Capacity-Constrained Wardrop Equilibria and Application to Multi-Connectivity in 5G Networks

F. Delli Priscoli, E. De Santis, A. Giuseppi*, A. Pietrabissa

Abstract— **In this paper, a distributed, non-cooperative and dynamic load-balancing algorithm is proposed in the context of multi-commodity adversarial network equilibria with constrained providers' capacities. The algorithm is proven to converge to a generalised Wardrop user-equilibrium, referred to as Beckmann equilibrium in the literature, in which, for each commodity, the latencies of the unsaturated providers are equalized. The algorithm is then used as a Multi-connectivity algorithm in the context of 5G heterogeneous networks, in which the user equipments are able to use different access networks simultaneously to increase the transmission capacity and/or to improve the transmission reliability. The proposed controller provides a solution for dynamic traffic steering by distributing the traffic load over the available heterogeneous access points, considered as capacity providers. Simulation results validate the approach. The developed network simulator is available as an open-source environment [1].**

Index Terms— **Load balancing, Lyapunov design, Beckmann equilibrium, 5G networks.**

Nomenclature

c_p	Maximum load of provider p
$\mathcal{I}$	Set of commodities
$l_p^i(x_p^i)$	Latency of commodity i on provider p under load x_p^i
$\mathcal{L}(\boldsymbol{x})$	Candidate Lyapunov function under flow $\boldsymbol{x}$
$r_{pq}^i[k]$	Migration rate of commodity i from provider p to provider q at time k
$\mathcal{P}, \mathcal{P}^i$	Set of providers, set of providers available to commodity i
$x_p^i[k]$	Load of commodity i over provider p at time k
$x_p[k] = \sum_{i\in\mathcal{I}} x_p^i[k]$	Total load over provider p at time k
$\boldsymbol{x} = (x_p)_{p\in\mathcal{P}}$	Flow vector at time k
$\mathcal{X}$	Feasible state space
$\mathcal{X}_{eq}, \mathcal{X}_{eq}^{\varepsilon}$	Set of Beckmann and ε-Beckmann equilibria
λ^i	Flow demand of commodity i
$\Phi(\boldsymbol{x})$	Beckmann, McGuire and Winsten potential under flow $\boldsymbol{x}$
$\mu_{pq}^i(l_p^i, l_q^i)$	Migration policy of commodity i from provider p to provider q
σ^i	Migration gain of commodity i

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I. Introduction

Load Balancing is a classic problem of network control and can be interpreted as a particular case of traffic routing with providers representing unitary paths and latency functions describing the performance of each provider. In adversarial (or selfish) routing, the control algorithms are aimed at leading the network into convenient equilibrium states without the cooperation of its agents. One of such states is known in mean-field game theory as Wardrop equilibrium (which can be regarded as a Nash equilibrium for infinite players [2]): in such state, the latencies experienced by the agents that constitute the traffic flows are equalised over all their available routes, and, as a consequence, no agent may improve its routing unilaterally. In this paper, we study a particular case of selfish capacitated load balancing, in which the capacities of the service providers are limited. Therefore, as it will be discussed, the proposed control law objective will be to equalize the latencies of all the providers which are not saturated. This network state is a generalization of the Wardrop equilibrium in capacitated networks and is known in the literature as the Beckmann user equilibrium [3].

Multi-connectivity is an emerging challenge in the heterogeneous network scenario envisaged by 5G, where multiple Radio Access Technologies (RATs), such as LTE, 5G and Satellite networks, are available to connect the network users to the core network [4]. According to the multi-connectivity paradigm, each User Equipment (UE) may be able to be served by several of the various Access Points (AP) of the available RATs, potentially at the same time. The problem, referred to in the 5G literature as *multi-connectivity*, consists in dynamically choosing which APs shall serve each UE and deciding how much traffic relevant to each UE shall be routed through each of the serving APs. This paper focuses on the downlink direction, i.e., it refers to the traffic transmitted from the core network to the UEs via the APs; nevertheless, similar considerations apply when considering the uplink direction.

In this paper, the performance of the network APs are measured in terms of *latency functions* that capture the amount of resources (in terms of resource blocks) required from each AP to serve the various *commodities*. In the considered 5G scenario, such commodities consist in the so-called QoS-Flows, which are streams of data toward a User Equipment (UE) that are characterised by standardised Quality of Service (QoS) requirements (e.g., bit error rate, maximum tolerated delay…). In general, the latency functions may account for different connection-specific performance indexes (e.g., amount of network resources utilised on a given AP, power consumption, service reliability), and may include additional factors, as operator preferences or different usage tariffs.

Overall, the objective of the proposed control law for load balancing is to dynamically *steer* the downlink traffic in such a way that the values of the latency functions are equalized.

The described scenario is typical in adversarial routing and load balancing problems, as the various connections are not concerned with the overall network state and aim at optimising their own, individual, performances. The two main problems in the algorithm development are i) the fact that the latency functions are not known apriori, but can be only measured, ii) the fact that a distributed approach is needed since a centralized approach would require too much control traffic to exchange information among the potentially thousands of UEs.

In this paper, a distributed, non-cooperative and dynamic load balancing algorithm is consequently developed in the context of adversarial network equilibria; specifically, the algorithm considers every single packet included in a QoS-Flow as an *agent*, able to make a decision regarding the AP it is assigned to. Such decisions are based on the measurements of the latency functions, obtained starting from the observation of the resource blocks allocated on the APs over which the commodity is routed to sustain the connection, and are made unilaterally in an adversarial framework, with no concern for the overall system performance.

The main motivations behind this work are then (i) to design a dynamic adversarial capacitated load balancing algorithm and to prove, using Lyapunov and Invariance Principle arguments, how the difference equation governing the global state of the system converges to an approximated Beckmann equilibrium, and (ii) to show the effectiveness of such an approach through its application to the multi-connectivity problem in a simulated 5G network scenario.

The work presented in this paper was carried out within the H2020 5G-ALLSTAR project (www.5g-allstar.eu), aimed at the seamless, reliable and ubiquitous provision of broadband services over heterogeneous 5G networks. However, we note that, since the algorithm is developed within the research framework of selfish routing, it can be applied to several problems and scenarios other than the one considered here.

The paper is organized as follows: Section II presents the state-of-the-art on multi-connectivity in 5G-networks and on Wardrop load balancing and the proposed novelties; Section III presents the algorithm and the convergence proof; Section IV introduces the open-source simulator and reports the simulation results; Section V draws the conclusions.

II. State of the Art and Proposed Innovations

Section II.A motivates the choice of a distributed adversarial load-balancing algorithm to address the multi-connectivity problem in 5G networks, whereas Section II.B summarizes the works in the literature relevant to dynamic selfish routing and load balancing and the proposed innovations.

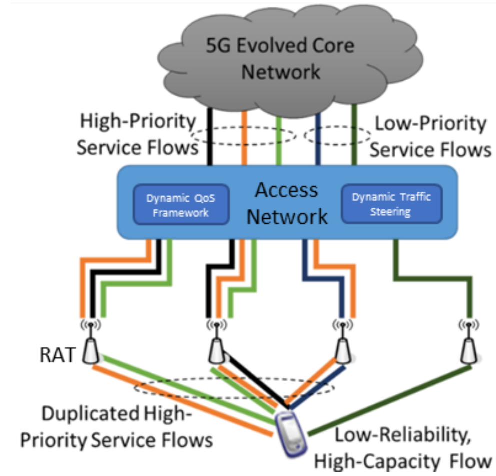

Figure 1. Dynamic Traffic Steering framework from [5]

A. Multi-Connectivity and Traffic Steering in 5G Networks

This work addresses the problem of traffic steering, i.e., of selecting which APs a QoS-Flow shall utilise to connect the UEs with the core network by modelling it as a load-balancing problem.

This vision is compliant with the latest developments of the 5G architecture (see Figure 1), as designed by 5GPPP in [5]. Multi-connectivity comprises the concept of dynamic traffic steering, which envisages the ability of dynamically steering the traffic, partitioned into QoS-Flows among the various available APs of the RATs, based on feedbacks on the current AP performances. In this framework, QoS-Flows may be duplicated over different APs to increase their resiliency, while other ones may be split over multiple RATs to increase their throughput or to better meet their QoS requirements.

Within the 5G architecture, the traffic steering problem is solved in three different ways: (i) with a User-Centric approach, where each UE decides its connection preferences according to local measures of some performance indicator; (ii) in a Radio Access Network (RAN)-Assisted fashion, in which the decision is still made by the UEs but the RAN provides them with additional information on the network state; (iii) with a RAN-Controlled approach, where all decisions are made by the RAN, which is a centralised unit by nature, or delegated to the distributed control units that govern the single APs.

Several works study the problem of multi-connectivity in the heterogeneous network framework proposed by 5G, from both architectural [6], [7] and algorithmic [8]–[10] points of view. Multi-connectivity enables the problem of optimally steering the network traffic over the available APs, in such a way that the QoS requirements of the various QoS-Flows are met [11], [12]. The problem of access network selection has been studied utilising several different approaches, spacing from fuzzy-logic control to multiple-attribute decision-making and combinatorial optimisation [8]. Common solutions utilise the concept of utility and latency functions, as in this work, to capture the network performances [8], [13], [14]. Several works in the literature also employ game-theoretic approaches for the AP selection, typically in adversarial frameworks, as [8], [15], [16], leading the networks to Nash equilibrium states.

Regarding game-theoretic solutions, one possible modelling choice is to have an adversarial game between the users, as in [15], [17] that envisage a setup similar to the one used in this work. In such scenarios, the users compete to attain the best connection quality while eventually also minimising their costs. An alternative approach is to set up a game between the various network operators, each controlling a set of APs as in [14], [16], and focusing on their economic performances.

The algorithm proposed in this work utilises *differential game theory*, a branch of game theory that studies dynamical systems, and shares some of the characteristics of the previously mentioned works, as the adversarial nature of its equilibrium. The control algorithm designed in this work will be proven to drive the communication network state to a convenient equilibrium state, and this convergence will be attained by following an explicit discrete-time control law, with no need for round-games or price/cost bidding auctions. Contrary to optimisation-based works, the proposed control law is also suitable to steer the traffic flows in real-time, and, being a distributed decision process, it does not require any significant control traffic overhead.

The previous aspects, together with the explicit inclusion of constraints on the available transmission capacity, makes the proposed approach a suitable candidate for the deployment in 5G scenarios implementing network slicing [18], in which the APs provide a limited quantity of resources to the QoS-Flows of a given service type or managed by third party tenants (e.g., video streaming, autonomous guidance, voice…). With reference to the mentioned Dynamic Traffic Steering framework [5], the algorithm can be implemented in the RAN-Assisted and in the RAN-Controlled configurations: in the former case, the algorithm would run in the UEs based on the information received by the RAN; in the latter case, the algorithm would run directly in the RAN and, in particular, for Non-StandAlone 5G systems (5G-NSA), in either the centralised unit (CU) or in the distributed units (DU) [19] of the next-generation-Node-Bs (gNodeBs or gNBs) [20] that govern the various APs.

B. Adversarial Load Balancing in 5G Networks and Beckmann Equilibria

The problem of optimally distributing the flow is one of the most fundamental and challenging aspects of any network operation. In the framework of selfish routing, the network flow is formed by a stream of infinitely-many decision-making agent [21] that compete for attaining the best performance, without consideration for the congestion, and consequent performance degradation, that their decisions cause to the other agents.

Wardrop equilibria [22] were then introduced to describe a network state in which no single agent can unilaterally improve its performances (e.g., in terms of travel time, as in the original Wardrop formulation). Being an adversarial kind of equilibria, the overall network performance is not optimised and the performance loss is referred to as the *price of anarchy* in the literature [23]. The concept of Wardrop equilibrium has been extended to various families of networks, among which the capacitated ones [3], [24]–[26], and problems, as the load balancing one [27]–[29]. Even if Wardrop equilibria can be computed by centralized algorithms in polynomial time [30], for the low connection latency promised by 5G – and the consequent agile and fast traffic steering requirements – distributed approaches are more suitable, motivating the development of a dynamic algorithm.

Based on a simple representation of the network dynamics in terms of difference equations derived from the flow conservations laws, this paper proposes a load balancing solution over the nodes of a dynamical network that represents the 5G infrastructure [31], [32], consisting in the connections between several APs and their users with the core network. In doing so, the algorithm takes into account that the amount of traffic each AP can support is limited, or *capacitated*, due to transmission power constraints and, in general, resource scarcity as in network slicing scenarios. This limitation implies that the user equilibrium to which the network will converge may not be in principle the Wardrop equilibrium [26], which is defined for unconstrained networks. Several works [3], [24]–[26] extended the original formulation of the Wardrop user equilibrium, which corresponds to a situation in which all the latencies of each commodity are equalised, to deal with capacitated networks. The resulting equilibrium, known as Beckman user equilibrium, is such that the latencies of all the unsaturated APs of each commodity are equalised. Differently from [3], [24]–[26], this work proposes a dynamic algorithm which will be proven to converge to a Beckmann equilibrium.

Regarding dynamic load balancing solutions for Wardrop equilibria in the literature, several works utilise the concepts of learning and exploration to cope with the limited feedback information that the decision-making agents have access to. To attain a better knowledge of the system state and dynamics, the agents sample different flow distribution strategies and then exploit the learned system characteristics to converge to optimal states. The authors of [33] present an asynchronous and distributed algorithm that employs reinforcement learning to update transmission probabilities, based on an estimation of the network edges latencies. In [34], an iterative and distributed learning solution is proven to converge to a Wardrop equilibrium state using Lyapunov arguments, as in this work.

An important contribution has been given by Fischer et al. in [35]–[37]. In [35] and [37], a round-based algorithm is developed to solve a game among the various commodities, aimed at redistributing the traffic flow and reaching an approximated Wardrop equilibrium. In [36], a similar set up is analysed assuming that the information available to the agents may be stale. In [38], a dynamic discrete-time load-balancing algorithm, later extended to the time-delayed case in [39], is presented in the context of Virtual Private Networks, which converges to an approximate Wardrop equilibrium.

The present work extends the results of previous works, starting from the algorithm in [38], mainly in two directions:

i) the convergence properties of the algorithm are studied in the multi-commodity case, a requirement for application in the 5G framework, that was not explicitly discussed in the cited works;

ii) the algorithm analysis and design are extended to the case of capacitated networks, not dealt with by the dynamic algorithms in the literature, enabling the application of the solution to more realistic case studies in several domains.

III. Proposed Wardrop Load Balancing Algorithm

Section III.A describes the basic definitions needed for the algorithm analysis; Section III.B presents the load balancing algorithm and the convergence proof; Section III.C. models the 5G traffic steering problem as a load balancing one.

A. Preliminaries on Wardrop and Beckmann Equilibria and on Lyapunov Stability

As anticipated in Section II, this paper further develops a well-known model for selfish routing [35], where an infinite population of agents carries an infinitesimal amount of load each and builds on the previous work [38] concerning distributed load balancing algorithms. The proposed control scheme relies on common assumptions on the latency functions. The considered network consists in a set of $\mathcal{P}$ providers, which serve a set $\mathcal{I}$ of commodities. Each commodity $i \in \mathcal{I}$ is characterised by a flow demand λ^i and is served by a subset of providers $\mathcal{P}^i \subset \mathcal{P}$. Each commodity i using provider p is characterised by a latency function l_p^i and each provider p is characterized by a capacity c_p.

Assumption 1. The latency functions $l_p^i(\xi)$ are positive, non-decreasing and Lipschitz continuous with constant β_p^i, for $\xi \in [0, c_p]$, where c_p is the capacity of provider p, for all $p \in \mathcal{P}$. Furthermore, the maximum Lipchitz constant of all the l_p^i's is denoted as $\bar{\beta} = \max_{p\in\mathcal{P}^i, i\in\mathcal{I}} \beta_p^i$.

The assumption is not restrictive in real use-cases since the provider performances decrease with their load.

In non-capacitated algorithms, if x_p^i indicates the amount of the flow of commodity i allocated on the provider p, the set of feasible states is defined as

$$\mathcal{X} = \Big\{\boldsymbol{x} = (x_p)_{p\in\mathcal{P}} | x_p = \textstyle\sum_{i\in\mathcal{I}} x_p^i\,,\ x_p^i \geq 0, \forall p \in \mathcal{P}^i, \sum_{p\in\mathcal{P}^i} x_p^i = \lambda^i, \forall i \in \mathcal{I}\Big\}, \tag{1}$$

and a flow $\boldsymbol{x} \in \mathcal{X}$ is at a Wardrop equilibrium if, for each commodity $i \in \mathcal{I}$, the latencies of the loaded providers are equalized, i.e., if $l_p^i(x_p^i) \leq l_q^i(x_q^i)$ for all $p \in \mathcal{P}^i$ such that $x_p^i > 0$, for all $q \in \mathcal{P}^i$ and for all $i \in \mathcal{I}$.

By defining the Beckmann-McGuire-Winsten potential

$$\Phi(\boldsymbol{x}) = \textstyle\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}^i}\int_0^{x_p^i} l_p^i(\xi)d\xi, \tag{2}$$

the Wardrop equilibria are the solutions of the optimization problem

$$\min_{\boldsymbol{x}\in\mathcal{X}} \Phi(\boldsymbol{x}). \tag{3}$$

Capacity-constrained networks are characterized by the additional capacity constraints

$$x_p \leq c_p, \forall p \in \mathcal{P}. \tag{4}$$

A flow $\boldsymbol{x} \in \mathcal{X}$ is feasible if constraints (4) hold, and the set of feasible states is defined as

$$\mathcal{X}_{CP} = \{\boldsymbol{x} \in \mathcal{X}\ | x_p \leq c_p, \forall p \in \mathcal{P}\}. \tag{5}$$

Considering a flow $\boldsymbol{x} \in \mathcal{X}_{CP}$, provider $p \in \mathcal{P}$ is defined as *capacity-constrained* or *saturated* if $x_p = c_p$.

A flow $\boldsymbol{x} \in \mathcal{X}_{CP}$ is at a Beckmann user equilibrium if, for each commodity, the latencies of the loaded and unconstrained providers are equalized, i.e., more precisely:

Definition 1 [3]. A flow $\boldsymbol{x} \in \mathcal{X}_{CP}$ is at a Beckmann user equilibrium if $l_p^i(x_p^i) \leq l_q^i(x_q^i)$ for all $p \in \mathcal{P}^i$ such that $x_p^i > 0$, for all $q \in \mathcal{P}^i$ such that $x_q < c_q$ and for all $i \in \mathcal{I}$.

The set of equilibria is then

$$\mathcal{X}_{eq} = \{\boldsymbol{x} \in \mathcal{X}_{CP} | l_p^i(\boldsymbol{x}) \leq l_q^i(\boldsymbol{x}), \forall p \in \mathcal{P}^i \text{ s.t. } x_p^i > 0, \forall q \in \mathcal{P}^i \text{ s.t.} x_q < c_q, \forall i \in \mathcal{I}\}. \tag{6}$$

Let us consider the minimization problem (3) with constraints (4), hereinafter referred to as capacity-constrained problem (CP). The Beckman user equilibria [25] are the optimal solutions of the CP.

Property 1 [3]. If the set of feasible solutions $\mathcal{X}_{CP}$ of the CP is nonempty, the optimization problem consists in minimizing a convex function over a nonempty polytope and, thus, the set of optimal flows $\mathcal{X}_{eq}$ is nonempty and convex.

The algorithm convergence proof of Section III.B relies on LaSalle invariance principle for discrete-time nonlinear systems [40], [41].

Definition 2. $\mathcal{L}: \mathcal{X} \to \mathbb{R}$ is a candidate Lyapunov function for a discrete-time nonlinear system $\boldsymbol{x}[k+1] = f(\boldsymbol{x}[k])$ if

i) $\mathcal{L} \in \mathcal{C}^1$ and is bounded from below;

ii) If $\boldsymbol{x}_{eq} \in \mathcal{X}_{eq}$, where $\mathcal{X}_{eq}$ is the set of equilibrium points, $\mathcal{L}(\boldsymbol{x}_{eq}) = 0$ and $\mathcal{L}(\boldsymbol{x}) > 0$ if $\boldsymbol{x} \notin \mathcal{X}_{eq}$;

iii) Along forward trajectories, $\mathcal{L}$ satisfies

$$\Delta\mathcal{L}(\boldsymbol{x}[k]) \coloneqq \mathcal{L}(f(\boldsymbol{x}[k])) - \mathcal{L}(\boldsymbol{x}[k]) \leq 0, k = 0,1,2, \dots$$

Theorem 1 ([40]). Let $\mathcal{L}(\boldsymbol{x})$ be a candidate Lyapunov function for the discrete-time nonlinear system $\boldsymbol{x}[k+1] = f(\boldsymbol{x}[k])$. Then, any bounded trajectory tends to the largest invariant subset $\mathcal{M}$ contained in the set of points defined by $\Delta\mathcal{L}(\boldsymbol{x}) = 0$.

B. Capacitated Load Balancing Algorithm and Convergence Proof

For each commodity $i \in \mathcal{I}$, the control action consists in the decision, at time k, of *migrating* part of the flow mapped onto a given provider p to another provider q, with $p, q \in \mathcal{P}^i$. By denoting the rate of such migration with $r_{pq}^i[k]$, the system dynamics is written as

$$\boldsymbol{x}[k+1] = f(\boldsymbol{x}[k]), k = 0,1,2, \dots \tag{7}$$

with

$$x_p[k] = \sum_{i \in \mathcal{I}} x_p^i[k], \tag{8}$$

$$x_p^i[k+1] = x_p^i[k] + \tau \sum_{q \in \mathcal{P}^i} \left(r_{qp}^i[k] - r_{pq}^i[k] \right), \tag{9}$$

and with feasible initial conditions

$$\boldsymbol{x}[0] \in \mathcal{X}_{CP}. \tag{10}$$

for all $p, q \in \mathcal{P}^i$ and $i \in \mathcal{I}$

The proposed controller builds on the dynamic algorithm in [38], which expresses the migration rate as

$$r_{pq}^i[k] = x_p^i[k] \sigma^i \mu_{pq}^i[k], \tag{11}$$

where σ^i is a positive migration gain and $\mu_{pq}^i[k]$ is the migration policy, representing the decision of whether (if it is positive) or not (if it is equal to zero) migrate some flow from provider p to provider q.

As in [38] for the Wardrop equilibria, approximated Beckmann user equilibria are defined.

Definition 3. The set of ε-Beckmann user equilibria is defined as

$$\mathcal{X}_{eq}^{\varepsilon} = \left\{ \boldsymbol{x} \in \mathcal{X}_{CP} \middle| l_p^i(x_p^i) \le l_q^i(x_q^i) + \varepsilon, \forall p \in \mathcal{P}^i \text{ s.t. } x_p^i > 0, \forall q \in \mathcal{P}^i \text{ s.t. } x_q \le c_q - \frac{\varepsilon}{2\bar{\beta}}, \forall i \in \mathcal{I} \right\}. \tag{12}$$

where $\varepsilon \ge 0$ represents a maximum tolerated latency mismatch.

Remark 1. The defined sets are such that $\mathcal{X}_{eq}^{\varepsilon} \xrightarrow[\varepsilon \to 0]{} \mathcal{X}_{eq}$ and $\mathcal{X}_{eq} \subseteq \mathcal{X}_{eq}^{\varepsilon} \subseteq \mathcal{X}_{CP}$: the objective of the controller is then, starting from a physically admissible state in $\mathcal{X}_{CP}$, to reach an approximated equilibrium state in $\mathcal{X}_{eq}^{\varepsilon}$, whose degree of approximation with respect to the equilibrium state in $\mathcal{X}_{eq}$ reduces with ε.

The tolerance ε is introduced since the kind of migration rates of equation (11) cannot guarantee convergence in the discrete-time case, however small the sampling period [36]. A flow $\boldsymbol{x} \in \mathcal{X}_{CP}$ is then at ε-Beckman equilibrium if, for each commodity i, the latencies of the loaded and ε-unconstrained providers are equalized, where we define a provider $p \in \mathcal{P}^i$ to be ε-unconstrained if $x_p < c_p - \frac{\varepsilon}{2\bar{\beta}}$.

In the proposed algorithm, the migration decision is defined as

$$\mu_{pq}^i[k] = \begin{cases} 0, & \text{if } l_p^i(x_p^i[k]) - l_q^i(x_q^i[k]) \le \varepsilon \text{ or if } x_q[k] \ge c_q - \frac{\varepsilon}{2\bar{\beta}}. \\ 1, & \text{otherwise} \end{cases} \tag{13}$$

The controlled system dynamics, hereafter denoted as load-balancing (20) dynamics, is then expressed by equations (9), (11), (13), with control gains set as

$$\sigma^i = \frac{\varepsilon}{2\tau\bar{\beta}\lambda^i(|\mathcal{P}^i|-1)|\mathcal{I}|}, \tag{14}$$

and with the tolerance set as

$$0 < \varepsilon \le \min_{i \in \mathcal{I}} \bar{\beta}\lambda^i|\mathcal{I}|. \tag{15}$$

Remark 2. The approximated capacity-constrained user equilibria are such that, for each commodity, the latencies of the loaded and ε-unconstrained providers are equalized within the tolerance ε. Then, for a given equilibrium flow $\boldsymbol{x} \in \mathcal{X}_{eq}^{\varepsilon}$ and for each commodity $i \in \mathcal{I}$, three classes of providers exist: the unloaded providers $p \in \mathcal{P}^i$ such that $x_p^i = 0$; the ε-constrained providers $p \in \mathcal{P}^i$ such that $x_p > c_p - \frac{\varepsilon}{2\bar{\beta}}$; the ε-unconstrained providers, whose latencies are equalized.

The convergence property of the algorithm relies on the following 3 lemmata.

Lemma 1. Under Assumption 1, considering the LB dynamics, the latency variation of a provider $p \in \mathcal{P}^i$ in one time-step is bounded by

$$\left| l_p^i(x_p^i[k+1]) - l_p^i(x_p^i[k]) \right| \le \frac{\varepsilon}{2|\mathcal{I}|}. \tag{16}$$

Proof: See Appendix A.

Lemma 2. $\mathcal{X}_{CP}$ is a positively invariant set for the LB dynamics.

Proof: See Appendix A.

Lemma 3. The function

$$\mathcal{L}(\boldsymbol{x}) \coloneqq \Phi(\boldsymbol{x}) - \Phi_{min},$$

where Φ_{min} is the minimum value of $\Phi(\boldsymbol{x})$ for all the minimizers of the CP, is a candidate Lyapunov function for the LB dynamics.

Proof: See Appendix A.

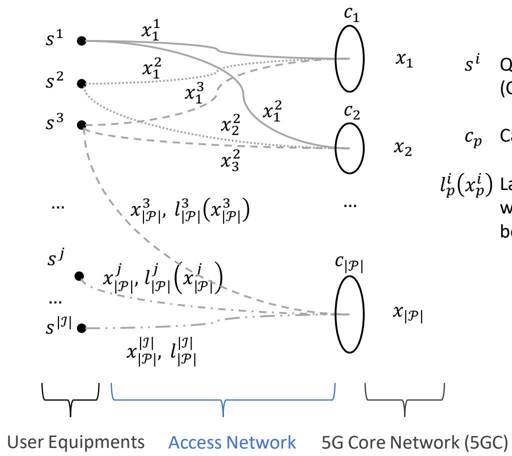

Figure 2 Load balancing graph

Finally, the following theorem proves the convergence towards an approximated Beckmann user equilibrium.

Theorem 2. The trajectories of the LB dynamics asymptotically tend to the set of equilibria $\mathcal{X}_{eq}^{\varepsilon}$.

Proof: See Appendix A.

C. 5G Traffic steering as a dynamic load-balancing problem

In the dynamic multi-connectivity framework of 5G networks [5], each UE selects the serving APs for its QoS-Flows. The network resources (capacity) are hence provided by the APs, and their efficient usage guides the design of traffic steering controllers. As introduced, in 5G systems, the dynamic management of such resources becomes of crucial importance in network slicing scenarios [18].

In order to model a multi-connectivity scenario in a network slicing environment as a dynamical network of the form (7-10), we regard the AP p as a provider in the set of providers $\mathcal{P}$, the QoS-Flow associated with a UE as a commodity i in the set of commodities $\mathcal{I}$ and we associate to the state variable $x_p^i[k]$ the amount of bitrate of the commodity i that is provided by the AP p at time k. The bitrate demand of the commodity i is then λ^i, which can be assumed, for limited time windows, to be constant.

In the following, we will consider a network slicing scenario in which the network operator dedicated a certain amount of bitrate c_p on each AP p to the controlled slice.

Regarding the latency functions, a natural choice is associating a different latency function l_p^i to the radio connection between the UE of commodity i and the AP $p \in \mathcal{P}^i$. This choice allows to capture quantities related to the specific connection performance, such as the resource blocks [42] usage, the power consumption of the single commodity i or its QoS degradation, but in turn implies that each commodity i is subject to a different latency from provider p, that may even depend only on x_p^i. We mention that, in this kind of scenarios, in general the network admits various equilibria characterised by different costs (latencies) [43]. Nevertheless, in the proposed framework depicted in Figure 2, the considered network is characterised by parallel arcs [43], implying that its equilibrium cost Φ_{min} is unique. In fact, with simple manipulations, the scenario of Figure 2 can be shown to be equivalent to a network in which the latencies are associated to the depicted radio links, each of which can only be used by a single commodity. The scenario is then equivalent to a standard adversarial routing scenario with an unique equilibrium cost.

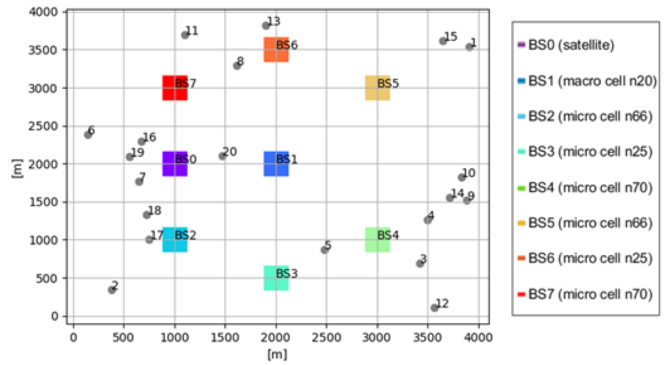

Figure 3 Network scenario

Regarding the mapping of the proposed control law onto the standard 5G architecture, we mention that Access Traffic Steering, Switching and Splitting (ATSSS) [44] decision rules for multi-connectivity are typically produced by a software module of the 5G core network, the PCF (Policy Control Function). The PFC configures the UEs and the UPF (User Plane Function, an entity directly connected to the gNodeBs of the RAN) to handle traffic steering based on local measurements, respectively for the uplink and downlink phase. Such ATSSS rules may define the set of APs $\mathcal{P}^i$ available to the user i, depending on its contract with the provider, their priority, and in general may define a control law to guide the steering of the QoS flows that constitute the considered PDU (Protocol Data Unit) session. The dynamic traffic steering functionalities [5] are taken at RAN level, as depicted in Figure 1, and so the proposed algorithm is designed to be deployed either in the distributed units (DU) of the gNodeBs that constitute the controlled RAN or in the UEs. The rules provided by the PCF can be included in the control logic by properly weighting or forbid the various AP connections.

IV. Numerical Simulation

This section reports the simulation setup and results in sections IV.A and IV.B, respectively.

A. Simulation Setup

For the validation of the proposed algorithm, in the scope of the 5G-ALLSTAR project, we developed an open-source network simulator available in [1], able to model different AP technologies, connection protocols and interference models in a multi-connectivity scenario. We consider the network depicted in Figure 3, consisting of a $4 \times 4\ Km$ area covered by a macro cell (provider BS1), a satellite (provider BS0) and six micro cells (BS2-BS7).

Table 1: Characteristics of micro and macro cells

Operating band N°	*Carrier frequency (GHz)*	*Bandwidth (MHz)*
$\boldsymbol{n20}$	0.8	20
$\boldsymbol{n25}$	1.9	40
$\boldsymbol{n66}$	1.7	40
$\boldsymbol{n70}$	2	25

A total of 20 UEs (grey dots in Figure 3) were randomly distributed in the area, each requiring a constant load $\lambda^i = 50\ Mbps$. The implemented interference model is taken from [45] and the frequency characteristics of the terrestrial APs are summarised in Table 1 [46], [47].

Regarding the satellite AP, we considered a Time Division Multiplexing (TDM) as in the example 6.6.2 of [48]. The satellite parameters are adapted in order to have at least 1bit per symbol with typical SNR values [49], [50]. According to the TDM frame structure used, it is possible to allocate only blocks of 64 symbols (1 μs). Moreover, each allocation must consider a header made of 288 symbols and a spacing between allocations of 64 symbols. Additional implementation details and updates can be found in [1].

We considered as latency functions l_p^i the number of resource blocks utilised by the commodity i on the access point p. This particular choice will drive the network towards a state in which each connection equalises the resource block usage over its available unsaturated APs $p \in \mathcal{P}^i$.

Assuming a stationary UE i (i.e., with constant path loss with all the access points p) and no interference, the amount of bitrate provided by a resource block on a given access point p is fixed. This implies that, in ideal conditions, l_p^i is linear, with a slope that depends on the utilised frequency bands, in line with Assumption 1. Note that several different choices could be made for the latency function, spacing from quantities that capture connection reliability, to transmission delay and user satisfaction, as the only requirements that such functions must satisfy are represented by Assumption 1, which open the possibility of considering a large family of functions (e.g., including polynomial or exponential ones).

To allow a fair comparison with the terrestrial AP resource blocks, the assumptions made for the satellite imply that its latency function is equal to the number of its allocated symbols divided by 64. Additionally, each AP was associated to a multiplicative scaling factor for their latency functions to model different operating costs. In particular, the satellite was given the highest factor (0.5), the macro cell was given a medium value (0.2) and the lowest weight was associated to micro cells (0.1). Regarding the capacitated nature of the considered network, we assume that the network operator dedicated to the controlled slice 200 $Mbps$ on all micro cells, save for BS4 that was capacitated at 55 $Mbps$.

Concerning the parameters of the controller, the choice of latency functions leads to an experimentally determined value $\bar{\beta} = 2.44$, the latency tolerance is selected as $\varepsilon = 0.5$ and the sampling time as $\tau = 10^{-3}s$. The resulting values for σ^i are in the range [0.02,0.05].

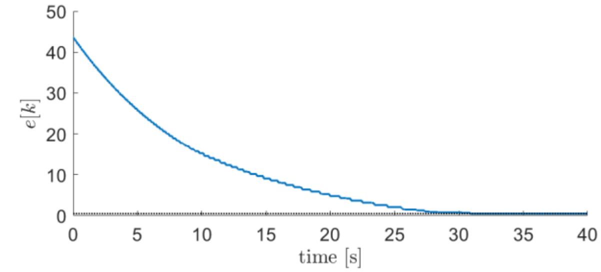

Figure 4: Maximum latency mismatch during the simulation (dotted line: tollerance ε).

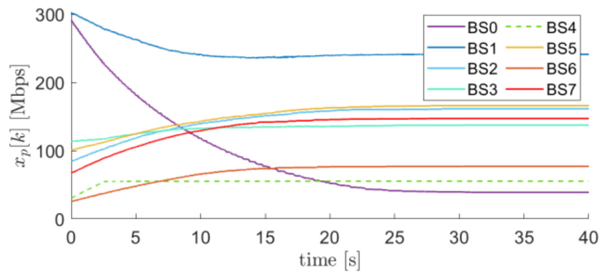

Figure 5 Network state in terms of total bitrate allocated on the various APs (solid lines: unconstrained providers, dashed line: constrained provider).

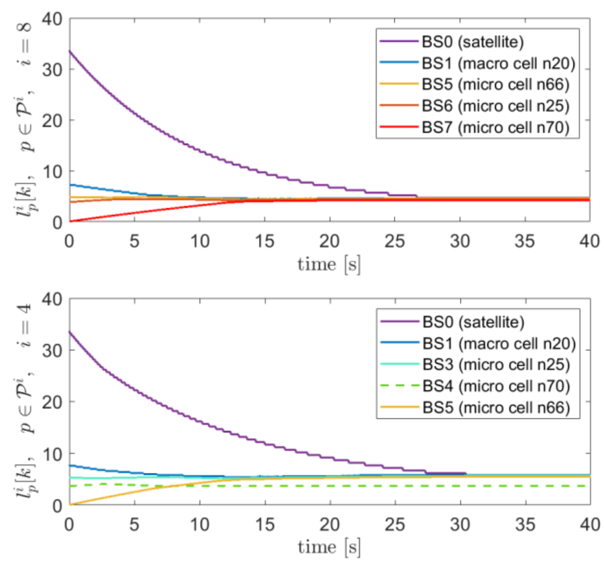

Figure 6: Commodity latency examples during the simulation (solid lines: unconstrained providers used by the commodity; dashed line: constrained providers).

B. Simulation Results

Simulation runs were initialized by distributing uniformly the load of the commodities over $|\mathcal{P}^i| - 1$ of their available APs, selected randomly.

The reported simulations showed a convergence time to an ε-Beckmann equilibrium in the order of $30s$, averaged over 25 runs. It is worth remarking that such convergence time is not related to the 5G QoS requirements, as it is assumed that the various access points are able to provide the proper QoS level (e.g., connection latency, average BER, reliability level,…) if their capacities are not violated.

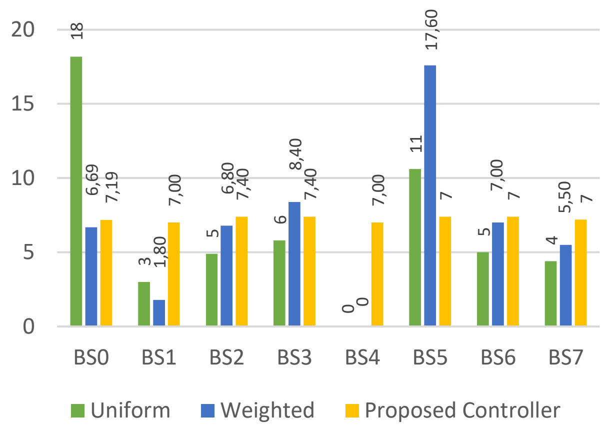

Figure 7 Comparison for the three considered algorithms of the latencies over APs for commodity $i = 20$.

Figure 4 shows, for an example run, how the maximum latency mismatch over all the commodities, defined as

$$e[k] = \max_{i\in\mathcal{I}}\left\{\max_{p\in\mathcal{P}^i|x_p^i[k]>0} l_p\big(x_p[k]\big) - \min_{q\in\mathcal{P}^i|x_q[k]<c_q-\frac{\varepsilon}{2\bar{\beta}}} l_q\big(x_q[k]\big)\right\},$$

decreases with time and, even if the initial conditions are quite unbalanced, with $e[k] > 40$, after $30s$ $e[k]$ is already below the threshold ε.As examples of simulation results, Figure 6 reports the evolution of the latencies that characterise the commodities 4, and 8, for all of their available APs.

The upper plot shows that the latencies of the APs available to QoS-Flow 8 converge to a common value, as expected, within the threshold ε; in particular, we can notice how the commodity rapidly starts using the (initially unused) micro cell BS7 and rapidly discharges the satellite.

The lower plot shows the latencies of the QoS-Flow 4 and highlights that the latency of micro-cell BS4 does not converge to the latencies of the other used APs: the reason is that the AP becomes ε-saturated after about $3s$ (see Figure 5) – thus, by definition, the population of QoS-Flow 4 still converges to an ε-Beckmann equilibrium. Note that the latency associated to BS4 starts higher than its final value, as the commodity migrates towards BS5, but remains the lowest latency for the commodity 4 from $10s$ onwards, as the other QoS flows already ε-saturated BS4 (i.e., no bitrate can be migrated to it).

Finally, Figure 5 shows the population dynamics over the APs, highlighting how the macro cell is the most utilised AP, while all the micro cells allocate a similar amount of bitrate. The satellite, whose latency was the most penalised as it is the most costly connection technology, is rapidly discharged.

For the sake of comparison, in Figure 7 we benchmark the proposed controller against two classic examples of load balancing solutions in heterogeneous networks. The figure reports the latency functions values experienced by the commodity $i = 20$ over the eight APs in the set $\mathcal{P}^{20}$. The first benchmarking algorithm ("uniform") uniformly distributes the bitrate demand over the various APs. The second algorithm ("weighted") distributes the bitrate considering the scaling factors associated to the latencies of the APs (i.e., 0.5 for the satellite, 0.2 for the macro cell and 0.1 for micro cells). From the analysis of the figure, one can note that the proposed controller – in the figure, the values are the ones achieved after convergence (~30s) – successfully equalises the latencies up to the threshold $\varepsilon = 0.5$. Furthermore, the other two controllers fail to allocate any bitrate on BS4, as it was already saturated by the other commodities. The uniform distribution causes the first controller to experience a very high latency on the satellite (BS0), while the distance and consequent low signal-to-noise-ratio causes the weighted controller to allocate too much bitrate on BS5 (this behaviour is further amplified by the fact that BS5 is a micro cell associated to a scaling factor of 0.1). Overall, we can conclude that the proposed controller, being a feedback-based solution that steers the traffic flow based on measurements of the latency functions, better balances the overall usage of network resources. The main limitation of the proposed approach is related to the availability of the measurements needed to compute the steering decisions (i.e., the latency values in terms of assigned resource blocks), whose impact on the control traffic overhead is to be evaluated considering the control traffic already necessary for the different access technologies, and the estimation of $\bar{\beta}$ which, however, can be performed starting from the channel models and the expected traffic that the network is designed to support. Regarding the complexity of the algorithm, the computation overhead is negligible since the control law (11) only involves basic operations (summations, multiplications and comparison between real numbers) that remain limited in number even for RANs with a high number of APs.

To conclude, we mention that the two benchmarking algorithms discussed above could be used to initialise the network resource allocation (we recall that, to stress the algorithm asymptotic properties, in Figure 4-6 the network was initialised with all UEs distributing their bitrate over $|\mathcal{P}^i| - 1$ APs), speeding up the convergence time.

V. Conclusions

This paper develops a distributed, non-cooperative and dynamic load balancing algorithm in the framework of adversarial selfish routing with link capacities. Each provider is associated to a latency function which represents its performance as a function of the provider's load. By using Lyapunov arguments, the proposed algorithm is proved to converge to an approximate Beckmann user equilibrium, in which the latencies of the non-saturated providers are equalized up to a tolerated latency mismatch.

The algorithm is then applied to the problem of multi-connectivity, one of the key features of 5G networks, which enables the user equipment to simultaneously transmit/receive traffic flows over different access networks, with the aim of increasing the transmission rate and/or to improve the transmission reliability. In multi-connectivity, the *traffic steering* functionality is in charge of distributing the traffic load of each flow over the different access network. This paper models the traffic steering problem as a capacitated load-balancing problem by associating a latency function to each access point/user equipment radio link. The problem is then solved by means of the developed algorithm. An open-source simulation environment was proposed, and some numerical simulation results validate the approach.

Beside the modelling of the 5G Multi-connectivity problem as a dynamic load-balancing one, this paper presents, up to the authors' knowledge, the first multi-commodity, dynamic and adversarial load-balancing algorithm which explicitly considers capacitated providers.

Future work is aimed i) at introducing latency constraints in the problem formulation in order to model more Quality-of-Service constraints of the 5G services and ii) at considering time-varying loads.

VI. Acknowledgements

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Appendix A

Proof of Lemma 1: Considering the generic commodity $i \in \mathcal{I}$, provider $p \in \mathcal{P}^i$ and time k, the maximum latency decrease occurs when no commodities migrate their populations from the other providers to provider p:

$$\begin{aligned} & l_p^i\left(x_p^i[k+1]\right) \\ & = l_p^i\left(x_p^i[k] + \tau \sum_{q\in\mathcal{P}^i}\left(r_{qp}^i[k] - r_{pq}^i[k]\right)\right) \\ & \geq l_p^i\left(x_p^i[k] - \tau \sum_{p\in\mathcal{P}^i} r_{pq}^i[k]\right). \end{aligned} \tag{17}$$

Since β_p^i is the Lipschitz constant of the function $l_p^i(\cdot)$ between 0 and c_p, it follows that

$$l_p^i\left(x_p^i[k+1]\right) \geq l_p^i\left(x_p^i[k]\right) - \tau\beta_p^i \sum_{q\in\mathcal{P}^i} r_{pq}^i[k]. \tag{18}$$

Considering equations (11) and (14), the last term of equation (18) is written as

$$\begin{aligned} & \tau\beta_p^i \sum_{q\in\mathcal{P}^i} r_{pq}^i[k] = \\ & = \tau\beta_p^i \sum_{q\in\mathcal{P}^i} x_p^i[k]\sigma^i\mu_{pq}^i[k] = \\ & = \tau\beta_p^i x_p^i[k]\sigma^i \sum_{q\in\mathcal{P}^i} \mu_{pq}^i[k] = \\ & = \tau\beta_p^i x_p^i[k] \frac{\varepsilon}{2\tau\bar{\beta}\lambda^i(|\mathcal{P}^i|-1)|\mathcal{I}|} \sum_{q\in\mathcal{P}^i} \mu_{pq}^i[k] = \\ & \leq \frac{\varepsilon}{2|\mathcal{I}|}, \end{aligned} \tag{19}$$

where the inequality holds since $x_p^j[k] \leq \lambda^i$, $\beta_p^i \leq \bar{\beta}$ and since, recalling equation (13), there are at most $\left(|\mathcal{P}^j| - 1\right)$ terms equal to 1 in $\sum_{q\in\mathcal{P}} \mu_{pq}^j[k]$. It follows that

$$l_p^i\left(x_p^i[k+1]\right) \geq l_p^i\left(x_p^i[k]\right) - \frac{\varepsilon}{2|\mathcal{I}|}. \tag{20}$$

Similarly, the maximum latency increase occurs when no commodities migrate their populations from provider p to other providers:

$$l_p^i\left(x_p^i[k+1]\right) \leq l_p^i\left(x_p^i[k]\right) + \tau\beta_p^i \sum_{q\in\mathcal{P}^i} r_{qp}^i[k], \tag{21}$$

which yields

$$l_p^i\left(x_p^i[k+1]\right) \leq l_p^i\left(x_p^i[k]\right) + \frac{\varepsilon}{2|\mathcal{I}|}. \tag{22}$$

■

Proof of Lemma 2: We need to show that, for all $k \geq 0$, for all $p \in \mathcal{P}^i$ and for all $i \in \mathcal{I}$, i) $\sum_{p\in\mathcal{P}^i} x_p^i[k] = \lambda^i$, ii) $x_p^i[k] \geq 0$, iii) $x_p[k] \leq c_p$.

Considering that $x[0] \in \mathcal{X}_{CP}$, equations (9), (11) and (8) yield that the population remains constant, since

$$\begin{aligned} & x_p^i[k+1] - x_p^i[k] = \sum_{p\in\mathcal{P}^i}\sum_{q\in\mathcal{P}^i}\left(r_{qp}^i[k] - r_{pq}^i[k]\right) = \\ & = \sum_{p\in\mathcal{P}^i}\sum_{q\in\mathcal{P}^i} r_{qp}^i[k] - \sum_{q\in\mathcal{P}^i}\sum_{p\in\mathcal{P}^i} r_{qp}^i[k] = 0, \end{aligned} \tag{23}$$

and thus that $\sum_{p\in\mathcal{P}^i} x_p^i[k] = \sum_{p\in\mathcal{P}^i} x_p^i[0] = \lambda^i, \forall k \geq 0$.

i) Given that $x_p^i[0] \geq 0$, it is proven below by induction that $x_p^i[k] \geq 0, \forall k \geq 0$. Assuming that $x_p^i[k] \geq 0$, for a given k, it is sufficient to prove that

$$x_p^i[k+1] = x_p^i[k] + \tau\sum_{q\in\mathcal{P}^i}\left(r_{qp}^i[k] - r_{pq}^i[k]\right) \geq 0, \forall p \in \mathcal{P}^i. \tag{24}$$

If $x_p^i[k] = 0$, it follows that $r_{pq}^i[k] = 0$ and thus equation (24) yields $x_p^i[k+1] \geq 0$.

If $x_p^i[k] > 0$, from equation (11) it follows that $r_{pq}^i[k] \geq 0$. Thus, the following inequality holds (in the worst case, no providers migrate part of their population to a provider p):

$$x_p^i[k+1] \geq x_p^i[k] - \tau\sum_{q\in\mathcal{P}^i} r_{pq}^i[k]. \tag{25}$$

A sufficient condition for inequality (24) to hold is then

$$x_p^i[k] - \tau\sum_{q\in\mathcal{P}^i} r_{pq}^i[k] \geq 0. \tag{26}$$

Recalling equations (11) and (13), eq. (26) is written as

$$\begin{aligned} & x_p^i[k] - \tau\sum_{q\in\mathcal{P}^i} r_{pq}^i[k] = x_p^i[k] - \tau\sum_{q\in\mathcal{P}^i} x_p^i[k]\sigma^i\mu_{pq}^i[k] = \\ & = x_p^i[k]\left(1 - \tau\sigma^i\sum_{q\in\mathcal{P}^i}\mu_{pq}^i[k]\right) \\ & \geq x_p^i[k]\left(1 - \tau\sigma^i\left(|\mathcal{P}^i| - 1\right)\right), \end{aligned} \tag{27}$$

where the inequality holds since the summation has at most $\left(|\mathcal{P}^i| - 1\right)$ terms equal to 1. In the case $x_p^i[k] > 0$, equations (14) and (15) are sufficient for equation (27) to be non-negative;

ii) Given that $x_p[0] \leq c_p$, it is proven below by induction that $x_p[k] \leq c_p, \forall k \geq 0$. Assuming that $x_p[k] \leq c_p$, for a given k, it is sufficient to prove that

$$x_p[k+1] = x_p[k] + \tau \sum_{i\in\mathcal{I}} \sum_{q\in\mathcal{P}^i} \left(r^i_{qp}[k] - r^i_{pq}[k]\right) \leq c_p, \forall p \in \mathcal{P}^i. \quad (28)$$

If $x_p[k] \geq c_p - \frac{\varepsilon}{2\bar{\beta}}$ equation (13) entails that $r^i_{qp}[k] = 0$ for all $q \in \mathcal{P}^i$ and $i \in \mathcal{I}$ and, thus, from equation (9), that $x_p[k+1] \leq x_p[k]$.

Otherwise, if $x_p[k] < c_p - \frac{\varepsilon}{2\bar{\beta}}$, we consider that

$$\begin{aligned} x_p[k+1] &\leq x_p[k] + \tau \sum_{i\in\mathcal{I}} \sum_{q\in\mathcal{P}^i} r^i_{qp}[k] = \\ &= x_p[k] + \tau \sum_{i\in\mathcal{I}} x^i[k]\sigma^i \sum_{q\in\mathcal{P}^i} \mu^i_{qp}[k] = \\ &\leq x_p[k] + \sum_{i\in\mathcal{I}} \frac{\varepsilon}{2\bar{\beta}|\mathcal{I}|} = x_p[k] + \frac{\varepsilon}{2\bar{\beta}} \end{aligned} \quad (29)$$

■

Proof of Lemma 3: For the definition of Φ_{min}, the function $\mathcal{L}(\boldsymbol{x})$ is positive definite in $\mathcal{X}_{CP}$.

Let $\Delta\mathcal{L}(\boldsymbol{x}[k])$ denote the difference of the Lyapunov function $\mathcal{L}(\boldsymbol{x})$ along the solutions of the controlled system:

$$\begin{aligned} \Delta\mathcal{L}(\boldsymbol{x}[k]) &= \mathcal{L}(\boldsymbol{x}[k+1]) - \mathcal{L}(\boldsymbol{x}[k]) \\ &= \sum_{p\in\mathcal{P}} \int_{x_p[k]}^{x_p[k+1]} l_p(\xi)d\xi \\ &\leq \sum_{p\in\mathcal{P}} \left(x_p[k+1] - x_p[k]\right) l_p\left(x_p[k+1]\right) \\ &= \tau \sum_{p\in\mathcal{P}} \sum_{i\in\mathcal{I}} \left(\sum_{q\in\mathcal{P}} r^i_{qp}[k] - \sum_{q\in\mathcal{P}} r^i_{pq}[k]\right) l_p\left(x_p[k+1]\right) \\ &= \tau \sum_{i\in\mathcal{I}} \sum_{p\in\mathcal{P}} \sum_{q\in\mathcal{P}} r^i_{pq}[k] \left(l_q\left(x_q[k+1]\right) - l_p\left(x_p[k+1]\right)\right). \end{aligned} \quad (30)$$

where the inequality holds from geometric considerations: If $x_p[k+1] > x_p[k]$, recalling that the l_p's are nondecreasing functions, the definite integral $\int_{x_p[k]}^{x_p[k+1]} l_p(\xi)d\xi$ is smaller than the quantity $\left(x_p[k+1] - x_p[k]\right) l_p\left(x_p[k+1]\right)$; conversely, if $x_p[k+1] < x_p[k]$, the integral $\int_{x_p[k+1]}^{x_p[k]} l_p(\xi)d\xi$ is larger than the quantity $\left(x_p[k] - x_p[k+1]\right) l_p\left(x_p[k+1]\right)$.

Analysing each term of the inner summation, two cases hold: if $r^i_{pq}(t) = 0$ the term is null, otherwise, if $r^i_{pq}(t) > 0$, the term is negative. In fact, it is shown below that, if $r^i_{pq}[k] > 0$, it holds that $l_p\left(x_p[k+1]\right) - l_q\left(x_q[k+1]\right) > 0$.

Lemma 1 states that

$$\begin{aligned} & l_p\left(x_p[k+1]\right) - l_q\left(x_q[k+1]\right) \\ &\geq \left(l_p\left(x_p[k]\right) - \frac{\varepsilon}{2}\right) - \left(l_q\left(x_q[k]\right) + \frac{\varepsilon}{2}\right) \\ &= l_p\left(x_p[k]\right) - l_q\left(x_q[k]\right) - \varepsilon > 0, \end{aligned} \quad (31)$$

where the inequality holds since a necessary condition for $r^i_{pq}[k] > 0$ is that $l_p(x_p[k]) - l_q(x_q[k]) > \varepsilon$ (see equation (13)). ■

Proof of Theorem 2: Given that Lemma 2 states that $\mathcal{L}(\boldsymbol{x})$ is a candidate Lyapunov function for the LB dynamics, the proof relies on the LaSalle invariance principle of Theorem 1, i.e., on showing that $\mathcal{X}^\varepsilon_{eq}$ is the maximum invariant set where $\Delta\mathcal{L} = 0$.

Let $\boldsymbol{x} \in \mathcal{X}^\varepsilon_{eq}$ and $\boldsymbol{x}[0] = \boldsymbol{x}$. By comparing definition (6) and equation (13), it holds that $r^i_{pq}[k] = 0$ for all $p, q \in \mathcal{P}^i$ and $i \in \mathcal{I}$, which entails i) that $\boldsymbol{x}[k] = \boldsymbol{x}[0] = \boldsymbol{x}_{eq} \in \mathcal{X}^\varepsilon_{eq}$ for all $k > 0$, i.e., that $\mathcal{X}^\varepsilon_{eq}$ is a positively invariant set, and ii) that $\Delta\mathcal{L}(\boldsymbol{x}[k]) = 0$ in $\mathcal{X}^\varepsilon_{eq}$ (see equation (30)).

To show that $\mathcal{X}^\varepsilon_{eq}$ is the maximum set where $\Delta\mathcal{L}(\boldsymbol{x}[k]) = 0$, it is proven below that $\Delta\mathcal{L}(\boldsymbol{x}[k]) < 0$ if $\boldsymbol{x}[k] = \boldsymbol{x}$, with $\boldsymbol{x} \notin \mathcal{X}_{eq}$. In fact, by definition (12), in this case there exist at least one pair of providers $p, q \in \mathcal{P}^i$ and a commodity $i \in \mathcal{I}$ such that $l_p\left(x_p[k]\right) - l_q\left(x_q[k]\right) > \varepsilon$, with $x^i_p[k] > 0$ and $x_q[k] < c_q - \frac{\varepsilon}{2\bar{\beta}}$, which, in turn, yields $r^i_{pq}[k] > 0$ (see equations (11), (14) and (13)). Having established that $r^i_{pq}[k] > 0$ with $l_p\left(x_p(t)\right) - l_q\left(x_q(t)\right) > \varepsilon$, it follows that the corresponding term of the inner summation of equation (30) is negative, which is a sufficient condition for $\Delta\mathcal{L}(\boldsymbol{x}[k]) < 0$ (recalling that, in the proof of Lemma 3, it is shown that the terms of equation (30) are non-positive).

■

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