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χSF near the electroweak scale

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We employ the chirally rotated Schrödinger functional (χSF) to study two-point fermion bilinear correlation functions used in the determination of $Z_{A,V,S,P,T}$ on a series of well-tuned ensembles. The gauge configurations, which span renormalisation scales from 4 to 70 GeV, are generated with $N_\mathrm{f} = 3$ massless flavors and Schrödinger Functional (SF) boundary conditions. Valence quarks are computed with χSF boundary conditions. We show preliminary results on the tuning of the χSF Symanzik coefficient z_f and the scaling of the axial current normalization Z_A. Moreover we carry out a detailed comparison with the expectations from one-loop perturbation theory. Finally we outline how automatically $\mathrm{O}(a)$-improved B_K matrix elements, including BSM contributions, can be computed in a χSF renormalization scheme.

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The chirally rotated Schrödinger functional (χSF) with massless Wilson fermions is a lattice regularization which endows the Schrödinger functional (SF) with the property of automatic O(a)-improvement. The χSF framework is effective in reducing lattice artefacts in correlation and step scaling functions, but especially it offers new strategies to study and simplify the pattern of renormalization. The price to pay for the automatic O(a)-improvement is the nonperturbative tuning of coefficients of new boundary counterterms. This tuning is the first phase of a long-term project, aiming at the computation of $B_{\rm K}$ low-energy contributions beyond the Standard Model (BSM), with Wilson fermion $N_{\rm f} = 2+1$ lattice QCD in a non-unitary (mixed-action) framework.

1. The χSF setup

Following ref. [1], the fermion flavour doublet $\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$ satisfies χSF boundary conditions

$$\tilde{Q}_{\pm} \equiv \frac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3) \qquad \begin{cases} \tilde{Q}_+\psi(x)|_{x_0=0} = 0 & \tilde{Q}_-\psi(x)|_{x_0=T} = 0 \\ \bar{\psi}(x)\tilde{Q}_+|_{x_0=0} = 0 & \bar{\psi}(x)\tilde{Q}_-|_{x_0=T} = 0 \end{cases}$$

in time and periodic ones in space. The massless fermion action is

$$S_f = a^4 \sum_{x_0=0}^{T} \sum_{\mathbf{x}} \bar{\psi}(x)(\mathscr{D}_W + \delta\mathscr{D}_W)\psi(x)\,,$$

with $\mathscr{D}_W$ the standard Wilson fermion matrix and the boundary term

$$\delta\mathscr{D}_W\,\psi(x) \;=\; (\delta_{x_0,0} + \delta_{x_0,T})\left[(z_f - 1) + (d_s - 1)\,a\mathbf{D}_s\right]\psi(x)\,.$$

The χSF boundary conditions can be derived from the standard SF boundary conditions by applying a non-anomalous chiral flavour rotation on the fermion doublet as follows:

$$R = \exp\left(i\frac{\alpha}{2}\gamma_5\tau^3\right)\Big|_{\alpha=\pi/2} \begin{cases} \psi & \to \psi' = R\psi \\ \bar{\psi} & \to \bar{\psi}' = \bar{\psi}R\,. \end{cases}$$

Consequently composite operators, which depend on fermion fields, are also rotated:

$$O[\psi,\bar{\psi}] \to Q[\psi,\bar{\psi}] = O[R\psi,\bar{\psi}R]\,.$$

SF and χSF correlation functions of composite operators O, Q defined in the bulk and $\mathscr{O}, \mathscr{Q}$ defined on a time boundary obey the following universality relation:

$$\begin{aligned} \langle O\,\mathscr{O}\rangle^{\rm cont}_{\rm (SF)} &= \lim_{a\to 0}[Z_O Z_{\mathscr{O}}\,\langle O\,\mathscr{O}\rangle_{\rm (SF)} + {\rm O}(a)] \\ &= \lim_{a\to 0}[Z_Q Z_{\mathscr{Q}}\,\langle Q\,\mathscr{Q}\rangle_{(\chi{\rm SF})} + {\rm O}(a^2)]\,. \end{aligned}$$

Note that χSF incorporates automatic O(a) improvement. The price to pay is the introduction of boundary counterterms. We tune non-perturbatively the boundary counterterm coefficient z_f, while we fix the others at their tree-level value.

1.1 Correlation functions

As in ref. [2], we consider the set of fermion bilinear operators; e.g.

$$V_\mu^{f_1 f_2}(x) = \overline{\psi}_{f_1}(x)\gamma_\mu \psi_{f_2}(x), \qquad A_\mu^{f_1 f_2}(x) = \overline{\psi}_{f_1}(x)\gamma_\mu\gamma_5 \psi_{f_2}(x),$$

with flavours $f_1, f_2 \in \{u, d, u', d'\}$, and determine the χSF bulk-to-boundary correlation functions

$$g_{\rm X}^{f_1 f_2}(x_0) = -\frac{1}{2}\left\langle X^{f_1 f_2}(x)\mathscr{Q}_5^{f_2 f_1}\right\rangle, \qquad X = V_0, A_0, S, P,$$
$$l_{\rm Y}^{f_1 f_2}(x_0) = -\frac{1}{6}\sum_{k=1}^{3}\left\langle Y_k^{f_1 f_2}(x)\mathscr{Q}_k^{f_2 f_1}\right\rangle, \qquad Y_k = V_k, A_k, T_{k0}, \widetilde{T}_{k0}.$$

The complete list of boundary operators $\mathscr{Q}_5^{f_2 f_1}, \mathscr{Q}_k^{f_2 f_1}$ can be found in [5].
Up to discretization effects and boundary fields renormalization they are related to the standard SF correlation functions $f_{\rm X}$ and $k_{\rm Y}$ by universality

$$f_{\rm A}^{\rm cont} = Z_{\rm A} g_{\rm A}^{uu'} = Z_{\rm A} g_{\rm A}^{dd'} = -iZ_{\rm V} g_{\rm V}^{ud} = iZ_{\rm V} g_{\rm V}^{du}, \tag{1.1}$$
$$f_{\rm V}^{\rm cont} = Z_{\rm V} g_{\rm V}^{uu'} = Z_{\rm V} g_{\rm V}^{dd'} = -iZ_{\rm A} g_{\rm A}^{ud} = iZ_{\rm A} g_{\rm A}^{du}, \tag{1.2}$$

$$k_{\rm V}^{\rm cont} = Z_{\rm V} l_{\rm V}^{uu'} = Z_{\rm V} l_{\rm V}^{dd'} = -iZ_{\rm A} l_{\rm A}^{ud} = iZ_{\rm A} l_{\rm A}^{du}, \tag{1.3}$$
$$k_{\rm A}^{\rm cont} = Z_{\rm A} l_{\rm A}^{uu'} = Z_{\rm A} l_{\rm A}^{dd'} = -iZ_{\rm V} l_{\rm V}^{ud} = iZ_{\rm V} l_{\rm V}^{du}. \tag{1.4}$$

The χSF correlation functions in eqs. (1.2), (1.4) are O(a), since they become $f_{\rm V}^{\rm cont}, k_{\rm A}^{\rm cont}$ in the continuum, which are parity odd. The local vector current can be replaced by the exactly conserved one $\widetilde{V}_\mu(x)$ with normalization $Z_{\widetilde{\rm V}} = 1$. Therefore $Z_{\rm A}$ may be obtained from the ratios

$$Z_{\rm A}^g = \left.\frac{-ig_{\widetilde{\rm V}}^{ud}(x_0)}{g_{\rm A}^{uu'}(x_0)}\right|_{x_0=L/2} \quad \text{or} \quad Z_{\rm A}^l = \left.\frac{il_{\widetilde{\rm V}}^{uu'}(x_0)}{l_{\rm A}^{ud}(x_0)}\right|_{x_0=L/2}. \tag{1.5}$$

2. Computational setup and results

We obtain results for $N_{\rm f} = 3$ QCD in a non-unitary setup. Valence quark propagators are inverted with χSF boundaries on the configuration ensembles of [3], generated on lattices with standard SF boundary conditions. These configurations have been used for the RG-running of the quark mass in a range of scales $2\,\text{GeV} \lesssim \mu \lesssim 128\,\text{GeV}$, in the standard framework of finite-size scaling $L \to 2L$.

2.1 Tuning

We must ensure that massless QCD with χSF boundary conditions is correctly regularized. This is achieved by tuning the bare mass parameter m_0 to its critical value, $m_{\rm cr}$, where the axial current is conserved, and by tuning the boundary counterterm coefficient z_f so that physical parity is restored. At present we choose to set the PCAC mass to zero in terms of the SF correlation functions, so taking the $m_{\rm cr}$ value from [3], and to set the χSF correlation function $g_{\rm A}^{ud}$ to zero:

$$m = \left. \frac{\tilde{\partial}_0 f_{\rm A}^{ud}(x_0)}{2 f_{\rm P}^{ud}(x_0)} \right|_{x_0=L/2} = 0, \qquad m_{\rm cr} \text{ tuning}\,,$$
$$\left. g_{\rm A}^{ud}(x_0) \right|_{x_0=L/2} = 0\,, \qquad z_f \text{ tuning}\,. \tag{2.1}$$

By requiring that eq. (2.1) be satisfied, z_f is tuned for each ensemble as shown in Fig. 1.

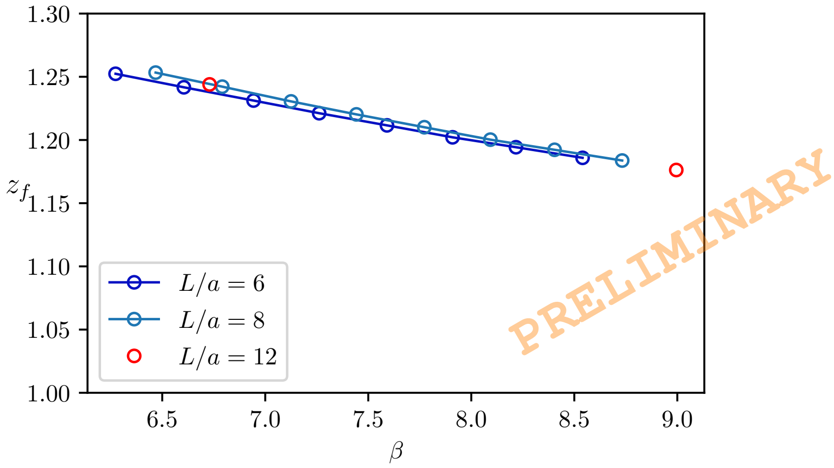

Figure 1: Results of nonperturbative tuning of z_f, according to eq. (2.1).

2.2 O(a)-improvement

Using eqs. (1.5) we obtain two estimates for $Z_{\rm A}$

$$Z_{\rm A}^g(\beta) = Z_{\rm A}^l(\beta) + {\rm O}(a^2)$$

which differ by discretization errors. In Fig. 2 we show the ratio of these two definitions and we confirm that, after tuning z_f, the ratio scales as a^2 and goes to 1 in the continuum.

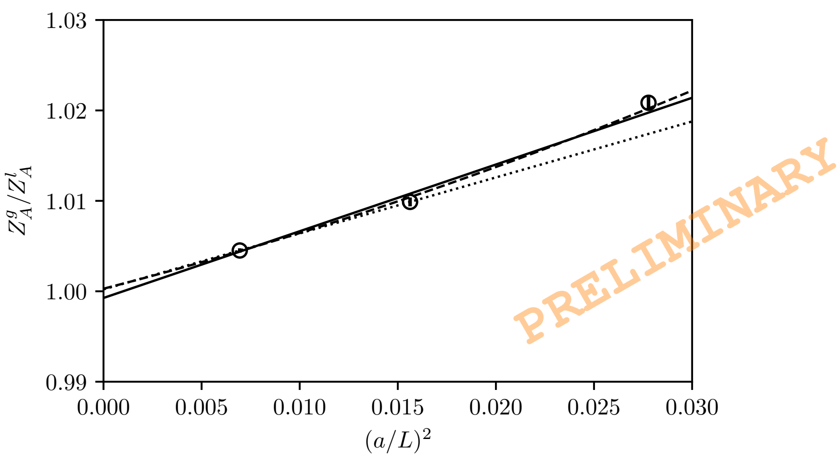

Figure 2: Ratio of two different definitions of Z_A (see eq. (1.5)) calculated on our ensembles with $1/L = 4$ GeV.

Finally we study $Z_A^l(g_0^2)$ over the full range of ensembles. As seen in Fig. 3 a fit to the data matches onto the asympotic perturbative result in the limit $g_0^2 \to 0$.

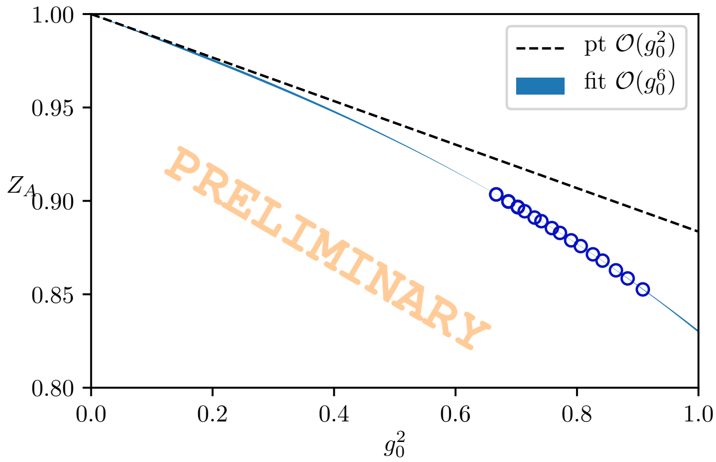

Figure 3: $Z_A^l(g_0^2)$ calculated across the full range of ensembles available.

3. Outlook for 4 fermions

3.1 Renormalization

χSF framework is especially valuable in simplifying the renormalization of four fermion operators. They enter the most general expression of the effective Hamiltonian which describes flavour physics processes at low energy in the Standard Model (SM) and its extensions (BSM). Here we focus on $\Delta F = 2$ transitions. The 4-quark operators with four distinct flavours

$$O_{XY}^{\pm} \equiv \frac{1}{2}\left[(\overline{\psi}_1 \Gamma_X \psi_2)(\overline{\psi}_3 \Gamma_Y \psi_4) \pm (2 \leftrightarrow 4)\right]$$

can be classified as parity even and parity odd:

$$O_k^{e,\pm} \in \left\{O_{VV+AA}^{\pm}, O_{VV-AA}^{\pm}, O_{SS-PP}^{\pm}, O_{SS+PP}^{\pm}, O_{TT}^{\pm}\right\},$$
$$O_k^{o,\pm} \in \left\{O_{VA+AV}^{\pm}, O_{VA-AV}^{\pm}, O_{SP-PS}^{\pm}, O_{SP+PS}^{\pm}, O_{T\tilde{T}}^{\pm}\right\}.$$

Due to the explicit breaking of chiral symmetry of the Wilson regularisation, the operators in general mix as follows:

$$O_i^{e,\pm} = \sum_{jm} Z_{ij}^{e,\pm}\left(\delta_{jm} + \Delta_{jm}^{e,\pm}\right) O_m^{e,\pm},$$
$$O_i^{o,\pm} = \sum_{jm} Z_{ij}^{o,\pm}\left(\delta_{jm} + \cancel{\Delta_{jm}^{o,\pm}}\right) O_m^{o,\pm}.$$

The parity-odd sector has a simpler, continuum-like mixing pattern ($\Delta_{jm}^{o,\pm} = 0$) [6]. In previous Wilson fermion computations [7, 8], standard SF renormalization conditions were imposed on parity-odd operators by setting suitable renormalized correlation functions equal to their tree level values at the scale $\mu = L^{-1}$:

$$F_i(x_0) = \langle \mathcal{O}_5'^{45} O_i^{o,1234}(x_0) \mathcal{O}_5^{21} \mathcal{O}_5'^{53} \rangle. \tag{3.1}$$

Note that five distinct valence flavours are required; see the left diagram in Fig. 4. These four-point correlation functions are parity even, but suffer from large statistical fluctuations. Moreover, they have bulk $\mathrm{O}(a)$ discretization errors.

Alternatively we plan to employ a new renormalization scheme on χSF three-point correlation functions

$$G_i(x_0) = \langle \mathcal{Q}_5'^{21} Q_i^{o,1234}(x_0) \mathcal{Q}_5^{43} \rangle \,, \tag{3.2}$$

which are statistically less noisy and automatically $\mathrm{O}(a)$ improved in the bulk. Performing suitable chiral rotations, the χSF flavours are rotated into the physical flavours [4]. Thus we can map the renormalized $[G_i(x_0)]_\mathrm{R}$ into the continuum correlation function

$$[G_i(x_0)]_\mathrm{R} \to \langle \mathcal{O}_5'^{21} O_i^{e,1234}(x_0) \mathcal{O}_5^{43} \rangle^{\mathrm{cont}} .$$

Figure 4: Correlation functions used in two renormalization schemes. Left: parity odd operators with standard SF boundary conditions; right: parity odd operators with χSF boundary conditions. Different colors stand for different valence flavors.

3.2 Physical determinations

The renormalization program will be employed in the lattice computation of the physical B_K-parameter which controls the $\bar{K}^0 - K^0$ meson oscillations, towards a better understanding of the physics of CP violation in the SM and BSM. Whereas there is general agreement among various collaborations on B_K in the SM, the situation is somewhat unclear for the BSM contributions [11].

In the SF approach, B_K has been computed by combining the renormalization parameters based on eq. (3.1) with bare four-point correlation functions of the parity odd operators in a twisted mass QCD setup [9, 10].

We plan to use the χSF renormalization conditions based on eq. (3.2). For the bare matrix elements we will employ the CLS $N_\mathrm{f} = 2+1$ ensembles, characterised by *large* physical volumes with open boundary conditions and by *non-zero* quark masses [12, 13]. The sea quarks are Wilson/Clover. Valence fermions are fully twisted [14], with three flavours tuned at twisted angle $\alpha = \pi/2$ and the fourth one at $\alpha = -\pi/2$. Unitarity is lost at finite lattice spacing, but it is recovered in the continuum limit. Performing distinct *Osterwalder-Seiler* chiral rotations for each flavour, correlation functions with parity-odd operators (renormalized in the χSF scheme as outlined above) are mapped onto the 3-point correlation functions of parity even operators with pseudoscalar sources, from which the B-parameters are readily extracted:

$$B_{\mathrm{K}i}(\mu) \propto \langle \bar{K}^0 | \, [O_i^e(\mu)]_R \, | K^0 \rangle \,.$$

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