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Optimal Stochastic Control of Energy Storage System Based on Pontryagin Minimum Principle for Flattening PEV Fast Charging in a Service Area

Francesco Liberati, *Member, IEEE*, Alessandro Di Giorgio, *Member, IEEE* and Giorgio Koch

***Abstract*— This letter discusses stochastic optimal control of an energy storage system (ESS) for reducing the impact on the grid of fast charging of electric vehicles in a charging area. A trade off is achieved between the objectives of limiting the charging power exchanged with the grid, and the one of limiting the fluctuation, around a given reference, of the ESS energy. We show that the solution of the problem can be derived from the one of a related deterministic problem, requiring the realistic assumption that the charging area operator knows an estimate of the aggregated charging power demand over the day. In addition, two alternative configurations of the charging area are discussed, and it is shown that, while they share the same solution, one better mitigates the demand uncertainty. Numeric simulations are provided to validate the proposed approach.**

***Index Terms*— Optimal control, Pontryagin minimum principle, energy storage system, plug-in electric vehicles, fast charging control.**

I. INTRODUCTION

THIS paper deals with the problem of controlling a grid-connected microgrid equipped with an electric energy storage system (ESS) and a set of charging stations (CSs), providing fast recharging service to plug-in electric vehicles (PEVs). We call such microgrid a "service area". The service area concept is relevant both for fast charging in urban scenarios and during long-range trips. Given the high power rates of PEV charging (up to several tens of kW), service area operators are investigating strategies to make the charging service convenient and, secondarily, to reduce its impact on the grid. Since, in a fast charging scenario, there is limited possibility to modulate the single charging sessions (as the priority is to serve customers at maximum power and in the minimum possible time), in this paper we focus on the optimal control of the ESS in order to balance and control the aggregated service area power exchange with the grid. Two possible ESS configurations in the service area are discussed, and a series of incrementally complex deterministic and stochastic optimal control problems are discussed. The control goals and requirements are:

i) control the ESS to flatten, namely to keep as low and smooth as possible, the power flow at the point of connection (POC) of the service area with the grid. This lowers the operation cost of the service area, currently one of the main barriers (the higher the power flow at the POC, the higher the grid connection fees and demand charges [1]). This action should be transparent to PEV users (i.e., charging at the CSs should still take place at maximum power);

ii) to keep the level of energy (LOE) of the ESS [kWh] close to a desired reference (usually, 50% of the ESS maximum capacity), to make sure the ESS has always a reserve of energy to charge or discharge.

These requirements work in opposition each other, giving raise to the need of formalizing an optimal control problem, here addressed through customization of the linear quadratic regulator framework and application of Pontryagin minimum principle (PMP). Similar scenarios, where the energy flows and the operation of an ESS need to be optimized, have been tackled in literature with various methodologies. Among them, calculus of variations and the PMP are emerging, especially for the optimization of microgrids (see, e.g., [2] and [3], where typical objectives are to minimize power flows, minimize cost of operation, flatten the power profiles, etc.), and hybrid vehicles (see, e.g., [4] and [5], for minimizing fuel consumption, total consumption cost, etc.). These techniques allow defining a continuous-time optimal solution, which is easy to implement in real time, with no demanding hardware/software requirements. However, in complex cases, finding a causal or a closed loop solution might be difficult, or impossible. In [2], the authors minimize the energy flows in a network of microgrids hosting ESSs and renewables. They show that the costate of the system (i.e., variable λ in the following) is constant, which simplifies the derivation of the optimal control. In [3], the authors use PMP to derive the optimal control for a lossy ESS in a microgrid (with the goal of minimizing fuel consumption); inclusion of losses is an element of innovation compared to previous works. In [4], the authors address minimization of fuel consumption in a hybrid vehicle. They show that, with good approximation, the costate can be considered constant, and show that the fuel economy achieved is within 1% of the optimal one (derived numerically offline). In [5], the authors combine PMP and model predictive control (MPC) for energy management of a plug-in hybrid electric bus.

In more complex scenarios, the costate is not constant and, in addition, the derivation of the optimal solution is only possible when some key information about the problem is available *a priori* (for example, in hybrid vehicles applications, the information on the entire driving cycle). In these cases, several references (see e.g., [6], [7]) introduce the so called

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The authors are with the Department of Computer, Control, and Management Engineering (DIAG) "Antonio Ruberti", University of Rome "La Sapienza", Via Ariosto, 25, 00185 Rome, Italy, e-mail: (e-mail: surname@diag.uniroma1.it).

λ-control strategy, where the costate λ is adjusted online in a feedback fashion.

Another relevant approach used in literature is MPC (see, e.g., [8], [9]). MPC is conceptually simpler and it allows to easily embed constraints and detailed models. Classic MPC repeatedly solves a discrete time open loop control problem through optimization, which often requires more advanced hardware and software (e.g., a solver). In case of complex (e.g., nonlinear) formulations, solving times might be high (especially when a long prediction horizon is used), which limits the control resolution. For these reasons, PMP appears as a natural choice whenever an analytic solution to the problem can be found, and its integration into a receding horizon approach (like MPC) a winning strategy to combine the strengths of both methods.

The distinctive features of this work are: *i)* a stochastic formulation of the service area fast charging control problem is presented, which only assumes the knowledge of the *expected* charging power demand in the service area over the control window; *ii)* the solution is derived from the one of a counterpart deterministic problem, which instead requires the knowledge of the exact charging power demand over the control window; *iii)* two alternative service area configurations are presented, and it is shown that the associated control problems have the same solution, but one is preferable over the other in terms of reduced impact of uncertainties; *iv)* to the best of our knowledge, this is one of the first applications of PMP to ESS control in a smart charging area.

The reminder of the paper is organized as follows. Sections II and III discuss the proposed optimal charging control problem for two possible infrastructure setups (i.e., different configurations of the involved devices). The sections present incrementally complex, both deterministic and stochastic, optimal control problems, whose solutions build one on top of the other. Section IV presents simple simulations to compare the performance under the two infrastructure setups. Section V concludes the work and discusses future works.

II. Infrastructure Setup 1

The first analysed infrastructure setup is in Fig. 1. The ESS and the CSs are directly connected to the POC with the grid, each through its own power electronics. This is a standard connection scheme for the ESS, used in many applications at different power levels [10] [11] [12]. Let $p(t)$ denote the power flowing at time t at the POC, $u(t)$ the ESS charging/discharging power, $w(t)$ the cumulative power absorbed by the PEVs (non-negative value at every time), $x(t)$ the difference of the ESS LOE from a reference value (typically, 50% of the full charge).

A. Deterministic problem statement and solution

Problem 1. *(Setup 1 - Deterministic optimal control problem). Given an initial time (t_i) and a final time (t_f) of problem definition, and given $\{w(t), t \in [t_i, t_f]\}$, find*

$$\min_u \left\{ J(u) := \underbrace{\frac{1}{2} s x(t_f)^2}_{:=S(x(t_f))} + \int_{t_i}^{t_f} \underbrace{\frac{1}{2}[q x(t)^2 + r p(t)^2]}_{:=L(x(t),u(t),t)} dt \right\}, \quad (1)$$

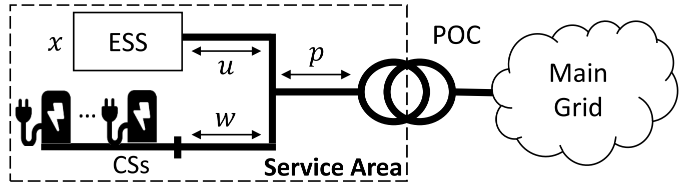

Fig. 1. System architecture, infrastructure setup I.

subject to

$$x(t_i) = x_i, \quad (2)$$

$$\dot{x}(t) := f(x(t), u(t), t) = u(t), \quad \forall t \in [t_i, t_f], \quad (3)$$

$$p(t) = u(t) + w(t), \quad \forall t \in [t_i, t_f], \quad (4)$$

with $s, q, r > 0$.

Problem 1 can be solved via PMP theory. The Hamiltonian of the system is $\mathcal{H} := L(x(t), u(t), t) + \lambda(t) f(x(t), u(t), t)$. The necessary optimality conditions are: i) $\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial x(t)}$, ii) $\frac{\partial \mathcal{H}}{\partial u(t)} = 0$, iii) $\lambda(t_f) = \frac{\partial S}{\partial x(t_f)}$. They define a two-point boundary value problem, which can be solved as described in [13, section 2.3.4] (in particular, the key variable to find is $\lambda(t_i)$). The optimal solution has the implicit form: $\dot{\lambda}(t) = -q x(t)$, $\dot{x}(t) = u(t)$, $u(t) = -\frac{1}{r}\lambda(t) - w(t)$. The initial costate can be written as: $\lambda(t_i) = c_1^\lambda x(t_i) + \int_{t_i}^{t_f} c_2^\lambda(\tau) w(\tau) d\tau$ (with c_1^λ a given constant and $c_2^\lambda(\tau)$ a given function). Consequently, the optimal trajectories can be written as:

$$\lambda(t) = c_3^\lambda(t) x(t_i) + \int_{t_i}^{t_f} c_4^\lambda(t, \tau) w(\tau) d\tau \ , \quad (5)$$

$$x(t) = c_1^x(t) x(t_i) + \int_{t_i}^{t_f} c_2^x(t, \tau) w(\tau) d\tau \ , \quad (6)$$

$$p(t) = -\frac{1}{r}\lambda(t) \ , \quad (7)$$

with c_3^λ, c_4^λ, c_1^x, and c_2^x known functions. In the following, for any given $w := \{w(t), t \in [t_i, t_f]\}$, we denote the optimal control of Problem 1 at time t as $u^{(1)}(w, t)$, and the related system trajectories as $x^{(1)}(w, t)$ and $p^{(1)}(w, t)$, where the superscript reminds that these are the solutions of Problem 1. Notice that, at any given time, these quantities depend on the entire signal w, from t_i to t_f. Similar notation is used for the control problems presented in the next sections.

Remark 1.1 (Constrained, lossy ESS model)**.** *The ESS model can be extended to include the ESS losses [3]: $\dot{x} = \eta(u(t))u(t)$, with $\eta(u(t))$ the charging/discharging efficiency function. For example, with constant charging and discharging efficiencies (respectively, $\eta_{ch}, \eta_{dis} \in (0, 1)$), it is $\eta(u(t))u(t) = \eta_{ch} u(t) h(u(t)) + \frac{u(t)}{\eta_{dis}} h(-u(t))$, where h is the Heaviside function. Though more complex, Problem 1 can still be solved as outlined above. The result is a two point boundary value problem defined on a switching system (the efficiency changes depending on the sign of $u(t)$). The initial costate in this case can be found with numerical integration. About constraints, it is immediate to include the box constraint $u^{min} \le u(t) \le u^{max}$, using the PMP (see, e.g., [13, Section 2.3, Theorem C]: it is enough to saturate the control found* *to the upper and lower limits, if they are surpassed). More complex is the inclusion of the box constraints on x, which can be carried out following the procedure outlined, e.g., in [13, Section 2.5, Theorem D]. Still more complex is the inclusion of the constraint $p^{min} \leq p(t) \leq p^{max}$, since it defines a time-varying constraint on $u(t)$ (i.e., $p^{min} - w(t) \leq u(t) \leq p^{max} - w(t)$). A practical approach is to include in the objective function an additional barrier term penalizing the violations of the constraint. This is illustrated, e.g., in [3] and [4], where it is used to deal with the constraints on x.*

The analysis carried out in the rest of the paper refers to the model adopted in Problem 1. The extension to the model in Remark 1.1 is subject of future works.

B. Stochastic problem statement and solution

In this section, we consider a more realistic case in which the signal w is a stochastic process, and we design a deterministic control u, based on the information available at t_i, able to fulfill the above mentioned requirements in a probabilistic sense. Specifically, we assume we are given, at time t_i, the distribution of $w(t)$ over the time interval (t_i, t_f). This is a valid assumption since, through the charging management system, the service area operator can perform analytics to characterize the charging demand. We present in the following two different possible stochastic problem formulations and discuss the relation among them.

Problem 2. *(Setup 1 - Stochastic optimal control problem a).*

$$\min_u \left\{ J_2(u) := \frac{1}{2} s x(t_f)^2 + \int_{t_i}^{t_f} \frac{1}{2} [q x(t)^2 + r E[p(t)^2]] dt \right\}, \quad (8)$$

subject to dynamics (2) - (3) and constraint (4).

Problem 2 can be seen as the natural evolution of Problem 1 to the case in which the lack of knowledge about the future evolution of w makes the power flow p uncertain, as resulting from the power balance (4); consequently, the term $p(t)^2$ appearing in the cost function (1) is here replaced by its expected value. The solution of Problem 2 can be found through comparison between the cost functions J_1 and J_2 appearing, respectively, in (1) and (8), as described below.

Lemma 2.1. *The optimal control of Problem 2 is given by*

$$u^{(2)}(t) = u^{(1)}(E[w], t). \quad (9)$$

The resulting trajectories of x and $E[p]$ are given by

$$x^{(2)}(t) = x^{(1)}(E[w], t), \quad (10)$$

$$E[p^{(2)}(t)] = p^{(1)}(E[w], t). \quad (11)$$

Proof. Problems 1 and 2 include the same constraints. The cost functions J_1 and J_2 differ for the presence of terms $p(t)^2$ and $E[p(t)^2]$ which, using (4), can be written as $p(t)^2 = u(t)^2 + 2u(t)w(t) + w(t)^2$ and $E[p(t)^2] = u(t)^2 + 2u(t)E[w(t)] + E[w(t)^2]$. The terms $w(t)^2$ and $E[w(t)^2]$ do not depend on the control u and, consequently, do not affect its determination. Comparing the second terms in the above expressions, the optimal control (9) follows. As (3) does not explicitly depends on $w(t)$, and the optimal control $u^{(2)}(t)$ is given by $u^{(1)}(E[w], t)$, i.e., the solution of Problem 1, where signal w is replaced by its expected value, then $x^{(2)}(t)$ is straightforwardly given by $x^{(2)}(t) = x^{(1)}(E[w], t)$. Finally, applying the expected value to both sides of (4) and using (9), $E[p^{(2)}(t)]$ is calculated as $E[p^{(2)}(t)] = u^{(2)}(t) + E[w(t)] = u^{(1)}(E[w], t) + E[w(t)] = p^{(1)}(E[w], t)$. □

In order to give a further insight into the significance of Problem 2, we now consider and discuss an alternative problem formulation, characterized by the same solution.

Problem 3. *(Setup 1 - Stochastic optimal control problem b).*

$$\min_u \left\{ J_3(u) = \frac{1}{2} E\big[s\big(x(t_f) - x^{(1)}(w, t_f)\big)^2\big] + \frac{1}{2} \int_{t_i}^{t_f} E\big[q\big(x(t) - x^{(1)}(w,t)\big)^2 + r\big(p(t) - p^{(1)}(w,t)\big)^2\big] dt \right\}, \quad (12)$$

subject to dynamics (2) - (3) and constraint (4).

Lemma 3.1. *The optimal control and system trajectories related to Problems 2 and 3 are the same, namely*

$$u^{(3)}(t) = u^{(1)}(E[w], t), \quad (13)$$

$$x^{(3)}(t) = x^{(1)}(E[w], t), \quad (14)$$

$$E[p^{(3)}(t)] = p^{(1)}(E[w], t). \quad (15)$$

Proof. Consider the unconstrained version of Problem 3. The optimal values (denoted with $*$) of x and $E[p]$ minimizing J_3 are easily found noting that the latter can be decomposed in separate functions of x and p, and consequently calculating their stationary points. This gives $x^*(t) = E[x^{(1)}(w, t)]$ and $E[p]^*(t) = E[p^{(1)}(w, t)]$. Using (6) to expand $E[x^{(1)}(w, t)]$ gives

$$x^*(t) = \int_\Omega \left[c_1^x(t) x(t_i) + \int_{t_i}^{t_f} c_2^x(t, \tau) w(\tau) d\tau \right] \pi(\omega) d\omega = \\ = c_1^x(t) x(t_i) + \int_{t_i}^{t_f} c_2^x(t, \tau) E[w(\tau)] d\tau = x^{(1)}(E[w], t). \quad (16)$$

The same calculation applies to $E[p]^*(t)$, using (7)

$$E[p]^*(t) = p^{(1)}(E[w], t) \quad (17)$$

Comparing (16)(17) and (10)(11), equations (14)(15), and consequently (13), follow. □

Remark 3.1. *In Problem 3, the cost function J_3 is designed so as to penalize the square deviation of the controlled trajectories from the optimal ones resulting from Problem 1; since w is a stochastic process, this deviation is considered on "average" through its expected value. In the light of Lemma 3.1, the control (9) keeps this deviation minimum, taking into account all the possible realizations of w.*

Remark 3.2. *The optimal control of Problems 2 and 3 is formally the same of Problem 1, with the peculiarity of being computed as a function of $E[w]$ instead of w. Hence the plant is controlled based on the average behaviour of the charging*

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power demand and, consequently, the realization of the power flow p is generally affected by the uncertainty of the stochastic process w. Indeed notice that

$$p^{(2)}(t)=u^{(2)}(t)+w(t)=u^{(1)}(E[w],t)+w(t)= \\ =p^{(1)}(E[w],t)+(w(t)-E[w(t)]) \tag{18}$$

and, consequently, the smaller the deviation of w from $E[w]$ is, the higher the significance of Problem 2 is.

As resulting from Remark 3.2, the uncertainty in the charging demand w, including the presence of spikes due to the activation of new charging sessions, can significantly affect plant performances and, specifically, the possibility of satisfying requirement 1. We analyse in the following an alternative setup which, based on the same available knowledge at t_i and previous results, can guarantee better performances in practical terms, even when the charging demand w is affected by a high degree of uncertainty.

III. Infrastructure Setup 2

In this infrastructure setup, the ESS is placed in between the CSs and the grid, as shown in Fig. 2 (refer to this figure for the nomenclature used in this section). This is also a standard connection scheme for the ESS, mainly used in applications where the main requirement is avoiding interruptions of the power supply. Though it can be realized in several ways, the key difference from setup 1 is the presence of a common direct current bus for connecting the batteries and the load, together with a grid connected inverter able to control the power flow [14] [15, Fig. 2]. The resulting ESS dynamics is given by $\dot{x}(t)=u(t)-w(t)$, where u represents both the controlled component of the total ESS charging/discharging power and the power flow at the POC, to be minimized.

A. Deterministic problem statement and solution

Problem 4. *(Setup 2 - Deterministic optimal control problem).*

$$\min_u \left\{ J_4(u) := \frac{1}{2}sx(t_f)^2 + \int_{t_i}^{t_f} \frac{1}{2}[qx(t)^2+ru(t)^2]dt \right\}, \tag{19}$$

subject to:

$$\dot{x}(t)=u(t)-w(t), \quad \forall t\in[t_i,t_f], \quad x(t_i)=x_i. \tag{20}$$

It is easy to check that the necessary optimality conditions for Problem 4 define the same two-point boundary value problem as for Problem 1, and consequently equations (5) and (6) still hold. We summarize this result as

$$x^{(4)}(t)=x^{(1)}(w,t), \tag{21}$$

$$\lambda^{(4)}(t)=\lambda^{(1)}(w,t). \tag{22}$$

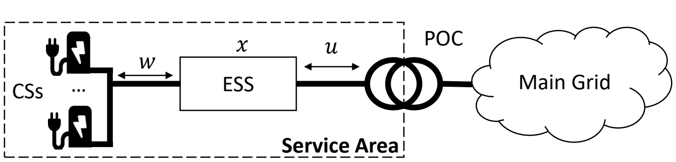

Fig. 2. System architecture, infrastructure setup II.

For the optimal control we have: $\frac{\partial\mathcal{H}}{\partial u(t)}=0$, i.e., $u(t)=-\frac{1}{r}\lambda(t)$ which, using (22) and (7), can be written as

$$u^{(4)}(t)=p^{(1)}(w,t)=u^{(1)}(w,t)+w(t). \tag{23}$$

The form of the optimal control (23), as well as the trajectory of the state (21), should not surprise the reader: in the infrastructure setup 2 the ESS control u represents the power flowing at the POC, the same role played by the variable p in the infrastructure setup 1. Problems 1 and 4 describe, in the deterministic case, the same control problem with reference to two different charging infrastructure configurations. We discuss in the following the peculiarity of setup 2 with respect to setup 1, through the natural evolution of Problem 4 to the stochastic case presented hereinafter.

B. Stochastic problem statement and solution

Consider the following control problem, in which, w is subject to the same assumptions considered in section II-B.

Problem 5. *(Setup 2 - Stochastic optimal control problem).*

$$\min_u \left\{ J_4(u) := \frac{1}{2}sE[x^2(t_f)] + \int_{t_i}^{t_f} \frac{1}{2}[qE[x^2(t)]+ru^2(t)]dt \right\}, \tag{24}$$

subject to dynamics (20).

Following the logic used for introducing Problem 2, Problem 5 can be seen as the natural evolution of Problem 4 to the case in which the lack of knowledge about the future behavior of w makes the trajectory of the state x uncertain, as resulting from the dynamics (20); consequently, the term x^2 appearing in the cost function (19) is here replaced by its expected value. Again, the solution of Problem 5 can be found making a comparison between the cost functions J_4 and J_5 (respectively, (19) and (24)), as reported below.

Lemma 5.1. *The optimal control of Problem 5 is given by*

$$u^{(5)}(t)=p^{(1)}(E[w],t). \tag{25}$$

The resulting trajectory of $E[x]$ is given by

$$E[x^{(5)}(t)]=x^{(1)}(E[w],t). \tag{26}$$

Proof. Problems 4 and 5 are characterized by the same dynamics which, in its explicit form, is given by

$$x(t)=x(t_i)+\int_{t_i}^{t}[u(\tau)-w(\tau)]d\tau. \tag{27}$$

Cost functions J_4 and J_5 appearing in (19) and (24) differ each other respectively by the presence of terms $x(t)^2$ and $E[x(t)^2]$. Inspection of these two terms using (27) gives

$$\begin{aligned} x(t)^2 =& x(t_i)^2 + 2x(t_i)\int_{t_i}^{t}u(\tau)d\tau + \left(\int_{t_i}^{t}u(\tau)d\tau\right)^2 + \\ & -2\int_{t_i}^{t}u(\tau)d\tau\int_{t_i}^{t}w(\tau)d\tau + \\ & +\left(\int_{t_i}^{t}w(\tau)d\tau\right)^2 - 2x(t_i)\int_{t_i}^{t}w(\tau)d\tau, \end{aligned} \tag{28}$$

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$$\begin{aligned} E[x(t)^2] =& x(t_i)^2 + 2x(t_i)\int_{t_i}^{t} u(\tau)d\tau + \left(\int_{t_i}^{t} u(\tau)d\tau\right)^2 + \\ & -2\int_{t_i}^{t} u(\tau)d\tau \int_{t_i}^{t} E[w(\tau)]d\tau + \\ & +E\left[\left(\int_{t_i}^{t} w(\tau)d\tau\right)^2\right] - 2x(t_i)\int_{t_i}^{t} E[w(\tau)]d\tau. \end{aligned} \tag{29}$$

The terms in the third row of (28) and (29) are contributions to J_4 and J_5 that can not be reduced via $u(t)$. Comparing the terms in the second rows of the equations results in

$$u^{(5)}(t) = u^{(4)}(E[w], t) \tag{30}$$

and, remembering (23), the optimal control (25) follows. Finally, applying the expected value to both sides of (27), using (30) and (21), $E[x^{(5)}(t)]$ is calculated as

$$\begin{aligned} E[x^{(5)}(t)] &= E[x(t_i) + \int_{t_i}^{t} [u^{(5)}(\tau) - w(\tau)]d\tau] = \\ &= x(t_i) + \int_{t_i}^{t} [u^{(4)}(E[w], \tau) - E[w(\tau)]d\tau] = \\ &= x^{(4)}(E[w], t) = x^{(1)}(E[w], t). \end{aligned} \tag{31}$$

□

Remark 5.1. *The optimal control of Problem 5 is obtained calculating the optimal control of Problem 4 as a function of $E[w]$ instead of the real signal w. The optimal control evolves as the expected value of power p in the infrastructure setup 1 (see* (25) *and* (11)*). The plant is controlled based on the average behaviour of the charging demand w and, consequently, any deviation of the realization of w with respect to its expected value affects the actual trajectory of the state x. Indeed notice that*

$$\begin{aligned} x^{(5)}(t) &= x(t_i) + \int_{t_i}^{t} [u^{(5)}(\tau) - w(\tau)]d\tau = \\ &= x(t_i) + \int_{t_i}^{t} [u^{(4)}(E[w], \tau) - w(\tau)]d\tau = \\ &= x^{(4)}(E[w], t) - \int_{t_i}^{t} (w(\tau) - E[w(\tau)])d\tau = \\ &= x^{(1)}(E[w], t) - \int_{t_i}^{t} (w(\tau) - E[w(\tau)])d\tau. \end{aligned} \tag{32}$$

Looking at Problems 2 and 5, it is straightforward to realize that they give rise to the same average behavior of variables x and p, obtained when $w = E[w]$ (compare (11)(10) and (25)(26), and notice that u in Problem 5 plays the role of p in Problem 2). The peculiarity of the infrastructure setup 2, with respect to setup 1, is that the uncertainty appearing in $p^{(2)}$ in (18) is moved to the state $x^{(5)}$ in (32). This uncertainty affects the state through the smoothing action of an integral, instead of directly affecting, without any filtering action, the power flow at the POC between the charging service area and the power network. Consequently, the application of the optimal control (25) to the plant characterizing the infrastructure setup 2 allows to better fulfill the control requirements, provided that the ESS converter is properly sized to be able to manage the mismatch between $E[w]$ and the realization w. Further

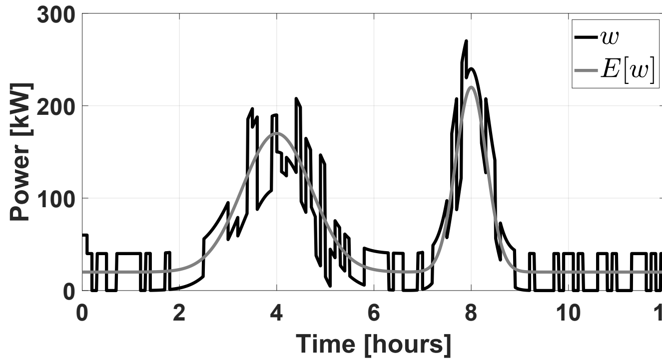

Fig. 3. Actual (black line) and expected charging power.

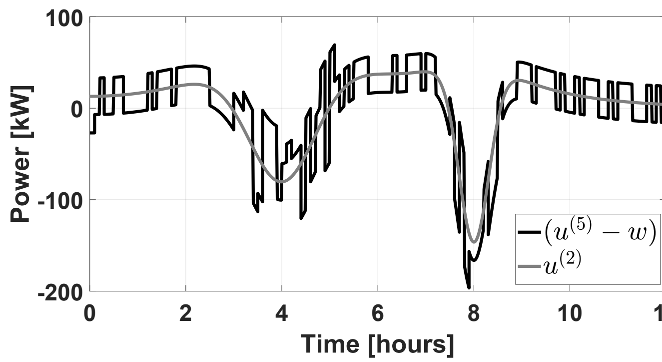

Fig. 4. ESS charging/discharging power.

this control setup poses the basis for a future extension of Problem 5, aimed at minimizing the probability of state saturation, while keeping the size of the ESS properly small.

IV. Numerical Simulations

Simulations aim to compare the evolution of the service area according to the control proposed in Problem 2 and Problem 5, i.e., the proposed stochastic control problems for the two addressed infrastructure setups. We consider a service area that can deliver a maximum of 280 kW of charging power. This in practice can reflect different numbers of CSs, depending on the adopted charging standards. Simulations were performed in Matlab and span 12 hours. We chose $q = 1$, $r = 1$ and $s = 10$. Figure 3 displays $E(w)$ and the realization w considered in the simulation. They reflect a realistic scenario with two peaks of charging requests during the day. Figures 4 and 5 report, in black line, respectively, the ESS charging/discharging power (i.e., $u^{(5)} - w$) and the ESS LOE trajectory resulting from Problem 5. The gray lines instead refer to Problem 2. In Problem 5 (i.e., in the second setup), it is the ESS that absorbs the mismatches between the expected charging power and the actual one, as visible from Fig. 4. On the other hand, from Fig. 6, which reports the power flow at the POC with the grid, it can be seen that Problem 5 results in a power flow (black line) that is smooth and in line with the desired requirement, while Problem 2, i.e., setup 1, results in large power variations injected into the grid. In addition, it can be shown that, in the figures, the smooth lines also represent the evolutions that would result from all the Problems in case the realization w coincided with the expected value $E[w]$ (looking

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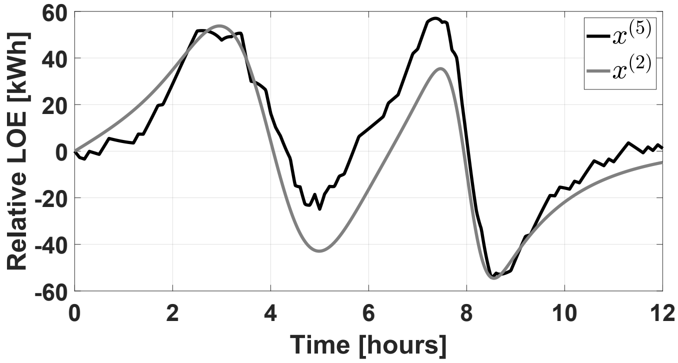

Fig. 5. ESS relative level of energy (LOE).

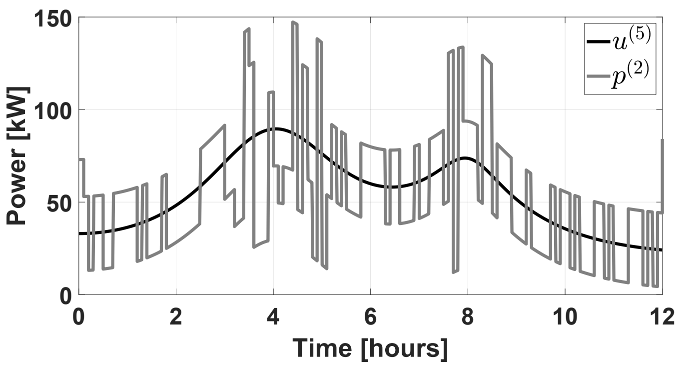

Fig. 6. Power flow at the connection with the grid.

at Fig. 4 for instance, this can be seen from (18) and (25), considering $w = E[w]$). The above results confirm that setup 2 can effectively meet the requirement on power flow smoothing, with a limited deviation (thanks to the integral action of the ESS) of the ESS SOC curve from the ideal profile obtained when $w = E[w]$. Setup 2 is consequently preferable over setup 1, for the reduced impact of uncertainties on the POC.

For completeness we remark that the choice of one setup over the other should also take into account additional electrical and economic aspects, such as the size and cost of the ESS, the inefficiencies of power converters, and the reliability of the equipment. Regarding the ESS, in principle setup 1 may be implemented using an ESS having smaller capacity than in setup 2, as the CSs are directly fed both by the ESS and the grid; however, in case uninterruptible power supply is required, the size of the ESS has to be same in both configurations. For what concerns inefficiencies, it is important to highlight that both the setups may be realized in several different ways, in terms of number, typology and connection schemes of power converters [10] [14]; this is the reason why this electrotechnical insight is left to future works. However we remark that, among the aspects to be evaluated for choosing a setup over the other, the impact of uncertainty appears relevant compared to inefficiencies; indeed while different connection schemes may results in different losses of energy, large deviations of w from $E[w]$ may actually compromise the possibility of achieving the desired system behaviour at the POC when using setup 1.

V. Conclusions and Future Works

This paper has presented deterministic and stochastic optimal control strategies for control of a PEV charging area equipped with CSs and an ESS. The main control goal is to use the ESS to flatten the power profile at the point of connection with the grid, which lowers the operation costs of the charging area. Future works will focus on the detailed modelling of w as a stochastic process, and on the design of an updated optimal stochastic version of the problem [16], to better model the ESS, to include a probabilistic formulation of constraints (extending Remark 1.1), and to integrate the proposed approach into a receding horizon scheme.

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