial distribution of characters within the subpopulation of taxpayers is such that $\bar{\beta} = 0$, therefore all individuals are ghosts, the interactions with fiscal authorities may lead to a different distribution of characters at the end of the game, as the payoff of taxpayers hiding income highly depends on whether they are controlled or not. By contrast, the case in which initially $\bar{\beta} = 1$ refers to the case of a subpopulation of taxpayers formed by all fully honest individuals.

- τ is the tax rate applied on the income declared βY, such that $0 < \tau < 1$ and proportional to the income declared βY.
- $C\tau Y$ is the total cost of inspections for the tax agency, which we assume to be linear on tax revenue τY, where the parameter $C > 0$ is the (marginal and average) cost of inspections. We assume that C is unknown to taxpayers, whereas it is well known to fiscal agencies.
- α is the proportion of taxes and fines recovered from inspections, and it measures the effort of fiscal authorities in their auditing activities. Particularly, as in Chapter 2, $\alpha \in (0; 1]$ and $\alpha = 0$ corresponds to the case of non-inspecting fiscal agencies. When $\alpha = 1$, the fiscal agency recovers the entire amount of taxes and of fines applied, indicating the full success of the inspection. When, by contrast, $0 < \alpha < 1$, tax agencies' inspecting activities are partial, due to a lack of effort in case of one taxpayer inspected at each time, or to a selection of the individuals to be inspected in case of insufficient resources.
- φ is the penalty rate applied on the evaded tax when a cheater is caught by an inspecting tax agency, such that $\varphi > 0$, assuming $\varphi > \tau$.
- Here we introduce $m\tau\beta Y$, the total cost faced by taxpayers once inspected by a fiscal agency, where $0 \leq m < 1$, and linear in the amount of taxes paid by the individual. When $\beta = 1$, in presence of honest taxpayers, the cost becomes $m\tau Y$.

- Moreover, $n[\left(\varphi\,\underbrace{\tau(1-\beta)Y}\right)+\tau(1-\beta)Y]$ is the opportunity cost faced by the non inspecting fiscal agency, in case the latter is matched with a cheating taxpayer. It is linear in the evaded fine, and in the evaded tax, and such that $0 \leq n < 1$.

We now consider the specific payoffs' structure of the game.

According to (1) and (2) below, when an <u>inspecting agency is matched with a honest taxpayer</u>, the inspecting agency payoff $\pi_A\left(s_A^i, s_T^h\right)$ is given by tax revenue on the entire income (τY) less the costs of control ($C\tau Y$), whereas the taxpayer payoff $\pi_T\left(s_A^i, s_T^h\right)$ is given by his/her disposable income after taxes $(1-\tau)Y$, less the cost associated with the circumstance of being inspected. In this case, α is not a relevant parameter, as the amount of taxes due has already been paid by the fully compliant taxpayer (as the honest taxpayer declares the entire income).

(7) $\pi_A\left(s_A^i, s_T^h\right) = \tau Y - C\tau Y$

(8) $\pi_T\left(s_A^i, s_T^h\right) = (1-\tau)Y - m\tau Y$

Suppose then an <u>inspecting agency is matched with a cheating taxpayer</u>. The payoff related to the tax agency $\pi_A\left(s_A^i, s_T^c\right)$ is given by the recovered amount of taxes applied on the true income earned τY and fines applied on the evaded tax, $\varphi\tau(1-\beta)Y$, multiplied by α, indicating the effort in recovering the entire amount, less the tax agency's costs of detection, $C\tau Y$, as it can be noticed in (3). The cheating taxpayer's payoff, $\pi_T\left(s_A^i, s_T^c\right)$, is given by the disposable income after taxes $(1-\tau)Y$ less the fine on the evaded part of income, $\varphi\tau\beta Y$ linear in the tax paid (4), multiplied by $1-\alpha$, indicating the part not recovered during the inspection, less the cost faced by the inspected taxpayer for the inspecting activities, $m\tau\beta Y$.

$$(9)\ \pi_A\left(s_A^i, s_T^c\right) = \left[\tau Y + \underbrace{\varphi\tau(1-\beta)Y}_{}\right]\alpha - C\tau Y$$

$$(10)\quad \pi_T\left(s_A^i, s_T^c\right) = [(1-\tau)Y \underbrace{- \varphi\tau(1-\beta)Y}_{}]\,(1-\alpha) - m\tau\beta Y$$

In case a non inspecting agency is matched with an honest taxpayer, (5) shows that tax agencies obtain the due tax revenues, without facing any cost, whereas the taxpayer gets his/her disposable income (6)[34].

$$(11)\quad \pi_A\left(s_A^{ni}, s_T^h\right) = \tau Y$$

$$(12)\quad \pi_T\left(s_A^{ni}, s_T^h\right) = (1-\tau)Y$$

When a matching between a non-inspecting agency and a cheater occurs, fiscal agency's payoff $\pi_A\left(s_A^{ni}, s_T^c\right)$ is the revenue coming from the under-declared income, $\tau\beta Y$, less the opportunity cost associated with the circumstance of non-inspecting a cheating taxpayer, $n[\left(\varphi\,\underbrace{\tau(1-\beta)Y}_{}\right) + \tau(1-\beta)Y]$, where $0 \leq \beta < 1$. The taxpayer gets $\pi_T\left(s_A^{ni}, s_T^c\right)$, given by the disposable income Y after taxation on the under-declared income $\tau\beta Y$, where the tax rate is applied on the amount of declared outcome. It should be noticed here that equation (7) is negative for $\beta < 0.5$[35].

$$(13)\quad \pi_A\left(s_A^{ni}, s_T^c\right) = \underbrace{\tau\beta Y}_{} - n[\left(\varphi\,\underbrace{\tau(1-\beta)Y}_{}\right) + \tau(1-\beta)Y]$$

$$(14)\quad \pi_T\left(s_A^{ni}, s_T^c\right) = Y(1 - \underbrace{\tau\beta)}_{}$$

[34] The case of non-inspecting fiscal agencies includes also the circumstance under which there might be no "one-to-one" matching between taxpayers and fiscal agencies, as the number of taxpayers might be higher than the number of fiscal agencies in the population, as it happens in most real contexts.

[35] In case fiscal agencies do not inspect and the taxpayer declares less than 50% of his/her income, fiscal agencies' payoff becomes negative.

As described for the model presented in Chapter 2, the game represents continuous interactions between individuals pertaining to the two subpopulations, where each member of the subpopulation of taxpayers is randomly matched with a member of tax agencies' subpopulation, over a continuous time-horizon. As in the previous chapter, Y is known only to the subpopulation of taxpayers, and unknown to fiscal authorities, whereas β is unknown to fiscal authorities before each interaction takes place.

Recalling what explained in Chapter 2, each individual pertaining to the taxpayer or tax agency subpopulation chooses a character within the subset, before the actual matching takes place. Each tax agency is then matched with a taxpayer. Neither of them knows *ex ante* which character is taken by the opponent. Thus, a non-inspecting agency may be matched either with a honest or a cheating taxpayer, without knowing it in advance.

We also recall briefly the concept of "fitness function", representing the expected payoffs for each taxpayer being honest or cheating, and for each tax agency inspecting or non inspecting, respectively, as given by the weighted average payoffs associated with a character. Weights are probabilities of finding a given character in the subpopulation. We indicate with $(\boldsymbol{e^a}, \boldsymbol{T})$ the state of the world for each tax agency, given the behaviour of the subpopulation of taxpayers, where *a*= *i* (inspect), *ni* (not inspect), and with $(\boldsymbol{e^b}, \boldsymbol{A})$ the state of the world for each taxpayer, given the behaviour of the subpopulation of tax agencies, where *b*=*h* (honest), and *c* (cheating).

We denote with p_I the proportion of inspecting agencies in the population and with p_{NI} the proportion of non-inspecting agencies. Denoting with q_H the proportion of honest and with q_C the proportion of cheating within the subpopulation of taxpayers, we can write the generic fitness functions associated to each strategy (or character) of each population under examination (using the notation by Friedman, 1991) as:

$$\boldsymbol{f_A(e^a, T) = q_H\left[\pi_A\left(s_A^a, s_T^h\right)\right] + q_C[\pi_A(s_A^a, s_T^c)]} \quad \textbf{(9)}$$

$$f_T(e^b, A) = p_I[\pi_T(s_A^i, s_T^b)] + p_{NI}[\pi_T(s_A^{ni}, s_T^b)] \quad \textbf{(10)}$$

which are displayed below:

(9.1) $$f_A(e^i, T) = q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]$$

(9.2) $$f_A(e^{ni}, T) = q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]$$

(10.1) $$f_T(e^h, A) = p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]$$

(10.2) $$f_T(e^c, A) = p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]$$

Each individual's fitness function depends on the distribution of characters within the other subpopulation, not known to subjects. Each taxpayer thus assigns a probability to the event "matching with an inspecting agency" and "matching with a non-inspecting agency", and each fiscal agency assigns a probability to the circumstance of matching with a honest or a cheating taxpayer. At each time *t*, a member of the taxpayers' population calculates its expected payoff, by assuming a certain distribution of characters within the subpopulation of fiscal agencies. The same, in turn, occurs for tax agencies, thus at each *t*, fiscal agencies calculate their expected payoff taking the distribution of the honest and cheating subjects as a given.

At each time *t*, interactions take place between each member of the subpopulation of taxpayers with a member of the fiscal agencies population, and at time *t+1* a memory is formed on the distribution of characters with respect to the other subpopulation, based on the overall interactions that took place in time *t* (a collective and individual memory).

Interactions at each time *t* therefore lead to a reshape in the distribution of characters, as individuals adjust both their subjective probabilities on the proportion of cheating and honest individuals, as well as on inspecting and non inspecting agencies in (9) and (10). Both taxpayers' and tax agencies' expected payoffs (fitness functions) change over time as a result of adjustments in the probability of finding characters within each subpopulation. Cheating taxpayers inspected may choose not to be cheating at the time $t+1$, and by contrast, honest taxpayers may choose to become cheating as a result of an encounter with a non-inspecting fiscal agency. At the end of the game, all these small changes produced at each time t - when interactions occur between members of the two subpopulations - reshape the subpopulations' distributions.

Over time, the distribution of characters within the taxpayers' and tax agencies' subpopulation is adjusted on the basis of experience. At a population level, there is a reshape towards a new distribution of characters after all the interactions for each time t. The distribution of a character depends on how the subpopulation as a whole reacts to the matching occurring between each pair of individuals pertaining to the two subpopulations, in terms of speed of reproduction.

Also in this case, by referring to Taylor and Yonker's dynamic replicator law[36] (Sandholm, 2010; Taylor and Yonker, 1978), as in Chapter 2, we assume that individuals share the same behaviour, but with heterogeneous intrinsic characteristics. As all individuals behave the same way, and their adaptive behaviour in choosing a strategy can be described as a comparison between the payoff associated with the specific strategy with the overall payoff of the subpopulation, the law of motion of each character of each subpopulation can approximate well the behaviour of the individual. Thus, the law of motion displays both the individual and collective perspective, although the relevant parameters differ from one individual to the other within the subpopulations.

[36] The replicator equation, defined in literature for the first time by Schuster and Sigmund (1983), is a mathematical description of the evolutionary game dynamics explaining the evolution of characters within a population as the result of the re-distribution of characters over time as a result of a mechanism of learning from the payoffs obtained. As well-described by Cressmann and Tao (2014) "*With payoff translated as fitness (i.e., reproductive success), the frequency of a strategy in a large, well-mixed single species changes under the (continuous-time) replicator equation at a per capita rate equal to the difference between its expected payoff and the average payoff of the population*".

Moving to the description of the dynamic setting, which is the same as the one described in Chapter 2[37], we can set up the replicator equations for the game as in Taylor and Yonker (1978) and Sandholm (2010), as follows:

$$(11)\ \frac{\dot{q_H}}{q_H} = f_T(e^1, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\left(f_T(e^2, s)\right)\right]$$

$$(12)\ \frac{\dot{q_C}}{q_C} = f_T(e^2, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\left(f_T(e^2, s)\right)\right]$$

$$(13)\ \frac{\dot{p_I}}{p_I} = f_A(e^1, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}\left(f_A(e^2, s)\right)\right]$$

$$(14)\ \frac{\dot{p_{NI}}}{p_{NI}} = f_A(e^2, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}\left(f_A(e^2, s)\right)\right]$$

The dynamic replicator law equations (11)-(14) represent the law of motion[38] of each character within the subpopulation over time. In other words, (11)-(14) represent how characters change over time for the subpopulation of taxpayers and of fiscal agencies. Recalling Chapter 2, dynamic replicator equations describe the growth rate of each character as given by the difference with respect to the average expected payoff of the population (also defined as average fitness function). The speed of growth of a character depends on the difference between the payoff associated with the character, and the average payoff of

[37] We let $q_H(t)$, $q_C(t)$, $p_I(t)$ and $p_{NI}(t)$ be the proportion of honest and cheating taxpayers, inspecting and non inspecting fiscal agencies respectively at time *t*. Assuming continuous time, the growth rate of each variable in a continuous time setting is calculated as the derivative of the logarithm with respect to time, following Taylor and Yonker (1978), Friedman (1991) and Barro and Sala-i-Martin (2004) as, in the baseline case, it is assumed a logarithmic growth of the characters. Therefore, by taking the logarithm of $q_H(t)$, $\ln(q_H)$, and differentiating with respect to time, we get $\frac{d \ln q_H(t)}{dt} = \frac{1}{q_H}\frac{dq_H(t)}{dt} = \frac{\dot{q_H}}{q_H}$, where we then denote $\dot{q_H} = \frac{dq_H(t)}{dt}$, where the dot over q_H, q_C, p_I and p_{NI}, indicate their change over time. The same can be derived for all the remaining variables $q_C(t)$, $p_I(t)$ and $p_{NI}(t)$.

[38] The law of motion for a character describes the evolution of a character over a continuous time, based on a set of relevant variables, as payoffs associated to each character, and the proportion of each character within the subpopulation. In the context of replicator equation, the law of motion of each character is given by the difference between the expected payoff associated to a character, and the average expected payoff the subpopulation (Friedman, 1998; Sandholm, 2010). For example, the replicator equation explaining the evolution over time of honest taxpayers is given by the expected payoff associated to the character "honest", less the average expected payoff of the entire subpopulation of taxpayers.

the subpopulation (Cressman and Tao, 2014). The latter is given by the average fitness functions associated to each character within the subpopulation, weighted for the distribution of characters within each subpopulation. The dynamics of a character does not depend on individuals' payoff maximization, but instead on the difference between the payoff associated to the character itself and the average expected payoff of all characters. If this difference is positive (therefore, if the payoff associated to the character is higher than the average expected payoff among all characters), the proportion of individuals choosing the character tends to grow. By contrast, if the difference is negative, then the proportion of individuals choosing the character will decrease.

By making equations (11) - (14) explicit, the evolution of characters associated with each subpopulation depends also on the distribution of characters within the other subpopulation. This feature represents strategic interactions among individuals, as explained in the equations (11a) – (14a) below

(11a) $\frac{\dot{q_H}}{q_H} = \{p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]\} - \{q_H[p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]] + q_C[p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]]\}$

(12a) $\frac{\dot{q_C}}{q_C} = \{p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]\} - \{q_H[p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]] + q_C[p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]]\}$

(13a) $\frac{\dot{p_I}}{p_I} = \{q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]\} - \left\{p_I\left[q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]\right] + p_{NI}\left[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]\right]\right\}$

(14a) $$\frac{\dot{p_{NI}}}{p_{NI}} = \{q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]\} - \{p_I\left[q_H[\pi_A(s_A^i, s_T^h)] + q_C[\pi_A(s_A^i, s_T^c)]\right] + p_{NI}\left[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_C[\pi_A(s_A^{ni}, s_T^c)]\right]\}$$

As shown in (11a) and (12a), characters "honest" and "cheating" evolve depending also on the distribution of characters "inspecting" and "not inspecting" within the subpopulation of fiscal agencies. The subpopulation of fiscal agencies evolves according to both the distribution of "cheating" and "honest" taxpayers and "inspecting" and "non inspecting" tax agencies, as represented in (13a) and (14a). Taxpayers, as described by (11) and (12), choose their character by comparing the fitness function of a strategy with the average fitness function associated to all strategies within the strategy set. The same happens for fiscal agencies, choosing whether to inspect or not by comparing the fitness functions (expected payoffs) associated to characters with the average fitness function of all characters within their set. At each time *t*, interactions take place between members of the subpopulation of taxpayers and fiscal agencies. After each interaction between the members of the two subpopulations, the distribution given by q_H, q_C, p_I and p_{NI} is fine-tuned by the population experience[39]. It occurs because a review of the distribution of characters for each subpopulation takes place as a result of the interactions among members of the two subpopulations. The choice between characters is driven by the comparison between the expected payoff associated to a character, and the average payoff of the overall population (Cressman and Tao, 2014).

At each time t, members of the two subpopulations form their subjective probabilities on the distribution of characters within the other subpopulation, as suggested by (9) and (10). The ESS equilibrium[40] is

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[39] In fact, as each individual belonging to the subpopulation of taxpayers or fiscal agencies, after each interaction, revises at time *t* its strategy as a result of the interaction at the previous time *t-1*, the distribution of characters changes at each time of the game, as a result of the previous time, driven by experience.

[40] According to Maynard Smith (1982), an evolutionary stable strategy is a "*strategy such that, if all members of a population adopt it, then no mutant strategy could invade the population under the influence of natural selection.*". In economic terms, a strategy is evolutionary stable if there is no other strategy associated with a higher

given by the fine-tuning in the distribution of characters happening at the end of each time *t*, after several interactions between the two subpopulation members. As described, individuals choose their character on the basis of experience, after interactions at each time *t*, and the distribution of characters varies accordingly.

In the ESS equilibrium, the rate of growth associated to each character is zero, which means that the distribution of each character stops varying over time, and that individuals stop changing their strategy (Friedman, 1991). Therefore, since $\frac{\dot{q_H}}{q_H}, \frac{\dot{q_C}}{q_C}, \frac{\dot{p_I}}{p_I}$ and $\frac{\dot{p_{NI}}}{p_{NI}}$ represent the rate of growth of the proportion of characters within the subpopulations over time, by solving the system (11)- (14) above for $\frac{\dot{q_H}}{q_H} = 0, \frac{\dot{q_c}}{q_C} = 0, \frac{\dot{p_I}}{p_I} = 0$ and $\frac{\dot{p_{NI}}}{p_{NI}} = 0$, we find the long run solutions of q_H, q_C, $p_I$and p_{NI}, which we will express as $\overline{q_H}$, $\overline{q_C}$, $\overline{p_I}$and $\overline{p_{NI}}$. The values $\overline{q_H}$, $\overline{q_C}$, $\overline{p_I}$and $\overline{p_{NI}}$ represent the point at which characters are distributed within each subpopulation after all interactions. The equilibrium related to the solution of the system of differential equations presented above approximates the concept of equilibrium defined as evolutionary stable strategy equilibrium (ESS), characterized by the fact that any "perturbation" over time (in other words, any exogenous change) does not change the distribution of characters over the population. Literature has shown that the ESS is both locally and asymptotically stable under certain conditions (Hines, 1980)[41], as the Nash Equilibrium. Therefore, the ESS is reached either from a starting point within its neighbourhood, or starting from a state farther from the neighbourhood of the ESS. Nonetheless, the presence of an infinite population of players is a relevant condition for the ESS equilibrium, which *per se* does not depict a real world situation (Fogel et al. 1997). We then assume for

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or equal expected payoff. This latter concept associates the ESS equilibrium to the Nash Equilibrium, as already explained in the introductory Chapter 1.

[41] Particularly, according to Takada and Kigami (1990), the stable solution coinciding with the Nash solution is always attainable when interactions among members of the populations are purely competitive and in case of a population formed by a large number of individuals.

our game a very large population formed by taxpayers and fiscal agencies, playing with a different set of strategies and evolving according to the law of motions described above.

2. Results

The solution of the game at the equilibrium, found by solving the system (11)-(14), is given by the following values $\overline{q_H}$, $\overline{q_C}$, $\overline{p_I}$and $\overline{p_{NI}}$ in (17)-(20):

$$(17)\ \overline{q_H} = 1 - \frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})}$$

$$(18)\ \overline{q_C} = \frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})}$$

$$(19)\ \overline{p_I} = \frac{\tau(1-\bar{\beta})}{\bar{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\bar{\alpha}\varphi\tau)(1-\bar{\beta})}$$

$$(20)\ \overline{p_{NI}} = \frac{\bar{\alpha}(1-\tau)+(\varphi\tau-\bar{\alpha}\varphi\tau-m\tau)(1-\bar{\beta})}{\bar{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\bar{\alpha}\varphi\tau)(1-\bar{\beta})}$$

where $\bar{\beta} \in [0,1]$ is the *ex-ante* average proportion of income declared by individuals pertaining to the subpopulation of taxpayers, and $\bar{\alpha}$ is the average effort within the subpopulation of fiscal agencies[42].

The equilibrium proportion of honest, cheating, inspecting and non inspecting individuals is studied considering individuals' propensity to declare their income to fiscal authorities, as well as the agencies' effort with respect to inspections. As in the game presented in Chapter 2, the level of income is not relevant for the equilibrium of the game, whereas tax rate and penalty rate are.

Here, with respect to the model presented in Chapter 2, we have equilibrium solutions $\overline{q_H}$ and $\overline{q_C}$ - (17) and (18) respectively - depending not only on φ, τ, C $\bar{\alpha}$, $\bar{\beta}$, but also on n.

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[42] For the calculations and methodology for the solution of the model, see Annex I.

As shown in (18), the proportion of cheating individuals within the subpopulation of taxpayers is given by the ratio between τC and $(\tau + \bar{\alpha}\varphi + n\varphi + n\tau)(1 - \bar{\beta})$. The latter is the sum of average unpaid taxes, $\tau(1 - \bar{\beta})$, average fine recovered from inspections, $\bar{\alpha}\varphi(1 - \bar{\beta})$, and average opportunity costs faced by tax agencies when non-inspecting a cheating taxpayer, $(n\varphi + n\tau)(1 - \bar{\beta})$. We can thus identify $(\tau + \bar{\alpha}\varphi + n\varphi + n\tau)(1 - \bar{\beta})$ as the average amount potentially recovered by fiscal agencies when inspecting a cheating individual. Here we can see that opportunity costs increase fiscal agencies' gain when inspecting a cheating taxpayer.

In this context, the fine-tuning of the distribution of characters on the basis of experience within the population is an approximation of individual behaviour, as individuals are characterized by homogeneity in behaviour and intrinsic heterogeneity (β varies among members). The equilibrium at the population level, which expresses $\overline{q_H}$ and $\overline{q_C}$, depends on $\bar{\beta}$, which is the arithmetic average proportion of income declared within the population.

As in Chapter 2, the parameter β (the proportion of income declared, thus individual preference towards evasion or compliance) is taken as given, and different for each individual. Neither tax agencies, nor taxpayers, do actually know ex ante the exact β of each member of the subpopulation of taxpayers. The choice on whether to inspect or not is taken assuming that β is a constant out of control, "guessing" the value of β. In fiscal agencies' perspective, $\bar{\beta}$ is the average guess made on the subpopulation of taxpayers. The same reasoning applies to α and $\overline{\alpha}.$

In the analysis of the results presented in the following pages, honest individuals are the ones who declare their entire income, and they are characterized by $\beta = 1$. Cheating taxpayers declare less than their actual income, with $0 \leq \beta < 1$, and the case $\beta = 0$ represents the "ghosts". The average $\bar{\beta}$ should also be distinguished by the proportion of honest and cheating individuals, as $\overline{q_H}$ and $\overline{q_C}$ represent all individuals with $\beta = 1$ and $0 \leq \beta < 1$, respectively.

We summarize in the following table I the direction of the effect of *ceteris paribus* changes of parameters on the equilibrium results. In Annex I, we shall report the exercises of comparative statics carried out at equilibrium values.

Table I: Direction of the effects on the equilibrium values of the ceteris paribus change of the relevant parameters

	Direction of the effects on the equilibrium values of the *ceteris paribus* change of the relevant parameter					
Equilibrium value	$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\boldsymbol{C}$	$\boldsymbol{\bar{\alpha}}$	$\boldsymbol{\bar{\beta}}$	$\boldsymbol{n}$	$\boldsymbol{m}$
$\overline{q_H}$	>0	<0	<0	>0	<0	>0	-
$\overline{q_C}$	<0	>0	>0	<0	>0	<0	-
$\overline{p_I}$	<0	>0	-	<0	<0	-	>0
$\overline{p_{NI}}$	>0	<0	-	>0	>0	-	<0

When $0 < \bar{\beta} < 1$ and $0 < \bar{\alpha} < 1$, we see in **(17)** above that equilibrium results for the proportion of honest and cheating taxpayers depend on $1 - \frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})} \geq 0$, thus, on

$\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})} \leq 1$ (21).

Looking at (21), we shall distinguish between two cases, *i)* $\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})} < 1$ in which $\overline{q_H} > 0$ and $\overline{q_C}$ >0; and *ii)* $\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})} = 1$, where $\overline{q_H} = 0$ and $\overline{q_C}$ =1.

In case *i)*, the long run equilibrium for the character "honest" $\overline{q_H}$ is strictly positive: honest individuals can be found as long as $\tau C < (\tau + \bar{\alpha}\varphi + n\varphi + n\tau)(1-\bar{\beta})$. Honest taxpayers can be found as long as they perceive that cost of control remains strictly less than the amount recovered in inspections, which includes the opportunity cost faced by the fiscal agency under the circumstance it does not inspect a cheating taxpayer. By contrast, if $\tau C > \tau + \bar{\alpha}\varphi + n\varphi + n\tau)(1-\bar{\beta})$, no honest individual can be found within the population, as audits become unprofitable to fiscal agencies, in taxpayers' perception.

By writing (17) as (17.b) below, we see clearly the implications of the role of opportunity costs:

$$(\tau + \bar{\alpha}\varphi)(1-\bar{\beta}) > \tau C - n(\tau+\varphi)(1-\bar{\beta}) \qquad (17.b)$$

which shows that, in equilibrium, in order to find a positive proportion of honest individuals, the amount recovered in auditing activities must be perceived to be higher than the costs of inspections, less the opportunity costs of missing the right target.

The second case *ii)*, where $\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})} = 1$, determines $\overline{q_H} = 0$ and and $\overline{q_C}$ =1, i.e. all taxpayers are cheating in equilibrium. This requires $\tau CY = (\tau + \bar{\alpha}\varphi + n\varphi + n\tau)(1-\bar{\beta})$, or $\bar{\alpha} = \frac{\tau C-(\tau+n\varphi+n\tau)(1-\bar{\beta})}{[\varphi(1-\bar{\beta})]}$. When the cost of audits is perceived as exactly equal to the net amount of fine and taxes recovered from inspections, plus the opportunity cost for the tax agencies, or the perceived effectiveness of control is equal to the ratio between the costs of inspections less tax agencies' opportunity costs and the unpaid taxes, it is worthy for taxpayers to be cheating (and thus the probability of cheating among the population increases).

Results show that taxpayers' perception of $\overline{\alpha}$, of the cost of inspection and of the opportunity costs faced by non inspecting fiscal agencies impacts the proportion of fully compliant and cheating individuals, with higher levels of fiscal agencies' efficiency in inspections being associated with a higher proportion of fully compliant individuals, *ceteris paribus*, and with higher costs of auditing activities being associated with a higher level of cheating individuals.

With respect to the results presented in Chapter 2, here we see that honest individuals can be found in equilibrium only in case $\tau CY \leq (\tau + \bar{\alpha}\varphi + n\varphi + n\tau)(1 - \bar{\beta})$. In this case, the proportion of honest individuals is different from zero and positive, as long as costs of audits are perceived as lower than the amount recovered from inspections, plus the opportunity costs faced by the non-inspecting agencies, which represent the loss associated with the circumstance of missing the right target (which is non-inspecting a cheating individual). Taxpayers perceive that fiscal agencies might be oriented only to the recovery of the amount due by cheating taxpayers, by organizing strategically their inspecting activities, not contemplating the situation where the amount of fines and taxes recovered from inspections does not cover the costs of inspections.

Moreover, looking at whether $\overline{q_h} \gtreqless \overline{q_C}$, $\overline{q_h} > \overline{q_C}$ when $\tau < \frac{\varphi(\bar{\alpha}+n)(1-\bar{\beta})}{2C-(1+n)(1-\bar{\beta})}$ and $\varphi > \frac{\tau[2C-(1+n)(1-\bar{\beta})]}{(\bar{\alpha}+n)(1-\bar{\beta})}$.

As in the results presented in Chapter 2, we see from table I that a higher penalty rate increases the proportion of honest individuals, *ceteris paribus*. In addition, a higher level of effort in inspections perceived by taxpayers increases the proportion of honest individuals. By contrast, higher tax rates and costs of audits are associated with an increase in the proportion of cheating taxpayers, *ceteris paribus*.

In this model, in addition to the findings of Chapter 2, we also show the role played by taxpayers' perception about fiscal agencies' opportunity costs associated with the circumstance of missing the right target. As taxpayers perceive that fiscal agencies' loss is higher when missing the right target in inspections (and particularly, when choosing not to inspect a cheating individual), they tend to adjust their behaviour towards honesty, and therefore the proportion of honest taxpayers increases, while the proportion of cheating decreases. In this respect, we make the assumption that fiscal agencies with higher opportunity costs are characterized by an "aggressive" strategy of inspection. In fact, they associate a loss to the circumstance of missing the amount of taxes and penalties during inspections. Taxpayers' perception of such "aggressiveness" might therefore drive compliance, in addition to the parameters already presented in Chapter 2.

With respect to (19) and (20), from table I, the proportion of inspecting fiscal agencies is increasing in tax rate, *ceteris paribus*, and decreasing in the effort put in inspections by fiscal authorities. Moreover, higher penalty rates are associated with a lower proportion of inspecting fiscal agencies. The proportion of inspecting agencies is decreasing in the level of $\overline{\alpha}$, since higher effort would require a lower proportion of inspecting fiscal agencies. Fiscal authorities can induce taxpayers towards compliance both with modifications in fiscal parameters such as tax and penalty rate, as well as through enhancing individuals' perception of effort put in enforcement. This, in turn, leads to the need of a lower proportion of inspecting agencies. In addition to the effects described in Chapter 2, here we see the role of fiscal agencies' perception of costs borne by taxpayers in case of inspection. As fiscal agencies perceive that taxpayers face a higher cost associated with inspections, they tend to inspect more. This result is related to the fact that taxpayers' cost of inspection is applied on the amount of paid tax, $m\tau\beta Y$, which becomes $m\tau Y$ when a honest taxpayer is inspected. Therefore, as $\beta < 1$ in case of cheating individuals, the cost related to inspections is lower for a cheating than for a honest individual, as $m\tau\beta Y < m\tau Y$ when $\beta < 1$. The latter implies that, as an increase in the cost of being inspected leads to a decrease in the inspected taxpayers' payoff in (2) and (4) above, that decrease is lower in case of cheating individuals with respect to honest individuals. Thus, an increase in the cost faced by taxpayers when subject to inspections – by decreasing taxpayers' payoffs with a higher magnitude for honest than for cheating – induces individuals to reconsider their strategy and to choose to become cheating. As a consequence, when fiscal agencies perceive an increase in m, they tend to increase their inspecting activities, as they expect taxpayers would choose the strategy "cheating" over "honest" when m increases.

Looking at (19) and (20), $\overline{p_I} \gtreqless \overline{p_{NI}}$, depending on whether $\tau(1-\bar{\beta}) \gtreqless \bar{\alpha}(1-\tau) + (\varphi\tau - \bar{\alpha}\varphi\tau - m\tau)(1-\bar{\beta})$. As we can see from simulations below, $\overline{p_I}$ tends to increase as the levels of m and τ increase, whereas higher levels of φ are associated with a lower $\overline{p_I}$. Therefore, $\overline{p_I} > \overline{p_{NI}}$ for higher levels of m and τ, and particularly for m>0.6 and $\tau > 0.3$.

As for the role of $\bar{\alpha}$, when $\bar{\alpha} \to 1$, and $0 < \bar{\beta} < 1$, (19) and (20) become $\overline{p_I} = \frac{\tau(1-\bar{\beta})}{(1-\tau)+[\tau(1-m)](1-\bar{\beta})}$ and $\overline{p_{NI}} = \frac{(1-\tau)-m\tau(1-\bar{\beta})}{(1-\tau)+[\tau(1-m)](1-\bar{\beta})}$, which implies that $\overline{p_I} \gtreqless \overline{p_{NI}}$ depending on whether $\tau[(1+m)(1-\bar{\beta})+1] \gtreqless 1$, thus depending on whether the sum of the evaded tax $\tau(1-\bar{\beta})$, plus the costs of inspection applied on the proportion of income evaded $\tau m(1-\bar{\beta})$ and the tax rate τ, is higher, equal or lower than 1.

In this case, (17) and (18) become $\overline{q_H} = 1 - \frac{\tau C}{(\tau+\varphi+n\varphi+n\tau)(1-\bar{\beta})}$ and $\overline{q_C} = \frac{\tau C}{(\tau+\varphi+n\varphi+n\tau)(1-\bar{\beta})}$, and as $\overline{q_C}$ is always a positive quantity (unless either tax rate, or cost of control, or both are equal to zero), the proportion of honest individuals $\overline{q_H}$ will always be less than 1. Moreover, $\overline{q_H} > \overline{q_C}$ as long as $\tau < \frac{\varphi(1+n)(1-\bar{\beta})}{2C-(1+n)(1-\bar{\beta})}$.

As for the *ceteris paribus* effects of by $\bar{\alpha}, \bar{\beta}$, τ and φ on the equilibrium solutions of the proportion of honest and cheating taxpayers, and of inspecting and non-inspecting tax agencies, we briefly recall the results presented in Chapter 2, as here we found similar results for the effects of $\bar{\alpha}, \bar{\beta}$, τ and φ. Particularly, higher levels of penalty increase – *ceteris paribus* – the proportion of honest individuals, and decrease the proportion of cheating, whereas, by contrast, higher levels of tax rate – *ceteris paribus* – induce an increase in the proportion of cheating taxpayers, lowering the proportion of honest ones. The level of effort put in inspections by fiscal authorities affects the equilibrium proportion of honest individuals, as well as costs. As regards fiscal authorities, higher levels of effort are associated with a lower proportion of inspecting agencies, as both the direct effect of effort and the indirect effect (through taxpayers' perception) lead to the need of a lower proportion of inspecting fiscal agencies (as the perception of effort is *per se* a policy tool to increase the proportion of fully honest taxpayers).

Higher levels of effort in enforcement and inspecting activities induce both a decrease in the proportion of cheating individuals within the subpopulation of taxpayers through the latters' perception, which in turn is associated with an indirect effect on the proportion of inspecting fiscal agencies, and a decrease in the proportion of inspecting agencies (direct effect on the proportion of inspecting fiscal agencies) due to a better allocation of resources.

Looking then at the effect of the opportunity cost n, the latter affects only the equilibrium proportion of honest and cheating taxpayers. Particularly, taxpayers tend to behave as "honest", as they perceive higher opportunity costs for fiscal agencies, in case the latter do not inspect a cheating individual. This happens as taxpayers perceive that fiscal agencies would put a higher focus in audits in order not to miss the opportunity of catching the cheating taxpayers. This explains why, as n increases, $\overline{q_H}$ increases and $\overline{q_C}$ decreases. Therefore, another driver of individuals' compliance, in addition to the ones already discussed in Chapter 2 (taxpayers' perception of effort put in inspection and the level of penalty rate) is taxpayers' perception on the opportunity costs faced by fiscal agencies missing the right target (not inspecting a cheating taxpayer).

Moreover, from table I, as m (the cost faced by the inspected taxpayers for the inspections carried out by fiscal agencies) increases, $\overline{p_I}$ increases, whereas the proportion of non-inspecting agencies decreases, as taxpayers tend to become "cheating" when m increases.

2.1 Equilibrium results for $\overline{\beta} = 0$ and $\overline{\beta} = 1$

We shall comment the other results by referring to table II below that summarizes the game equilibria for different levels of $\overline{\alpha}$ and $\overline{\beta}$. The case $\overline{\alpha} = 0$ is not relevant, as it corresponds to the case when all fiscal agencies do not inspect.

Table II: equilibrium results of the model, for different combinations of parameters, in case $\bar{\beta} = 0$ and $\bar{\beta} = 1$

	$\bar{\beta} = 0$	$\bar{\beta} = 1$
$0 < \bar{\alpha} < 1$	**(A)** $(1.a)\quad \overline{q_H} = 1 - \frac{\tau C}{(\tau + \bar{\alpha}\varphi + n\varphi + n\tau)}$ $(2.a)\quad \overline{q_C} = \frac{\tau C}{(\tau + \bar{\alpha}\varphi + n\varphi + n\tau)}$ $(3.a)\quad \overline{p_I} = \frac{\tau}{\bar{\alpha}(1-\tau) + (\tau - m\tau + \varphi\tau - \bar{\alpha}\varphi\tau)}$ $(4.a)\quad \overline{p_{NI}} = \frac{\bar{\alpha}(1-\tau) + (\varphi\tau - \bar{\alpha}\varphi\tau - m\tau)}{\bar{\alpha}(1-\tau) + (\tau - m\tau + \varphi\tau - \bar{\alpha}\varphi\tau)}$	**(C)** (17.b) $\overline{q_H} = 1$ (18.b) $\overline{q_C} = 0$ (19.b) $\overline{p_I} = 0$ (20.b) $\overline{p_{NI}} = 1$
$\bar{\alpha} \to 1$	**(D)** (17.d) $\overline{q_H} = 1 - \frac{\tau C}{(\tau+\varphi+n\varphi+n\tau)}$ (18.d) $\overline{q_C} = \frac{\tau C}{(\tau+\varphi+n\varphi+n\tau)}$ (19.d) $\overline{p_I} = \frac{\tau}{1-m\tau}$ (20.d) $\overline{p_{NI}} = \frac{1-\tau-m\tau}{1-m\tau}$	

When $\bar{\beta} = 1$, as represented <u>**in case C)**</u> in the table above, all individuals within the subpopulation of taxpayers are honest (they declare the entire amount of their income). Both cases where $\bar{\beta} = 1$ joint with $\bar{\alpha} \to 1$, and $\bar{\beta} = 1$ joint with $0 < \bar{\alpha} < 1$, are characterized by the solution $\overline{q_H} = +\infty = 1, \overline{q_C} = -\infty = 0, \overline{p_I} = 0$ and $\overline{p_{NI}} = 1$. Both cases in **C)** are therefore associated with solutions for $\overline{q_H}$ and $\overline{q_C}$ that are outside of the space of the game, converging to 1 and 0, respectively, being thus non-stable.

When $\bar{\beta} = 0$, all individuals within the subpopulation of taxpayers are perceived as ghosts (they evade the entire amount of their income). In <u>**case A**</u>, when $\bar{\beta} = 0$ and $0 < \bar{\alpha} < 1$, the equilibrium values for taxpayers, $\overline{q_C}$ and $\overline{q_H}$ depend on $1 - \frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)} \geq 0$ starting from (*1.a*), therefore on

$\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)} \leq 1.$

In the population of taxpayers, the level of $\bar{\alpha}$ and n make the difference in determining a positive proportion of honest taxpayers in the population. Also in this case, we shall distinguish between $\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)} < 1$ and $\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)} = 1$. The first case, $\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)} < 1$, qualifies the long run equilibrium value for "honest" $\overline{q_H}$ as stricly positive. This requires $\tau C < (\tau + \bar{\alpha}\varphi + n\varphi + n\tau)$ and means that honest individuals can be found in the population as long as the cost of audit activity remains strictly less than the amount of tax and fines recovered (at a rate $\bar{\alpha}$), plus the opportunity costs associated with the circumstance of missing the right target in inspections. The second case mentioned, $\frac{\tau C}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)} = 1$, qualifyes $\overline{q_H} = 0$, $\overline{q_C} = 1$, where it is worthy for taxpayers to be cheating (probability of honest among the population tends to zero). This requires $\tau C = (\tau + \bar{\alpha}\varphi + n\varphi + n\tau)$ or $\bar{\alpha} = \frac{\tau(C-n-1)}{\varphi} - n$. Therefore, $\overline{q_H} = 0$ as long as $\bar{\alpha} + n = \frac{\tau(C-n-1)}{\varphi}$, thus as long as fiscal agencies' level of effort and the opportunity cost associated with missing the right target are equal to the ratio between overall costs associated with inspections and the amount of penalty applied once a cheater is caught.

As for tax agencies, we can see that $\overline{p_I} > 0$ as long as $\tau > 0$ (tax rate is higher than zero, which is always verified), as the denominator is always positive, for each value of $0 < \bar{\alpha} < 1$, τ, φ and m. Moreover, $\overline{p_I} = \overline{p_{NI}}$ when $\tau = \frac{\bar{\alpha}}{(1+\bar{\alpha}+\bar{\alpha}\varphi-\varphi+m\tau)}$, whereas $\overline{p_I} > \overline{p_{NI}}$ as $\tau > \frac{\bar{\alpha}}{(1+\bar{\alpha}+\bar{\alpha}\varphi-\varphi+m\tau)}$.

Inspecting activities on tax declarations are normally carried out by fiscal authorities regardless of whether the amount recovered from inspections is higher or lower than their costs. Fiscal controls are in fact aimed at both pursuing an illegal behaviour and recovering the amount due by the cheating taxpayers. In this model, as in the one presented in Chapter 2, we focus on the case where fiscal agencies are mainly concentrated on the activity of recovering the amount of taxes evaded, also in light of education of cheating taxpayers.

Looking then at the case where the population is formed by ghosts and fiscal authorities put all effort in inspecting activities (*ex ante* $\bar{\beta}$ is equal to 0 and $\bar{\alpha}$ tends to 1), **case (D)** in table II above, $\overline{q_H}$ and $\overline{q_C}$ depend on the cost of control, tax and penalty rate, as well as on n. In this case, $\overline{q_H}$ is positive as long as $\tau C < (\varphi + \tau)(1 + n)$, which is verified for all values of $n > 0$, $\varphi > 0$ and $\tau > 0$. Moreover, in this case $\overline{q_H} > \overline{q_C}$ only when $C < \frac{(\varphi+\tau)(1+n)}{2\tau}$.

The equilibrium proportions of inspecting and non inspecting fiscal agencies, $\overline{p_I}$ and $\overline{p_{NI}}$, depend only on the tax rate and on the taxpayers' cost associated with inspections m. In case $\bar{\beta}$ tends to 0 and $\bar{\alpha}$ tends to 1 *ex ante*, $\overline{p_I} \gtreqless \overline{p_{NI}}$ depends on whether $\tau \gtreqless \frac{1}{2+m}$, therefore on the relation between tax rate and costs faced by taxpayers when subject to inspections.

2.2 Simulations

By summing up the results for the game in the long run, the level of effort put in inspections by fiscal authorities affects the equilibrium proportion of honest individuals, as well as taxpayers' perception of inspection costs faced by fiscal authorities and their opportunity costs when missing their right target. As long as the average effort of inspections is perceived as higher than the ratio between costs and amount recovered plus the amount of taxes and fines unrecovered, including opportunity costs, the equilibrium solution will lead to a positive proportion of honest individuals within the subpopulation of taxpayers, $\overline{q_H} > 0$.

Higher tax rates induce a higher proportion of both cheating taxpayers, and consequently, of inspecting agencies, whereas on the contrary, higher penalty rates induce a higher proportion of compliant taxpayers, and therefore of non inspecting fiscal agencies. Higher levels of effort in enforcement and inspecting activities induce both a decrease in the proportion of cheating individuals within the subpopulation of taxpayers through perception, which in turn is associated with an indirect effect on the proportion of inspecting fiscal agencies, and a decrease in the proportion of inspecting agencies (direct effect on the proportion of inspecting fiscal agencies) due to a better allocation of resources. Effort put in inspections and enforcement is therefore crucial, together with tax and penalty rate, to enhance compliance.

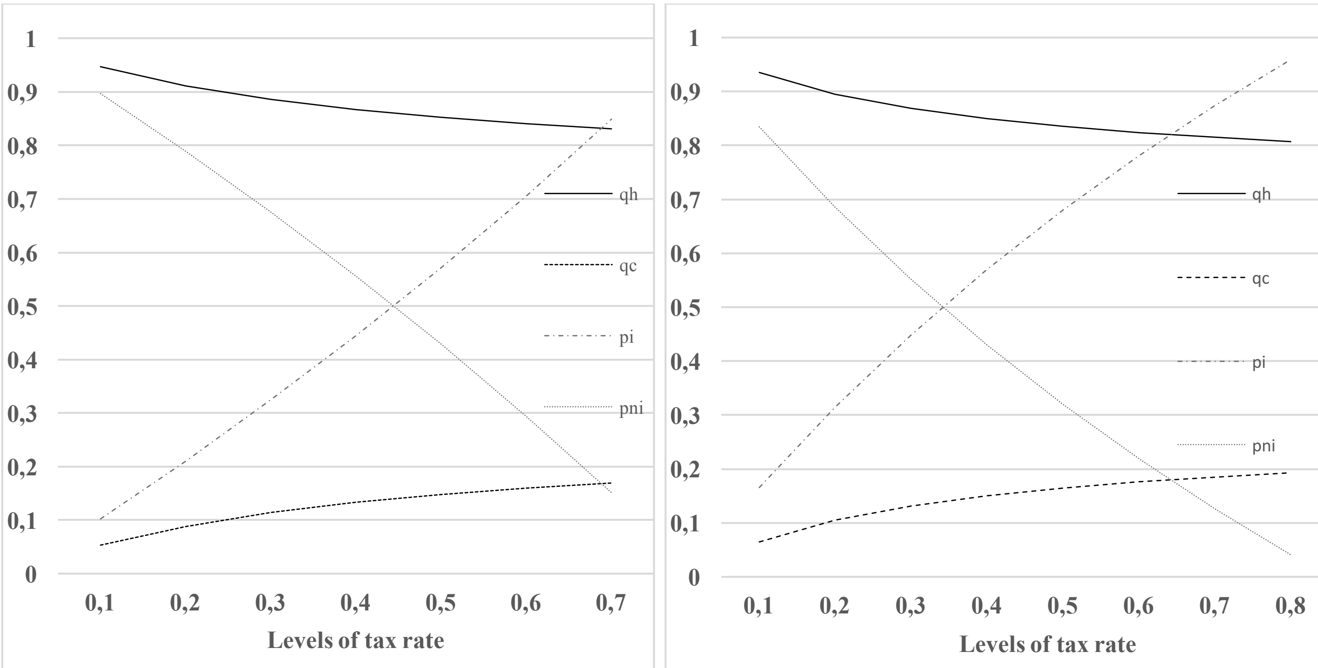
Moreover, taxpayers' perception of higher fiscal agencies' opportunity costs associated with the circumstance of non inspecting a cheating individual increases the equilibrium proportion of honest taxpayers, *ceteris paribus*, while fiscal agencies' perception of higher costs faced by taxpayers when inspected increases the equilibrium proportion of inspecting individuals.

As we recall from the previous chapter 2, Taylor and Jonker (1978), and Hines (1980), proved the local stability and asymptotically stability of the ESS equilibrium with replicator dynamics, under certain conditions, one of them being the population being formed by a very large number of individuals. Thus, both the solutions presented above are locally stable, meaning that, starting from each point in their neighbourhood, the solution after a high number of interactions will always end at that point. Additionally, the asymptotic stability allows the solutions above to be attractive also from starting points that do not fall in the neighbourhood of the ESS equilibrium itself.

An evolutionary stable state (ESS), as already described in the previous chapters 1 and 2, is a set of solutions $x^* \in X$ such that for each set $y \in X$, $(y - x^*)'F(x^*) < 0$. This condition implies that an ESS is a Nash equilibrium (Sandholm, 2010).

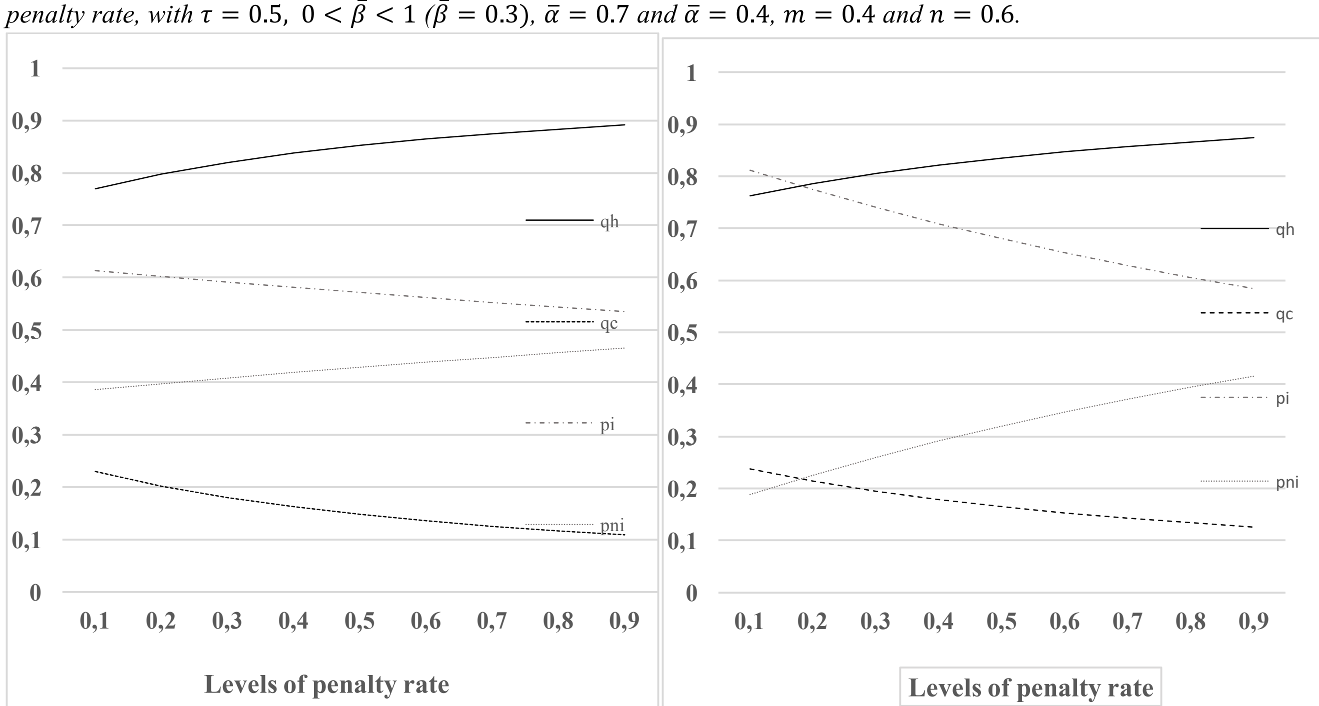
Charts 1.a-1.b and 2.a-2.b below show the different simulations of equilibrium solutions for different levels of tax rate and penalty rate, in case where $0 < \bar{\beta} < 1$. Charts 1.a, 1.b and 2.a and 2.b in Annex I, Section III, represent the simulations for the case $\bar{\beta} = 0$.

Chart 1.a and 1.b: distribution of characters within the subpopulations of taxpayers and fiscal agencies for different levels of tax rate, with $\varphi = 0.5 \;\; 0 < \bar{\beta} < 1$ ($\bar{\beta} = 0.3$), $\bar{\alpha} = 0.7$ and $\bar{\alpha} = 0.4$, $m = 0.4$ and $n = 0.6$



Distribution of characters within the subpopulation of tax agencies and taxpayers , for different levels of tax rate, **with $\bar{\alpha} = 0.7$ (left) and $\bar{\alpha} = 0.4$ (right).**

Chart 2.a and 2.b: distribution of characters within the subpopulations of taxpayers and fiscal agencies for different levels of penalty rate, with $\tau = 0.5,\; 0 < \bar{\beta} < 1$ ($\bar{\beta} = 0.3$), $\bar{\alpha} = 0.7$ and $\bar{\alpha} = 0.4$, $m = 0.4$ and $n = 0.6$.



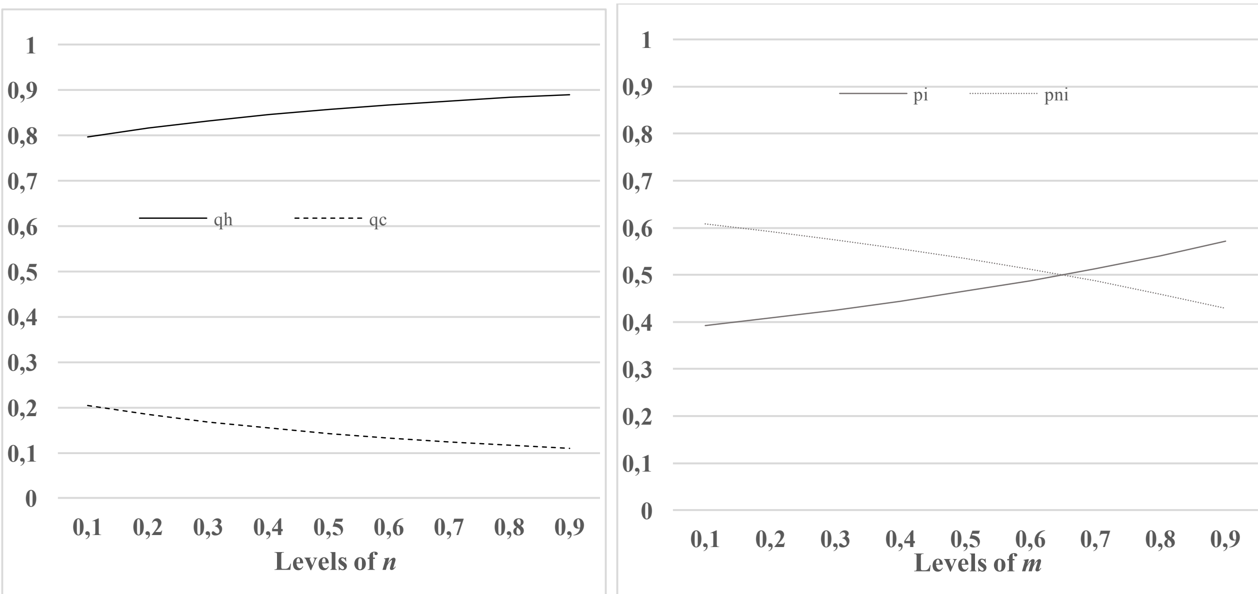
Distribution of characters within the subpopulation of tax agencies and taxpayers, for different levels of penalty rate, **with $\bar{\alpha} = 0.7$ (left) and $\bar{\alpha} = 0.4$ (right).**

As we can see in figures 1 and 2, when $0 < \overline{\beta} < 1$ and specifically $\overline{\beta} = 0.3$, the proportion of honest individuals is higher for higher levels of penalty rate, and lower for higher levels of tax rate. By contrast, the equilibrium proportion of cheating taxpayers increases for higher levels of tax rate, and decreases for higher levels of penalty rate, for both levels of $\overline{\alpha} = 0.7$ and $\overline{\alpha} = 0.4$. Moreover, for all levels of $\overline{\alpha}$, the proportion of honest is always higher than the proportion of cheating individuals.

As regards the subpopulation of fiscal agencies, when $0 < \overline{\beta} < 1$ ($\overline{\beta} = 0.3$), figures above show that the equilibrium proportion of inspecting agencies is higher for high levels of tax rate and lower for high levels of penalty rate, for both high and low levels of $\bar{\alpha}$. Moreover, when tax rate is high (holding constant the penalty rate), the proportion of inspecting fiscal agencies is higher than the proportion of non inspecting agencies. By contrast, for lower levels of tax rate, and particularly for $\tau < 0.45$ (for $\overline{\alpha} = 0.7$) and $\tau < 0.35$ (for $\overline{\alpha} = 0.4$), the proportion of non-inspecting agencies is higher than the proportion of inspecting fiscal agencies. Moreover, for each level of penalty rate – holding constant tax rate – the proportion of inspecting fiscal agencies is higher than the proportion non non-inspecting ones.

Figures 3.a and 3.b below show different simulations of equilibrium solutions for $\overline{q_H}$ and $\overline{q_C}$, for different levels of fiscal agencies' opportunity costs perceived, and for $\overline{p_I}$ and $\overline{p_{NI}}$, for different levels of taxpayers' costs associated with the circumstance of being inspected.

Chart 3.a and 3.b: distribution of characters within the subpopulations of taxpayers and fiscal agencies for different levels of n *and* m *respectively, with* $\tau = 0.4, \varphi = 0.5$ $0 < \bar{\beta} < 1$ $(\bar{\beta} = 0.3)$, *and* $\bar{\alpha} = 0.7$



Distribution of characters within the subpopulation of tax agencies (right) and taxpayers (left), for different levels of penalty rate, **with** $\bar{\alpha} = 0.7$.

The proportion of honest taxpayers increases as *n* increases, thus for higher opportunity costs, as shown in the left-hand side graph 3.a, whereas the proportion of cheating individuals decreases as the level of opportunity cost increases. Looking at the fiscal agencies' subpopulation, we see that the proportion of inspecting agencies increases as the inspection costs for taxpayers increase.

The table below summarizes the considerations above about what happens to the equilibrium solutions for both taxpayers and fiscal agencies subpopulation for different levels of tax and penalty rates, m and n.

Table III: summary of the results of simulations performed on the model results for $0 < \bar{\beta} < 1$

	Levels of τ, holding $\varphi = 0.5$ (constant)	Levels of φ, holding $\tau = 0.5$ (constant)	$\overline{\alpha}$=0.7	$\overline{\alpha}$=0.4	Levels of m	Levels of n
$\overline{q_H}$	Decreasing (for both low and high levels of $\overline{\alpha}$)	Increasing (for both low and high levels of $\overline{\alpha}$)	$\overline{q_H} > \overline{q_C}$	$\overline{q_H} > \overline{q_C}$	-	Increasing
$\overline{q_C}$	Increasing (for both low and high levels of $\overline{\alpha}$)	Decreasing (for both low and high levels of $\overline{\alpha}$)			-	Decreasing
$\overline{p_I}$	Increasing $\overline{p_I} > \overline{p_{NI}}$ for high levels of τ	Decreasing	–	–	Increasing $\overline{p_I} > \overline{p_{NI}}$ for high levels of m	-
$\overline{p_{NI}}$	Decreasing	Increasing	–	–	Decreasing	-

3. Concluding remarks

In this chapter, we presented an evolutionary game similar to the one presented in the previous Chapter 2. We introduced an opportunity cost faced by non inspecting fiscal agencies, under the circumstance of being matched with a cheating taxpayer, which represents a cost associated with the circumstance of “missing the right target” in inspections. We also introduced a cost faced by taxpayers when they are subject to fiscal authorities’ inspections.

Also for this model, we have shown the presence of a long run equilibrium where the proportion of characters chosen by the members of each subpopulation does not change over time. Particularly, we have shown that in equilibrium the subpopulation of taxpayers is characterized by the presence of honest individuals as long as the costs of inspections are less than the amount of taxes and fines recovered from inspecting activities, plus the potentially unrecovered taxes and fines when fiscal agencies do not inspect a cheating taxpayer.

As this equilibrium approximates the ESS, it is both locally and asymptotically stable, and it is reached regardless of whether the starting state is close or far from it. When one or more parameters are changed, another stable equilibrium is reached, with a different distribution of characters within the population.

Also for this model, higher tax rates lead to a higher proportion of cheating individuals (which in turn, can hide a certain proportion or their entire income), but also a higher proportion of inspecting tax agencies with respect to non-inspecting ones.

Higher penalty rates are instead associated with a higher proportion of honest individuals with respect to the cheating taxpayers, and with a lower proportion of inspecting fiscal agencies. As lower costs of controls produce a higher proportion of honest with respect to cheating, it seems the effect of penalty and the perception of penalty leads to an adjustment in taxpayers' behaviour towards honesty.

Higher opportunity costs faced by fiscal agencies when missing their right target in inspection are associated with a higher proportion of honest individuals. As taxpayers perceive that fiscal agencies' loss is higher when missing the right target in inspections (and particularly, when choosing not to inspect a cheating individual), they tend to choose the character "honest", and therefore the proportion of honest taxpayers increases, while the proportion of cheating individuals decreases. Thus, taxpayers' perception of fiscal authorities' aggressive strategy of inspection is also a driver of compliance.

Higher inspection costs faced by taxpayers once subject to inspections are associated with a higher proportion of inspecting and a lower proportion of non inspecting tax agencies. As an increase in the cost of being inspected leads to a decrease in the inspected taxpayers' payoff in (2) and (4), the decrease in payoff is lower in case of cheating individuals with respect to honest individuals. An increase in the cost faced by taxpayers when subject to inspections – by decreasing taxpayers' payoffs with a higher magnitude for honest than for cheating – induces individuals to reconsider their strategy and to choose the character “cheating”. As a consequence, when fiscal agencies perceive an increase in m, they tend to increase their auditing activities, as they expect taxpayers would choose the strategy “cheating” over “honest” when m increases.

Moreover, as long as individuals perceive that costs of controls are lower than the amount of revenues and penalty recovered through inspections, plus the amount potentially unrecovered when non-inspecting a cheating, they choose to play the strategy honest, as they know fiscal agencies have a strong cost-benefit imbalance towards inspecting activities. In other words, in this case, inspecting activities are economically reasonable for tax authorities. As the cost of controls become equal to the amount of taxes due recovered through inspections, although there is still a proportion of inspecting tax agencies, the latter are in lower number, and some tax agencies would decide to become non inspecting.

In conclusion, costs of detection, perception of penalty, level of effort and taxpayers’ perception of the opportunity costs faced by fiscal agencies when non-inspecting a cheating individual, are important parameters in explaining the distribution of honest and cheating individuals within the subpopulation of taxpayers, as well as tax rates, with the perception of the imbalance between the costs of inspecting activities and the amount of taxes due recovered through them being the main driver of adjustments in individuals’ behaviour, both with respect to the subpopulation of taxpayers and of fiscal agencies.

The next Chapter 4 will present an experiment aimed at verifying the role played by learning from past experience on individuals’ compliance level, in the context of a multiple-stage choice where individuals are randomly inspected with a fixed probability of inspection.

Chapter 4: Learning to evade

1. Introduction

This chapter presents an experiment aimed at verifying the role of learning in individuals' tax evasion decision making process. Particularly, the experiment simplifies the structure of the models presented in Chapters 2 and 3, where taxpayers choose how much of their income to declare for taxation under the condition that fiscal authorities' behaviour consists in a fixed probability of inspection equal to 0.5.

Participants were subject to a 45 rounds experiment, where they had to decide how much of their income to declare for taxation, after having carried out a risk aversion test. They were divided into two groups, each subject to a different set of combinations of tax and penalty rate. In total, participants in each group were subject to 5 different combinations of tax and penalty rate, 9 rounds each. After each 9 rounds of the experiment, the combination of tax and penalty rate (which we will call also the "scenario" in the following pages) changed.

The experiment focuses on verifying three features outlined in the previous chapters. First, it is aimed at verifying whether, *ceteris paribus*, individuals' choice of being honest or cheating varies from round 1 to round 9 of each combination of tax and penalty rate, as a result of learning occurred at the previous round. Second, it investigates whether and how, when there is a change in scenario, and a new combination of tax and penalty rate is shown to participants, the latter learn to adapt to the new scenario, and use the information gained during the previous rounds (related to the previous combination of tax and penalty rate) to choose whether to evade or not, and how much to evade. Finally, it analyses whether an increase in tax rate is related to a higher proportion of cheating individuals and a lower proportion of honest taxpayers, whereas a higher penalty rate is associated with a higher proportion of honest taxpayers and a lower proportion of cheating taxpayers.

Tax evasion behaviour has been extensively treated in economic literature both from a theoretic and experimental perspective. Particularly, starting from the pioneering contribution by Allingham and Sandmo (1972), the main research question addressing the topic was related to the effect of tax rate, penalty rate and probability of detection on taxpayers' behaviour. In this respect, not only a slight difference between theoretical and empirical studies arises, but theoretical literature itself did not find unique answers.

In fact, looking at the effect of tax rate on tax declaration decision, both theoretic and empirical literature highlight two different and contrasting forces. On one hand, Yitzhaki (1974) points out that an increase in tax rate might have a negative effect on the tax declaration decision, as high tax rates affect positively absolute risk aversion. Conversely, according to the Expected Utility models, higher tax rates make tax evasion convenient for individuals. Experimental literature supports both these sides of the story, since income effect and substitution effect may be overlapping in empirical studies on tax declaration decision. In fact, a wide part of experimental literature supports the negative impact of high tax rates on the level of compliance, both within multi-period lab experiments and within field-experiments. Particularly, in a multi-period public good game lab experiment, Alm, Jackson, and McKee (1992) highlight the negative impact of high tax rates on tax compliance decision, whereas indicating the increase in the compliance level as auditing activities and level of income increase.

Moreover, in a lab experimental setting using University students, Alm, Sanchez, and De Juan (1995) find an increase in the compliance levels as a response to an increase in both audit rate and fine.

Experimental literature also investigated how different magnitudes in tax rates affect compliance by comparing individuals' different sources of income. In this respect, Boylan and Sprinkle (2001), by comparing the situation where participants were allowed to earn their income with the one characterized by an effortless endowment within a lab experiment, find that participants directly endowed by experimenters were less compliant in case of a tax rate increase, whereas in the scenario where the income is earned with effort, an increase in tax rate produces an increase in the compliance behaviour. Effort in gaining an endowment or income therefore plays a role in individuals' compliance decision in case of a change in tax policies, as well audit probabilities, as supported empirically. Therefore, it seems that detection and punishment, especially if investigated together with moral suasion, may play an important role in designing a policy strategy against evasion.

Using an econometric approach based on a dataset of the Internal Revenue Service, Ali, Cecil, and Knoblett (2001) explore the effects of penalty rates and audit probabilities on individuals' compliance behaviour. Their findings suggest that the effectiveness of these policy tools slightly depends on income level, and the magnitude of the effects is higher for high-income earners, at a decreasing rate. Interestingly, once looking at the increases in the marginal tax rate, individuals are less compliant as tax rate increases, and the effect is higher for high-income earners. In line with other studies on tax detection policy tools, Alm, McKee and Beck (1990) experimentally investigate the impact of amnesty on compliance in the long run, showing that compliance levels slightly decreases after amnesty, if no effective and massive inspections are carried out right after the amnesty. In this latter case, however, when high efforts are put into enforcement, amnesty may be a useful tool to increase the level of compliance.

Different findings have been highlighted by other empirical investigations, with respect to the relation between income earned and compliance. Fishlow and Friedman (1994) investigate the role of tax evasion and tax avoidance with respect to economic fluctuations, within the context of an inter-temporal model "à la Allingham-Sandmo", testing empirically their results with data from Argentina, Brazil and Chile. According to their findings, the level of tax compliance is positively correlated with income and expectations about future, meaning that as individuals expect a future improvement in the economic cycle and an increase in income, the level of compliance increases. This result, which is consistent with other developing countries, is rarely observable in many developed contexts, although an interesting case study is represented by Greece. Koumanakos, Theodoros, and Yorgos (2017) find a negative relationship between the GDP rate of growth and corporate tax avoidance, using a sample of macroeconomic data combined with firm level data of Greek firms. According to their results, tax avoidance increases during downturns.

In addition to survey studies and empirical econometric analyses, even lab experiments have been used to investigate the relation between the level of income and individuals' tax compliance. Park and Hyun (2003), using experimental data, find tax evasion decision to be substantially independent from the level of income or endowment.

On the other hand, tax evasion might play an important role when evaluating redistribution effects associated with public policy measures, as described by Frire-Seren and Panades (2013). The latter, by analytically studying the role played by tax evasion in the context of redistribution effects associated with progressive tax rates, highlight a decrease in the redistribution effect associated with progressive tax rates. Therefore, in settings characterized by a higher proportion of cheating taxpayers, redistribution effects associated with progressive tax rates might be completely displaced, as a result of the massive taxpayers' cheating behaviour.

All in all, empirical investigations have been slightly concentrated on the Allingham and Sandmo's setting, and broadly speaking, on the tax compliance decision, finding controversial, and not yet definitive results.

Kleven et al. (2011) find a high level of compliance in their sample, as in many modern tax systems, even in presence of low audit rates and fairly modest penalties. Particularly, by using a dataset originating from a field experiment conducted in 2007 and 2008, they find three main results, relevant in the context of this chapter. First, as tax evasion appears to be very low in case of third party reporting - which represented the highest proportion of subjects in the sample - whereas it becomes higher when looking at self-reporting cases, overall level of compliance appears to be high among participants, according to their results. Second, the positive effect of marginal tax rates on tax evasion is relevant for self-reported income. Third, according to their findings, individuals' memory about audits and of "*threat-of-audit* letters" has a significant effect on self-reported income.

Moreover, Dhami and Nowahiri (2007) underline two other relevant aspects when comparing theoretical paradigms and empirical studies. First, when assuming a positive expected return to tax evasion, all taxpayers should hide a certain proportion of income, whereas empirical evidence suggests a lower percentage of cheating taxpayers with respect to theoretical results. Second, and most importantly, beyond economic motivations, the effect of tax rate on compliance decision may be also shaped by interactions with fiscal authorities and with peers.

As regards the interactions with peers, individuals' perception of fairness, as well as peer pressures, are drivers for tax compliance, as underlined by Torgler (2007). In this respect, time consistency of fiscal auditing strategies and a clear representation of different audit scenarios to taxpayers appear to be crucial. In the context of fiscal inspections' credibility, investigations by Kastlunger, Kirchler, Mittone and Pitters (2009) by comparing the immediate reactions of participants to audits with their long-term tax plans within a lab experimental setting, highlight that inspections performed at an earlier stage were highly effective in increasing compliance levels compared to later stage audits. Their results suggest also a positive correlation between the level of tax evasion and timespan at which individuals are not inspected.

Moreover, experimental results by Mittone (2006) stress the importance of "context" and psychological pressures on taxpayers' compliance decisions, suggesting their role as potential tools for detection. The term "context" is widely used by literature to indicate the role played by social interactions in individuals' decision making process. Manski (2000), after a review of the key terminologies and fields of application of both theory and empirical analyses on social interactions in economics, highlights the role of experimental data in investigating social interactions. The latter might be in fact very important to address tax compliance, although empirical analyses show that their role as drivers of economic development and decisions is still controversial. One direct consequence of social interactions is the peers' moral suasion with respect to actions considered not morally acceptable (or, in other words, less acceptable).

With data gained from a controlled field experiment in Switzerland, Torgler (2004) tests the effects of moral suasion on tax compliance. Results suggest moral suasion doesn't act as driver in taxpayers' compliance behaviour.

Significant effects can be nevertheless observed for tax payments. Alm, Blomqvist and McKee (2017), within an experimental setting, stress the importance of peer effects on the level of compliance. According to their results, tax compliance decision is affected by peers through interactions that may take place between individuals within the same social context, although they find a weak correlation between peer interactions and the level of compliance (the amount of endowment declared).

Furthermore, Alm, Bruner and McKee (2016) examine individuals' exchanges of thoughts and messages to their peers regarding their own audit outcome and their own compliance behaviour, indicating that – statistically speaking – a large part of individuals' own audit outcomes and compliance behaviours' messages are accurate. Nonetheless, their results highlight that many individuals appear to be systematically dishonest about being audited, pointing out also a high correlation between individuals' audit outcomes and their compliance behaviours (thus, cheating individuals subject to audits were more likely to tell the truth in their messages with respect to cheating taxpayers not subject to auditing activities).

Battiston and Gamba (2016) investigate the role of social pressures on individuals' attitudes towards compliance, within a field experiment conducted among bakeries in Italy. Peers' social pressure was represented in the experiment by the explicit request of receipts by customers. Their results suggest a positive effect of social moralization on the level of compliance, as well as the persistence of compliance decision over time, as a result of customers' receipt requests.

Geeroms and Wilmots (1985) explore the interdependency between taxpayers' behaviour when other individuals (i.e. peers) are believed to evade, revealing the importance of imitation in tax evasion behaviour and moral effects.

Peer effects have also been studied in light of individual perception of economic status compared to other members of a community, and the support perceived from the latter. Vogel (1974) measures the effect of perception of the improvement in economic status on individuals' compliance behaviour with a survey on Swedish taxpayers' attitudes, as well as the role of "group support" on attitudes towards tax compliance decision. Results suggest two interesting outcomes worth highlighting: first, peer effects matter, as individuals' perception on their reference group's orientation towards tax evasion has effects on individuals' decisions. Second, tax avoidance practices are unequally distributed, as they require a certain level of education, which is not equally distributed within a population. As a result, tax compliance levels tend to decrease, as economic and social status improves.

Finally, another relevant aspect emerging from recent studies on individuals' tax compliance decision is the effect of learning, not only from peers, but most importantly from a memory formed through previous experience. Broadly speaking, learning has been widely used in experimental economics, to understand a variety of theoretical axioms in games, from the stability of mixed strategies equilibria over time (Erev and Roth, 1998) to individuals' initial predisposition towards certain strategies' updating rules (Camerer and Ho, 1999).

However, recent studies show also a certain interest in the learning process while investigating tax compliance choice. As Soliman, Jones and Cullis (2014) point out, the dynamic interaction between audits performed and penalty rates might prove quite useful in building a memory in taxpayers, and learning from memory represents a potential tool in addressing tax compliance, as also their experimental data suggest. Accordingly, O'Doherty (2014), by comparing different audit strategies, suggested that predictive analytics' strategies – used in many OECD countries - are highly effective in reducing the tax gap, and that effectiveness is highly related to the taxpayers' learning process estimation by fiscal authorities.

In the experiment presented in this paper, the evolutionary setting of the models presented in both Chapters 2 and 3 has been simplified to a repeated individual's lottery choice, where the evolution of taxpayers' behaviour over time is studied, when exposed to a fiscal authority exogenously set up to behave as a 50-50 lottery. Inspection probability is in fact fixed at 0.5, and known to participants, thus the experimental setting assumes randomly performed inspections after each round of the experiment, characterized by the 50%-50% proportions of inspected and non inspected taxpayers.

The experimental design, where taxpayers' decisions are studied in a context of individual choice under different tax and penalty rate scenarios, and under the fixed auditing strategy of fiscal authorities, allowed to simplify the dynamic games presented in both the models of Chapters 2 and 3, where instead the '*population of players'* is divided into two sub-populations, taxpayers and tax agencies, and individuals in the two sub-populations are matched randomly pairwise.

In the models presented in Chapters 2 and 3, over time, the distribution of characters within the taxpayers' and tax agencies' subpopulations is thus adjusted on the basis of overall experience, and at a population level there is a reshape towards a new distribution of characters after all the interactions for each time *t*. The spread of a character depends therefore on how the subpopulation as a whole reacts to the exposure to the behaviour carried out by the other subpopulation, in terms of speed of reproduction.

In this context, we simplify the models presented in Chapters 2 and 3, to allow the behaviour of fiscal authorities to be fixed, and taken as given. We therefore simplify the payoffs presented in Chapter 2, by eliminating the level of effort and letting payoffs depend only on tax and penalty rate, proportion of income declared and cost of inspections. The following payoffs are therefore relevant for the purpose of this chapter.

By using the same notation and symbols as in the previous Chapters 2 and 3, we see in (1) and (2) that when an inspecting agency is matched with a honest taxpayer, the inspecting agency payoff $\pi_A(s_A^i, s_T^h)$ is given by tax revenue on the entire income (τY) less the costs of control ($C\tau Y$), whereas the taxpayer payoff $\pi_T(s_A^i, s_T^h)$ is given by his/her disposable income after taxes $(1-\tau)Y$.

1) $\pi_A(s_A^i, s_T^h) = \tau Y - C\tau Y$

2) $\pi_T(s_A^i, s_T^h) = (1-\tau)Y$

When an inspecting agency is matched with a cheating taxpayer, the payoff related to the tax agency $\pi_A(s_A^i, s_T^c)$ is given by the recovered amount of taxes applied on the true income earned τY and fines applied on the evaded income, $\varphi(1-\beta)Y$, less the tax agency's costs of detection, $C\tau Y$ as in equation 3). The cheating taxpayer's payoff, $\pi_T(s_A^i, s_T^c)$, is given by the disposable income after taxes, $(1-\tau)Y$, less the fine on the evaded part of income, $\varphi(1-\beta)Y$, as indicated in equation 4).

3) $\pi_A(s_A^i, s_T^c) = \left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right] - C\tau Y$

4) $\pi_T(s_A^i, s_T^c) = [(1-\tau)Y \underbrace{- \varphi(1-\beta)Y}]$

Equations 5) and 6) represent the circumstance when a non-inspecting fiscal agency is matched with a honest taxpayer. The latter will obtain his/her disposable income after taxes $(1-\tau)Y$, as in equation 6), whereas the fiscal agency will obtain the amount of taxes due, equation 5).

5) $\pi_A(s_A^{ni}, s_T^h) = \tau Y$

6) $\pi_T(s_A^{ni}, s_T^h) = (1-\tau)Y$

Finally, when a non-inspecting fiscal agency is matched with a cheating taxpayer, the latter will obtain his/her disposable income Y after taxation on the under-declared income τβY, as shown in 8), whereas fiscal agency's payoff $\pi_A(s_A^{ni}, s_T^c)$ is the revenue coming from the under-declared income, τβY, less the amount of taxes undiscovered, taken as a sort of opportunity costs for missed inspection, as in equation 8) .

7) $\pi_A(s_A^{ni}, s_T^c) = \underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}$

8) $\pi_T(s_A^{ni}, s_T^c) = Y(1 - \underbrace{\tau\beta})$

Using the replicator equations dynamic approach presented in Chapters 2 and 3, we find the ESS equilibrium solutions for the proportion of honest, cheating taxpayers, and for inspecting, non-inspecting fiscal agencies:

$$\overline{q_h} = 1 - \frac{\tau C}{(\varphi + 2\tau)(1 - \bar{\beta})}$$

$$\overline{q_C} = \frac{\tau C}{(\varphi + 2\tau)(1 - \bar{\beta})}$$

$$\overline{p_I} = \frac{\tau}{(\varphi + \tau)}$$

$$\overline{p_{NI}} = \frac{\varphi}{(\varphi + \tau)}$$

As we can see, $\overline{q_h}$ and $\overline{q_C}$, representing the equilibrium solutions of the system of four replicator equations presented in Chapters 2 and 3 for honest and cheating taxpayers, depend on tax and penalty rate, proportion of income declared and cost of inspections. The solutions for the subpopulation of taxpayers $\overline{p_I}$ and $\overline{p_{NI}}$ depend only on tax and penalty rate, and therefore are fixed, as long as τ and φ don't change.

Table 1 below presents the direction of the effect of *ceteris paribus* changes of parameters on the equilibrium results.

Table 1: Direction of the effects on the equilibrium values of the ceteris paribus change of the relevant parameters

	Parameters affecting the equilibrium results and direction of the effects on the equilibrium values of the ceteris paribus change of the relevant parameter		
Equilibrium values	$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\boldsymbol{C}$
$\overline{q_H}$	>0	<0	<0
$\overline{q_C}$	<0	>0	>0
$\overline{p_I}$	<0	>0	-
$\overline{p_{NI}}$	>0	<0	-

Results suggest – in line with the findings presented for the previous chapters - that an increase in tax rate, *ceteris paribus*, leads to a decrease in the equilibrium proportion of honest individuals and to an increase in the proportion of cheating. When looking at the fiscal agencies' subpopulation, an increase in tax rate, *ceteris paribus*, leads to a decrease in the equilibrium proportion of inspecting agencies, while decreasing the proportion of non inspecting fiscal agencies.

Considering an increase in penalty rate, *ceteris paribus*, the equilibrium proportion of honest individuals increases, whereas the equilibrium proportion of cheating taxpayers decreases. Within the subpopulation of fiscal agencies, moreover, an increase in penalty rate is associated with a higher proportion of non inspecting agencies, and a lower proportion of inspecting fiscal agencies.

Finally, we see that costs of inspection and average proportion of income declared affect only the proportion of honest and cheating taxpayers. Particularly, an increase in the costs of inspection – *ceteris paribus* – leads to a decrease in the proportion of honest taxpayers, and to an increase in the proportion of cheating individuals. An increase in the average proportion of income declared is associated with a decrease in the proportion of honest taxpayers, and with an increase in the proportion of cheating individuals.

The experimental design focuses only on the individuals' choice and learning, thus only on taxpayers' behaviour, without considering fiscal agencies' behaviour, but rather treating them as a 50-50 chance of performing an audit. The reasoning behind the choice of focusing only on taxpayers' decision making and learning is mainly due to the need of a clear set up of the experimental hypotheses presented in the following pages. A simplified structure of the experiment - focusing only on taxpayers' learning process – allows to better control the key variables, which would not be possible if also fiscal agencies were introduced within the experimental design.

The experiment consisted of 45 rounds, 5 scenarios for each group of participants such that after each 9 rounds, participants were subject to a different scenario, representing a combination of tax and penalty rate. Participants were divided in two groups, and each group was subject to a different set of 5 scenarios (combinations of tax and penalty rate). Therefore, 5 combinations of tax and penalty rate were proposed to each group of participants, and for each combination of tax and penalty rate, participants played 9 rounds.

The expected results of the experiment might be summarized as follows. For each combination of tax and penalty rate, individuals' propensity to be honest at round 9 of each scenario (combination of tax and penalty rate) will vary with respect to the first round of each scenario, as individuals learn to become honest or cheating from the repeated exposure to the probability of being audited by fiscal authorities. Moreover, from one scenario to another, we expect also a learning effect related to the circumstance of being exposed to different combinations of tax and penalty rate, and consequently, a different individuals' propensity towards honesty. Finally, we expect that the individuals' propensity towards honesty is decreasing as tax rate increases, and increasing as penalty rate increases.

This chapter is organised as follows: Section 2 presents the experimental design, Section 3 describes the procedures followed during the experimental sessions. Section 4 presents the results of the experiment, whereas Section 5 contains the conclusions.

2. Tax compliance as a result of learning within the lab: experimental design

We set up the experimental design, which adapts and simplifies the model presented in Section 1, to a 45 rounds individuals' repeated choice. Each participant received an endowment of 1000 euros at each round, and fiscal authorities auditing activities were exogenously set up, to behave as a 50-50 lottery.
During each round, participants had to decide how much to declare of their 1000 euros income to pay taxes, used to finance a public good, and specifically national health care expenditure. After each round, each participant was told whether it was inspected or not by fiscal agencies, with probability 0.5.
Participants were told that the Society where they lived was the entire community of students at the Faculty of Economics at the University "La Sapienza" – Rome. Participants were also clearly told that, at each round, they could choose to declare the entire amount of their endowment, a part of it, or nothing.
After each 9 rounds, another scenario was introduced, characterized by a modified tax rate, penalty rate, or both, as explained in Table 2 below.
Departing from the structure of the models presented in the previous Chapters 2 and 3 and in Section 1, the experimental design proposed focuses on the sub-population of taxpayers, and particularly on their choice to be honest, cheating or ghost under a probability of being subject to inspections equal to 0.5. As individuals were told that the probability of being inspected was 0.5, the game between fiscal agencies and taxpayers is represented here as situation where fiscal agencies do not adapt their behaviour over

time or to changes of scenarios, but instead fiscal agencies' auditing strategy remains fixed (the fixed probability of inspection at 0.5).

The main feature and novelty of this experiment is that individuals' decision making processes are monitored through multiple rounds, for a measurement of the effects of learning from experience on taxpayers' choice of being honest, cheating or ghost. Multiple rounds are aimed at verifying whether a memory is formed at individual level as a result of a repeated multi-period relation between taxpayers and fiscal authorities under the same auditing rule, as represented in the evolutionary game of paragraph 1.

In the experimental setting, participants were told about the actual probability of being inspected, as well as about their endowment and tax and penalty rate applied. Participants' knowledge and awareness of the actual inspecting activities allowed to better simulate the situation where individuals' perception of inspections is *ex-ante* well known and included in individuals' decision making, in an attempt to replicate what happens outside the lab in terms of perception of audits.

Further to the results presented above, and specifically in Table 1, we aim at testing the following hypotheses with this experiment:

Hypothesis 1: *ceteris paribus*, the proportion of honest and cheating individuals, as well as individuals' propensity to be honest, varies from round 1 to round 9 for each combination of tax and penalty rate, as a result of learning from being exposed to fiscal agencies' auditing activities occurred at the previous rounds. We therefore expect that the distribution of characters among taxpayers changes between round 1 and round 9 as a result of individuals' exposure to fiscal agencies' audits.

Hypothesis 2: when there is a change between a scenario (a combination of tax and penalty rate) and another, individuals learn to adapt to the new scenario, and use the information gained during the previous combination of tax and penalty rate to choose whether to evade or not, and how much to evade. As a result, when there is a change of scenario, we expect that the proportion of honest and cheating individuals changes, as well as individuals' propensity to be honest.

Hypothesis 3: scenarios where there is an increase in tax rate are related to a higher proportion of cheating individuals and a lower proportion of honest taxpayers, whereas scenarios characterized by a higher penalty rate are associated with a higher proportion of honest taxpayers and a lower proportion of cheating taxpayers, in line with the results presented above in Section 1.

The experiment was conducted entirely online through Qualtrics[43], and the platforms "Zoom" and "Google Meet", used by the Faculty of Economics at "University La Sapienza" for online lessons. Particularly, 106 students of the Faculty of Economics at University "La Sapienza" in Rome participated in the experiments in 5 sessions.

Individuals were equally divided in two groups (Group 1 and Group 2), with a well-balanced gender representation, as it can be noticed in table 2 below.

Table 2

Total number of participants	**106**
Female	50
Male	56

Before describing the experimental design, two specific features of the experiment should be mentioned. First, we presented each of the combinations of penalty and tax rate only once, without considering the order at which they were presented to participants, as the interest here, in line with the results of the

[43] A detailed description of how the experiment was set up in Qualtrics can be found in Annex II.

model presented in Section 1, and with the Hypotheses above, was related to the learning process behind the change in the proportion of honest, cheating and ghost participants, "for and within" each setting, therefore considering each setting proposed separately, and all settings proposed. Nonetheless, in Section 4 we present also a Directional Learning analysis to investigate the role of learning in light of mitigating order effects.

Moreover, when comparing both the average proportion of income declared across rounds and across scenarios, we should consider the fact that observations are not mutually independent. To address it, we will use a non-parametric Wilcoxon signed rank test on average session's proportion of income declared by participants.

At the beginning of the experiment, the experimenter explained basic instructions.

First, it was explained to participants the structure of the experiment, divided in two parts, with different endowments and questions asked. Moreover, the payment calculation was described, as extracted from the results obtained during all rounds. Payments, in the form of online book store vouchers, were delivered through email upon the submission of an ID number assigned to each participant during the experiment.

As represented in table 3 below, before the second part of the experiment consisting of 45 rounds, a risk aversion test took place. This first part of the experiment, which was identical for group 1 and group 2 individuals, consisted in a risk aversion test in line with most standard tests for risk aversion (Quattrone and Tversky, 1988; Levy, 1994). Participants were asked how much would they choose to invest of an endowment of 50 euros in a coin toss, considering that they would receive the amount invested multiplied by 2,5 if head came out, and 0 in case tails came out (see the Annex II for the full text of the experiment). After this first part, the second part started, with different instructions. Participants were told they were part of a community formed by all students of the Faculty of Economics at the University "La Sapienza", with an endowment of 1000 euros for each round, and they had to pay taxes to finance public expenditure on national healthcare. Experiment instructions contained also information about how taxes were calculated starting from the income declared, and about the fine applied in case cheating participants were subject to inspections. Participants were also told that controls would occur in 50% of cases. Before the experiment started, and for any change of scenario (in case of any change in the combination of tax and penalty rate), participants were subject to a test session for simulating different scenarios of income declared, and the consequent amount of disposable income remaining from taxes (and eventually penalties) in case they were inspected or not.

During the 45 rounds of the experiment, participants were asked to declare the amount of money subject to taxes. At each round, 50% of individuals were inspected and 50% were not inspected, as inspection activities were performed at random by the software, programmed to guarantee that at each round 50% of participants were inspected. After each round, participants were told whether they were subject to inspections and, if inspected and caught cheating, the disposable income after taxes and penalty.

As described in Table 3, the first 9 rounds, common to both Group 1 and Group 2, were characterized by a tax and penalty rate both equal to 45%. After the first 9 rounds, a message was shown to participants, to warn them that either the tax rate (for Group 1) or penalty rate (for Group 2) changed. Before the subsequent 9 rounds began - and in case of each change in the combination of tax and penalty rate - subjects were then given a test session, to check different scenarios depending on their declaration. After the completion of these 9 rounds, the subsequent 9 rounds started, with the same structure of instruction and test session as the previous ones.

Group 1 individuals where therefore subject to a change in tax rate after each 9 rounds, and a change in tax rate and penalty rate within the last 18 rounds. Group 2 individuals, by contrast, were subject to a change in penalty rate after each 9 rounds, except from the last 18 rounds, were both tax and penalty rate changed, at different levels with respect to Group 1 setting, as described in table 4 below.

Table 3: the experimental design

		Group 1	Group 2
Part 1		Risk aversion test	Risk aversion test
Instructions for Part 2			
Test session for rounds 1 to 9			
Part 2	Rounds 1 to 9	Tax rate: 45% Penalty rate: 45%	Tax rate: 45% Penalty rate: 45%
	Instructions for rounds 10 to 18		
	Test session for rounds 10 to 18		
	Rounds 10 to 18	Tax rate: 60% Penalty rate: 45%	Tax rate: 45% Penalty rate: 60%
	Instructions for rounds 19 to 27		
	Test session for rounds 19 to 27		
	Rounds 19-27	Tax rate: 25% Penalty rate: 45%	Tax rate: 45% Penalty rate: 25%
	Instructions for rounds 28 to 36		
	Test session for rounds 28 to 36		
	Rounds 28-36	Tax rate: 60% Penalty rate: 25%	Tax rate: 25% Penalty rate: 60%
	Instructions for rounds 37 to 45		
	Test session for rounds 37 to 45		
	Rounds 37-45	Tax rate: 25% Penalty rate: 25%	Tax rate: 60% Penalty rate: 60%
ID Assignation to each participant			

Table 4: combinations of tax rate and penalty rate displayed within the rounds of the experiment for Group 1 and Group2 participants

Round	**Combinations of tax and penalty rate for participants in Group 1**		**Combinations of tax and penalty rate for participants in Group 2**	
	Tax rate	**Penalty rate**	**Tax rate**	**Penalty rate**
Rounds 1 to 9	45%	45%	45%	45%
Rounds 10 to 18	60%	45%	45%	60%
Rounds 19-27	25%	45%	45%	25%

Rounds 28-36	60%	25%	25%	60%
Rounds 37-45	25%	25%	60%	60%

Finally, table 5 shows the overall number of participants allocated to each group within all the sessions of the experiment. As we can see, they were equally distributed.

Table 5: number of participants in two groups

Total number of participants	**106**
Group 1: tax rate change	53
Group 2: penalty rate change	53

A single, personalized link was generated and sent through the Qualtrics platform to each participant. During the experiment, students were all connected during each session, communicating with the experimenters through a chat-room. Students were allowed to ask the experimenters to repeat the instructions or part of them (and the experimenter could repeat them partially or totally) or in case of technical issues. No communication between participants was allowed. The full translation of the experiment from Italian can be found in the Annex II.

3. Procedures

Participants were recruited from the students of the courses of Public Finance and Economic Policy at the Faculty of Economics of the University "La Sapienza" in Rome. All students registered to the courses received an e-mail with an invitation to participate in the experiment, and the instructions to take part in the experiment. Within the e-mail, participants were invited to send in advance their contact details to the experimenters to be registered. Sessions were conducted before/after lectures of Public Finance, after which the registered participants were invited to remain connected to the virtual room, and a single, customized link was sent to each registered participant through Qualtrics. Before the experiment started, experimenters explained the instructions of the experiment and asked participants to take note of the personal ID number shown to each of them on the screen at the end, to get the payment upon completion. After all students received the link, they were told to begin, and the experiment started. Once finished, participants left the virtual room[44].

4. Results

4.1 Descriptive statistics[45]

In this subsection, we will first present descriptive statistics on the experimental results, with respect to the three hypotheses described above, to show the effects of learning over rounds and of changes in scenario on individuals' choice to be honest or cheating, from a descriptive point of view. We will then test the significance of the effects of tax and penalty rate, as well as of learning by using two types of tests.

First, the Kruskal and Wallis (KW) test is used to measure the effect of different combinations of tax and penalty rate on the proportion of honest and cheating individuals. The KW test has been used as observations did not follow the normal distribution, and therefore a parametric t-test or ANOVA test were not applicable. As the null hypothesis tested with the KW test represents the circumstance under which the mean ranks of two subsamples are the same, the test addresses whether two subsamples come from the same distribution. In this context, therefore, the null hypothesis represents the circumstance

[44] Section III in Annex II explains the details regarding the payment of participants.
[45] I wish to thank one of the referees for his helpful comments on this subsection.

where the proportion of honest, cheating and ghosts does not vary across scenarios, whereas the alternative to be tested represents the case in which a change in scenario affects the proportion of honest, cheating and ghost participants.

To complement the results of the KW test, as subsamples subject to comparison with the KW test – with respect to the different scenarios proposed - might not be mutually independent, and thus observations at individual level might be not independent across rounds but instead might be characterized by different distributions, we also used the non parametric Wilcoxon signed rank test on average observations at session level, and particularly, on the average proportion of income declared, to compare the within-subjects treatment effects, and the learning effects between round 1 and 9. As Wilcoxon signed rank approach compares pairs of observations within a sample, we use this approach to test: *(i)* whether the average income declared under the base scenario across all rounds is significantly different from the case of alternative scenarios proposed to participants, and *(ii)* whether the average income declared across scenarios is significantly different between round 1 and all other rounds played by participants, to test whether individuals learn from past experience. In both cases, the Wilcoxon signed rank test ranks absolute differences in the average proportion of income declared from the lowest to the highest, and then ranks are summed separately for positive and different differences. If the treatments and learning do not affect the outcome, the rank sums for positive and negative differences are equal, meaning that the average proportion of income declared (thus, average propensity towards honesty) is not affected by treatment and learning.

Starting with descriptive statistics, charts 1 and 2 below represent number of honest and cheating individuals, for each of the 9 rounds of each scenario proposed to Group 1 and Group 2 participants, respectively.

Looking at Group 1 participants in Chart 1 when both tax and penalty rate are equal to 0.45, we notice that the number of honest individuals is higher than the number of cheating participants in rounds 1 and

2, whereas it becomes lower than the number of cheating individuals in round 3. From round 6 to round 9, moreover, the number of honest participants increases, whereas the number of cheating participants decreases. All in all, in the first scenario when $\tau = \varphi = 0.45$, the number of honest and cheating individuals varies over the first 9 rounds, and in the last 4 rounds we see a decline in the number of participants choosing the strategy "cheating", and an increase in the number of honest individuals. By comparing the first scenario, where $\tau = \varphi = 0.45$, with the second one, where $\tau = 0.6$ and $\varphi = 0.45$, we see that in the latter scenario, the number of cheating individuals increases between round 10 and round 12, becoming higher than the number of honest participants, with a decrease between round 15 and round 16. For this combination of tax and penalty rate, the number of cheating individuals remains slightly higher than the number of honest individuals in several rounds, but in the last three rounds (16 to 18), the number of participants choosing to play "honest" becomes higher than the number of cheating ones.

Conversely, when $\tau = 0.25$ and $\varphi = 0.45$, the majority of individuals chooses to play the strategy "honest" in all rounds 19-27, with the exception of round 23. An interesting scenario is the fourth one proposed, where $\tau = 0.6$ and $\varphi = 0.25$. We see that, starting from round 30, the number of cheating individuals remains higher than the number of honest participants, although there is an increase in the number of honest individuals between rounds 33 and 35.

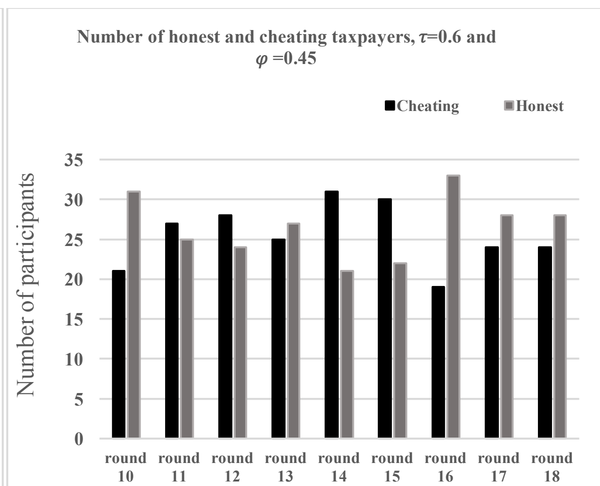
Finally, the majority of participants chose to be "honest" also in the last scenario, where both tax and penalty rate were 0.25.

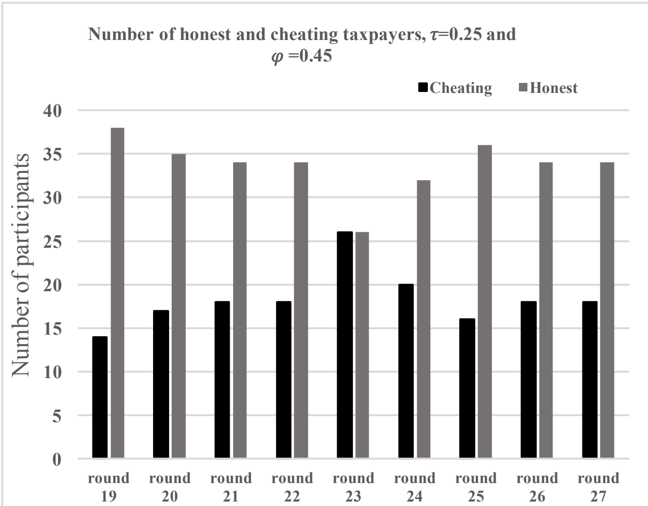
We see three main results for Group 1 individuals. First, the number of honest and cheating participants varies across rounds, for each scenario proposed, where all participants were subject to a 0.5 probability of being inspected. Second, the last rounds of each scenario show a clear pattern for both the number of honest and cheating individuals, meaning that the latter adjust their behaviour after several rounds where they experience the inspections carried out with the fixed probability at 0.5. By combining these findings, we see that the data seem to be in line Hypothesis 1, as we see a certain level of adjustment of individuals' behaviour to the tax authorities auditing activities.

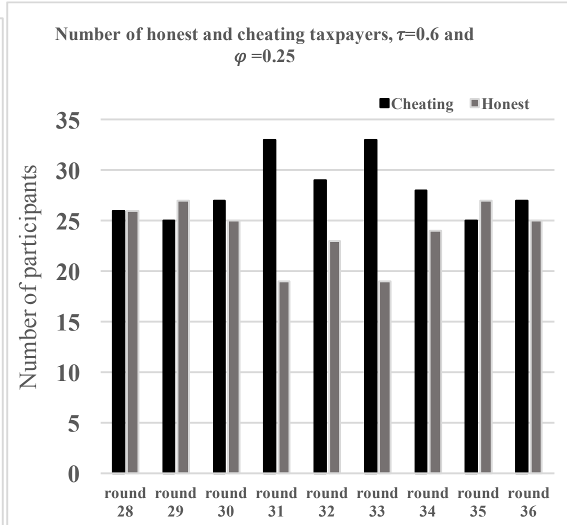
Third, we see that higher levels of tax rate are associated with an increase in the number of cheating individuals with respect to honest individuals, whereas lower levels of tax rate are associated with a decrease in the number of cheating participants, which appears to be coherent with Hypothesis 3.

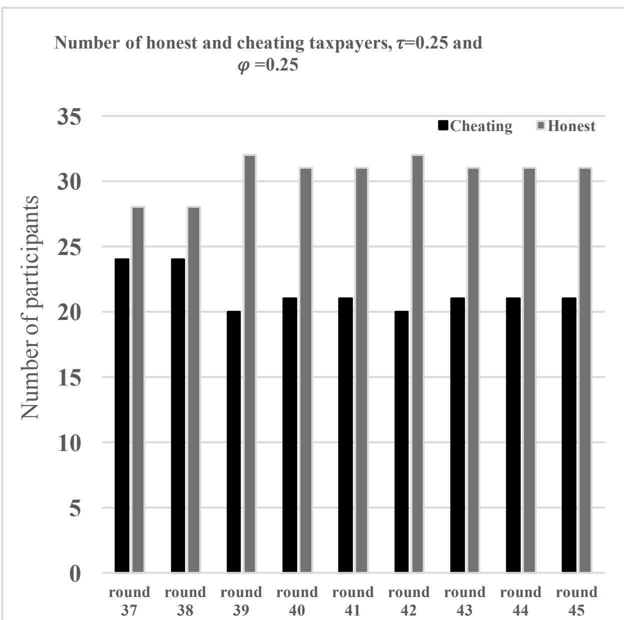
Lastly, even in case of an increase in tax rate, we see that individuals choose to play the strategy "honest" over the strategy "cheating" in the last rounds of each scenario.

Chart 1: Number of honest and cheating individuals for each of the 9 rounds of each combination of tax and penalty rate, Group1





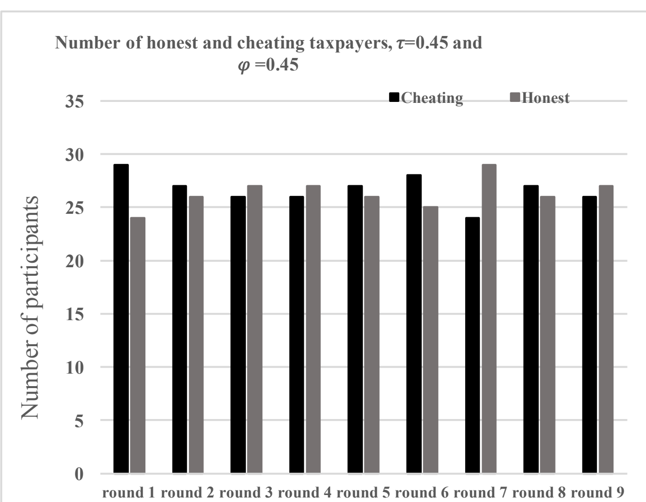


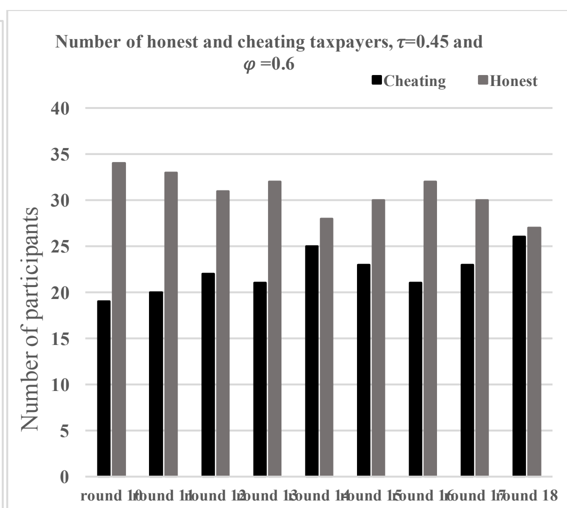


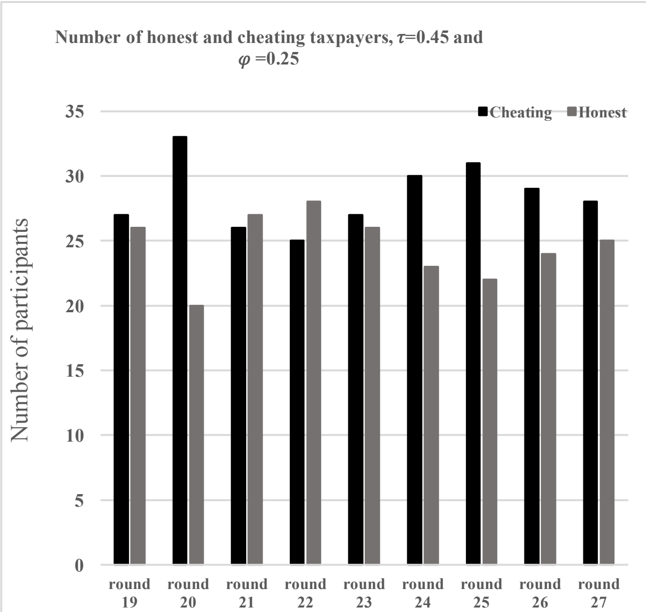
Looking then at Group 2 participants in the following Chart 2, we see a slight decrease in the number of cheating taxpayers when penalty rate increases (τ= 0.45 and φ =0.6 – rounds 10 to 18 - vs τ= 0.45 and φ =0.45, for rounds 1 to 9). By contrast, a decrease in penalty rate (τ= 0.45 and φ =0.25 scenario of rounds 19 to 27 vs the τ= 0.45 and φ =0.45 scenario of rounds 1 to 9) induces a slight increase in the number of cheating individuals, in line with Hypothesis 3. Interestingly, when analysing the scenario related to τ= 0.25 and φ =0.6 of rounds 28 to 36, we see that the number of honest participants is slightly higher than the number of cheating participants, except from rounds 31 to 33, where the number of honest and cheating participants is almost the same. Finally, in case $\tau = \varphi = 0.6$, the number of honest individuals remains lower than the number of participants choosing to play the strategy "cheating" for rounds 38 to 40, and 43, whereas it becomes higher for the remaining rounds 37 and 41 to 45.

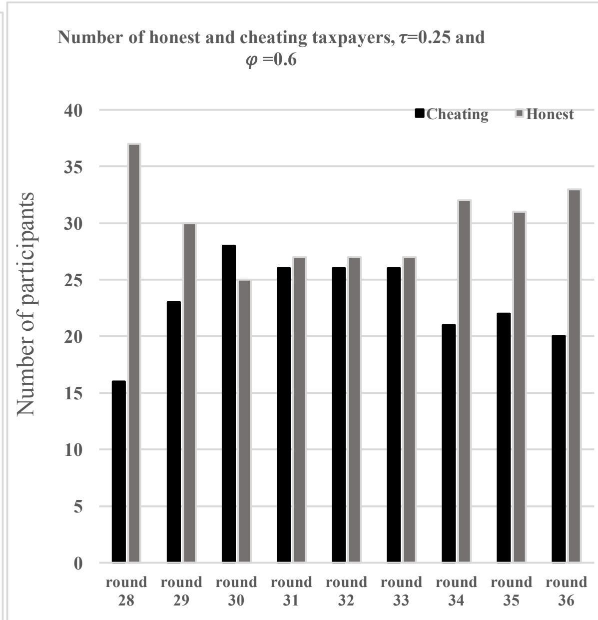
All in all, we observe that data seem to be in line with both Hypothesis 1 and 3 for both Group 1 and Group 2 participants. In fact, in both groups the number of honest and cheating individuals varies over the 45 rounds and over each 9 rounds of each scenario, and for Group 1 individuals, a pattern in the last rounds of each scenario can be identified. Moreover, in light of Hypothesis 3, we see that data seem clearly to be in line with the results of the comparative statics of model with respect to penalty rate, whereas for tax rate the effect seems less clear, as the effect of a decrease in tax rate seems to certainly induce an increase in the number of honest taxpayers, whereas the effect of an increase in tax rate on the number of honest individuals seems to be lower in magnitude.

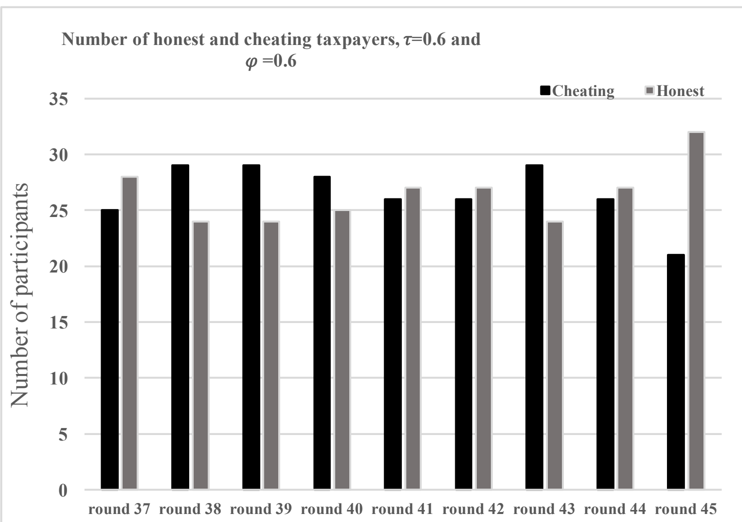
Chart 2: Number of honest and cheating individuals for each of the 9 rounds of each combination of tax and penalty rate, Group 2









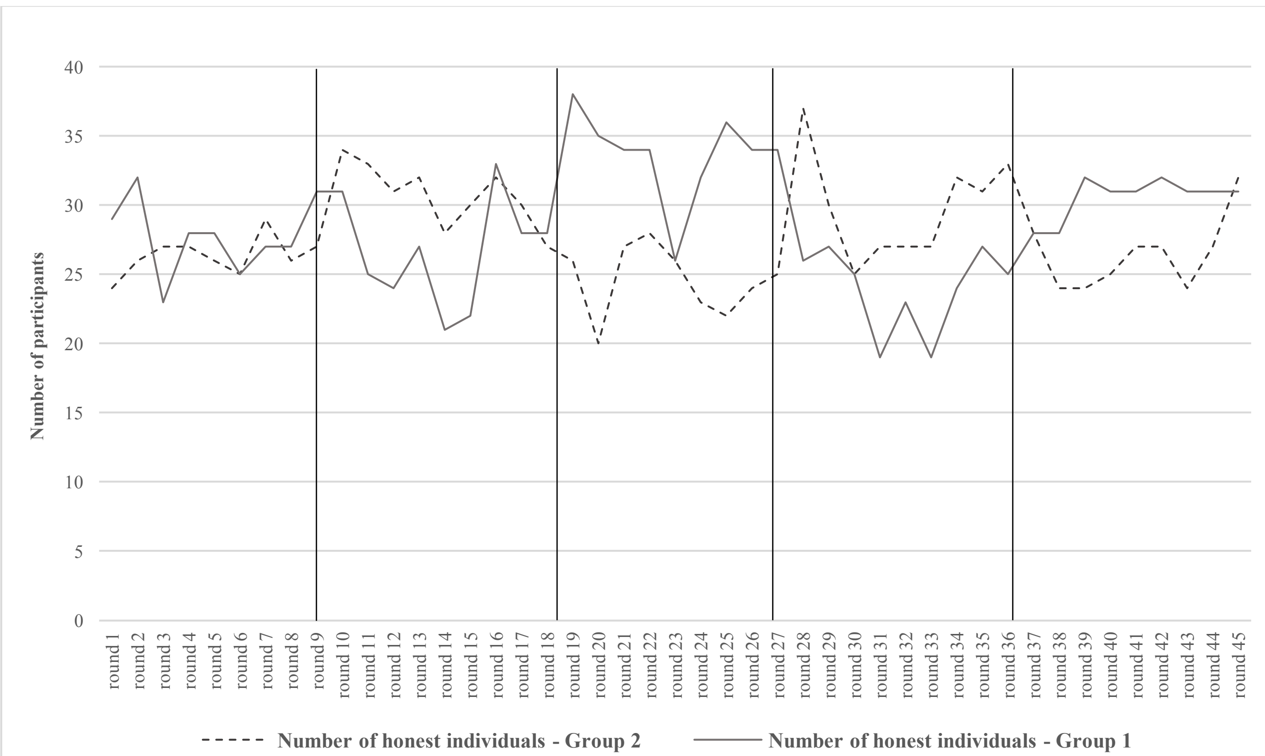


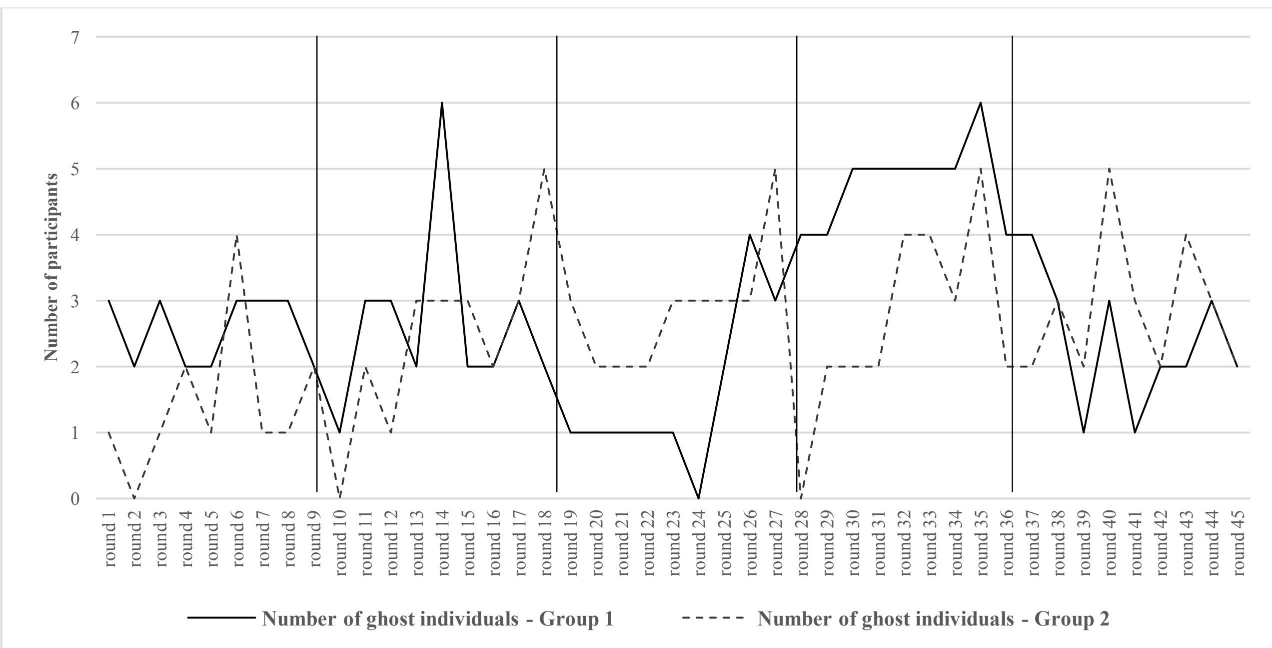
Charts 3.a, 3.b and 3.c below compare the number of honest, cheating and ghost participants over all 45 rounds in Group 1 and 2. Vertical lines indicate the change in the scenario (from a combination of tax and penalty rate to another). We can see that data seem to be partially in line with the Hypothesis 2, as for Group 1 individuals there seem to be no change in the number of honest and cheating participants between round 9 and 10 (from the scenario where τ= φ =0.45 to the scenario τ= 0.6 and φ =0.45), whereas for Group 2 individuals, the number of honest and cheating individuals changes between round 9 and 10 (from the scenario where τ= φ =0.45 to the scenario τ= 0.45 and φ =0.6). For both groups,

nonetheless, changes of scenario in rounds 19, 28 and 37 are characterized by a change in the proportion of honest and cheating individuals with respect to the previous rounds 18, 27 and 36, respectively, associated with different combinations of tax and penalty rate.

Therefore, changes in scenarios lead to a different distribution of characters among participants of both groups, but only in case of later rounds, and specifically, from round 19 onwards, which is partially in line with Hypothesis 2.

Chart 3.a, 3.b and 3.c: Number of honest, cheating and ghost participants over the 45 rounds of the game, Group 1 and 2





Tables 6.a and 6.b below compare the proportion of honest, cheating and ghosts between round 1 and round 9 for the first scenario proposed (where both tax and penalty rates were equal to 0.45) and all other scenarios for group 1 and 2 individuals.

Looking at the case when tax and penalty rate are both equal to 0.45, we see an increase in the proportion of honest individuals, and a decrease in the proportion of cheating participants, between round 1 and round 9 (therefore, after several exposures to fiscal authorities' inspecting activities) for both groups.

Moving then to the first two scenarios of group 1, where we keep penalty rate constant, we notice a decrease in the proportion of honest individuals between round 1 and round 9 for both higher and lower levels of tax rate. Nonetheless, looking only at round 9, and comparing the benchmark scenario (where tax and penalty rate are both equal to 0.45) with all changes in tax rate (holding penalty rate constant at 0.45), we see that for a higher level of tax rate (tax rate at 0.6), the proportion of honest individuals decreases. By contrast, for a lower level of tax rate (tax rate at 0.25), the proportion of honest participants increases, in line with the model presented in Section 1. Moreover, for all three cases (benchmark scenario, high and low tax rate scenarios), the proportion of honest individuals is always higher than the proportion of cheating individuals. Comparing then the benchmark scenario with the fourth one (where tax rate is equal to 0.6 and penalty rate is equal to 0.25), we see that in the latter case there is a small decrease in the proportion of honest individuals between round 1 and round 9, but the proportion of honest is always higher than the proportion of cheating. Moreover, looking only at round 9 (which represents the choices made after several rounds where audits were performed with probability 0.5), we still see that the proportion of honest individuals decreases as the tax rate increases, whereas the proportion of cheating is higher for the scenario characterized by a higher tax rate (0.6) together with a low penalty (0.25) with respect to the benchmark case (both tax and penalty rates at 0.45).

Finally, looking at the last scenario proposed to group 1, we see an increase in the proportion of honest participants between round 1 and round 9, whereas by comparing that scenario with the benchmark, we see no significant differences between the proportion of honest and cheating in round 9 in last scenario with respect to the benchmark scenario.

By comparing the scenario related to tax and penalty rate equal to 0.45 with the other four scenarios presented to group 2, we see that - for lower levels of penalty rate - the proportion of honest in round 9 decreases, while remaining unchanged for higher levels of penalty. Moreover, for all scenarios presented to group 2 participants, the proportion of honest is always higher than the proportion of cheating, for both rounds 1 and 9. Nonetheless, there is a decrease in the proportion of honest individuals between round 1 and round 9 for both cases where penalty rate increases at 0.6 and decreases at 0.25 (holding tax rate constant). Looking then at the case where tax rate is particularly low (0.25) and penalty is high at 0.6, we see a decrease in the proportion of honest between round 1 and round 9, whereas in case of both tax and penalty rates equal to 0.6, the proportion of honest individuals increases over time. The comparison between round 9 for the benchmark scenario and round 9 for the scenarios corresponding to 25%-60% and to both tax and penalty rate at 60% suggests that the proportion of honest individuals increases with respect to the benchmark scenario. This happens also in case of a decrease in tax rate from 0.45 to 0.25 (case 1) and a high level of tax and penalty rates.

Table 6.a: proportion of honest, cheating and ghost individuals between round 1 and round 9, and percentage variations, for all scenarios proposed in the experiment to Group 1 individuals.

	Round 1			**Round 9**			**percentage variation**		
	qh	**qc**	**qg**	**qh**	**qc**	**qg**	**qh**	**qc**	**qg**
τ=0.45, φ=0.45	0,55	0,40	0,06	0,58	0,38	0,04	7%	-5%	0%
τ=0.6, φ=0.45	0,58	0,40	0,02	0,53	0,43	0,04	-10%	10%	0%
τ=0.25, φ=0.45	0,72	0,26	0,02	0,64	0,30	0,06	-11%	14%	0%
τ=0.6, φ=0.25	0,49	0,43	0,08	0,47	0,45	0,08	-4%	4%	0%
τ=0.25, φ=0.25	0,53	0,40	0,08	0,58	0,38	0,04	11%	-5%	0%

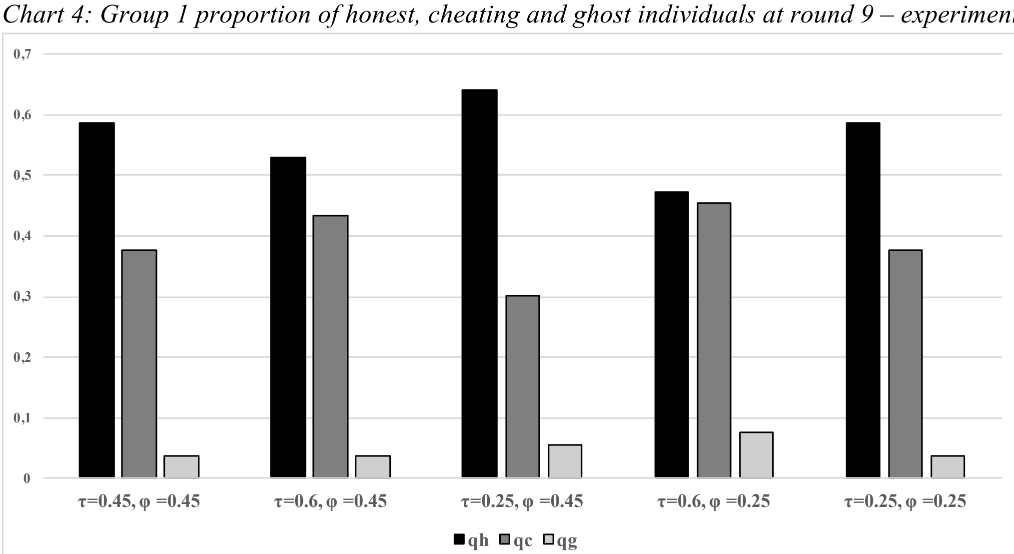
Table 6.b: proportion of honest, cheating and ghost individuals between round 1 and round 9, and percentage variations, for all scenarios proposed in the experiment to Group 2 individuals.

	Round 1			**Round 9**			**percentage variation**		
	qh	**qc**	**qg**	**qh**	**qc**	**qg**	**qh**	**qc**	**qg**
τ=0.45, φ =0.45	0,45	0,53	0,02	0,51	0,45	0,04	13%	-14%	100%
τ=0.45, φ=0.6	0,64	0,36	0,00	0,51	0,40	0,09	-21%	11%	0%
τ=0.45, φ=0.25	0,49	0,45	0,06	0,47	0,43	0,09	-4%	-4%	67%
τ=0.25, φ=0.6	0,70	0,30	0,00	0,62	0,34	0,04	-11%	13%	0%
τ=0.6, φ=0.6	0,53	0,43	0,04	0,60	0,36	0,04	14%	-17%	0%

Looking at the change in the proportion of honest, cheating and ghost participants with respect to a change in tax rate (group 1), chart 4 below shows that, as tax rate increases, the proportion of honest decreases, whereas the proportion of cheating increases, and vice-versa in case of a tax decrease. According to the results presented in chart 4, the proportion of honest, cheating and ghosts vary according to variations in tax and penalty rate, which is also confirmed by the Kruskal and Wallis test results (*p-value*=0.0001 and *chi-squared with ties* = 26.083 with 4 d.f. for the proportion of honest, *p-value*=0.0005 and *chi-squared with ties* = 20.130 with 4 d.f. for the proportion of cheating, and *p-value*=0.0001 and *chi-squared with ties* = 22.768 with 4 d.f. for the proportion of ghost). We can see that, for a higher level of tax rate, the proportion of honest individuals decreases, whereas it increases for the scenario with tax rate at 25%. Moreover, the decrease in $\overline{q_h}$ is higher for the case where both tax rate increases at 60% and penalty rate goes at 25%, with respect to the case of an increase only in tax rate (scenario where tax rate is at 60% and penalty rate at 45%).

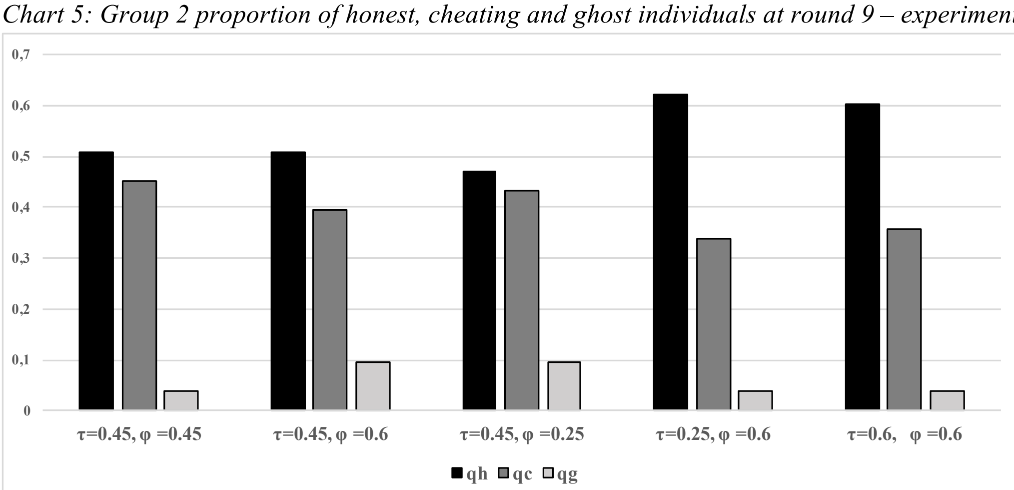
Finally, we should notice that when the tax rate is extremely high and the penalty rate extremely low, $\overline{q_c}$ and $\overline{q_h}$ tend to be similar, whereas in case of both a low tax and penalty rate, $\overline{q_h} > \overline{q_c}$.

Chart 4: Group 1 proportion of honest, cheating and ghost individuals at round 9 – experimental results.



Moving to chart 5, which represents the proportion of honest, cheating and ghosts for round 9 of group 2, we can see that the difference between the proportion of honest and cheating individuals is quite low, for the first three cases displayed in the graph (baseline case, where penalty and tax rate are both equal to 0.45, when the penalty rate increases to 0.6 and decreases to 0.25), whereas in both cases where the penalty rate is 0.6 (high penalty rate scenario) and tax rate is equal to 0.25 and 0.6 respectively, the proportion of honest is slightly higher than the proportion of cheating. Penalty rate variation produces also a variation in the proportion of honest, cheating and ghost taxpayers and this result is also confirmed by Kruskal and Wallis test results (*p-value*=0.0003 and *chi-squared with ties* = 21.170 with 4 d.f. for the proportion of honest, *p-value*=0.0001 and *chi-squared with ties* = 24.699 with 4 d.f. for the proportion of cheating, and *p-value*=0.0579 and *chi-squared with ties* = 9.131 with 4 d.f. for the proportion of ghost).

Chart 5: Group 2 proportion of honest, cheating and ghost individuals at round 9 – experimental results.



As explained, samples subject to comparison – with respect to different scenarios proposed - might not be mutually independent, and thus observations at individual level might be not independent across rounds. To address this issue, the Wilcoxon signed rank test has been performed on average observations at session level, and particularly on the average proportion of income declared to compare the within-subjects treatment effects and learning effects between round 1 and 9, as represented in Tables 7.a and 7.b below.

Table 7.a: Wilcoxon signed-rank test results on average proportion of income declared at session level, for the comparison of the different tax-penalty rates scenarios proposed to subjects participating in the experiment

Group 1	**Average income declared Scenario 60%-45%**	**Average income declared Scenario 25%-45%**	**Average income declared Scenario 60%-25%**	**Average income declared Scenario 25%-25%**
Average income dec Scenario 45%-45%	z = 1.836 p-value = 0.0663*	z = -1.007 p-value = 0.3139	z = 2.666 p-value = 0.0077***	z = -0.178 p-value = 0.8590
Group 2	**Average income declared Scenario 45%-60%**	**Average income declared Scenario 45%-25%**	**Average income declared Scenario 25%-60%**	**Average income declared Scenario 60%-60%**
Average income dec Scenario 45%-45%	z = -1.836 p-value = 0.0663*	z = 2.547 p-value = 0.0109**	z = -1.836 p-value = 0.0663*	z = 0.178 p-value= 0.8590

where we note that ***p<0.01; **p<0.05; *p<0.10.

Table 7.b: Wilcoxon signed-rank test results on average proportion of income declared across scenarios proposed to participants (tax-penalty rates combinations)

	Average income declared Round 1	**Average income declared Round 2**	**Average income declared Round 3**	**Average income declared Round 4**	**Average income declared Round 5**	**Average income declared Round 6**	**Average income declared Round 7**	**Average income declared Round 8**	**Average income declared Round 9**
Average income declared Round 1	--	z = 2.191 p-value=0.0284**	z = 1.172 p-value= 0.2411	z = 1.886 p-value=0.0593*	z = 1.784 p-value=0.0745*	z = 2.395 p-value=0.0166**	z = 1.070 p-value=0.2845	z = 1.886 p-value=0.0593*	z = 1.274 p-value=0.2026
Average income declared Round 2	--	--	z=0.764 p-value=0.4446	z = 1.172 p-value= 0.2411	z = 1.274 p-value= 0.2026	z = 2.090 p-value= 0.0367**	z = 0.153 p-value= 0.8785	z = 1.376 p-value= 0.1688	z = 0.459 p-value= 0.6465
Average income declared Round 3	--	--	--	z = 0.866 p-value= 0.3863	z = 1.070 p-value= 0.2845	z = 1.988 p-value= 0.0469**	z = -0.051 p-value= 0.9594	z = 1.070 p-value= 0.2845	z = -0.255 p-value= 0.7989
Average income declared Round 4	--	--	--	--	z = 0.764 p-value= 0.4446	z = 1.682 p-value= 0.0926*	z = -0.968 p-value = 0.3329	z = 0.153 p-value= 0.8785	z = -0.561 p-value= 0.5751
Average income declared Round 5	--	--	--	--	--	z = 0.764 p-value= 0.4446	z = -0.663 p-value= 0.5076	z = -0.051 p-value= 0.9594	z = -0.968 p-value= 0.3329
Average income declared Round 6	--	--	--	--	--	--	z = -2.090 p-value= 0.0367**	z = -1.376 p-value= 0.1688	z = -1.886 p-value= 0.0593*
Average income declared Round 7	--	--	--	--	--	--	--	z = 1.886 p-value= 0.0593*	z = 0.153 p-value= 0.8785
Average income declared Round 8	--	--	--	--	--	--	--	--	z = -0.968 p-value= 0.3329
Average income declared Round 9	--	--	--	--	--	--	--	--	--

where we note that ***p<0.01; **p<0.05; *p<0.10.

As we can see in Table 7a, presenting Wilcoxon signed-rank test results on average proportion of income declared at session level, both tax and penalty rate affect the average proportion of income declared across rounds, and especially when comparing the baseline scenario (with tax and penalty rate both fixed at 45%) and the scenarios where tax rate and penalty rate increase at 60%, respectively.

By looking then at the results of the test presented in 7.b, which are conducted on the average proportion of income declared across the scenarios proposed to participants, we see that there is a statistically significant difference in the average proportion of income declared between rounds 1,2,3,4 and round 6, indicating that the latter was the point at which individuals (on average) changed their tax declaration to fiscal authorities. Thus, both tests proposed in Tables 7.a and 7.b above underline two elements: first, treatment manipulations show a statistically significant effect on individuals' average proportion of income declared, therefore that changes in both tax and penalty rates produce statistically significant changes in individuals' tax declaration to fiscal authorities. This result is particularly significant when comparing the baseline scenario to the scenario 2 in both groups (increase in tax rate or in penalty rate to 60%, respectively), as this comparison does not suffer from order effects. Second, that – after a certain period of time - individuals adjust their tax declaration as a consequence of the experience in previous rounds, irrespective of the combination of tax and penalty rate they are subject to.

4.2 Empirical analysis and results[46]

In order to study the effects of the experience gained by participants for being controlled by fiscal authorities on individual propensity to be honest, our analytical strategy will follow two empirical methodologies. First, we will study the effect of experience of being subject to controls on individuals' propensity to be honest through a fixed and random effects OLS estimation, controlling for tax and penalty rate, as well as for gender and risk aversion attitude. In the context of this first empirical analysis,

[46] I wish to thank one of the referees for the helpful comments and suggestions with respect to the empirical analysis of the experiment, and particularly, on the methodology regarding the measurement of learning.

we will also check whether some of the results of the models presented in Section 1 are verified from the experimental data (namely, the effects of tax and penalty rate on individuals' propensity to be honest). Secondly, we will investigate the learning effect generated by the circumstance that participants play several rounds in the experiment under different scenarios, by adapting the Directional Learning model (DL model) as represented in Selten and Stocker (1986) and Moffatt (2015), to the context under investigation. This method allows also to mitigate the potential order effects arising from the experimental design.

With respect to the first approach used - described in detail in the following pages - aimed at verifying whether experience of being controlled - gained by participants during the rounds of the experiment - affects the individual propensity to be honest, we constructed 9 binary variables, indicating whether the participant has been controlled or not during each round, taking value 1 if the participant has been subject to controls, and 0 otherwise. We then run the OLS estimation of the fixed and random effects models as represented in (I) below:

$$(I)\ Y = X\beta + Rb + (Z) + \varepsilon$$

where, for each *i=1,....., 530* individual, and *j=1,....,9* round:

Y is the vector representing the outcome variable "*Propensity to be honest*";

X is a matrix representing the set of explanatory variables (tax rate, penalty, gender, risk aversion and experience of being controlled at each round *j,* described in further detail in table 8 below);

β is the vector of model coefficients, including the constant term;

Rb is the matrix of random effects, capturing individuals' unobserved heterogeneity features;

Z represents the vector of fixed effect, invariant with respect to the scenarios;

ε is the vector representing the random erratic component of the model.

We therefore estimate whether the binary variables – created for each participant and indicating whether the participant has been subject to controls or not, have a significant impact the individuals' propensity of being honest, controlling for gender, tax and penalty rate and risk aversion. Moreover, we check some of the results of the models presented in the previous chapters.

Table 8 below presents a summary of the variables that will be used for the statistical estimation of model (I), presented in the following pages.

Table 8: summary statistics for the variables used in the random and fixed effects models.

Variable		Number of observations	Mean	Std. Dev.	Min	Max
Individuals' propensity to be honest	**Average income declared over the 9 rounds of each scenario**	**530**	**.7464419**	**.2688263**	**0**	**1**
τ=0.45	**Binary variable representing the benchmark tax rate, it takes value 1 when tax rate is equal to 0.45**	**530**	**.3**	**.4586905**	**0**	**1**
τ=0.60	**Binary variable representing the increase in tax rate, taking value 1 when tax rate is equal to 0.6**	**530**	**.3**	**.4586905**	**0**	**1**
τ=0.25	**Binary variable representing the decrease in tax rate, taking value 1 when tax rate is equal to 0.25**	**530**	**.3**	**.4586905**	**0**	**1**
Risk aversion	**Risk aversion measurement for each *i-th* individual, where 50 is the maximum level of risk love**	**530**	**22.8**	**11.8**	**0**	**50**
gender	**Binary variable for gender (0 if female, 1 if male)**	**530**	**.5283019**	**.49967**	**0**	**1**
penalty due	**It is given by (τ*φ) - where τ and φ are the tax rate and the penalty rate associated with each scenario - since penalty rate was applied to the amount of evaded taxes once individuals were caught cheating within the game**	**530**	**.18925**	**.0857989**	**.0625**	**.36**
Experience of being controlled at round 1	**Binary variables for each individual, representing whether the participant has been controlled during each of the j=9 rounds, where "Experience of being controlled at round j"=0 if the participant was not subject to control at round j, whereas "Experience of being controlled at round j"=1 if the participant was subject to control at round j**	**530**	**.5018868**	**.5004688**	**0**	**1**
Experience of being controlled at round 2		**530**	**.4962264**	**.5004581**	**0**	**1**
Experience of being controlled at round 3		**530**	**.4981132**	**.5004688**	**0**	**1**
Experience of being controlled at round 4		**530**	**.5018868**	**.5004688**	**0**	**1**
Experience of being controlled at round 5		**530**	**.4858223**	**.500272**	**0**	**1**
Experience of being controlled at round 6		**530**	**.489603**	**.500365**	**0**	**1**
Experience of being controlled at round 7		**530**	**.5018868**	**.5004688**	**0**	**1**
Experience of being controlled at round 8		**530**	**.5018868**	**.5004688**	**0**	**1**
Experience of being controlled at round 9		**530**	**.509434**	**.5003833**	**0**	**1**

We first notice a potential issue regarding the circumstance that observations are related to the same individuals answering questions several times. Therefore, there might be specific, individual-related features not captured by the variables of the model, which might be nonetheless related to some independent variables in the model, for example each individual propensity to comply with rules. To address the individuals' specific unobserved components which might be related to some variables presented in table 9 above, we estimated both random and fixed effects models, as it will be shown in the following pages, controlling and not controlling for the different scenarios proposed to participants, where the dependent variable is the individuals' propensity to be honest over the 9 rounds of each scenario, whereas the independent variables are tax rate (represented by two binary variables representing high and low levels of tax rate, τ=0.6 and τ=0.25 respectively, where τ=0.45 is omitted to avoid multicollinearity, as we also relate both τ=0.6 and τ=0.25 to τ=0.45), penalty rate due, given by tax rate multiplied by penalty rate, for each scenario (as in the experiment the penalty was applied on the amount of evaded tax, the "true" penalty applied was given by the penalty rate multiplied by tax rate), gender (a binary variable taking value 0 if female, and 1 if male), the risk aversion measurement gained with the first question at the beginning of the experiment, which takes values between 0 and 50 (where 50 is maximum level of risk love), and 9 dummies representing whether inspections were carried out in the corresponding round of the 9 rounds of each scenario, for both random and fixed effects respectively.

Table 10 below shows the estimation results, for the model estimated with random effects, fixed effects, controlling and not controlling for the scenarios proposed to participants.

Table 9: regression results using random and fixed effects (model (I) above), controlling and not controlling for the different scenarios proposed to participants (standard errors in parentheses)

Dependent variable: individuals' propensity to be honest	Coefficients (robust standard errors in parentheses)			
	Random effects	Fixed Effects	Random Effects	Fixed effects
	N. of observations=528		N. of groups=106	
R-squared	0.1227	0.1272	0.1228	0.1273
	Wald chi-sq = 50.45	F(13,409) = 4.58	Wald chi-sq = 50.46	F(14,408) = 4.25
τ=0.6	-.0754008*** (.0218597)	-.0783617 *** (.0218733)	-.0710234*** (.0246518)	-.0739773*** (.024303)
τ=0.25	.0672251*** (.0247946)	.0656287*** (.0249336)	-.0710234*** (.0246518)	.0694706*** (.0252244)
penalty due	.5119477*** (.1255427)	.5092931*** (.1279535)	.5082277*** (.1267468)	.5028807*** (.130147)
Risk aversion	.0006673 (.0022119)	.1279535*** (.0040657)	.0006263 (.0021995)	.0295792*** (.0040778)
gender	-.0437574 (.0425612)	--	-.0432454 (.0424258)	--
inspection				
Experience of being controlled at round 1	-.0222959 (.017938)	-.0250191 (.0182279)	-.0222883 (.0179484)	-.0250256 (.0182403)
Experience of being controlled at round 2	-.0118473 (.0140506)	-.0103589 (.0139655)	-.0118543 (.0140829)	-.0103497 (.0139911)
Experience of being controlled at round 3	-.0132739 (.0159392)	-.0165968 (.0160915)	-.0132265 (.0159192)	-.0165646 (.0160859)
Experience of being controlled at round 4	-.0164266 (.0176566)	-.0169586 (.0179859)	-.0164386 (.0176497)	-.0169624 (.0179752)
Experience of being controlled at round 5	-.0234408 (.0148978)	-.0241753 (.015377)	-.0236112 (.0149727)	-.024345 (.0154768)
Experience of being controlled at round 6	-.0367323** (.016196)	-.0363702** (.016146)	-.0367297** (.0162134)	-.0363282** (.0161493)
Experience of being controlled at round 7	-.0100826 (.0187037)	-.0071115 (.0192793)	-.0101016 (.0187218)	-.0071473 (.0193199)
Experience of being controlled at round 8	-.0585935*** (.0153097)	-.0573694*** (.0159274)	-.0585384*** (.0153124)	-.0573047*** (.0159379)
Experience of being controlled at round 9	-.0147199 (.0156235)	-.0173705 (.0159804)	-.0146801 (.0156488)	-.0173548 (.0160095)
Controlling for scenarios	No	No	Yes	Yes
Constant	.7630001*** (.0798795)	.087842 (.089073)	.7681668*** (.0810261)	.0875949 (.0890186)

where we note that ***p<0.01; **p<0.05; *p<0.10.

As we can see from Table 9, we observe consistency in the results with respect to all the variables included in the different models, both controlling and not controlling for the scenarios.

First, to complement the analysis on the fixed and random effects, we performed the Hausman test on the results, also with the aim of choosing between random and fixed effects. The results seem to suggest the use of random effects for both cases of controlling and not controlling for scenarios (respectively, *Chi-sq*= 6.70 and *Prob>chi2* =0.9458; *Chi-sq*=6.72 and *Prob>chi2* = 0.9158).

We then analyse whether controls carried out at each round affect the individuals' propensity to be honest. Results suggest that inspections performed during the first five rounds do not significantly affect individuals' propensity towards honesty, whereas inspections carried out at rounds 6 and 8 show a statistically significant relationship with the participants' propensity to be honest, and particularly, the latter is negatively affected by inspections. This result might be interpreted as if individuals' experience after the first five rounds, in which they make attempts and experience the circumstance of being inspected, drives their propensity to be honest in the later rounds of each scenario. In this respect, their learning process through experience is shown at later rounds of each scenario, after several attempts and controls.

Moving then to the analysis of the coefficients indicating the effects of tax rates, penalty, gender and risk aversion on the participants' propensity to be honest, we see that a higher level of tax rate with respect to the baseline scenario ($\tau = 0.6$) induces a decrease in the propensity towards honesty, whereas lower levels of tax rate ($\tau = 0.25$) affect it positively, both coefficients being statistically significant at 0.01 level. Moreover, the penalty due (given by the penalty rate multiplied by the tax rate, as also in the experiment the penalty was applied on the evaded tax) is statistically significant and affects positively the outcome variable, therefore – according to the results - an increase in the penalty rate due leads to an increase in the participants' average propensity to declare. These results appear in line with the outcome of the model presented in Section 1 of this chapter, and in the previous chapters.

Gender does not show a statistically significant effect on the propensity to be honest, as well as the measure of risk aversion within the random effects model, whereas in the fixed effects model it appears to be statistically significant.

By summarizing the regression results, we see that individuals' experience gained during the first five rounds of each scenario significantly affects individuals' propensity to be honest, and particularly, it induces participants to decrease their propensity towards honesty in rounds 6 and 8, by producing a learning effect over time.

Moreover, both the levels of tax rate and penalty significantly affect individuals' propensity towards honesty, in the same direction suggested by the model presented in Section 1.

Inspections performed at round 9 do not affect individuals' declaration, but it might be due to the fact that after round 9, another scenario was proposed to each participant.

As mentioned, the experimental design presented might imply the existence of order effects in rounds 19-45, for which an additional analysis has been performed to disentangle such order effects from treatment and learning effects. We therefore assess the effect of learning on individuals' choice to change the amount of income declared to fiscal authorities over time by using a Directional Learning model, as in Selten and Stocker (1986).

Generally speaking, the Directional Learning model investigates intuitively whether subjects change and adjust their behaviour as as result of past experience, and particularly, of the outcome related to previous periods. We should mention that, even though the experiment presented here does not involve a proper game between pairs of players as in the original version of the model, the latter has been adapted to the experimental design proposed. This approach allows the comparison between rounds, by disentangling order effects from the measurement of learning within different scenarios proposed.

As for the DL model in this context, the reasoning behind was the following. In order to mitigate the order effects associated with the experimental design, we study the learning process across rounds 1-45 from the perspective of the observed deviation periods across rounds 1-45. Particularly, to estimate an adapted version of the DLM, we introduce a new variable, which we will call '*self*' as in the notation used by Moffatt (2015), created on the basis of the observations of the dataset.

We therefore construct the variable '*self*', modelled viewing the individuals' choice over rounds as a Markov chain, and therefore taking values between 2 and 11, as described in detail by Selten and Stocker

(1986, p. 56). In this setting, '*self*' indicates the observed period at which subjects choose to change their amount of income declared to fiscal authorities with respect to the amount previously chosen, therefore at which subjects choose to deviate from their previous choice, within each scenario. More specifically, for each scenario proposed in the game, after round 1 - in which the participant chooses a specific amount of income to be declared to fiscal authorities - the participant can either choose to declare to fiscal authorities the exact amount of income declared in round 1, or deviate from his/her previous choice and declare a higher or lower amount of income, in rounds 2 to 9, or after round 9 of the specific scenario (thus, in the subsequent scenario). It can also happen that for 2 or 3 subsequent scenarios (hence, for 18 or 27 rounds) the participant does not change the amount of income declared. In other words, the change in the amount of income declared might happen in any of the rounds 2 to 9 of the scenario proposed, or at the subsequent scenario, or never. As an example, consider a participant who plays round 1 of scenario 1, and declares, for example, an income of 500 euros. Suppose that at round 2 of the scenario, the participant again declares an income of 500 euros to fiscal authorities, whereas at the third round of the first scenario, he/she declares an income of 499 euros. In this case, '*self*' takes value 3 for the first scenario, since the deviation of his/her amount of income declared occurred at round 3. We should then mention two additional specific cases that might happen. First, we consider the case in which the subject deviates from his/her choice right after the change of scenario, thus at round 1 of the subsequent scenario. Consider, in the example above, that the participant declares an income of 500 euros for all 9 rounds of the first scenario, and he/she declares 499 euros at round 1 of the second scenario. In this case, the variable '*self*' takes value 10 for the first scenario, whereas for the second scenario it takes the value corresponding to the first deviation from the amount of income declared of 499 euros. Second, we should consider the case where the change in the amount of income declared does not happen neither in any of the rounds of the scenario, nor in the first round of the subsequent scenario. By referring to the same example as above, consider the case where the subject declares an income of 500 euros for all 9 rounds of the first scenario and for the first two rounds of the second scenario, and he/she declares an income of 499 euros in the third round of the second scenario. In this case, '*self*' takes value 11 for the first scenario, whereas for the second scenario it takes the value corresponding to the first deviation from the amount of income declared of 499 euros. Therefore, the variable '*self*' is associated with 5 observations for each participant, each corresponding to the observed deviation period associated with each scenario.

Additionally, and before moving forward, we should mention an important feature of the model, which represents also its limitation. Particularly, when considering the change in subjects' choices, we only consider the first change occurring with respect to individuals' income declared, and not all the subsequent changes occurring after the first deviation. Therefore, we only measure the effect of learning on the first change occurring in the amount of income declared to fiscal authorities, without considering repeated changes over time.

From the variable '*self*', then, three additional binary variables have been created, '*before*', '*same*' and '*after*'[47]. In the original version of the DLM, '*before*', '*same*' and '*after*' are binary variables taking value 1 if the observed deviation period from the cooperative behaviour in the Prisoner's Dilemma game of player *i* is before, the same, or after the observed deviation period of his/her opponent. In this setting, we do not have paired players, but instead an individual choice under risk, where we aim at a within-subjects comparison under different scenarios. To compare therefore the effects of learning over rounds across the different scenarios proposed, we pair - for each subject – each individual's income declared under the baseline scenario, involving both tax and penalty rate at 45%, with the income declared under the alternative scenarios proposed. Thus, for each participant, we create 4 pairs, involving comparisons between the responses given by each individual in the baseline scenario (tax and penalty rate at 0.45) and the ones given in the other scenarios proposed. In this respect, '*before*', '*same*' and '*after*' are created – for each individual – by pairing observed amount of income declared in round 1-9, to the observed

[47] We use here the same variable names used in Moffatt (2015).

income declared in rounds 10-18, 19-27, 28-37, 38-45. For example, consider the case where the variable '*self*' associated with participant *i-th* takes values 2, 4, 6, 11, 2. We construct the 4 observations for each of the binary variables '*before*', '*same*' and '*after*', associated with the four scenarios presented after the first one (which is baseline, and common to both Group 1 and 2). Particularly, considering the second scenario, we see that the observed deviation in the amount of income declared took place at round 4, whereas in the first scenario it took place at round 2. Therefore, variables '*before*' and '*same*' will both take value 0, whereas '*after*' will take value 1, since the observed deviation in the second scenario took place after the observed deviation in the first scenario. Similarly, for the third scenario, '*before*' and '*same*' will both take value 0, whereas '*after*' will take value 1, as the observed deviation in the third scenario took place after the observed deviation in the first scenario. Finally, looking at the fifth scenario, '*before*' and '*after*' will take value 0, whereas '*same*' will be equal to 1, considering that the observed deviation in the fifth scenario took place in the same round as in the first scenario.

We then create two datasets for the two distinct groups (Group 1 where tax rate varies across the first 27 rounds) and Group 2 (where penalty rate varies across the first 27 rounds), where we have, for each individual, 5 observations of the variable '*self*' and 4 observations for the variables '*before*', '*same*' and '*after*', which represent the pairs created with the aim of measuring the learning effect under the different four alternative scenarios, with a mitigation of potential order effects (in fact, we compare choices made in each of the four alternative scenarios directly with the baseline, as the individual's observed deviation period in scenario 45%-45% is paired with all other four treatment manipulations proposed).

The estimation is a simple OLS regression, where the outcome variable is the first difference of the variable '*self*', and the explanatory variables are the lagged binary variables '*before*', '*same*' and '*after*'. The DL model suggest that, if the coefficients, especially for the variables '*before*' and '*after*', are statistically significant, and particularly if the sign of the coefficient for the variable '*after*' is negative, then when the observed deviation period in one scenario is later than the participant observed deviation period in the baseline scenario, the participant adjusts its own deviation period earlier in the subsequent scenarios. Accordingly, if the coefficient of the variable '*before*' is statistically significant and positive, then the results suggest that if the participant's observed deviation in a scenario happens before with respect to the observed deviation in the baseline scenario, the participant adjusts its own deviation period earlier in the subsequent scenarios. Both these results suggest that individuals learn from experience, as their choice depends on the outcome of previous rounds of the game.

The aim of the original DL model was basically to prove that, in repeated Supergames of Prisoner's Dilemma, individuals learn how to deviate from the cooperation as a result of what happened in the previous round. In this case, we similarly aim at verifying whether the choice made in the previous round affects the choice made in the subsequent ones, under the different scenarios proposed.

Thus, by using the same notation as Moffatt (2015), we define the variables and the DL model used to estimate the effect of learning on the individuals' choice on the income declared to fiscal authorities:

- '*d. self*' is the difference operator applied on the variable '*self*' described above, which we will indicate also as *d.self*=$\Delta self_i$. Thus, *d.self*=$\Delta self_i = self_{i,s} - self_{i,s-1}$, where i represent the *i-th* observation for the variable, and *s* represents the scenario (*s-1* represents the previous scenario). The variable *d.self*, therefore, represents the change in the deviation period (the latter being the round at which a participant chooses to change the amount of income to be declared to fiscal authorities) over time.

- '*l.before*', '*l.same*' and '*l.after*' represent the lag operator applied on the three binary variables '*before*', '*same*' and '*after*'. Thus, *l.before*=$before_{i,s-1}$, *l.same*=$same_{i,s-1}$ and *l.after*=$after_{i,s-1}$ where $s = 1, \dots, 5$ represent the 5 scenarios of the experiment represent the *i-th* observation for the variable.

Thus, following Selten and Stocker (1986), we estimate the OLS model as described by (II) as follows:

(II) $\Delta self_{i,s} = \beta_1 before_{i,s-1} + \beta_2\ same_{i,s-1} + \beta_3 after_{i,s-1} + \varepsilon_i$

Table 10 below represents the outcome of the estimated Directional Learning models in both Group 1 and Group 2.

Table 10: DLM estimation results, for both Groups 1 and 2

Group 1		Group 2	
Dependent variable: 'd.self'	***Coefficients (standard errors in parentheses)***	***Dependent variable: 'd.self'***	***Coefficients (standard errors in parentheses)***
Adjusted R-squared	***0.2257***	***Adjusted R-squared***	***0.1071***
N. of observations	***159***	***N. of observations***	***159***
F-stat	***16.45***	***F-stat***	***7.36***
Prob > F	***0.0000***	***Prob > F***	***0.0001***
l.before	***2.79****** ***(0.4661794)***	***l.before***	***1.51***** ***(0.6181044)***
l.same	***0.29*** ***(0.4395181)***	***l.same***	***- 0.16*** ***(0.3847669)***
l.after	***- 1.54****** ***(0.4277955)***	***l.after***	***- 1.97****** ***(0.4967319)***

where ***p<0.01; **p<0.05; *p<0.10.

Looking at table 10, estimation results show that variables '*l.before*' and '*l.after*' coefficients are both statistically significant, and different from zero, whereas the coefficient of the variable '*l.same*' is not statistically significant. It happens for both Group 1 and Group 2 estimation. We should then interpret the results in line with the DL model prediction, as also the sign of the coefficients for '*l.before*' and '*l.after*' is consistent with the DL interpretation. Particularly, from the estimation results for the coefficient of '*l.before*', for Group 1, we see that if the participant's observed deviation in rounds 10-18 happens before the participant's observed deviation in the baseline scenario proposed (rounds 1-9), the participant adjusts its own deviation period by 2.79 times earlier in the following scenarios (that is, from rounds 19-45, the individuals' deviation from the first choice occurs 2.9 periods earlier). Moreover, also looking at the '*l.after'* coefficient, we see that if the participant observed deviation period in rounds 10-18 (representing the scenario that is subsequent with respect to the baseline scenario of tax and penalty rate at 45%) is after the participant observed deviation period in the baseline scenario (rounds 1-9), the participant adjusts its own deviation period by 1.54 times earlier in the following scenarios (that is, from rounds 19-45, the individuals' deviation from the first choice occurs 1.54 periods earlier). Therefore, results suggest that individuals tend to anticipate the point in time at which they change (increase or decrease) the amount of income declared to fiscal authorities with respect to the first scenario. The same happens for Group 2 individuals, where, nonetheless, the magnitude of the coefficient for the variable '*l.after'* is higher than the one of the coefficient of '*l.before'*. All in all, DL model estimation results show consistency with DL assumption, as a result of the exposure of individuals to fiscal authorities' auditing activities (or, in a broader sense, to fiscal authorities' auditing strategy), regardless of the combination of tax and penalty rate proposed. Therefore, the results depicted in table 10 suggest that participants tend to adjust the amount of income declared as a result of the experience in the first scenario proposed. Particularly, their experience in the first 9 rounds of the first scenario allows them to increase or decrease the amount of income declared to fiscal authorities at an earlier stage in the subsequent scenarios. In other words, results show that participants tend to anticipate the point in time at which they change the amount of income they decide to declare to fiscal authorities, with respect to what happens in the first scenario.

5. Concluding remarks

In this chapter, we presented an experiment sharing a similar conceptual framework as the models presented in Chapters 2 and 3, but within a simplified setting as proposed in paragraph 1, where individuals choose how much to declare of their income, subject to several rounds where they are inspected with probability 0.5. Particularly, participants were given an endowment, and they were asked to declare their endowment to fiscal authorities, knowing that after each round they could be inspected with a probability of 0.5. Moreover, after each round, participants received notice on whether they were inspected, and on their final payoff after inspections took place. The experiment was aimed at verifying three hypotheses. First, it was aimed at verifying whether, for each combination of tax and penalty rate, the proportion of honest and cheating individuals was subject to a change from round 1 to round 9, as a result of learning from interactions occurred at the previous round. Second, it was aimed at investigating if, in case of a change between a scenario (a combination of tax and penalty rate) and another one, individuals learn to adapt to the new scenario, and use the information gained during the previous combination of tax and penalty rate to choose whether to evade or not, and how much to evade. Third, it analysed whether scenarios characterized by an increase in tax rate are related to a higher proportion of cheating individuals and a lower proportion of honest taxpayers, whereas scenarios characterized by a higher penalty rate are associated with a higher proportion of honest taxpayers and a lower proportion of cheating taxpayers, in line with the findings of the model presented in paragraph 1.

Results show that Hypotheses 1 and 3 are verified, whereas Hypothesis 2 is partially verified. Moreover, we see a stronger effect of a change in penalty rate over a change in tax rate on the proportion of honest and cheating individuals.

The use of Directional Learning Model (DLM) allowed to measure the extent of individuals' change in the proportion of income declared across rounds of the experiment, as a result of the previous rounds' outcome. Results show that the participants' learning process was significantly influenced by the outcome of the previous rounds, which in turn depends on the fiscal authorities' auditing activities, occurring with probability 0.5, but randomly distributed over rounds.

Finally, estimation results show a partial statistically significant effect of inspections (and of participants' awareness of inspections) on the individuals' tax compliance decision, displaying therefore the effect of learning on taxpayers' behaviour and decision making process at the latest rounds of each scenario.

In future versions of the analysis, the experimental design could be modified in two main directions. First, individuals' choice under risk design might be extended to allow individuals' outcome (and payment) in each round to depend on the other participants' choices, for studying the concept of learning from peers. Additionally, the experimental design might be extended to let fiscal authorities' auditing strategy vary in the experiment, instead of being a fixed probability set equal to 0.5, to let individuals play a proper "game" with fiscal authorities, even though it would add complexity in the analysis of the results.

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Annex I

Section I: mathematical and computational features of the model presented in Chapter 2

We start from differential equations (11) to (14) in Chapter 2

(11) $\frac{\dot{q_H}}{q_H} = f_T(e^1, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\,(f_T(e^2, s))\right]$

(12) $\frac{\dot{q_C}}{q_C} = f_T(e^2, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\,(f_T(e^2, s))\right]$

(13) $\frac{\dot{p_I}}{p_I} = f_A(e^1, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}(f_A(e^2, s))\right]$

(14) $\frac{\dot{p_{NI}}}{p_{NI}} = f_A(e^2, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}(f_A(e^2, s))\right]$

and we substitute (9) and (10) inside

$$\boldsymbol{f_A(e^a, T) = q_H\left[\pi_A\left(s_A^a, s_T^h\right)\right] + q_C[\pi_A(s_A^a, s_T^c)]} \quad \textbf{(9)}$$

$$\boldsymbol{f_T(e^b, A) = p_I[\pi_T\left(s_A^i, s_T^b\right)] + p_{NI}[\pi_T\left(s_A^{ni}, s_T^b\right)]} \quad \textbf{(10)}$$

leading to (11a) to (14a)

(11a) $\frac{\dot{q_H}}{q_H} = \left\{p_I[\pi_T\left(s_A^i, s_T^h\right)] + p_{NI}[\pi_T\left(s_A^{ni}, s_T^h\right)]\right\} - \left\{q_H\left[p_I[\pi_T\left(s_A^i, s_T^h\right)] + p_{NI}[\pi_T\left(s_A^{ni}, s_T^h\right)]\right] + \right.$

$\left. q_C\left[p_I[\pi_T\left(s_A^i, s_T^c\right)] + p_{NI}[\pi_T\left(s_A^{ni}, s_T^c\right)]\right]\right\}$

(12a) $\frac{\dot{q_C}}{q_C} = \{p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]\} - \{q_H[p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]] + q_C[p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]]\}$

(13a) $\frac{\dot{p_I}}{p_I} = \{q_H[\pi_A(s_A^i, s_T^h)] + q_c[\pi_A(s_A^i, s_T^c)]\} - \left\{p_I\left[q_H[\pi_A(s_A^i, s_T^h)] + q_c[\pi_A(s_A^i, s_T^c)]\right] + p_{NI}\left[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_c[\pi_A(s_A^{ni}, s_T^c)]\right]\right\}$

(14a) $\frac{\dot{p_{NI}}}{p_{NI}} = \{q_H[\pi_A(s_A^{ni}, s_T^h)] + q_c[\pi_A(s_A^{ni}, s_T^c)]\} - \left\{p_I\left[q_H[\pi_A(s_A^i, s_T^h)] + q_c[\pi_A(s_A^i, s_T^c)]\right] + p_{NI}\left[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_c[\pi_A(s_A^{ni}, s_T^c)]\right]\right\}$

We then substitute in (11a) to (14a) the payoffs (1) to (8), obtaining

(11a.1) $\frac{\dot{q_H}}{q_H} = \{p_I[(1-\tau)Y] + p_{NI}[(1-\tau)Y]\} - \left\{q_H[p_I[(1-\tau)Y] + p_{NI}[(1-\tau)Y]] + q_C\left[p_I\left[\left[(1-\tau)Y \underbrace{-\varphi(1-\beta)Y}\right](1-\alpha)\right] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\right]\right\}$

(12a.1) $\frac{\dot{q_C}}{q_C} = \left\{p_I[[(1-\tau)Y \underbrace{-\varphi(1-\beta)Y}](1-\alpha)] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\right\} - \left\{q_H[p_I[(1-\tau)Y] + p_{NI}[(1-\tau)Y]] + q_C\left[p_I\left[\left[(1-\tau)Y \underbrace{-\varphi(1-\beta)Y}\right](1-\alpha)\right] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\right]\right\}$

(13a.1) $\frac{\dot{p_I}}{p_I} = \left\{q_h[\tau Y - C\tau Y] + q_c\left[\left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha - C\tau Y\right]\right\} - \left\{p_I\left[q_h[\tau Y - C\tau Y] + q_c\left[\left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha - C\tau Y\right]\right] + p_{NI}\left[q_h[\tau Y] + q_c\left[\underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}\right]\right]\right\}$

$$(14a.1)\ \frac{\dot{p}_{NI}}{p_{NI}} = \left\{q_H[\tau Y] + q_c\left[\underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}\right]\right\} - \left\{p_I\left[q_H[\tau Y - C\tau Y] + q_c\left[\left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha - C\tau Y\right]\right] + p_{NI}\left[q_H[\tau Y] + q_c\left[\underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}\right]\right]\right\}$$

We then set $\frac{\dot{q}_H}{q_H} = \frac{\dot{q}_C}{q_C} = \frac{\dot{p}_I}{p_I} = \frac{\dot{p}_{NI}}{p_{NI}} = 0$, to find the equilibrium(s), which are represented by the situation where the distribution of characters in the population does not change. To find the solutions in equilibrium we have therefore to solve the following system of four equations (11a.1) to (14a.1) for p_I, p_{NI}, q_H and q_C

$$(11a.1)\quad \{p_I[(1-\tau)Y] + p_{NI}[(1-\tau)Y]\} - \left\{q_H[p_I[(1-\tau)Y] + p_{NI}[(1-\tau)Y]] + q_C\left[p_I\left[\left[(1-\tau)Y \underbrace{- \varphi(1-\beta)Y}\right](1-\alpha)\right] + p_{NI}[Y(1-\underbrace{\tau\beta})]\right]\right\} = 0$$

$$(12a.1)\quad \left\{p_I[[(1-\tau)Y \underbrace{- \varphi(1-\beta)Y}](1-\alpha)] + p_{NI}[Y(1-\underbrace{\tau\beta})]\right\} - \left\{q_H[p_I[(1-\tau)Y] + p_{NI}[(1-\tau)Y]] + q_C\left[p_I\left[\left[(1-\tau)Y \underbrace{- \varphi(1-\beta)Y}\right](1-\alpha)\right] + p_{NI}[Y(1-\underbrace{\tau\beta})]\right]\right\} = 0$$

$$(13a.1)\ \left\{q_H[\tau Y - C\tau Y] + q_c\left[\left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha - C\tau Y\right]\right\} - \left\{p_I\left[q_H[\tau Y - C\tau Y] + q_c\left[\left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha - C\tau Y\right]\right] + p_{NI}\left[q_H[\tau Y] + q_c\left[\underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}\right]\right]\right\} = 0$$

$$(14a.1)\quad \left\{q_H[\tau Y] + q_c\left[\underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}\right]\right\} - \left\{p_I\left[q_H[\tau Y - C\tau Y] + q_c\left[\left[\tau Y + \underbrace{\varphi(1-\beta)Y}\right]\alpha - C\tau Y\right]\right] + p_{NI}\left[q_H[\tau Y] + q_c\left[\underbrace{\tau\beta Y} - \underbrace{\tau(1-\beta)Y}\right]\right]\right\} = 0$$

For solving the system of equations, the model has been written in Matlab. First, the set of variables and parameters has been declared, together with fitness functions (9) and (10). Then the system of differential equations (11a.1)-(14a.1) above has been presented, therefore setting them equal to zero. It has been declared in the script to solve the system for q_H, q_c, p_I and p_{NI}.

Section II: simulations on the solutions of the model presented in Chapter 2

Table 1: simulations for $\overline{p_I}$ and $\overline{p_{NI}}$ in case $\bar{\beta} = 0$, for different values of φ, τ and $\bar{\alpha}$

$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\bar{\boldsymbol{\alpha}}$	$\overline{\boldsymbol{p_I}}$	$\overline{\boldsymbol{p_{NI}}}$
0.1	0.2	0.6	0.28	0.72
0.2	0.2	0.6	0.26	0.74
0.3	0.2	0.6	0.25	0.75
0.4	0.2	0.6	0.24	0.76
0.2	0.1	0.3	0.20	0.80
0.3	0.2	0.3	0.31	0.69
0.4	0.3	0.3	0.38	0.62
0.5	0.4	0.3	0.43	0.57
0.4	0.3	0.4	0.37	0.63
0.4	0.3	0.4	0.37	0.63
0.4	0.3	0.4	0.37	0.63
0.4	0.3	0.4	0.37	0.63
0.7	0.6	0.8	0.57	0.43
0.8	0.6	0.8	0.56	0.44
0.85	0.6	0.8	0.55	0.45
0.9	0.6	0.8	0.55	0.45
0.75	0.55	0.8	0.52	0.48
0.8	0.55	0.8	0.51	0.49
0.85	0.55	0.8	0.51	0.49

0.9	0.55	0.8	0.50	0.50
0.75	0.5	0.6	0.45	0.55
0.8	0.5	0.6	0.45	0.55
0.85	0.5	0.6	0.44	0.56
0.9	0.5	0.6	0.43	0.57

Table 2: simulations for $\overline{p_I}$ *and* $\overline{p_{NI}}$ *in case* $0 < \bar{\beta} < 1$ *and* $0 < \bar{\alpha} < 1$*, for different values of* φ, τ *and* $\bar{\alpha}$

φ	τ	$\bar{\alpha}$	$\bar{\beta}$	$\overline{p_I}$	$\overline{p_{NI}}$
0.7	0.55	0.4	0.3	0.45	0.55
0.7	0.55	0.3	0.4	0.43	0.57
0.7	0.55	0.3	0.2	0.46	0.54
0.7	0.55	0.2	0.3	0.44	0.56
0.7	0.55	0.7	0.2	0.48	0.52
0.7	0.55	0.7	0.25	0.47	0.53
0.7	0.55	0.7	0.3	0.45	0.55
0.7	0.55	0.7	0.4	0.43	0.57
0.7	0.55	0.3	0.6	0.40	0.60
0.7	0.55	0.3	0.7	0.37	0.63
0.7	0.55	0.3	0.8	0.32	0.68
0.7	0.55	0.3	0.9	0.23	0.77
0.7	0.55	0.7	0.6	0.36	0.64
0.7	0.55	0.6	0.7	0.32	0.68
0.7	0.55	0.7	0.8	0.24	0.76
0.7	0.55	0.8	0.7	0.29	0.71
0.4	0.25	0.4	0.3	0.27	0.73
0.4	0.25	0.3	0.4	0.28	0.72
0.4	0.25	0.3	0.2	0.31	0.69
0.4	0.25	0.2	0.3	0.32	0.68

0.4	0.25	0.7	0.2	0.24	0.76
0.4	0.25	0.7	0.25	0.23	0.77
0.4	0.25	0.7	0.3	0.22	0.78
0.4	0.25	0.7	0.4	0.20	0.80
0.4	0.25	0.3	0.6	0.23	0.77
0.4	0.25	0.3	0.7	0.20	0.80
0.4	0.25	0.3	0.8	0.15	0.85
0.4	0.25	0.3	0.9	0.09	0.91
0.4	0.25	0.7	0.6	0.15	0.85
0.4	0.25	0.6	0.7	0.13	0.87
0.4	0.25	0.7	0.8	0.08	0.92
0.4	0.25	0.8	0.7	0.11	0.89
0.7	0.55	0.5	0.5	0.41	0.59
0.7	0.55	0.5	0.5	0.41	0.59
0.7	0.55	0.5	0.5	0.41	0.59
0.7	0.55	0.5	0.5	0.41	0.59
0.4	0.25	0.5	0.5	0.21	0.79
0.4	0.25	0.5	0.5	0.21	0.79
0.4	0.25	0.5	0.5	0.21	0.79
0.4	0.25	0.5	0.5	0.21	0.79
0.7	0.55	0.3	0.3	0.45	0.55
0.7	0.55	0.3	0.3	0.45	0.55
0.7	0.55	0.3	0.3	0.45	0.55
0.7	0.55	0.3	0.3	0.45	0.55
0.7	0.55	0.7	0.7	0.30	0.70
0.7	0.55	0.7	0.7	0.30	0.70
0.7	0.55	0.7	0.7	0.30	0.70
0.7	0.55	0.7	0.7	0.30	0.70

0.4	0.25	0.3	0.3	0.29	0.71
0.4	0.25	0.3	0.3	0.29	0.71
0.4	0.25	0.3	0.3	0.29	0.71
0.4	0.25	0.3	0.3	0.29	0.71
0.4	0.25	0.7	0.7	0.12	0.88
0.4	0.25	0.7	0.7	0.12	0.88
0.4	0.25	0.7	0.7	0.12	0.88
0.4	0.25	0.7	0.7	0.12	0.88

Table 3: simulations for $\overline{q_h}$ and $\bar{q}_c$ in case $\bar{\beta} = 0$, for different values of φ, τ , $\bar{\alpha}$

$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\bar{\boldsymbol{\alpha}}$	$\overline{\boldsymbol{q_h}}$	$\overline{\boldsymbol{q_c}}$
0.5	0.1	0.7	0.94	0.06
0.5	0.2	0.7	0.9	0.1
0.5	0.3	0.7	0.87	0.14
0.5	0.4	0.7	0.84	0.16
0.5	0.5	0.7	0.82	0.18
0.5	0.6	0.7	0.8	0.2
0.5	0.7	0.7	0.8	0.2
0.5	0.8	0.7	0.78	0.22
0.5	0.9	0.7	0.77	0.23
0.1	0.45	0.7	0.86	0.14
0.2	0.45	0.7	0.76	0.24
0.3	0.45	0.7	0.71	0.29
0.4	0.45	0.7	0.75	0.25
0.5	0.45	0.7	0.79	0.21
0.6	0.45	0.7	0.81	0.19
0.7	0.45	0.7	0.83	0.17
0.8	0.45	0.7	0.85	0.15

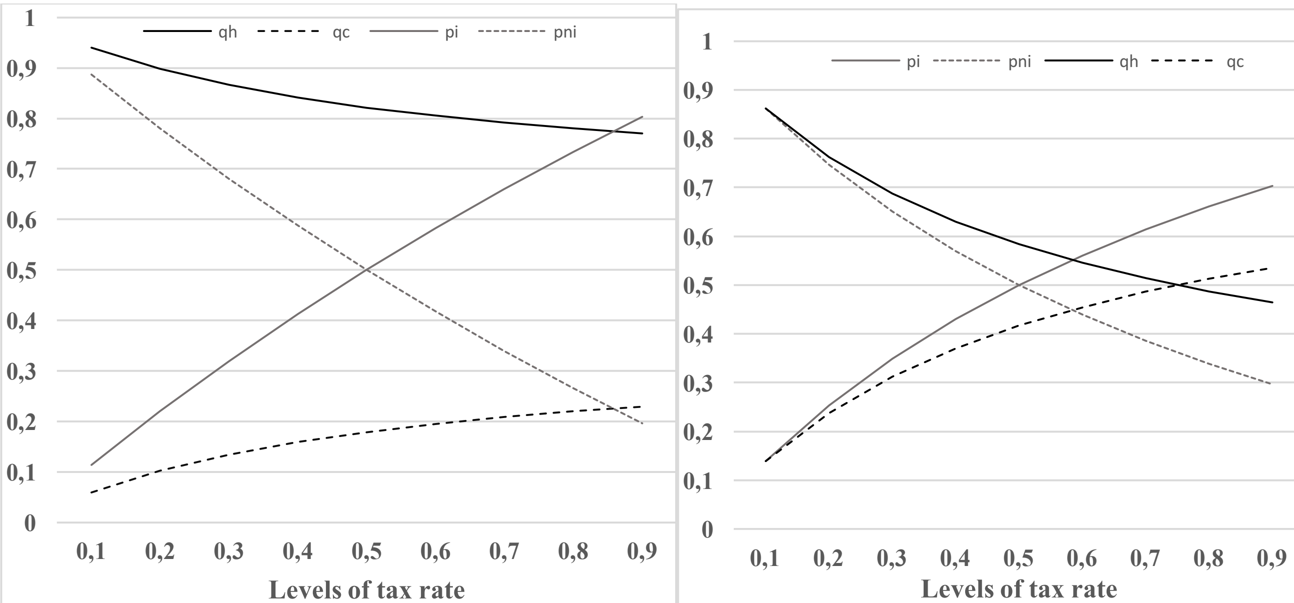
0.9	0.45	0.7	0.86	0.14

Table 4: simulations for $\overline{q_h}$ and $\bar{q}_c$ in case $0 < \bar{\beta} < 1$, for different values of φ, τ, $\bar{\alpha}$

$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\bar{\boldsymbol{\alpha}}$	$\overline{\boldsymbol{q_h}}$	$\overline{\boldsymbol{q_c}}$
0.5	0.1	0.7	0.91	0.09
0.5	0.2	0.7	0.85	0.15
0.5	0.3	0.7	0.79	0.21
0.5	0.4	0.7	0.75	0.25
0.5	0.5	0.7	0.72	0.28
0.5	0.6	0.7	0.69	0.31
0.5	0.7	0.7	0.67	0.33
0.5	0.8	0.7	0.65	0.35
0.5	0.9	0.7	0.63	0.37
0.1	0.45	0.7	0.51	0.49
0.2	0.45	0.7	0.60	0.40
0.3	0.45	0.7	0.66	0.34
0.4	0.45	0.7	0.70	0.30
0.5	0.45	0.7	0.74	0.26
0.6	0.45	0.7	0.76	0.24
0.7	0.45	0.7	0.78	0.22
0.8	0.45	0.7	0.80	0.20
0.9	0.45	0.7	0.82	0.18

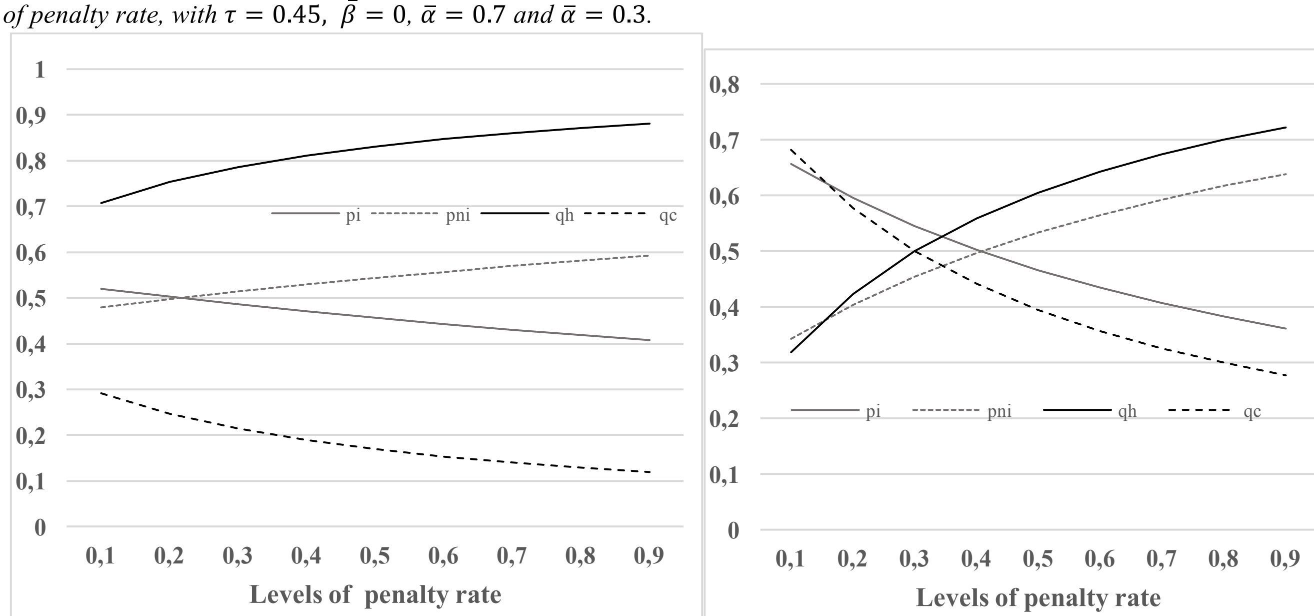
Section III: Simulations for the distribution of characters within the subpopulation of taxpayers and fiscal agencies for different levels of tax and penalty rate for $\overline{\beta} = 0$

Chart 1.a and 1.b: distribution of characters within the subpopulations of taxpayers and fiscal agencies for different levels of tax rate, with $\varphi = 0.5$, $\bar{\beta} = 0$, $\bar{\alpha} = 0.7$ and $\bar{\alpha} = 0.3$.



Distribution of characters within the subpopulation of tax agencies and taxpayers, for different levels of tax rate, with $\overline{\alpha} = 0.7$ (left) and $\overline{\alpha} = 0.3$ (right).

Chart 2.a and 2.b: distribution of characters within the subpopulations of taxpayers and fiscal agencies for different levels of penalty rate, with $\tau = 0.45$, $\bar{\beta} = 0$, $\bar{\alpha} = 0.7$ and $\bar{\alpha} = 0.3$.



Distribution of characters within the subpopulation of tax agencies and taxpayers, for different levels of penalty rate, with $\overline{\alpha} = 0.7$ (left) and $\overline{\alpha} = 0.3$ (right).

Section IV: comparative statics exercise on the equilibrium results for the model presented in Chapter 2

$$\frac{\partial \overline{q_h}}{\partial \tau} = \frac{-\varphi\overline{\alpha}C(1-\overline{\beta})}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} < 0$$

$$\frac{\partial \overline{q_c}}{\partial \tau} = \frac{\varphi\overline{\alpha}(C-\overline{\beta})}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} > 0$$

$$\frac{\partial \overline{q_h}}{\partial \varphi} = \frac{\tau\overline{\alpha}C(1-\overline{\beta})}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} > 0$$

$$\frac{\partial \overline{q_c}}{\partial \varphi} = \frac{-\tau\overline{\alpha}C(1-\overline{\beta})}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} < 0$$

$$\frac{\partial \overline{q_h}}{\partial C} = -\frac{\tau}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} < 0$$

$$\frac{\partial \overline{q_c}}{\partial C} = \frac{\tau}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} > 0$$

$$\frac{\partial \overline{q_h}}{\partial \overline{\alpha}} = \frac{\tau C[\tau+\varphi(1-\overline{\beta})]}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} > 0$$

$$\frac{\partial \overline{q_c}}{\partial \overline{\alpha}} = -\frac{\tau C[\tau+\varphi(1-\overline{\beta})]}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} < 0$$

$$\frac{\partial \overline{q_h}}{\partial \overline{\beta}} = -\frac{\tau C(\tau+\overline{\alpha}\varphi)}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} < 0$$

$$\frac{\partial \overline{q_c}}{\partial \overline{\beta}} = \frac{\tau C(\tau+\overline{\alpha}\varphi)}{[\overline{\alpha}(\tau+\varphi)-\overline{\beta}(\tau+\overline{\alpha}\varphi)]^2} > 0$$

$$\frac{\partial \overline{p_I}}{\partial \tau} = \frac{(1-\overline{\beta})[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}-\tau(1-\overline{\beta})]+\overline{\alpha}\tau(1-\overline{\beta})}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}]^2} > 0$$

$$\frac{\partial \overline{p_I}}{\partial \varphi} = \frac{-\tau(1-\overline{\beta})\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}]^2} < 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \tau} = \frac{-\overline{\alpha}[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}]-\left[\left(\varphi(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}(1-\tau+\overline{\beta}\varphi)\right)(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})\right]}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}]^2} < 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \varphi} = \frac{(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})[\tau(1-\overline{\beta}))]}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}]^2} > 0 \; for\; \bar{\alpha}\; and\; \bar{\beta} \neq 1$$

$$\frac{\partial \overline{p_I}}{\partial \overline{\alpha}} = -\frac{\tau(1-\overline{\beta})(1-\tau-\varphi(1-\overline{\beta}))}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}]^2} < 0$$

$$\frac{\partial \overline{p_I}}{\partial \overline{\beta}} = \frac{-\tau(\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}))-\tau(1-\overline{\beta})[-\tau-\varphi(1-\overline{\alpha})]}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau(1-\overline{\alpha}-\overline{\beta})+\overline{\alpha}]^2} < 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \overline{\alpha}} = \frac{\tau\left(1-\overline{\beta}\right)(1-\tau-\varphi(1-\overline{\beta})}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau\left(1-\overline{\alpha}-\overline{\beta}\right)+\overline{\alpha}]^2} > 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \overline{\beta}} = \frac{\tau\left(\varphi\left(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta}\right)+\tau\left(1-\overline{\alpha}-\overline{\beta}\right)+\overline{\alpha}\right)+\tau\left(1-\overline{\beta}\right)[-\tau-\varphi(1-\overline{\alpha})]}{[\varphi(1-\overline{\alpha}-\overline{\beta}+\overline{\alpha}\overline{\beta})+\tau\left(1-\overline{\alpha}-\overline{\beta}\right)+\overline{\alpha}]^2} > 0$$

Section V: mathematical and computational features of the model presented in Chapter 3

We start from differential equations (11) to (14) in Chapter 3

(11) $\frac{\dot{q_H}}{q_H} = f_T(e^1, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\,(f_T(e^2, s))\right]$

(12) $\frac{\dot{q_C}}{q_C} = f_T(e^2, s) - \left[q_H\left(f_T(e^1, s)\right) + q_C\,(f_T(e^2, s))\right]$

(13) $\frac{\dot{p_I}}{p_I} = f_A(e^1, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}(f_A(e^2, s))\right]$

(14) $\frac{\dot{p_{NI}}}{p_{NI}} = f_A(e^2, s) - \left[p_I\left(f_A(e^1, s)\right) + p_{NI}(f_A(e^2, s))\right]$

and we substitute (9) and (10) inside

$$\boldsymbol{f_A(e^a, T) = q_H\left[\pi_A\left(s_A^a, s_T^h\right)\right] + q_C[\pi_A(s_A^a, s_T^c)]} \quad \textbf{(9)}$$

$$\boldsymbol{f_T(e^b, A) = p_I[\pi_T\left(s_A^i, s_T^b\right)] + p_{NI}[\pi_T\left(s_A^{ni}, s_T^b\right)]} \quad \textbf{(10)}$$

leading to (11a) to (14a)

(11a) $\frac{\dot{q_H}}{q_H} = \{p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]\} - \{q_H[p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]] + q_C[p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]]\}$

(12a) $\frac{\dot{q_C}}{q_C} = \{p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]\} - \{q_H[p_I[\pi_T(s_A^i, s_T^h)] + p_{NI}[\pi_T(s_A^{ni}, s_T^h)]] + q_C[p_I[\pi_T(s_A^i, s_T^c)] + p_{NI}[\pi_T(s_A^{ni}, s_T^c)]]\}$

(13a) $\frac{\dot{p_I}}{p_I} = \{q_H[\pi_A(s_A^i, s_T^h)] + q_c[\pi_A(s_A^i, s_T^c)]\} - \Big\{p_I\Big[q_H[\pi_A(s_A^i, s_T^h)] + q_c[\pi_A(s_A^i, s_T^c)]\Big] +$

$p_{NI}\Big[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_c[\pi_A(s_A^{ni}, s_T^c)]\Big]\Big\}$

(14a) $\frac{\dot{p_{NI}}}{p_{NI}} = \{q_H[\pi_A(s_A^{ni}, s_T^h)] + q_c[\pi_A(s_A^{ni}, s_T^c)]\} - \Big\{p_I\Big[q_H[\pi_A(s_A^i, s_T^h)] + q_c[\pi_A(s_A^i, s_T^c)]\Big] +$

$p_{NI}\Big[q_H[\pi_A(s_A^{ni}, s_T^h)] + q_c[\pi_A(s_A^{ni}, s_T^c)]\Big]\Big\}$

We then substitute in (11a) to (14a) the payoffs (1) to (8), obtaining

(11a.1) $\frac{\dot{q_H}}{q_H} = \{p_I[(1-\tau)Y - m\tau Y] + p_{NI}[(1-\tau)Y]\}$–$\Big\{q_H[p_I[(1-\tau)Y - m\tau Y] + p_{NI}[(1-$

$\tau)Y]] + q_C\Big[p_I[\Big[[(1-\tau)Y \underbrace{- \varphi\tau\beta Y}]\ (1-\ \alpha) - m\tau\beta Y\Big]] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\Big]\Big\}$

(12a.1) $\frac{\dot{q_C}}{q_C} = \Big\{p_I[\Big[[(1-\tau)Y \underbrace{- \varphi\tau\beta Y}]\ (1-\ \alpha) - m\tau\beta Y\Big]] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\Big\}$–$\Big\{q_H[p_I[(1-\tau)Y -$

$m\tau Y] + p_{NI}[(1-\tau)Y]] + q_C\Big[p_I[\Big[[(1-\tau)Y \underbrace{- \varphi\tau\beta Y}]\ (1-\ \alpha) - m\tau\beta Y\Big]] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\Big]\Big\}$

(13a.1) $\frac{\dot{p_I}}{p_I} = \Big\{q_h[\tau Y - C\tau Y\ ] + q_c\Big[\Big[\tau Y + \underbrace{\varphi(1-\beta)Y}\Big]\alpha - C\tau Y\ \Big]\Big\} - \Big\{p_I\Big[q_h[\tau Y - C\tau Y] + q_c\Big[\ \Big[\tau Y +$

$\underbrace{\varphi(1-\beta)Y}\Big]\alpha - C\tau Y\Big]\Big] + p_{NI}\Big[q_h[\tau Y] + q_c\Big[\underbrace{\tau\beta Y} - n[\Big(\varphi\,\underbrace{\tau(1-\beta)Y}\Big) + \tau(1-\beta)Y]\Big]\Big]\Big\}$

(14a.1) $\frac{\dot{p_{NI}}}{p_{NI}} = \Big\{q_H[\tau Y] + q_c\Big[\underbrace{\tau\beta Y} - n[\Big(\varphi\,\underbrace{\tau(1-\beta)Y}\Big) + \tau(1-\beta)Y]\Big]\Big\} - \Big\{p_I\Big[q_h[\tau Y - C\tau Y] +$

$q_c\Big[\ \Big[\tau Y + \underbrace{\varphi(1-\beta)Y}\Big]\alpha - C\tau Y\Big]\Big] + p_{NI}\Big[q_h[\tau Y] + q_c\Big[\underbrace{\tau\beta Y} - n[\Big(\varphi\,\underbrace{\tau(1-\beta)Y}\Big) + \tau(1-\beta)Y]\Big]\Big]\Big\}$

We then set $\frac{\dot{q_H}}{q_H} = \frac{\dot{q_C}}{q_C} = \frac{\dot{p_I}}{p_I} = \frac{\dot{p_{NI}}}{p_{NI}} = 0$, to find the equilibrium(s), which are represented by the situation where the distribution of characters in the population does not change. To find the solutions in equilibrium we have therefore to solve the following system of four equations (11a.1) to (14a.1) for p_I, p_{NI}, q_H and q_C

(11a.1) $$\{p_I[(1-\tau)Y - m\tau Y] + p_{NI}[(1-\tau)Y]\} - \Big\{q_H[p_I[(1-\tau)Y - m\tau Y] + p_{NI}[(1-\tau)Y]] + q_C\Big[p_I\Big[\Big[[(1-\tau)Y \underbrace{- \varphi\tau\beta Y}]\,(1-\alpha) - m\tau\beta Y\Big]\Big] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\Big]\Big\} = 0$$

(12a.1) $$\Big\{p_I\Big[\Big[[(1-\tau)Y \underbrace{- \varphi\tau\beta Y}]\,(1-\alpha) - m\tau\beta Y\Big]\Big] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\Big\} - \Big\{q_H[p_I[(1-\tau)Y - m\tau Y] + p_{NI}[(1-\tau)Y]] + q_C\Big[p_I\Big[\Big[[(1-\tau)Y \underbrace{- \varphi\tau\beta Y}]\,(1-\alpha) - m\tau\beta Y\Big]\Big] + p_{NI}[Y(1-\underbrace{\tau\beta)}]\Big]\Big\} = 0$$

(13a.1) $$\Big\{q_h[\tau Y - C\tau Y] + q_c\Big[\Big[\tau Y + \underbrace{\varphi(1-\beta)Y}\Big]\alpha - C\tau Y\Big]\Big\} - \Big\{p_I\Big[q_h[\tau Y - C\tau Y] + q_c\Big[\Big[\tau Y + \underbrace{\varphi(1-\beta)Y}\Big]\alpha - C\tau Y\Big]\Big] + p_{NI}\Big[q_h[\tau Y] + q_c\Big[\underbrace{\tau\beta Y} - n\Big[\Big(\varphi\,\underbrace{\tau(1-\beta)Y}\Big) + \tau(1-\beta)Y\Big]\Big]\Big]\Big\} = 0$$

(14a.1) $$\Big\{q_H[\tau Y] + q_c\Big[\underbrace{\tau\beta Y} - n\Big[\Big(\varphi\,\underbrace{\tau(1-\beta)Y}\Big) + \tau(1-\beta)Y\Big]\Big]\Big\} - \Big\{p_I\Big[q_h[\tau Y - C\tau Y] + q_c\Big[\Big[\tau Y + \underbrace{\varphi(1-\beta)Y}\Big]\alpha - C\tau Y\Big]\Big] + p_{NI}\Big[q_h[\tau Y] + q_c\Big[\underbrace{\tau\beta Y} - n\Big[\Big(\varphi\,\underbrace{\tau(1-\beta)Y}\Big) + \tau(1-\beta)Y\Big]\Big]\Big]\Big\} = 0$$

For solving the system of equations, the model has been written in Matlab. First, the set of variables and parameters has been declared, together with fitness functions (9) and (10). Then the system of differential equations (11a.1)-(14a.1) above has been presented, therefore setting them equal to zero. It has been declared in the script to solve the system for q_H, q_c, p_I and p_{NI}.

Section VI: simulations on the solutions of the model presented in Chapter 3

Table 1: simulations for $\overline{p_I}$ and $\overline{p_{NI}}$ in case $\bar{\beta} = 0$, for different values of φ, τ and $\bar{\alpha}$

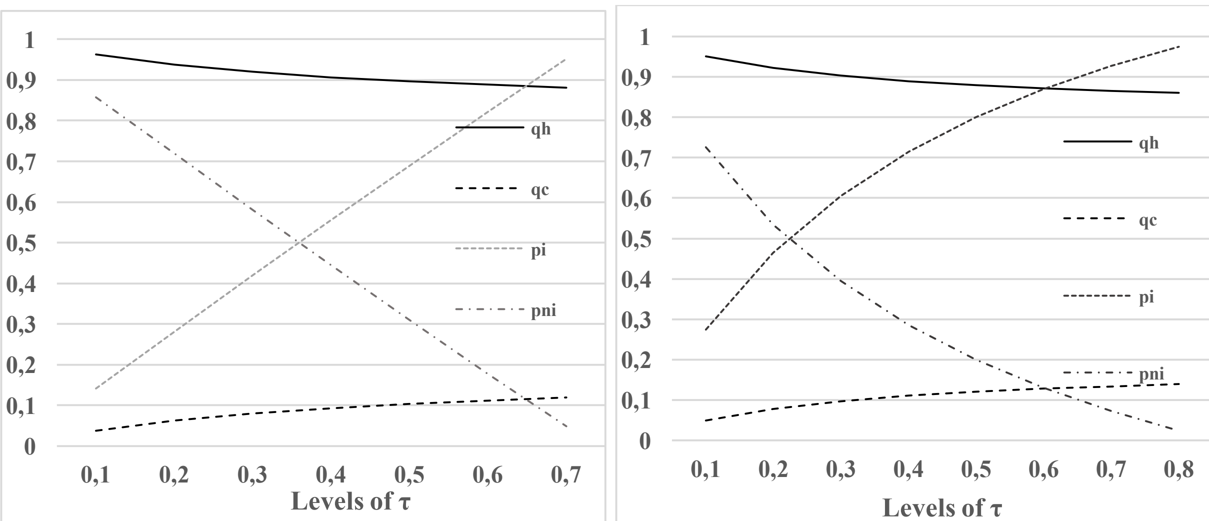
$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\bar{\boldsymbol{\alpha}}$	$\boldsymbol{m}$	$\boldsymbol{n}$	$\overline{\boldsymbol{p_I}}$	$\overline{\boldsymbol{p_{NI}}}$
0.4	0.1	0.7	0.1	0.3	0.14	0.86
0.4	0.2	0.7	0.2	0.3	0.27	0.73
0.4	0.3	0.7	0.5	0.3	0.44	0.56
0.4	0.4	0.7	0.6	0.3	0.64	0.36
0.2	0.4	0.5	0.3	0.5	0.65	0.35
0.3	0.5	0.5	0.3	0.5	0.74	0.26
0.4	0.2	0.5	0.3	0.5	0.34	0.66

Table 2: simulations for $\overline{p_I}$ and $\overline{p_{NI}}$ in case $0 < \bar{\beta} < 1$ and $0 < \bar{\alpha} < 1$, for different values of $\varphi, \tau, \bar{\alpha}$, m and n

$\boldsymbol{\varphi}$	$\boldsymbol{\tau}$	$\bar{\boldsymbol{\alpha}}$	$\bar{\boldsymbol{\beta}}$	$\boldsymbol{m}$	$\boldsymbol{n}$	$\overline{\boldsymbol{p_I}}$	$\overline{\boldsymbol{p_{NI}}}$
0.5	0.1	0.7	0.3	0.4	0.6	0.10	0.90
0.5	0.2	0.7	0.3	0.4	0.6	0.21	0.79
0.5	0.3	0.7	0.3	0.4	0.6	0.32	0.68
0.5	0.4	0.7	0.3	0.4	0.6	0.44	0.56
0.5	0.5	0.7	0.3	0.4	0.6	0.57	0.43
0.5	0.6	0.7	0.3	0.4	0.6	0.71	0.29
0.5	0.7	0.7	0.3	0.4	0.6	0.85	0.15
0.5	0.4	0.4	0.3	0.4	0.6	0.57	0.43
0.5	0.5	0.4	0.3	0.4	0.6	0.68	0.32
0.5	0.6	0.4	0.3	0.4	0.6	0.78	0.22
0.5	0.7	0.4	0.3	0.4	0.6	0.87	0.13

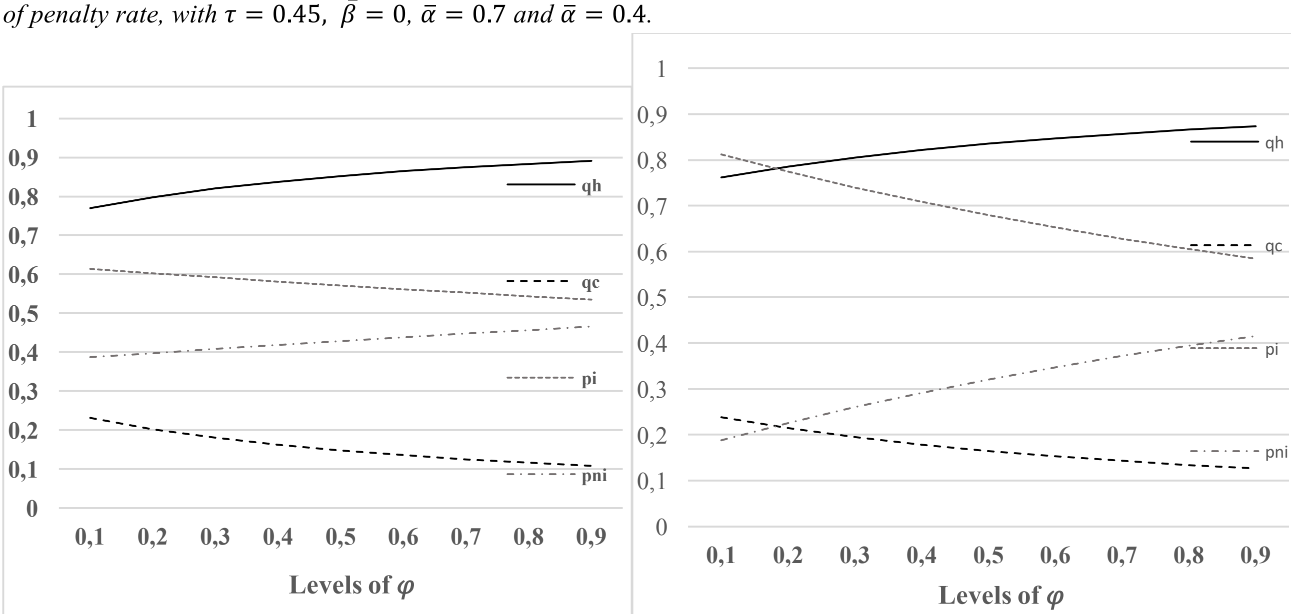
Section VII: Simulations for the distribution of characters within the sub-population of taxpayers and fiscal agencies for different levels of tax and penalty rate for $\bar{\beta} = 0$

Chart 1.a and 1.b: distribution of characters within the sub-populations of taxpayers and fiscal agencies for different levels of tax rate, with $\varphi = 0.5$, $\bar{\beta} = 0$, $\bar{\alpha} = 0.7$ and $\bar{\alpha} = 0.3$.



Distribution of characters within the sub-population of tax agencies and taxpayers, for different levels of tax rate, with $\overline{\alpha} = 0.7$ (left) and $\overline{\alpha} = 0.3$ (right).

Chart 2.a and 2.b: distribution of characters within the sub-populations of taxpayers and fiscal agencies for different levels of penalty rate, with $\tau = 0.45$, $\bar{\beta} = 0$, $\bar{\alpha} = 0.7$ and $\bar{\alpha} = 0.4$.



Distribution of characters within the sub-population of tax agencies and taxpayers, for different levels of penalty rate, with $\overline{\alpha} = 0.7$ (left) and $\overline{\alpha} = 0.4$ (right).

Section VIII: comparative statics exercise on the equilibrium results for the model presented in Chapter 3

$$\frac{\partial \overline{q_h}}{\partial \tau} = -\frac{C(\bar{\alpha}\varphi + n\varphi)(1-\bar{\beta})}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} < 0$$

$$\frac{\partial \overline{q_c}}{\partial \tau} = \frac{C(\bar{\alpha}\varphi + n\varphi)(1-\bar{\beta})}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} > 0$$

$$\frac{\partial \overline{q_h}}{\partial \varphi} = \frac{\tau C(1-\bar{\beta})(\bar{\alpha}+n)}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} > 0$$

$$\frac{\partial \overline{q_c}}{\partial \varphi} = \frac{-\tau C(1-\bar{\beta})(\bar{\alpha}+n)}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} < 0$$

$$\frac{\partial \overline{q_h}}{\partial C} = -\frac{\tau}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})} < 0$$

$$\frac{\partial \overline{q_c}}{\partial C} = \frac{\tau}{(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})} > 0$$

$$\frac{\partial \overline{q_h}}{\partial \bar{\alpha}} = \frac{\tau C[\varphi(1-\bar{\beta})]}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} > 0$$

$$\frac{\partial \overline{q_c}}{\partial \bar{\alpha}} = -\frac{\tau C[\varphi(1-\bar{\beta})]}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} < 0$$

$$\frac{\partial \overline{q_h}}{\partial \bar{\beta}} = \frac{-\tau C(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} < 0$$

$$\frac{\partial \overline{q_c}}{\partial \bar{\beta}} = \frac{\tau C(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} > 0$$

$$\frac{\partial \overline{q_h}}{\partial n} = \frac{\tau C\left((\tau+\varphi)(1-\bar{\beta})\right)}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} > 0$$

$$\frac{\partial \overline{q_c}}{\partial n} = \frac{-\tau C\left((\tau+\varphi)(1-\bar{\beta})\right)}{[(\tau+\bar{\alpha}\varphi+n\varphi+n\tau)(1-\bar{\beta})]^2} < 0$$

$$\frac{\partial \overline{p_I}}{\partial \tau} = \frac{\bar{\alpha}(1-\bar{\beta})}{[\bar{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\bar{\alpha}\varphi\tau)(1-\bar{\beta})]^2} > 0$$

$$\frac{\partial \overline{p_I}}{\partial \varphi} = \frac{-\tau^2(1-\bar{\beta})^2(1-\bar{\alpha})}{[\bar{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\bar{\alpha}\varphi\tau)(1-\bar{\beta})]^2} < 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \tau} = -\frac{\bar{\alpha}(1-\bar{\beta})}{[\bar{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\bar{\alpha}\varphi\tau)(1-\bar{\beta})]^2} < 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \varphi} = \frac{\tau^2(1-\bar{\beta})^2(1-\bar{\alpha})}{[\bar{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\bar{\alpha}\varphi\tau)(1-\bar{\beta})]^2} > 0$$

$$\frac{\partial \overline{p_I}}{\partial \overline{\alpha}} = \frac{-\tau(1-\overline{\beta})(1-\tau-\varphi\tau(1-\overline{\beta}))}{[\overline{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\overline{\alpha}\varphi\tau)(1-\overline{\beta})]^2} < 0$$

$$\frac{\partial \overline{p_I}}{\partial \overline{\beta}} = \frac{-\tau(1-\tau)\overline{\alpha}}{[\overline{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\overline{\alpha}\varphi\tau)(1-\overline{\beta})]^2} < 0$$

$$\frac{\partial \overline{p_I}}{\partial m} = \frac{\tau^2(1-\overline{\beta})^2}{[\overline{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\overline{\alpha}\varphi\tau)(1-\overline{\beta})]^2} > 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \overline{\alpha}} = \frac{\tau(1-\overline{\beta})(1-\tau-\varphi\tau(1-\overline{\beta}))}{[\overline{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\overline{\alpha}\varphi\tau)(1-\overline{\beta})]^2} > 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial \overline{\beta}} = \frac{\tau(1-\tau)\overline{\alpha}}{[\overline{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\overline{\alpha}\varphi\tau)(1-\overline{\beta})]^2} > 0$$

$$\frac{\partial \overline{p_{NI}}}{\partial m} = \frac{-\tau^2(1-\overline{\beta})^2}{[\overline{\alpha}(1-\tau)+(\tau-m\tau+\varphi\tau-\overline{\alpha}\varphi\tau)(1-\overline{\beta})]^2} < 0$$

Annex II

Section I

Translation of Part 1 of the experiment – Groups 1 and 2

Introduction

Welcome!
Thank you for signing up to the experiment. It will occur only few minutes to complete it.
For the participation, you will earn a voucher to be spent at the Bookstore LaFeltrinelli. The amount of money rewarded will be proportional to one extracted payoff out of all payoffs obtained in the game.
Enjoy it!

Part 1

Suppose you have 50 euros, and a lottery where you have 50% chances to win. For example, consider the toss of a coin. If heads comes out, you win, otherwise you lose.
If heads comes out, you will earn the amount given multiplied by 2,5.
If tails comes out, you will loose the entire amount spent in the lottery.
How much will you spend in the lottery? Please enter the amount in the blank space below.
Thank you very much for your answer.
Now the second part begins!

Translation of Part 2 of the experiment – Group 1

Instructions – rounds 1 to 9

In this game, you are part of a community formed by all students at the Faculty of Economics at the University "La Sapienza" in Rome.
All members in your community have an income of 1000 euros. All members of the community are subject to a tax, based on the income declared to fiscal authorities, to finance public expenditure in national health care.
In the experiment, behave as a member of this community.
The payment of the taxes, and therefore your tax declaration, will be repeated over time. Thus, you will be asked to declare your income to fiscal authorities for multiple, subsequent rounds.

Each 9 rounds, the tax rate or the penalty rate would change (increase or decrease). Each time the tax rate or the penalty rate or both change, the game is repeated 9 rounds, in each of them you will have to declare your income to pay taxes, and finance public healthcare system.
The tax will be calculated by subtracting 45% of the income declared.
All members of the community may be inspected by fiscal authorities, with a probability of 0.5, to avoid tax evasion and similar behaviour.
If you declare less than your actual income and you are inspected, you will have to pay ALL the tax due, PLUS a penalty of 45% of the amount evaded.

Test session – rounds 1 to 9

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared}) - \mathbf{0.45}\ ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$

Click on "BACK" for return to the test session.

Round 1

Now the game starts!
Write in the blank space below how much you want to declare for the tax payment.

The tax is calculated by taking the 45% of the income declared.

Inspections take place in 50% of cases, to avoid evasion.
In case of inspection, and caught cheating, the fine applied will be 45%.

Please remind that inspections take place for 50% of the taxpayers, to avoid evasion and elusive behavior.

In case you are inspected and caught cheating, you will pay a fine equal to 45% of the evaded tax, and you will have to pay ALL the due tax.

To confirm your income declared, and proceed, click on the "NEXT" Button.

After Round 1 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.45} * \mathbf{1000}\}$**.** Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.45} * \mathbf{1000 - 0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$**.** Please click on the "NEXT" Button to continue the game.

Rounds 2 to 9

Write in the blank space below how much you want to declare for the tax payment.

Please remind that the tax is calculated by taking the 45% of the income declared.

Remind also that inspections take place in 50% of cases, to avoid evasion.

In case of inspection, and caught cheating, the fine applied will be 45% of the evaded tax.

In case you are inspected and caught cheating, you will pay a fine equal to 45% of the tax evaded, and you will have to pay ALL the due tax.

To confirm your income declared, and proceed, click on the "NEXT" Button.

After each Round from 2 to 9 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 -\ 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 -\ 0.45} * \mathbf{1000}\}$**.** Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 -\ 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 -\ 0.45} * \mathbf{1000 - 0.45} * (\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45}\}$**.** Please click on the "NEXT" Button to continue the game.

Before round 10 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the “NEXT” Button to continue the game!

- *After the “NEXT” Button –*

For you the tax rate increases from 45% to 60%, whereas the penalty rate remains at 45%.

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 -\ 0.6} * (\boldsymbol{income\ declared}) - \mathbf{0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.6})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 -\ 0.6} * (\boldsymbol{income\ declared})\}$

Click on “BACK” for return to the test session.

Rounds 10 to 18

Suppose now that the tax is calculated by taking the 60% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 45% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the “NEXT” Button.

After each Round from 10 to 18 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 -\ 0.6} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6 * 1000}\}$**.** Please click on the “NEXT” Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6 * 1000 - 0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.6})\}$**.** Please click on the “NEXT” Button to continue the game.

Before round 19 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the “NEXT” Button to continue the game!

- *After the “NEXT” Button –*

For you the tax rate decreases from 45% to 25%, whereas the penalty rate remains at 45%.

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - \ 0.25} * (\boldsymbol{income\ declared}) - \mathbf{0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.25})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - \ 0.25} * (\boldsymbol{income\ declared})\}$

Click on “BACK” for return to the test session.

Rounds 19 to 27

Suppose now that the tax is calculated by taking the 25% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 45% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the “NEXT” Button.

After each Round from 19 to 27 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following was displayed

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.25} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.25 * 1000}\}$**.** Please click on the “NEXT” Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following answers was shown

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.25} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.25} * \mathbf{1000 - 0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.25})\}$. Please click on the "NEXT" Button to continue the game.

Before round 28 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the "NEXT" Button to continue the game!

- *After the "NEXT" Button –*

For you the both the tax rate increases from 45% to 60%, and the penalty rate goes at 25%

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - 0.6} * (\boldsymbol{income\ declared}) - \mathbf{0.25} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.6})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - 0.6} * (\boldsymbol{income\ declared})\}$

Click on "BACK" for return to the test session.

Rounds 28 to 36

Suppose now that the tax is calculated by taking the 60% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 25% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the "NEXT" Button.

After each Round from 28 to 36 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.25} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.25} * \mathbf{1000}\}$**.** Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.6} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6 * 1000 - 0.25 * ((1000 - \mathit{income\ declared}) * 0.6)\}}$. Please click on the "NEXT" Button to continue the game.

Before round 37 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the "NEXT" Button to continue the game!

- *After the "NEXT" Button –*

For you the both the tax rate decreases from 45% to 25%, and the penalty rate goes at 25%

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - \ 0.25 * (\mathit{income\ declared}) - 0.25 * ((1000 - \mathit{income\ declared}) * 0.25)}\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - \ 0.25 * (\mathit{income\ declared})}\}$

Click on "BACK" for return to the test session.

Rounds 37 to 45

Suppose now that the tax is calculated by taking the 25% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 25% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the "NEXT" Button.

After each Round from 27 to 45 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.25} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.25} * \mathbf{1000}\}$**.** Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.25} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.25} * \mathbf{1000} - \mathbf{0.25} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.25})\}$**.** Please click on the "NEXT" Button to continue the game.

Translation of Part 2 of the experiment – Group 2

Instructions – rounds 1 to 9

In this game, you are part of a community formed by all students at the Faculty of Economics at the University "La Sapienza" in Rome.

All members in your community have an income of 1000 euros. All members of the community are subject to a tax, based on the income declared to fiscal authorities, to finance public expenditure in national health care.
In the experiment, behave as a member of this community.
The payment of the taxes, and therefore your tax declaration, will be repeated over time. Thus, you will be asked to declare your income to fiscal authorities for multiple, subsequent rounds.
Each 9 rounds, the tax rate or the penalty rate would change (increase or decrease). Each time the tax rate or the penalty rate or both change, the game is repeated 9 rounds, in each of them you will have to declare your income to pay taxes, and finance public healthcare system.
The tax will be calculated by subtracting 45% of the income declared.
All members of the community may be inspected by fiscal authorities, with a probability of 0.5, to avoid tax evasion and similar behaviour.
If you declare less than your actual income and you are inspected, you will have to pay ALL the tax due, PLUS a penalty of 45% of the amount of taxes evaded.

Test session – rounds 1 to 9

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared}) - \mathbf{0.45}\ ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$

Click on “BACK” for return to the test session.

Round 1

Now the game starts!
Write in the blank space below how much you want to declare for the tax payment.

The tax is calculated by taking the 45% of the income declared.

Inspections take place in 50% of cases, to avoid evasion.
In case of inspection, and caught cheating, the fine applied will be 45% of the evaded tax.

Please remind that inspections take place for 50% of the taxpayers, to avoid evasion and elusive behavior.

In case you are inspected and caught cheating, you will pay a fine equal to 45% of the tax evaded, and you will have to pay ALL the due tax.

To confirm your income declared, and proceed, click on the “NEXT” Button.

After Round 1 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45 * 1000}\}$**.** Please click on the “NEXT” Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45 * 1000 - 0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$**.** Please click on the “NEXT” Button to continue the game.

Rounds 2 to 9

Write in the blank space below how much you want to declare for the tax payment.

Please remind that the tax is calculated by taking the 45% of the income declared.

Remind also that inspections take place in 50% of cases, to avoid evasion.
In case of inspection, and caught cheating, the fine applied will be 45% on the tax evaded.

In case you are inspected and caught cheating, you will pay a fine equal to 45% of the tax evaded, and you will have to pay ALL the due tax.

To confirm your income declared, and proceed, click on the “NEXT” Button.

After each Round from 2 to 9 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45 * 1000}\}$**.** Please click on the “NEXT” Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45 * 1000 - 0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$**.** Please click on the “NEXT” Button to continue the game.

Before round 10 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the “NEXT” Button to continue the game!

- *After the “NEXT” Button* –

For you the penalty rate increases from 45% to 60%, whereas the tax rate remains at 45%.

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared}) - \mathbf{0.6} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$

Click on “BACK” for return to the test session.

Rounds 10 to 18

Suppose now that the tax is calculated by taking the 45% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 60% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the “NEXT” Button.

After each Round from 10 to 18 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.45} * \mathbf{1000}\}$**.** Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000-\ 0.45} * \mathbf{1000} - \mathbf{0.6} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$**.** Please click on the "NEXT" Button to continue the game.

Before round 19 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the "NEXT" Button to continue the game!

- *After the "NEXT" Button –*

For you the penalty rate decreases from 45% to 25%, whereas the tax rate remains at 45%.

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - \ 0.45} * (\boldsymbol{income\ declared}) - \mathbf{0.25} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - \ 0.45} * (\boldsymbol{income\ declared})\}$

Click on "BACK" for return to the test session.

Rounds 19 to 27

Suppose now that the tax is calculated by taking the 45% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 25% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the "NEXT" Button.

After each Round from 19 to 27 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * \mathbf{1000}\}$. Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * \mathbf{1000} - \mathbf{0.25} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$. Please click on the "NEXT" Button to continue the game.

Before round 28 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the "NEXT" Button to continue the game!

- *After the "NEXT" Button –*

For you the both the tax rate decreases from 45% to 25%, and the penalty rate goes at 60%.

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - 0.25} * (\boldsymbol{income\ declared}) - \mathbf{0.6} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.25})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - 0.25} * (\boldsymbol{income\ declared})\}$

Click on "BACK" for return to the test session.

Rounds 28 to 36

Suppose now that the tax is calculated by taking the 25% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 60% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the "NEXT" Button.

After each Round from 28 to 36 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.25} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.25 * 1000}\}$**.** Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.25} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.25} * \mathbf{1000 - 0.6} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.25})\}$. Please click on the "NEXT" Button to continue the game.

Before round 37 started, this messages were displayed to each participant in Group 1

Now a new game begins.

The tax rate applied on income for financing public health expenditure, or the penalty rate may vary.

Please click on the "NEXT" Button to continue the game!

- *After the "NEXT" Button* –

For you both the tax rate increases from 45% to 60%, and the penalty rate goes at 60%.

Test it! You can try different scenarios for different levels of income declared, in case you are controlled or in case you are not subject to controls. Please, fill the blank space with the income you want to declare.

****

If you are inspected, your disposable income will be $\{\mathbf{1000 - \ 0.6} * (\boldsymbol{income\ declared}) - \mathbf{0.6} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.6})\}$

If you are not inspected, your disposable income will be $\{\mathbf{1000 - \ 0.6} * (\boldsymbol{income\ declared})\}$

Click on “BACK” for return to the test session.

Rounds 37 to 45

Suppose now that the tax is calculated by taking the 60% of the income declared.

Please remind that inspections take place in 50% of cases, to avoid evasion.

Please also remind that, in case you are inspected and caught cheating, you will pay a fine equal to 60% of the tax evaded, and you will have to pay ALL the due tax.

Write in the blank space below how much you want to declare for the tax payment.

To confirm your income declared, and proceed, click on the “NEXT” Button.

After each Round from 27 to 45 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6} * \mathbf{1000}\}$**.** Please click on the “NEXT” Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6} * (\boldsymbol{income\ declared})\}$. Please click on the “NEXT” Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - \ 0.6 * 1000 - 0.6 * ((1000 - \mathit{income\ declared}) * 0.6)\}}$. Please click on the "NEXT" Button to continue the game.

Section II – Experiment set up in Qualtrics

The experiment was entirely programmed and conducted in Qualtrics. Particularly, both the tables *a* and *b* below represent a section of the survey flow that has been created for each round, and for the two groups of the experimental design.

Table a: Qualtrics survey flow for Group 1

EmbeddedData
    Random ID = ${rand://int/1:10000}

Block: INTRO (2 Questions)
Standard: BLOCCO_INTRO2 (5 Questions)
Standard: ROUND 1 - 1 (1 Question)

BlockRandomizer: 1 - Evenly Present Elements

- Group: CONTROL_1
  - Block: ROUND 1 -1.1 (1 Question)
  - EmbeddedData
    Group = Control_1
- Group: TREATMENT_1
  - EmbeddedData
    Group = Treatment_1
  - Branch: New Branch
    If
    If Inizia il gioco! Scrivi nello spazio qui sotto quanto vuoi dichiarare ai fini del pagamento dell... Text Response Is Equal to 1000
    - Block: ROUND 1 -1.2 (1 Question)
  - Branch: New Branch
    If
    If Inizia il gioco! Scrivi nello spazio qui sotto quanto vuoi dichiarare ai fini del pagamento dell... Text Response Is Less Than 1000
    - Block: ROUND 1 -1.3 (1 Question)
    - To be repeated for all rounds, for all combinations of tax and penalty rates

Standard: BLOCCO RANDOMID (1 Question)

Table b: Qualtrics survey flow for Group 2

EmbeddedData
    Random ID = ${rand://int/1:10000}

Block: INTRO (2 Questions)
Standard: BLOCCO_INTRO2 (5 Questions)
Standard: ROUND 1 - 1 (1 Question)

BlockRandomizer: 1 - Evenly Present Elements

- **Group: CONTROL_1**
  - **Block: ROUND 1 -1.1 (1 Question)**
  - **EmbeddedData**
    Group = Control_1
- **Group: TREATMENT_1**
  - **EmbeddedData**
    Group = Treatment_1
  - **Branch: New Branch**
    If
    If Inizia il gioco! Scrivi nello spazio qui sotto quanto vuoi dichiarare ai fini del pagamento del... Text Response Is Equal to 1000
    - **Block: ROUND 1 -1.2 (1 Question)**
  - **Branch: New Branch**
    If
    If Inizia il gioco! Scrivi nello spazio qui sotto quanto vuoi dichiarare ai fini del pagamento del... Text Response Is Less Than 1000
    - **Block: ROUND 1 -1.3 (1 Question) Block: ROUND 1 -1.3 (1 Question)**
    - **To be repeated for all rounds, for all combinations of tax and penalty rates**

Standard: BLOCCO RANDOMID (1 Question)

As it can be easily seen, survey flows for Groups 1 and 2 of the experiments are characterized by the same structure, therefore the only change between the two groups survey flow is only related to the combinations of tax and penalty rates presented to individuals over the rounds.

We now explain the programming within Qualtrics represented in both tables above.

First, both the commands

```
EmbeddedData
    Random ID = ${rand://int/1:10000}
```

and at the end of the table

```
Standard: BLOCCO RANDOMID (1 Question)
```

are the ones related to the assignment of the random ID number used for payments.

We then see

```
Block: INTRO (2 Questions)
Standard: BLOCCO_INTRO2 (5 Questions)
```

These two blocks are common to both tables *a)* and *b)*, and include the *Introduction*, *Part 1* of the experiment, and the "*Test it!*" session of *Part 2* of the experiment.

BlockRandomizer: 1 - Evenly Present Elements
Group: CONTROL_1
Block: ROUND 1 -1.1 (1 Question)
EmbeddedData Group = Control_1
Group: TREATMENT_1
EmbeddedData Group = Treatment_1
Branch: New Branch If If Inizia il gioco! Scrivi nello spazio qui sotto quanto vuoi dichiarare ai fini del pagamento del... Text Response Is Equal to 1000
Block: ROUND 1 -1.2 (1 Question)
Branch: New Branch If If Inizia il gioco! Scrivi nello spazio qui sotto quanto vuoi dichiarare ai fini del pagamento del... Text Response Is Less Than 1000
Block: ROUND 1 -1.3 (1 Question) Block: ROUND 1 -1.3 (1 Question)

The blocks above represent instead:

"Round 1

Now the game starts!
Write in the blank space below how much you want to declare for the tax payment.

The tax is calculated by taking the 45% of the income declared.

Inspections take place in 50% of cases, to avoid evasion.
In case of inspection, and caught cheating, the fine applied will be 45%.

_

Please remind that inspections take place for 50% of the taxpayers, to avoid evasion and elusive behavior.

In case you are inspected and caught cheating, you will pay a fine equal to 45% of the evaded tax, and you will have to pay ALL the due tax.

To confirm your income declared, and proceed, click on the "NEXT" Button.

After Round 1 – to each participant was shown the following

In case of honest individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * \mathbf{1000}\}$. Please click on the "NEXT" Button to continue the game.

In case of cheating individuals, with probability 0.5 each, one of the following

You were not subject to inspections. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * (\boldsymbol{income\ declared})\}$. Please click on the "NEXT" Button to continue the game.

You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is $\{\mathbf{1000 - 0.45} * \mathbf{1000} - \mathbf{0.45} * ((\mathbf{1000} - \boldsymbol{income\ declared}) * \mathbf{0.45})\}$. Please click on the "NEXT" Button to continue the game."

In practice, to create the setting where after each round the participant would be inspected with the exact probability 0.5 (and accordingly informed with a message), the commands "*Randomizer*", "Group" and "*Embedded data*" have been used. Particularly, at each round, first the randomizer was introduced, allowing each of the two scenarios (inspect or not inspect) to happen with probability 0.5. Then, the command "*Group*" has been used, dividing into "Control" and "Treatment", where "Control" is displayed with probability 0.5 to the participant, and it contains the text "*You were not subject to inspections. Therefore, your disposable income after taxes is* $\{1000 - tax\,rate * (income\ declared)\}$. *Please click on the "NEXT" Button to continue the game*". "*Treatment*", instead, also shown with probability 0.5 to the participant was divided into two sub-cases:

- <u>In case of honest individuals</u>: in this case, the text contained was "*You were subject to inspections. Because you declared the entire amount given to you, only the due tax will be subtracted from your income. Therefore, your disposable income after taxes is* $\{1000 - \ tax\,rate * 1000\}$. *Please click on the "NEXT" Button to continue the game*".

- <u>In case of cheating individuals</u>: the text contained was "*You were subject to inspections. Because you declared less than your income, you will have to pay 45% of the evaded tax, plus the tax of 45% on your income. Therefore, your disposable income after taxes is* $\{1000 - \ tax\,rate * 1000 - penalty\,rate * ((1000 - income\ declared) * tax\,rate)\}$. *Please click on the "NEXT" Button to continue the game*".

This procedure has been followed for all rounds of the experiment, for each combination of tax and penalty rate.

Moreover, each time the tax rate or penalty rate or both were changed, a block was introduced, containing the message informing the participant of the changes in tax rate, penalty rate, or both.

Section III – Payments

According to what has been declared to participants during the experiment, the payment for the participation is an amount of money proportional to the results obtained during the rounds of the experiment, randomly extracted among the results.

Practically, for each participant, the results of all rounds have been taken, and among them, one result was chosen.

Afterwards, ranges have been created to reduce all possible amounts of money coming from the selection to three voucher amounts (taking into account the maximum amount permitted by the University "La Sapienza" – Rome for a single voucher, which is 20 euros).

Particularly, Table *c* below shows the range created for each voucher.

Table c: range of amounts extracted corresponding to each voucher assigned

Voucher amount	***Range (results in the experiment)***
5 euros	0-550 euros
10 euros	551-700 euros
20 euros	701-1000 euros

Table *d* below shows the distribution of payments assigned to the 106 participants to the experiment.

Table d: number of participants paid, corresponding to each voucher assigned

Voucher amount	***Number of participants obtaining the voucher***
5 euros	56
10 euros	33

20 euros	17

Participants have been then asked through e-mail to submit a receipt for acceptance of the payment, and upon submission of the receipt, they received the voucher.